# Numerical Methods Homework 6

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## 1 Netwons Forward Interpolation Formula

We need to interpolate the table of the cubic polynomial $f(x)$ and look to then calculate $f(4)$.

To do so we use Newtons Forward Interpolation formula in eq. 1.

$$P(x) = \sum_{i=0}^{n} c_i N_i(x) = c_0 + c_1(x - x_0) + c_2(x - x_0)(x - x_1) + ... + c_n(x - x_0)\dots(x - x_{n-1}) \tag{1}$$

Where $N_i(x) = \prod_{j=0}^{i-1}(x - x_j) = (x - x_0)...(x - x_{i-1})$ in eq. 1 and the approximation $P(x_i) = f_i$ has the form:

$$\begin{pmatrix} 1 & ... & & 0 \\ 1 & (x_1 - x_0) & & 0 \\ 1 & (x_2 - x_0) & (x_2 - x_0)(x_2 - x_1) & 0 \\ \vdots & \vdots & \ddots & \\ 1 & (x_n - x_0) & \dots & \prod_0^{n-1}(x_n - x_i) \end{pmatrix} \begin{pmatrix} c_0 \\ \vdots \\ c_n \end{pmatrix} = \begin{pmatrix} f_0 \\ \vdots \\ f_n \end{pmatrix}$$

So in our case, the function $P(x)$ is looking like so:

$$P(x) = 1 + \frac{2-1}{1-0}(x-0) + \frac{\frac{1-2}{2-1} - \frac{2-1}{1-0}}{2-0}(x-0)(x-1) + \frac{\frac{\frac{10-1}{3-2} - \frac{1-2}{2-1}}{3-1} - \frac{\frac{1-2}{2-1} - \frac{2-1}{1-0}}{2-0}}{3-0}(x-0)(x-1)(x-2)$$

Plugging in $x = 4$ we get $f(4) = 131$.

## 2 Income

This doesn't really count towards the solution, but I spent some time on it so I thought I might as well include it. One can solve using the following code found in the file *interpolatingNN.py* in this repository. See the README for a proper explanation of how one can use a simple node network to minimize a cost function and fit a line.

## 3 Polynomial Function Fit

One can do this using the lagrange basis and the polynomial approximation $P(x)$ is represented as $P(x) = \sum_i l_i(x)f(x_i)$, where $l_i$ are described below.

$$l_i(x) = \prod_{j=0, j\neq i}^{n} \frac{x - x_j}{x_i - x_j} = \frac{x - x_0}{x_i - x_0} \dots \frac{x - x_{i-1}}{x_i - x_{i-1}} \frac{x - x_{i+1}}{x_i - x_{i+1}} \dots \frac{x - x_n}{x_i - x_n}$$

Our function $P(x)$ then resembles

$$P(x) = 4x^3 - 30x^2 + 6x - 18$$

this then has a derivative of

$$P'(x) = 12x^2 - 60x + 6 \Rightarrow P'(2) = 48 - 120 + 6 = -66$$

# 4 Viscosity

We use the newton divide difference polynomial to estimate the viscosity in this. The divided difference table looks like:

$$\begin{array}{ccccc}
x_i & f(x_i) & \Delta & \Delta^2 & \Delta^3 \\
110 & 10.8 & & & \\
 & & -\frac{27}{200} & & \\
130 & 8.1 & & \frac{29}{30000} & \\
 & & -\frac{26}{300} & & -\frac{7}{3200000} \\
160 & 5.50 & & \frac{19}{24000} & \\
 & & -\frac{7}{300} & & \\
190 & 4.8 & & &
\end{array}$$

this results in the interpolating polynomial:

$$P(x) = 10.8 - \frac{27}{200}(x - 110) + \frac{29}{30000}(x - 110)(x - 130) - \frac{7}{3200000}(x - 110)(x - 130)(x - 160)$$

Which when we plug in $x = 140$ we find $P(140) \approx 7.053125$ which makes sense since it is right between the two. A graphical representation is seen in 1.

# 5 Lagrange Interpolation Formula

We do the same as section 3 ( Polynomial function fit) and find the following approximation:

$$P(x) = -\frac{1}{140}(x - 658)(x - 659)(x - 661)2.8156 + \frac{1}{12}(x - 654)(x - 659)(x - 661)2.8182+$$

$$-\frac{1}{10}(x - 654)(x - 658)(x - 661)2.8189 + \frac{1}{42}(x - 654)(x - 658)(x - 659)2.8202$$

Which when we plug in $x = 656$, we find $P(656) = 2.81681$ which fits fairly well with our table. A graphical representation is seen in 2.

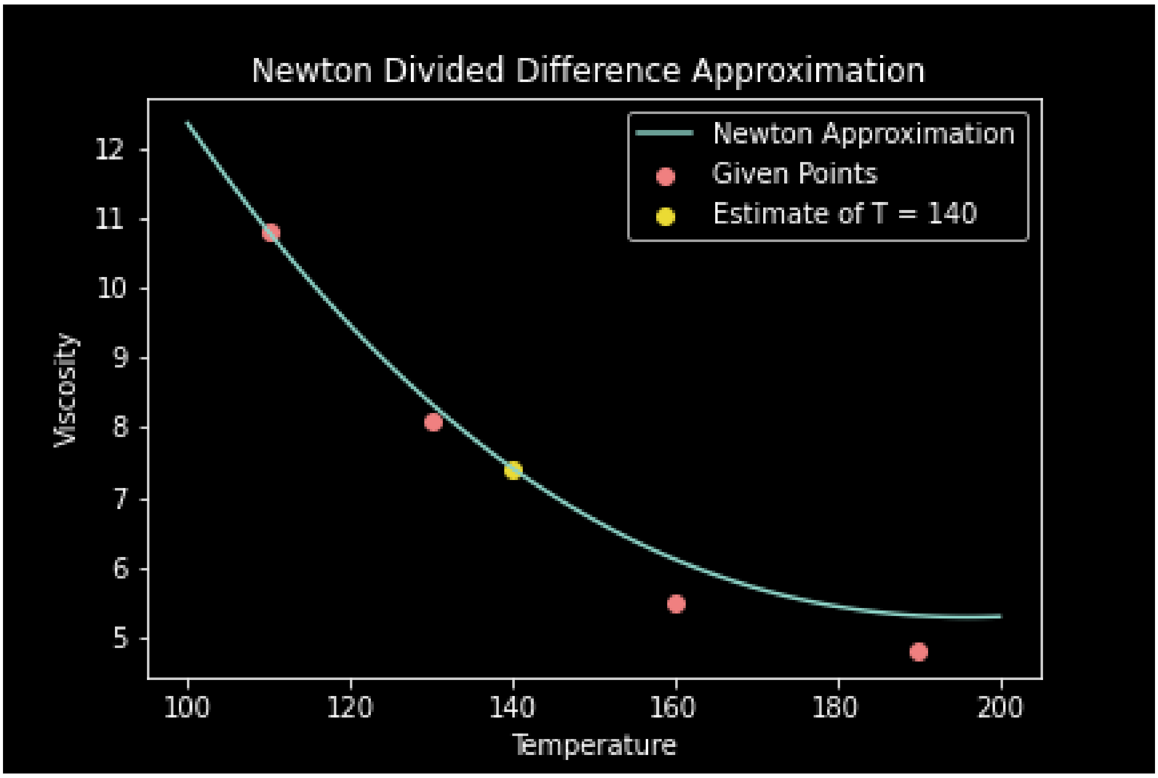


Figure 1: The newton divided difference metthod for viscosity given in sheet

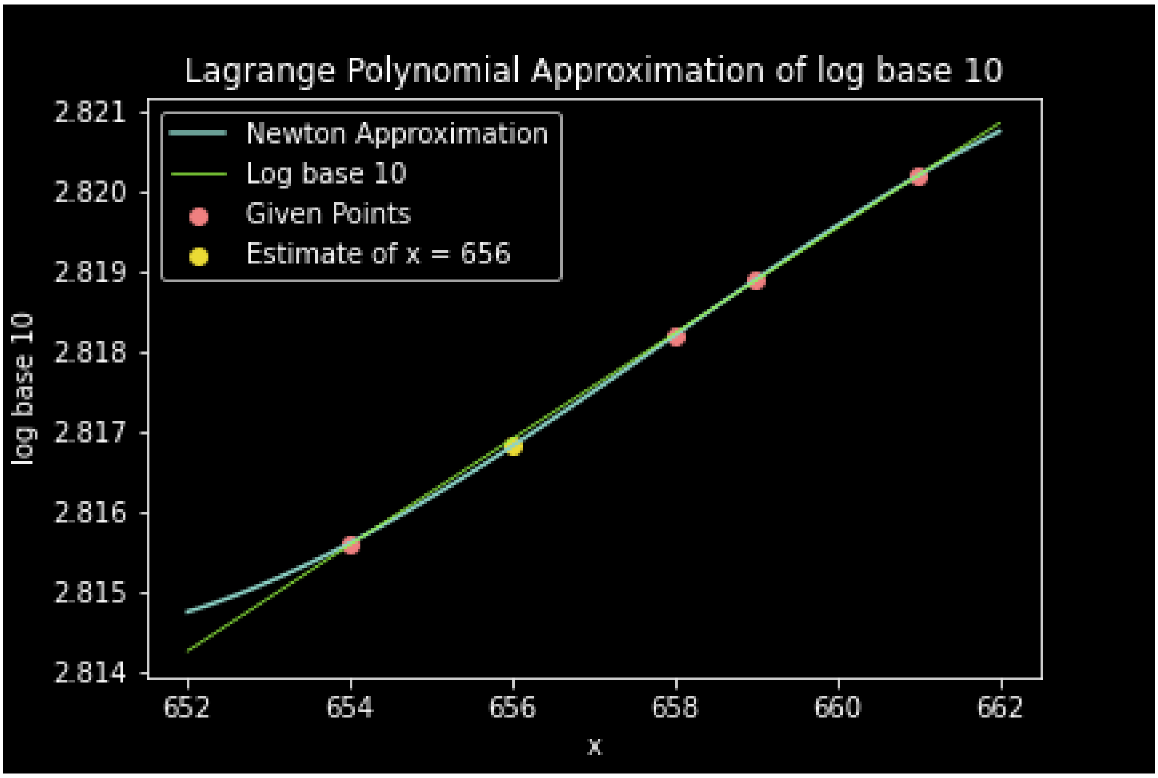


Figure 2: The Lagrange Polynomial Interpolation for $log_{10}$