# FIZIKA ENAČBE

**ODVODI** $\left(f'(x) = \frac{df}{dx}\right)$

$f \pm g = f' \pm g'$

$c \cdot f = c \cdot f'$

$f \cdot g = f' \cdot g + f \cdot g'$

$\frac{f}{g} = \frac{f' \cdot g - f \cdot g'}{g^2} \quad (g \neq 0)$

$f \circ g = (f' \circ g) \cdot g' = f'(g) \cdot g'$

$c' = 0$

$x' = 1$

$(x^n)' = n \cdot x^{n-1}$

$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}, \ x \neq 0$

$(\sqrt{x})' = \frac{1}{2\sqrt{x}}, \ x > 0$

$(\sqrt[n]{x})' = \frac{1}{n\sqrt[n]{x^{n-1}}}, \ x > 0$

$(\sin x)' = \cos x$

$(\sin(ax))' = a\cos(ax)$

$(\cos x)' = -\sin x$

$(\cos(ax))' = -a \cdot \sin(ax)$

$(\tan x)' = \frac{1}{\cos^2 x}, \ x \neq \frac{\pi}{2} + k\pi, \ k \in \mathbb{Z}$

$(\mathrm{ctan}\, x)' = -\frac{1}{\sin^2 x}, \ x \neq k\pi, \ k \in \mathbb{Z}$

$(e^x)' = e^x$

$(e^{kx})' = ke^{kx}$

$(a^x)' = a^x \ln a, \ a > 0$

$(x^x)' = x^x(1 + \ln x), \ x > 0$

$(\ln x)' = \frac{1}{x}, \ x > 0$

$(\log_a x)' = \frac{1}{x \ln a}, \ a, x > 0$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$ $\Big\}\ |x| < 1$

$(\arctan x)' = \frac{1}{1+x^2}$

$(\mathrm{arcctan}\, x)' = -\frac{1}{1+x^2}$

**INTEGRALI**

$\int f(x)dx = F(x) + C$

$\frac{dF}{dx} = f(x)$

$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a) = P$

$\int k\,dx = k \cdot x + C$

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$

$\int \frac{1}{x} dx = \ln|x| + C$

$\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C$

$\int e^x dx = e^x + C$

$\int e^{nx} dx = \frac{e^{nx}}{n} + C$

$\int x \cdot e^x dx = e^x(x-1) + C$

$\int a^x dx = \frac{a^x}{\ln a} + C$

$\int \ln x\, dx = x \cdot \ln x - x + C$

$\int \log_a x\, dx = x \cdot \log_a x - \frac{x}{\ln a} + C$

$\int \sin x\, dx = -\cos x + C$

$\int \sin(nx) dx = -\frac{\cos(nx)}{n} + C$

$\int \cos x\, dx = \sin x + C$

$\int \cos(nx) dx = \frac{\sin(nx)}{n} + C$

$\int \tan x\, dx = -\ln|\cos x| + C$

$\int \mathrm{ctan}\, x\, dx = \ln|\sin x| + C$

## PREMO GIBANJE

$v = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$

$a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$

**enakomerno**

$x = v \cdot t + x_0, \ v = \text{konst.}, a = 0$

**enakomerno pospešeno**

$\Delta x = v_0 t \pm \frac{at^2}{2} + x_0 = \frac{v_k^2 - v_z^2}{2a}$

$\Delta v = v_0 + at, \ v^2 = v_0^2 \pm 2a\Delta x$

$v(t) = \dot{x}(t) = \frac{dx}{dt}$

$a(t) = \dot{v}(t) = \frac{dv}{dt} = \ddot{x}(t) = \frac{d^2x}{dt^2}$

$x(t) = x(t_0) + \int_{t_0}^{t} v(t')dt'$

$v(t) = v(t_0) + \int_{t_0}^{t} a(t')dt'$

**prosti pad**

$v = v_0 + g \cdot t, \ v = \sqrt{2gh}$

$h = \frac{gt^2}{2} = h_0 + v_0 t$

**navpični met**

$v = v_0 - gt, \ v = \sqrt{2gh}$

$h = v_0 t - \frac{gt^2}{2} + h_0$

$v^2 = v_0^2 - 2gh, \ t = \sqrt{\frac{2h}{g}}$

$h_{max.} = \frac{v_0^2}{2g}, \ v(t) = \dot{h}$

**kroženje**

$l = R \cdot \varphi = \frac{v_k^2 - v_z^2}{2a_t}, \ v = R \cdot \omega$

$\omega = \frac{\varphi}{t} = 2\pi\nu \,[\mathrm{Hz}] = \dot{\varphi}, \ \alpha = \frac{d\omega}{dt}$

$a_r = R \cdot \omega^2 = \frac{v^2}{R} = \omega \cdot v$

$a = \sqrt{a_r^2 + a_t^2}, \ a_t = R \cdot \alpha$

**enakomerno kroženje**

$\alpha = 0, \ \omega = \omega_0 = \text{konst.}, a_r = R\omega_0^2$

$\varphi = \omega_0 \cdot t = \frac{\omega_k^2 - \omega_z^2}{2\alpha}$

$v = \frac{\omega_0}{2\pi}, \ t_0 = \frac{l}{v} = \frac{2\pi}{\omega}$

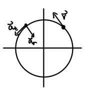
**enakomerno pospešeno kroženje**

kotni posp. ... a(t)    kotna hitrost ... v(t)

$\alpha = \frac{d\omega}{dt}, \ \omega = \omega_0 + \alpha \cdot t$

... x(t)

$\varphi = \omega_0 \cdot t + \frac{\alpha t^2}{2}$



$a_t = R \cdot \alpha = \text{konst.}$

$v = v_0 + a_t \cdot t$

$l = v_0 \cdot t + \frac{a_t \cdot t^2}{2}$ $\quad \omega = \dot{\varphi} \quad v = \dot{l}$, $\quad \alpha = \dot{\omega} \quad a_t = \dot{v}$

$\omega^2 = \omega_0^2 + 2\alpha\varphi, \ v_1^2 = v_0^2 + 2a_t \cdot l$

$\vec{r}(t) = (R \cdot \cos\varphi, R \cdot \sin\varphi), \quad \vec{v}(t) = (-R \cdot \omega \cdot \sin\varphi, R \cdot \omega \cdot \cos\varphi)$

$\vec{a}(t) = (-R \cdot \omega^2 \cdot \cos\varphi - R \cdot \alpha \cdot \sin\varphi,$
$\quad -R \cdot \omega^2 \cdot \sin\varphi + R \cdot \alpha \cdot \cos\varphi)$

**vodoravni met**

$x = v_x \cdot t, \ v_x = \text{konst.}$

$y = y_0 - \frac{1}{2}gt^2, \ v_y = gt$

**poševni met**

$x = (v_0 \cdot \cos\alpha) \cdot t, \ x_{max} = \frac{v_0^2 \cdot \sin(2x)}{g}$

$y = (v_0 \cdot \sin\alpha) \cdot t - \frac{1}{2}gt^2$

$y_{max} = \frac{v_0^2 \cdot \sin^2\alpha}{2g}$

**krivo gibanje v 2D**

$\vec{r} = (x, y)$

$\vec{v} = \frac{d\vec{r}}{dt} = (v_x, v_y) = \left(\frac{dx}{dt}, \frac{dy}{dt}\right)$

$\vec{a} = \frac{d\vec{v}}{dt} = (a_x, a_y) = \left(\frac{dv_x}{dt}, \frac{dv_y}{dt}\right)$

$x = \int_0^t |\vec{v}| dt = \int_0^t \sqrt{v_x^2(t) + v_y^2(t)}\, dt$

## SILE

1.N.Z.: $\sum F = 0, \ \vec{a} = 0, \ \vec{v} = 0 \parallel \vec{v} = \text{konst.}$

2.N.Z.: $\sum \vec{F} = m \cdot \vec{a}$

3.N.Z.: $\vec{F}_{12} = -\vec{F}_{21}$

centripetalna sila

$F_c = m \cdot a_r = m \cdot \frac{v^2}{r} = m \cdot \omega^2 \cdot R$

$F_{tr} = F_n \cdot k_t, \ k_e > k_t$

$F_x = F \cdot \cos\varphi, \ F_y = F \cdot \sin\varphi$

$\sin\varphi = \frac{a}{c}, \ \cos\varphi = \frac{b}{c}$

vleka po $\parallel$ podlagi

$a = \frac{F(\cos\varphi + k_{tr} \cdot \sin\varphi)}{m} - k_{tr} \cdot g$

$k_e = \tan(\varphi_{max.})$

**energija in delo** $\quad A_g = -F_g \cdot \Delta h$

$A = \vec{F} \cdot \vec{r} = \vec{F} \cdot \vec{r} \cdot \cos\varphi, \ \Delta W = W_k - W_z$

$A = \Delta W = \Delta W_k + \Delta W_p + \Delta W_{pr} = \Delta W_{meh.}$

$W_k = \frac{mv^2}{2}, \ W_{pr} = \frac{kx}{2}, \ W_p = m \cdot g \cdot h$

$\boxed{\Delta W_{meh.} = 0 \quad W_k = W_z}$ ohranitev mehanske energije (prožni trk)

$P = \frac{dA}{dt} = \dot{A} = \vec{F} \cdot \vec{v} = M \cdot \frac{d\varphi}{dt} = M \cdot \dot{\varphi}$

**en. vlečenje po klancu brez trenja**

$F = F_d = m \cdot g \cdot \sin\varphi, \ F < F_g$

$A = F \cdot x = F_g \cdot h$

**en. pri vrtiljaku**

$W_k = \frac{mv^2}{2} = \frac{m(R\omega)^2}{2} = \frac{m(l \cdot \omega \cdot \sin\varphi)^2}{2}$

$l$ - dolžina vrvi

$W_p = m \cdot g \cdot h = mg(l - l \cdot \cos\varphi)$

$\quad = m \cdot g \cdot l(1 - \cos\varphi)$

**vrtljiva lahka prečka s težko kroglo**

$v_2 = \sqrt{v_1^2 + 4gl}$ $\quad$ os, $l$

$v_2(\varphi) = \sqrt{v_1^2 + 4gl(1 - \cos\varphi)}$

**gravitacija**

$F_g = G \cdot \frac{m_1 \cdot m_2}{r^2} = m \cdot a = m \cdot g$

$W_g = -G \cdot \frac{m_1 \cdot m_2}{r^2}, \ A = \frac{mv^2}{2}$

$g_0 = \frac{G \cdot M}{R^2}, \ F_{g_0} = G\frac{mM}{R^2}$

g v planetovi globini $\quad$ odd. telesa od središča planeta $\quad$ radij

$g(r) = g_0 \frac{r}{R}, \ r < R$

$g(h) = \frac{GM}{(R+h)^2} = g_0\left(\frac{R}{R+h}\right)^2$

* $M_{zemlje} = 6 \cdot 10^{24}\,\mathrm{kg}$

* $R_{zemlje} = 6370\,\mathrm{km}$

* $G = 6{,}67 \cdot 10^{-11} \frac{\mathrm{Nm}^2}{\mathrm{kg}^2}$

**gibanje satelita**

$F_c = F_g$

$\omega = \sqrt{\frac{g_0}{R}} \cdot \left(\frac{R}{R+h}\right)^{\frac{3}{2}}$

$W_k = m \cdot g_0 \frac{R^2}{2r}$

$W_g = -m \cdot g_0 \cdot \frac{R^2}{r} = -2W_k$

$W = W_k + W_g = -W_k = \frac{W_g}{2}$

**Keplerjevi zakoni**

1. $G_1 \quad G_2$, $G = (\pm e, 0)$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \ e^2 = a^2 - b^2$

ploščinska hitrost

2. $\mu = \frac{dS}{dt} = \dot{S} = \frac{\pi ab}{t_0}, \ \varepsilon = \frac{e}{a}$

masa sonca

3. $m_s = \frac{4\pi^2 \cdot r^3}{t_0^2}$ $\quad r$ – razd. med planetom in soncem

**sunek sile in gibalna količina**

$\text{sunek sile} = \vec{F} \cdot t = \int_0^t \vec{F} dt = \vec{G} = m \cdot \Delta\vec{v}$

**neprožni trk** $\quad$ ohranitev G, NE pa Wk

$\vec{v}_k = \frac{m_1 \cdot \vec{v}_1 + m_2 \cdot \vec{v}_2}{m_1 + m_2}, \quad G_{po} = (m_1 + m_2) \cdot v'$

$v' = v \cdot \frac{m_1}{m_1 + m_2}$

**eksplozija**

$G_{pred} = 0, \ G_{po} = m_1 \cdot v_1 - m_2 \cdot v_2, \ \frac{v_1}{v_2} = \frac{m_2}{m_1}$

**prožni trk** $\quad$ ohranitev G in Wk

$v_{1k} = \frac{(m_1 - m_2)v_{1z} + 2m_2 v_{2z}}{m_1 + m_2}$

$v_{2k} = \frac{(m_2 - m_1)v_{2z} + 2m_1 v_{1z}}{m_1 + m_2}$

"dovujeni plastični trk"

**odriv**

$m_1 \cdot v_1 = m_2 \cdot v_2, \quad \frac{W_{k1}}{W_{k2}} = \frac{m_2}{m_1}$

**masno središče in težišče**

$\vec{r}^* = \frac{\sum_{i=1}^N m_i \vec{r}_i}{\sum_{i=1}^N m_i}$ $\quad$ lega skupnega masnega središča

$\vec{v}^* = \frac{\sum_{i=1}^N m_i \vec{v}_i}{\sum_{i=1}^N m_i}$ $\quad$ hitrost in pospešek masnega sr. telesa

$\vec{a}^* = \frac{\sum_{i=1}^N m_i \vec{a}_i}{\sum_{i=1}^N m_i}$

$\vec{r}^* = \frac{1}{m}\int_m \vec{r}\, dm = \frac{1}{m}\int_V \vec{r} \rho\, dV$

$\vec{r}^* = \frac{1}{V}\int_V \vec{r}\, dV$ $\quad$ homogeno telo

$y^* = \frac{1}{S}\int_S y\, dS$ $\quad$ y koord. masn. težišča 2D lika

**gibanje težišča sistema teles**

$\sum \vec{F}_{zun.} = m \cdot \vec{a}$

**navor**

$\vec{M} = \vec{r} \times \vec{F}$ $\quad$ (x - vektorski produkt)

$M = F \cdot r \cdot \sin\varphi$ $\quad$ kot med $\vec{r}$ in $\vec{F}$

$A = F \cdot r \cdot \varphi = \vec{M} \cdot \vec{\varphi}$

$A = \int_0^{\varphi} \vec{M}(\varphi) d\varphi$

$\vec{M}_g = \vec{r} \times \vec{F}_{mg} = m \cdot g \cdot r \cdot \sin\varphi$

**vztrajnostni moment & rotacijska en.**

p vztrajnostni moment

$M = J \cdot \alpha = (m \cdot r^2) \cdot \alpha$

$J = \int dm\, r^2$

$W_k = \frac{J\omega^2}{2}$ $\quad$ rotacijska Wk

$W_k = \frac{mv^2 + J\omega^2}{2}$ $\quad$ celotna Wk

$J = J^* + md^2$ $\quad$ Steinerjev izrek: $\parallel$ premik osi za d → J se poveča

| | |
|---|---|
| tanka palica, dolžine $l$ | $\frac{ml^2}{12}$ |
| polna krogla, radij R | $\frac{2mR^2}{5}$ |
| tanka krogelna lupina, radij R | $\frac{2mR^2}{3}$ |
| poln valj, radij R | $\frac{mR^2}{2}$ |
| poln valj, radij R, višina h | $\frac{mR^2}{4} + \frac{mh^2}{12}$ |
| tanka valjna lupina, radij R | $mR^2$ |
| stožec, radij R | $\frac{3mR^2}{10}$ |

**navor dvojice naspr. enakih sil**

$\alpha = \frac{M}{J^*} = \frac{F \cdot r}{J^*}$ $\quad$ težišče miruje, telo se posp. vrti $\quad$ $-\vec{F}$, m.s., $\vec{F}$

**vrtilna količina**

$\vec{\Gamma} = \vec{r} \times \vec{G} = \vec{r} \times m \cdot \vec{v} \cdot \sin\varphi$ $\quad$ točkasto telo

$\Gamma = J \cdot \omega$ $\quad$ togo telo

$\Delta\vec{\Gamma} = \int_0^t \vec{M} dt = J \cdot \Delta\vec{\omega} = \vec{\Gamma}_k - \vec{\Gamma}_z$

**ravnovesje togega telesa**

$\sum_i \vec{F}_i = 0, \ \sum_i \vec{M}_i = 0$

**kotaljenje**

$v_t = R \cdot \omega$

$W_k = \frac{1}{2}\left(m + \frac{J}{x^2}\right) \cdot v_t^2$ ?

| | |
|---|---|
| tera (T) : $10^{12}$ | deci (d) : $10^{-1}$ |
| giga (G) : $10^9$ | centi (c) : $10^{-2}$ |
| mega (M) : $10^6$ | mili (m) : $10^{-3}$ |
| kilo (k) : $10^3$ | mikro (μ) : $10^{-6}$ |
| hekto (h) : $10^2$ | nano (n) : $10^{-9}$ |
| deka (da) : $10^1$ | piko (p) : $10^{-12}$ |

## MEHANSKO NIHANJE IN VALOVANJE

**sinusno nihanje**

$x = x_0 \cdot \sin(\omega t + \delta)$ $\quad$ fazni zamik $\delta = \frac{\pi}{2} \to$ cos. fja $\quad$ Simbol$_0$ = max.

$v = \frac{dx}{dt} = x_0 \cdot \omega \cdot \cos(\omega \cdot t + \delta) = v_0 \cdot \cos(\omega t + \delta)$

$a = \frac{dv}{dt} = -x_0 \cdot \omega^2 \cdot \sin(\omega t + \delta) = -a_0 \cdot \sin(\omega t + \delta)$

$a = -\omega^2 \cdot x = \ddot{x}$

$F = m \cdot a = -m \cdot \omega^2 \cdot x$

za dovolj majhen kot $\varphi$:

$\cos\varphi = 1 - \frac{\varphi^2}{2}$

$\sin 2\varphi = 2 \sin\varphi \cdot \cos\varphi$

$\cos 2\varphi = \cos^2\varphi - \sin^2\varphi$

$\sin\varphi = \cos\left(\frac{\pi}{2} - \varphi\right)$

$\cos(-x) = \cos x$

$\sin(-x) = -\sin x$

$\sin^2\varphi + \cos^2\varphi = 1$

$\tan\varphi \cdot \mathrm{ctan}\,\varphi = 1$

**vzmetno nihalo**

$F_v = -k \cdot x$

$a = -\frac{kx}{m}$

$\omega = \sqrt{\frac{k}{m}}$

$t_0 = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

$W_{pr} = \frac{kx^2}{2}$

$W_k + W_{pr} = \frac{m \cdot x_0^2 \omega^2}{2}$

sučno nihalo, navor sorazmeren z zasukom iz vodoravne lege

**nihalo na polžasto vzmet**

$M = -D\varphi$ $\quad$ koef. vzmeti [Nm]

$\alpha = -\frac{D}{J} \cdot \varphi = -\Omega^2 \cdot \varphi$ $\quad$ krožna frekvenca

$\Omega = \sqrt{\frac{D}{J}}$

$t_0 = \frac{2\pi}{\Omega} = 2\pi\sqrt{\frac{J}{D}}$

$W_k = \frac{J\omega^2}{2} = \frac{J\varphi_0^2\Omega^2\cos^2(\Omega t)}{2}$

$W_p = \frac{D\varphi^2}{2} = \frac{D\varphi_0^2 \cdot \sin^2(\Omega t)}{2}$

$W_k + W_{pr} = \frac{J\varphi_0^2 \cdot \Omega^2}{2}$

**fizično nihalo** → razsežno togo telo niha okoli vodoravne osi

$M = J \cdot \alpha = -m \cdot g \cdot r^* \cdot \sin\varphi$

$\alpha = -\frac{m \cdot g \cdot r^*}{J} \cdot \varphi$

$\Omega = \sqrt{\frac{m \cdot g \cdot r^*}{J}}$

$t_0 = \frac{2\pi}{\Omega} = 2\pi\sqrt{\frac{J}{m \cdot g \cdot r^*}}$

* $W_p = \frac{m \cdot g \cdot r^* \cdot \varphi^2}{2}$

* $W_k = \frac{J\omega^2}{2}$

* $W_k + W_p = \frac{J \cdot \varphi_0^2 \cdot \Omega^2}{2}$

* **matematično nihalo** → fizično nihalo: niha točkasto telo

$\Omega = \sqrt{\frac{g}{l}}$

$t_0 = \frac{2\pi}{\Omega} = 2\pi\sqrt{\frac{l}{g}}$

## VALOVANJE

$y(x,t) = y_0 \cdot \sin(\omega \cdot t - k \cdot x + \delta)$ $\quad$ → fazni odmik $\quad$ faza

$k = \frac{2\pi}{\lambda}$

$y(x,t) = y_0 \cdot \sin\left(2\pi\left(\frac{t}{t_0} - \frac{x}{\lambda}\right) + \delta\right)$

$y(x,t) = 2y_0 \cdot \sin(\omega t) \cdot \cos(kx)$ $\quad$ → združimo dve valovanji, potujoči v naspr. smer

$c = \frac{\lambda}{t_0} = \lambda \cdot \nu, \ c_0 = 3{,}0 \cdot 10^8 \frac{\mathrm{m}}{\mathrm{s}}$

$v = \frac{c}{\lambda}$

$W = \frac{m \cdot \omega^2 \cdot x_0^2}{2}$

$v_0 = \omega \cdot x_0$

gostota en. $[\frac{\mathrm{J}}{\mathrm{m}^3}]$

$w = \frac{W}{V} = \frac{\rho \cdot \omega^2 \cdot x_0^2}{2}$

gostota en. toka

$j = c \cdot w = \frac{c \cdot \rho \cdot \omega^2 \cdot x_0^2}{2} = \frac{W}{S \cdot t} \qquad j = \frac{P}{4\pi r^2}$

**nihanje**

$\nu = \frac{N}{t} = \frac{1}{t_0} = \frac{\omega_0}{2\pi}\ [\mathrm{Hz}]$

$v = \frac{2\pi r}{t_0} = 2\pi r\nu = r \cdot \omega \ [\frac{\mathrm{m}}{\mathrm{s}}]$

$\omega = 2\pi\nu = \frac{2\pi}{t_0}$

$c = \sqrt{\frac{F \cdot l}{m}}$

$\nu = \frac{\nu_0}{1 \pm \frac{v}{c}}$ $\quad$ premika se odd. zvoka; −: bližanje; +: odd.

$\nu = \nu_0\left(1 \pm \frac{v}{c}\right)$ $\quad$ premika se sprejemnik zvoka; −: odd.; +: bližanje

## EL. NABOJ IN SILA

$J = Nm = VAs$, $V = \frac{kg \cdot m^2}{A \cdot s^3}$

$\varepsilon_0 = 8{,}85 \cdot 10^{-12} \frac{As}{Vm}$

$F_e = \frac{e_1 \cdot e_2}{4\pi \cdot \varepsilon_0 \cdot r^2}$ $[As = C]$

$\vec{F}_e = e \cdot \vec{E}$

$E = \frac{e}{4\pi \cdot \varepsilon_0 \cdot r^2}$

## EL. POTENCIALNA EN.

$A_e = \int_r^{r'} \vec{F}_e\, d\vec{r} = \int_r^{r'} e \vec{E}\, d\vec{r}$

$A = \Delta W_e + \Delta W_k$

$W_e = \frac{e_1 \cdot e_2}{4\pi \cdot \varepsilon_0 \cdot r}$

## EL. POTENCIAL

$V = \frac{W}{e} = -\int \vec{E}\, d\vec{r} = \frac{e}{4\pi \cdot \varepsilon_0 \cdot r}$

## EL. POLJE NA OSI NABITEGA OBROČA

$V = \frac{e}{4\pi \cdot \varepsilon_0 \cdot \sqrt{R^2 + z^2}}$

$z \approx r$ (zelo vel. razd.)

$V = \frac{e}{4\pi \cdot \varepsilon_0 \cdot z} \approx \frac{e}{4\pi \cdot \varepsilon_0 \cdot r}$

$E_z = \frac{e \cdot \cos\varphi}{4\pi \cdot \varepsilon_0 \cdot r^2} = \frac{e \cdot z}{4\pi \cdot \varepsilon_0 (R^2 + z^2)^{3/2}}$

max. $z_0 = \frac{R}{\sqrt{2}} \rightarrow E_0 = \frac{e}{6\sqrt{3}\pi \cdot \varepsilon_0 \cdot R^2}$

## EL. POLJE NABITE KROGLE

r<R: $E = \frac{er}{4\pi \varepsilon_0 R^3}$ "znotraj krogle"

$E_0 = \frac{e}{4\pi \varepsilon_0 R^2}$ "na lupini, r=R"

$E = E_0 \cdot \frac{r}{R}$

r>R: $E = E_0 \left(\frac{R}{r}\right)^2$

## EL. POLJE NABITE PLOŠČE

$E = \frac{e_1}{2\pi \varepsilon_0 \cdot R^2}\left(1 - \frac{z}{\sqrt{R^2+z^2}}\right)$

$F = \frac{e_1 \cdot e_2}{2\pi \varepsilon_0 \cdot R^2}\left(1 - \frac{z}{\sqrt{R^2+z^2}}\right)$

velike plošče: $\sigma = \frac{e}{S} = \frac{e}{\pi R^2}$

$\hookrightarrow E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{R^2+z^2}}\right)$

$z \ll R$: $E = \frac{e}{2\varepsilon_0 \cdot S} = \frac{\sigma}{2\varepsilon_0}$

## EL. POLJE NABITEGA VALJA

r=R: $E_0 = \frac{\mu}{2\pi \varepsilon_0 \cdot R}$

r<R: $E = E_0 \cdot \frac{r}{R}$

r>R: $E = E_0 \cdot \frac{R}{r}$

## JAKOST EL. POLJA

$\oint_S \vec{E} \cdot d\vec{S} = \frac{e}{\varepsilon_0}$

npr. $\int_{plašč} E\, dS = E\, 2\pi r d$

$e = \mu \cdot d \longrightarrow E = \frac{\mu}{2\pi \varepsilon_0 r}$

## EKVIPOTENCIALNE PLOSKVE

↳ "točke v prostoru z enakim U"

↳ "pravokotne na silnice"

## EL. NAPETOST

$U_{12} = V_2 - V_1 = -\int_{x_1}^{x_2} \vec{E}\, d\vec{r} = -E(x_2 - x_1) = -Ed$

$U = E \cdot d$

## EL. KONDENZATOR

$E = \frac{e}{\varepsilon_0 S}$, $U = E \cdot d = \frac{A_e}{e}$

$e = \varepsilon_0 \cdot S \cdot \frac{U}{d} = C \cdot U$

$C = \frac{\varepsilon_0 \cdot S}{d}$

## EN. KONDENZ. IN EL. POLJA

$A_e = F_e \cdot x = eEx = -e \cdot \Delta V = -eU$

$dA_e = U de'$

$A_e = \int U de' = \int \frac{e'}{C} de' = \frac{e^2}{2C}$

$W_e = \frac{e^2}{2C} = \frac{1}{2} CU^2 = \int w_e\, dV$ (gost. en.)

$w_e = \frac{W_e}{V} = \frac{CU^2}{2V} = \frac{\varepsilon_0 S U^2}{2 d S d} =$ ↳ gost. en.

$= \frac{1}{2}\varepsilon_0 \left(\frac{U}{d}\right)^2 = \frac{1}{2}\varepsilon_0 E^2$

## EL. SILA MED PLOŠČAMA KOND.

$F_e = e \cdot E = \frac{e^2}{2\varepsilon_0 S}$

## A PRI RAZMIKU PLOŠČ S KONST. e

$A = \frac{e^2}{2\varepsilon_0 S}(d_2 - d_1) = \frac{e^2}{2C_2} - \frac{e^2}{2C_1} = \Delta W_e$

## A PRI RAZMIKU PLOŠČ S KONST. U

$A = \int_{d_1}^{d_2} \frac{e^2}{2\varepsilon_0 S} dx = \dots = -\Delta W_e$

## GIBANJE EL. NABOJA V KOND.

$v = \sqrt{v_0^2 \pm \left|\frac{2eUx}{md}\right|}$ x - pospešena razd., d - dol. razd.

$F_e = e \cdot E = \cancel{m} \cdot a$ zanemarimo

$\Delta W_k = e_0 \cdot U$

$v_x = v_0 = \sqrt{\frac{2e_0 U}{m}} \longrightarrow t = \frac{l}{v_0}$

$F_y = e \cdot E = e \cdot \frac{U}{d}$, $a_y = \frac{F_y}{m} = \frac{eU}{md}$

$t = \frac{l}{v_0} \rightarrow v_y = a_y \cdot t = \frac{eUl}{mdv_0}$

$y = \frac{a_y \cdot t^2}{2} = \frac{eUl^2}{2mdv_0^2}$ premik proti nasp. nabiti plošči

$\varphi = \arctan\frac{v_y}{v_x} = \arctan\frac{eUL}{dmv_0^2}$

## DIELEKTRIK V KONDENZATORJU

$\vec{p} = e \cdot \vec{r}$ $[Asm]$ dipolni moment

$\vec{P} = \frac{\vec{p}}{V}$ $\left[\frac{As}{m^2}\right]$ polarizacija - volumska gost. induciranega el. dipolnega momenta

$E = \frac{E_0}{\varepsilon}$ — E brez dielektrika — dielektričnost

$C = \frac{\varepsilon \cdot \varepsilon_0 \cdot S}{d}$

$w_e = \frac{1}{2}\varepsilon \cdot \varepsilon_0 \cdot E^2$ volumska gostota el. potencialne en.

## GOST. EL. POLJA / GAUSS LAW

$\vec{D} = \varepsilon \varepsilon_0 \vec{E}$

$\vec{P} = \varepsilon_0(\varepsilon - 1)\vec{E}$

$\vec{D} = \frac{e}{4\pi r^2} \cdot \frac{\vec{r}}{r}$ gost. polja v okolici točkastega naboja v središču koord. sistema

$\int D\, dS = D \cdot 4\pi r^2 = e$

$\oint_S \vec{D} \cdot d\vec{S} = \int_S d\Phi_e = e$ GAUSSOV ZAKON

## ELEKTRIČNI TOK

$I = \frac{de}{dt} = \frac{e}{t}$ $[A]$

$j = \frac{dI}{dS}$ $\left[\frac{A}{m^2}\right]$ ploskovna gostota toka

$n = \frac{N}{V}$ gostota nosilcev naboja

$j = \frac{I}{S} = \frac{e}{S \cdot t} = \frac{N \cdot e_0}{S \cdot t}$ homogen. in konst. tok

$j = \frac{n \cdot V \cdot e_0}{S \cdot t} = \frac{n \cdot S \cdot x \cdot e_0}{S \cdot t} = \frac{n \cdot v \cdot t \cdot e_0}{t} = n \cdot v \cdot e_0$

$v = \mu \cdot E$ $\left[\frac{m^2}{Vs}\right]$

$j = n \cdot v \cdot e_0 = n \cdot \mu \cdot E \cdot e_0 = n \cdot e_0 \cdot \mu \cdot \frac{U}{L}$

$I = n \cdot e_0 \cdot \mu \cdot \frac{S}{L} \cdot U = \frac{U}{R}$

## EL. UPORNOST IN EL. MOČ

$U = R \cdot I$, $R = \frac{L}{n \cdot e_0 \cdot \mu \cdot S} = \frac{\zeta L}{S}$ $[\Omega]$

$\zeta = \frac{1}{n \cdot e_0 \cdot \mu}$

ZAP: $R_s = R_1 + R_2 + R_3 + \dots$

VZP: $\frac{1}{R_s} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$

$R\ [\Omega = \frac{V}{A}]$, $\zeta\ [\Omega m = \frac{\Omega mm^2}{m}]$

spec. prevodnost snovi "siemens"

$\sigma = \frac{1}{\zeta} = n \cdot e_0 \cdot \mu$ $\left[\frac{1}{\Omega m} = S\right]$

$P_e = U \cdot I$ $[W = V \cdot A]$

$P = R \cdot I^2 = \frac{U^2}{R}$

## KIRCHHOFFOVA ZAKONA

1) v razvejišču: $I_{vhodni} = I_{izhodni}$

2) v sklenjeni zanki v vezju: $\Sigma U = 0$
pazimo na predznak, če je rezultat
↳ ⊕ je prav, če pa ⊖, sklepamo na pravo smer toka

## PRAZNJENJE/POLNJENJE KOND.

praznjenje

$e_0 = C \cdot U_0$, $\tau = R \cdot C$ → karakteristični čas, čas praznjenja

$I = I_0 \cdot e^{-\frac{t}{\tau}}$, $I_0 = \frac{U_0}{R} = \frac{e_0}{\tau}$ zač. tok in upor

$e = e_0 \cdot \exp(-\frac{t}{\tau})$, $U_c = U_0 \cdot \exp(-\frac{t}{\tau})$

polnjenje

$e = \cancel{e_0 - e_0^{-\frac{t}{\tau}}}$ $e_0(1 - \exp(-\frac{t}{\tau}))$

$U_c = \cancel{U_0 - U_0^{-\frac{t}{\tau}}}$ $U_0(1 - \exp(-\frac{t}{\tau}))$

↳ $e^x$, $e = 2{,}7\dots$

## MAGNETNI DIPOLI

$\vec{F}_{12} = \frac{3\mu_0}{4\pi r^5}\left((\vec{p}_{m1} \cdot \vec{r}_{12})\vec{p}_{m2} + (\vec{p}_{m2} \cdot \vec{r}_{12})\vec{p}_{m1} + (\vec{p}_{m1} \cdot \vec{p}_{m2})\vec{r}_{12} - \frac{5(\vec{p}_{m1} \cdot \vec{r}_{12})(\vec{p}_{m2} \cdot \vec{r}_{12})}{r^2}\vec{r}_{12}\right).$

↳ sila med dvema dipoloma

## MAGNETNO POLJE

$\vec{F} = I \cdot \vec{l} \times \vec{B}$ $\sin\alpha$

$\vec{B}(r) = \frac{\mu_0}{4\pi}\left(\frac{3(\vec{p}_m \cdot \vec{r}) \cdot \vec{r}}{r^5} - \frac{\vec{p}_m}{r^3}\right)$

↳ gost. mag. polja okoli magn. dipola v (0,0), za razd., velike v primerjavi z dimenz. dipola

## IZVORI MAGNETNEGA POLJA

polje okoli ravnega vodnika

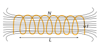
$B = \frac{\mu_0 \cdot I}{2\pi r}$

$\vec{p}_m = I \cdot \vec{S}$ → ⊥ na ravnino zanke

↳ žična zanka, po kateri teče tok I, povzroča na velikih razdaljah enako magnetno polje kot točkasti dipol z $\vec{p}_m$

polje znotraj in okoli tuljave

$B = \frac{\mu_0 \cdot N \cdot I}{l}$



$\vec{p}_{mt} = N \cdot I \cdot \vec{S}$

## MAGNETNA SILA

$\vec{F}_m = e \cdot \vec{v} \times \vec{B}$ → vloga centripetalne sile, $F_m$ ⊥ le smer od v nabit delec vstopi v hom. $\vec{B}$,

$m \cdot v = e \cdot R \cdot B$ → ⊥ na smer polja

$\vec{F} = e \cdot \vec{E} + e \cdot \vec{v} \times \vec{B}$ → Lorentzova sila: el. + mag. sila

## MAG. SILA NA VODNIK S TOKOM

$F_m = N \cdot e_0 \cdot v \cdot B = n \cdot V \cdot e_0\, v \cdot B = n \cdot S \cdot L \cdot e_0 \cdot v \cdot B$

↳ privzamemo: hitrost ⊥ polje

gost. el.: $n = \frac{N}{V} = \frac{N}{S \cdot L}$

→ gost. toka

$j = n \cdot e_0 \cdot v \longrightarrow v = \frac{j}{n \cdot e_0}$

$F_m = I \cdot l \cdot B$

$F = \frac{\mu_0 \cdot I_1 \cdot I_2 \cdot L}{2\pi R}$ → sila med žicama

$I_1 \uparrow\uparrow I_2$ → privlak; $I_1 \uparrow\downarrow I_2$ → odboj

## MAG. NAVOR IN MAG. $W_p$

$M = N \cdot I \cdot S \cdot B \cdot \sin\varphi$

$\vec{M} = \vec{p}_m \times \vec{B}$

$W_m = -\vec{p}_m \cdot \vec{B}$

## SNOV V MAG. POLJU

tuljava

$B = \frac{\mu_0 \cdot N \cdot I}{l}$

$\vec{p}_m = N \cdot I \cdot \vec{S}$ pl. enega ovoja

$L = \frac{\mu_0 \cdot N^2 \cdot S}{l}$ $\left[\frac{Vs}{A} = H\right]$ induktivnost tuljave

$W_m = \frac{LI^2}{2}$

$W_m = \frac{B^2}{2\mu_0}$ → volumska gost. mag. potenc. energije

## MAGNETIZACIJA

magnetizacija

$\vec{M} = \frac{\vec{p}_m}{V}$ → volumska gostota induciranih mag. momentov v snovi

$\vec{B} = \mu_0 \cdot \vec{H} + \mu_0 \cdot \vec{M} = \mu \cdot \mu_0 \cdot H$ (permeabilnost, jakost mag. polja)

$\vec{M} = (\mu - 1)\vec{H}$

$L = \frac{\mu \cdot \mu_0 \cdot N^2 \cdot S}{l}$ → indukt. snovi

$w_m = \frac{B^2}{2\mu\mu_0} = \frac{H \cdot B}{2}$ → gost. en. mag. polja

## FEROMAGNETNE SNOVI

$\mu < 1$: diamagnetne

$\mu > 1$: paramagnetne

feromagnetne: zveza med $\vec{M}$ in $\vec{H}$ ni linearna

$\vec{B} = \mu(H) \cdot \mu_0 \cdot \vec{H}$

histereznost: $\vec{H} = 0$, $\vec{M} \neq 0$

## MAGNETNI PRETOK

$\Phi_m = \vec{B} \cdot \vec{S} = \int_S \vec{B}\, d\vec{S}$ $[Vs = Wb]$ "veber"

$\oint \vec{B}\, d\vec{S} = \oint d\Phi_m = 0$

## INDUKCIJA

$U_i = -\frac{d\Phi_m}{dt}$

↳ $U_i$ nasprotuje $\Delta\Phi_m$

$F_m = I \cdot l \cdot B$

$P = F_m \cdot v = IlBv = U_i \cdot I$

$U_i = B \cdot l \cdot v = \omega \cdot S \cdot B \cdot \sin\omega t$

$\Phi_m = N \cdot \Phi_1 = N \cdot \vec{B} \cdot \vec{S}$

$\Phi_m = N \cdot \frac{\mu_0 \cdot N \cdot I}{l} \cdot S = \frac{\mu_0 \cdot N^2 \cdot S}{l} \cdot I$

$\Phi_m = L \cdot I$, $U_i = -\frac{d\Phi_m}{dt} = -L \cdot \frac{dI}{dt}$

## SUNEK $U_i$

$\Delta\Phi_m = \int U_i\, dt$

$\int_0^t U_i(t)\, dt = \Delta\Phi_m = NSB \cdot \cos\varphi$

## IZMENIČNI U, I IN $P_e$

$U = U_0 \cdot \cos(\omega t)$
$I = I_0 \cdot \cos(\omega t)$
amplituda; $\omega = 2\pi\nu$ "krožna frekvenca"

$U = U_0 \cdot \cos(\omega t + \delta)$ → fazni premik: $+\delta < \pi$ → U prehiteva I, else → zaostaja

$\bar{P} = \frac{U_0 \cdot I_0}{2} \cdot \cos\delta = I_{ef} \cdot U_{ef} \cdot \cos\delta$ $\delta = 0$ → max. P, $\delta = \frac{\pi}{2}$ → P=0

$I_{ef} = \frac{I_0}{\sqrt{2}}$, $U_{ef} = \frac{U_0}{\sqrt{2}}$

## IMPEDANCA (izmenični/sinusni tok)

$e^{i\varphi} = \cos\varphi + i \cdot \sin\varphi$

$I = I_0 \cdot e^{i\omega t}$

$U(t) = R \cdot I(t)$

ohmski upornik

$U_0 \cdot e^{i\omega t} = R \cdot I_0 \cdot e^{i\omega t}$, $\delta = 0$

kondenzator

$e = \frac{I_0}{i\omega} e^{i\omega t}$

$U = \frac{e}{C} = \frac{I_0}{i\omega C} \cdot e^{i\omega t}$

impedanca

$Z_c = \frac{1}{i\omega C}$

$I = Re(I_0 \cdot e^{i\omega t})$

$U = Re(\frac{I_0}{i\omega C} e^{i\omega t}) = \frac{I_0}{\omega C}\sin(\omega t) = \frac{I_0}{\omega C}\cos(\omega t - \frac{\pi}{2})$

tuljava

$U = L \cdot \frac{dI}{dt} = L \cdot \frac{d}{dt}(I_0 \cdot e^{i\omega t}) = i\omega L \cdot I_0 \cdot e^{i\omega t}$

$Z_L = i\omega L$

$U = Re(i\omega L \cdot I_0 \cdot e^{i\omega t}) = -\omega L I_0 \sin(\omega t)$

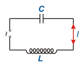
$= \omega L I_0 \cos(\omega t + \frac{\pi}{2})$, $\delta = \frac{\pi}{2}$

## VEKTORJI

$|\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \cdot \sin\varphi$

vezava kondenzator & tuljava

$Z = \frac{1}{i\omega C} + i\omega L = i(\omega L - \frac{1}{\omega C})$



IF $\omega^2 = \frac{1}{LC}$ $\Rightarrow Z = Z_L + Z_c = 0$

vezava ohmski upornik in kondenzator

$U = I \cdot R + I \cdot Z_c$

↳ $I = \frac{U}{R + Z_c} = \frac{U_0 \cdot e^{i\omega t}}{R + \frac{1}{i\omega C}}$

$I = U_0 \cdot \frac{\cos(\omega t + \delta)}{\sqrt{R^2 + \frac{1}{(\omega C)^2}}}$, $\delta = \arctan(\frac{1}{\omega RC})$

$Z = \frac{U_0}{I_0} = \sqrt{R^2 + Z_c^2} = \sqrt{R^2 + \frac{1}{(\omega C)^2}}$

$Z_r = R$

vezava ohmski upornik, kondenzator in tuljava

$|Z| = \sqrt{R^2 + (\frac{1}{\omega C} - \omega L)^2}$

$I = I_0 \cdot \cos(\omega t)$

$U = U_0 \cdot \cos(\omega t + \delta) = |Z| \cdot I_0 \cdot \cos(\omega t + \delta)$

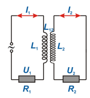
$\delta = \arctan\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$

## INDUKTIVNA SKLOPITEV TOKOKROGOV

$I_1 = \frac{U_1}{R_1} = \frac{U_0}{R_1}\cos(\omega t) = I_{01}\cos(\omega t)$

$I_2 = I_{02}\sin(\omega t)$

↳ oba sinusna, med seboj fazno zamaknjena, razl. amplitudi



## TRANSFORMATOR

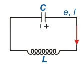
$\frac{U_1}{U_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1}$

$P_1 = P_2(\eta \cdot P_1)$ ?

## ELEKTRIČNI NIHAJNI KROG

$U_c + U_L = -L\frac{dI}{dt} - \frac{e}{C} = 0$

$I = I_0 \cdot \sin(\omega t)$



$\omega = \frac{1}{\sqrt{L \cdot C}}$

$W_m = \frac{LI^2}{2} = \frac{LI_0^2}{2}\sin^2(\omega t)$

$W_e = \frac{CU^2}{2} = \frac{CU_0^2}{2}\cos^2(\omega t) = \frac{LI_0^2}{2} \cdot \cos(\omega t)$

$U_0 = \sqrt{\frac{L}{C}} I_0$

$W_m + W_e = \frac{LI_0^2}{2} = \frac{CU_0^2}{2}$

## KONSTANTE

$c = 3{,}00 \cdot 10^8 \frac{m}{s}$

$e_0 = 1{,}6 \cdot 10^{-19}$ As

$\varepsilon_0 = 8{,}85 \cdot 10^{-12} \frac{As}{Vm}$

$\mu_0 = 4\pi \cdot 10^{-7} \frac{Vs}{Am}$

$m_u = u = 1{,}66054 \cdot 10^{-27}$ kg $= 931{,}494$ MeV/$c^2$

$m_e = 9{,}109 \cdot 10^{-31}$ kg $= \frac{1u}{1823}$

$m_p = 1{,}67262 \cdot 10^{-27}$ kg $= 1{,}00728u$

$m_n = 1{,}67493 \cdot 10^{-27}$ kg $= 1{,}00866u$

## KIRCHHOFF ADDENDUM

- Izberemo zanko za 2.K.Z., določimo ⊕ smer
- gremo skozi upornik: $\cdot$ v⊕: $-I \cdot R$ $\cdot$ v⊖: $+I \cdot R$
- gremo skozi baterijo v ⊕ smeri: $\cdot$ iz - v +: +U $\cdot$ iz + v -: -U
- zapišemo enačbo $\xrightarrow{+1.K.Z}$ sistem enačb

## VZP. / ZAP. VEZAVA

| | KONDENZATOR | UPORNIK |
|---|---|---|
| V | $U = U_1 = U_2$<br>$e_s = e_1 + e_2$<br>$C_s = C_1 + C_2$ | $U = U_1 = U_2$<br>$I = I_1 + I_2$<br>$\frac{1}{R_s} = \frac{1}{R_1} + \frac{1}{R_2}$ |
| Z | $U = U_1 + U_2$<br>$e = e_1 = e_2$<br>$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$ | $U = U_1 + U_2$<br>$I = I_1 = I_2$<br>$R_s = R_1 + R_2$ |

## ODVODI VS. INTEGRALI

1) Majhna Δ
2) naklon premice
3) deljenje

1) S pod krivuljo
2) Akumulacija
3) Množenje