Database - HW2

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1-> $A_1A_3 \rightarrow A_2$

2-> $A_4 \rightarrow A_5A_2$

$A_4 \rightarrow A_5$

$A_4 \rightarrow A_2$

3-> $A_2A_3 \rightarrow A_4$

4-> $A_3A_2 \rightarrow A_2A_4$

$A_3A_2 \rightarrow A_2$

$A_3A_2 \rightarrow A_4$

5-> $A_1A_3A_4 \rightarrow A_2$

6-> $A_3A_5 \rightarrow A_1A_2$

$A_3A_5 \rightarrow A_1$

$A_3A_5 \rightarrow A_2$

Check 1: $A_1A_3 \rightarrow A_2$

A_1 without A_3 $\{A_1\}^+ = \{A_1\}$ we cant get A_2

A_3 without A_1 $\{A_3\}^+ = \{A_3\}$ we cant get A_2

So these attributes arent redundant

Check 2: $A_4 \rightarrow A_5A_2$

$A_4 \rightarrow A_5$ and $A_4 \rightarrow A_2$ are neccessary because there is only one attribute right and left side.

Check 3: $A_2A_3 \rightarrow A_4$

A_2 without A3: $A_3^+ = A_3 \not\supseteq A_4$ so A_2 is necessary

A_3 without A_2: $A_2^+ = A_2 \not\supseteq A_4$ so A_3 is necessary

So these attributes aren't redundant

Check 4: $A_3A_2 \rightarrow A_2A_4$

A_3 without A_2: $A_2^+ = A_2 \not\supseteq A_2$

A_2 without A_3: $A_3^+ = A_3 \not\supseteq A_2$

} we cant get A_2 so these attributes aren't redundant

So, $A_3A_7 \rightarrow A_4$ like $A_2A_7 \rightarrow A_2$!

$A_3^+ = A_3 \not\supseteq A_4$ So we cant get A_4
$A_7^+ = A_7 \not\supseteq A_4$ these attributes aren't redundant

Check 5 ! $A_1A_3A_4 \rightarrow A_2$

$A_1A_3^+ \rightarrow A_1A_3$
$(A_1A_3 \rightarrow A_7)$ $A_1A_3A_7$
$(A_3A_7 \rightarrow A_1)$ $A_1A_2A_7$ $A_2^+ \supseteq A_2$
So A_4 is extraneous

Check 6 ! $A_3A_5 \rightarrow A_1A_7$

$A_3A_5 \rightarrow A_1$ and $A_3A_5 \rightarrow A_7$

A_3 without A_5 ! $\{A_3\}^+ = \{A_3\}$ and $A_5^+ = A_5 \not\supseteq A_1$
A_5 without A_3 ! $\{A_5\}^+ = \{A_5\}$ and $A_3^+ = A_3 \not\supseteq A_1$

We can't get A_1 and A_2, A_3 and A_5 are necessary.
These attributes are not redundant

Lets find redundant functional dependency and remove them.

FDs! $A_1A_3 \rightarrow A_2$ (removed)
$A_4 \rightarrow A_5$
$A_4 \rightarrow A_7$
$A_2A_3 \rightarrow A_4$
$A_3A_7 \rightarrow A_2$ (removed)
$A_3A_7 \rightarrow A_4$
$A_1A_3 \rightarrow A_2$
$A_3A_5 \rightarrow A_1$
$A_3A_5 \rightarrow A_7$ (removed)

1) $A_1 A_3 \rightarrow A_7$

$\{A_1, A_3\}^+ = \{A_1, A_3, A_2, A_4, A_5, A_7\}$

we have A_7 so its removed.

$A_1 A_3 A_2$ ($A_1 A_3 \rightarrow A_2$)

$A_1 A_3 A_2 A_4$ ($A_2 A_3 \rightarrow A_4$)

⋮

So all cover A_7, is redundant

2) $A_4 \rightarrow A_7$ and $A_4 \rightarrow A_5$ are necessary

$A_4^+ = A_4 A_7$ $\quad A_4 A_7 \not\supseteq A_5$

$A_4^+ = A_4 A_5$ $\quad A_4 A_5 \not\supseteq A_7$

3) $A_2, A_3 \rightarrow A_4$

$\{A_2, A_3\}^+ = \{A_2 A_3\}$ we don't get A_4.

$A_2 A_3^+ = A_2 A_3 \not\supseteq A_4$ it is necessary

4) $A_3 A_7 \rightarrow A_2$

$\{A_3, A_7\}^+ = \{A_3, A_7, A_4, A_5, A_1, A_2\}$

$A_3 A_7^+ = A_3 A_7$

$A_3 A_7 A_4$ ($A_3 A_7 \rightarrow A_4$)

$A_3 A_7 A_4 A_5$ ($A_4 \rightarrow A_5$)

So all cover A_2 ($A_1 A_3 \rightarrow A_2$) so its redundant

so its removed

5) $A_3 A_7 \rightarrow A_4$

$\{A_3 A_7\}^+ = \{A_3 A_7\}$ we removed ($A_3 A_7 \rightarrow A_2$)

so we didn't take A_2. So this is not redundant

$A_3 A_7^+ = A_3 A_7 \not\supseteq A_4$

6) $A_1 A_3 \rightarrow A_2$

$A_1 A_3^+ = A_1 A_3 \not\supseteq A_2$

$\{A_1 A_3\}^+ = \{A_1, A_3\}$ we don't get A_2

7→ $A_3A_5 \to A_1$

$A_3A_5^+ = A_3A_5$

$A_3A_5A_2$ $(A_3A_5 \to A_2)$

$A_3A_5A_2A_4 \notin A_1 (A_3A_2 \to A_4)$

$\{A_1A_3\}^+ = \{A_1A_3\}$ we don't get A_1

8→ $A_3A_5 \to A_2$

$\{A_3, A_5\}^+ = \{A_3, A_5, A_1, A_2, A_4, A_2\}$ we get A_2

$A_3A_5^+ = A_3A_5$

$A_3A_5A_1$ $(A_3A_5 \to A_1)$

$A_3A_5A_1A_2$ $(A_1A_3 \to A_2)$

⋮

$A_3A_5A_1A_2A_4A_2 \supseteq A_2$ $(A_4 \to A_2)$

So $A_3A_5 \to A_2$ is redundant

removing functional dependency, apply union

$A_4 \to A_5$ $A_4 \to A_2$ $A_2A_3 \to A_4$

$A_3A_2 \to A_4$ $A_1A_3 \to A_2$ $A_3A_5 \to A_1$

A_4 in 2 FD, Lets union them.

So, $A_4 \to A_5A_2$

$A_1A_3 \to A_2$

$A_2A_3 \to A_4$

$A_3A_5 \to A_1$

$A_3A_2 \to A_4$

} Canonical Cover

2-) $R(A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8)$

$F = \{ A_1A_2 \to A_3,$
$A_1, A_4 \to A_5,$
$A_2 \to A_4,$
$A_1A_6 \to A_2 \}$

a) $\{A_1, A_2\}^+ = \{A_1, A_2, A_3, A_4, A_5\}$ $(A_1A_2 \to A_3,\ A_2 \to A_4,\ A_1A_4 \to A_5)$

$\{A_1, A_6\}^+ = \{A_1, A_6, A_2, A_4, A_3, A_5\} = \{A_1, A_2, A_3, A_4, A_5, A_6\}$

candidate key $(A_1A_4 \to A_5,\ A_1A_2 \to A_3,\ A_2 \to A_4,\ A_1A_6 \to A_2)$

b) We use closure more than one purpose.

- Testing for super key:
  Determine which is super key, check contains all attributes of R.
  we compute α^+ and check α^+ contains if α is super key.
- Testing functional dependency:
  If a functional dependency $\alpha \to \beta$ holds, then we have to just find α^+ and check if β is in α^+ or not.
  $\alpha \to \beta$ holds, check if $\beta \subseteq \alpha^+$
  Compute α^+ by using closure, then check if contains
- Computing closure of F.

  For each $Y \subseteq R$, we find the closure Y^+ and for each $S \subseteq Y^+$ we output a functional dependency $Y \to S$.

3-) $R(A_1, A_2, A_3, A_4)$ $F(A_1 \to A_2,\ A_2 \to A_3\}$ A_1 and A_2 are super keys

a) The relation is BCNF for every functional dependency like $X \to Y$ (X: superkey) (Y: attribute set)
So A_1 and A_2 are satisfying those condition
Hence it is in BCNF.

b) $R(A_1, A_2, A_3, A_4)$

$R_1(A_1 A_2)$ $A_1 \to A_2$

$R_2(A_1 A_3)$ $A_1^+ \{A_1 A_2 A_3\}$ $A_1 \to A_3$

$R_3(A_1 A_4)$ No FD

Check relations are in BCNF

for R_1, we can find all attributes from A_1.
Because A_1 is superkey. R_1 is in BCNF

for R_2, we can find all attributes from A_1
because A_1 is superkey. and $A_1 \to A_2$, $A_2 \to A_3$
So R_2 is in BCNF

for R_3, we can't find all attributes from A_1 or A_4.
functional dependency not enough
But IF A_1 is superkey in relation R, then definitely
FD $A_1 \to A_4$ implies, hence in BCNF
So R_3 is in BCNF

c)

$R_1(A_1, A_2)$

$A_1^+ = \{A_1, A_2\} \Rightarrow A_1 \to A_2$
$A_2^+ = \{A_2\}$

$R_2(A_1 A_3)$

$A_1^+ = \{A_1, A_2, A_3\} \Rightarrow A_1 \to A_3$
$A_3^+ = \{A_3\}$

A_2 is not in R_2

Since $A_2 \to A_3$ also possible

$R_3(A_1 A_4)$

$A_1^+ = \{A_1 A_2 A_3\}$
$A_4^+ = \{A_4\}$

No FD

Now $f_1 \cup f_2 = \{A_1 \to A_2, A_1 \to A_3\}$
$f = \{A_1 \to A_2, A_2 \to A_3\}$

But $f \neq f_1 \cup f_2$
$f_1 \cup f_2$ doesn't contains $A_2 \to A_3$

So decomposition dependency is not preserving