Fuzzy Modelling

LECTURE 5

T-norm fuzzy logics

- **Algebraic product**

$$\begin{aligned}&\forall x \in X:\ \mu_{A\cap B}(x) = \\ &= \mathrm{prod}\left(\mu_A(x), \mu_B(x)\right) = \\ &= \mu_A(x) \cdot \mu_B(x)\end{aligned} \tag{5.1}$$

- **Drastic product**

$$\begin{aligned}&\forall x \in X:\ \mu_{A\cap B}(x) = \\ &= \mathrm{prod}_{\mathrm{dras}}\left(\mu_A(x), \mu_B(x)\right) = \\ &= \begin{cases} \mu_A(x) \ \text{ for } \mu_B(x) = 1 \\ \mu_B(x) \ \text{ for } \mu_A(x) = 1 \\ 0 \ \text{ in other cases} \end{cases}\end{aligned} \tag{5.2}$$

- **Łukasiewicz product**

$$\begin{aligned}&\forall x \in X:\ \mu_{A\cap B}(x) = \\ &= \mathrm{prod}_{\mathrm{Łuk}}\left(\mu_A(x), \mu_B(x)\right) = \\ &= \max\left(0, \mu_A(x) + \mu_B(x) - 1\right)\end{aligned} \tag{5.3}$$

- **Einstein product**

$$\begin{aligned}&\forall x \in X:\ \mu_{A\cap B}(x) = \\ &= \mathrm{prod}_{\mathrm{Ein}}\left(\mu_A(x), \mu_B(x)\right) = \\ &= \frac{\mu_A(x)\cdot\mu_B(x)}{2 - \left(\mu_A(x) + \mu_B(x) - \mu_A(x)\cdot\mu_B(x)\right)}\end{aligned} \tag{5.4}$$

- **Hamacher product**

$$\begin{aligned}&\forall x \in X:\ \mu_{A\cap B}(x) = \\ &= \mathrm{prod}_{\mathrm{Ham}}\left(\mu_A(x), \mu_B(x)\right) = \\ &= \frac{\mu_A(x)\cdot\mu_B(x)}{\mu_A(x) + \mu_B(x) - \mu_A(x)\cdot\mu_B(x)}\end{aligned} \tag{5.5}$$

Example 5.1.

Script:

```
% Sigmoidal membership functions
x1=[-2:0.2:10]
y1=(1)./(1+exp(-2.*(x1-2)))
y2=(1)./(1+exp(-2.*(x1-6)))
y3=(1)./(1+exp(-4.*(x1-6)))

% plot , color, parameters, line width, markersize
plot(x1,y1, 'c-*','linewidth', 2,'markersize', 12,
     x1,y2, 'm-o','linewidth', 2,'markersize', 12,
     x1,y3, 'k-d','linewidth', 2,'markersize',12)

% axis , line width, font size, grid, legend
set(gca,'linewidth',2, 'fontsize',18)
grid on
legend('MFA', 'MFB', 'MFC')
```

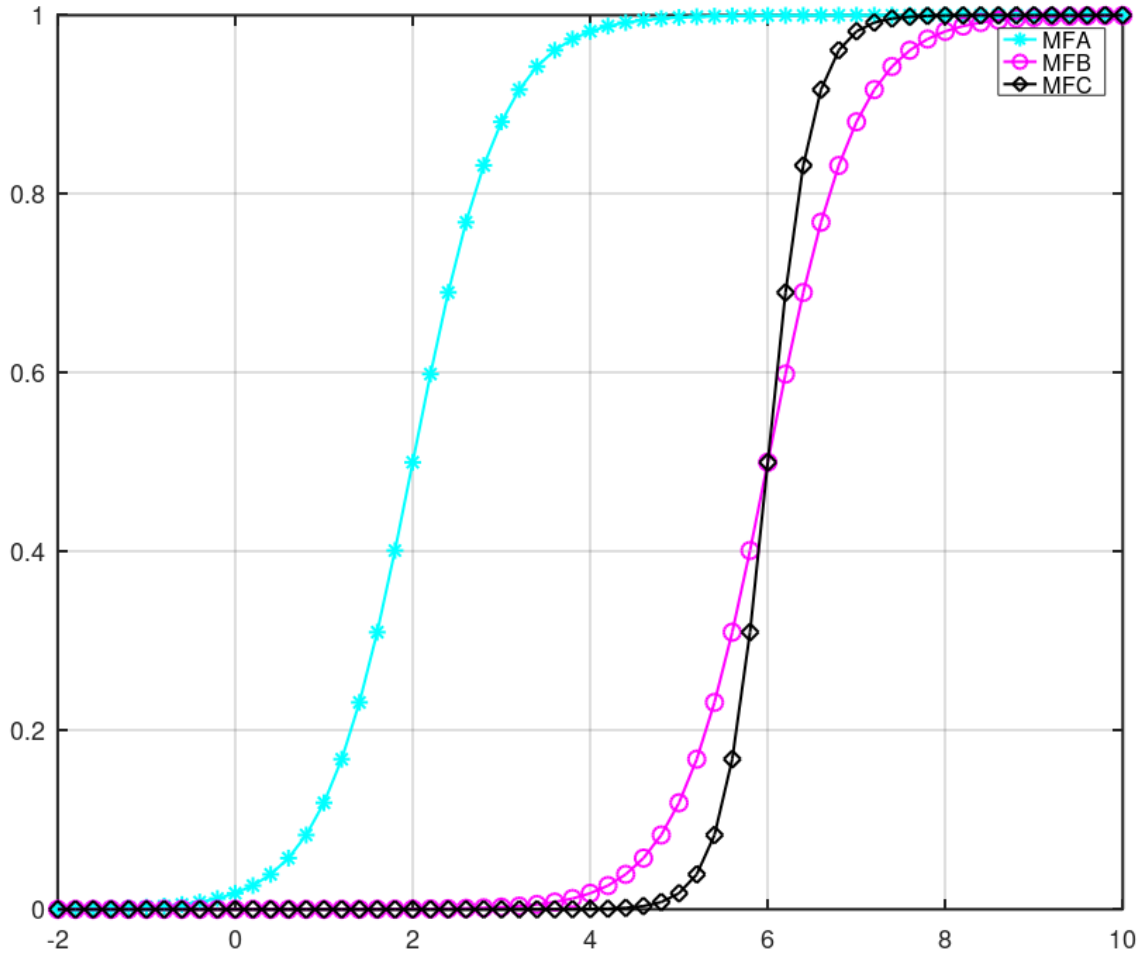

Fig. 5.1. The sigmoidal membership functions MFA, MFB and MFC

α -cut(A) for α=0.3

α -cut(A) = { 1.6 1.8 2 2.2 2.4
2.6 2.8 3 3.2 3.4 3.6 3.8 4
4.2 4.4 4.6 4.8 5 5.2 5.4

5.6 5.8 6 6.2 6.4 6.6 6.8 7 7.2
7.4 7.6 7.8 8 8.2 8.4 8.6
8.8 9 9.2 9.4 9.6 9.8 10 }

α -cut(B) for α=0.6

α -cut(B) = { 6.2 6.4 6.6 6.8 7 7.2
7.4 7.6 7.8 8 8.2 8.4 8.6
8.8 9 9.2 9.4 9.6 9.8 10 }