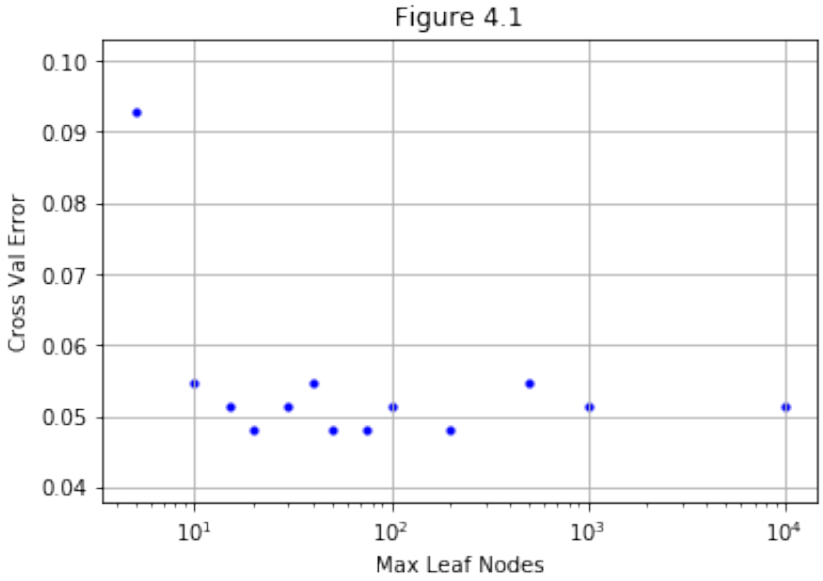
In [67]:
```
# Import required packages here (after they are installed)
import numpy as np
import matplotlib.pyplot as mp
from pylab import show

# Load data. csv file should be in the same folder as the notebo ok fo
r this to work, otherwise
# give data path.
data = np.loadtxt("data.csv")
```

In [68]:
```
#shuffle the data and select training and test data
np.random.seed(100)
np.random.shuffle(data)

features = []
digits = []

for row in data:
#import the data and select only the 1's and 5's
    if(row[0]==1 or row[0]==5):
        features.append(row[1:])
        digits.append(str(row[0]))

#Select the proportion of data to use for training. #Notice that we ha
ve set aside 80% of the data for testing
numTrain = int(len(features)*.2)

trainFeatures = features[:numTrain]
testFeatures = features[numTrain:]
trainDigits = digits[:numTrain]
testDigits = digits[numTrain:]
```

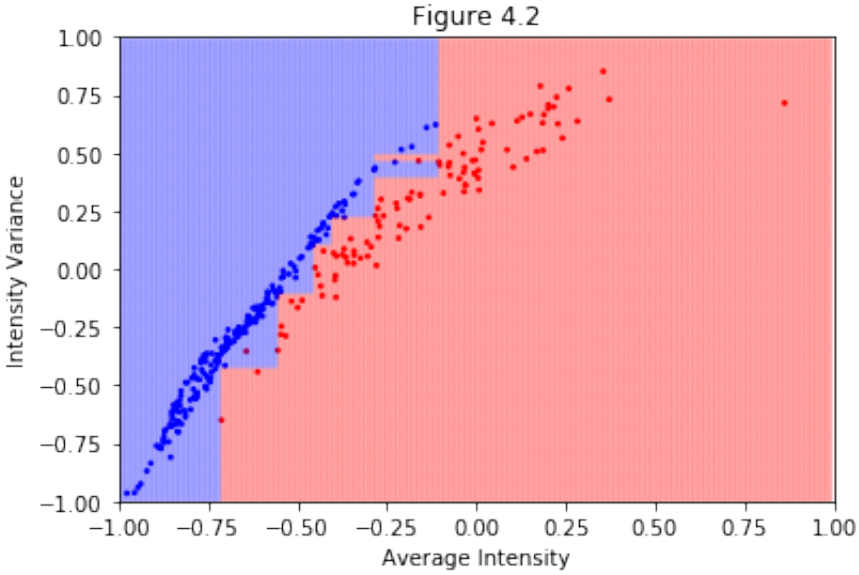
In [69]:
```
#Convert the 256D data (trainFeatures) to 2D data
#We need X and Y for plotting and simpleTrain for building the model.
#They contain the same points in a different arrangement

X = []
Y = []
simpleTrain = []
colors = []

for index in range(len(trainFeatures)):
    #produce the 2D dataset for graphing/training and scale the data s
o it is in the [-1,1] square
    xNew = 2*np.average(trainFeatures[index])+.75
    yNew = 3*np.var(trainFeatures[index])-1.5
    X.append(xNew)
    Y.append(yNew)
    simpleTrain.append([xNew,yNew])

    if(trainDigits[index]=="1.0"):
        colors.append("b")
    else:
        colors.append("r")
```

In [70]:
```
from sklearn.tree import DecisionTreeClassifier
from sklearn.model_selection import cross_val_score

maxLeafNodes = [5,10,15,20,30,40,50,75,100,200,500,1000,10000]
errors = []

for i in maxLeafNodes:
    model = DecisionTreeClassifier(criterion = "entropy", max_leaf_nod
es = i)
    errors.append( np.average( 1 - cross_val_score( model, simpleTrain
, trainDigits, cv = 10)))

#plot the points
mp.scatter(maxLeafNodes,errors,s=10,c="b")

#specify the axes
mp.xlabel("Max Leaf Nodes")
mp.ylabel("Cross Val Error")

mp.grid()
mp.xscale("log")

#label the figure
mp.title("Figure 4.1")

#display the current graph
show()
```



There was evidence of underfitting for the value of 5 for max leaf nodes as it has significantly error. This is likely because this is too few nodes to make the model complex enough to fit the data well.

In [71]:
```
model = DecisionTreeClassifier(criterion = "entropy", max_leaf_nodes =
maxLeafNodes[ np.argmin( errors)])

model.fit( simpleTrain, trainDigits)


# Lists to hold inpoints, predictions and assigned colors
xPred = []
yPred = []
cPred = []
# Use input points to get predictions here
for xP in range(-100,100):
    xP = xP/100.0
    for yP in range(-100,100):
        yP = yP/100.0
        xPred.append(xP)
        yPred.append(yP)
        if(model.predict([[xP,yP]])=="1.0"):
            cPred.append("b")
        else:
            cPred.append("r")


## Visualize Results
#plot the points
mp.scatter(X,Y,s=3,c=colors)

#plot the regions
mp.scatter(xPred,yPred,s=3,c=cPred,alpha=.1)

#setup the axes
mp.xlim(-1,1)
mp.xlabel("Average Intensity")
mp.ylim(-1,1)
mp.ylabel("Intensity Variance")

#label the figure
mp.title("Figure 4.2")

show()
```



In [73]:
```
from sklearn.ensemble import RandomForestClassifier

maxLeafNodes = [10, 100, 1000]
numEstimators = [5,10,15,20,30,40,50,75,100,200,500,1000]#,10000]

errors = [ [], [], [] ]

i = 0
for nNodes in maxLeafNodes:
    for nEstimators in numEstimators:
        model = RandomForestClassifier( n_estimators = nEstimators, ma
x_leaf_nodes = nNodes, n_jobs = -1)
        errors[i].append( np.average( 1 - cross_val_score( model, simp
leTrain, trainDigits, cv = 10)))
    i+=1


#plot the points
mp.scatter(numEstimators,errors[0],s=10,c="b")

#specify the axes
mp.xlabel("Number of Estimators")
mp.ylabel("Cross Val Error")

mp.grid()
mp.xscale("log")

#label the figure
mp.title("Figure 4.3")

#display the current graph
show()
```

```
#plot the points
mp.scatter(numEstimators,errors[1],s=10,c="b")

#specify the axes
mp.xlabel("Number of Estimators")
mp.ylabel("Cross Val Error")

mp.grid()
mp.xscale("log")

#label the figure
mp.title("Figure 4.4")

#display the current graph
show()


#plot the points
mp.scatter(numEstimators,errors[2],s=10,c="b")

#specify the axes
mp.xlabel("Number of Estimators")
mp.ylabel("Cross Val Error")

mp.grid()
mp.xscale("log")

#label the figure
mp.title("Figure 4.5")

#display the current graph
show()
```

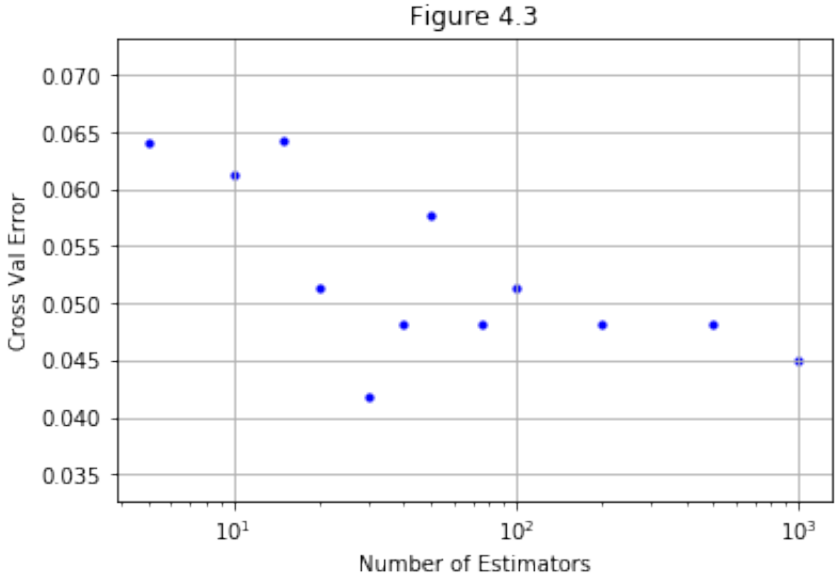


Figure 4.3
Cross Val Error
0.070
0.065
0.060
0.055
0.050
0.045
0.040
0.035
10^1
10^2
10^3
Number of Estimators

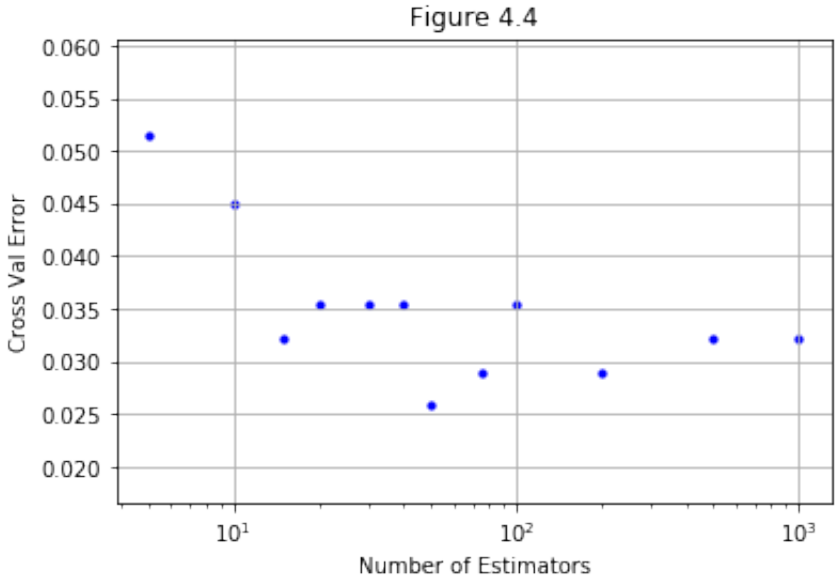
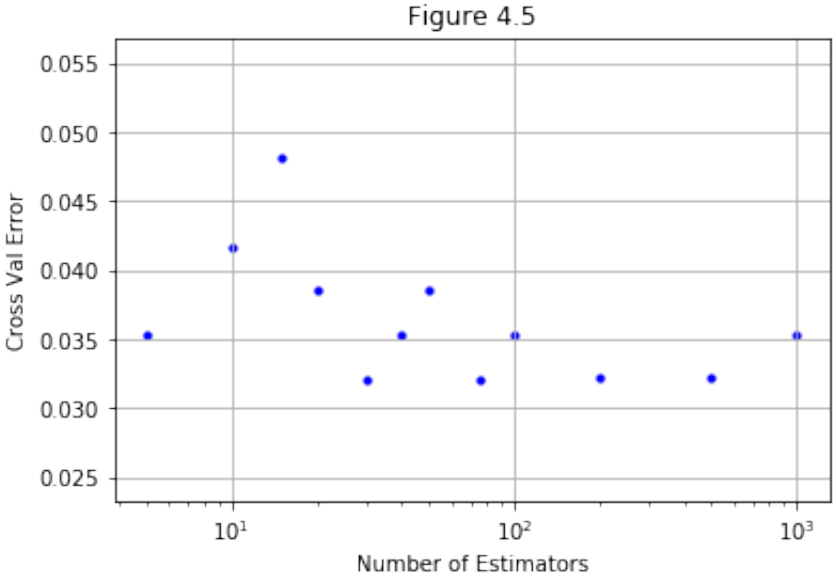


Figure 4.4
Cross Val Error
0.060
0.055
0.050
0.045
0.040
0.035
0.030
0.025
0.020
10^1
10^2
10^3
Number of Estimators



The value of 100 for max leaf nodes was most impacted. Unlike the other models, the accuracy for max_leaf_nodes=100 consistently decreases then steadies as n_estimators is increased. I believe this is the case because having only 10 leaf nodes likely causes the model to underfit the data whereas having 1000 leaf nodes causes the model to overfit the data. 100 leaf nodes likely is a nice fit for the data.

For the model where max_leaf_nodes=100, n_estimators impacted the result quite significantly, bringing error down from 0.45 to under 0.35. Increasing estimators no longer made an impact after having about 40 estimators.

In [87]:
```
RandomForestClassifier( n_estimators = 50, max_leaf_nodes = 100, n_jobs = -1)

model.fit( simpleTrain, trainDigits)

# Lists to hold inpoints, predictions and assigned colors
xPred = []
yPred = []
cPred = []
# Use input points to get predictions here
for xP in range(-100,100):
    xP = xP/100.0
    for yP in range(-100,100):
        yP = yP/100.0
        xPred.append(xP)
        yPred.append(yP)
        if(model.predict([[xP,yP]])=="1.0"):
            cPred.append("b")
        else:
            cPred.append("r")
    print(xP, end='|')


## Visualize Results
#plot the points
mp.scatter(X,Y,s=3,c=colors)

#plot the regions
mp.scatter(xPred,yPred,s=3,c=cPred,alpha=.1)

#setup the axes
mp.xlim(-1,1)
mp.xlabel("Average Intensity")
mp.ylim(-1,1)
mp.ylabel("Intensity Variance")

#label the figure
mp.title("Figure 4.6")

show()
```

FIGURE 4.6 I let my machine run this code for a little under an hour and it did not complete so I added print statements to see how fast each coordinate is calculated. After recording the time and making some calculations, it seems as if figure 4.6 would take my machine about 2 and a half hours to finish executing, which is why I am not able to produce the figure. Above is the code, though.

In [91]:
```
from sklearn.ensemble import AdaBoostClassifier
```

```
numEstimators = [1,5,10,100,1000,10000]
errors = []

for i in numEstimators:
    model = AdaBoostClassifier( base_estimator = DecisionTreeClassifie
r( max_depth = 1), n_estimators = i)
    errors.append( np.average( 1 - cross_val_score( model, simpleTrain
, trainDigits, cv = 10)))

#plot the points
mp.scatter(numEstimators,errors,s=10,c="b")

#specify the axes
mp.xlabel("Number of Estimators")
mp.ylabel("Cross Val Error")

mp.grid()
mp.xscale("log")

#label the figure
mp.title("Figure 4.7")

#display the current graph
show()


errors = []

for i in numEstimators:
    model = AdaBoostClassifier( base_estimator = DecisionTreeClassifie
r( max_depth = 10), n_estimators = i)
    errors.append( np.average( 1 - cross_val_score( model, simpleTrain
, trainDigits, cv = 10)))

#plot the points
mp.scatter(numEstimators,errors,s=10,c="b")

#specify the axes
mp.xlabel("Number of Estimators")
mp.ylabel("Cross Val Error")

mp.grid()
mp.xscale("log")

#label the figure
mp.title("Figure 4.8")

#display the current graph
show()
```

```
errors = []

for i in numEstimators:
    model = AdaBoostClassifier( base_estimator = DecisionTreeClassifie
r( max_depth = 1000), n_estimators = i)
    errors.append( np.average( 1 - cross_val_score( model, simpleTrain
, trainDigits, cv = 10)))

#plot the points
mp.scatter(numEstimators,errors,s=10,c="b")

#specify the axes
mp.xlabel("Number of Estimators")
mp.ylabel("Cross Val Error")

mp.grid()
mp.xscale("log")

#label the figure
mp.title("Figure 4.9")

#display the current graph
show()
```

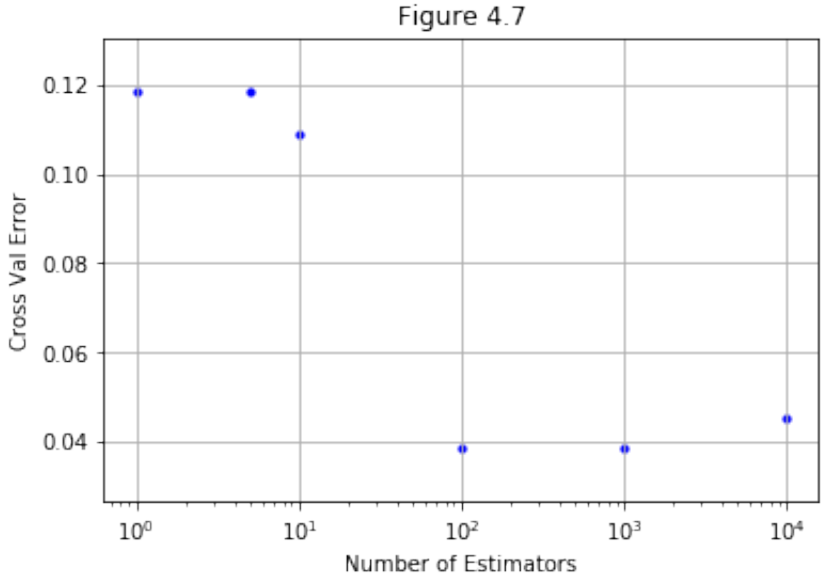


Figure 4.8

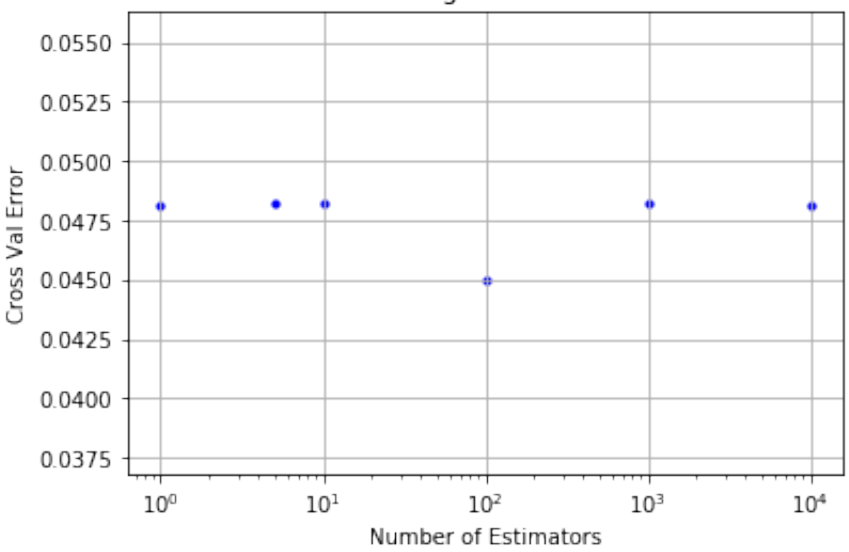
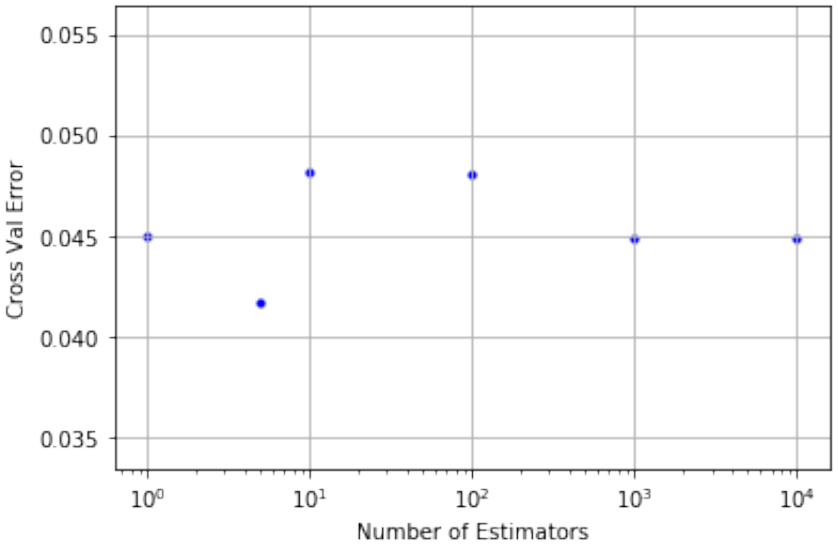
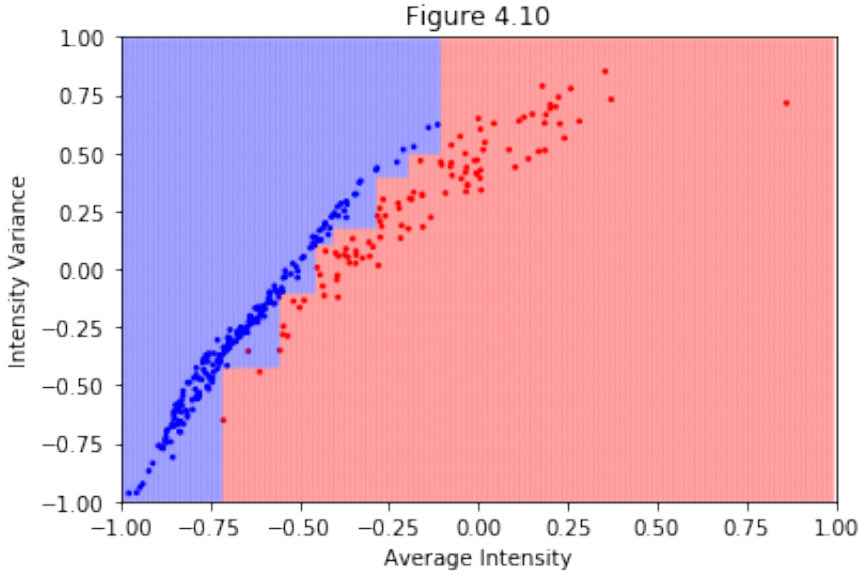


Figure 4.9



In [15]:
```
model = AdaBoostClassifier( base_estimator = DecisionTreeClassifier( m
ax_depth = 1000), n_estimators = 5)

model.fit( simpleTrain, trainDigits)


# Lists to hold inpoints, predictions and assigned colors
xPred = []
yPred = []
cPred = []
# Use input points to get predictions here
for xP in range(-100,100):
    xP = xP/100.0
    for yP in range(-100,100):
        yP = yP/100.0
        xPred.append(xP)
        yPred.append(yP)
        if(model.predict([[xP,yP]])=="1.0"):
            cPred.append("b")
        else:
            cPred.append("r")


## Visualize Results
#plot the points
mp.scatter(X,Y,s=3,c=colors)

#plot the regions
mp.scatter(xPred,yPred,s=3,c=cPred,alpha=.1)

#setup the axes
mp.xlim(-1,1)
mp.xlabel("Average Intensity")
mp.ylim(-1,1)
mp.ylabel("Intensity Variance")

#label the figure
mp.title("Figure 4.10")

show()
```



In [17]:
```
simpleTest = []

for index in range(len(testFeatures)):
    #produce the 2D dataset for graphing/training and scale the data s
o it is in the [-1,1] square
    xNew = 2*np.average(testFeatures[index])+.75
    yNew = 3*np.var(testFeatures[index])-1.5
    X.append(xNew)
    Y.append(yNew)
    simpleTest.append([xNew,yNew])
```

In [28]:
```
from sklearn.svm import SVC
from sklearn.neural_network import MLPClassifier

polySVM = SVC( C = 4, kernel = "poly", degree = 3)

neuralNetwork = MLPClassifier(learning_rate_init=0.01, hidden_layer_si
zes=([50]*10), activation='relu', epsilon=0.001, max_iter=10000, alpha
=0, solver="adam")

randomForest = RandomForestClassifier( n_estimators = 4, max_leaf_node
s = 100)

adaBoost = AdaBoostClassifier( base_estimator = DecisionTreeClassifier
( max_depth = 10), n_estimators = 100)


polySVM.fit( simpleTrain, trainDigits)
neuralNetwork.fit( simpleTrain, trainDigits)
randomForest.fit( simpleTrain, trainDigits)
adaBoost.fit( simpleTrain, trainDigits)

print( polySVM.score( simpleTest, testDigits))
print( neuralNetwork.score( simpleTest, testDigits))
print( randomForest.score( simpleTest, testDigits))
print( adaBoost.score( simpleTest, testDigits))
```

```
0.9903923138510808
0.9511609287429944
0.9455564451561249
0.967173738991193
```

Polynomial SVM error = 1 - 0.9903923138510808 = 0.00960768614 Neural Network error = 1 - 0.9511609287429944 = 0.04883907125 Random Forest error = 1 - 0.9455564451561249 = 0.05444355484 AdaBoost error = 1 - 0.967173738991193 = 0.032826261

Markov:

```
If the confidence interval is 99%, then delta = 1-0.99 = 0.01.
This means that E(X)/a = 0.01 which means a must equal 1/0.01*E(X) = 100*E
(X)
    Polynomial SVM error upper bound at 99% = 100 * 0.00960768614 = 0.9607
68614
    Neural Network error upper bound at 99% = 100 * 0.04883907125 = 4.8839
07125
    Random Forest error upper bound at 99% = 100 * 0.05444355484 = 5.44435
5484
    AdaBoost error upper bound at 99% = 100 * 0.032826261 = 3.2826261

If the confidence interval is 95%, then delta = 1-0.95 = 0.05.
This means that E(X)/a = 0.05 which means a must equal 1/0.05*E(X) = 20*E(
X)
    Polynomial SVM error upper bound at 95% = 20 * 0.00960768614 = 0.19215
37228
    Neural Network error upper bound at 95% = 20 * 0.04883907125 = 0.97678
1425
    Random Forest error upper bound at 95% = 20 * 0.05444355484 = 1.088871
0968
    AdaBoost error upper bound at 95% = 20 * 0.032826261 = 0.65652522

If the confidence interval is 75%, then delta = 1-0.75 = 0.25.
This means that E(X)/a = 0.05 which means a must equal 1/0.25*E(X) = 4*E(X
)
    Polynomial SVM error upper bound at 75% = 4 * 0.00960768614 = 0.038430
74456
    Neural Network error upper bound at 75% = 4 * 0.04883907125 = 0.195356
285
    Random Forest error upper bound at 75% = 4 * 0.05444355484 = 0.2177742
1936
    AdaBoost error upper bound at 75% = 4 * 0.032826261 = 0.131305044
```

In [61]:
```
models = ["Polynomial SVM", "Neural Network", "Random Forest", "AdaBoo
st"]
scores = [0.9903923138510808, 0.9511609287429944, 0.9455564451561249,
0.967173738991193]
confidences = [0.99, 0.95, 0.75]

print("Chebyshev:")

for confidence in confidences:
    print("\tConfidence Interval: ", confidence)
    for i in range( len ( models)):
        print("\t\t", models[ i], "error upper bound at", confidence,
"=", ( 1 - scores[ i] + 0.25 / ( len( testDigits) * ( 1 - confidence)
* ( 1 - confidence))))
```

```
Chebyshev:
        Confidence Interval:  0.99
                Polynomial SVM error upper bound at 0.99 = 2.011208
9671737357
                Neural Network error upper bound at 0.99 = 2.050440
352281822
                Random Forest error upper bound at 0.99 = 2.0560448
358686916
                AdaBoost error upper bound at 0.99 = 2.034427542033
6233
        Confidence Interval:  0.95
                Polynomial SVM error upper bound at 0.95 = 0.089671
73738991181
                Neural Network error upper bound at 0.95 = 0.128903
12249799823
                Random Forest error upper bound at 0.95 = 0.1345076
0608486773
                AdaBoost error upper bound at 0.95 = 0.112890312249
79969
        Confidence Interval:  0.75
                Polynomial SVM error upper bound at 0.75 = 0.012810
24819855887
                Neural Network error upper bound at 0.75 = 0.052041
63330664531
                Random Forest error upper bound at 0.75 = 0.0576461
168935148
                AdaBoost error upper bound at 0.75 = 0.036028823058
44675
```

In [66]:
```
models = ["Polynomial SVM", "Neural Network", "Random Forest", "AdaBoo
st"]
scores = [0.9903923138510808, 0.9511609287429944, 0.9455564451561249,
0.967173738991193]
confidences = [0.99, 0.95, 0.75]

print("Hoeffding:")

for confidence in confidences:
    print("\tConfidence Interval: ", confidence)
    for i in range( len ( models)):
        value = (1 / (2 * len( testDigits)) * np.log( 2 / ( 1 - confid
ence)))
        value = value**0.5
        print("\t\t", models[ i], "error upper bound at", confidence,
"=", value)
```

```
Hoeffding:
        Confidence Interval:  0.99
                Polynomial SVM error upper bound at 0.99 = 0.046054
57377537783
                Neural Network error upper bound at 0.99 = 0.046054
57377537783
                Random Forest error upper bound at 0.99 = 0.0460545
7377537783
                AdaBoost error upper bound at 0.99 = 0.046054573775
37783
        Confidence Interval:  0.95
                Polynomial SVM error upper bound at 0.95 = 0.038428
28604270711
                Neural Network error upper bound at 0.95 = 0.038428
28604270711
                Random Forest error upper bound at 0.95 = 0.0384282
8604270711
                AdaBoost error upper bound at 0.95 = 0.038428286042
70711
        Confidence Interval:  0.75
                Polynomial SVM error upper bound at 0.75 = 0.028852
08087345724
                Neural Network error upper bound at 0.75 = 0.028852
08087345724
                Random Forest error upper bound at 0.75 = 0.0288520
8087345724
                AdaBoost error upper bound at 0.75 = 0.028852080873
45724
```

a) The model that changed most dramatically with respect to confidence interval was the Random Forest Classifier

b) I would choose the polynomial support vector machine because along with scoring the highest on the testing data, it also (resultingly) had the least errors with the different error bounds.

c) Some other considerations could be since the data is quite easily separable, we do not need as complex models and a SVM would fit the data well.