$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

For B(x,y) = [1,2] and F(x',y') =[2,2]

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

$a_{11} + 2a_{12} + a_{13} = 2$

$a_{21} + 2a_{22} + a_{23} = 2$

For B(x,y) = [2,1] and F(x',y') =[-1,4]

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix}$$

$2a_{11} + a_{12} + a_{13} = -1$

$2a_{21} + a_{22} + a_{23} = 4$

For B(x,y) = [3,1] and F(x',y') =[-4,4]

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ 4 \\ 1 \end{bmatrix}$$

$3a_{11} + a_{12} + a_{13} = -4$

$3a_{21} + a_{22} + a_{23} = 4$

Then, Let's look at these equations collectively

For first equations,

$a_{11} + 2a_{12} + a_{13} = 2$

$2a_{11} + a_{12} + a_{13} = -1$

$3a_{11} + a_{12} + a_{13} = -4$

We have the matrix:

$$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -1 \\ 3 & 1 & 1 & -4 \end{bmatrix}$$

For second equations,

$a_{21} + 2a_{22} + a_{23} = 2$

$2a_{21} + a_{22} + a_{23} = 4$

$3a_{21} + a_{22} + a_{23} = 4$

We have the matrix:

$$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 1 & 4 \\ 3 & 1 & 1 & 4 \end{bmatrix}$$

Let's solve first equation with Gaussian elimination with scaled partial pivoting

$$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 1 & -1 \\ 3 & 1 & 1 & -4 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 1 & -1 \\ 0 & 1.5 & 0.5 & 2.5 \\ 0 & -0.5 & -0.5 & -2.5 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 1 & -1 \\ 0 & 1.5 & 0.5 & 2.5 \\ 0 & 0 & -0.333 & -1.666 \end{bmatrix}$$

a_{13} = 5.0

a_{12} = 0.0

a_{11} =-3.0

Then, solve second equation with Gaussian elimination with scaled partial pivoting

$$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 1 & 4 \\ 3 & 1 & 1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 1 & 4 \\ 0 & 1.5 & 0.5 & 0 \\ 0 & -0.5 & -0.5 & -2.0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 1 & 4 \\ 0 & 1.5 & 0.5 & 0 \\ 0 & 0 & -0.3 & -2.0 \end{bmatrix}$$

a_{23} = 5.999

a_{22} =-1.999

a_{21} = 4.441 * 10^{-16}

So A matrix is

$$A = \begin{bmatrix} -3.000 & 0.000 & 5.000 \\ 4.441 * 10^{-16} & -1.999 & 5.999 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$$

That's mean how to transform B into F.

Let's take inverse of A matrix to find the opposite (F->B)

First, matrix of Minors:

$$\begin{bmatrix} -1.999 & 4.441 * 10^{-16} & 0.000 \\ 0.000 & -3.000 & 0.000 \\ 9.995 & -17.997 & 5.997 \end{bmatrix}$$

And cofactor

$$\begin{bmatrix} -1.999 & -4.441 * 10^{-16} & 0.000 \\ 0.000 & -3.000 & 0.000 \\ 9.995 & 17.997 & 5.997 \end{bmatrix}$$

Transpoze of cofactor

$$\begin{bmatrix} -1.999 & 0.000 & 9.995 \\ -4.441 . 10^{-16} & -3.000 & 17.997 \\ 0.000 & 0.000 & 5.997 \end{bmatrix}$$

Determinant of the matrix is = 5.97

A^{-1} = $\frac{1}{det(A)}$. (Transpoze Cofactor matrix)

So,

$$A^{-1} = \begin{bmatrix} -0.333 & 0.000 & 1.666 \\ -7.439 . 10^{-7} & -0.502 & 3.010 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}$$

So we find the (F->B) matrix.

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