Fuzzy Modelling

LECTURE 8

Fuzzy Union (Fuzzy Sum)

Fuzzy Union of two sets, A and B, is interpreted as "A or B", which takes the maximum value of two membership functions

$$\forall x \in X : \ \mu_{A \cup B}(x) = \max\left(\mu_A(x), \mu_B(x)\right) \tag{8.1}$$

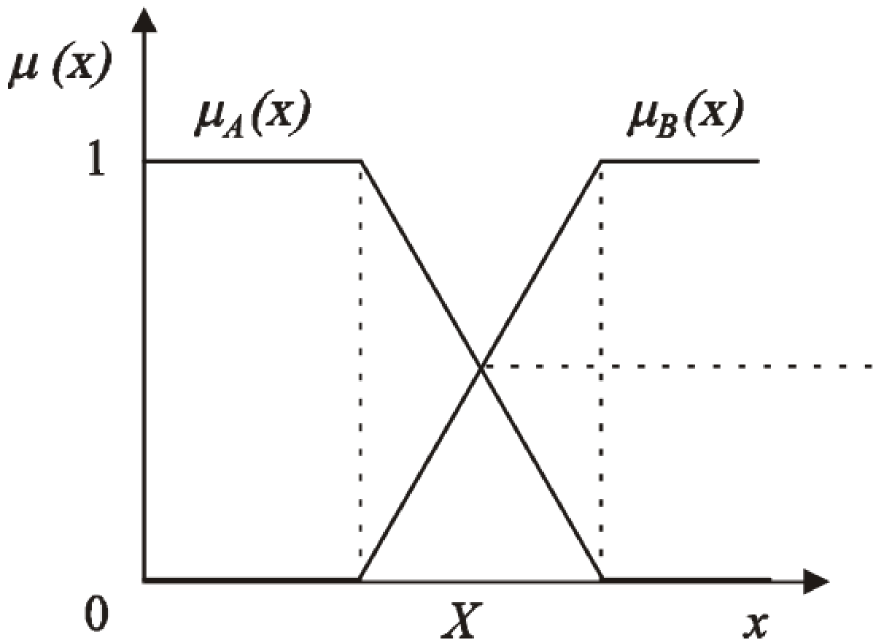

Fig. 8.1. The membership functions of fuzzy sets A and B

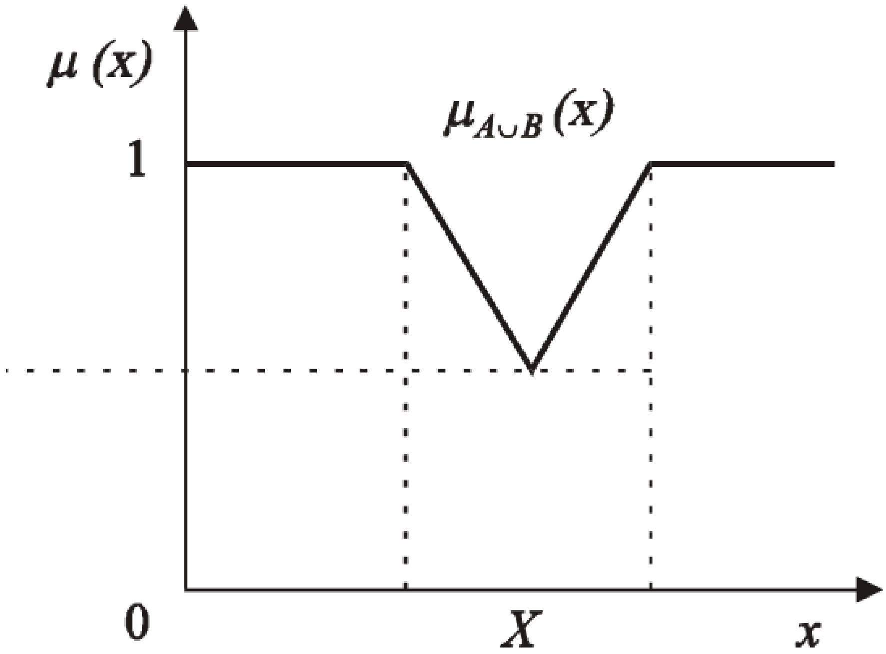

Fig. 8.2. The membership function of fuzzy sum of two sets: A and B

S-norm fuzzy logics

S-norm fuzzy logics belong in broader classes of fuzzy logics and many-valued logics.

The conditions for S-norm operators:

$$1)\quad S:[0,1]\,x\,[0,1]\rightarrow[0,1]$$

-The identity of representation,

$$2)\quad S\left(\mu_A(x),\mu_B(x)\right)=S\left(\mu_B(x),\mu_A(x)\right)$$

- Commutativity,

$$3)\ \ \mathrm{S}\big(\mu_A(x), \mathrm{S}\,(\mu_B(x), \mu_C(x))\big) =$$
$$= \mathrm{S}\big(\mathrm{S}\,(\mu_A(x), \mu_B(x)), \mu_C(x)\big)$$

\- Associativity,

$$4)\ \ \mu_A(x) \geq \mu_C(x)\ \ \wedge\ \ \mu_B(x) \geq \mu_D(x)$$
$$\Rightarrow \mathrm{S}\,\big(\mu_A(x), \mu_B(x)\big) \geq \mathrm{S}\,\big(\mu_C(x), \mu_D(x)\big)$$

\- Monotony,

$$5)\ \ \mathrm{S}\,\big(\mu_A(x), 0\big) = \mu_A(x)$$

\- Neutrality,

S-norm fuzzy logics

- **Probabilistic sum**

$$\forall x \in X:\ \mu_{A \cup B}(x) =$$
$$= \mathrm{sum}_{\mathrm{prob}}\big(\mu_A(x), \mu_B(x)\big) =$$
$$= \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x) \qquad (8.2)$$

- **Drastic sum**

$$\forall x \in X: \ \mu_{A\cup B}(x) =$$
$$= \mathrm{sum}_{\mathrm{dras}}\left(\mu_A(x), \mu_B(x)\right) =$$
$$= \begin{cases} \mu_A(x) & \text{dla } \mu_B(x) = 0 \\ \mu_B(x) & \text{dla } \mu_A(x) = 0 \\ 1 & \text{in other cases} \end{cases} \tag{8.3}$$

- **Łukasiewicz sum**

$$\forall x \in X: \ \mu_{A\cup B}(x) =$$
$$= \mathrm{sum}_{\text{Łuk}}\left(\mu_A(x), \mu_B(x)\right) =$$
$$= \min\left(1, \mu_A(x) + \mu_B(x)\right) \tag{8.4}$$

- **Einstein sum**

$$
\begin{aligned}
&\forall x \in X: \ \mu_{A\cup B}(x) = \\
&= \operatorname{sum}_{\mathrm{Ein}}\left(\mu_A(x), \mu_B(x)\right) = \\
&= \frac{\mu_A(x) + \mu_B(x)}{1 + \mu_A(x) \cdot \mu_B(x)}
\end{aligned}
\tag{8.5}
$$

- **Hamacher sum**

$$
\begin{aligned}
&\forall x \in X: \ \mu_{A\cup B}(x) = \\
&= \operatorname{sum}_{\mathrm{Ham}}\left(\mu_A(x), \mu_B(x)\right) = \\
&= \frac{\mu_A(x) + \mu_B(x) - 2 \cdot \mu_A(x) \cdot \mu_B(x)}{1 - \mu_A(x) \cdot \mu_B(x)}
\end{aligned}
\tag{8.6}
$$

Example 8.1

Probabilistic sum, Łukasiewicz sum

```
x1=[-12:0.2:0];
y1=exp(-((x1+4)/2).^2);
y2=exp(-((x1+6)/2).^2);
```

```
y3=exp(-((x1+8)/2).^2);

y4=y1+y2-y1.*y2

% probabilistic sum for y1 and y2
y4=y1+y2-y1.*y2

plot(x1,y1,'r-', x1,y2,'m-', x1,y3,'g-', x1, y4, 'b-o')
grid on
hold on
text(5.7, 0.95, 'MFA')
```

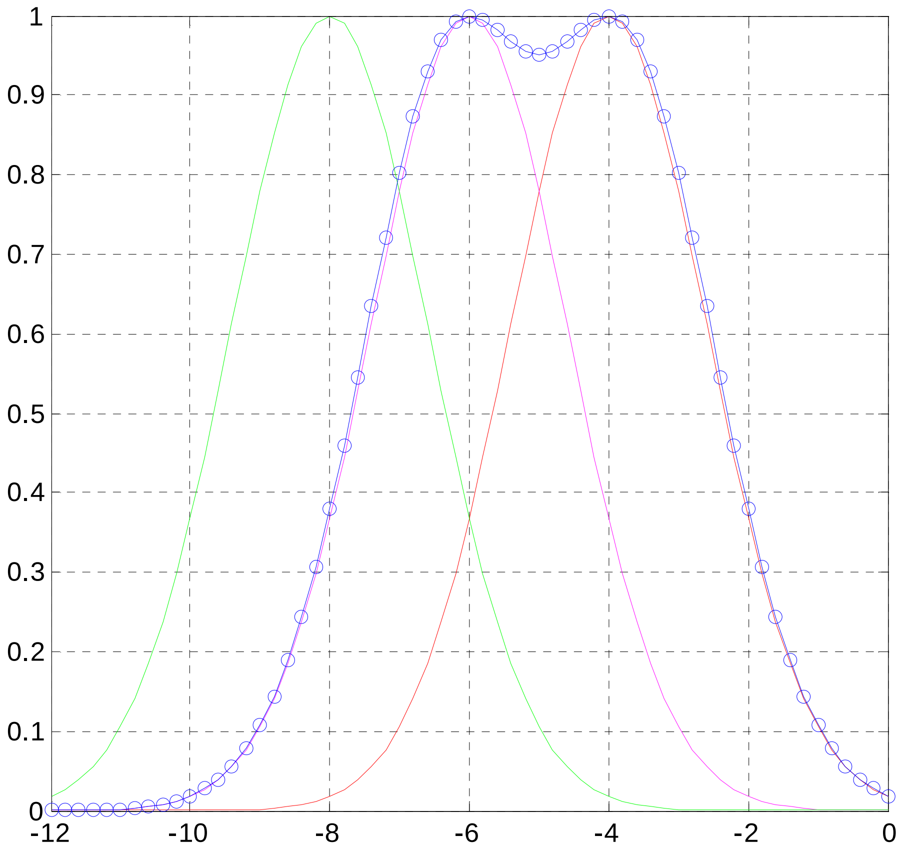

Fig. 8.3. Probabilistic sum of fuzzy sets