Project: Monte Carlo Simulation

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Work division: Zhao works on the First problem and the final report, Shuyue works on the second and the Second and the Third problems.

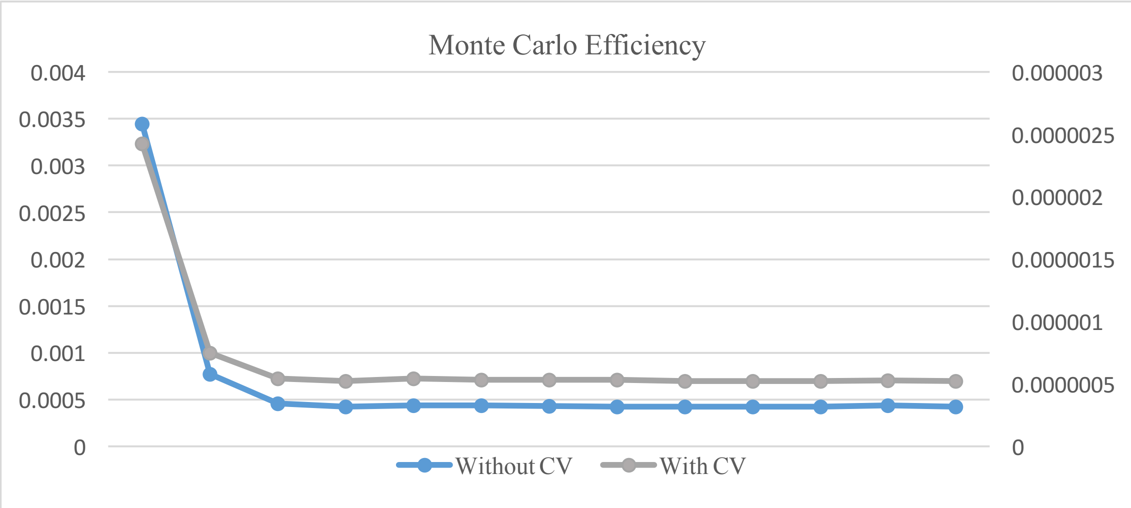
Work to reduce computational time:

1. Use one function and same random number to generate result for both with or without control variate.
2. Use 1000 trials to generate b in the control variate method, hugely increase the efficiency and save a lot of time.
3. Use dynamic array to save storage space
4. Use the algorithm described in lecture notes to calculate mean and standard error

1. Monte Carlo Simulation on Asian call

N	Monte Carlo							
	Price		Standard Error		Efficiency		Time	
	without control	**with control**	**without control**	**with control**	**without control**	**with control**	**without control**	**with control**
100	7.90672	7.18317	0.928408	0.0284262	0.003447766	2.42415E-06	0.004	0.003
1000	6.91939	7.15841	0.293147	0.00866385	0.000773416	7.50623E-07	0.009	0.01
10000	7.18186	7.16226	0.0957359	0.00278404	0.000458268	5.42562E-07	0.05	0.07
100000	7.14409	7.16338	0.0302445	0.000889747	0.000426264	5.21697E-07	0.466	0.659
200000	7.15594	7.1639	0.0214437	0.000632379	0.000436381	5.41869E-07	0.949	1.355
300000	7.16517	7.1648	0.017541	0.000518419	0.000435069	5.35635E-07	1.414	1.993
400000	7.16474	7.16442	0.0151875	0.000447764	0.000432718	5.36318E-07	1.876	2.675
500000	7.15556	7.16459	0.013581	0.000400218	0.000421454	5.31619E-07	2.285	3.319
600000	7.15743	7.16464	0.0123962	0.000365724	0.000426576	5.24182E-07	2.776	3.919
700000	7.15751	7.16455	0.0114771	0.000338431	0.000421648	5.25718E-07	3.201	4.59
800000	7.16057	7.16466	0.0107375	0.000316792	0.000423129	5.25771E-07	3.67	5.239
900000	7.16398	7.16456	0.0101243	0.000298343	0.000440756	5.27465E-07	4.3	5.926
1000000	7.1603	7.16457	0.0095985	0.000282812	0.000421224	5.24766E-07	4.572	6.561

Table 1.1



Note: the grey line which is the efficiency of normal Monte Carlo method uses a secondary axis.

According to the table and plots shown above, we can easily find out that if the number of sample path we generated is less than 1000, in fact the Monte Carlo without Variate Control has a higher efficiency. But since the number of paths is too small, the result we could get is not accurate, which means a higher absolute error, we shall not take this part of data into our conclusion.

When the number of paths is greater than 10000, we can see a huge lead of Control Variate, ***it converges much faster to the real mean and has a much higher efficiency***. (With the increase of sample size, their efficiency will stay in the same level)

Although with the increase of sample paths, the Method without Variate Control indeed will cost us less time than the one with Variate Control, but if we take the Standard Error and Efficiency into consideration, we still believe that the Monte Carlo Method with Variate Control has a much better performance.

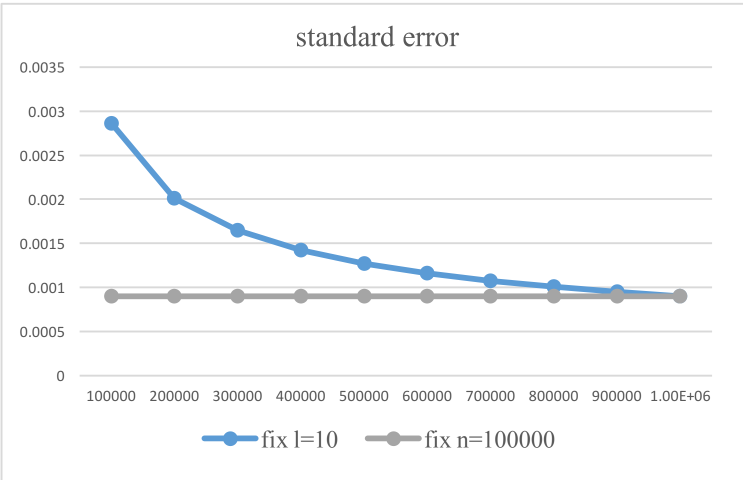
2. Quasi-Monte Carlo Simulation on Asian call

1) the factors that affect the efficiency of Quasi-Monte Carlo.

N=100000					L=10				
N*L	Price	SE	Time	Efficiency	N*L	Price	SE	Time	Efficiency
100000	7.16371	0.000894244	0.775302	6.20E-07	100000	7.16489	0.00286144	0.732829	6.00E-06
200000	7.16419	0.000894223	1.44746	1.16E-06	200000	7.16463	0.00200902	1.39611	5.63E-06
300000	7.16443	0.00089708	2.21946	1.79E-06	300000	7.16461	0.00164269	2.09451	5.65E-06
400000	7.16462	0.000898781	2.98683	2.41E-06	400000	7.16436	0.00142359	2.80026	5.68E-06
500000	7.16468	0.000899035	3.52302	2.85E-06	500000	7.16435	0.00126835	3.52104	5.66E-06
600000	7.16448	0.000897447	4.27749	3.45E-06	600000	7.16459	0.0011591	4.14469	5.57E-06
700000	7.16447	0.000897647	5.13626	4.14E-06	700000	7.16435	0.00107377	4.82508	5.56E-06
800000	7.16454	0.000897845	5.65715	4.56E-06	800000	7.16468	0.00100478	5.48808	5.54E-06
900000	7.16457	0.000898413	6.33105	5.11E-06	900000	7.16431	0.000946165	6.18448	5.54E-06
1.00E+06	7.16467	0.000898231	7.00138	5.65E-06	1.00E+06	7.16467	0.000898231	6.8598	5.53E-06

Table 2.1

First we try to look into the factors that affect the efficiency of Quasi-Monte Carlo.



We first set N = 100000, and change L from 1 to 10. The result we got is shown in table 2.1 and the grey line of the plot shows the standard error when L is increasing. We can see very clear that increasing ***L has very little effect on increasing efficiency*** and decreasing standard error.

Then we set L fix to 10 and change N from 10000 to 100000, which makes N*L equal to the first plot. We found out that when n is small, the standard error is large, and ***it decreases when n increases***, which can be seen from the blue line in the above plot.

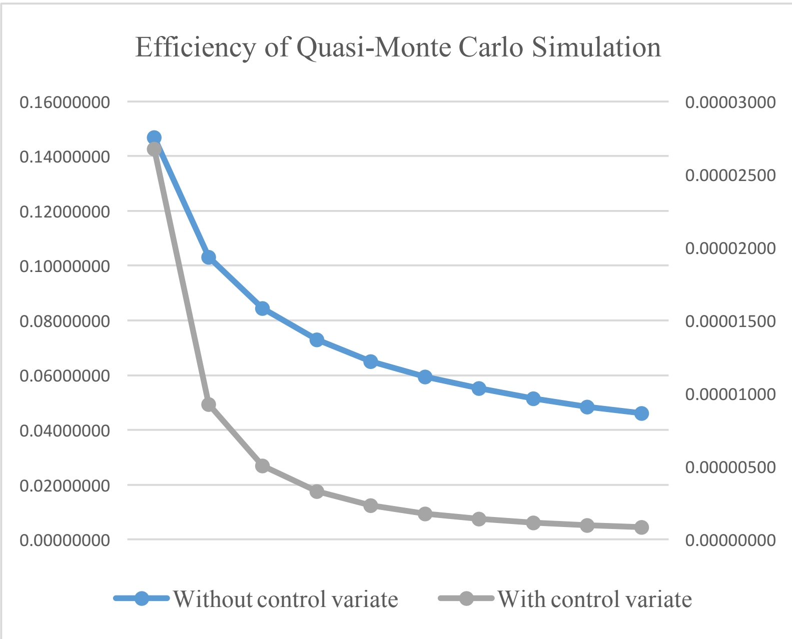
Therefore, we can conclude that when N*L is fixed, ***N is the only deterministic factor that affects the performance of Quasi-Monte Carlo simulation***. L has little effect on the efficiency, it only changes the way how these N set of numbers are distributed in [0,1]. But since they are all normal and their number are fixed, it has no effect in adding more randomness into the simulation.

2) the performance of Quasi- Monte Carlo method when using control variate

N	Price		Standard Error		Efficiency	
	without control	**with control**	**without control**	**with control**	**without control**	**with control**
100	6.72469	7.15132	2.77325	0.0672438	3.43894058	0.51716639
1000	7.46476	7.17169	0.999728	0.0294708	1.51577571	0.02945477
10000	7.13878	7.16434	0.306699	0.00910746	0.46746559	0.00085669
100000	7.2085	7.16489	0.0966535	0.00286144	0.14689388	0.00002673
200000	7.15079	7.16463	0.067938	0.00200902	0.10312686	0.00000927
300000	7.16836	7.16461	0.0555114	0.00164269	0.08432197	0.00000506
400000	7.18376	7.16436	0.0480526	0.00142359	0.07307010	0.00000329
500000	7.1723	7.16435	0.0429662	0.00126835	0.06510176	0.00000234
600000	7.159	7.16459	0.0391506	0.0011591	0.05949817	0.00000178
700000	7.17928	7.16435	0.0362986	0.00107377	0.05511437	0.00000141
800000	7.16758	7.16468	0.033916	0.00100478	0.05157801	0.00000116
900000	7.17433	7.16431	0.0320107	0.000946165	0.04856413	0.00000097
1.00E+06	7.16989	7.16467	0.0303863	0.000898231	0.04610844	0.00000083

Time		
N	with control	without control
100	0.003285	0.003242
1000	0.010864	0.010904
10000	0.081033	0.084898
100000	0.853730	0.783793
200000	1.677450	1.57322
300000	2.428340	2.37263
400000	3.258020	3.18792
500000	4.043480	3.96879
600000	4.840360	4.73088
700000	5.633000	5.5516
800000	6.400890	6.32514
900000	7.032950	7.11862
1.00E+06	8.460530	8.36577

Table 2.2



Note: the grey line which is the efficiency of control variate method uses a secondary axis.

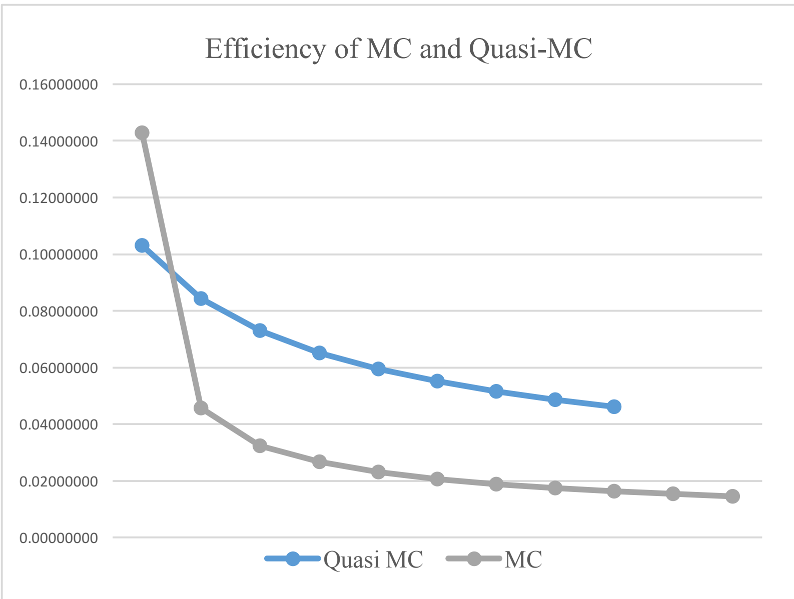
We next look into the performance of Quasi- Monte Carlo method when using control variate. As the table and the graph indicated, similar to normal Monte Carlo, Quasi-Monte Carlo simulation perform a lot better when using control variate method and efficiency increases when n increases for both methods. But compared to normal Monte Carlo, the efficiency of Quasi-Monte Carlo simulation increases more sharply when N increased.

3) Compare efficiency of Quasi-MC and control variate methods.

N	Without Control Variate					
	Price		Standard Error		Efficiency	
	Normal	Quasi	Normal	Quasi	Normal	Quasi
500000	7.15556	7.16489	0.013581	0.00127587	2.21026242	0.06510176
600000	7.15743	7.16533	0.0123962	0.00116803	2.00651840	0.05949817
700000	7.15751	7.16496	0.0114771	0.00107432	1.87090467	0.05511437
800000	7.16057	7.16481	0.0107375	0.00100706	1.74240747	0.05157801
900000	7.16398	7.16484	0.0101243	0.000946525	1.64762309	0.04856413
1.00E+06	7.1603	7.16432	0.0095985	0.00089757	1.56207833	0.04610844
N	With Control Variate					
	Price		Standard Error		Efficiency	
	Normal	Quasi	Normal	Quasi	Normal	Quasi
500000	7.16459	7.16435	0.000400218	0.00126835	0.02054235	0.00126835
600000	7.16464	7.16459	0.000365724	0.0011591	0.01877311	0.00115910
700000	7.16455	7.16435	0.000338431	0.00107377	0.01737096	0.00107377
800000	7.16466	7.16468	0.000316792	0.00100478	0.01626177	0.00100478
900000	7.16456	7.16431	0.000298343	0.000946165	0.01531315	0.00094617
1.00E+06	7.16457	7.16432	0.000282812	0.000898231	0.01451745	0.00089823

Table 2.3

As the table and plots shown above, if we compare the performance of Quasi-MC without control and Normal MC with control variates (which is the shaded area in table 2.3), we can see that the control variate method outperformed Quasi-Monte Carlo method.



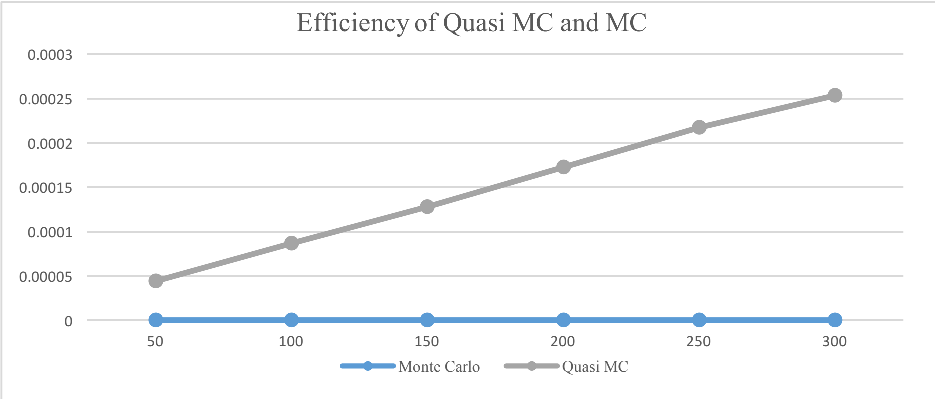
So in conclusion, ***Quasi Monte Carlo method with control variate is the best method*** which has highest efficiency and lowest standard error. Normal MC with control variate is the second best. And normal MC without control variate has the worst performance.

3. Asian call with continuous average

N	M	Price		Standard Error		Efficiency	
		Control Variate	Quasi MC	Control Variate	Quasi MC	Control Variate	Quasi MC
100000	50	0.355461	0.357469	0.000236672	0.00737768	4.14908E-08	4.45118E-05
100000	100	0.352718	0.352443	0.000249662	0.00726294	9.21366E-08	8.6693E-05
100000	150	0.35219	0.349605	0.00024609	0.00723507	1.32972E-07	0.000127938
100000	200	0.351707	0.350013	0.000242044	0.00725226	1.73557E-07	0.000172815
100000	250	0.351586	0.353966	0.00023764	0.00731564	2.08228E-07	0.000217491
100000	300	0.351018	0.351027	0.000232183	0.00721038	2.35476E-07	0.000253263

m	Time	
	Control Variate	Quasi MC
50	0.673	0.809
100	1.358	1.542
150	2.012	2.341
200	2.677	3.101
250	3.353	3.850
300	4.012	4.639

Table 3.1



According to the table and plot presented above, we are able to see the difference, pros and cons for both Quasi Monte Carlo and Normal Monte Carlo while increasing the dimension m.

With the increasing the number of m, both method will definitely increase the computational time. Meanwhile, both of them will have a lower efficiency with a bigger m. But we can see from the graph that ***Quasi Monte Carlo's efficiency will drop much more than Control Variate's method using normal Monte Carlo simulation***. And the efficiency of normal Monte Carlo simulation only slightly drops a little when increasing dimension, which is reflected from the flat blue line in the above graph.

Even if we put the computational and the standard error into consideration, we still believe that the control variate approach is a better choice than Quasi Monte Carlo when dimension is large.