

Tugas Pertemuan 5

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1. Diket = $u = (2, -2, 1)$, $v = (2, 1, 2)$

Ditanya dan dijawab:

a. $u \cdot v = (2)(2) + (-2)(1) + (1)(2)$
 $= 4 - 2 + 2 = 4$

b. Sudut antara vector u dan $v = \cos \theta = \frac{u \cdot v}{\|u\| \|v\|}$

$\|u\| = \sqrt{(2)^2 + (-2)^2 + 1^2} = \sqrt{9} = 3$

$\|v\| = \sqrt{2^2 + 1^2 + 2^2} = \sqrt{9} = 3$

$\cos \theta = \frac{4}{3 \cdot 3} = \frac{4}{9}$

$\theta = \cos^{-1}\left(\frac{4}{9}\right)$

$\theta = 63,615^\circ$

c. Jarak antara vector u dan v

$d = \sqrt{(2-2)^2 + (1+2)^2 + (2-1)^2} = \sqrt{0+9+1} = \sqrt{10}$

d. $u \times v = \left(\begin{vmatrix} -2 & 1 \\ 1 & 2 \end{vmatrix}, -\begin{vmatrix} 2 & 1 \\ 2 & 2 \end{vmatrix}, \begin{vmatrix} 2 & -2 \\ 2 & 1 \end{vmatrix} \right) = ((-4-1), -(4-2), (2+4))$
 $= (-5, -2, 6)$

e. $\|u \times v\| = \|u \times v\| = (-5, -2, 6)$

$\|u \times v\| = \sqrt{(-5)^2 + (-2)^2 + 6^2}$
 $= \sqrt{25 + 4 + 36}$
 $= \sqrt{65}$

2. Diket = $a = (1, k, k^2)$, $b = (-2, -1, 1)$

Ditanya dan dijawab:

a. nilai k agar vector a dan b tegak lurus

$\rightarrow \theta = 90^\circ$

$\cos 90^\circ = 0$

$\theta = \frac{u \cdot v}{\|u\| \|v\|}$

$0 = \frac{u \cdot v}{\|u\| \|v\|}$

$0 = u \cdot v$

$u \cdot v = (1)(-2) + (k)(-1) + (k^2)(1) = 0$

$= -2 - k + k^2 = 0$

$k^2 - k - 2 = 0$

$(k-2)(k+1)$

$k = 2 \quad k = -1$

$-2 = -2 \times 1$

$-1 = -2 + 1$

b. $\|a \times b\|$ untuk k positif

$a = (1, 2, a)$

$b = (-2, -1, 1)$

$a \times b = \left(\begin{vmatrix} 2 & a \\ -1 & 1 \end{vmatrix}, -\begin{vmatrix} 1 & a \\ -2 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 2 \\ -2 & -1 \end{vmatrix} \right) = ((2+a), -(1+a), (-1+a))$
 $= (6, -9, 3)$

$\|a \times b\| = \sqrt{(6)^2 + (-9)^2 + 3^2}$
 $= \sqrt{36 + 81 + 9} = \sqrt{126} = 3\sqrt{14}$

3. diketahui: $u = (k+1, k+1, 1)$

$v = (-k-1, -k-1, k)$

ditanya: u agar sudut 180°

jawab: $\theta = 180$

$\cos 180 = -1$

$$-1 = \frac{u \cdot v}{||u|| ||v||} = \frac{(k+1)(-k-1) + (k+1)(-k-1) + (1)(k)}{\sqrt{(k+1)^2 + (k+1)^2 + 1^2} \sqrt{(-k-1)^2 + (-k-1)^2 + k^2}}$$

$$-1 = \frac{(-k^2 - k - k - 1) + (-k^2 - k - k - 1) + k}{\sqrt{(k^2 + 2k + 1) + (k^2 + 2k + 1) + 1} \sqrt{(k^2 + 2k + 1)^2 + (k^2 + 2k + 1) + k^2}}$$

$$-1 = \frac{2(-k^2 - 2k - 1) + k}{\sqrt{(2k^2 + 2k + 1) + 1} \sqrt{(2k^2 + 2k + 1) + k^2}}$$

$$-1 = \frac{-2k^2 - 4k - 2 + k}{\sqrt{(2k^2 + 4k + 3)(3k^2 + 4k + 2)}}$$

$$-\sqrt{(6k^4 + 8k^3 + 4k^2 + 12k^3 + 16k^2 + 8k + 9k^2 + 12k + 6)} = -2k^2 - 3k - 2$$

$$\sqrt{6k^4 + 20k^3 + 29k^2 + 20k + 6} = 2k^2 + 3k + 2$$

$$6k^4 + 20k^3 + 29k^2 + 20k + 6 = (2k^2 + 3k + 2)^2$$

$$6k^4 + 20k^3 + 29k^2 + 20k + 6 = 4k^4 + 6k^3 + 4k^2 + 6k^3 + 9k^2 + 6k + 4k^2 + 6k + 4$$

$$6k^4 + 20k^3 + 29k^2 + 20k + 6 = 4k^4 + 12k^3 + 17k^2 + 12k + 4$$

$$2k^4 + 8k^3 + 12k^2 + 8k + 2 = 0$$

$$2(k^4 + 4k^3 + 6k^2 + 4k + 1) = 0$$

$$k^4 + 4k^3 + 6k^2 + 4k + 1 = 0$$

$$\begin{array}{r|rrrrr} 1 & 4 & 6 & 4 & 1 \\ -1 & & -1 & -3 & -3 & -1 \end{array}$$

$$-2k = -1$$

$$\begin{array}{r|rrrr} 1 & 3 & 3 & 1 & 0 \\ -1 & & -1 & -2 & -1 \end{array}$$

$$-2k = -1$$

$$k = -1$$

$$k^2 + 2k + 1$$

$$(k+1)(k+1)$$

$$k = -1 \quad k = -1$$