DEKOMPOSISI LU

TK13023
COMPUTATION II

KELAS A DAN C

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Sistem Persamaan Linear?

- SPL adalah sebuah himpunan berhingga dari persamaan-persamaan linear dalam peubah $x_1, x_2, \dots, x_n$

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\
\vdots \qquad\qquad \vdots \qquad\qquad & \vdots \\
a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n &= b_n
\end{aligned}
\;\Rightarrow\;
\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \dots \\ b_n \end{bmatrix}
\;\Rightarrow\; Ax = b
$$

A adalah matriks koefisien dari SPL

Solusi SPL

- Subtitusi
- Eliminasi: Gauss dan Gauss Jordan
- Invers Matriks
- Crammer

Dekomposisi LU

- Salah satu pendekatan untuk penyelesaian sistem persamaan linier.
  - Faktorisasi matriks A ke dua macam matriks, yaitu matriks segitiga bawah (L) dan matriks segitiga atas (U)

If solving a set of linear equations	$Ax = b$
If $A = LU$ then	$LUx = b$
Multiply by	L^{-1}
Which gives	$L^{-1}LUx = L^{-1}b$
Remember $L^{-1}L = I$ which leads to	$IUx = L^{-1}b$
Now, if $IU = U$ then	$Ux = L^{-1}b$
Now, let	$L^{-1}b = y$
Which ends with	$Ly = b \quad (1)$
and	$Ux = y \quad (2)$

$$L = \begin{bmatrix} l_{11} & 0 & \dots & 0 \\ l_{21} & l_{22} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ l_{n1} & l_{n2} & \dots & l_{nn} \end{bmatrix}$$

$$U = \begin{bmatrix} u_{11} & u_{21} & \dots & u_{1n} \\ 0 & u_{22} & \dots & u_{2n} \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & u_{nn} \end{bmatrix}$$

Sumber: https://nm.mathforcollege.com/mws/gen/04sle/mws_gen_sle_txt_ludecomp.pdf

Contoh: Diketahui matriks $\boldsymbol{L}$ dan $\boldsymbol{U}$

$$\begin{aligned} 2x_1 + 6x_2 + 2x_3 &= 2 \\ -3x_1 - 8x_2 \qquad &= 2 \\ 4x_1 + 9x_2 + 2x_3 &= 3 \end{aligned} \Rightarrow \begin{bmatrix} 2 & 6 & 2 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$$

$$A = LU \Rightarrow \begin{bmatrix} 2 & 6 & 2 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ -3 & 1 & 0 \\ 4 & -3 & 7 \end{bmatrix} \times \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$LUx = b \Rightarrow \begin{bmatrix} 2 & 0 & 0 \\ -3 & 1 & 0 \\ 4 & -3 & 7 \end{bmatrix} \times \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$$

$$Ly = b \Rightarrow \begin{bmatrix} 2 & 0 & 0 \\ -3 & 1 & 0 \\ 4 & -3 & 7 \end{bmatrix} \times \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} \Rightarrow y_1 = 1, y_2 = 5, y_3 = 2$$

$$Ux = y \Rightarrow \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix} \Rightarrow \boldsymbol{x_1 = 2, x_2 = -1, x_3 = 2}$$

Mencari Matriks $\boldsymbol{L}$ dan $\boldsymbol{U}$ dari Matriks $\boldsymbol{A}$

- Mencari matriks elementer

$$E_k \times \cdots \times E_2 \times E_1 \times A = U,$$

dimana

$$A = E_1^{-1} \times E_2^{-1} \times \cdots \times E_k^{-1} \times U,$$

sehingga

$$L = E_1^{-1} \times E_2^{-1} \times \cdots \times E_k^{-1}.$$

Konsep 1:

Mencari Matriks $\boldsymbol{L}$ dan $\boldsymbol{U}$ dari Matriks $\boldsymbol{A}$

- Operasi **reduksi baris** pada matriks $\boldsymbol{A}$ sampai membentuk matriks eselon baris $\boldsymbol{U}$
- **Contoh 1**:

$$A = \begin{bmatrix} 2 & 6 & 2 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix}$$

1. $H_{1(\mathbf{1/2})}$ $\begin{bmatrix} 1 & 3 & 1 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix}$ $\Rightarrow$ $\begin{bmatrix} \mathbf{1/2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 6 & 2 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 1 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix}$

$$E_1 = \begin{bmatrix} \mathbf{1/2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad E_1^{-1} = \begin{bmatrix} \mathbf{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Konsep 1:

Mencari Matriks $\boldsymbol{L}$ dan $\boldsymbol{U}$ dari Matriks $\boldsymbol{A}$

- Operasi **reduksi baris** pada matriks $\boldsymbol{A}$ sampai membentuk matriks eselon baris $\boldsymbol{U}$
- **Contoh 1**:

1. $H_{1(\mathbf{1/2})}$ $\begin{bmatrix} 1 & 3 & 1 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix}$

2. $H_{21(\mathbf{3})}$ $\begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 4 & 9 & 2 \end{bmatrix}$ $\Rightarrow$ $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 3 & 1 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 4 & 9 & 2 \end{bmatrix}$

$$E_2 = \begin{bmatrix} 1 & 0 & 0 \\ \mathbf{3} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad E_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ \mathbf{-3} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Konsep 1:

Mencari Matriks $\boldsymbol{L}$ dan $\boldsymbol{U}$ dari Matriks $\boldsymbol{A}$

- Operasi **reduksi baris** pada matriks $\boldsymbol{A}$ sampai membentuk matriks eselon baris $\boldsymbol{U}$
- **Contoh 1**:

2. $H_{21(\mathbf{3})}$ $\begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 4 & 9 & 2 \end{bmatrix}$

3. $H_{31(\mathbf{-4})}$ $\begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & -3 & -2 \end{bmatrix}$ ⇒ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 4 & 9 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & -3 & -2 \end{bmatrix}$

$$E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \mathbf{-4} & 0 & 1 \end{bmatrix} \qquad E_3^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \mathbf{4} & 0 & 1 \end{bmatrix}$$

Konsep 1:

Mencari Matriks $\boldsymbol{L}$ dan $\boldsymbol{U}$ dari Matriks $\boldsymbol{A}$

- Operasi **reduksi baris** pada matriks $\boldsymbol{A}$ sampai membentuk matriks eselon baris $\boldsymbol{U}$
- **Contoh 1**:

3. $H_{31(\boldsymbol{-4})}$ $\begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & -3 & -2 \end{bmatrix}$

4. $H_{32(\boldsymbol{3})}$ $\begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 7 \end{bmatrix}$ ⇨ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & -3 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 7 \end{bmatrix}$

$$E_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \boldsymbol{3} & 1 \end{bmatrix} \qquad E_4^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \boldsymbol{-3} & 1 \end{bmatrix}$$

Konsep 1:

Mencari Matriks $\boldsymbol{L}$ dan $\boldsymbol{U}$ dari Matriks $\boldsymbol{A}$

- Operasi **reduksi baris** pada matriks $\boldsymbol{A}$ sampai membentuk matriks eselon baris $\boldsymbol{U}$
- **Contoh 1**:

4. $H_{32(\mathbf{3})}$ $\begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 7 \end{bmatrix}$

5. $H_{3(\mathbf{1/7})}$ $\begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ $\Rightarrow$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/7 \end{bmatrix} \times \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

$$E_5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \mathbf{1/7} \end{bmatrix} \qquad E_5^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \mathbf{7} \end{bmatrix}$$

Konsep 1:

Mencari Matriks $\boldsymbol{L}$ dan $\boldsymbol{U}$ dari Matriks $\boldsymbol{A}$

5. $H_{3(\mathbf{1/7})}$ $\begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

maka:

$$U = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

dimana

$$L = E_1^{-1} \times E_2^{-1} \times E_3^{-1} \times E_4^{-1} \times E_5^{-1}$$

sehingga

$$L = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ -3 & 1 & 0 \\ 4 & -3 & 7 \end{bmatrix}$$

Konsep 2:

Mencari Matriks $\boldsymbol{L}$ dan $\boldsymbol{U}$ dari Matriks $\boldsymbol{A}$

- Operasi **reduksi baris** pada matriks $\boldsymbol{A}$ sampai membentuk matriks eselon baris $\boldsymbol{U}$
- **Contoh 1**:

$$A = \begin{bmatrix} 2 & 6 & 2 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix}$$

1. $H_{1(\mathbf{1/2})}$ $\begin{bmatrix} \mathbf{1} & 3 & 1 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix}$

2. $H_{21(\mathbf{3})}$ $\begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 4 & 9 & 2 \end{bmatrix}$

3. $H_{31(\mathbf{-4})}$ $\begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & -3 & -2 \end{bmatrix}$

4. $\begin{bmatrix} 1 & 3 & 1 \\ 0 & \mathbf{1} & 3 \\ 0 & -3 & -2 \end{bmatrix}$

5. $H_{32(\mathbf{3})}$ $\begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 7 \end{bmatrix}$

6. $H_{3(\mathbf{1/7})}$ $\begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & \mathbf{1} \end{bmatrix}$

$$\boldsymbol{U} = \begin{bmatrix} \mathbf{1} & \mathbf{3} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{3} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$$

$$\boldsymbol{L} = \begin{bmatrix} \mathbf{2} & \mathbf{0} & \mathbf{0} \\ \mathbf{-3} & \mathbf{1} & \mathbf{0} \\ \mathbf{4} & \mathbf{-3} & \mathbf{7} \end{bmatrix}$$

Konsep 2:

Mencari Matriks $\boldsymbol{L}$ dan $\boldsymbol{U}$ dari Matriks $\boldsymbol{A}$

- **Contoh 2**:

$$A = \begin{bmatrix} 6 & -2 & 0 \\ 9 & -1 & 1 \\ 3 & 7 & 5 \end{bmatrix}$$

1. $H_{1(\mathbf{1/6})}$ $\begin{bmatrix} \mathbf{1} & -1/3 & 0 \\ 9 & -1 & 1 \\ 3 & 7 & 5 \end{bmatrix}$

2. $H_{21(\mathbf{-9})}$ $\begin{bmatrix} 1 & -1/3 & 0 \\ 0 & 2 & 1 \\ 3 & 7 & 5 \end{bmatrix}$

3. $H_{31(\mathbf{-3})}$ $\begin{bmatrix} 1 & -1/3 & 0 \\ 0 & 2 & 1 \\ 0 & 8 & 5 \end{bmatrix}$

4. $H_{2(\mathbf{1/2})}$ $\begin{bmatrix} 1 & -1/3 & 0 \\ 0 & \mathbf{1} & 1/2 \\ 0 & 8 & 5 \end{bmatrix}$

5. $H_{32(\mathbf{-8})}$ $\begin{bmatrix} 1 & -1/3 & 0 \\ 0 & 1 & 1/2 \\ 0 & 0 & 1 \end{bmatrix}$

6. $\begin{bmatrix} 1 & -1/3 & 0 \\ 0 & 1 & 1/2 \\ 0 & 0 & \mathbf{1} \end{bmatrix}$

$$\boldsymbol{U} = \begin{bmatrix} \mathbf{1} & \mathbf{-1/3} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1/2} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$$

$$\boldsymbol{L} = \begin{bmatrix} \mathbf{6} & \mathbf{0} & \mathbf{0} \\ \mathbf{9} & \mathbf{2} & \mathbf{0} \\ \mathbf{3} & \mathbf{8} & \mathbf{1} \end{bmatrix}$$

Mencari Inverse Matriks $\boldsymbol{B} = \boldsymbol{A}^{-1}$ dari Matriks $\boldsymbol{L}$ dan $\boldsymbol{U}$

$AB = I$

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \dots & \dots & \dots & \dots \\ b_{n1} & b_{n2} & \dots & b_{nn} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

First Column **Second Column** **n-th Column**

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \times \begin{bmatrix} b_{11} \\ b_{21} \\ \dots \\ b_{n1} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \dots \\ 0 \end{bmatrix}, \quad \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \times \begin{bmatrix} b_{12} \\ b_{22} \\ \dots \\ b_{n2} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ \dots \\ 0 \end{bmatrix}, \dots \quad , \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \times \begin{bmatrix} b_{1n} \\ b_{2n} \\ \dots \\ b_{nn} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dots \\ 1 \end{bmatrix}$$

First Column

$$\begin{bmatrix} l_{11} & 0 & \dots & 0 \\ l_{21} & l_{22} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ l_{n1} & l_{n2} & \dots & l_{nn} \end{bmatrix} \times \begin{bmatrix} y_{11} \\ y_{21} \\ \dots \\ y_{n1} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \dots \\ 0 \end{bmatrix}, \quad \begin{bmatrix} u_{11} & u_{21} & \dots & u_{1n} \\ 0 & u_{22} & \dots & u_{2n} \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & u_{nn} \end{bmatrix} \times \begin{bmatrix} b_{11} \\ b_{21} \\ \dots \\ b_{n1} \end{bmatrix} = \begin{bmatrix} y_{11} \\ y_{21} \\ \dots \\ y_{n1} \end{bmatrix},$$

Mencari Inverse Matriks $\boldsymbol{B} = \boldsymbol{A}^{-1}$ dari Matriks $\boldsymbol{L}$ dan $\boldsymbol{U}$

Second Column

$$\begin{bmatrix} l_{11} & 0 & \dots & 0 \\ l_{21} & l_{22} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ l_{n1} & l_{n2} & \dots & l_{nn} \end{bmatrix} \times \begin{bmatrix} y_{12} \\ y_{22} \\ \dots \\ y_{n2} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ \dots \\ 0 \end{bmatrix}, \quad \begin{bmatrix} u_{11} & u_{21} & \dots & u_{1n} \\ 0 & u_{22} & \dots & u_{2n} \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & u_{nn} \end{bmatrix} \times \begin{bmatrix} b_{12} \\ b_{22} \\ \dots \\ b_{n2} \end{bmatrix} = \begin{bmatrix} y_{12} \\ y_{22} \\ \dots \\ y_{n2} \end{bmatrix},$$

…

n-th Column

$$\begin{bmatrix} l_{11} & 0 & \dots & 0 \\ l_{21} & l_{22} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ l_{n1} & l_{n2} & \dots & l_{nn} \end{bmatrix} \times \begin{bmatrix} y_{1n} \\ y_{2n} \\ \dots \\ y_{nn} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dots \\ 1 \end{bmatrix}, \quad \begin{bmatrix} u_{11} & u_{21} & \dots & u_{1n} \\ 0 & u_{22} & \dots & u_{2n} \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & u_{nn} \end{bmatrix} \times \begin{bmatrix} b_{1n} \\ b_{2n} \\ \dots \\ b_{nn} \end{bmatrix} = \begin{bmatrix} y_{1n} \\ y_{2n} \\ \dots \\ y_{nn} \end{bmatrix}$$

Mencari Inverse Matriks $\boldsymbol{B} = \boldsymbol{A}^{-1}$ dari Matriks $\boldsymbol{L}$ dan $\boldsymbol{U}$

- **Contoh**:

$$A = \begin{bmatrix} 2 & 6 & 2 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix} \Rightarrow U = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \quad L = \begin{bmatrix} 2 & 0 & 0 \\ -3 & 1 & 0 \\ 4 & -3 & 7 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 6 & 2 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix}$$

Kolom 1

1a

$$\begin{bmatrix} 2 & 0 & 0 \\ -3 & 1 & 0 \\ 4 & -3 & 7 \end{bmatrix} \times \begin{bmatrix} y_{11} \\ y_{21} \\ y_{31} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} 2y_{11} &= 1 \\ -3y_{11} + y_{21} &= 0 \\ 4y_{11} - 3y_{21} + 7y_{31} &= 0 \end{aligned}$$

$$y_{11} = 1/2, y_{21} = 3/2, y_{31} = 5/14$$

1b

$$\begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{bmatrix} = \begin{bmatrix} 1/2 \\ 3/2 \\ 5/14 \end{bmatrix}$$

$$\begin{aligned} b_{11} + 3b_{21} + b_{31} &= 1/2 \\ b_{21} + 3b_{31} &= 3/2 \\ b_{31} &= 5/14 \end{aligned}$$

$$\boldsymbol{b_{11} = -8/7, b_{21} = 3/7, b_{31} = 5/14}$$

Mencari Inverse Matriks $\boldsymbol{B} = \boldsymbol{A}^{-1}$ dari Matriks $\boldsymbol{L}$ dan $\boldsymbol{U}$

- **Contoh**:

2a $\begin{bmatrix} 2 & 0 & 0 \\ -3 & 1 & 0 \\ 4 & -3 & 7 \end{bmatrix} \times \begin{bmatrix} y_{12} \\ y_{22} \\ y_{32} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$$\begin{aligned} 2y_{12} &= 0 \\ -3y_{12} + y_{22} &= 1 \\ 4y_{12} - 3y_{22} + 7y_{32} &= 0 \end{aligned}$$

$y_{12} = 0, y_{22} = 1, y_{32} = 3/7$

2b $\begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} b_{12} \\ b_{22} \\ b_{32} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3/7 \end{bmatrix}$

$$\begin{aligned} b_{12} + 3b_{22} + b_{32} &= 0 \\ b_{22} + 3b_{32} &= 1 \\ b_{32} &= 3/7 \end{aligned}$$

$\boldsymbol{b_{12} = 3/7, b_{22} = -2/7, b_{32} = 3/7}$

Mencari Inverse Matriks $\boldsymbol{B} = \boldsymbol{A}^{-1}$ dari Matriks $\boldsymbol{L}$ dan $\boldsymbol{U}$

- **<u>Contoh</u>**:

3a

$$\begin{bmatrix} 2 & 0 & 0 \\ -3 & 1 & 0 \\ 4 & -3 & 7 \end{bmatrix} \times \begin{bmatrix} y_{13} \\ y_{23} \\ y_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$2y_{13} = 0$$
$$-3y_{13} + y_{23} = 0$$
$$4y_{13} - 3y_{23} + 7y_{33} = 1$$

$$y_{13} = 0, y_{23} = 0, y_{33} = 1/7$$

3b

$$\begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} b_{13} \\ b_{23} \\ b_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1/7 \end{bmatrix}$$

$$b_{13} + 3b_{23} + b_{33} = 0$$
$$b_{23} + 3b_{33} = 0$$
$$b_{33} = 1/7$$

$$\boldsymbol{b_{13} = 8/7, b_{23} = -3/7, b_{33} = 1/7}$$

$$\boldsymbol{b_{11} = -8/7, b_{21} = 3/7, b_{31} = 5/14}$$
$$\boldsymbol{b_{12} = 3/7, b_{22} = -2/7, b_{32} = 3/7}$$
$$\boldsymbol{b_{13} = 8/7, b_{23} = -3/7, b_{33} = 1/7}$$

$$\Rightarrow$$

$$A^{-1} = \begin{bmatrix} -8/7 & 3/7 & 8/7 \\ 3/7 & -2/7 & -3/7 \\ 5/14 & 3/7 & 1/7 \end{bmatrix}$$

Kompleksitas Waktu

Problem	Eliminasi Gauss	Dekomposisi LU
Mencari solusi SPL $Ax = b$	$T\times\left(\frac{8n^3}{3}+12n^2+\frac{4n}{3}\right)$	$T\times\left(\frac{8n^3}{3}+12n^2+\frac{4n}{3}\right)$
Mencari inverse A (A^{-1})	$T\times\left(\frac{8n^4}{3}+12n^3+\frac{4n^2}{3}\right)$	$T\times\left(\frac{32n^3}{3}+12n^2-\frac{20n}{3}\right)$

T adalah *clock cycle time.*
n adalah jumlah baris dan kolom dari matriks A.

Sumber: https://nm.mathforcollege.com/mws/gen/04sle/mws_gen_sle_txt_ludecomp.pdf

Soal Latihan (60 poin)

Diketahui SPL berikut ini:

$$
\begin{aligned}
3x_1 - 6x_2 - 3x_3 &= 2 \\
2x_1 \qquad\quad + 6x_3 &= 2 \\
-4x_1 + 7x_2 + 4x_3 &= 3
\end{aligned}
$$

1. Carilah matriks L dan U untuk matriks koefisien A dari SPL di atas!
2. Tentukan nilai x_1, x_2 dan x_3 menggunakan matriks L dan U!
3. Tentukan inverse dari matriks koefisien A dari SPL di atas menggunakan matriks L dan U!