

# Self-Driving Cars

## Lecture 7 – Odometry, SLAM and Localization

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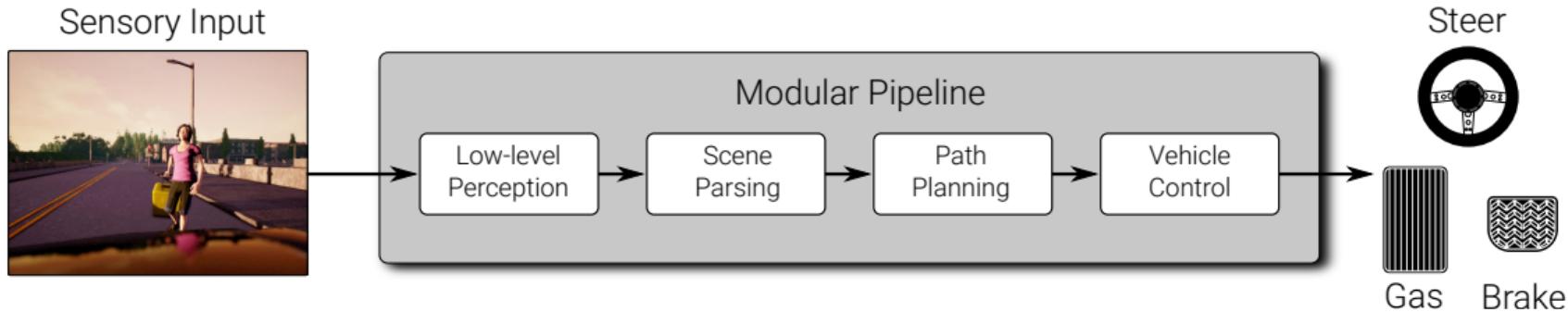
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European Laboratory for Learning and Intelligent Systems

# Modular Pipeline

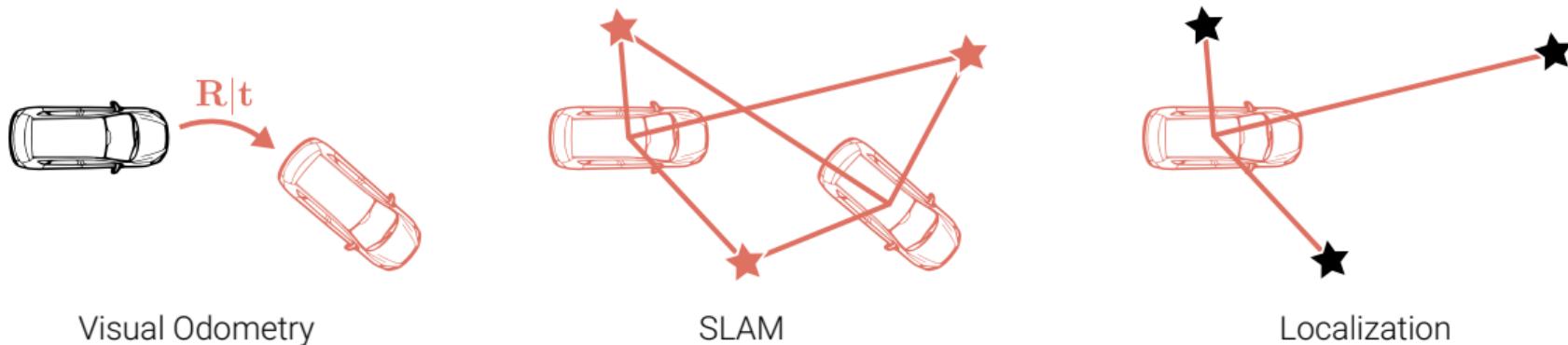


## Course Content:

- ▶ Lectures 5+6: Vehicle Dynamics and Control
- ▶ Lectures 7-11: Low-level Perception and Scene Parsing
- ▶ Lecture 12: Decision Making and Planning

# Ego-motion Estimation and Localization

Self-driving cars operate in a **dynamic environments** and hence must be aware of their **ego-motion** (own motion) and the motion of other traffic participants.



- ▶ **Visual odometry** refers to the estimation of **relative ego-motion** from images
- ▶ **SLAM algorithms** build a **map** and simultaneously **localize** in that map
- ▶ **Localization methods** find the **global pose** of a vehicle in a given map

# Agenda

**7.1** Visual Odometry

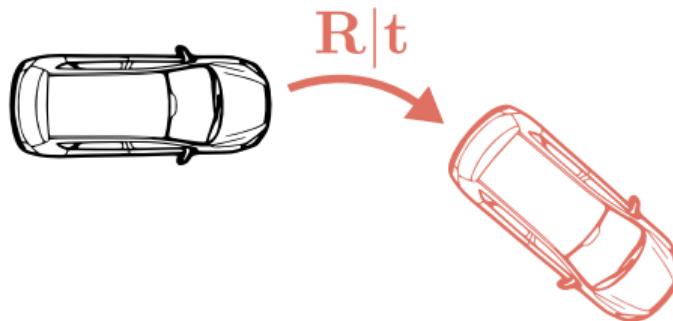
**7.2** Simultaneous Localization and Mapping

**7.3** Localization

# 7.1

## Visual Odometry

# Odometry



- ▶ **Odometry** is the use of sensors to estimate change in ego-position over time
- ▶ Greek origin: odos (=route) and metron (=measure)
- ▶ In the example above, the car undergoes a **rigid body motion** between 2 frames
- ▶ Odometry yields **relative motion estimates** (not global position wrt. a map)
- ▶ It is hence sensitive to **error accumulation** over time (only precise locally)
- ▶ Odometry information can be obtained using a variety of sensors ...

# Odometry



Wheel Odometry



Inertial Measurement Unit (IMU)

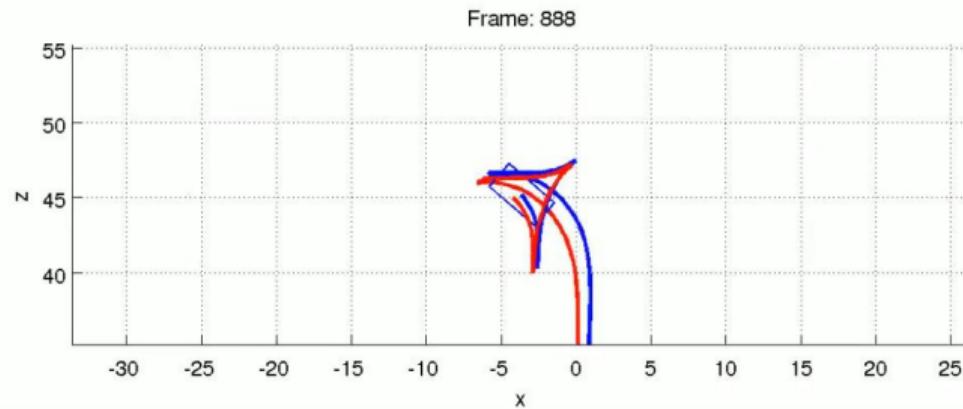
- ▶ **Wheel odometry systems** use wheel encoders to measure wheel rotation
- ▶ **Inertial measurement units** measure a body's forces ( $\Rightarrow$  acceleration)
- ▶ **Visual odometry algorithms** use camera images ( $\Rightarrow$  focus of this unit)
- ▶ Multiple systems often combined to complement each other, e.g., VIO

# Visual Odometry



- ▶ **Visual odometry** is concerned with **tracking the pose**, i.e., position and orientation, of the camera with respect to the environment from its images
- ▶ VO considers a limited set of recent images for **real-time** ⇒ error accumulation
- ▶ A (sparse) **local map** is often built as a by-product, but mapping is not the focus

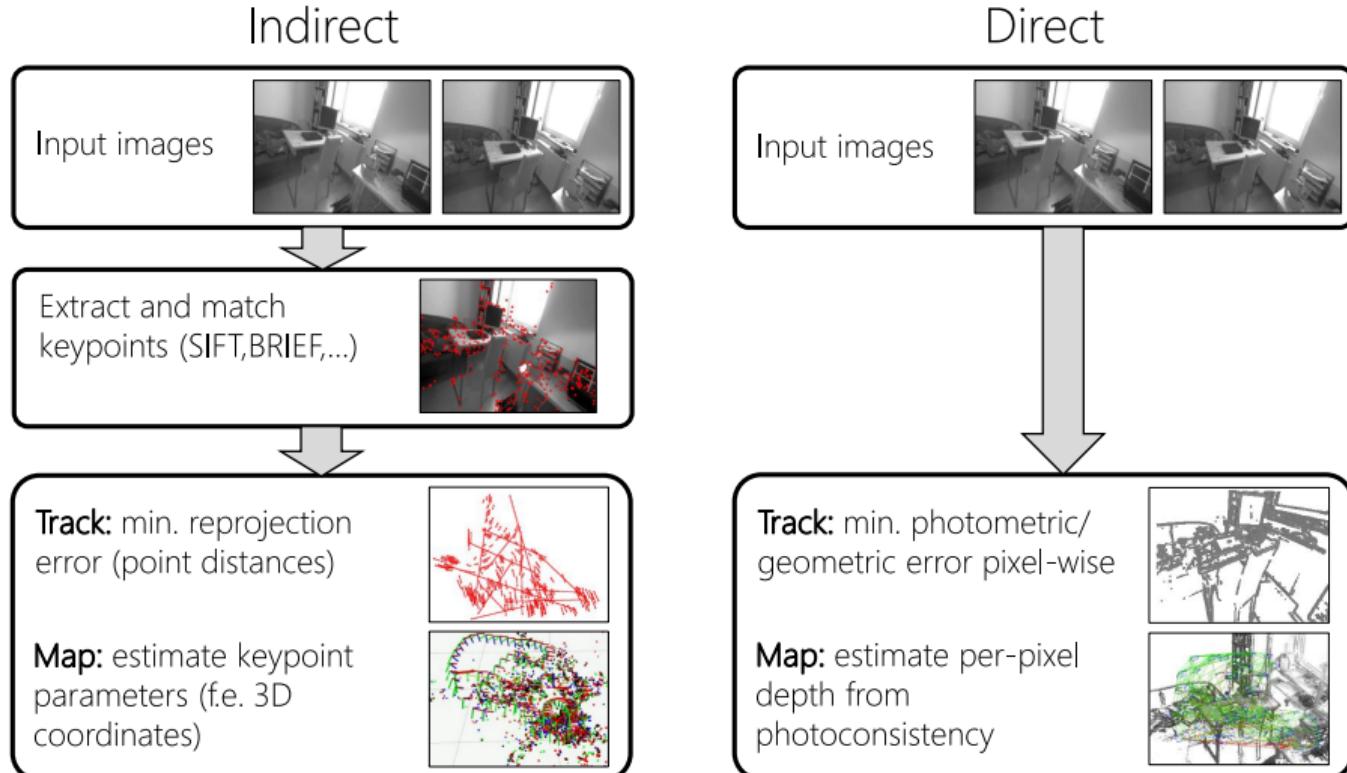
# Visual Odometry



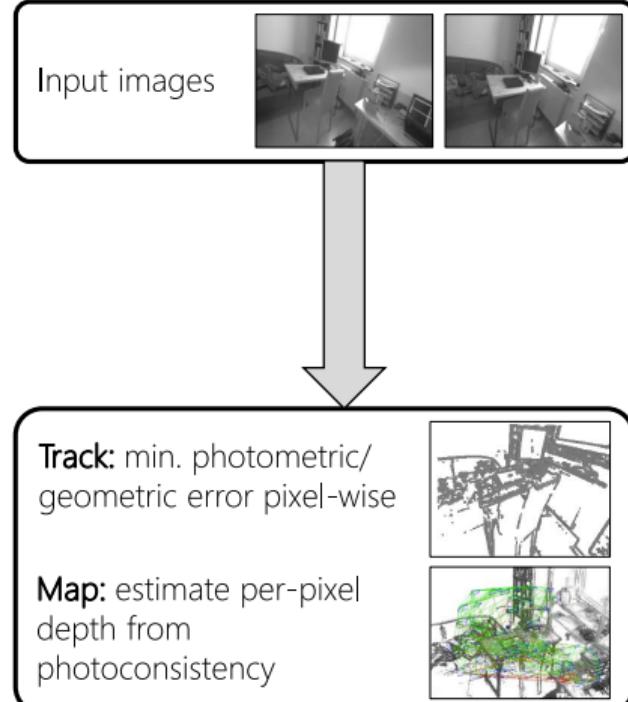
# Visual Odometry



# Indirect vs. Direct Methods



Direct



# Indirect Visual Odometry

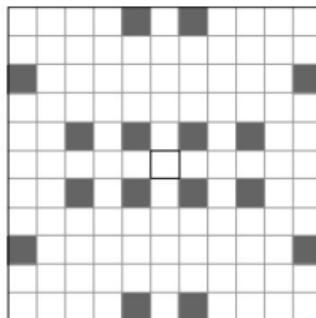
# Extract and Match Keypoints

$$\begin{array}{cccccc} -1 & -1 & -1 & -1 & -1 \\ -1 & +1 & +1 & +1 & -1 \\ -1 & +1 & +8 & +1 & -1 \\ -1 & +1 & +1 & +1 & -1 \\ -1 & -1 & -1 & -1 & -1 \end{array}$$

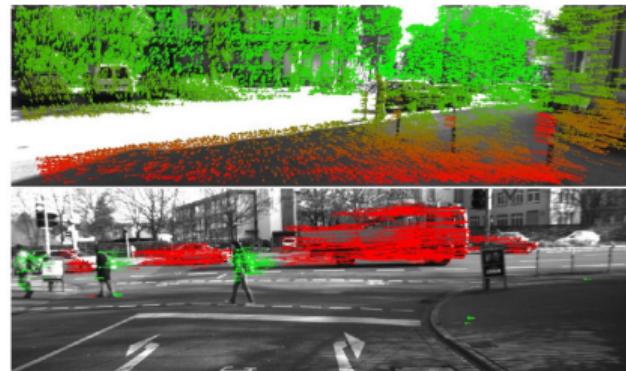
(a) Blob detector

$$\begin{array}{ccccc} -1 & -1 & 0 & +1 & +1 \\ -1 & -1 & 0 & +1 & +1 \\ 0 & 0 & 0 & 0 & 0 \\ +1 & +1 & 0 & -1 & -1 \\ +1 & +1 & 0 & -1 & -1 \end{array}$$

(b) Corner detector



(c) Feature descriptor



- ▶ Detect **salient points** in the image (blobs, corners) and extract **local features**
- ▶ Features should be invariant to perspective and illumination changes
- ▶ Many options (some faster than others): SIFT, SURF, U-SURF, BRISK, ORB, FAST
- ▶ **Match features** between two images by their similarity ⇒ correspondences

# Image Formation

## 2D Points

**2D points** can be written in **inhomogeneous coordinates** as

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2$$

or in **homogeneous coordinates** as

$$\tilde{\mathbf{x}} = \begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{w} \end{pmatrix} \in \mathbb{P}^2$$

where  $\mathbb{P}^2 = \mathbb{R}^3 \setminus \{(0, 0, 0)\}$  is called **projective space**.

**Remark:** Homogeneous vectors that differ only by scale are considered equivalent and define an equivalence class.  $\Rightarrow$  Homogeneous vectors are defined only up to scale.

## 2D Points

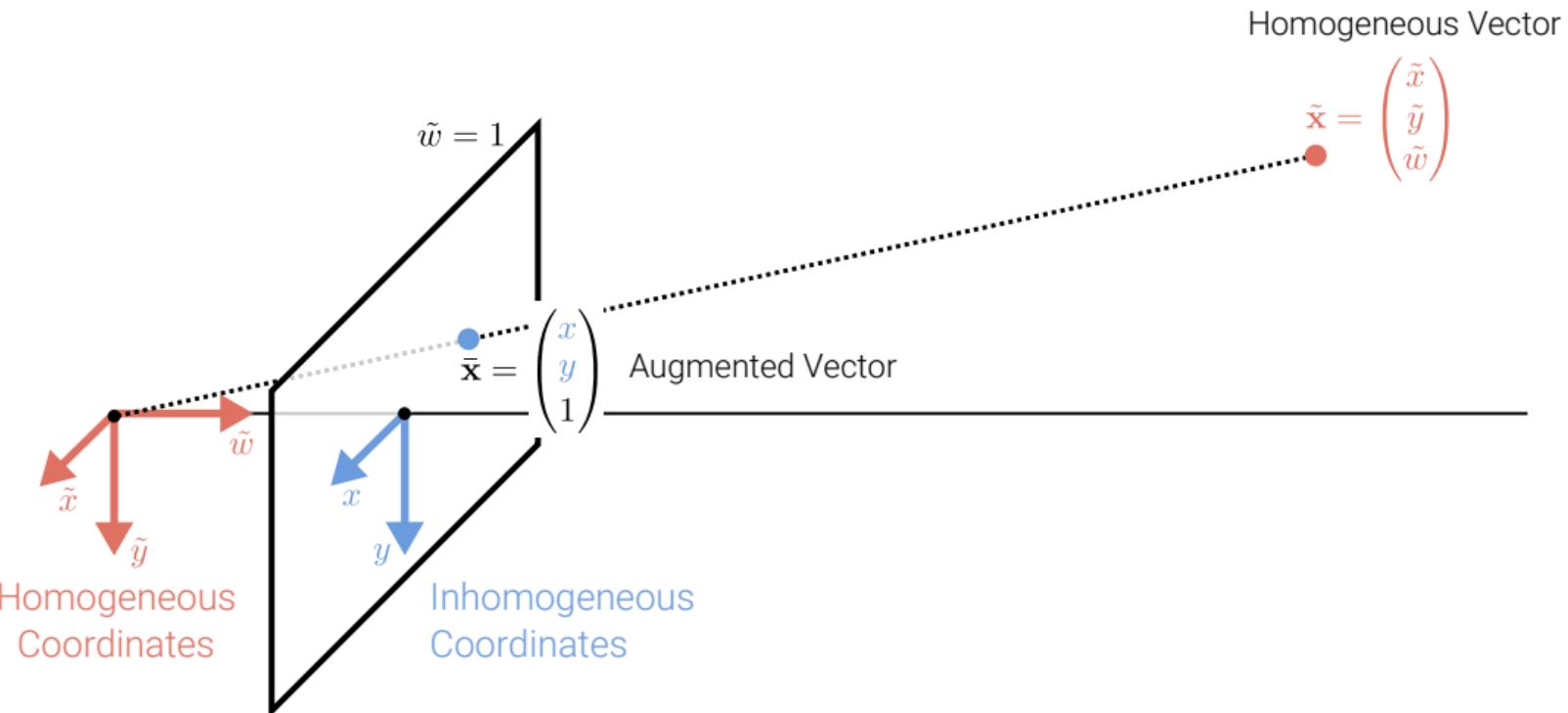
An **inhomogeneous vector**  $\mathbf{x}$  is converted to a **homogeneous vector**  $\tilde{\mathbf{x}}$  as follows

$$\tilde{\mathbf{x}} = \begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{w} \end{pmatrix} = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} \mathbf{x} \\ 1 \end{pmatrix} = \bar{\mathbf{x}}$$

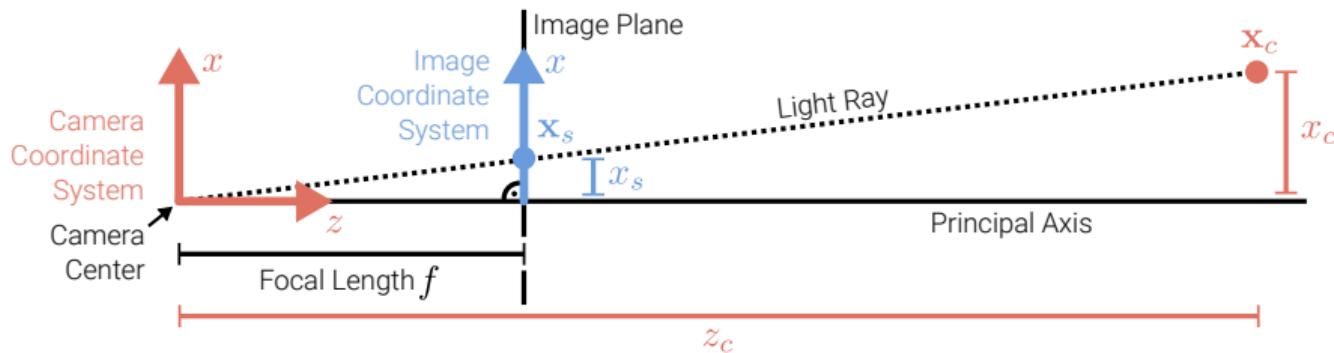
with **augmented vector**  $\bar{\mathbf{x}}$ . To convert in the opposite direction we divide by  $\tilde{w}$ :

$$\bar{\mathbf{x}} = \begin{pmatrix} \mathbf{x} \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \frac{1}{\tilde{w}} \tilde{\mathbf{x}} = \frac{1}{\tilde{w}} \begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{w} \end{pmatrix} = \begin{pmatrix} \tilde{x}/\tilde{w} \\ \tilde{y}/\tilde{w} \\ 1 \end{pmatrix}$$

# 2D Points



# Perspective Projection



$$\frac{x_s}{f} = \frac{x_c}{z_c}$$

**Perspective projection** of a 3D point  $\mathbf{x}_c \in \mathbb{R}^3$  to pixel coordinates  $\mathbf{x}_s \in \mathbb{R}^2$ :

- ▶ The light ray passes through the camera center, the pixel  $\mathbf{x}_s$  and the point  $\mathbf{x}_c$
- ▶ Convention: the principal axis (orthogonal to image plane) aligns with the z-axis
- ▶ Remark: the y coordinate is not shown here for clarity, but behaves similarly

# Perspective Projection

In **perspective projection**, 3D points in camera coordinates are mapped to the image plane by **dividing** them **by their z component** and multiplying with the focal length:

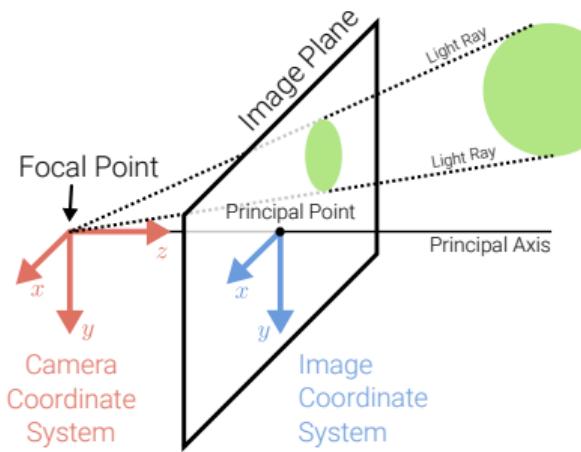
$$\begin{pmatrix} x_s \\ y_s \end{pmatrix} = \begin{pmatrix} fx_c/z_c \\ fy_c/z_c \end{pmatrix} \Leftrightarrow \tilde{\mathbf{x}}_s = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \bar{\mathbf{x}}_c$$

Note that this projection is **linear** when using **homogeneous coordinates**.

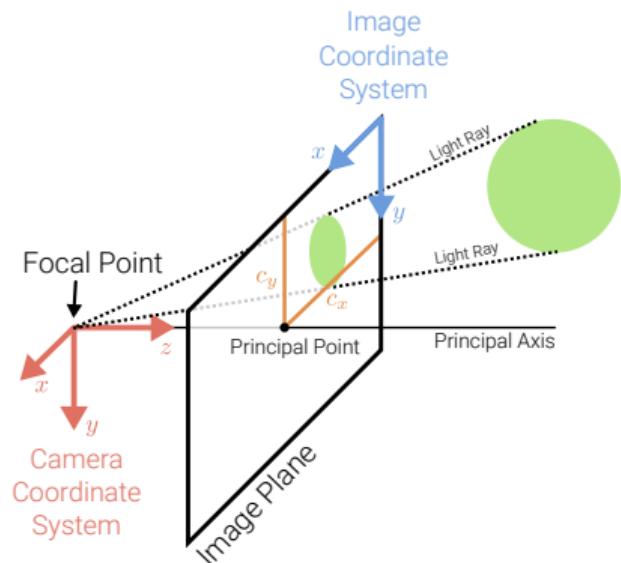
Remark: The unit for  $f$  is px (=pixels) to convert metric 3D points into pixels.

# Perspective Projection

Without Principal Point Offset



With Principal Point Offset



- To ensure positive pixel coordinates, a **principal point offset**  $c$  is usually added
- This moves the image coordinate system to the corner of the image plane

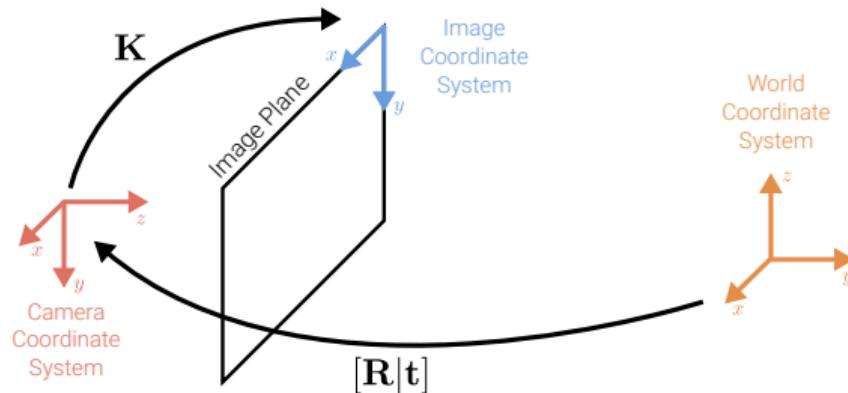
# Perspective Projection

The **complete perspective projection model** is given by:

$$\begin{pmatrix} x_s \\ y_s \end{pmatrix} = \begin{pmatrix} f_x x_c/z_c + s y_c/z_c + c_x \\ f_y y_c/z_c + c_y \end{pmatrix} \Leftrightarrow \tilde{\mathbf{x}}_s = \begin{bmatrix} f_x & s & c_x & 0 \\ 0 & f_y & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \bar{\mathbf{x}}_c$$

- ▶ The left  $3 \times 3$  submatrix of the projection matrix is called **calibration matrix  $\mathbf{K}$**
- ▶ The parameters of  $\mathbf{K}$  are called **camera intrinsics** (as opposed to extrinsic pose)
- ▶ Here,  $f_x$  and  $f_y$  are independent, allowing for different pixel aspect ratios
- ▶ The skew  $s$  arises due to the sensor not mounted perpendicular to the optical axis
- ▶ In practice, we often set  $f_x = f_y$  and  $s = 0$ , but model  $\mathbf{c} = (c_x, c_y)^\top$

# Chaining Transformations



Let  $\mathbf{K}$  be the calibration matrix (intrinsics) and  $[\mathbf{R}|\mathbf{t}]$  the camera pose (extrinsics). We **chain both transformations** to project a point in world coordinates to the image:

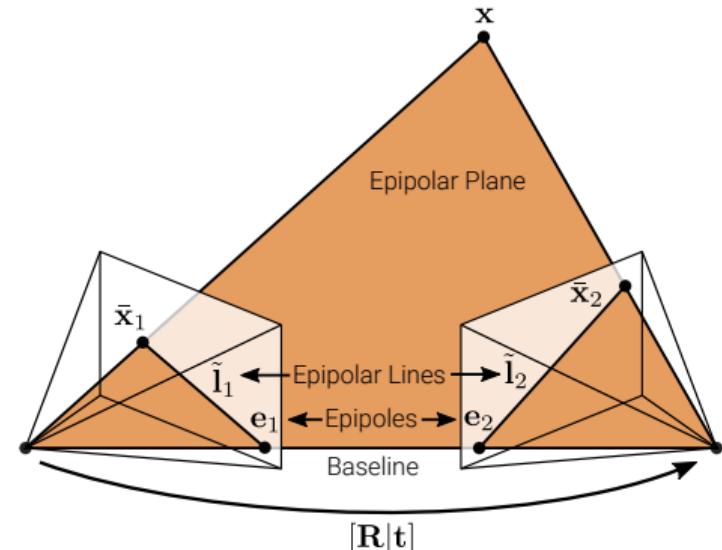
$$\tilde{\mathbf{x}}_s = \begin{bmatrix} \mathbf{K} & \mathbf{0} \end{bmatrix} \bar{\mathbf{x}}_c = \begin{bmatrix} \mathbf{K} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix} \bar{\mathbf{x}}_w = \mathbf{K} [\mathbf{R} \quad \mathbf{t}] \bar{\mathbf{x}}_w = \mathbf{P} \bar{\mathbf{x}}_w$$

# Epipolar Geometry

# Epipolar Geometry

**Goal:** Recovery of camera pose (and 3D structure) from image correspondences.  
The required relationships are described by the two-view **epipolar geometry**.

- ▶ Let  $\mathbf{R}$  and  $\mathbf{t}$  denote the relative pose between **two perspective cameras**
- ▶ A 3D point  $\mathbf{x}$  is projected to pixel  $\bar{\mathbf{x}}_1$  in image 1 and to pixel  $\bar{\mathbf{x}}_2$  in image 2
- ▶ The 3D point  $\mathbf{x}$  and the two camera centers span the **epipolar plane**
- ▶ The correspondence of pixel  $\bar{\mathbf{x}}_1$  in image 2 must lie on the **epipolar line**  $\tilde{\mathbf{l}}_2$  in image 2
- ▶ All epipolar lines pass through the **epipole**



# Epipolar Geometry

The **epipolar constraint** is given by

$$\bar{\mathbf{x}}_2^\top \tilde{\mathbf{E}} \bar{\mathbf{x}}_1 = 0$$

with **essential matrix**:

$$\tilde{\mathbf{E}} = [\mathbf{t}]_\times \mathbf{R}$$

Let  $\bar{\mathbf{x}}_1 = (x_1 \ y_1 \ 1)^\top$  and  $\bar{\mathbf{x}}_2 = (x_2 \ y_2 \ 1)^\top$ . This allows us to rewrite the epipolar constraint in terms of the elements of the essential matrix as follows ..

# Estimating the Epipolar Geometry

We can **recover the essential matrix  $\tilde{\mathbf{E}}$**  from  $N$  **image correspondences** forming  $N$  homogeneous equations in the nine elements of  $\tilde{\mathbf{E}}$ :

$$\begin{aligned}x_1 x_2 e_{11} &+ y_1 x_2 e_{12} + x_2 e_{13} \\x_1 y_2 e_{21} &+ y_1 y_2 e_{22} + y_2 e_{23} \\x_1 e_{31} &+ y_1 e_{32} + e_{33} = 0\end{aligned}$$

As  $\tilde{\mathbf{E}}$  is homogeneous we use **singular value decomposition** to constrain the scale.

Note that some terms are products of two image measurements and hence amplify measurement noise asymmetrically. Thus, the **normalized 8-point algorithm** whitens the observations to have zero-mean and unit variance before the calculation and back-transforms the matrix recovered by SVD accordingly.

# Estimating the Epipolar Geometry

From  $\tilde{\mathbf{E}}$ , we can recover the **direction**  $\hat{\mathbf{t}}$  of the **translation** vector  $\mathbf{t}$ . We have:

$$\hat{\mathbf{t}}^\top \tilde{\mathbf{E}} = \hat{\mathbf{t}}^\top [\mathbf{t}]_\times \mathbf{R} = \mathbf{0}$$

Thus,  $\tilde{\mathbf{E}}$  is singular and we obtain  $\hat{\mathbf{t}}$  as the left singular vector associated with singular value 0. In practice the singular value will not be exactly 0 due to measurement noise, and we choose the smallest one. The other two singular values are roughly equal.

The rotation matrix  $\mathbf{R}$  can also be calculated, see Szeliski, Section 11.3 (p. 683).

Remark: The essential matrix has 5 DoF (3 for rotation  $\mathbf{R}$ , 2 for translation direction  $\hat{\mathbf{t}}$ ). In other words, the **global scale cannot be determined** using monocular VO.

# Non-linear Optimization

The rigid body motion ( $\mathbf{R}, \mathbf{t}$ ) can be further refined via **non-linear optimization**.

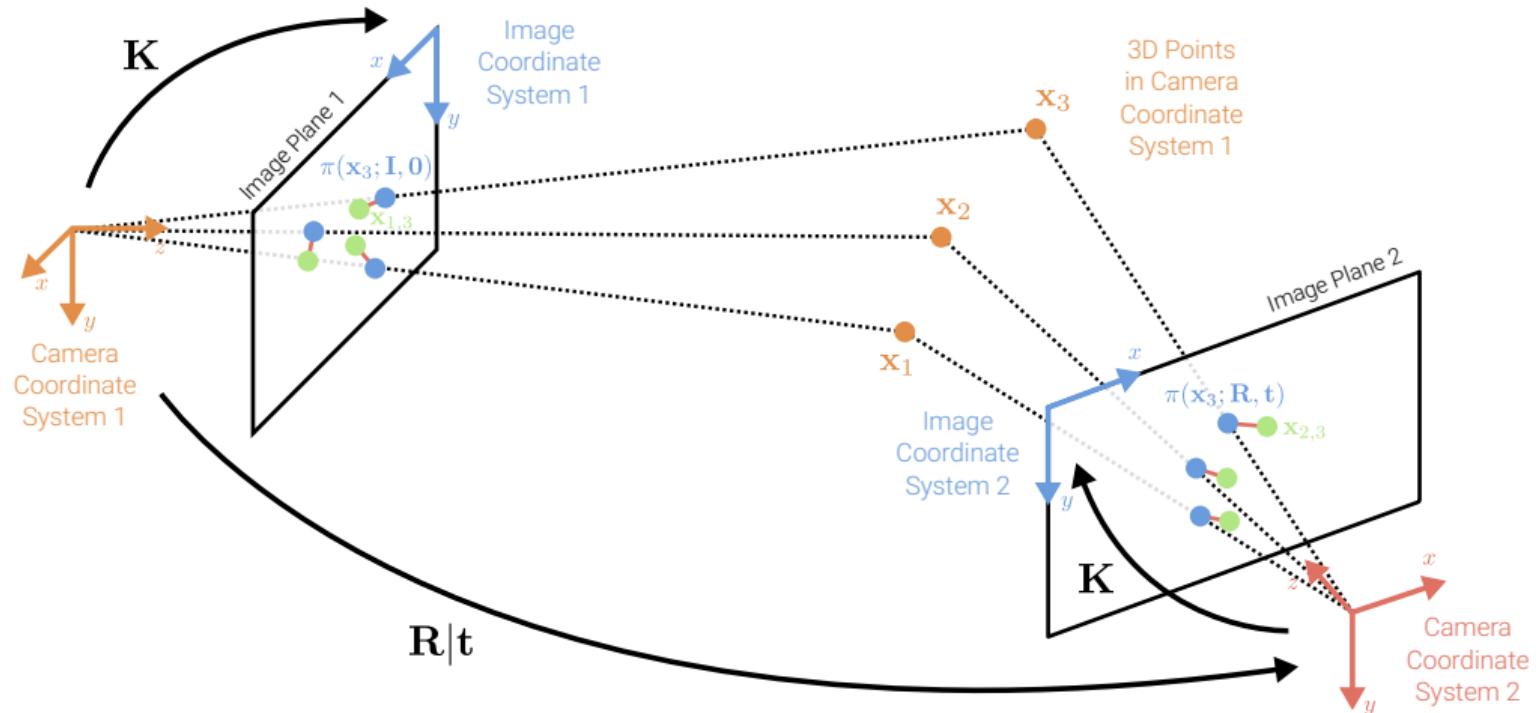
Let  $\pi(\mathbf{x}_i, \mathbf{R}, \mathbf{t})$  denote the projection of the  $i$ 'th 3D point onto the 2D image plane and let  $(\mathbf{x}_{1,i}, \mathbf{x}_{2,i})$  denote the 2D feature correspondences in the first and second frame.

Minimize the **reprojection error** of all correspondences wrt. motion and 3D location:

$$\mathbf{R}^*, \mathbf{t}^*(, \mathbf{X}^*) = \operatorname{argmin}_{\mathbf{R}, \mathbf{t}(, \mathbf{X})} \sum_{i=1}^N \|\mathbf{x}_{1,i} - \pi(\mathbf{x}_i; \mathbf{I}, \mathbf{0})\|_2^2 + \|\mathbf{x}_{2,i} - \pi(\mathbf{x}_i; \mathbf{R}, \mathbf{t})\|_2^2$$

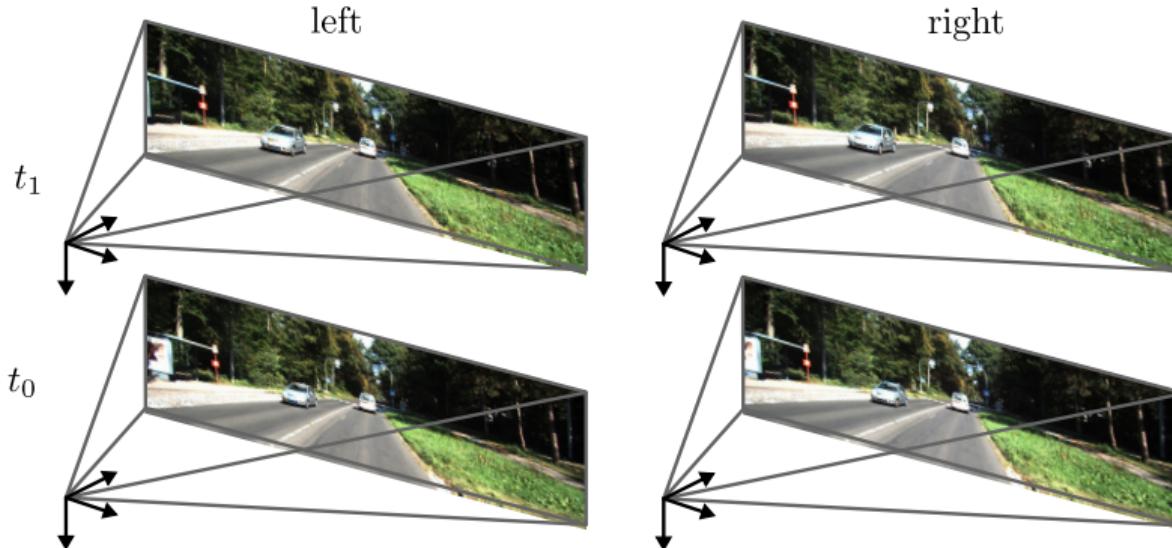
- ▶ Optimizing reprojection errors leads to better results (better noise model)
- ▶ Triangulation can be used to initialize  $\mathbf{X} = \{\mathbf{x}_i\}$ ; speed-up by not optimizing  $\mathbf{X}$
- ▶ Outliers must be removed to not affect the estimate (e.g., using RANSAC)
- ▶ For monocular VO, the length of the translation vector cannot be determined
- ▶ This ambiguity can be resolved by using stereo images or wheel odometry

# Non-linear Optimization



- ▶ Use off-the-shelf **non-linear least squares** solver, e.g., Levenberg-Marquardt

# Stereo Visual Odometry

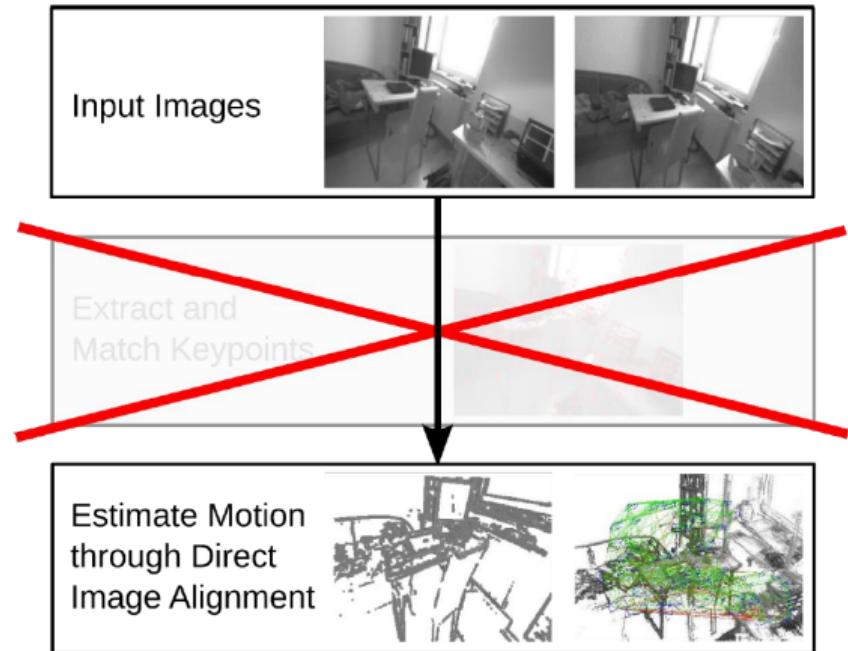


- ▶ If a **stereo camera** is available we can optimize with respect to all 4 images
- ▶ If the stereo camera is calibrated, we can estimate **global scale** (i.e.,  $\|t\|$ )
- ▶ Furthermore, stereo measurements allow for instantaneous triangulation

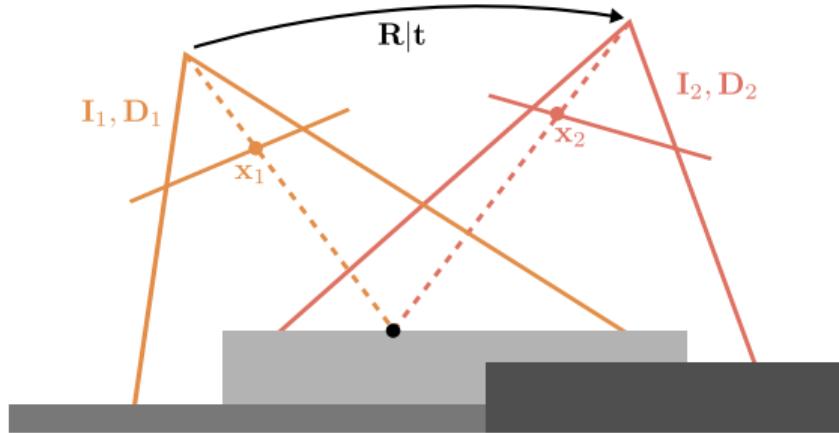
# Direct Visual Odometry

# Direct Visual Odometry

- ▶ Avoid hand-crafted keypoint detection, description and matching
- ▶ Instead: **direct image alignment** based on all image pixels
- ▶ Depth from sensor (RGB-D, Lidar) or through multi-view stereo



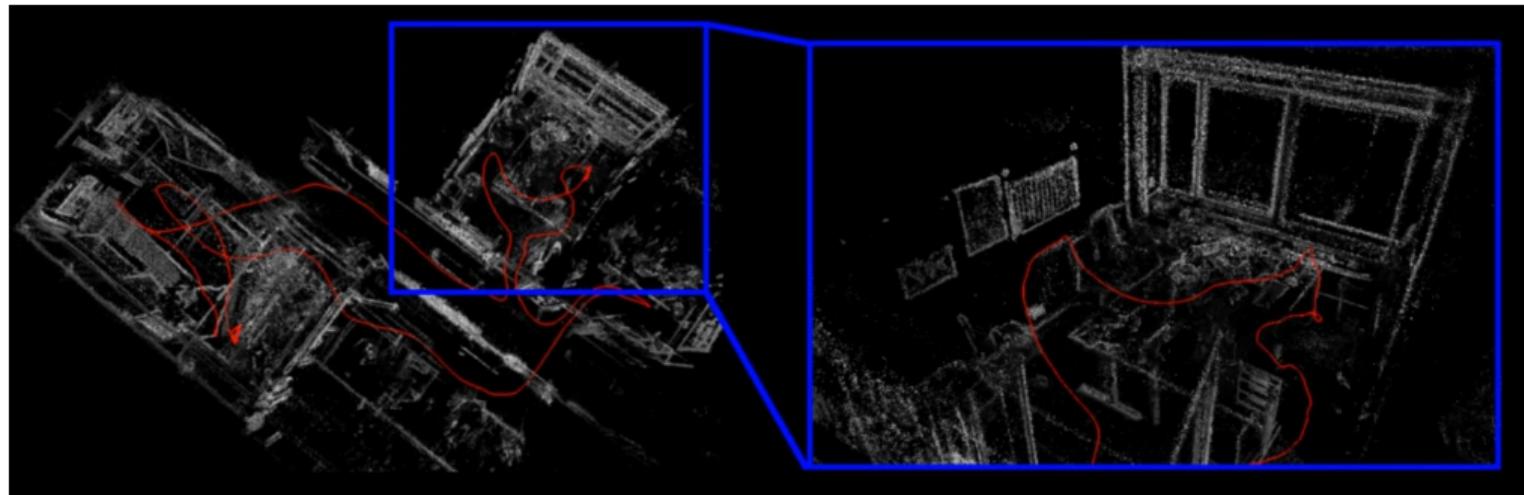
# Direct Visual Odometry



- If we know per-pixel depth, we can **simulate** an image from a different view point
- Ideally, image  $\mathbf{I}_2$  **warped** into the first camera's image plane is equal to image  $\mathbf{I}_1$ :

$$\mathbf{R}^*, \mathbf{t}^*, \mathbf{D}_1^* = \operatorname{argmin}_{\mathbf{R}, \mathbf{t}, \mathbf{D}_1} \sum_{\mathbf{x} \in \Omega} \|\mathbf{I}_1(\mathbf{x}) - \mathbf{I}_2(\pi(\mathbf{x}; \mathbf{R}, \mathbf{t}, \mathbf{D}_1))\|_2^2$$

# Direct Visual Odometry



- ▶ Direct VO methods often consider only parts of the image (e.g., non-saturated)
- ▶ Direct methods can also be combined with feature-based (indirect) methods

# Indirect vs. Direct Methods

## Some Remarks:

- ▶ Direct methods often lead to more accurate results as they exploit the full image
- ▶ However, satisfying real-time requirements requires efficient implementations
- ▶ Furthermore, they suffer from local minima and require accurate initialization
- ▶ Feature-based methods are faster but in general less accurate
- ▶ This is because often only few reliable feature correspondences can be found
- ▶ However, they are more robust to initialization and less prone to local minima
- ▶ Thus, often both techniques are combined (e.g., using features to initialize pose)



# Simultaneous Localization and Mapping (SLAM)

- ▶ So far: Optimization of 2 adjacent frames, no focus on map
- ▶ Now: optimize over larger windows (ideally: entire history)
- ▶ We optimize both poses and map (e.g., 3D feature locations)
- ▶ SLAM is a chicken-egg problem (localization requires mapping and vice-versa) ⇒ Joint optimization of poses and map is necessary
- ▶ Key feature of SLAM: Correct accumulation errors via loop-closure detection
- ▶ The resulting map can be used for localization
- ▶ There exist indirect (feature-based) and direct SLAM methods
- ▶ Many flavors: EKF SLAM, Bundle adjustment, Windowed BA, ...
- ▶ We will cover feature-based SLAM via bundle adjustment

# Feature-based SLAM



- ▶ **Goal: Optimize reprojection errors** (distance between observed feature and projected 3D point in image plane) **wrt. camera parameters and 3D point cloud**

# Bundle Adjustment

Let  $\Pi = \{\pi_i\}$  denote the  $N$  cameras including their intrinsic and extrinsic parameters.

Let  $\mathcal{X}_w = \{\mathbf{x}_p^w\}$  with  $\mathbf{x}_p^w \in \mathbb{R}^3$  denote the set of  $P$  3D points in world coordinates.

Let  $\mathcal{X}_s = \{\mathbf{x}_{ip}^s\}$  with  $\mathbf{x}_{ip}^s \in \mathbb{R}^2$  denote the image (screen) observations in all  $i$  cameras.

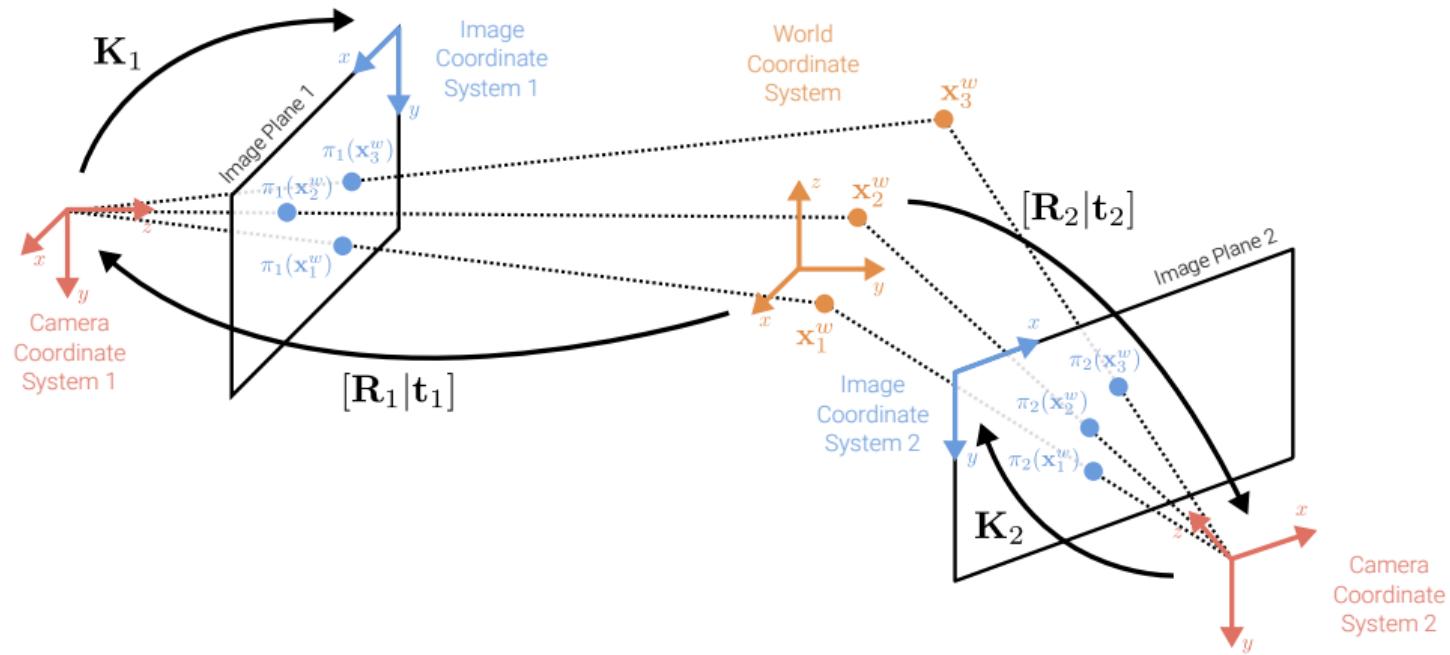
**Bundle adjustment** minimizes the reprojection error of all observations:

$$\Pi^*, \mathcal{X}_w^* = \underset{\Pi, \mathcal{X}_w}{\operatorname{argmin}} \sum_{i=1}^N \sum_{p=1}^P w_{ip} \|\mathbf{x}_{ip}^s - \pi_i(\mathbf{x}_p^w)\|_2^2$$

Here,  $w_{ip}$  indicates if point  $p$  is observed in image  $i$  and  $\pi_i(\mathbf{x}_p^w)$  is the 3D-to-2D projection of 3D world point  $\mathbf{x}_p^w$  onto the 2D image plane of the  $i$ 'th camera, i.e.:

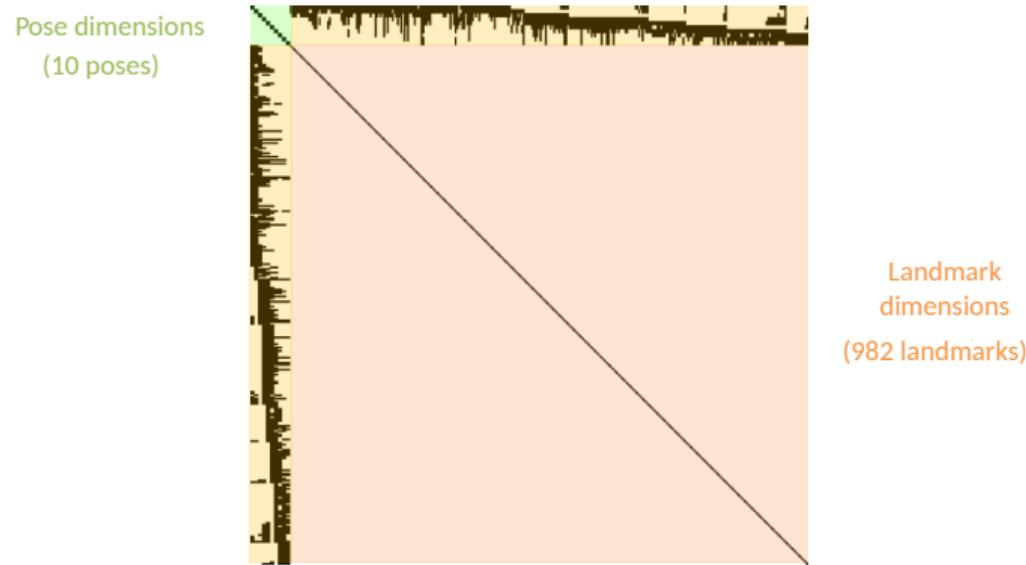
$$\pi_i(\mathbf{x}_p^w) = \begin{pmatrix} \tilde{x}_p^s / \tilde{w}_p^s \\ \tilde{y}_p^s / \tilde{w}_p^s \end{pmatrix} \quad \text{with} \quad \tilde{\mathbf{x}}_p^s = \mathbf{K}_i(\mathbf{R}_i \mathbf{x}_p^w + \mathbf{t}_i)$$

# Bundle Adjustment



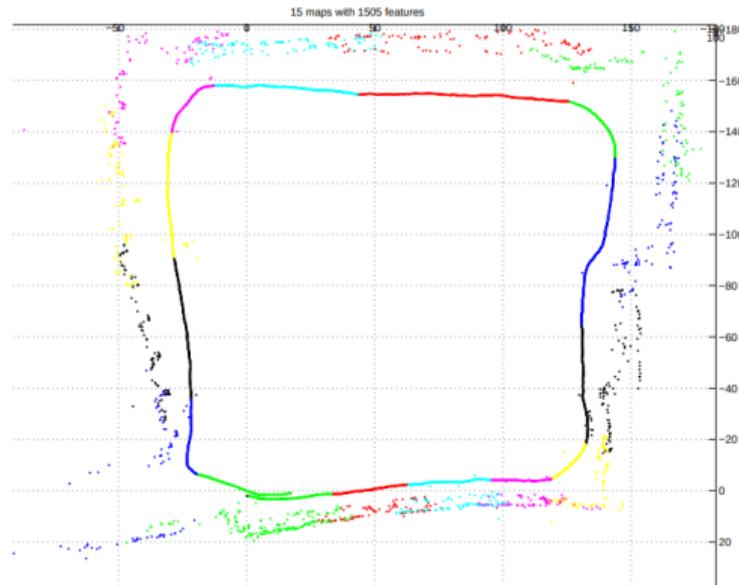
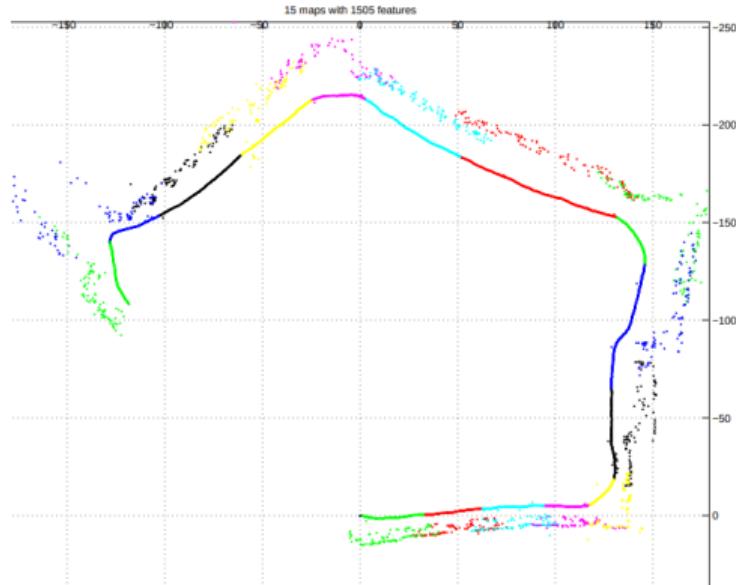
$\mathbf{K}_i$  and  $[\mathbf{R}_i|\mathbf{t}_i]$  are the intrinsic and extrinsic parameters of  $\pi_i$ , respectively. During bundle adjustment, we optimize  $\{(\mathbf{K}_i, \mathbf{R}_i, \mathbf{t}_i)\}$  and  $\{\mathbf{x}_p^w\}$  jointly.

# Exploiting Sparsity



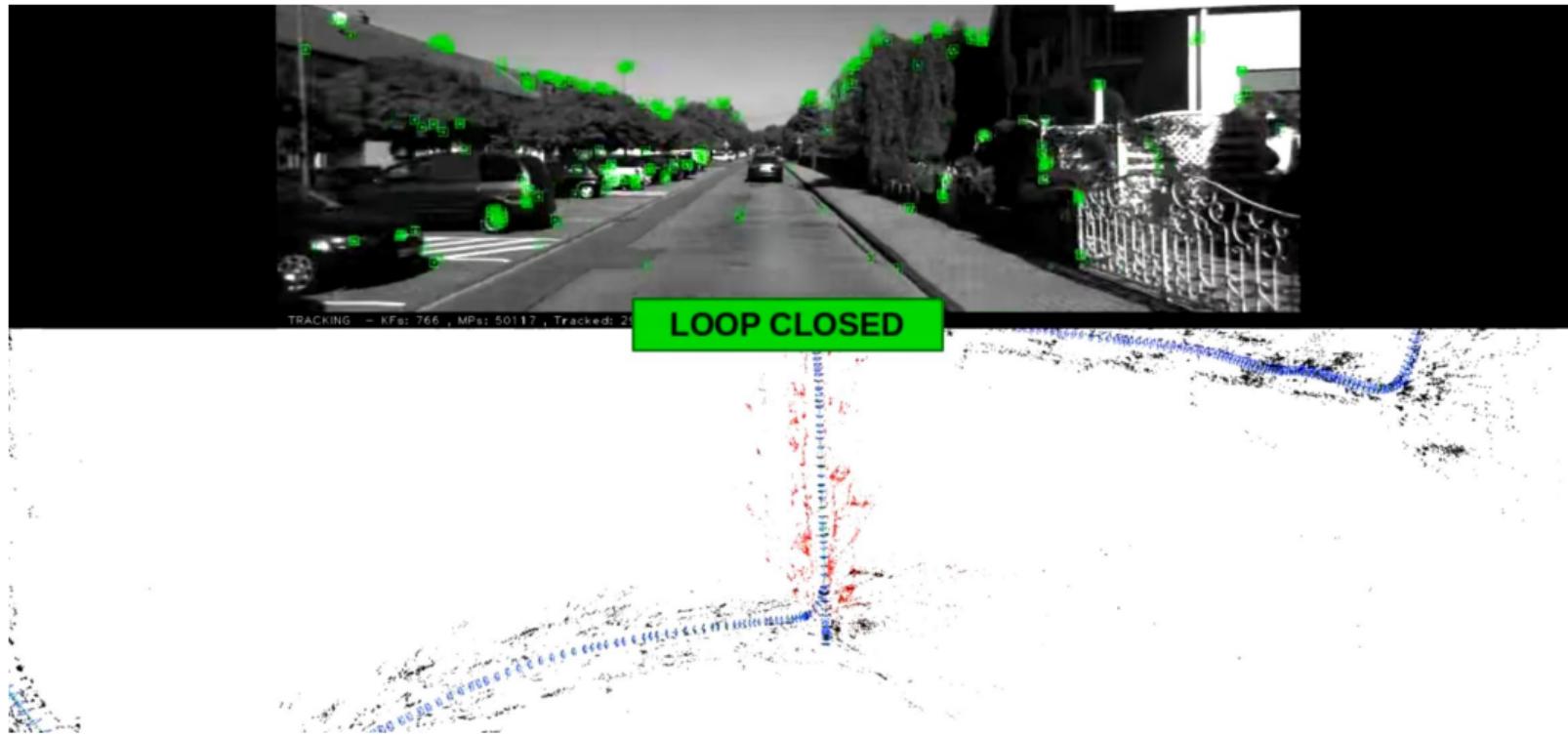
- ▶ The Hessian is typically **sparse** as landmarks don't interact with each other
- ▶ This leads to tremendous computational benefits when using sparse optimizers

# Loop Closure Detection



- ▶ Find **correspondences** between the current frame and all previous frames
- ▶ Use these correspondences as **additional constraints** during optimization

# ORB SLAM



# Indirect SLAM Methods

MonoSLAM [1]	PTAM [2]	ORB-SLAM 2 [3]
+ monocular cameras	+ monocular cameras	+ monocular cameras + stereo cameras + RGB-D cameras
- no global consistency	- no global consistency	+ global consistency
Extended Kalman filtering of camera pose and point coordinates. Includes a motion model	Mapping as BA over keyframes, real-time tracking towards keyframe	Mapping as BA over keyframes, real-time tracking towards keyframe, loop-closing via place recognition
Filtering	Bundle adjustment	Bundle adjustment
- local accuracy -- global accuracy	+ local accuracy - global accuracy	+ local accuracy ++ global accuracy

# Direct SLAM Methods

DVO-SLAM [4]	LSD-SLAM [5]	DSO [6]
+ RGB-D cameras	+ monocular cameras + stereo cameras	+ monocular cameras + stereo cameras
+ global consistency	+ global consistency	- no global consistency
camera pose tracking towards keyframe	camera pose tracking towards keyframe	camera pose tracking towards keyframe, camera pose optimization in local keyframe window
+ depth from sensor	+ depth from stereo comparisons & filtering	++ depth optimization in local keyframe window
tracking-only & pose graph optimization	tracking-and-mapping & pose graph optimization	tracking-and-mapping & direct sparse bundle adjustment in local keyframe window with marginalization
+ local accuracy	+ local accuracy	++ local accuracy

# 7.3

## Localization

# Vehicle Localization

## Motivation:

- ▶ Control wrt. a path requires knowledge about the position of the vehicle
- ▶ Sometimes local knowledge is sufficient (lateral position on highway)
- ▶ But often the global location is required (path planning, determining relevant elements that have been marked in a map such as street signs or lane markings)

## Localization Approaches:

- ▶ Satellite Localization (uses **infrastructure**)
- ▶ Visual Localization (uses **visual map**)
  - ▶ Topometric Localization
  - ▶ Learning-based Localization
  - ▶ Feature-based Localization
- ▶ Map-based Localization (uses **road map**)

# Satellite Localization

# Satellite Localization

## Satellite Systems:

- ▶ GPS: 24 satellites (United States)
  - ▶ First satellite launch: 1978
  - ▶ Full constellation: 1994
  - ▶ Today: 30 satellites
  - ▶ Since 2000 full accuracy for all users (not only military users)
- ▶ GLONASS: 24 satellites (Russia)
- ▶ Galileo: 30 satellites (Europe)
- ▶ BeiDou-2: 30 satellites (China)



GPS: 4-5 satellites per orbital plane

# Satellite Localization

## **Trilateration:**

- ▶ Determine location by measuring distances to satellites at known locations using the geometry of circles and spheres
- ▶ Distance to each satellite is measured as time delay between sent signal/code and received signal/code (receiver knows code!)
- ▶ Satellites equipped with onboard atomic clock
- ▶ Given one satellite, we know which sphere we are on
- ▶ Given two satellites, we know which circle we are on
- ▶ Given three satellites, we know the exact location
- ▶ In practice, a fourth satellite is used for time synchronization
- ▶ Differential GPS: Improve measurements using ground stations at fixed location

# Satellite Localization

## **Problems with Satellite Localization:**

- ▶ Availability
  - ▶ Satellites not visible in tunnels, narrow city streets
  - ▶ Dependency on national organizations / interests
- ▶ Accuracy
  - ▶ 5m - 15m for GPS and 0.5m-5m for DGPS
  - ▶ Only location, no rotation (not full pose)
- ▶ Frequency: max. 5-10 Hz
- ▶ Atmospheric variations (lead to errors)
  - ▶ Can be partially corrected by DGPS using signal from nearby base station
- ▶ DGPS requires communication and initialization
- ▶ Multipath effects (wrong signals due to reflections, e.g., at facades)

# Satellite Localization



[Danny Ilанд, Uber]

# Visual Localization

# Visual Localization

## Main Idea:

- ▶ Record map/database of known locations with associated features
- ▶ Features can be extracted from images or laser scans
- ▶ Mapping should be conducted under similar conditions as during localization  
(minimize domain shift wrt. sensor setup and environmental conditions)
- ▶ At localization time, try to retrieve features extracted from the current input image/scan in the map/database
- ▶ Either localize only at the image level or refine using triangulation or geometric pose estimation

# Challenges



## Geometry changes



## Appearance changes



# Visual Localization: Topometric Localization

# Topometric Localization



- ▶ Setup: Record sequence with cameras, Lidar and GPS as ground truth
- ▶ Goal: Localize new image (different day/season) wrt. recorded sequence
- ▶ Combination of metric and topological localization:
  - ▶ Metric: Optimize pose wrt. feature correspondences (discussed later in the lecture)
  - ▶ Topological: Estimate observer location qualitatively from finite set of locations
  - ▶ Here: Describe image with global feature vector; align query with recorded sequence
  - ▶ However, only localize wrt. node (=frame) in graph (no full 6D pose)

# Topometric Localization

## Map Creation:

- ▶ Directed graph, nodes (=frames) created based on distance threshold
- ▶ Features: SURF applied to whole image (64D) + average/std.dev. of range scan

## Bayesian Localization:

- ▶ Predict:  $P(x_t|z_{1:t-1}) = \sum_{x_{t-1}} P(x_t|x_{t-1})P(x_{t-1}|z_{1:t-1})$
- ▶ Update:  $P(x_t|z_{1:t}) \propto P(z_t|x_t)P(x_t|z_{1:t-1})$
- ▶ With:
  - ▶ Motion model  $P(x_t|x_{t-1}) = \mathcal{N}(x_t|x_{t-1} + v_t\Delta t, \sigma^2)$
  - ▶ Measurement model  $P(z_t|x_t) = \chi(\|z_t - d(x_t)\|, k)$
  - ▶ Location  $x_t \in \mathbb{R}$ , velocity  $v_t \in \mathbb{R}$ , observation  $z_t$ , map feature  $d(x_t)$  at location  $x_t$
  - ▶ Note: strictly speaking this formulation handles tracking, not localization!

# Topometric Localization



# Visual Localization: Learning-based Localization

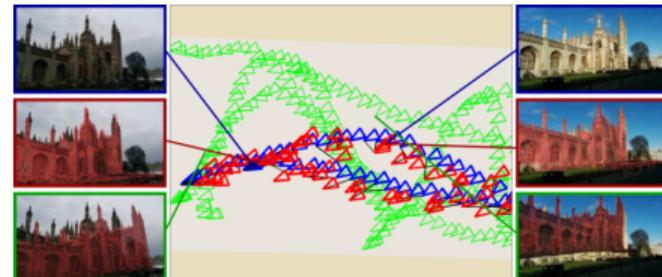
# Learning-based Localization

## PoseNet:

- ▶ Input: Single RGB image ( $224^2$  Px)
- ▶ Output: 6 DOF camera pose  $\mathbf{p} = (\mathbf{x}, \mathbf{q})$ 
  - ▶  $\mathbf{x}$ : 3D camera position
  - ▶  $\mathbf{q}$ : 3D camera rotation (quaternion)
- ▶ 23 Layer deep convolutional network based on GoogLeNet
- ▶ More robust than feature-based methods
- ▶ Less accurate than feature-based methods

Loss:

$$\mathcal{L} = \|\hat{\mathbf{x}} - \mathbf{x}\|_2 + \beta \left\| \hat{\mathbf{q}} - \frac{\mathbf{q}}{\|\mathbf{q}\|} \right\|_2$$



(green = train, red = pred, blue = GT)

# Learning-based Localization: Results



(b) Relocalization under difficult dusk and night lighting conditions. In the dusk sequences, the landmark is silhouetted against the backdrop however again the convnet seems to recognize the contours and estimate pose.

# Visual Localization

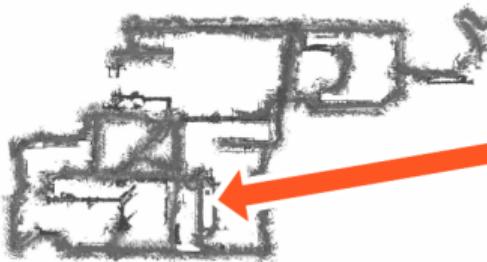
## Feature-based Localization

# Feature-based Localization Overview

Database Images



Database Map

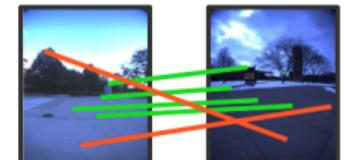


Query Image  
(+Depth Map)



Geometric Verification

- Image-to-image
- Image-to-3D
- 3D-to-3D

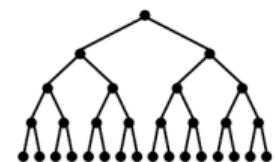


Descriptor Extraction

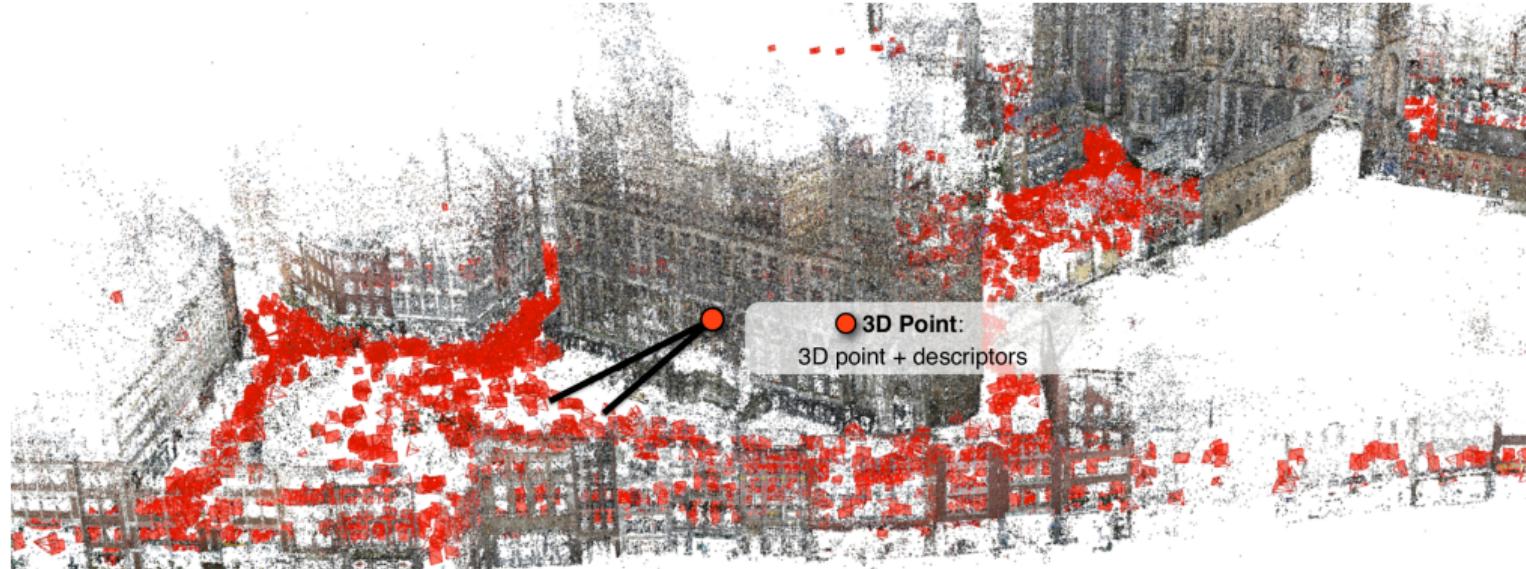
- Image Descriptors  
(SIFT, DeepDesc, ...)
- Shape Descriptors  
(FPFH, 3DMatch, CGF, ...)



Descriptor Matching

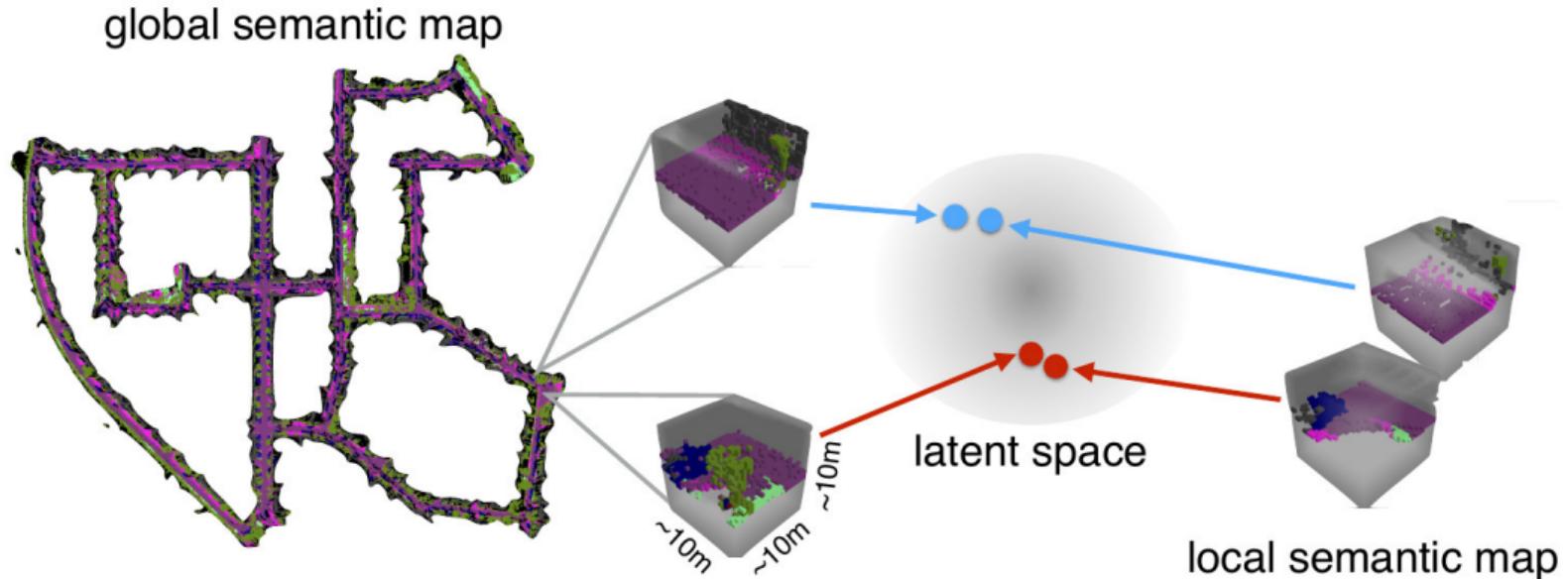


# Mapping



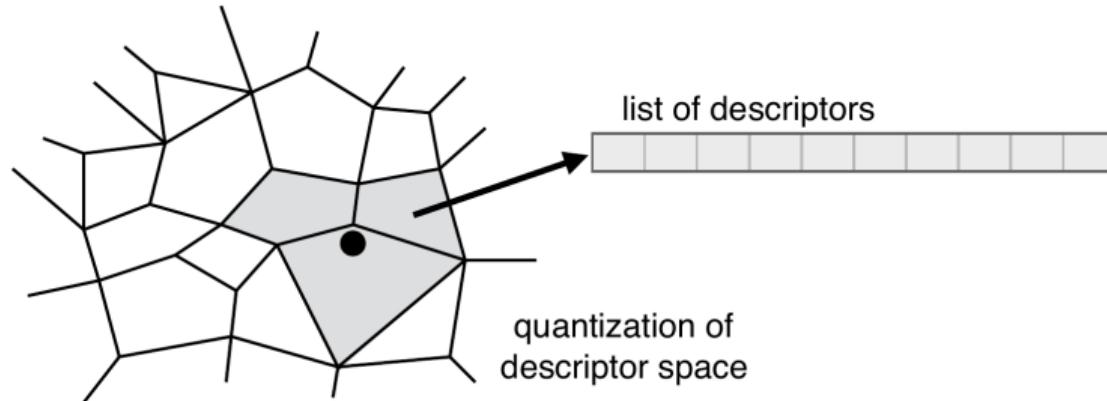
- ▶ Sparse 3D reconstruction based on sparse feature correspondences (SLAM)
- ▶ Associate each 3D point with a local image descriptor (e.g., SIFT, SURF)

# Mapping



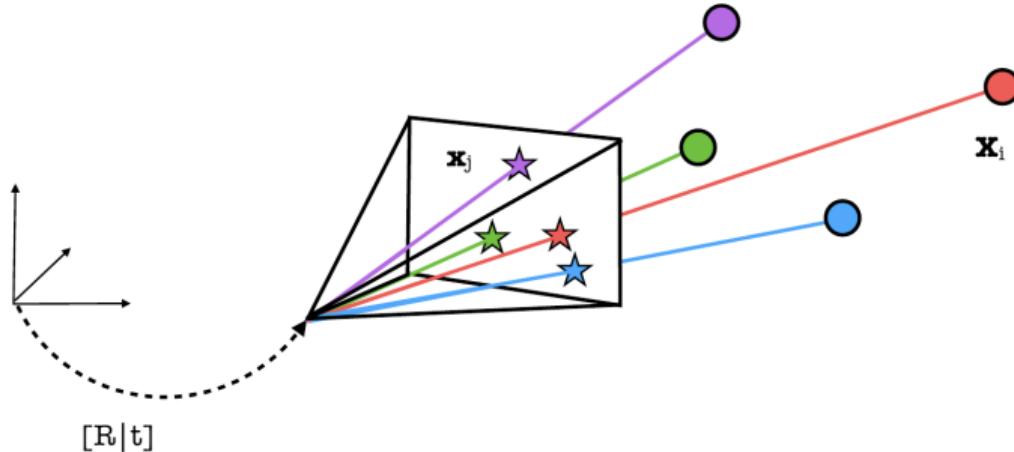
- Semantic viewpoint invariant feature representations can facilitate matching

# Descriptor Matching



- ▶ Search nearest neighbors of features from query image (in descriptor space)
- ▶ Approximate search due to curse of dimensionality
- ▶ Popular approaches: k-d trees, inverted index, ...
- ▶ Good software packages available: FLANN, FAISS, ...

# Pose Estimation



- ▶ Given:  $n$  2D-3D correspondences  $(\mathbf{x}_i, \mathbf{X}_i)$
- ▶ Goal: compute pose  $\mathbf{R}, \mathbf{t}$  such that  $\mathbf{x}_i = \mathbf{K}[\mathbf{R}|\mathbf{t}]\mathbf{X}_i$   
Here,  $\mathbf{K}$ ,  $\mathbf{R}$ ,  $\mathbf{t}$  denote calibration matrix, rotation matrix, and translation vector
- ▶ Very efficient methods ( $< 2\mu s$ ) exist if camera calibration is known ( $\Rightarrow$  PnP)

# Geometric Verification

**While** probability of missing correct model  $>\eta$

    Estimate model from  $n$  random data points

    Estimate support (= #**inliers**) of model     [\[Chum, Matas, Optimal Randomized RANSAC. PAMI 2008\]](#)

    If new best model

**Perform Local Optimization (LO)**

[\[Lebeda, Matas, Chum, Fixing the Locally Optimized RANSAC. BMVC 2012\]](#) [\[code\]](#)

        update best model,  $\eta$

**Return:** Model with most inliers

## RANdom SAMple Consensus (RANSAC):

- ▶ Problem: wrong feature matches corrupt pose estimation result
- ▶ Solution: solve minimal problem many times & return best solution

# Map-based Localization

# Map-based Localization



- ▶ Update location probability distribution on map using VO measurements

## Summary

- ▶ Visual odometry estimates relative ego-motion from images
- ▶ SLAM algorithms build a map and simultaneously localize in that map
- ▶ SLAM algorithms use loop closure detection to close loops
- ▶ Localization methods find the global pose in a given map
- ▶ Indirect and direct VO/SLAM methods exist
- ▶ Indirect methods are faster and converge better, but are less accurate
- ▶ Direct methods are slower but lead to more accurate results
- ▶ Direct and indirect methods can be advantageously combined
- ▶ Except for offline mapping, real-time computation is required