EP Homework 11

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1 Spectral Line Splitting

The normal Zeeman effect is can b observed when the coupling spin number S of an atom is 0, therefore there is no contribution to the magnetic moment of the atom. The anomalous Zeeman effect occurs when $S \neq 0$, and since our transition occurs between two states that both have $S = 0$, we see the normal Zeeman effect occur. The varius spectral lines can be seen through the selection rules of $\Delta m_j = \pm 1, 0$, and $m_j = -j, -j+1, ..., j$, and is drawn in Figure 1. We look at the spectral lines of the fundamental transition $^1D_2 \rightarrow ^1 F_3$ when detected in a magnetic field with $B = 1$T.

2 Splitting of D

My intial guess is yes, as the coupling term that dtermines the splitting of an orbital is found by the equation 1, and we can see that for $s = 3/2$, $l = 2$, $j = 1/2$, eq. 1 is nonzero!

3 Spin-Orbit Coupling

From the lecture notes we obtain the equation for the energy levels of an atomic system under the influence of spin-orbit interaction. Equation 1.296 is reproduced below:

$$E_{n,l,j} = E_n + \frac{a}{2}(j(j+1) - l(l+1) - s(s+1)) \tag{1}$$

We want to look at the splitting of the P orbital ($l = 1$), where due to spin-orbit coupling, the P level is split between $P_{1/2}$ and $P_{3/2}$, when $n = 6$, $s = 1/2$. My initial guess is to calculate the energy difference between the two levels, $E_{6,1,1/2}$ and $E_{6,1,3/2}$ from 1, such that $\Delta E = E_{6,1,3/2} - E_{6,1,1/2}$. I think the constant $a = \hbar^2$

4 Selection Rules for Quantum Harmonic Oscillator

To do so, we set up the transition dipole matrix M_{ik} of a transition $i \rightarrow k$ such that:

$$M_{ik} = e \int \psi_i^\dagger \vec{r} \psi_k d\tau$$

for the hydrogen atom, we can use the wave function for the state (n, l, m_l) in radial coordinates:

$$\psi_{n,l,m_l} = \frac{1}{\sqrt{2\pi}} R_{n,l}(r) Y_m^l(\vartheta) e^{im\phi}$$

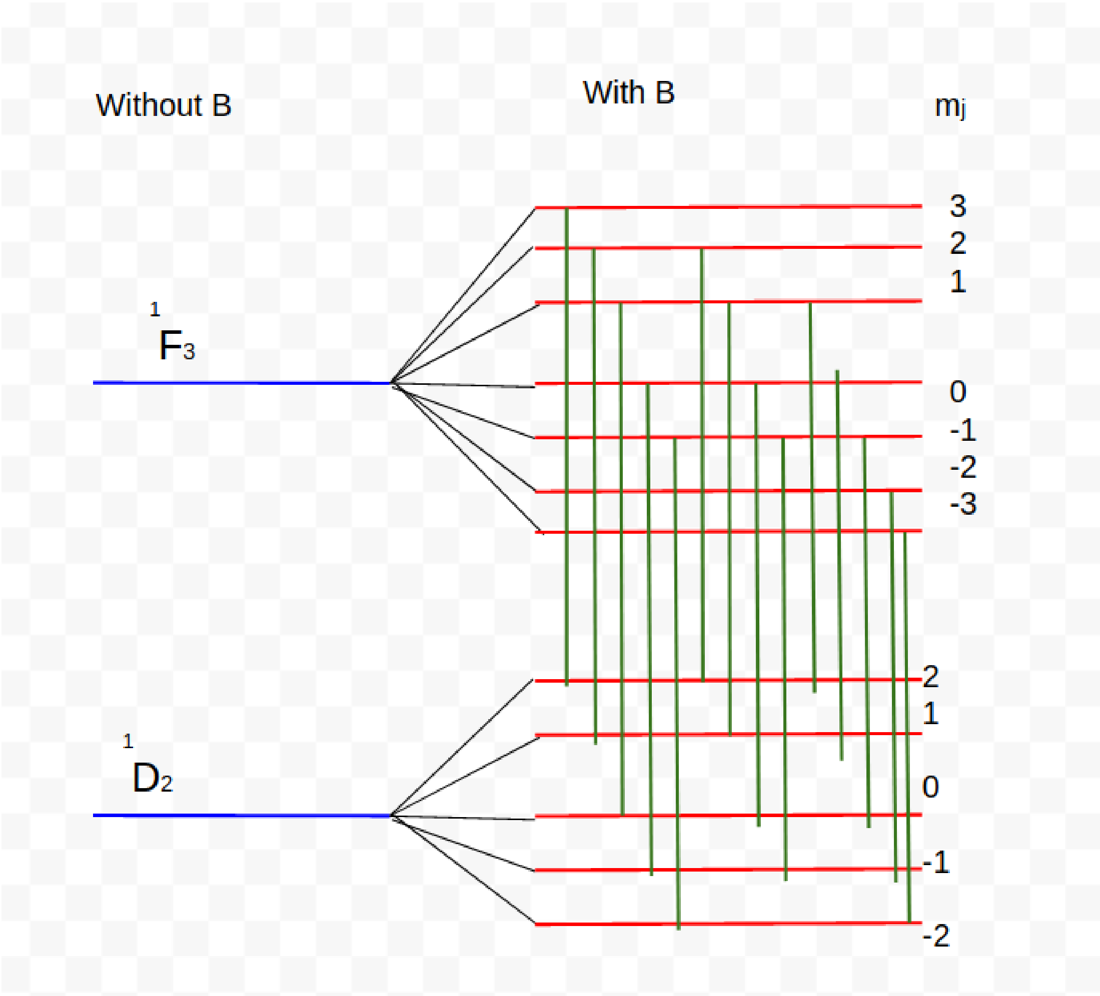

Figure 1: The normal Zeeman Effect of the transition between ${}^1D_2 \rightarrow {}^1F_3$

For a *linearly* polarized light wave with electric field vector $\vec{E} = (0, 0, E_0)$ on the atom, only the term $E_0 z$ from the scalar product $\vec{E}\vec{r}$ survives. So we can look at the z axis of our dipole matri element, and make the substitution $z = r\cos\vartheta$ such that the transition from $i \to k$ is written as:

$$(M_{ik})_k = \frac{e}{2\pi}\int_{r=0}^{\infty} R_i R_k r^3 dr \int_{\vartheta=0}^{\pi} Y_{m_k}^{l_k} Y_{m_i}^{l_i} \sin\vartheta\cos\vartheta d\vartheta \int_{\phi=0}^{2\pi} e^{i(m_k - m_i)\phi} d\phi$$

Now we can look at the three components of this integral, and by doing so, we inspect various quantum numbers. For example, in the last component, we see that outside of $m_i = m_k$, the integral will be evaluated to zero. Thus for linearly polarized light, the selection rule $(M_{ik})_z \neq 0$ only for $\Delta m = m_i - m_k = 0$. Would this hold for circularly polarized light? To find out we must look at the other components of the dipole matrix, namely the complex linear combination of the x and y components. Again making the polar coordinate substitutions of $x = r\sin\vartheta\cos\phi$ and $y = r\sin\vartheta\sin\phi$ we have the dipole matrix combination:

$$(M_{ik})_x + i(M_{ik})_y = \frac{1}{2\pi}\int_0^{\infty} R_i R_k r^3 dr \int_0^{\pi} Y_{m_k}^{l_k} Y_{m_i}^{l_i} \sin^2\vartheta d\vartheta \int_0^{2\pi} e^{i(m_k - m_i + 1)\phi} d\phi$$

and once again looking at the last component (ϕ dependent) of the integral, we find similar rules. Only when $m_k = m_i - 1$ or $m_k = m_i + 1$ does $(M_{ik})_x + i(M_{ik})_y \neq 0$. This means that the selection rules for circullarly polarized light are slightly different, i.e., $\Delta m = m_i - m_k = \pm 1$. As for the other components of the integrals, we now look at the ϑ dependence, which is also l dependence. So for linearlly polarized light:

$$\int_{\vartheta=0}^{\pi} Y_{m_k}^{l_k} Y_{m_i}^{l_i} \sin\vartheta\cos\vartheta d\vartheta$$

which from previous homeworks, we know that the Legendre polynomial for differing states are sometimes orthogonal, thus the integral is only nonzero for $m_k = m_i$ occurs when $l_k - l_i = \pm 1$. The same goes for circular light for when $m_k = m_i \pm 1$