Fuzzy Modelling

LECTURE 3

Operations on Fuzzy Sets

Fuzzy Complement

The complement of a fuzzy set A, which is understood as "NOT(A)," is defined by:

$$\forall x \in X : \mu_{\bar{A}}(x) = 1 - \mu_A(x) \tag{3.1}$$

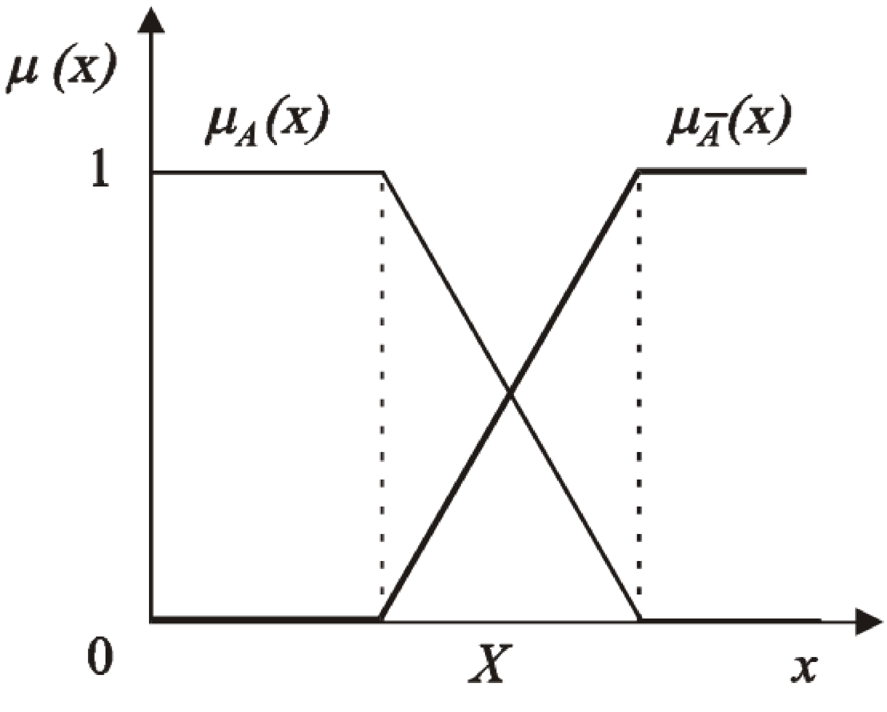

Fig. 3.1. The complement of a fuzzy set A

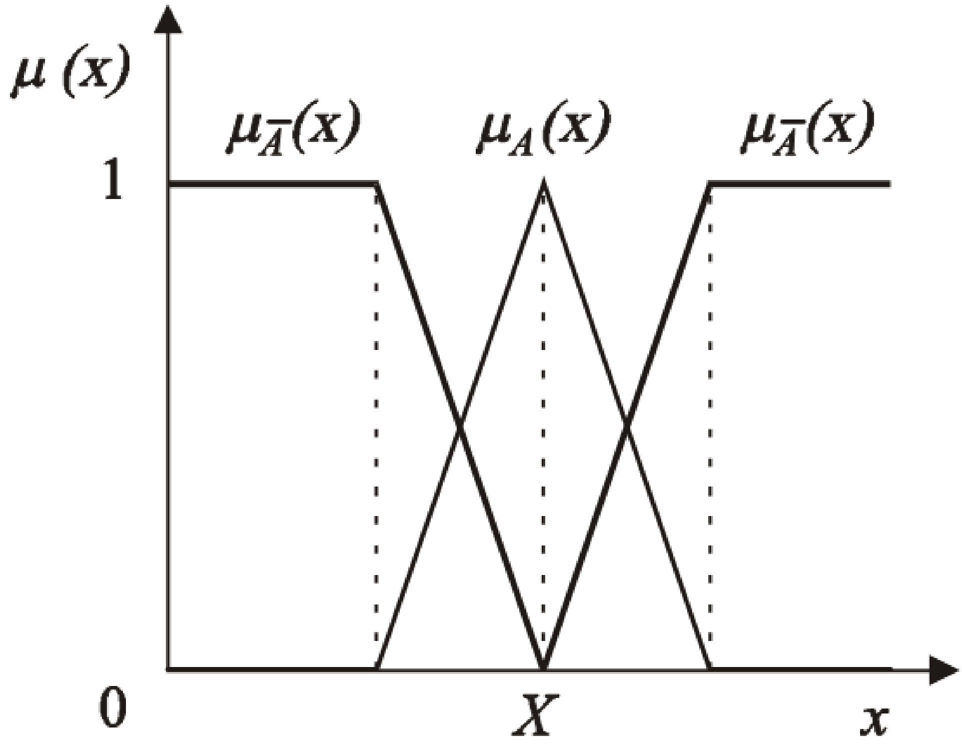

Fig. 3.2. The complement of a fuzzy set A

Fuzzy Intersection

Fuzzy intersection of two sets, A and B, is interpreted as "A and B", which takes the minimum value of two membership functions

$$\forall x \in X : \mu_{A\cap B}(x) = \min\left(\mu_A(x), \mu_B(x)\right)$$

(3.2)

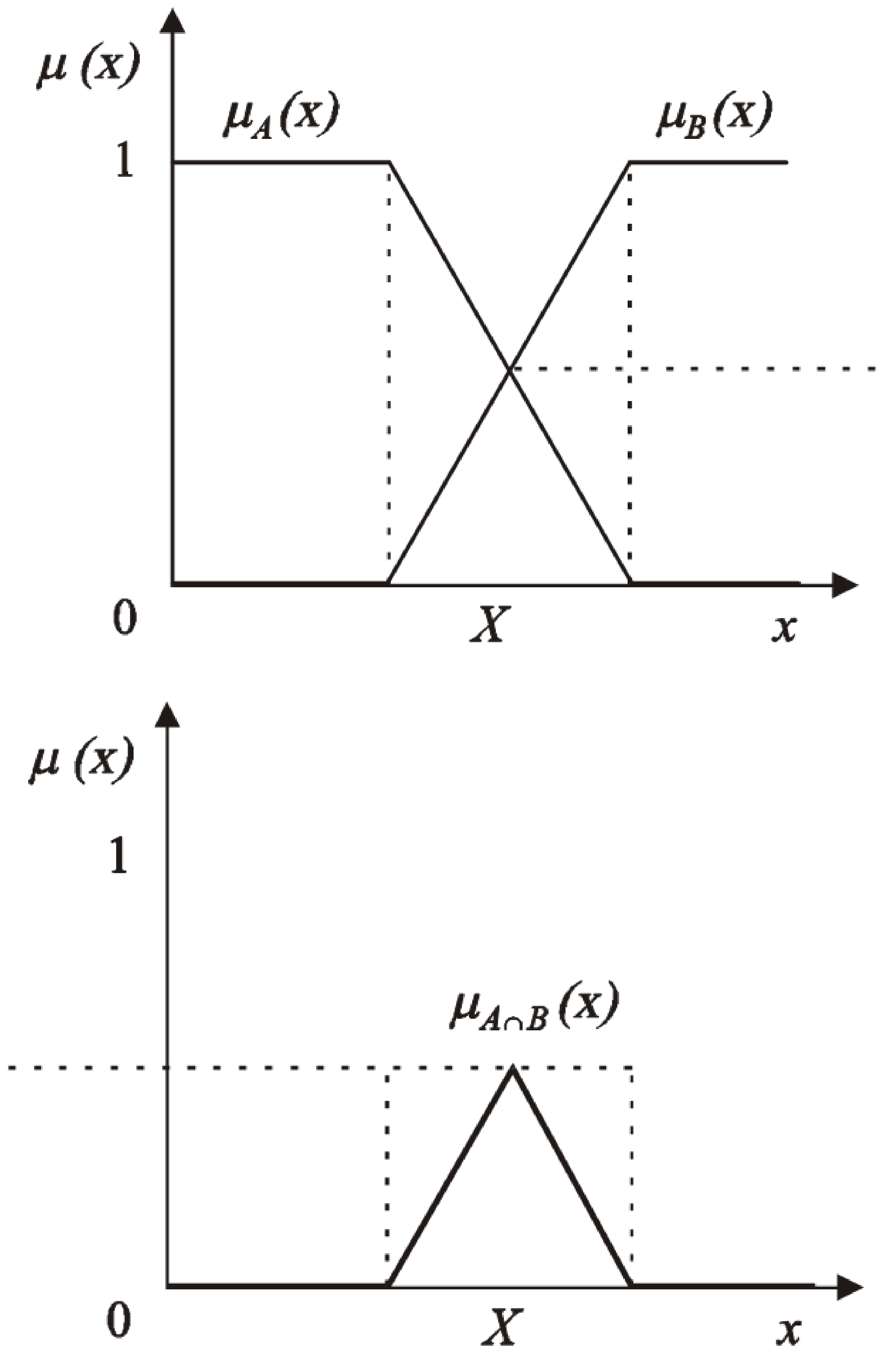

Fig. 3.3. Fuzzy intersection of two sets: A and B

Example 3.1.

```
x1=[-1:0.25:9]
y1=(1)./(1+exp(-4.*(x1-3)))
y2=(1)./(1+exp(-2.*(x1-5)))
y3=(1)./(1+exp(-8.*(x1-5)))
plot(x1,y1, 'r-*',x1,y2, 'g-+',x1,y3, 'b-d')
grid on
legend('MFA', 'MFB','MFC')
```

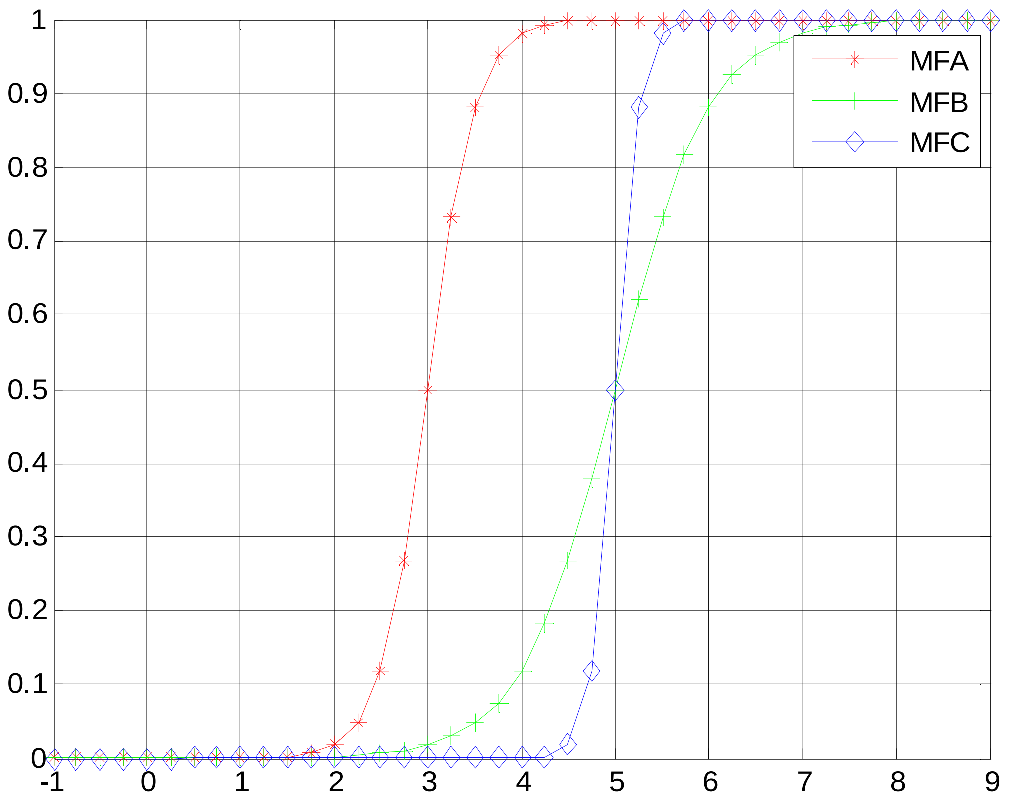

Fig. 3.4. Sigmoidal membership functions

$\alpha = 0.3$

α cut (A) = { 3 3,25000000000000 3,50000000000000 3,75000000000000 4 4,25000000000000 4,50000000000000 4,75000000000000 5 5,25000000000000 5,50000000000000 5,75000000000000 6 6,25000000000000 6,50000000000000 6,75000000000000 7 7,25000000000000 7,50000000000000 7,75000000000000 8 8,25000000000000 8,50000000000000 8,75000000000000 9}

$\alpha = 0.6$

α cut(B) = {5,25000000000000 5,50000000000000 5,75000000000000 6 6,25000000000000 6,50000000000000 6,75000000000000 7 7,25000000000000 7,50000000000000 7,75000000000000 8 8,25000000000000 8,50000000000000 8,75000000000000 9}

T-norm fuzzy logics

T-norm fuzzy logics belong in broader classes of fuzzy logics and many-valued logics.

The conditions for T-norm operators

$$1)\quad \mathrm{T}:[0,1]\,\mathrm{x}\,[0,1]\rightarrow[0,1]$$

-The identity of representation,

$$2)\quad \mathrm{T}\big(\mu_A(x),\mu_B(x)\big)=\mathrm{T}\big(\mu_B(x),\mu_A(x)\big)$$

- Commutativity,

$$3)\quad \mathrm{T}\Big(\mu_A(x),\mathrm{T}\big(\mu_B(x),\mu_C(x)\big)\Big)= \\ =\mathrm{T}\Big(\mathrm{T}\big(\mu_A(x),\mu_B(x)\big),\mu_C(x)\Big)$$

- Associativity,

$$4)\quad \mu_A(x)\geq\mu_C(x)\ \wedge\ \mu_B(x)\geq\mu_D(x) \\ \Rightarrow\mathrm{T}\big(\mu_A(x),\mu_B(x)\big)\geq\mathrm{T}\big(\mu_C(x),\mu_D(x)\big)$$

- Monotony,

5) $\mathrm{T}\left(\mu_A(x),1\right)=\mu_A(x)$

- Neutrality,