TP4 Homework 4

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Fourier Transform and Gaussian Wave Functions

Fourier transform and Gaussian wave functions.

1 Fourier Transform

Determine the Fourier transform of

$$\Psi(x) - (\pi\omega^2)^{-1/4} e^{ip_0 x/\hbar - (x-x_0)^2/(2\omega_0^2)}$$

where x_0, p_0, w_0 are real parameters.

Solution

2 Normaliziation

(b) Show that ψ is normalized

3 Mean Values

4 Adjoints

5 Variance

For the wave functionin (a), show that the variances of position and momentum oper-ator areˆx=—w0—2,ˆp= h—w0—2.(5)(

6 Differentiabl Wave Function

f) For any differentiable wave functionhaving sufficiently rapid decay as—x—(aswell as its derivative), show that (x)(k) =iddk (k), (iddx)(k) =k (k),(6)so Fourier transform exchanges position and momentum operators. Show that ifis anL2-wave function, then(x) =eik0x(xx0)has Fourier transform (k) =eix0k (kk0). How are these statements related?

7 Hamiltonian for Schrodinger Equation

We now consider the Hamilton operatorˆH=12mˆp2= h22md2dx2for a free particle on thereal line. We wish to solve the time-dependent Schrodinger equation hiddtt(x) =ˆHt(x),0(x) =(x)(7)with initial wave function(x)as in (a). Using (e), show that such a solution satisfiesddt t(k) =ik2 h2m t(k). Using this and the result of (a), find t(k). Applying the inverseFourier transform, findt(x).

8 General Solution Schrodinger Equation

Using the same type of method as in (g), show that the general solution to the time-dependent Schrodinger equation for our Hamiltonian may be written ast(x) =dy Kt(xy)0(y),Kt(x) =(mi h2t)1/2eimx22 ht(8)Hint: You may use the convolution theorem: Iff(x)and(x)are decaying sufficientlyrapidly, then (f)(k) =dp f(kp)(p),(9)together with a similar theorem for the inverse Fo