EP4 Homework 4

Adam Kit

4 May 2020

1 Task 1

An electron performs underdamped, but nearly harmonic, oscillations with a frequency of 1015 Hz. After which time period it will loose 90 percent of its initial energy E_0.

Lorentz Oscillator

We first describe aan electrons oscillation in terms of the classical spring harmonics, as if the elctron is attached to the nucleus by a string. To be simple, we also include a small damping force, $F_{damping} = -m_e \gamma \frac{dy}{dt}$, which is inserted in Newton's 2^{nd} law eq:

$$m_e \frac{d^2 y}{dt^2} = F_{driving} + F_{damping} + F_{spring} = F_{driving} - m_e \gamma \frac{dy}{dt} - Ky$$

$$\frac{d^2 y}{dt^2} + \gamma \frac{dy}{dt} + \omega_0^2 y = F_{driving}$$

With the mass of the electron $m_e = 9.11 \times 10^{-31}$ and K being the "spring" constant, we are able to reduce the coefficients to $\omega_0 = \sqrt{\frac{K}{m_e}}$. I decided that the driving force is a driving oscillating electric field $E = E_0 cos(-\omega t)$ where $F_{driving} = qE_0 cos(-\omega t)$ and the new form is then:

$$\frac{d^2 y}{dt^2} + \gamma \frac{dy}{dt} + \omega_0^2 y = qE_0 cos(-\omega t)$$

which has complex solutions that which can be described as having a complex amplitude of

$$A_0 = \frac{qE_0}{m_e} \frac{1}{\omega_0^2 - \omega^2 - i\omega\gamma}$$

and real amplitude

$$A_{real,0} = \frac{qE_0}{m_e} \frac{\omega_0^2 - \omega^2}{(\omega_0^2 - \omega^2)^2 + (\omega\gamma)^2}$$

and real solution:

$$y(t) = A_{real,0} e^{-\omega t}$$

where we have swapped the cosine for the exponential from eulers relationship. And an Energy function for the damped oscillator modled by $E_{total} = P = \frac{1}{2}ky^2 + eE_0 y$ so with underdamping: $\omega_0^2 >> \gamma^2$ we can see that $0.1 = \frac{1}{2}kA_{real,0}e^{-2\omega t} + eE_0 A_{real,0} e^{-\omega t}$ and the first term will be much less than the second, with further reduction to $0.1 \propto e^{-\omega t} \Rightarrow ln(0.1) = -\omega t$ with $\omega = 1015$Hz $t = .00226$ seconds.

1.1 Task 2

Find the average scattering angle $\bar{\theta}$ resulting for the Thomson model of atom. Use the fact that the scattering angles are small.

Plum Model

Thomson suggested that the electrons were distributed in a positively charged medium, thus he shot a material of atoms with lots of high-energy $\alpha-$particles and watched how they α-particles scattered. If the particle were to pass the atom, which has a radius R, at a distance b(collision parameter) from the center of the atom, we would see one of two different scenarios, depending on b. For the scenario of $b > R$: since the atom is neutral, the α-particle's trajectory is not affected. For the scenario $b < R$: there is an electrostatic interaction and the particle is deviated by a small angle $\theta(b)$ which will lead to an intensity profile $I(\theta)$ of the particle beam hitting a capturing screen. This can also be modeled by a 2D random walk, where the intensity is described by N number of steps of s step length, and $\bar{s}$ meaned net step length:

$$I(s) \propto e^{-\frac{s^2}{N\bar{s}^2}}$$

or in terms of random angle deviations θ and average angle deviation $\bar{\theta}$:

$$I(\theta) \propto e^{-\frac{\theta^2}{N\bar{\theta}^2}}$$

When modeling with a random walk, we can see that the meaned net step length will have to be around the size of the plum pudding atom. Thus the number of walks to get to $\bar{s}$ corresponds to the number of steps s. The same goes for the angles, where the scattering within the atom is going to go through many angles before finally coming out one of the other sides. Thus the average angle, should be proportional to equal to the $\sqrt{N}$ multiplied by $\frac{L}{2\pi}$ if L is the size of the plum pudding model.

2 Task 3

Find how the distance Δr between two adjacent electron orbits varies with increasing radius in the limit of large quantum numbers n.

Electron Radius

In the Figure 1 we see the relationship between the distance between adjacent electron orbits. This comes from the Bohr relationship of electron orbit radius $r_n = a_0\frac{n^2}{Z}$ where $a_0 = 0.0529 * 10^{-9}$m is the Bohr radius of the first electron orbit, and Z the atomic number of the atom in whose electrons are in question. We can then describe the distance between two adjacent orbits as:

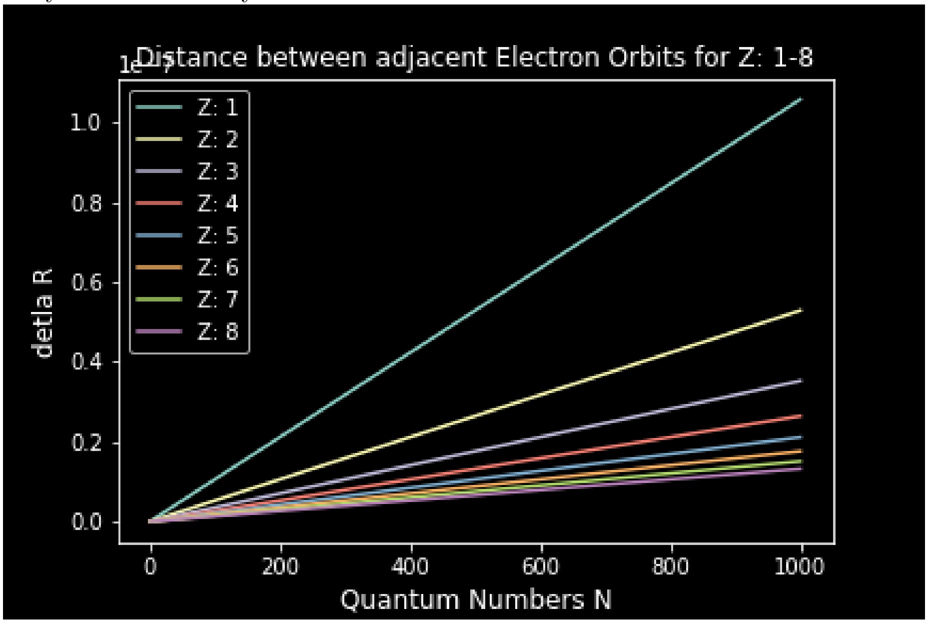
$$\Delta r = r_{n+1} - r_n = \frac{a_0}{Z}((n+1)^2 - n^2) = \frac{a_0}{Z}(2n+1)$$

Although linear, and seemingly increasing, note the size of a_0 and the division by Z. For the hydrogen atom (Z = 1) Even if $n = 1 * 10^{11}$, we find that $\Delta r = 10.58$ meters. !

3 Task 4

Show that the energy of the transitions between two adjacent orbit ΔE is proportional to n^{-3} in the limit of large quantum numbers.

Figure 1: Linear relationship of the distance between adjacent electron orbits for atomic numbers Z: 1-8. Note: the y axis is scaled by $1 * 10^{-7}$



Transition Energies

Just like for problem three, I have graphed the relationship between ΔE and n. The n^{-3} proportion is apparent in Figure 2. This comes from the energy an electron has in a given orbit n being $E_n = -E_0 \frac{Z^2}{n^2}$ and the relationship being:

$$\Delta E = E_{n+1} - E_n = -E_0 Z^2 [\frac{1}{(n+1)^2} - \frac{1}{n^2}] = -E_0 Z^2 \frac{-2n-1}{n^3 + 2n^2 + n} \propto -E_0 Z^2 n^{-3}$$

as for $n^3 >> n^2 >> n$ and the other terms will drop. The code for these graphs can be found here.

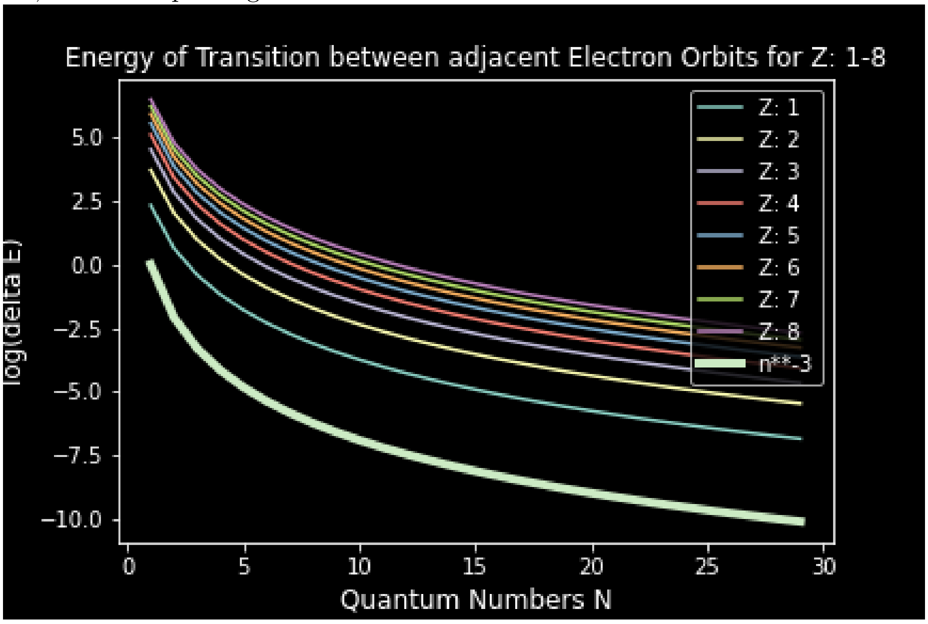
4 Task 5

Which spectral lines will appear in the emission spectrum of an atomic hydrogen gas upon irradiating it with ultraviolet light with the wave length of 100 nm?

4.1 Absorption vs Emission

From the lecture notes we know that changes of energy level can lead to the emission of a photon in a gas. The energy of the photon is equal to the energy difference between the atomic levels. For absorption, the same applies, but the incoming photon must have the energy to move the electron from one energy level to the next, and when it can, the energy difference between the two levels will be equal to the energy of the incoming photon. So if we have $\lambda = 100nm \rightarrow E_{incoming} = \Delta E = \frac{hc}{\lambda} = 12.4$ eV, then we seek the energy level transition which is the same as ΔE, and since we know the incoming light is UV

Figure 2: Relationship of the ΔE between adjacent electron orbits for atomic numbers Z: 1-8. Note the scaling is logarithmic, as these values are decreasing very very quickly as N is increasing, thus to properly show, I chose to plot logE instead



(80 - 150 nm) we can look only for transitions starting at n = 1 (Lyman Series).

$$\Delta E = (\frac{1}{n_{low}^2} - \frac{1}{n_{high}^2}) * 13.6\text{eV} \rightarrow (1 - \frac{12.4}{13.6})^{-1} = 11.01 \geq n_{final}^2 \rightarrow n_{final} = 3$$

Although n = 2 also works here, we know that if the electron has enough energy, it will jump to the highest possible level. So the transition is from the ground state to $n = 3$, and the resulting wavelength of the emitted photon from that transition is going to be:

$$\lambda = ((\frac{1}{1^2} - \frac{1}{3^2})\frac{13.6}{hc})^{-1} = 102 \text{ nm}$$

We have made use of plancks constant in terms of eV: $h = 4.135 \times 10^{-15}$eV s, and $c = 3 \times 10^8 \frac{m}{s}$.