Réécriture des équations différentielles sans dimension :

Barrière : ① $\ddot{q}_1 + \beta_1 \dot{q}_1 + \beta_{12} \dot{q}_2 - \omega_1^2 q_1 = x_1(t)$

Puit : ② $\ddot{q}_2 + \beta_2 \dot{q}_2 + \beta_{21} \dot{q}_1 - \omega_2^2 q_2 = x_2(t)$

On pose $2\xi_i = \dfrac{\beta_i}{\omega_i}$; $\lambda = \dfrac{\omega_2}{\omega_1}$; $\tau = \omega_1 t$

$$\frac{1}{\omega_1^2} \frac{d^2 q_1}{dt^2} + \frac{\beta_1}{\omega_1^2}\frac{dq_1}{dt} + \frac{\beta_{12}}{\omega_1^2}\frac{dq_2}{dt} - q_1 = \frac{x_1(t)}{\omega_1^2}$$

$$\frac{1}{\omega_2^2} \frac{d^2 q_2}{dt^2} + \frac{\beta_2}{\omega_2^2}\frac{dq_2}{dt} + \frac{\beta_{21}}{\omega_2^2}\frac{dq_1}{dt} - q_2 = \frac{x_2(t)}{\omega_2^2}$$

On pose $\begin{pmatrix} \beta_1 & \beta_{12} \\ \beta_{21} & \beta_2 \end{pmatrix} = \begin{pmatrix} \beta_1 & \beta_{12} \\ \beta_{21} & \sigma\beta_1 \end{pmatrix}$

et de plus, on a : $\beta_{12} = \beta_{21} = \beta$

et $\beta_{12}^2 < \beta_1\beta_2 \Rightarrow \beta_{12}^2 = c\,\beta_1\beta_2$

$$\beta^2 = c\sigma\beta_1^2$$

$$\beta = \sqrt{c\sigma}\,\beta_1$$

$2\xi_1 = \dfrac{\beta_1}{\omega_1}$ et $2\xi_2 = \dfrac{\beta_2}{\omega_2} = \dfrac{\sigma\beta_1}{\omega_1}\dfrac{\omega_1}{\omega_2}$

$$2\xi_2 = 2\sigma\lambda\xi_1$$

$$\frac{1}{\omega_1^2 Q_{01}} \frac{d^2 q_1}{dt^2} = \frac{d^2 Q_1}{d\tau^2}$$

$$\frac{\beta_1}{\omega_1^2 Q_{01}} \frac{dq_1}{dt} = 2x \frac{dQ_1}{d\tau}$$

$$\frac{\beta_{12}}{Q_{01}\omega_1^2}\frac{dq_2}{dt} \underset{\substack{\uparrow \\ Q_{01} = kQ_{02}}}{=} \frac{\sqrt{c\sigma}}{k}\frac{\beta_1}{\omega_1}\frac{dQ_2}{d\tau} = \frac{2\sqrt{c\sigma}}{k} x \frac{dQ_2}{d\tau}$$

$$\frac{r_1(t)}{Q_{01}\omega_1^2} = R_1(t).$$

Barrière $\Rightarrow$ $$\boxed{\frac{d^2Q_1}{d\tau^2} + 2x\frac{dQ_1}{d\tau} + \frac{2\sqrt{c\sigma}}{k} x \frac{dQ_2}{d\tau} - Q_1 = R_1(t)}$$

$$\frac{1}{Q_{02}\omega_2^2}\frac{d^2q_2}{dt^2} = \frac{1}{Q_{02}\omega_1^2\lambda^2}\frac{d^2q_2}{dt^2} = \frac{1}{\lambda^2}\frac{d^2Q_2}{d\tau^2}$$

$$\frac{\beta_2}{Q_{02}\omega_2^2}\frac{dq_2}{dt} = \frac{2\sigma\lambda x}{\lambda}\frac{dQ_2}{d\tau} = 2\sigma x\frac{dQ_2}{d\tau}$$

$$\frac{\beta_{21}}{Q_{02}\omega_2^2}\frac{dq_1}{dt} = \frac{k\sqrt{c\sigma}}{\lambda^2}\frac{\beta_1}{\omega_1}\frac{dQ_1}{d\tau} = \frac{2k\sqrt{c\sigma}}{\lambda^2} x \frac{dQ_1}{d\tau}$$

$$\frac{r_2(t)}{Q_{02}\omega_2^2} = R_2(t) \left(= \frac{r_2(t)}{kQ_{02}\lambda^2\omega_1^2}\right)$$

Puit $\Rightarrow$ $$\boxed{\frac{d^2Q_2}{d\tau^2} + 2\sigma x\frac{dQ_2}{d\tau} + \frac{2k\sqrt{c\sigma}}{\lambda^2} x \frac{dQ_1}{d\tau} + Q_2 = R_2(t)}$$