

EP Homework 10

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15 June 2020

1 Spectral Line Splitting

The Zeeman effect is the splitting of a spectral line into several other closed spaces lines when in the presence of a static external magnetic field. The splitting is attributed to the interaction between the magnetic field and the magnetic dipole moment μ associated with the orbital momentum L . So in the absence of a magnetic field, the hydrogen energies depend only on n since $l = 0, \dots, n - 1$

If the spin-orbit interaction is dominating, then only the total angular momentum of the atom is conserved $\vec{J} = \vec{L} + \vec{S}$ where \vec{L} and \vec{S} are the total orbital momentum and the spin angular momentum respectively. As derived in the lecture notes, the magnetic moment of the atom is

$$\vec{\mu} = -\frac{q}{2m_e} \vec{J}$$

So we can look at the magnetic dipole moment's projection:

$$\mu_z = -m \frac{q\hbar}{2m_e} = -m\gamma\hbar$$

where $m = -l, -l + 1, \dots, +l$ the magnetic quantum number and when inserting this into the magnetic potential energy from the torque exerted on a magnetic dipole:

$$U_b = -\vec{\mu} \vec{B} = -m\gamma\hbar \vec{B}$$

So the total energy of the atom in the magnetic field will be $E_{n,l,m} = E_n - \mu_B m B_0$ where μ_B is the bohr magneton, and the difference in energy is then

$$\Delta E = \mu_B m B_0$$

As the lecture notes states, we know that the energy levels will split into $(2l + 1)$ levels, so there will be a total of 3 level. The total angular momentum

For the Lyman series, the hydrogen atom emits an electron as it falls from the $n = 2$ orbital to the $n = 1$ orbital for the first spectral line. In addition, in the $n = 2$ orbital, there are two possible states due to the fine structure above and as the Hamiltonian is using the total angular momentum J and not just the orbital momentum l , the two states are $j = 1/2$, and $j = 3/2$. The $j = 3/2$ state will have a higher energy and will be energetically farther from the $n = 1$ orbital to which it transitions. So this state will be having a shorter wavelength (higher energy) spectral line. This will have a splitting found by equation 1.303 in the lecture notes.

$$\Delta E = -5.3 \times 10^{-5} \frac{Z^2 E_n}{nl(l+1)}$$

. For the first spectral line $l = 1$, so we have a wavelength of 1215.668 angstroms, as opposed to the smaller energy jump which would have 1215.674 angstroms.

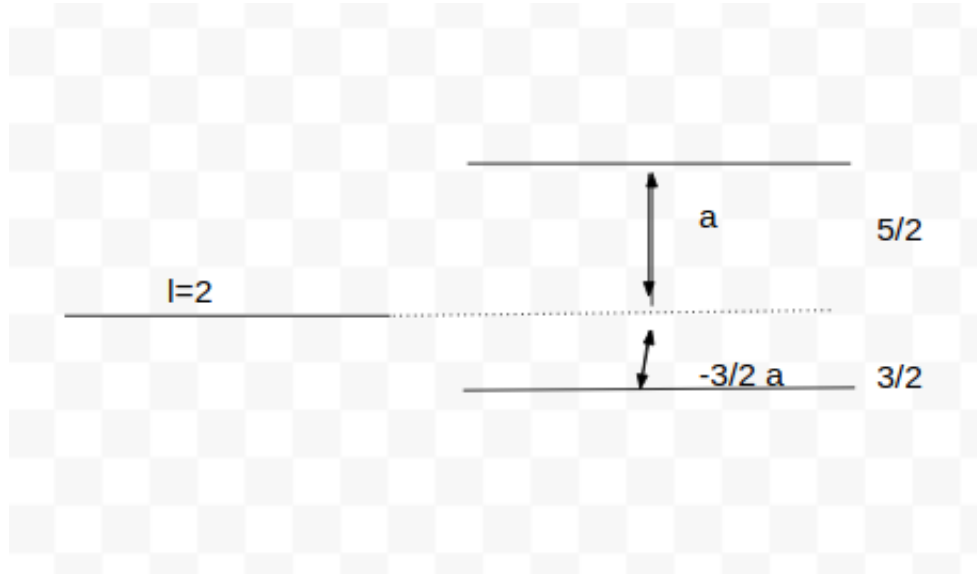


Figure 1: Spin Orbit Coupling

2 C_{60} fullerene Molecule

Due to spherical symmetry, we can say that the rotational energy of the molecule can be written as $E = BJ(J + 1)$, where $B = \frac{\hbar^2}{2I}$ and $I = \mu r^2$. Here we see the reduced mass μ of the carbon atom is , and r is the van-der-Waals diameter of 1.1nm. Once we have the energy, we can then find the

3 Spin-orbit coupling

For the d orbital, we have the total orbital angular momentum quantum number $L = 2$, meaning that there are 5 orbitals that correspond to magnetic quantum numbers $-2, -1, 0, 1, 2$. Since spin coupling numbers take on values of $J = l \pm 1$, we can have for $l = 2 \rightarrow J = \frac{5}{2}, \frac{3}{2}$. Using the equation 1.295 in the lecture notes and 1:

$$E_{n,l,j} = E_n + \frac{a}{2} (j(j + 1) - l(l + 1) - s(s + 1)) \quad (1)$$

Thus for the top, $j = 5/2$, we have Eq 1 becoming

$$E_{3,2,5/2} = E_n + a$$

and for $j = 3/2$ we have:

$$E_{3,2,3/2} = E_n - \frac{3}{2}a$$

We can then represent this graphically with Figure 1.