Częstochowa University of Technology
Faculty of Electrical Engineering

Fuzzy Modelling

Lecture

Dr inż. Krzysztof Olesiak
E-mail: kolesiak@el.pcz.czest.pl

LECTURE 1

Fuzzy sets

A fuzzy set is an extension of a classical set.

If X is the universe of discourse and its elements are denoted by x, then a fuzzy set A in X is defined as a set of ordered pairs.

$$\forall x \in X: \; A = \left\{ \left(x, \mu_A(x) \right) \right\} \tag{1.1}$$

The membership function μ_A maps each element of X to a membership value between 0 and 1.

Membership Functions

A membership function μ_A is a curve that defines how each point in the input space is mapped to a membership value (or degree of membership) between 0 and 1.

The input space is sometimes referred to as the universe of discourse, a fancy name for a simple concept.

Continuous fuzzy sets

A continuous fuzzy set is defined by means of a continuous membership function:

$$\forall x \in X: \quad A = \int_X \mu_A(x)/x \tag{1.2}$$

Discrete fuzzy set

A discrete fuzzy set is defined by ordered pairs:

$$\begin{aligned}
&\forall x \in X: \\
&A = \sum_{i=1}^{n} \mu_A(x_i)/x_i = \\
&= \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \ldots \\
&+ \mu_A(x_n)/x_n
\end{aligned} \tag{1.3}$$

The simplest membership functions are formed using straight lines.

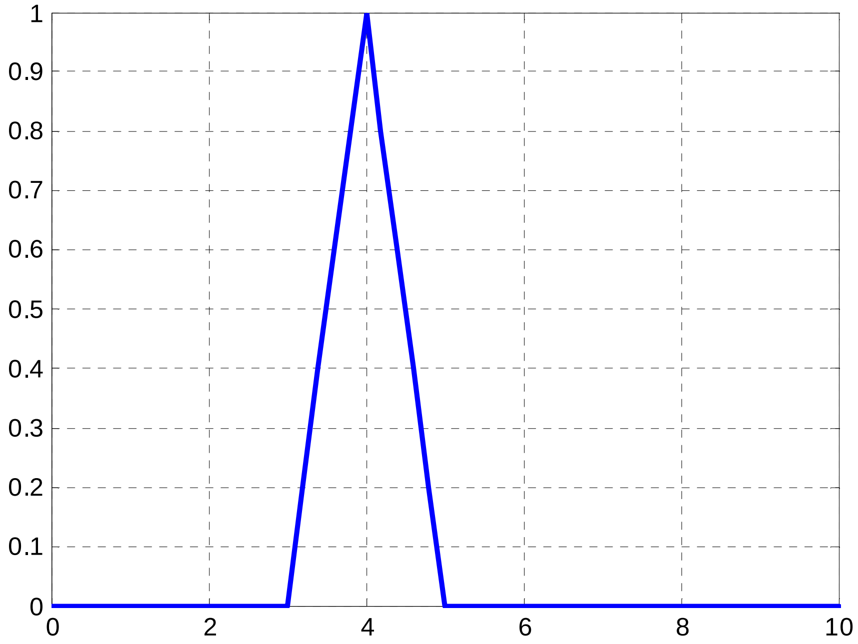

Fig. 1.1. The triangular membership function

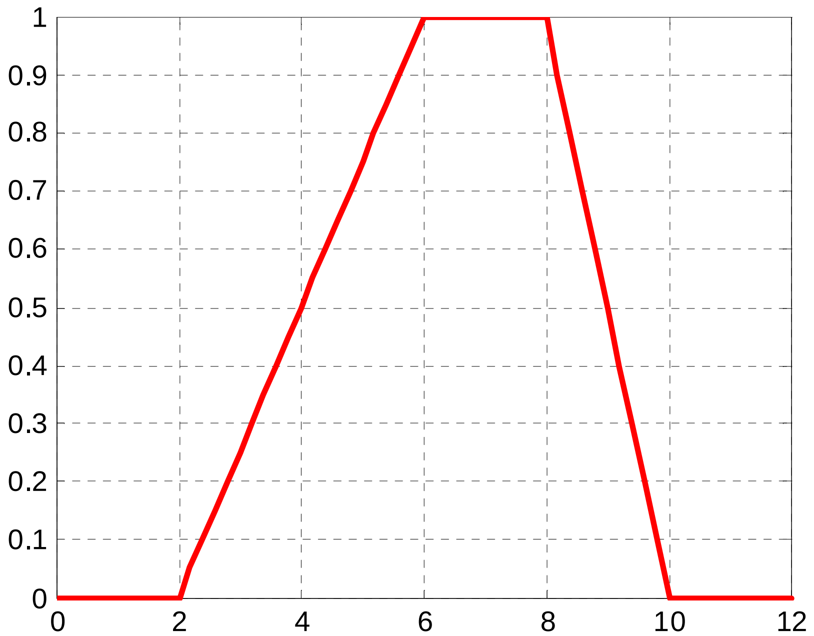

Fig. 1.2. The trapezoidal membership function

Example 1

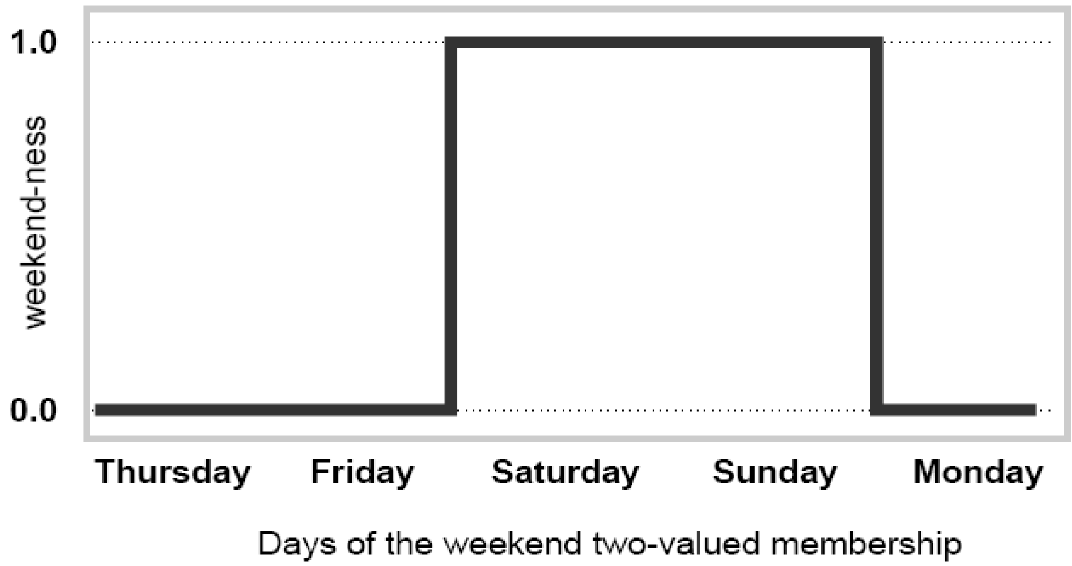

Fig. 1.3. The crisp set: days of the weekend

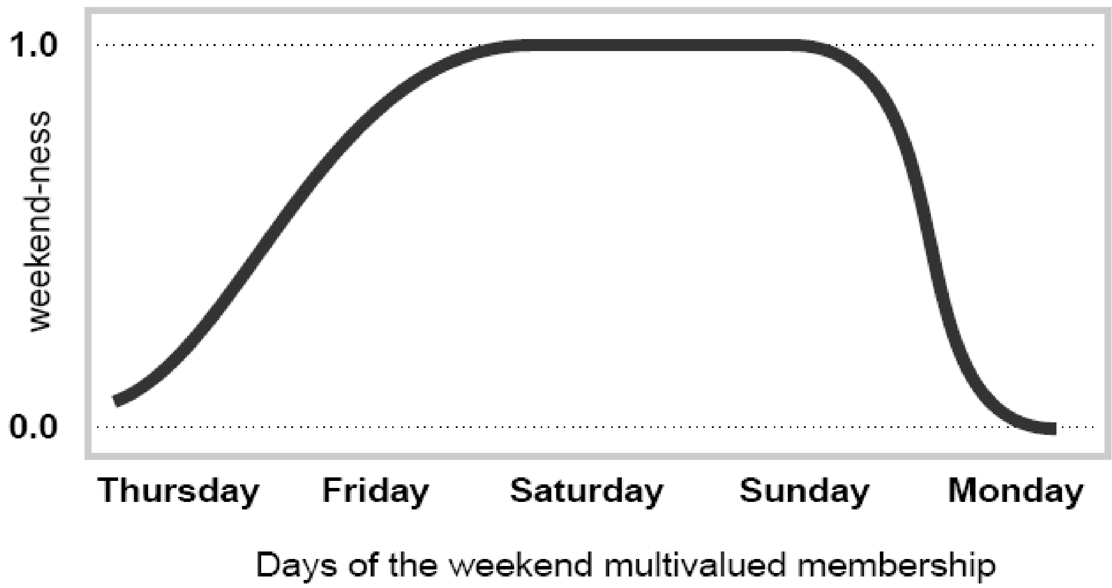

Fig. 1.4. The fuzzy set: days of the weekend

Example 2

Using the astronomical definitions for the season, we get sharp boundaries as shown on the left in the figure that follows.

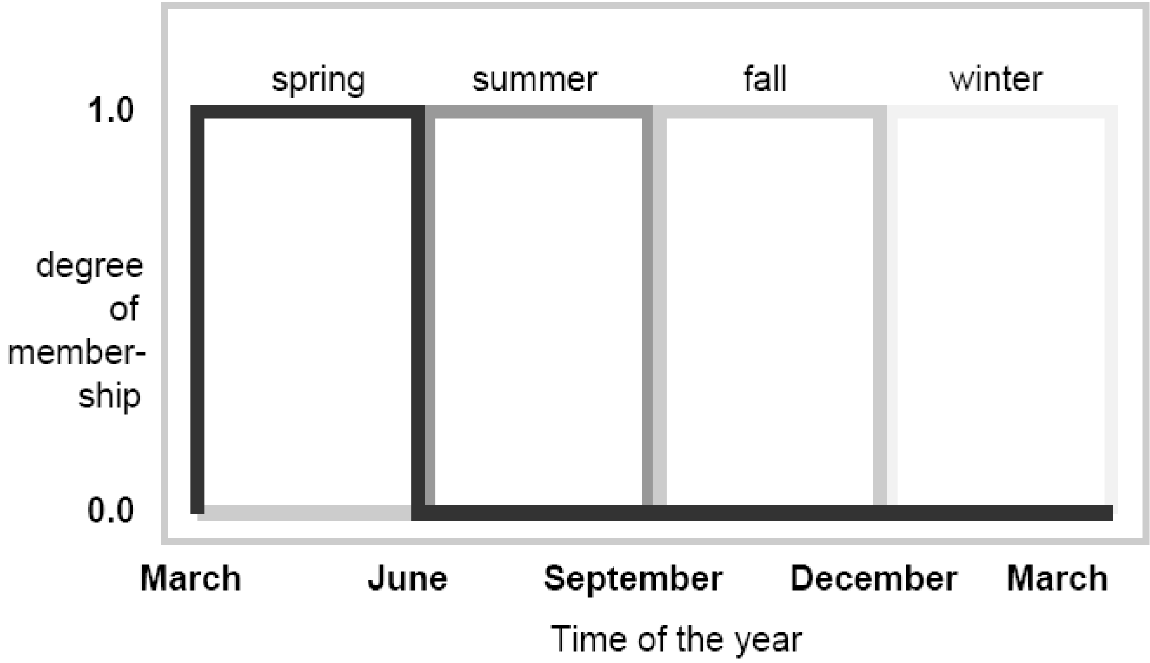

Fig. 1.5. The crisp set: time of the year

But what we experience as the seasons vary more or less continuously as shown on the right below (in temperate northern hemisphere climates).

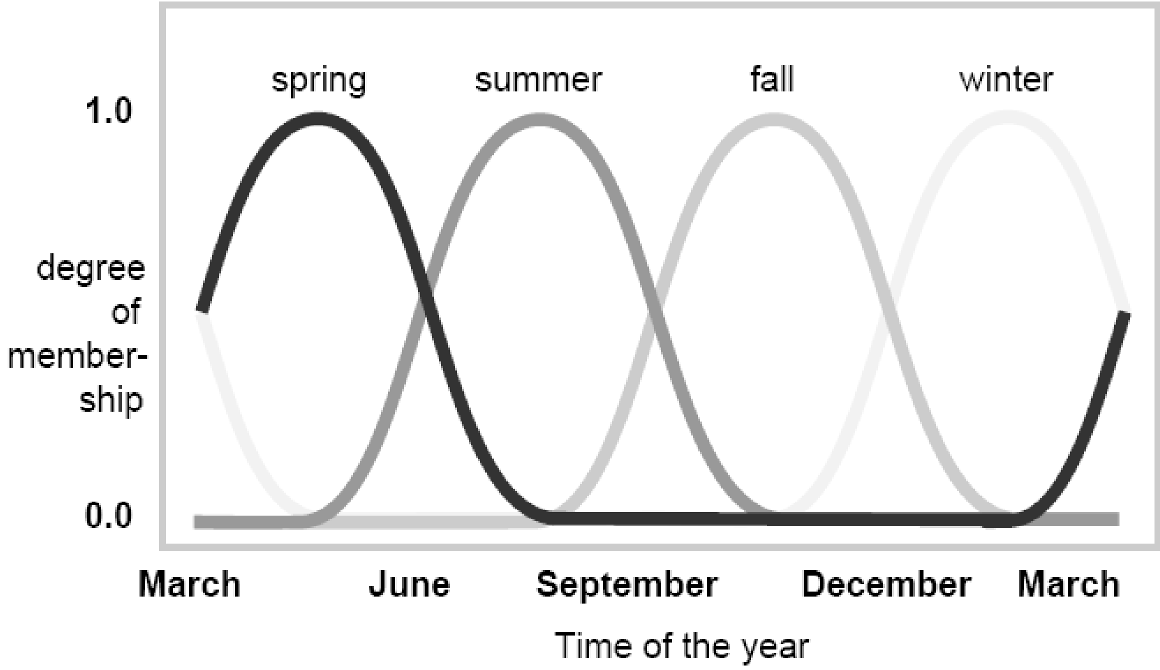

Fig. 1.6. The fuzzy set set: time of the year

Properties of fuzzy sets

1) The height of a fuzzy set A is defined by:

$$\mathrm{hgt}(A) = \mathrm{height}(A) = \sup_{x \in X} \mu_A(x) \qquad (1.4)$$

The fuzzy sets with a height equal to 1 are called normal sets.

2) The support of a fuzzy set A is defined by:

$$\mathrm{supp}(A) = \left\{ x \in X : \mu_A(x) > 0 \right\} \quad (1.5)$$

Notation: $\mathrm{supp}(A)$ or $\mathrm{support}(A)$

3) The core of a fuzzy set A is defined by:

$$\mathrm{core}\,(A) = \left\{ x \in X : \mu_A(x) = 1 \right\} \quad (1.6)$$

4) The α-cut of a fuzzy set A is defined by:

$$A_\alpha = \left\{ x \in X : \forall \alpha \in [0,1] \;\; \mu_A(x) \geq \alpha \right\} \quad (1.7)$$

Notation: A_α or $\alpha\, cut(A)$

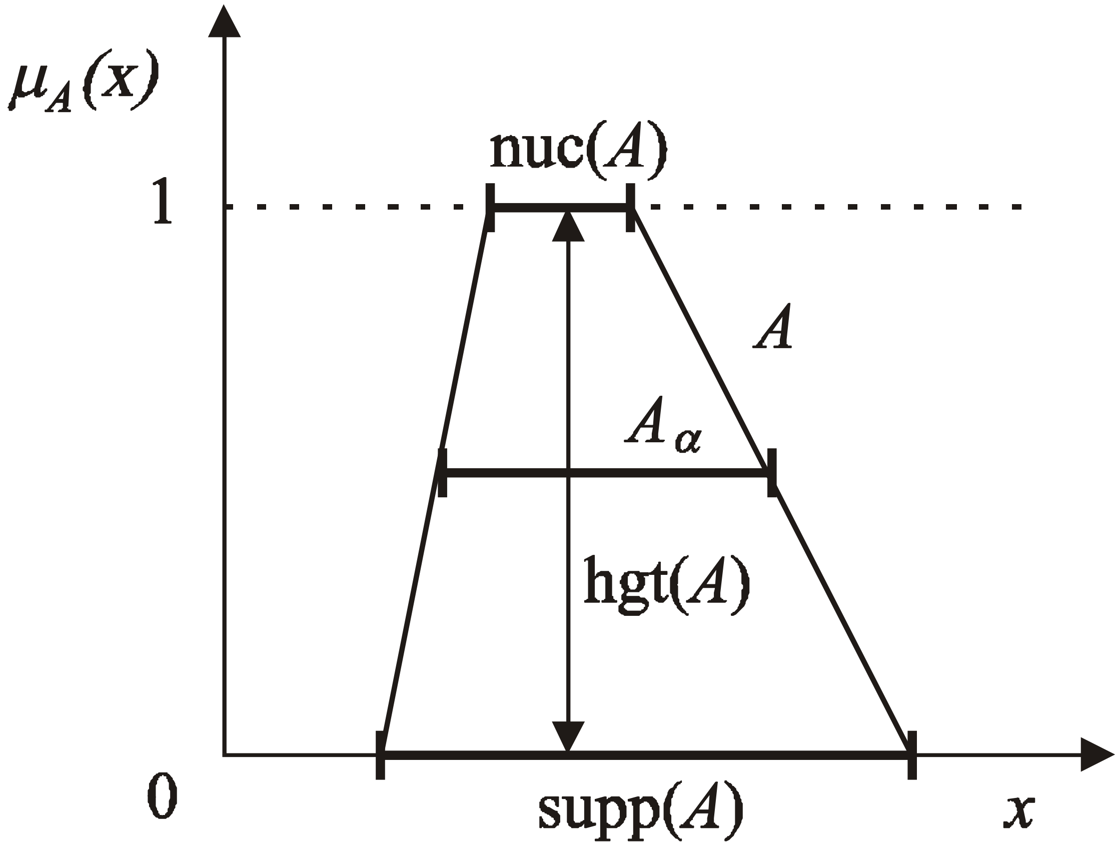

Fig. 1.7. Properties of fuzzy sets