# EP Homework 6

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## 1 Superposition

We are tasked with finding the probability density $|\psi|^2$ for the superposiition of two stationary wave-functions

$$\psi = C_1\psi_1 + C_2\psi_2 \tag{1}$$

When we take the norm of Equation 1, we get the following:

$$|\psi|^2 = |C_1\psi_1 + C_2\psi_2|^2 = |C_1\psi_1|^2 + |C_2\psi_2|^2 + C_1^*C_2\psi_1^*\psi_2 + C_1C_2^*\psi_1\psi_2^* \tag{2}$$

where in 2 the star represents the complex conjugate.
As seen in Figure 1, when $\psi_1$ and $\psi_2$ correspond to different atomic states, n=3 and n=4 respectively, the superposition is as one expects. The Wave Functions are simply added together, while the probabilities are a combination of the two. So depending on the atomic states of the two wave functions that are superimposed, the density and wavefunction are converging towards the larger n.

## 2 Potential Step 1

If the section is not filled in at time of submission, it means I did not finish the problem 2 in the homework. My initial thoughts are to solve the two regions with certain $E < U_0$ and prove that photons are emitted with energy $E_{photon} = U_0$

## 3 Potential Step 2

In the lecture notes we discuss both sides of this potential wall problem, however here we have the potential defined by:

$$U(x) = \begin{cases} 0, & \text{if x} < 0 \\ U_0, & \text{if x} \geq 0 \end{cases}$$

The 1D time independent Schroedinger Equation we aim to solve is:

$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi + U(x)\psi \tag{3}$$

The solution for $\psi(x)$ has the following form for Region 1:

$$\psi_I = Ae^{ikx} + B^{-ikx} \tag{4}$$

and for Region 2:

$$\psi_{II} = Ce^{\alpha x} + D^{-\alpha x} \tag{5}$$

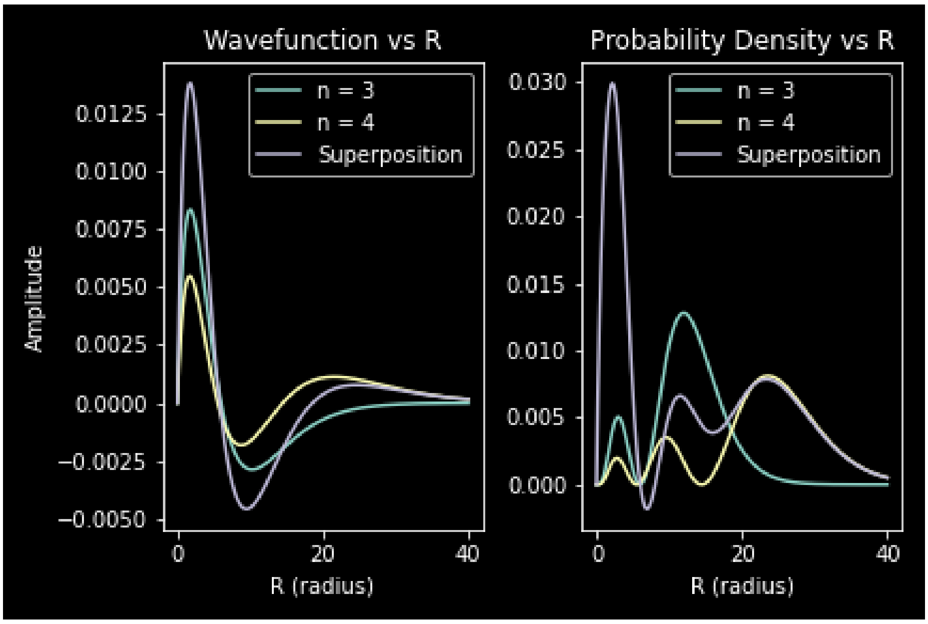


Figure 1: Radial Wave function $\psi(r)$ and Probability $|\psi(r)|^2$

Where $\alpha = \sqrt{\frac{2m(U_0 - E)}{\hbar^2}}$

One can note now, that for each $\psi$, there is both a forward propogating wave and a backwarg propogating wave, yet in our problem, only region II has both forward and backward components, while region I has only the transmitted (forward). Thus Eq 4 becomes:

$$\psi_I = Ae^{ikx} \tag{6}$$

And with continuity conditions

$$\psi_I(x=0) = \psi_{II}(x=0)$$

and

$$\frac{d\psi_I}{dx}_{x=0} = \frac{d\psi_{II}}{dx}_{x=0}$$

We find the system of equations when inserting $x = 0$ into Eqs: 6 and 5 and their first order derivitaives:

$$C + D = A \tag{7}$$

and

$$\alpha(C - D) = ikA \tag{8}$$

The coeffeicients are then found to be, when C = 1:

$$D = \frac{\alpha - ik}{\alpha + ik} \tag{9}$$

$$A = 1 + D = 1 + \frac{\alpha - ik}{\alpha + ik} = \frac{2\alpha}{\alpha + ik} \tag{10}$$

So the probability for reflection is:

$$P_{reflection} = R = |D|^2 = (\frac{\alpha - ik}{\alpha + ik})^2$$

and

$$P_{transmission} = 1 - P_{reflection} = T = \frac{4\alpha ik}{(\alpha + ik)^2}$$

These are the same as the fresnel coefficients for normal incidence, except the refractive indexes of the regions are replaced by the Total Energy in both regions, i.e $n_2 = \alpha$ and $n_1 = ik$