Numerical Methods Sheet 12

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1 Parabola Fitting

We are tasked with fitting a parabola of the type $y = p_0 + p_1 x + p_2 x^2$ to the the points found in table using the least squares method. For the least squares method, I describe below the procedure. To estimate the coefficeints p_k we must solve the foloowing system of linear equations.

$$\begin{bmatrix} N & \sum_{i=1}^N x_i & \dots & \sum_{i=1}^N x_i^k \\ \sum_{i=1}^N x_i & \sum_{i=1}^N x_i^2 & \dots & \sum_{i=1}^N x_i^{k+1} \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{i=1}^N x_i^k & \sum_{i=1}^N x_i^{k+1} & \dots & \sum_{i=1}^N x_i^{2k} \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ \vdots \\ p_k \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^N y_i \\ \sum_{i=1}^N x_i y_i \\ \vdots \\ \sum_{i=1}^N x_i^k y_i \end{bmatrix}$$

Here N respresents the number of datapoints, and k is the maximum order of the polynomial. For this problem, $N = 5$, $k = 3$. We can then use Cramer's rule to solve the system of equations using the following equation:

$$p_k = \frac{det(M_i)}{det(M)}$$

where M_i is the matrix M with the $i - th$ column replaced with the column vector b, i.e.,

$$M_0 = \begin{bmatrix} \sum_{i=1}^N y_i & \sum_{i=1}^N x_i & \dots & \sum_{i=1}^N x_i^k \\ \sum_{i=1}^N x_i y_i & \sum_{i=1}^N x_i^2 & \dots & \sum_{i=1}^N x_i^{k+1} \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{i=1}^N x_i^k y_i & \sum_{i=1}^N x_i^{k+1} & \dots & \sum_{i=1}^N x_i^{2k} \end{bmatrix}$$

For our problem, we must solve the following system of equations:

$$\begin{bmatrix} 6 & 108.5 & 2500.11 \\ 108.5 & 2500.11 & 62819.513 \\ 2500.11 & 62819.513 & 1638397.8195 \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 138.0 \\ 3216.31 \\ 81256.065 \end{bmatrix}$$

as well as having the following matricies for M:

$$M = \begin{bmatrix} 6 & 108.5 & 2500.11 \\ 108.5 & 2500.11 & 62819.513 \\ 2500.11 & 62819.513 & 1638397.8195 \end{bmatrix} \rightarrow det(M) = 65695336.63296$$

$$M_0 = \begin{bmatrix} 138.0 & 108.5 & 2500.11 \\ 3216.31 & 2500.11 & 62819.513 \\ 81256.065 & 62819.513 & 1638397.8195 \end{bmatrix} \rightarrow det(M_0) = 12146654.6941248$$

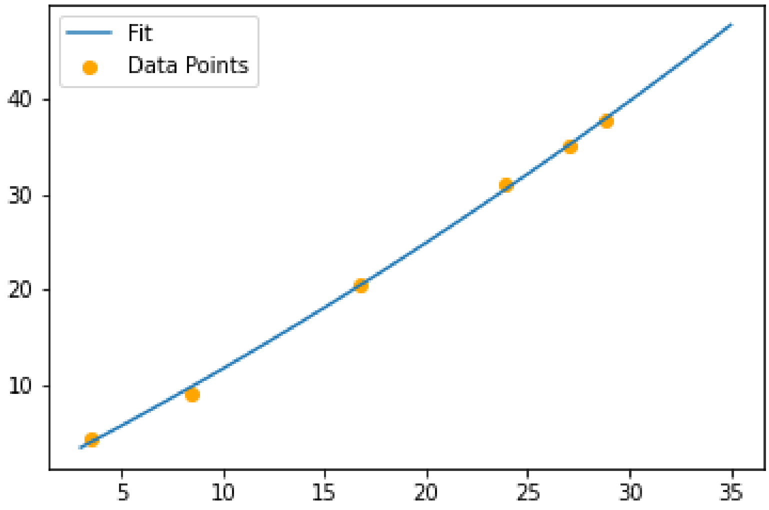

Figure 1: Graph of Curve Fitting

$$M_1 = \begin{bmatrix} 6 & 138.0 & 2500.11 \\ 108.5 & 3216.31 & 62819.513 \\ 2500.11 & 81256.065 & 1638397.8195 \end{bmatrix} \rightarrow det(M_1) = 70699102.240464$$

$$M_2 = \begin{bmatrix} 6 & 108.5 & 138.0 \\ 108.5 & 2500.11 & 3216.31 \\ 2500.11 & 62819.513 & 81256.065 \end{bmatrix} \rightarrow det(M_2) = 528866.30352$$

Therefore:

$$p_0 = 0.1849$$

$$p_1 = 1.0762$$

$$p_2 = 0.0081$$

Which is graphed in Figure 1