

# Tugas Pertemuan 7

Georgia Sugisandhea - 535230080

## Ruang Vektor

$$u = [1, -2, 3]$$

$$v = [5, 6, -1]$$

$$w = [3, 2, 1]$$

1. vektor  $z = [0, 16, -16]$  merupakan kombinasi linear dari  $u, v, w$

$$(0, 16, -16) = k_1(1, -2, 3) + k_2(5, 6, -1) + k_3(3, 2, 1)$$

$$(0, 16, -16) = (k_1 + 5k_2 + 3k_3, -2k_1 + 6k_2 + 2k_3, 3k_1 - 4k_2 + k_3)$$

$$\begin{cases} k_1 + 5k_2 + 3k_3 = 0 \\ -2k_1 + 6k_2 + 2k_3 = 16 \\ 3k_1 - 4k_2 + k_3 = -16 \end{cases}$$

$$\begin{array}{r} -2k_1 + 6k_2 + 2k_3 = 16 \\ -2k_1 - 10k_2 - 6k_3 = 0 \\ \hline 16k_2 + 8k_3 = 16 \\ \boxed{2k_2 + k_3 = 2} \end{array}$$

$$k_1 + 5k_2 + 3(2 - 2k_2) = 0$$

$$k_1 + 5k_2 + 6 - 6k_2 = 0$$

$$\boxed{k_1 - k_2 = -6}$$

$$2k_1 + 10k_2 + 6k_3 = 0$$

$$-6k_1 + 18k_2 + 6k_3 = 48$$

$$8k_1 - 8k_2 = -48$$

$$\boxed{k_1 - k_2 = -6}$$

$$\begin{array}{r} 3k_1 - 4k_2 + k_3 = -16 \\ 3k_1 + 15k_2 + 9k_3 = 0 \\ \hline -16k_2 - 8k_3 = -16 \\ \boxed{2k_2 + k_3 = 2} \\ \hline 16k_2 + 8k_3 = 16 \\ \boxed{2k_2 + k_3 = 2} \\ \hline k_3 = 2 - 2k_2 \end{array}$$

$z$  merupakan kombinasi linear karena hasil SPL nya adalah konsisten

2. apakah vektor  $u, v, w$  bebas linier

$$k_1 + 5k_2 + 3k_3 = 0$$

$$-2k_1 + 6k_2 + 2k_3 = 0$$

$$3k_1 - k_2 + k_3 = 0$$

$$-2k_1 - 10k_2 - 6k_3 = 0$$

$$-2k_1 + 6k_2 + 2k_3 = 0$$

$$-16k_2 - 8k_3 = 0$$

$$k_2 = -\frac{1}{2}k_3$$

$$9k_1 - 3k_2 + 3k_3 = 0$$

$$k_1 + 5k_2 + 3k_3 = 0$$

$$8k_1 - 8k_2 = 0$$

$$8k_1 - 8(-\frac{1}{2}k_3) = 0 \rightarrow k_1 = -\frac{1}{2}k_3$$

$$-6k_1 + 18k_2 + 6k_3 = 0$$

$$-6k_1 + 2k_2 - 2k_3 = 0$$

$$16k_2 + 8k_3 = 0$$

$$-16k_2 - 8k_3 = 0$$

$$6k_1 - 2k_2 + 2k_3 = 0$$

$$-2k_1 + 6k_2 + 2k_3 = 0$$

$$8k_1 - 8k_2 = 0$$

ketika  $k_3$  dimasukkan sembarang angka, hasilnya tidak selalu 0, jadi tidak bebas linier

3. merentang  $R^3$

$$k_1 - 2k_2 + 3k_3 = b_1$$

$$5k_1 + 6k_2 - k_3 = b_2$$

$$3k_1 + 2k_2 + k_3 = b_3$$

$$\left| \begin{array}{ccc|cc} 1 & -2 & 3 & 1 & -2 \\ 5 & 6 & -1 & 5 & 6 \\ 3 & 2 & 1 & 3 & 2 \end{array} \right|$$

$$((1) + (6) + (30)) - ((54) + (-2) + (-10))$$

$$42 - 42 = 0$$

$\rightarrow$  maka tidak merentang  $R^3$



4. Dimensi dan rank dari ruang  $S = \{u, v, w\}$

$$\begin{pmatrix} 1 & -2 & 3 \\ 5 & 6 & -1 \\ 3 & 2 & 1 \end{pmatrix} \rightarrow H_{21}(-5) \begin{pmatrix} 1 & -2 & 3 \\ 0 & 16 & -16 \\ 3 & 2 & 1 \end{pmatrix}$$

$$H_{31}(-3) \begin{pmatrix} 1 & -2 & 3 \\ 0 & 16 & -16 \\ 0 & 8 & -8 \end{pmatrix}$$

dimensi = 2  
rang  
baris

$$H_{32}\left(-\frac{1}{2}\right) \begin{pmatrix} 1 & -2 & 3 \\ 0 & 16 & -16 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 5 & 3 \\ -2 & 6 & 2 \\ 3 & -1 & 1 \end{pmatrix} \rightarrow H_{21}(2) \begin{pmatrix} 1 & 5 & 3 \\ 0 & 16 & 8 \\ 3 & -1 & 1 \end{pmatrix}$$

$$H_{31}(-3) \begin{pmatrix} 1 & 5 & 3 \\ 0 & 16 & 8 \\ 0 & -16 & -8 \end{pmatrix}$$

dimensi = 2  
rang  
kolom

$$H_{32}(1) \begin{pmatrix} 1 & 5 & 3 \\ 0 & 16 & 8 \\ 0 & 0 & 0 \end{pmatrix}$$

rank = 2

Transformasi Linier

1.  $f: \mathbb{R}^2 \rightarrow \mathbb{R}^3$

$$f(x, y) = (y, -5x + 13y, -7x + 16y)$$

a. Diketahui  $u = (1, 2)$ ,  $v = (3, 4)$ ,  $k = 2$

b. cek  $T(u+v) = T(u) + T(v)$

$$T(1+3, 2+4) = T(4, 6) = (6, -5(4) + 13(6), -7(4) + 16(6)) \\ = (6, 58, 68)$$

$$T(1, 2) + T(3, 4) = (2, -5(1) + 13(2), -7(1) + 16(2)) + \\ (4, -5(3) + 13(4), -7(3) + 16(4)) \\ = (2, 21, 25) + (4, 37, 43) \\ = (6, 58, 68)$$

→ Syarat pertama memenuhi

c.  $T(ku) = kT(u)$

$$T(2(1, 2)) = T(2, 4) = (4, -5(2) + 13(4), -7(2) + 16(4)) \\ = (4, 42, 50)$$

$$2(T(1, 2)) = 2(2, -5(1) + 13(2), -7(1) + 16(2)) \\ = 2(2, 21, 25) \\ = (4, 42, 50) \rightarrow \text{Syarat kedua memenuhi}$$

$f(x, y) = (y, -5x + 13y, -7x + 16y)$  adalah transformasi linier



$$\begin{aligned}
 2. \quad P_1 &= (0, 1) \rightarrow (1, 13, 16) \\
 P_2 &= (2, 1) \rightarrow (1, 3, 2) \\
 P_3 &= (1, 3) \rightarrow (3, 34, 41)
 \end{aligned}$$

→ dicerminkan terhadap bidang  $xz$

$$P_1 = (1, 13, 16) \rightarrow (1, -13, 16)$$

$$P_2 = (1, 3, 2) \rightarrow (1, -3, 2)$$

$$P_3 = (3, 34, 41) \rightarrow (3, -34, 41)$$

→ dirotasikan searah lawan jarum jam terhadap sumbu  $y$  dengan sudut  $45^\circ$

$$\begin{aligned}
 P_1 &= (1, -13, 16) \quad \left. \begin{aligned} x' &= 1 \cos 45^\circ + 16 \sin 45^\circ \\ &= 1 \cdot \frac{1}{2}\sqrt{2} + 16 \cdot \frac{1}{2}\sqrt{2} = \frac{1}{2}\sqrt{2} + 8\sqrt{2} = \frac{17}{2}\sqrt{2} \\ y' &= -13 \\ z' &= -1 \sin 45^\circ + 16 \cos 45^\circ \\ &= -\frac{1}{2}\sqrt{2} + 16 \cdot \frac{1}{2}\sqrt{2} = -\frac{1}{2}\sqrt{2} + 8\sqrt{2} = \frac{15}{2}\sqrt{2} \end{aligned} \right\} \\
 &\rightarrow \left( \frac{17}{2}\sqrt{2}, -13, \frac{15}{2}\sqrt{2} \right)
 \end{aligned}$$

$$\begin{aligned}
 P_2 &= (1, -3, 2) \quad \left. \begin{aligned} x' &= 1 \cos 45^\circ + 2 \sin 45^\circ \\ &= \frac{1}{2}\sqrt{2} + 2 \cdot \frac{1}{2}\sqrt{2} = \frac{1}{2}\sqrt{2} + \sqrt{2} = \frac{3}{2}\sqrt{2} \\ y' &= -3 \\ z' &= -1 \sin 45^\circ + 2 \cos 45^\circ \\ &= -\frac{1}{2}\sqrt{2} + 2 \cdot \frac{1}{2}\sqrt{2} = -\frac{1}{2}\sqrt{2} + \sqrt{2} = \frac{1}{2}\sqrt{2} \end{aligned} \right\} \\
 &\rightarrow \left( \frac{3}{2}\sqrt{2}, -3, \frac{1}{2}\sqrt{2} \right)
 \end{aligned}$$

$$\begin{aligned}
 P_3 &= (3, -34, 41) \quad \left. \begin{aligned} x' &= 3 \cos 45^\circ + 41 \sin 45^\circ \\ &= 3 \cdot \frac{1}{2}\sqrt{2} + 41 \cdot \frac{1}{2}\sqrt{2} = \frac{3}{2}\sqrt{2} + \frac{41}{2}\sqrt{2} = 22\sqrt{2} \\ y' &= -34 \\ z' &= -3 \sin 45^\circ + 41 \cos 45^\circ \\ &= -3 \cdot \frac{1}{2}\sqrt{2} + 41 \cdot \frac{1}{2}\sqrt{2} = -\frac{3}{2}\sqrt{2} + \frac{41}{2}\sqrt{2} = 19\sqrt{2} \end{aligned} \right\} \\
 &\rightarrow (22\sqrt{2}, -34, 19\sqrt{2})
 \end{aligned}$$

sehingga hasil akhir =  $P_1 \left( \frac{17}{2}\sqrt{2}, -13, \frac{15}{2}\sqrt{2} \right)$

$$P_2 \left( \frac{3}{2}\sqrt{2}, -3, \frac{1}{2}\sqrt{2} \right)$$

$$P_3 (22\sqrt{2}, -34, 19\sqrt{2})$$