Self-Driving Cars

Lecture 6 – Vehicle Control

Prof. Dr.-Ing. Andreas Geiger

Autonomous Vision Group

University of Tübingen / MPI-IS

EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

ellis
European Laboratory for Learning and Intelligent Systems

Agenda

6.1 Introduction

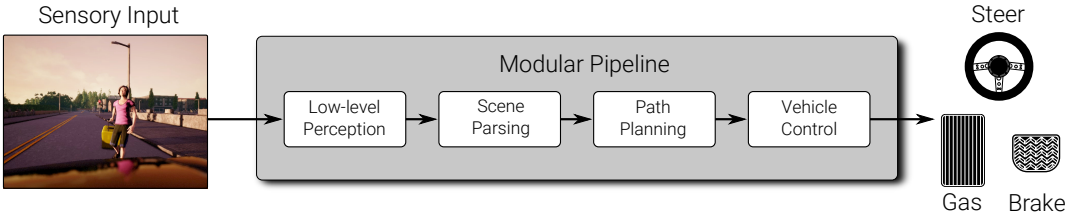
6.2 Black Box Control

6.3 Geometric Control

6.4 Optimal Control

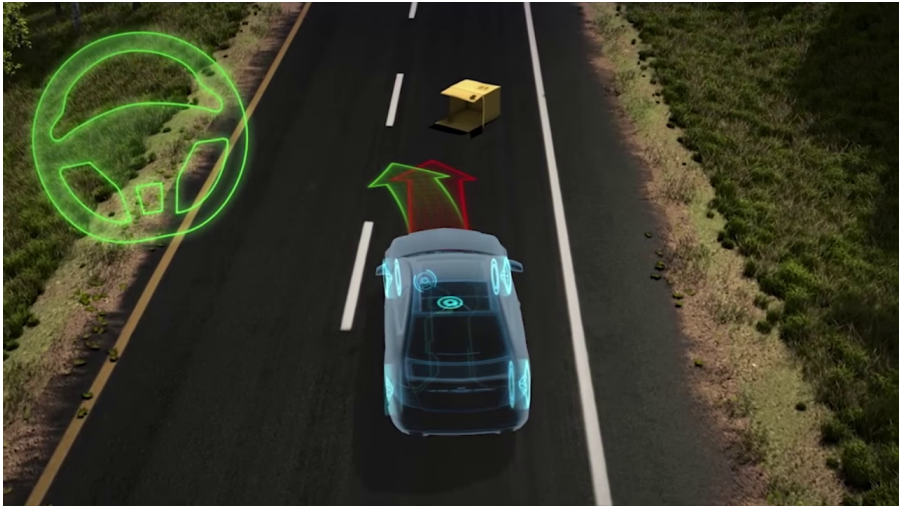
6.1
Introduction

Approaches to Self-Driving



- Self-driving cars and driver assistance systems require **vehicle control**

Electronic Stability Program

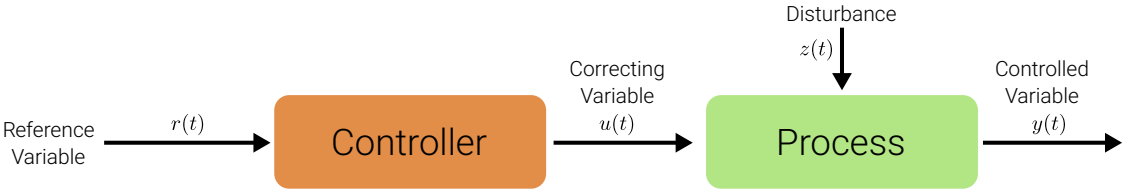


Sophisticated controllers are already in your car today!

Brief History of Driver Assistance Systems

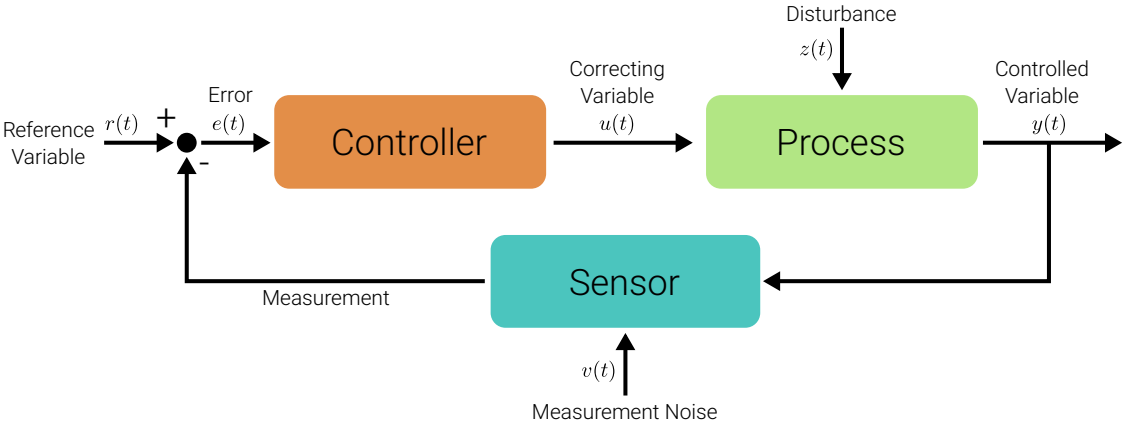
- 1926: Servo braking (Pierce-Arrow)
- 1951: Servo steering (Chrysler)
- 1958: Cruise control (Chrysler)
- 1978: Anti-lock braking system ABS (Bosch)
- 1986: Traction control system ASR (Bosch)
- 1995: Electronic stability program ESP (Bosch/BMW)
- 2000: Adaptive cruise control ACC (Mitsubishi/Toyota/Bosch)
- 2002: Emergency brake assistant (Mercedes Benz)
- 2003: Lane-keeping assistant (Honda)
- 2007: Automatic park assistant (Valeo)

Open-Loop Control



- Requires **precise knowledge** of the plant and the influence factors
- **No feedback** about the controlled variable
- Cannot handle unknown disturbances, resulting in **drift**

Closed-Loop Control



- Exploit feedback loop to minimize error between reference and measurement

Centrifugal Governor

https://www.youtube.com/watch?v=B01LgS8S5C8

Closed-Loop Control

- We will be considering **closed-loop control** in this lecture
- A vehicle needs to be controlled both **longitudinally** and **laterally**
- We consider 3 different types of controllers:
  - **Black box controllers** don't require knowledge about the process
  - **Geometric controllers** exploit geometric relationships between the vehicle and the path, resulting in compact control laws for path tracking
  - **Optimal controllers** use knowledge of the system and minimize an objective function over future time steps

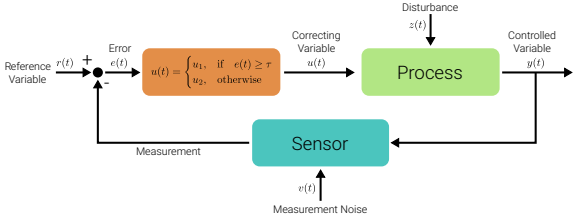
6.2
Black Box Control

Bang-Bang Control

Bang-Bang Control

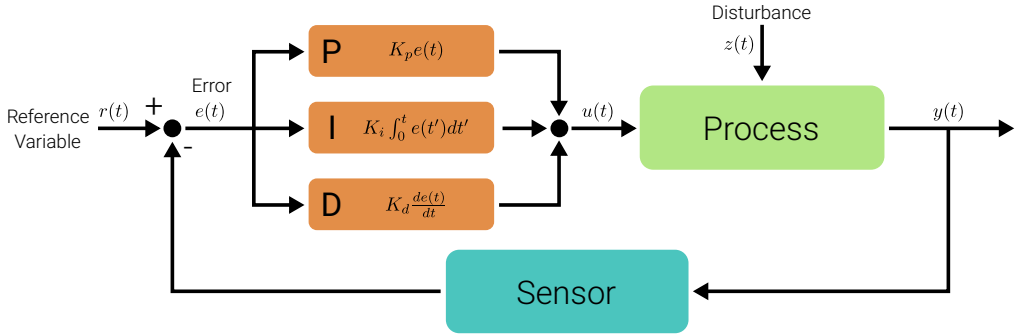
- Also called: hysteresis controller
- Often applied, e.g. in household thermostats
- Switches abruptly between two states
- Mathematical formulation:

$$u(t) = \begin{cases} u_1, & \text{if} \quad e(t) \geq \tau \\ u_2, & \text{otherwise} \end{cases}$$



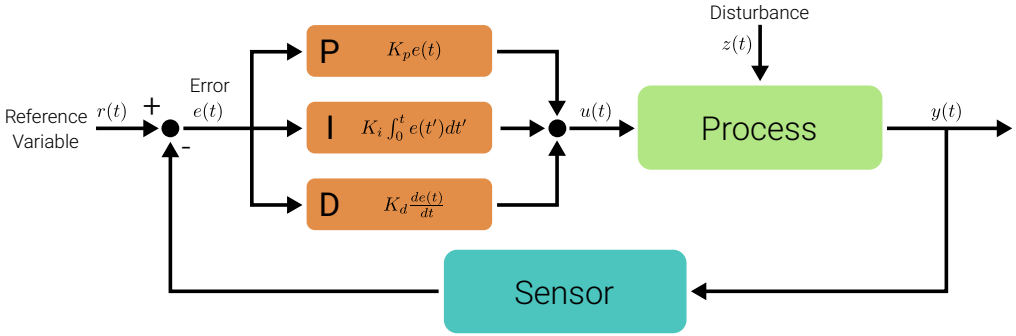
PID Control

PID Control



- Most common controller in current industrial applications
- Doesn't require knowledge about plant/process to be controlled
- Comprises proportional (**P**), integration (**I**) and differential (**D**) element
- Reference variable $r(t)$, correcting variable $u(t)$, controlled variable $y(t)$, error $e(t)$

PID Control

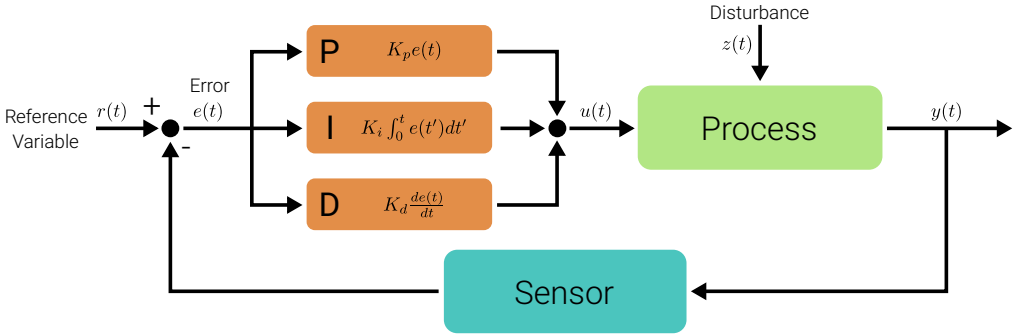


Mathematical formulation:

$$u(t) = K_p\, e(t) + K_i \int_0^t e(t')dt' + K_d \frac{de(t)}{dt}$$

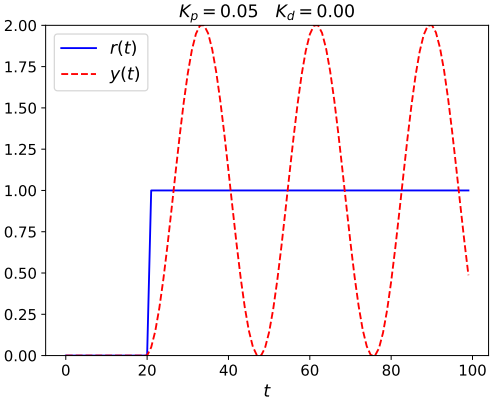
with parameters K_p, K_i and K_d

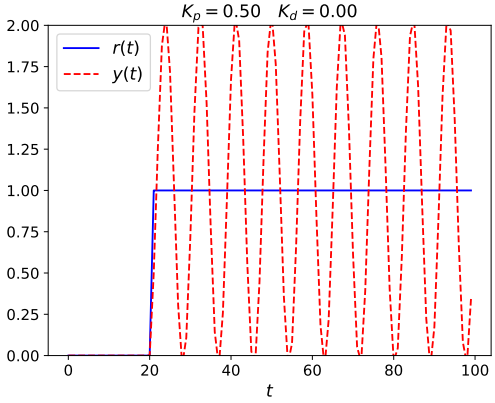
PID Control



- Using the **P** element alone leads to overshooting / oscillation
- Adding a **D** element alleviates this problem by introducing damping behavior
- The **I** element corrects residual errors by integrating past error measurements

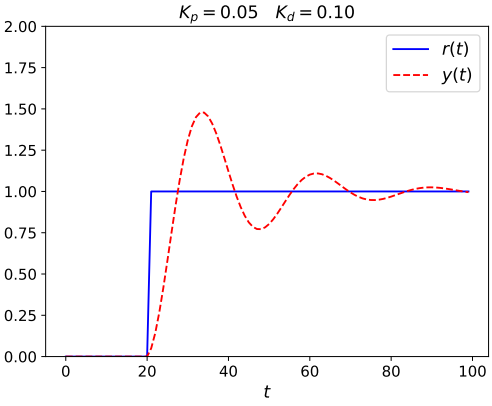
P Control

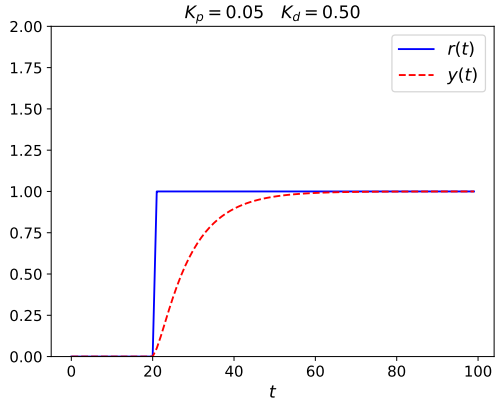




- Controlled Variable: Position $y(t) = x(t)$
- Correcting Variable: Acceleration $u(t) = a(t) = \ddot{x}(t)$

PD Control





- Controlled Variable: Position $y(t) = x(t)$
- Correcting Variable: Acceleration $u(t) = a(t) = \ddot{x}(t)$

PID Control

As PID controllers are often used to control black box processes, various heuristics have been proposed for setting the parameters. **Ziegler-Nichols** is most common:

- Set $K_i = K_d = 0$
- Increase K_p until the ultimate gain $K_p = K_u$ where the system oscillates
- Measure the oscillation period T_u at K_u
- Set $K_p = 0.6K_u$, $K_i = 1.2K_u/T_u$ and $K_d = 3K_uT_u/40$

These heuristics have been derived empirically using computer simulations. While they provide a good starting point, manual fine-tuning is often required in practice. Note: Ziegler-Nichols does not apply to double integrators as in the previous example.

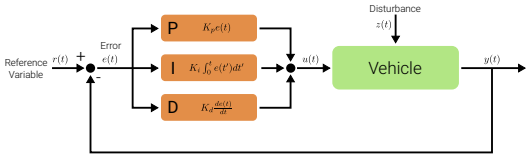
Ziegler and Nichols: Optimum settings for automatic controllers. Transactions of the ASME, 1942.

PID Control

Example: Longitudinal Vehicle Control

$$v(t) = v_{\max}\left(1 - \exp\left(-\theta_1\, d(t) - \theta_2\right)\right)$$

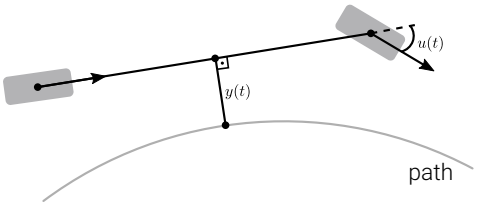
- $v(t)$: target velocity at time t
- $d(t)$: distance to preceding car

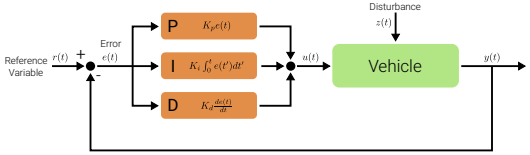


- Reference variable: $r(t) = v(t) =$ target velocity
- Correcting variable: $u(t) =$ gas/brake pedal
- Controlled variable: $y(t) =$ current velocity
- Error: $e(t) = v(t) - y(t)$

PID Control

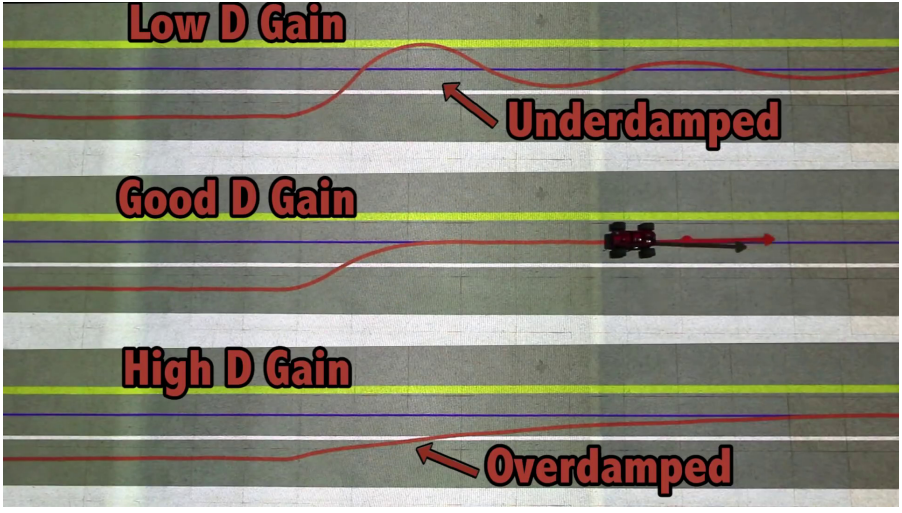
Example: Lateral Vehicle Control





- Reference variable: $r(t) = 0 =$ no cross track error
- Correcting variable: $u(t) = \delta =$ steering angle
- Controlled variable: $y(t) =$ cross track error
- Error: $e(t) = -y(t) =$ cross track error

PID Control



https://www.youtube.com/watch?v=4Y7zG48uHRo

PID Control

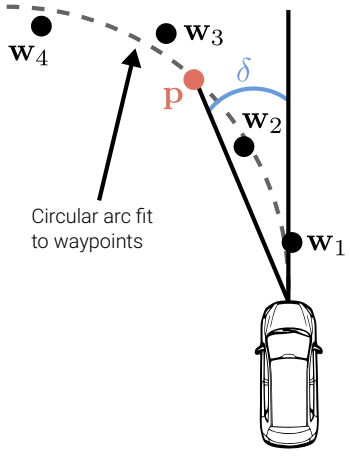
Example: Waypoint-based Vehicle Control

- **Input:** Waypoints $\mathbf{w} = \{\mathbf{w}_1, \ldots, \mathbf{w}_K\}$
- **Velocity:** (Longitudinal PID control)

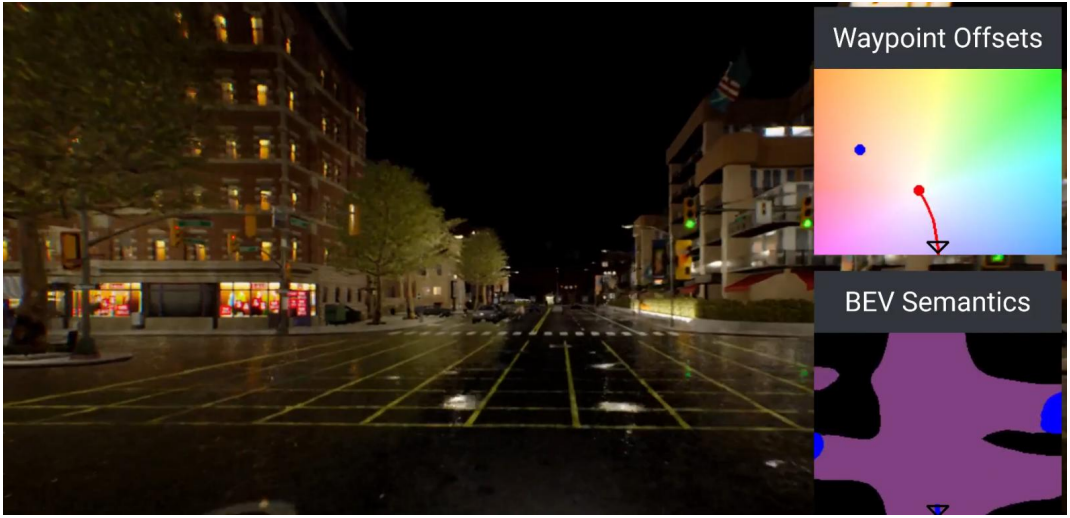
$$v = \frac{1}{K} \sum_{k=1}^{K} \frac{\|\mathbf{w}_k - \mathbf{w}_{k-1}\|_2}{\Delta t}$$

- **Steering angle:** (Lateral PID control)

$$\delta = \tan^{-1}\left(\frac{p_y}{p_x}\right)$$



PID Control



Chitta, Prakash and Geiger: Neural Attention Fields for End-to-End Autonomous Driving. ICCV, 2021.

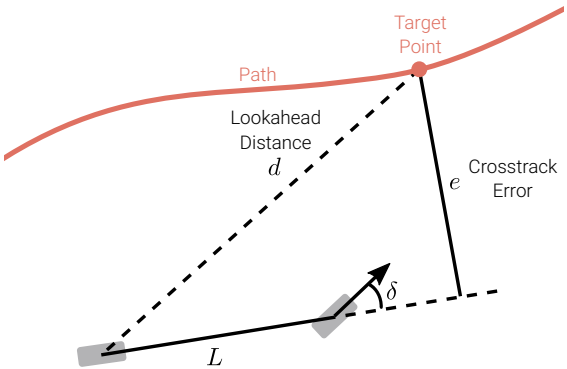
6.3
Geometric Control

Pure Pursuit Control

Pure Pursuit Control

Goal:

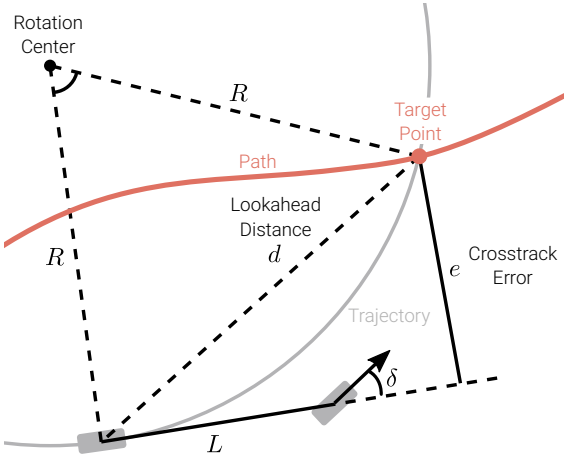
- Track **target point** at **lookahead distance** d to follow path
- Exploit geometric relationship between vehicle and path to follow



Pure Pursuit Control

Goal:

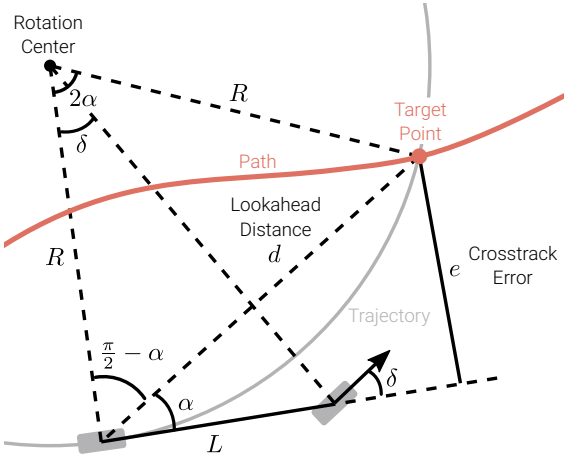
- Track **target point** at **lookahead distance** d to follow path
- Exploit geometric relationship between vehicle and path to follow
- Minimize **crosstrack error** e by following circular trajectory



Pure Pursuit Control

Goal:

- Track **target point** at **lookahead distance** d to follow path
- Exploit geometric relationship between vehicle and path to follow
- Minimize **crosstrack error** e by following circular trajectory
- Steering angle δ determined by angle α between vehicle heading direction and lookahead direction: $\delta(\alpha) = ?$



Pure Pursuit Control

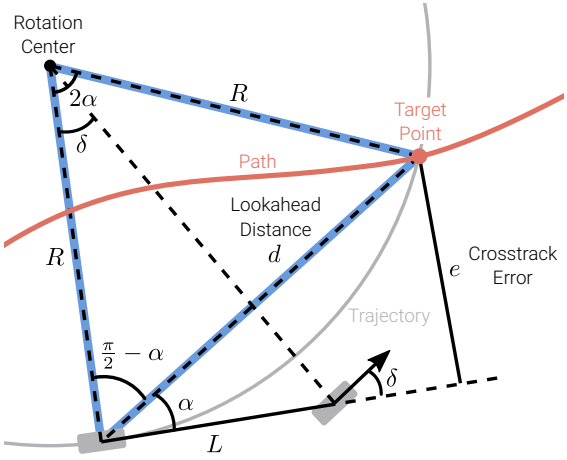
From the **law of sines:**

$$\frac{d}{\sin(2\alpha)} = \frac{R}{\sin\left(\frac{\pi}{2} - \alpha\right)}$$

$$\frac{d}{2\sin\alpha\cos\alpha} = \frac{R}{\cos\alpha}$$

$$\kappa = \frac{1}{R} = \frac{2\sin\alpha}{d}$$

with κ the curvature of trajectory.



Pure Pursuit Control

The **steering angle** is calculated as:

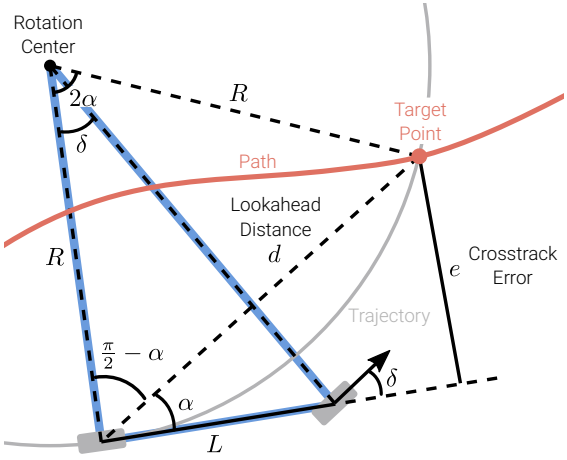
$$\tan(\delta) = \frac{L}{R} = \frac{2L\sin(\alpha)}{d}$$

$$\delta = \tan^{-1}\left(\frac{2L\sin(\alpha)}{d}\right)$$

$$\delta \approx \frac{2L\sin(\alpha)}{d}$$

- d is often based on vehicle speed v:

$$d = Kv \quad \text{with constant } K$$



Pure Pursuit Control

In terms of **cross track error** we obtain:

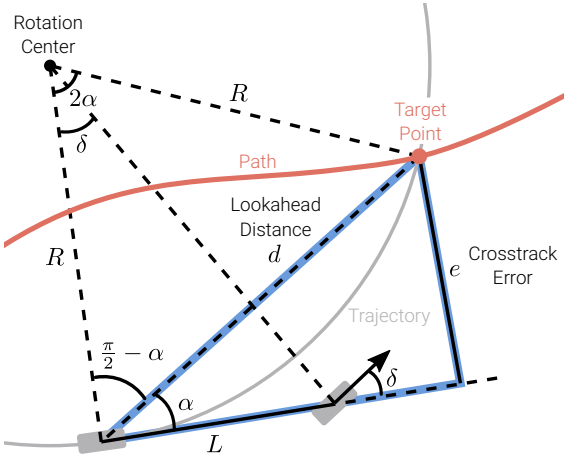
$$\sin\alpha = \frac{e}{d}$$

$$\delta = \tan^{-1}\left(\frac{2L\sin(\alpha)}{d}\right)$$

$$\delta = \tan^{-1}\left(\frac{2Le}{d^2}\right) \approx \frac{2L}{d^2}e$$

- Pure pursuit acts as a **proportional** controller wrt. the crosstrack error
- d is often based on vehicle speed v:

$$d = Kv \quad \text{with constant } K$$



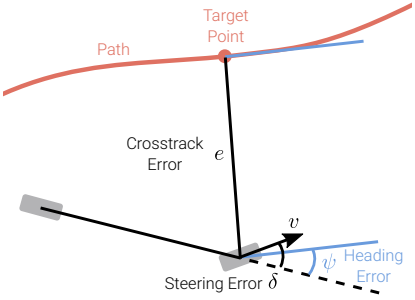
Stanley Control

Stanley Control

Control Law used by **Stanley** in **DARPA Challenge:**

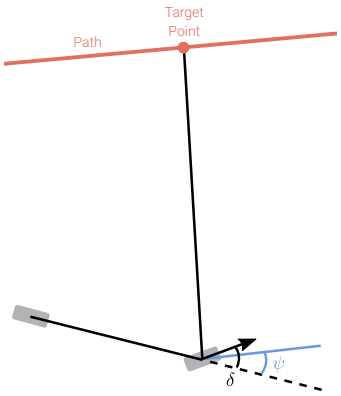
$$\delta = \psi + \tan^{-1}\left(\frac{k\,e}{v}\right)$$

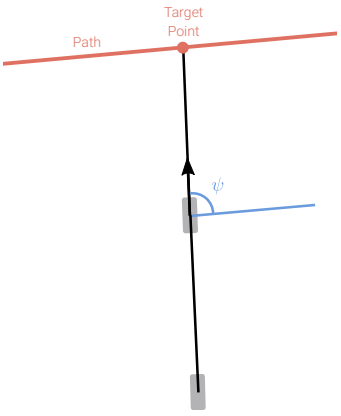
- v = speed, ψ = heading err., e = crosstrack err.
- Reference at **front axle, no lookahead**
- Combines **heading** and **crosstrack error**
- It can be shown that the crosstrack error **converges** exponentially to 0 (indep. of v)
- Works for small velocities w/o disturbances

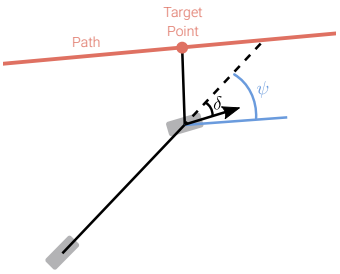


Thrun et al.: Stanley: The robot that won the DARPA Grand Challenge. Journal of Field Robotics, 2006.

Stanley Control



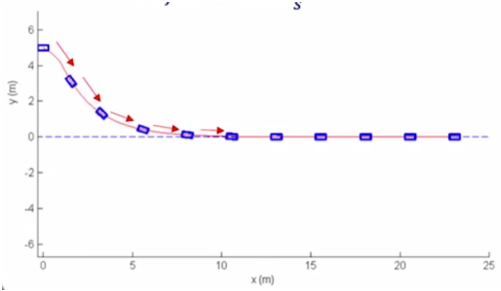


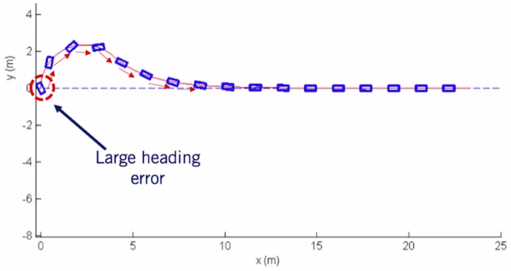


- As heading changes, heading correction **counteracts** crosstrack correction

Thrun et al.: Stanley: The robot that won the DARPA Grand Challenge. Journal of Field Robotics, 2006.

Stanley Control





- The Stanley controller can correct **large crosstrack** and **large heading errors**
- **Global stability:** Independent of initial conditions guides vehicle back (proven)
- But does not consider noisy observations, actuator dynamics, tire force effects
- Softening/dampening terms and curvature information can be added

Slide Credits: Steven Waslander

Stanley Control

https://www.youtube.com/watch?v=M2AcMnfzpNg

6.4
Optimal Control

Optimal Control

Recap: Dynamic Bicycle Model

$$\begin{bmatrix} \dot{v}_y \\ \dot{\psi} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} -\frac{c_r+c_f}{mv_x} & 0 & \frac{c_r l_r - c_f l_f}{mv_x} - v_x \\ 0 & 0 & 1 \\ \frac{l_r c_r - l_f c_f}{I_z v_x} & 0 & -\frac{l_f^2 c_f + l_r^2 c_r}{I_z v_x} \end{bmatrix} \underbrace{\begin{bmatrix} v_y \\ \psi \\ \omega \end{bmatrix}}_{\text{State } \mathbf{x}} + \begin{bmatrix} \frac{c_f}{m} \\ 0 \\ \frac{c_f}{I_z} l_f \end{bmatrix} \underbrace{\delta}_{\text{Input}}$$

With state $\mathbf{x}$ and front steering angle δ.

We can rewrite this equation as the following linear system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}\delta \qquad \text{with} \qquad \mathbf{x} = (v_y, \psi, \omega)^T$$

Optimal Control

Linear Quadratic Regulator (LQR): For the continuous-time linear system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}\delta \qquad \text{with} \qquad \mathbf{x}(0) = \mathbf{x}_{\text{init}}$$

and quadratic cost functional defined as ($\mathbf{Q}$ is a diagonal weight matrix)

$$J = \frac{1}{2}\int_0^\infty \mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) + q\delta(t)^2\, dt$$

the feedback control $\delta(t)$ that minimizes J is given by

$$\delta(t) = -\mathbf{k}^T(t)\mathbf{x}(t)$$

with $\mathbf{k}(t) = \frac{1}{q}\mathbf{b}^T\mathbf{P}(t)$ and $\mathbf{P}(t)$ the solution to a Ricatti equation (no details here).

Model Predictive Control

Model Predictive Control

Generalizes LQR to:

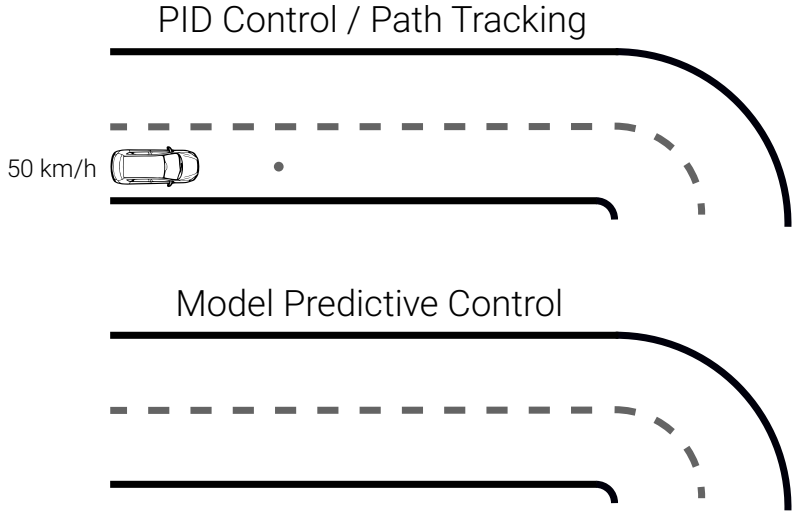
- **Non-linear** cost function and dynamics (consider straight road leading into turn)
- **Flexible:** allows for receding window & incorporation of constraints
- **Expensive:** non-linear optimization required at every iteration

Formally:

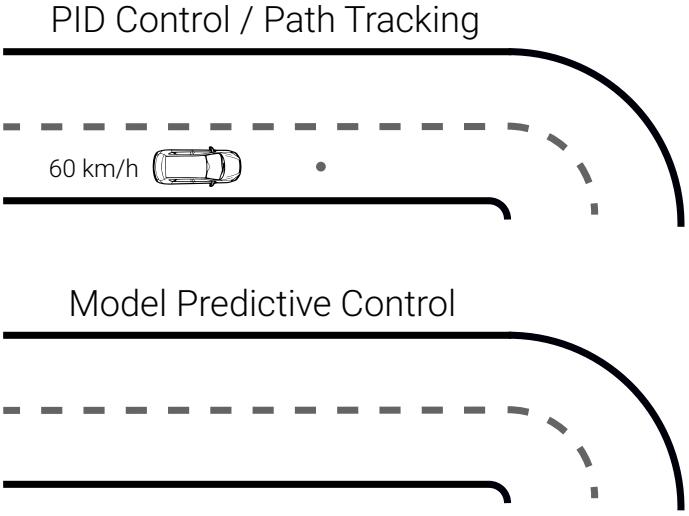
$$
\begin{aligned}
\underset{\delta_1,\ldots,\delta_T}{\operatorname{argmin}} \quad & \sum_{t=1}^{T} C_t(\mathbf{x}_t, \delta_t) && \text{(sum of costs)} \\
s.t. \quad & \mathbf{x}_1 = \mathbf{x}_{\text{init}} && \text{(initialization)} \\
& \mathbf{x}_{t+1} = f(\mathbf{x}_t, \delta_t) && \text{(dynamics model)} \\
& \underline{\delta} \leq \delta_t \leq \overline{\delta} && \text{(constraints)}
\end{aligned}
$$

- Unroll dynamic model T times $\Rightarrow$ apply non-linear optimization to find $\delta_1, \ldots \delta_T$

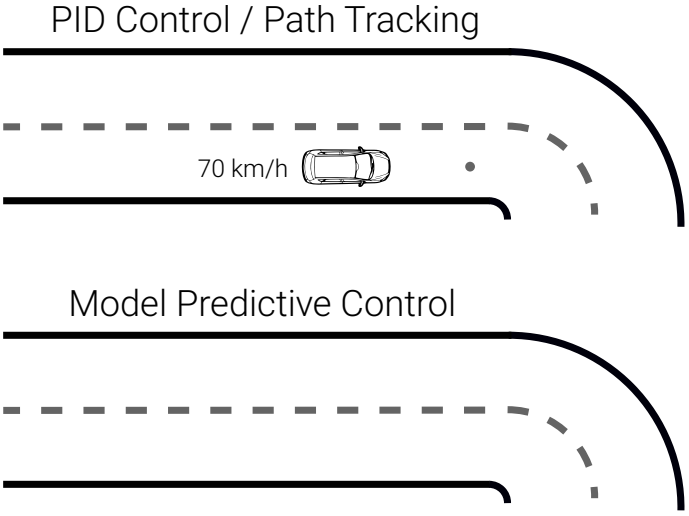
Model Predictive Control



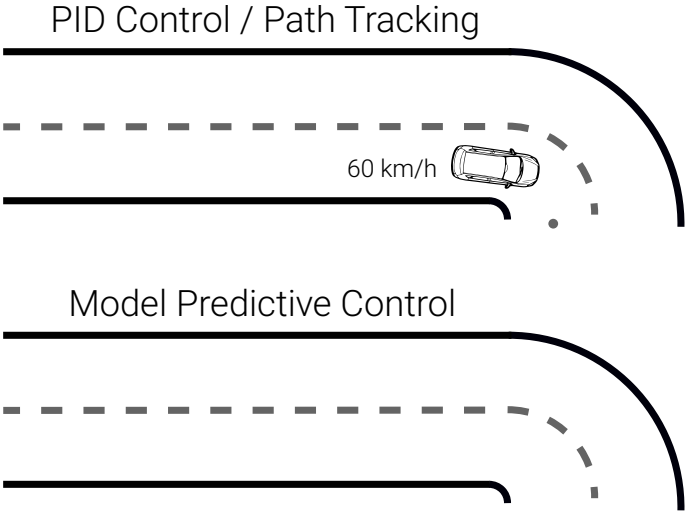
Model Predictive Control



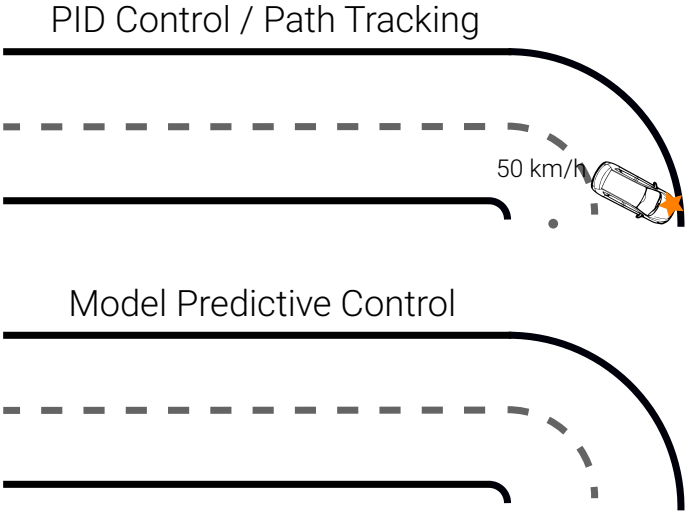
Model Predictive Control



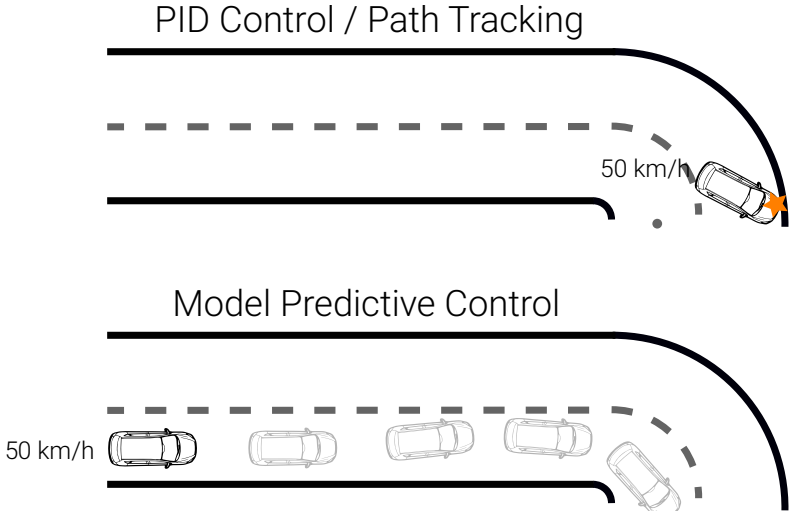
Model Predictive Control



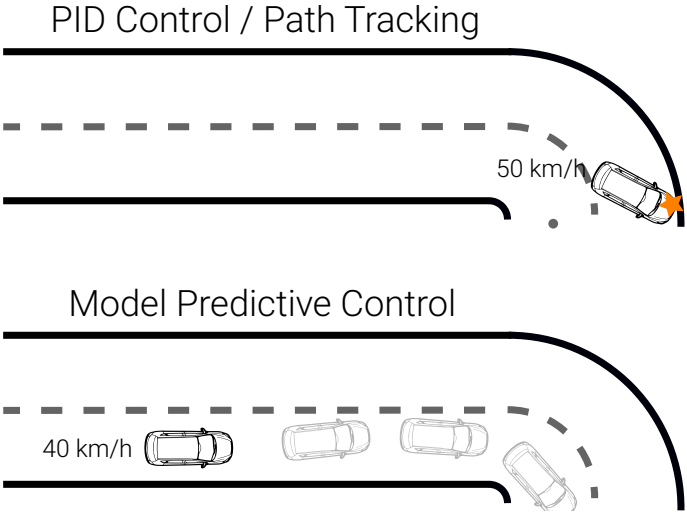
Model Predictive Control



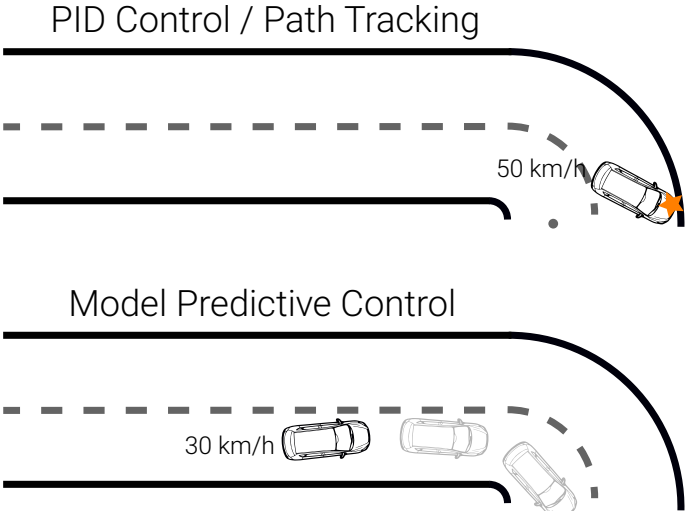
Model Predictive Control



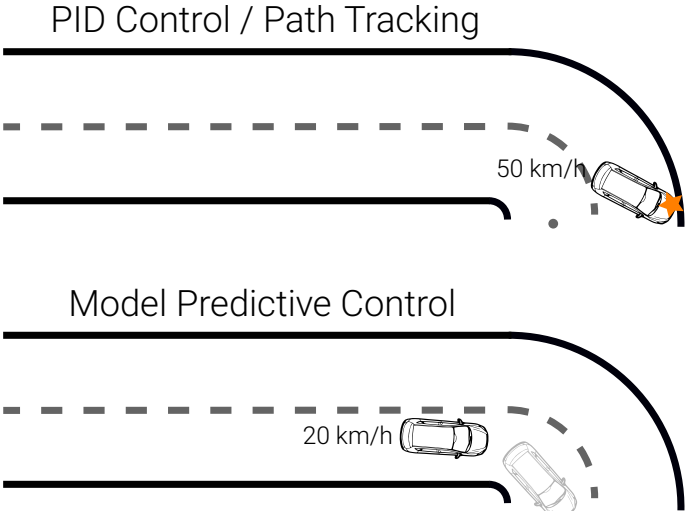
Model Predictive Control



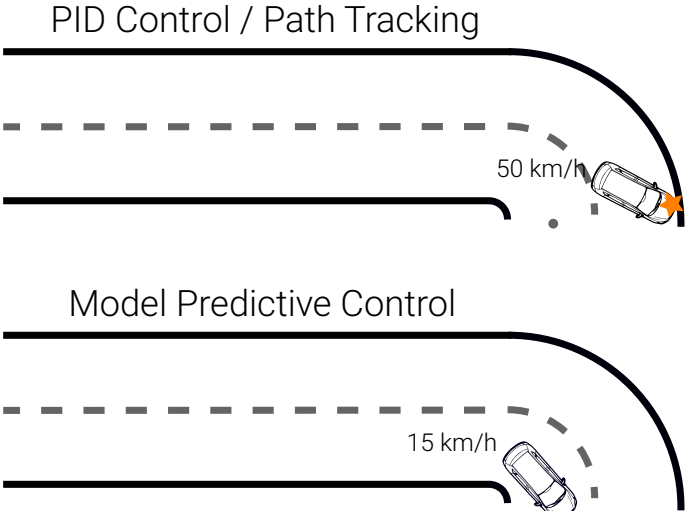
Model Predictive Control



Model Predictive Control



Model Predictive Control



Summary

- Open-loop controllers cannot handle unknown disturbances
- In practice, we thus require closed-loop control with sensor feedback
- Black box controllers don’t require knowledge about the process
- Most popular black box controller: PID controller
- Geometric controllers exploit geometric relationships for path tracking
- Optimal controllers use a vehicle model and optimize a cost function
- MPC is the most flexible and powerful approach
- However, MPC requires solving an optimization problem at every time step