Tugas Pertemuan 4

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1. Diket = 3 angka → x, y, z

$x + y + z = 11$

$2x + y = z \rightarrow 2x + y - z = 0$

$x + y - z = -1$

Ditanya : x, y, z ? dengan metode invers matriks

Dijawab :

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 1 & 1 & -1 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 11 \\ 0 \\ -1 \end{bmatrix}$$

matriks kofaktor A : $M_{11} = \begin{vmatrix} 1 & -1 \\ 1 & -1 \end{vmatrix}$ $M_{12} = \begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix}$ $M_{13} = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}$

$M_{21} = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}$ $M_{22} = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}$ $M_{23} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}$

$M_{31} = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}$ $M_{32} = \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix}$ $M_{33} = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}$

matriks kofaktor $A = \begin{pmatrix} 0 & 1 & 1 \\ 2 & -2 & 0 \\ -2 & 3 & -1 \end{pmatrix}$

$adj(A) = C^T = \begin{bmatrix} 0 & 2 & -2 \\ 1 & -2 & 3 \\ 1 & 0 & -1 \end{bmatrix}$ → $det(A) = 1 \cdot (-2) + 1(3) + (-1)(-1)$

$= -2 + 3 + 1 = 2$

$A^{-1} = \frac{1}{det(A)} adj(A)$

$A^{-1} = \frac{1}{2} \begin{bmatrix} 0 & 2 & -2 \\ 1 & -2 & 3 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ \frac{1}{2} & -1 & \frac{3}{2} \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix}$

$x = A^{-1} B = \begin{bmatrix} 0 & 1 & -1 \\ \frac{1}{2} & -1 & \frac{3}{2} \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 11 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix}$ → maka $x = 1, y = 4, z = 6$

2. diket = tiket dewasa = 33.000

tiket remaja = 24.000

tiket anak = 9.000

$A + R = 30 + \frac{1}{2}D \rightarrow -\frac{1}{2}D + A + R = 30$

$R = 5(4A) \rightarrow R = 20A \rightarrow -20A + R = 0$

$33D + 24R + 9A = 89820 \rightarrow 11D + 8R + 3A = 29940$

$A = \begin{bmatrix} -\frac{1}{2} & 1 & 1 \\ 0 & -20 & 1 \\ 11 & 8 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 30 \\ 0 \\ 29940 \end{bmatrix}$ $A_1 = \begin{bmatrix} 30 & 1 & 1 \\ 0 & -20 & 1 \\ 29940 & 8 & 3 \end{bmatrix}$ $A_2 = \begin{bmatrix} -\frac{1}{2} & 30 & 1 \\ 0 & 0 & 1 \\ 11 & 29940 & 3 \end{bmatrix}$

$A_3 = \begin{bmatrix} -\frac{1}{2} & 1 & 30 \\ 0 & -20 & 0 \\ 11 & 8 & 29940 \end{bmatrix}$

$\det(A) = \begin{vmatrix} -\frac{1}{2} & 1 & -1 \\ 0 & -20 & 1 \\ 11 & 3 & 8 \end{vmatrix} \begin{matrix} -\frac{1}{2} & 1 \\ 0 & -20 \\ 11 & 8 \end{matrix}$

$= 312.5$

$\det(A_2) = \begin{vmatrix} -\frac{1}{2} & 1 & 30 \\ 0 & -20 & 0 \\ 11 & 8 & 299\frac{1}{2} \end{vmatrix} \begin{matrix} -\frac{1}{2} & 1 \\ 0 & -20 \\ 11 & 8 \end{matrix}$

$= 306000$

$I2 = \frac{306\,000}{312.5} = 979.2$

Dekomposisi LU

$3x_1 - 6x_2 - 3x_3 = 2$
$2x_2 + 6x_3 = 2$
$-4x_1 + 7x_2 + 4x_3 = 3$

$A = \begin{bmatrix} 3 & -6 & -3 \\ 2 & 0 & 6 \\ -4 & 7 & 4 \end{bmatrix}$

1. Matriks LU

1. $H_1(\frac{1}{3}) \begin{bmatrix} 1 & -2 & -1 \\ 2 & 0 & 6 \\ -4 & 7 & 4 \end{bmatrix}$

2. $H_{21}(-2) \begin{bmatrix} 1 & -2 & -1 \\ 0 & 4 & 8 \\ -4 & 7 & 4 \end{bmatrix}$

3. $H_{31}(4) \begin{bmatrix} 1 & -2 & -1 \\ 0 & 4 & 8 \\ 0 & -1 & 0 \end{bmatrix}$

4. $H_2(\frac{1}{4}) \begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & 2 \\ 0 & -1 & 0 \end{bmatrix}$

5. $H_{32}(1) \begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$

6. $H_3(\frac{1}{2}) \begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$L = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 3 & 0 & 0 \\ 2 & 4 & 0 \\ -4 & -1 & 2 \end{bmatrix}$

2. $Ly = b \quad \begin{bmatrix} 3 & 0 & 0 \\ 2 & 4 & 0 \\ -4 & -1 & 2 \end{bmatrix} \times \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} \rightarrow y_1 = \frac{2}{3}$

$2 \cdot y_1 + 4 y_2 = 2$
$2(\frac{2}{3}) + 4 y_2 = 2$
$4 y_2 = 2 - \frac{4}{3}$
$4 y_2 = \frac{2}{3}$
$y_2 = \frac{1}{6}$

$-4 y_1 + (-1) y_2 + 2 y_3 = 3$
$-4(\frac{2}{3}) - 1(\frac{1}{6}) + 2 y_3 = 3$
$2 y_3 = 3 + \frac{8}{3} + \frac{1}{6}$
$y_3 = \frac{35}{6} \cdot \frac{1}{2} = \frac{35}{12}$

$y_1 = \frac{2}{3}, \; y_2 = \frac{1}{6}, \; y_3 = \frac{35}{12}$

$$Ux = y \rightarrow \begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{6} \\ \frac{35}{12} \end{bmatrix}$$

- $x_3 = \frac{35}{12}$
- $x_2 + 2x_3 = \frac{1}{6}$

$$x_2 + 2\left(\frac{35}{12}\right) = \frac{1}{6}$$

$$x_2 + \frac{35}{6} = \frac{1}{6}$$

$$x_2 = -\frac{34}{6}$$

- $x_1 - 2x_2 - x_3 = \frac{2}{3}$

$$x_1 - 2\left(-\frac{34}{6}\right) - \frac{35}{12} = \frac{2}{3}$$

$$x_1 + \frac{34}{3} - \frac{35}{12} = \frac{2}{3}$$

$$x_1 = -\frac{31}{4}$$

$$x_1 = -\frac{31}{4}, \quad x_2 = -\frac{34}{6}, \quad x_3 = \frac{35}{12}$$

3. $A = \begin{bmatrix} 3 & -6 & -3 \\ 2 & 0 & 6 \\ -9 & 7 & 9 \end{bmatrix}$, $L = \begin{bmatrix} 3 & 0 & 0 \\ 2 & 9 & 0 \\ -9 & -1 & 2 \end{bmatrix}$, $U = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

kolom 1

1a $\begin{bmatrix} 3 & 0 & 0 \\ 2 & 9 & 0 \\ -9 & -1 & 2 \end{bmatrix} \times \begin{bmatrix} y_{11} \\ y_{21} \\ y_{31} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

- $y_{11} = \frac{1}{3}$
- $2y_{11} + 9y_{21} = 0$
  $\frac{2}{3} + 9y_{21} = 0$
  $9y_{21} = -\frac{2}{3}$
  $y_{21} = -\frac{2}{3} \cdot \frac{1}{9} = -\frac{1}{6}$
- $-9y_{11} + -1y_{21} + 2y_{31} = 0$
  $-\frac{9}{3} + \frac{1}{6} + 2y_{31} = 0$
  $2y_{31} = 1$
  $y_{31} = \frac{1}{2}$

$y_{11} = \frac{1}{3}, y_{21} = -\frac{1}{6}, y_{31} = \frac{1}{2}$

1b $\begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ -\frac{1}{6} \\ \frac{1}{2} \end{bmatrix}$

- $b_{31} = \frac{1}{2}$
- $b_{21} + 2b_{31} = -\frac{1}{6}$
  $b_{21} + \frac{2}{6} = -\frac{1}{6}$
  $b_{21} = -\frac{1}{6} - \frac{2}{6} = -\frac{3}{6} = -\frac{1}{2}$
- $b_{11} - 2b_{21} - b_{31} = \frac{1}{3}$
  $b_{11} + \frac{2}{3} - \frac{1}{2} = \frac{1}{3}$
  $b_{11} = \frac{1}{3} - \frac{2}{3} + \frac{1}{2} = -\frac{1}{2}$

$b_{11} = -\frac{1}{2}, b_{21} = -\frac{1}{2}, b_{31} = \frac{1}{2}$

kolom 2

2a $\begin{bmatrix} 3 & 0 & 0 \\ 2 & 9 & 0 \\ -9 & -1 & 2 \end{bmatrix} \times \begin{bmatrix} y_{12} \\ y_{22} \\ y_{32} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

- $y_{12} = 0$
- $2y_{12} + 9y_{32} = 1$
  $0 + 9y_{32} = 1$
  $y_{32} = \frac{1}{9}$
- $-9y_{12} - 1y_{32} + 2y_{32} = 0$
  $0 - \frac{1}{9} + 2y_{32} = 0$
  $2y_{32} = \frac{1}{9}$
  $y_{32} = \frac{1}{18}$

$y_{11} = 0, y_{22} = \frac{1}{9}, y_{32} = \frac{1}{18}$

2b $\begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} b_{12} \\ b_{22} \\ b_{32} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{9} \\ \frac{1}{18} \end{bmatrix}$

- $b_{32} = \frac{1}{18}$
- $1 \cdot b_{22} + 2b_{32} = \frac{1}{9}$
  $b_{22} + 2 \cdot \frac{1}{18} = \frac{1}{9}$
  $b_{22} = \frac{1}{9} - \frac{1}{9} = 0$
- $b_{12} - 2b_{22} - b_{32} = 0$
  $b_{12} = 0 - \frac{1}{18} = 0$
  $b_{12} = 0 + \frac{1}{18} = \frac{1}{18}$

$b_{12} = \frac{1}{18}, b_{22} = 0, b_{32} = \frac{1}{18}$

kolom 3

3a. $\begin{bmatrix} 3 & 0 & 0 \\ 2 & 4 & 0 \\ -4 & 1 & 2 \end{bmatrix} \times \begin{bmatrix} Y_{13} \\ Y_{23} \\ Y_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

- $3Y_{13} = 0$
- $2Y_{13} + 4Y_{23} = 0$
  $0 + 4Y_{23} = 0$
  $Y_{23} = 0$
- $-4Y_{13} + 1Y_{23} + 2Y_{33} = 1$
  $0 + 0 + 2Y_{33} = 1$
  $Y_{33} = \frac{1}{2}$

$\boxed{Y_{13} = 0,\ Y_{23} = 0,\ Y_{33} = \frac{1}{2}}$

3b. $\begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} b_{13} \\ b_{23} \\ b_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{2} \end{bmatrix}$

- $b_{33} = \frac{1}{2}$
- $1b_{23} + 2b_{33} = 0$
  $b_{23} + 2 \cdot \frac{1}{2} = 0$
  $b_{23} = -1$
- $b_{13} - 2(b_{23}) - b_{33} = 0$
  $b_{13} - 2(-1) - \frac{1}{2} = 0$
  $b_{13} + 2 - \frac{1}{2} = 0$
  $b_{13} + \frac{4}{2} - \frac{1}{2} = 0$
  $b_{13} = -\frac{3}{2}$

$\boxed{b_{13} = -\frac{3}{2},\ b_{23} = -1,\ b_{33} = \frac{1}{2}}$

hasil

$A^{-1} = \begin{bmatrix} -\frac{7}{4} & \frac{1}{8} & -\frac{3}{2} \\ -\frac{4}{3} & 0 & -1 \\ \frac{7}{12} & \frac{1}{8} & \frac{1}{2} \end{bmatrix}$