MATH 118: Statistics and Probability **(Due: 26/04/20)**

Homework #2

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Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr.
- Submit your homework (both your latex and pdf files in a zip file) into the course page of Moodle.
- Save your latex, pdf and zip files as "Name_Surname_StudentId".{tex, pdf, zip}.
- The deadline of the homework is 26/04/20 23:55.

Problem 1: (10 points)

Suppose that you are inspecting a lot of 1000 surgical masks, among which 20 are used. You choose two masks randomly from the lot without replacement. Let

$$X_1 = \begin{cases} 1, & \text{if the 1st surgical mask is used,} \\ 0, & \text{otherwise} \end{cases}$$

$$X_2 = \begin{cases} 1, & \text{if the 2nd surgical mask is used,} \\ 0, & \text{otherwise} \end{cases}$$

Find the probability that at least one surgical mask chosen is used.

(Solution)

Problem 2: (8+8=16 points)

Suppose X and Y are random variables with P(X = 1) = P(X = -1) = $\frac{1}{2}$; P(Y = 1) = P(Y = -1) = $\frac{1}{2}$. Let c = P(X = 1 and Y = 1).

(a) Determine the joint distribution of X and Y, Cov(X, Y), and Cor(X, Y).

(Solution)

(b) For what value(s) of c are X and Y independent? For what value(s) of c are X and Y 100% correlated?

(Solution)

Problem 3: (4+4+4+4+4+4=24 points)

In the information security department in a software company, a single crucial program works only 85% of the time. In order to enhance the reliability of the system, it is decided that 3 programs will be executed in parallel such that the system fails only if they all fail. Assume the programs act independently and that they are equivalent in the sense that all 3 of them have an 85% success rate. Consider the random variable X as the number of programs out of 3 that fail.

(a) Write out a probability function for the random variable X.

(Solution)

(b) What is $E(X)$ (i.e., the mean number of programs out of 3 that fail)?

(Solution)

(c) What is Var(X)?

(Solution)

(d) What is the probability that the entire system is successful?

(Solution)

(e) What is the probability that the system fails?

(Solution)

(f) If the desire is to have the system be successful with probability 0.99, are three programs sufficient? If not, how many are required?

(Solution)

Problem 4: (10+10=20 points)

According to World Health Organization (WHO), approximately 30% of all treatment failures in Covid-19 are caused by lack of available respirators.

(a) What is the probability that out of the next 20 treatment failures at least 10 are due to lack of available respirators?

(Solution)

(b) What is the probability that no more than 4 out of 20 such failures are due to lack of available respirators?

(Solution)

Problem 5: (7+7=14 points)

A manufacturing company uses an acceptance scheme on items from a production line before they are shipped. The plan is a two-stage one. Boxes of 25 items are ready for shipment, and a sample of 3 items is tested for defectives. If any defectives are found, the entire box is sent back for 100% screening. If no defectives are found, the box is shipped.

(a) What is the probability that a box containing 3 defectives will be shipped?

(Solution)

(b) What is the probability that a box containing only 1 defective will be sent back for screening?

(Solution)

Problem 6: Probability Distributions of Random Variables (8+8=16 points)

Suppose the probability that any given person will believe a tale about the transgressions of a famous actress is 0.8. What is the probability that

(a) the sixth person to hear this tale is the fourth one to believe it?

(Solution)

(b) the third person to hear this tale is the first one to believe it?

(Solution)