Title:

A Proposed Framework for Applying CHARM to EEG Data

Abstract:

We propose a novel framework for applying the Complex Harmonics Decomposition (CHARM) method to EEG data, adapted from the work of Deco, Sanz Perl, and Kringelbach (2025). CHARM, originally designed to uncover low-dimensional, nonlocal structures in fMRI BOLD signals using Schrödinger-based kernel operators, is here extended for the analysis of high temporal resolution EEG data. This framework incorporates EEG-specific preprocessing, time-frequency decomposition, and nonlocal kernel construction to extract latent manifolds reflective of critical neural dynamics.

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1. Introduction

The CHARM framework provides a dimensionality reduction approach based on a complex kernel derived from the Schrödinger equation:

$$\left(i\frac{\partial}{\partial t} - \mathcal{L}\right)\hat{u} = 0,$$

where $\mathcal{L}$ is the Laplace-Beltrami operator acting over a Riemannian manifold $\mathcal{M}$, and $\hat{u}(x,t)$ evolves according to nonlocal interference patterns. CHARM's kernel enables the projection of high-dimensional data $X \in \mathbb{R}^{M\times N}$ (with M spatial channels and N time samples) into a reduced non-Euclidean space that preserves long-range temporal-spatial interactions.

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2. Motivation for EEG Adaptation

Unlike fMRI, EEG provides millisecond-level temporal resolution but is susceptible to noise, artefacts, and reference-dependence. To adapt CHARM effectively:

- EEG preprocessing must include filtering, referencing, artefact rejection.
- Envelope dynamics of oscillatory bands (e.g., α, β) are more suitable analogues to BOLD fluctuations.

- Dimensionality reduction must preserve spatial dependencies while respecting the temporal structure of neural oscillations.

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3. Proposed CHARM-EEG Framework

3.1 Preprocessing:

- Apply spatial referencing (e.g., common average, REST, Laplacian).
- Bandpass filter EEG into narrow bands $B = [b_1, b_2] \subset \mathbb{R}$.
- Extract analytic envelope via Hilbert transform:

$$a(t) = |x(t) + i\mathcal{H}[x(t)]|,$$

  where $\mathcal{H}$ is the Hilbert transform.
- Downsample and segment into windows of length T_w with stride s.

3.2 Feature Representation:

Each window $w_j \in \mathbb{R}^M$ represents the mean envelope activity over electrodes for window j.

Construct a data matrix:

$$X = [w_1, w_2, \ldots, w_N] \in \mathbb{R}^{M\times N}.$$

3.3 Complex Kernel (Schrödinger):

Define pairwise distances between columns of X:

$$D_{ij} = \|w_i - w_j\|^2,$$

then build the complex kernel:

$$\hat{W}_{ij} = e^{iD_{ij}/\sigma},$$

with scale parameter $\sigma > 0$.

3.4 Transition Matrix and Diffusion:

Define:

$$Q(t) = |\hat{W}^t|^2, \quad D_{ii} = \sum_j Q_{ij}, \quad P(t) = D^{-1}Q(t).$$

3.5 Manifold Reduction:

Perform eigendecomposition:

$$P(t) = \Psi \Lambda \Psi^T,$$

and define reduced coordinates:

$$\hat{y}_i = [\lambda_1 \psi_1(i), \ldots, \lambda_k \psi_k(i)],$$

where λ_j and ψ_j are the j-th eigenvalue/vector.

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4. Validation Metrics

- **Edge-Centric Metastability (ECM):** Quantify dynamic fluctuations in manifold space vs. source.
- **Spectral Gap:** Determine optimal k via eigenspectrum.
- **Temporal Smoothness:** Assess clustering and transitions across $\hat{y}_i$.

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5. Conclusion

The CHARM-EEG framework adapts a powerful nonlocal manifold learning method to time-resolved EEG data. By focusing on band-limited envelope representations and leveraging complex diffusion kernels, this method offers a principled way to extract low-dimensional, critical, and interpretable spatiotemporal patterns.

References:

Deco, G., Sanz Perl, Y., & Kringelbach, M. L. (2025). Complex harmonics reveal low-dimensional manifolds of critical brain dynamics. *Physical Review E, 111*(1), 014410.