Electric Machinery

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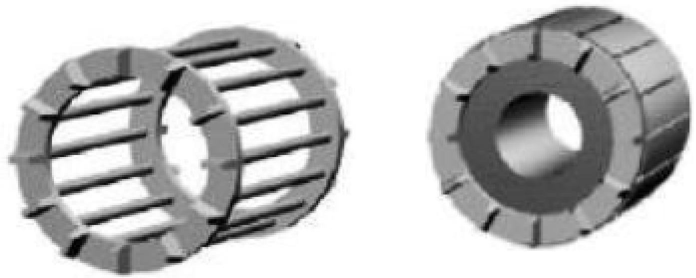
Induction machines are mostly used as motors.

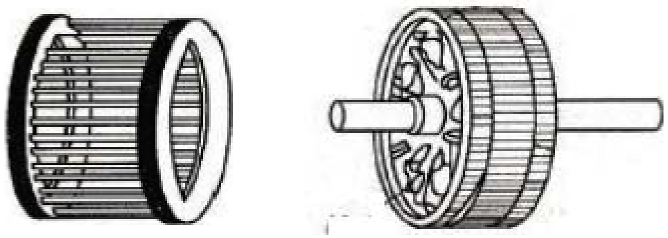
Structures and Basic Principle

Squirrel-Cage Rotor

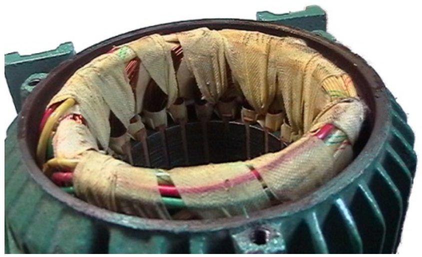
Cast Aluminum

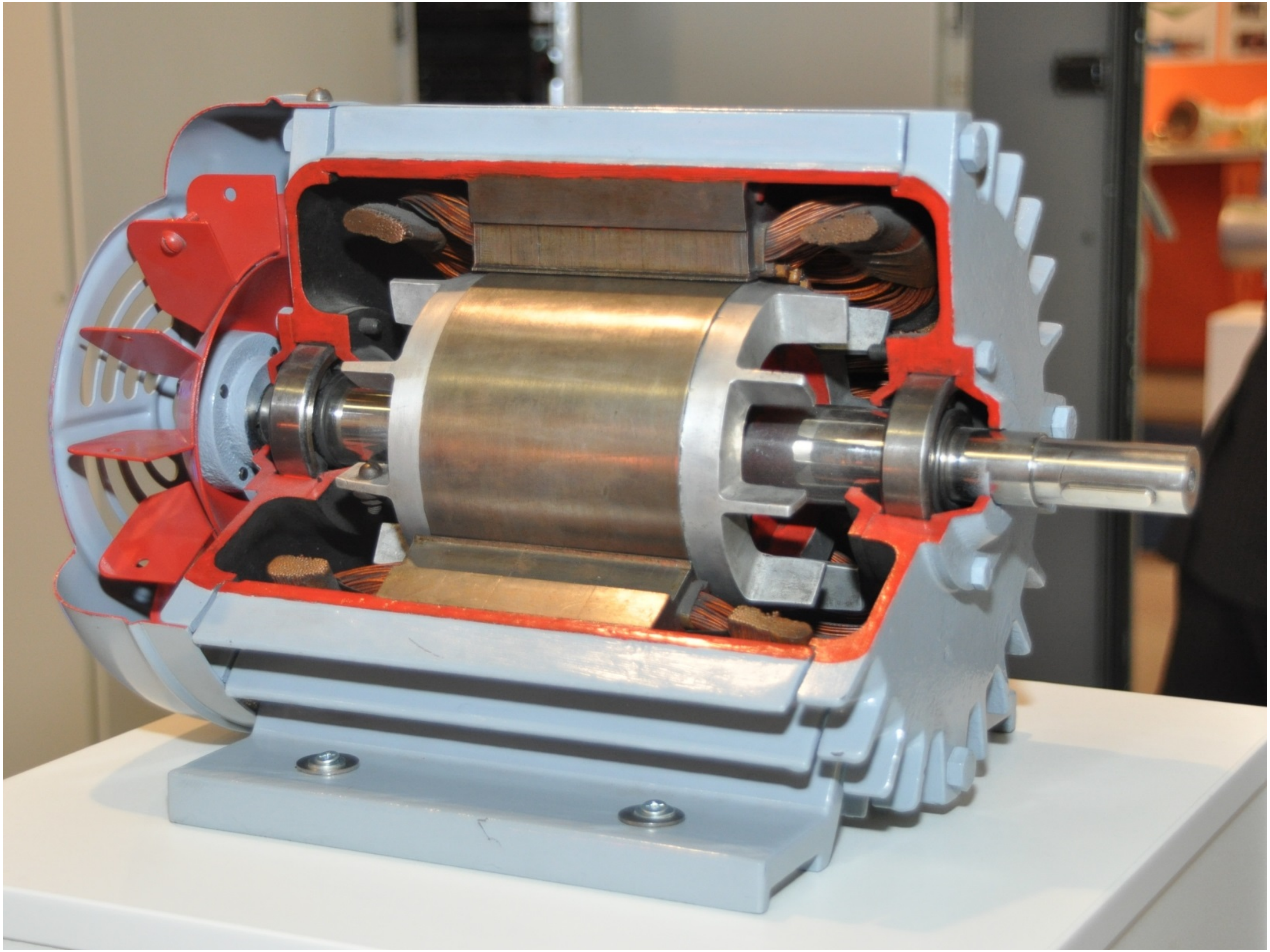


Copper Bar



Wound Rotor





Structures and Basic Principle

Feature: mechanical speed $n \neq$ synchronous speed n_s

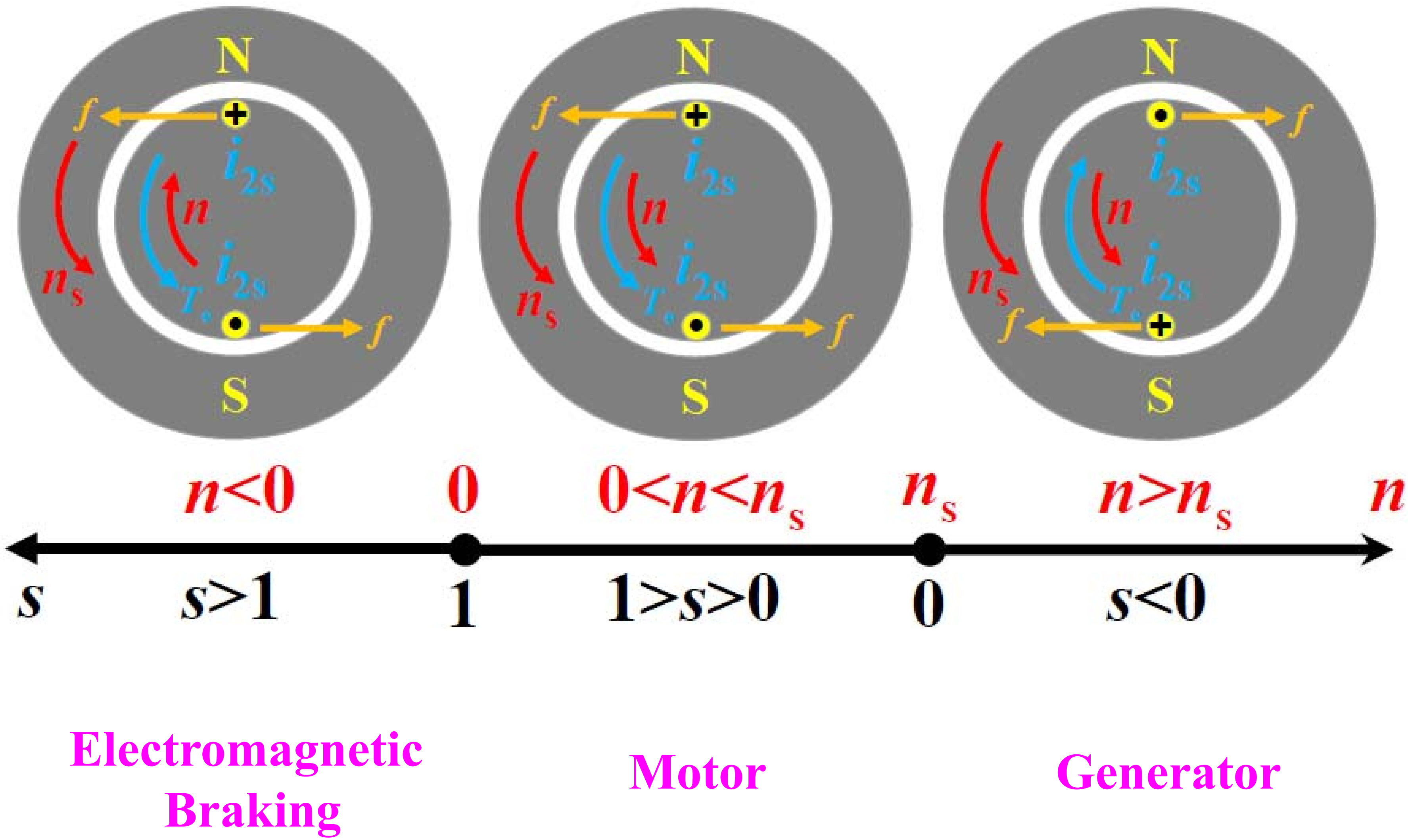
Alias: Asynchronous Machine

Slip Ratio:

$$s = \frac{n_s - n}{n_s}$$

Slip Ratio is an important parameter and can represents the different operation states of induction machine.

Structures and Basic Principle



Rated Values of Induction Machine

Power: P_N(W, kW)

Voltage: U_{1N}(V, kV)

Current: I_{1N}(A, kA)

Frequency: f_N(Hz)

Speed: n_N(r/min)

Power Factor: $\cos\varphi_N$

Efficiency: η_N

Generator

$$P_N = \sqrt{3} U_{1N} I_{1N} \cos\varphi_N$$

Motor

$$P_N = \sqrt{3} U_{1N} I_{1N} \cos\varphi_N \eta_N$$

MMF and Magnetic Field

MMF and magnetic field on no-load

- **Main flux and exciting impedance**
- **Leakage flux and leakage reactance**

MMF and magnetic field on load

- **Rotor MMF**
- **MMF equation**

No-Load

- **MMF on no-load: is produced by stator 3-phase symmetrical currents fed in 3-phase symmetrical windings. Rotor current and its MMF are ignorable.**
- **Magnetic Field on no-load: is produced by stator rotating MMF.**

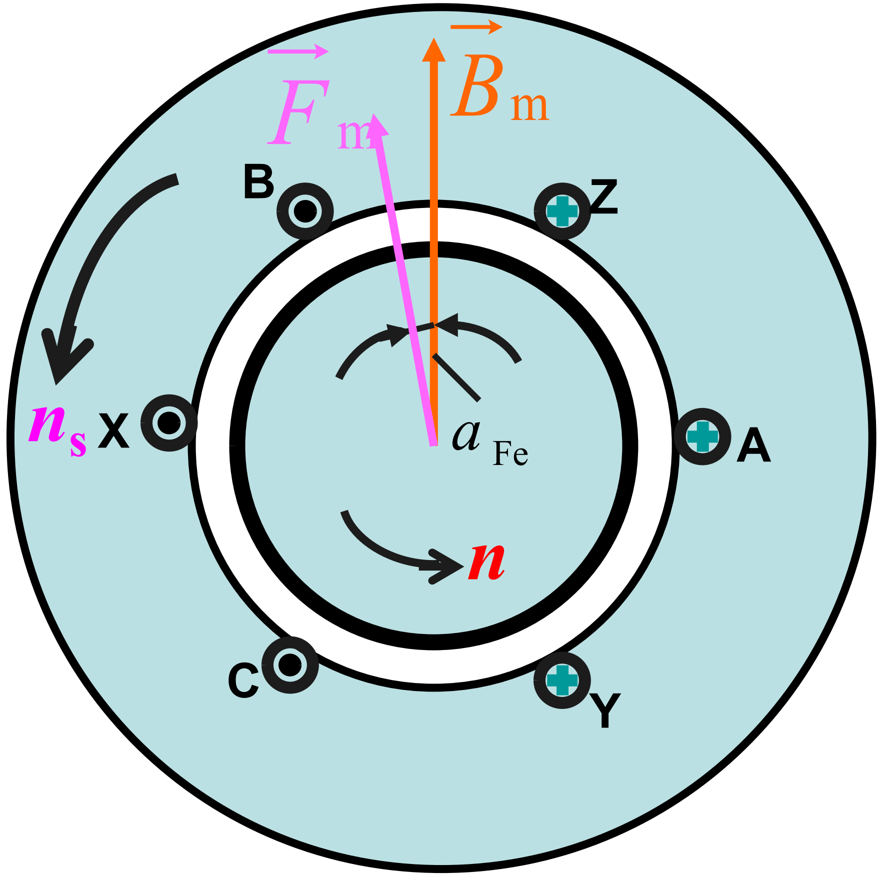
Main Flux Φ_m: goes through the air-gap, couples with stator and rotor windings, and contributes to the energy conversion.

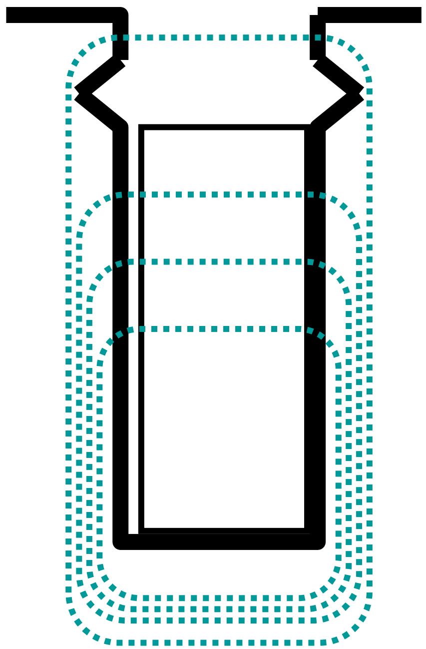
Stator Leakage Flux $\Phi_{1\sigma}$: only couples with stator winding, does not contribute to the energy conversion, but leads to the voltage drop.

$$\dot{I}_2 \approx \mathbf{0}$$

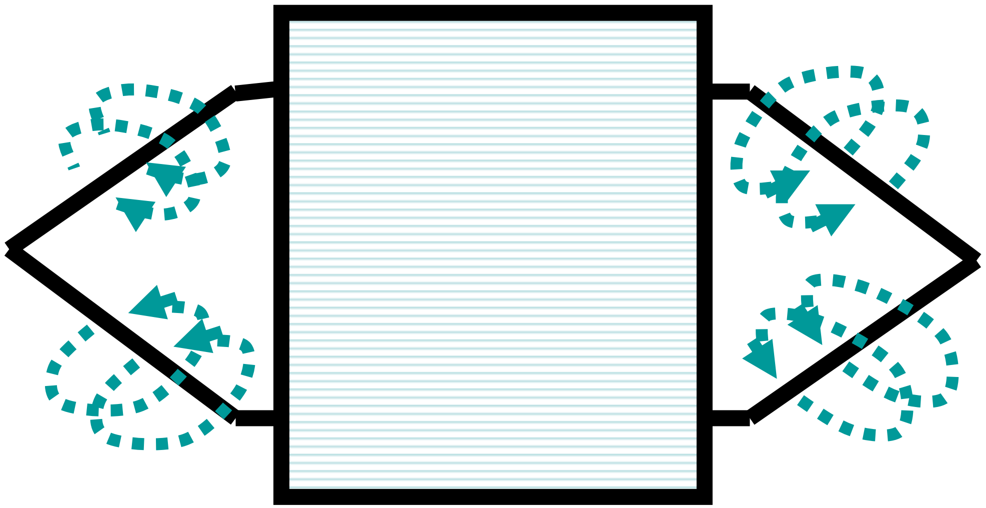
$$\vec{F}_1 = \vec{F}_m$$

$$\dot{I}_{10} = \dot{I}_m$$





1. Slot leakage Flux



2. End Leakage Flux

3. Harmonic leakage Flux

Main Flux and Exciting Impedance

The model of induction machine is similar to that of transformer.

Main Flux and Exciting Impedance

$$\dot{E}_1 = -\mathrm{j}4.44 f_1 N k_{\mathrm{w1}} \dot{\Phi}_{\mathrm{m}}$$

$$\dot{E}_1 = -\dot{I}_{\mathrm{m}} Z_{\mathrm{m}} = -\dot{I}_{\mathrm{m}} (R_{\mathrm{m}} + \mathrm{j}X_{\mathrm{m}})$$

Leakage Flux and Leakage Reactance

$$\dot{E}_{1\sigma} = -\mathrm{j}\dot{I}_1 X_{1\sigma}$$

$$X_{1\sigma} = 2\pi f_1 L_{1\sigma} = 2\pi f_1 N_1^2 \Lambda_{1\sigma}$$

On Load

Besides F_1, F_2 occurs now.

Analyze the speed of F_2

Stator speed: 0 (motionless)

Speed of F_1: n_s(synchronous speed)

Speed of rotor: n(same direction with ns).

Speed of F_1 versus rotor: $\Delta n=n_s$-n=s×n_s

Frequency of rotor: $f_2=p\Delta n/60=s\times f_1$

Speed of F_2 versus rotor: Δn

Speed of F_2 versus stator: Δn +n=n_s

Conclusion: stator and rotor MMFs F_1 and F_2 have the same speeds and rotating directions, they keep motionless, no matter what rotor speed is.

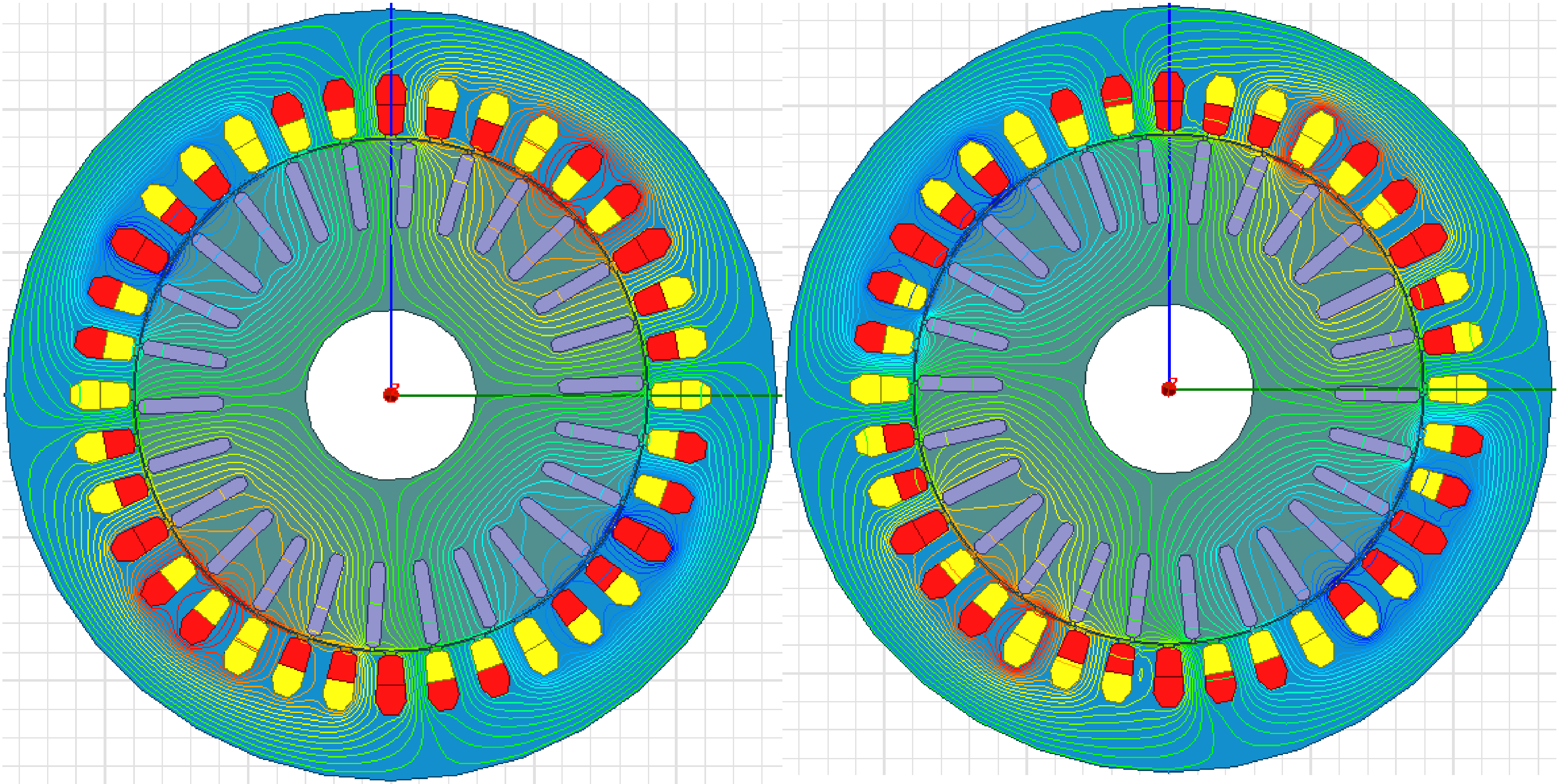
$$\dot{\boldsymbol{I}}_1 = \dot{\boldsymbol{I}}_{\mathrm{m}} + \dot{\boldsymbol{I}}_{1\mathrm{L}} \qquad \overrightarrow{\boldsymbol{F}}_{1\mathrm{L}} = -\overrightarrow{\boldsymbol{F}}_2$$

$$\overrightarrow{\boldsymbol{F}}_1 = \overrightarrow{\boldsymbol{F}}_{\mathrm{m}} + \overrightarrow{\boldsymbol{F}}_{1\mathrm{L}} = \overrightarrow{\boldsymbol{F}}_{\mathrm{m}} + (-\overrightarrow{\boldsymbol{F}}_2)$$

$$\overrightarrow{\boldsymbol{F}}_1 + \overrightarrow{\boldsymbol{F}}_2 = \overrightarrow{\boldsymbol{F}}_{\mathrm{m}}$$

$$\boldsymbol{F}_1 = \frac{\boldsymbol{m}_1}{2} 0.9 \frac{\boldsymbol{N}_1 \boldsymbol{k}_{\mathrm{w}1} \boldsymbol{I}_1}{\boldsymbol{p}} \qquad \boldsymbol{F}_2 = \frac{\boldsymbol{m}_2}{2} 0.9 \frac{\boldsymbol{N}_2 \boldsymbol{k}_{\mathrm{w}2} \boldsymbol{I}_2}{\boldsymbol{p}}$$

$$\boldsymbol{F}_{\mathrm{m}} = \frac{\boldsymbol{m}_1}{2} 0.9 \frac{\boldsymbol{N}_1 \boldsymbol{k}_{\mathrm{w}1} \boldsymbol{I}_{\mathrm{m}}}{\boldsymbol{p}}$$



Discussion: **An induction motor connected to 50Hz AC power is on no-load and the rotor speed is 980 r/min. Find: (1) the pole numbers of the motor, (2) the frequency of rotor current, (3) the speed of stator MMF vs rotor, (4) the speed of stator MMF vs stator, and (5) the speed of rotor MMF vs stator MMF.**

Answer:

(1) Rotor speed n is most approximate to the synchronous 1000 r/min.

So the pole number should be $2p=6$

(2) $s=(n_s-n)/n_s=(1000-980)/1000=2\%$, $f_2=sf_1=2\%\times 50=1$ (Hz)

(3) $\Delta n_1=n_s-n=1000-980=20$ (r/min)

(4) $\Delta n_2=n_s=1000$ (r/min)

(5) $\Delta n_3=n_s-n_s=1000-1000=0$ (r/min)

Equivalent Circuits & Mathematical Models

Voltage Equations

Stator

$$\dot{U}_1 e^{j\omega_1 t} = \dot{I}_1 e^{j\omega_1 t}(R_1 + jX_{1\sigma}) - \dot{E}_1 e^{j\omega_1 t}$$

$$\dot{U}_1 = \dot{I}_1(R_1 + jX_{1\sigma}) - \dot{E}_1$$

$$\dot{E}_1 = -\dot{I}_m Z_m$$

Rotor

$$\dot{E}_{2\sigma s} = -j\dot{I}_2 2\pi s f_1 L_{2\sigma}$$

$$X_{2\sigma s} = 2\pi s f_1 L_{2\sigma} = s \cdot 2\pi f_1 L_{2\sigma} = sX_{2\sigma}$$

$$\dot{E}_{2s} e^{j\omega_2 t} = \dot{I}_{2s} e^{j\omega_2 t}(R_2 + jsX_{2\sigma})$$

$$\dot{E}_{2s} = \dot{I}_{2s}(R_2 + jsX_{2\sigma})$$

Rotor Analysis of Motor

When Rotor is motionless:

$n=0$, $s=1$, $f_2=f_1$

$E_2=4.44f_1N_2k_{w2}\Phi_m$

$X_{2\sigma}=2\pi f_1L_{2\sigma}$

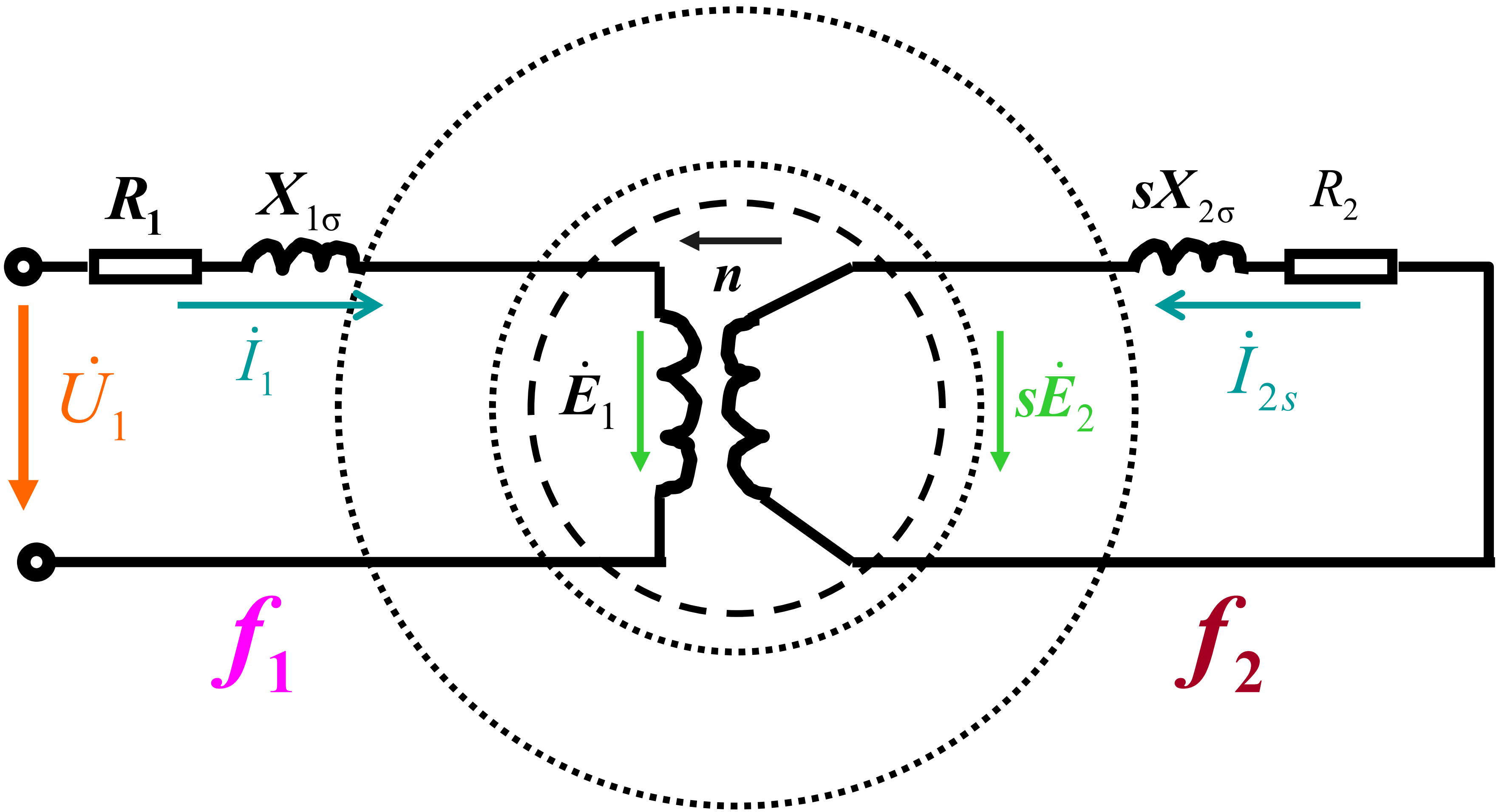
When Rotor is rotating:

$n>0$, $s<1$, $f_2=sf_1$

$E_{2s}=4.44sf_1N_2k_{w2}\Phi_m=sE_2$

$X_{2\sigma s}=sX_{2\sigma}$

The rotor current lags the induced voltage by the power-factor angle φ_2 of the rotor leakage impedance. The rotor leakage reactance, equal to $s\omega_s$ times the rotor leakage inductance, is very small compared with the rotor resistance (which is typically the case at the small slips corresponding to normal operation).



R_1
$X_{1\sigma}$
$sX_{2\sigma}$
R_2
n
$\dot{I}_1$
$\dot{U}_1$
$\dot{E}_1$
$s\dot{E}_2$
$\dot{I}_{2s}$
f_1
f_2

Frequency Referred

$$\dot{E}_{2s}e^{j\omega_2 t}=\dot{I}_{2s}e^{j\omega_2 t}(R_2+jsX_{2\sigma})$$

$$s\dot{E}_2 e^{j\omega_2 t}\frac{e^{j(\omega_1-\omega_2)t}}{s}=\dot{I}_{2s}e^{j\omega_2 t}\frac{e^{j(\omega_1-\omega_2)t}}{s}(R_2+jsX_{2\sigma})$$

$$\dot{E}_2 e^{j\omega_1 t}=\dot{I}_2 e^{j\omega_1 t}(\frac{R_2}{s}+jX_{2\sigma})$$

$$\dot{E}_2=\dot{I}_2(\frac{R_2}{s}+jX_{2\sigma})$$

f_2 **system**

$$\dot{E}_{2s} = \dot{I}_{2s}(R_2 + \mathrm{j}sX_{2\sigma})$$

$$\Downarrow$$

$$\dot{E}_2 = \dot{I}_2(\frac{R_2}{s} + \mathrm{j}X_{2\sigma})$$

f_1 **system**

Copper Loss

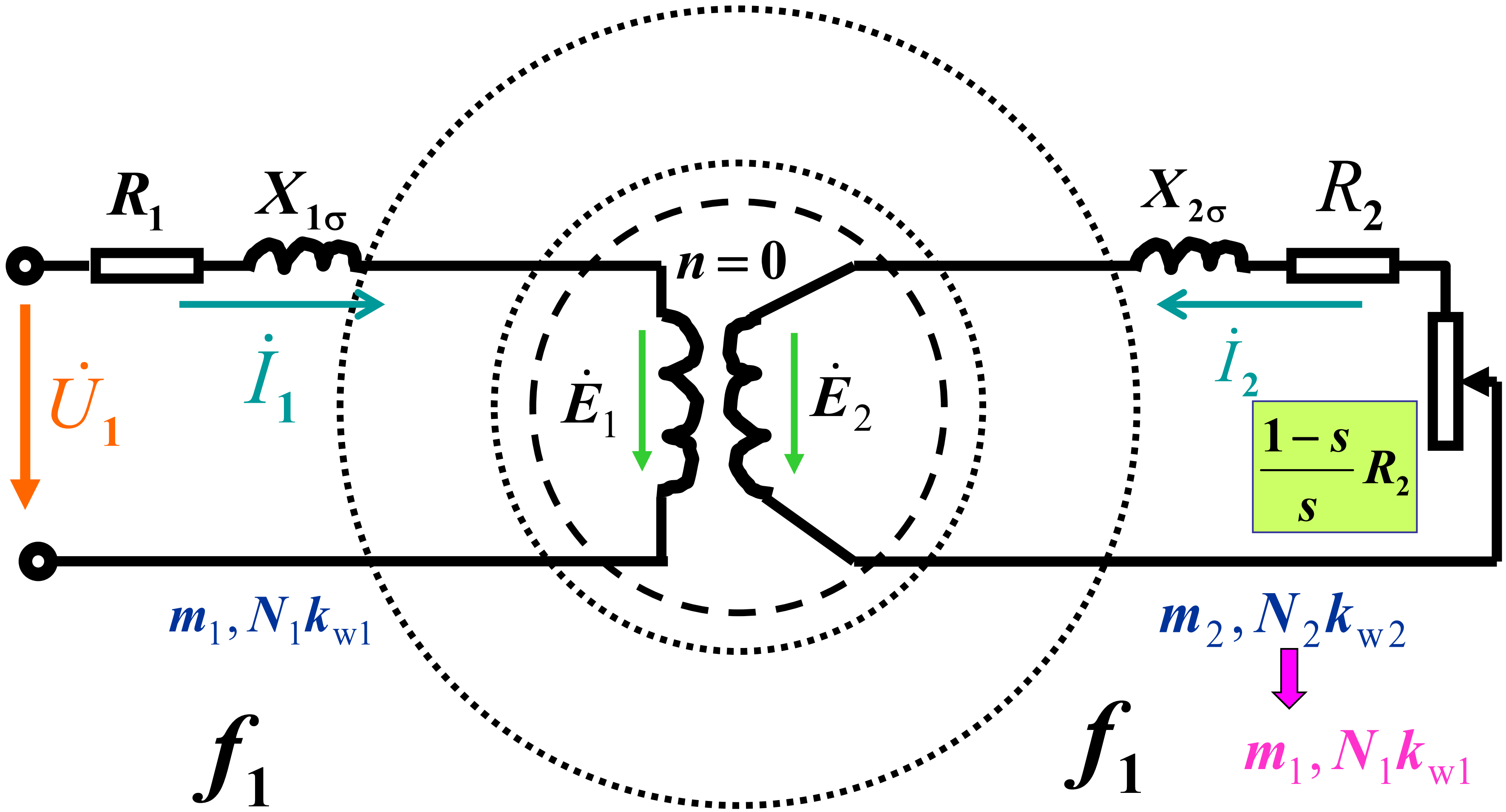
$$\frac{R_2}{s} = R_2 + \boxed{\frac{1-s}{s}R_2}$$

Mechanical Power

$$\frac{P_\Omega}{P_e} = 1 - s \qquad \frac{p_{cu2}}{P_e} = s$$

Distribution Law of Eletromagnetic Power

After Frequency Transformation



The rotor is motionless after frequency transformation

Winding Referred

$$\frac{m_1}{2}0.9\frac{N_1 k_{w1}\dot{I}_2'}{p}=\frac{m_2}{2}0.9\frac{N_2 k_{w2}\dot{I}_2}{p}$$

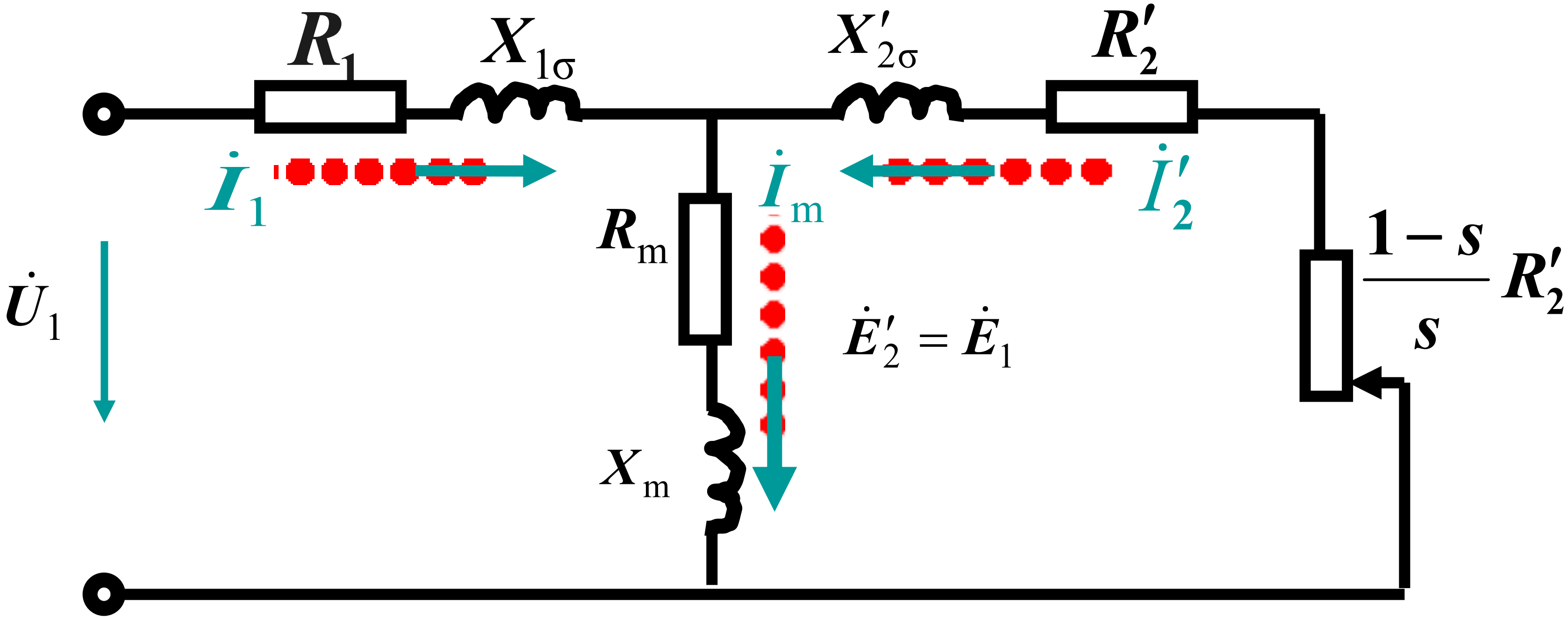
$$\dot{I}_2'=\frac{m_2 N_2 k_{w2}}{m_1 N_1 k_{w1}}\dot{I}_2=\frac{\dot{I}_2}{k_i},k_i=\frac{m_1 N_1 k_{w1}}{m_2 N_2 k_{w2}}$$

$$\dot{\boldsymbol{E}}_2'=\frac{\boldsymbol{N}_1\boldsymbol{k}_{w1}}{\boldsymbol{N}_2\boldsymbol{k}_{w2}}\dot{\boldsymbol{E}}_2=\boldsymbol{k}_e\dot{\boldsymbol{E}}_2 \qquad k_e=\frac{N_1 k_{w1}}{N_2 k_{w2}}$$

$$\boldsymbol{R}_2'=\boldsymbol{k}_e\boldsymbol{k}_i\boldsymbol{R}_2=\frac{\boldsymbol{m}_1}{\boldsymbol{m}_2}\left(\frac{\boldsymbol{N}_1\boldsymbol{k}_{w1}}{\boldsymbol{N}_2\boldsymbol{k}_{w2}}\right)^2\boldsymbol{R}_2$$

$$\boldsymbol{X}_{2\sigma}'=\boldsymbol{k}_e\boldsymbol{k}_i\boldsymbol{X}_{2\sigma}=\frac{\boldsymbol{m}_1}{\boldsymbol{m}_2}\left(\frac{\boldsymbol{N}_1\boldsymbol{k}_{w1}}{\boldsymbol{N}_2\boldsymbol{k}_{w2}}\right)^2\boldsymbol{X}_{2\sigma}$$

Equivalent Circuit of Induction Motor



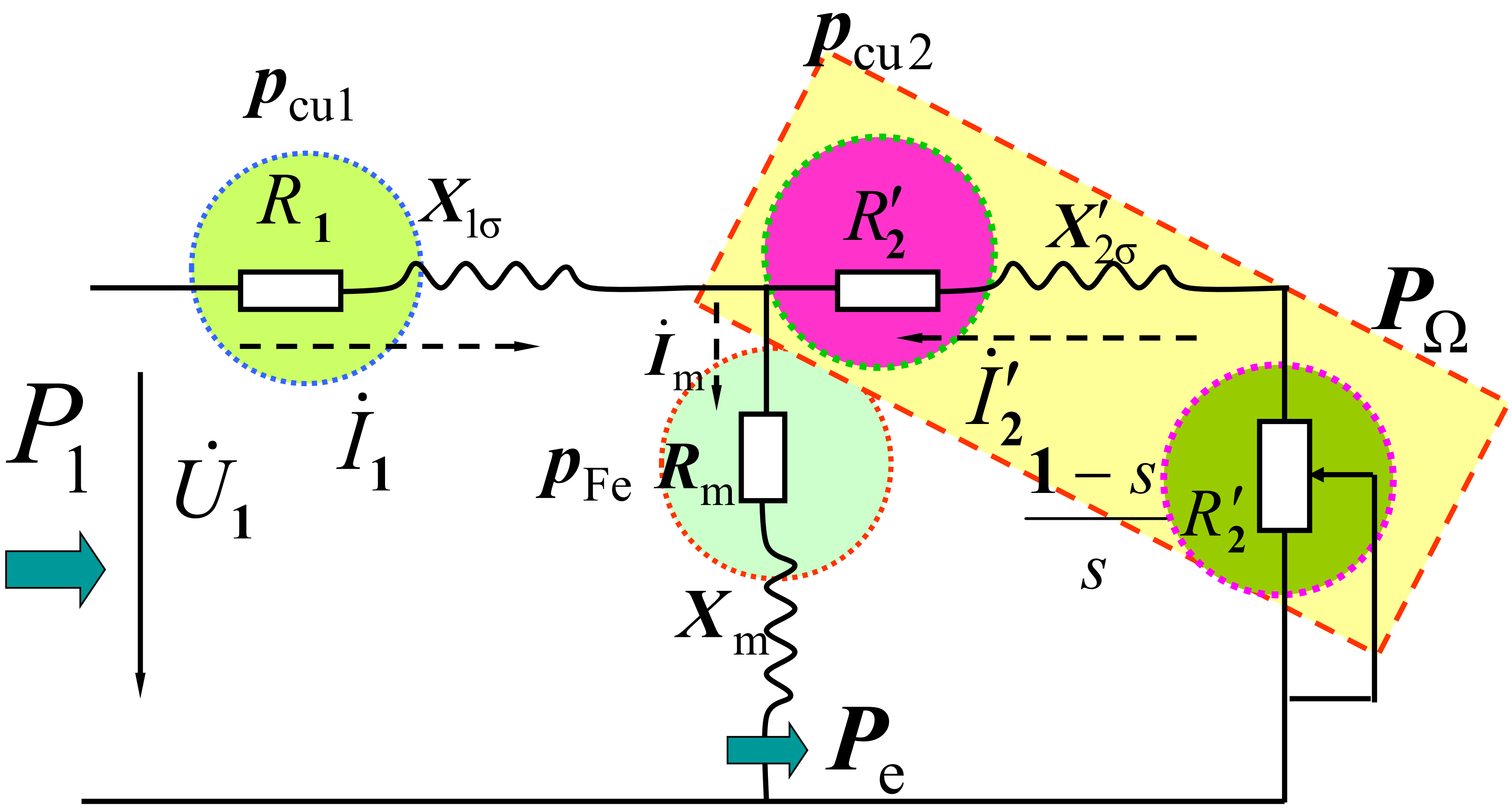
$$\dot{U}_1 = \dot{I}_1(R_1 + \mathrm{j}X_{1\sigma}) - \dot{E}_1$$

$$\dot{E}'_2 = \dot{I}'_2\left(\frac{R'_2}{s} + \mathrm{j}X'_{2\sigma}\right)$$

$$\dot{E}_1 = \dot{E}'_2 = -\dot{I}_m Z_m$$

$$\dot{I}_1 + \dot{I}'_2 = \dot{I}_m$$

Power Flow of Induction Motor



Power Equations and Power Flow

$$P_1 = m_1 U_1 I_1 \cos\varphi_1$$

$$p_{\mathrm{cu1}} = m_1 I_1^2 R_1$$

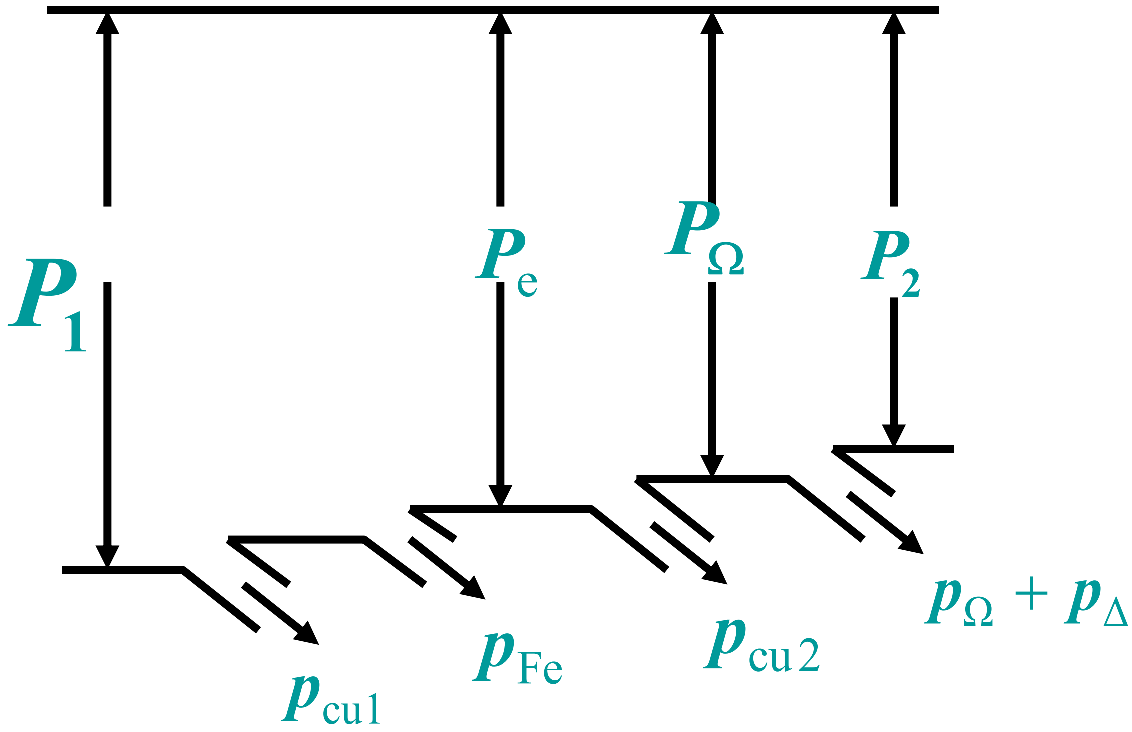
$$p_{\mathrm{Fe}} = m_1 I_{\mathrm{m}}^2 R_{\mathrm{m}}$$

$$P_{\mathrm{e}} = m_1 E_2' I_2' \cos\psi_2'$$

$$= m_1 I_2'^2 \frac{R_2'}{s} = T_{\mathrm{e}} \Omega_{\mathrm{s}}$$

$$p_{\mathrm{cu2}} = m_1 I_2'^2 R_2' = s P_{\mathrm{e}}$$

$$P_\Omega = P_{\mathrm{e}} - p_{\mathrm{cu2}} = m_1 I_2'^2 \frac{1-s}{s} R_2' = (1-s) P_{\mathrm{e}}$$



Torque Equation

$$\frac{P_\Omega}{\boldsymbol{\Omega}} = \frac{p_\Omega + p_\Delta}{\boldsymbol{\Omega}} + \frac{P_2}{\boldsymbol{\Omega}}$$

$$T_e = T_0 + T_2$$

T_e is driving torque for motor

Discussion: **With the increase of output power, analyze the changes of mechanical loss p_{Ω}, stator core loss p_{Fe1}, rotor core loss p_{Fe2}, stator copper loss p_{Cu1} and rotor copper loss p_{Cu2} of an induction motor.**

Analyze: p_{Ω}

$P_2 \nearrow \rightarrow n \searrow \rightarrow p_{\Omega} \searrow$

$n \searrow \rightarrow s \nearrow \rightarrow I_2 \nearrow$ **and** $I_1 \nearrow \rightarrow I_1\,|Z_{1\sigma}| \nearrow$,

when U_1=C **and** f_1=C, $E_1 \searrow$ **slightly** $\rightarrow \Phi_m \searrow$ **slightly** $\rightarrow B_m \searrow$ **slightly** $\rightarrow p_{Fe1} \searrow$ **slightly** (or $p_{Fe1} \approx$ C)

$s \nearrow \rightarrow f_2 \nearrow \rightarrow p_{Fe2} \nearrow$ (usually $p_{Fe2} \ll p_{Fe1}$)

$I_1 \nearrow$ **and** $I_2 \nearrow \rightarrow p_{Cu1} \nearrow$ **and** $p_{Cu2} \nearrow$

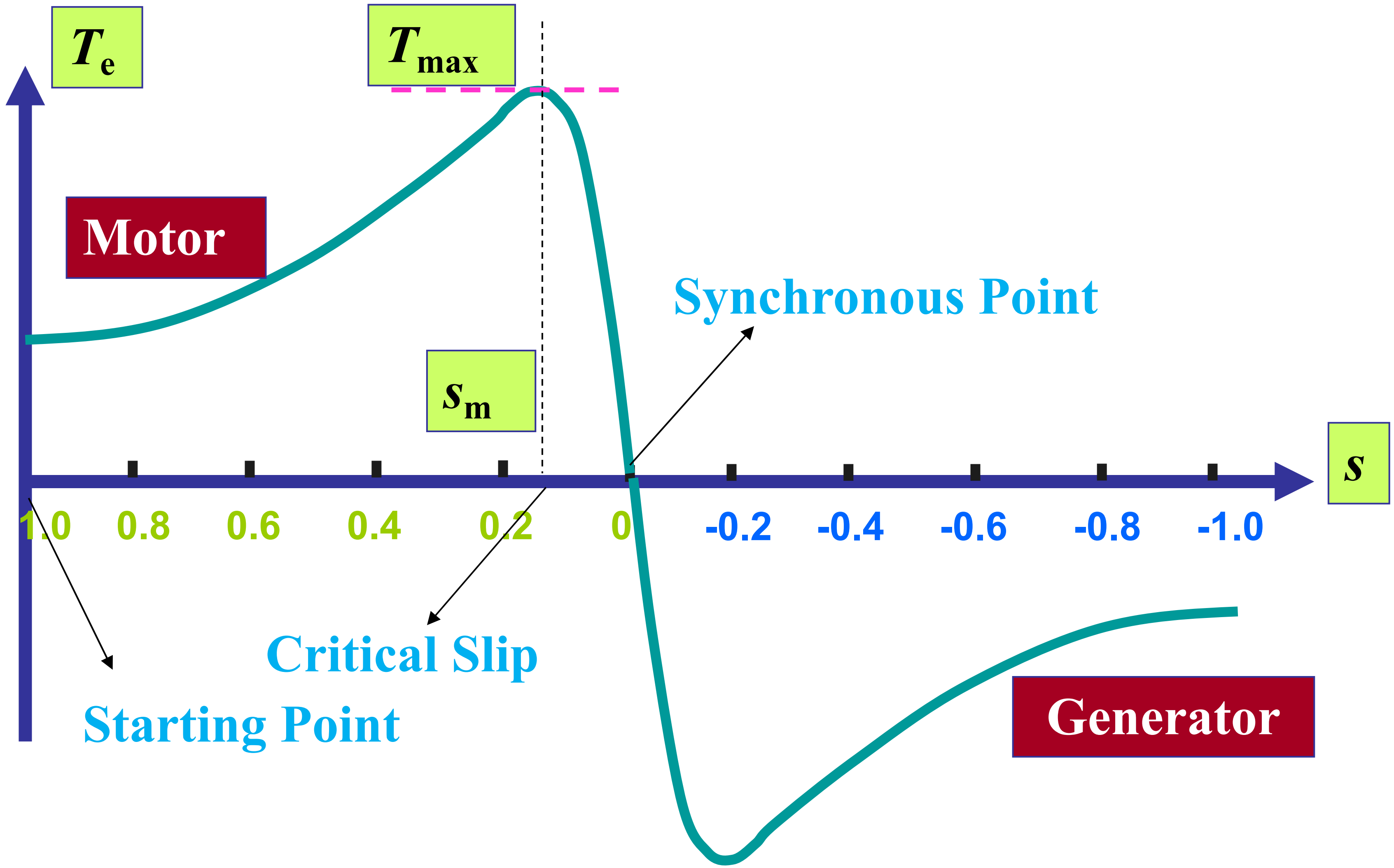
Torque-Slip Curve

$$T_e = \frac{m_1}{\Omega_s} \frac{U_1^2 \dfrac{R_2'}{s}}{\left(R_1 + c\dfrac{R_2'}{s}\right)^2 + \left(X_{1\sigma} + cX_{2\sigma}'\right)^2}$$

T_e is related with m_1, p, U_1, f_1, R_1, $X_{1\sigma}$, R_2, $X_{2\sigma}$ and s.

When m_1, p, U_1, f_1, R_1, $X_{1\sigma}$, R_2 and $X_{2\sigma}$ keep constant, $T_e = f(s)$

Torque-Slip Curve



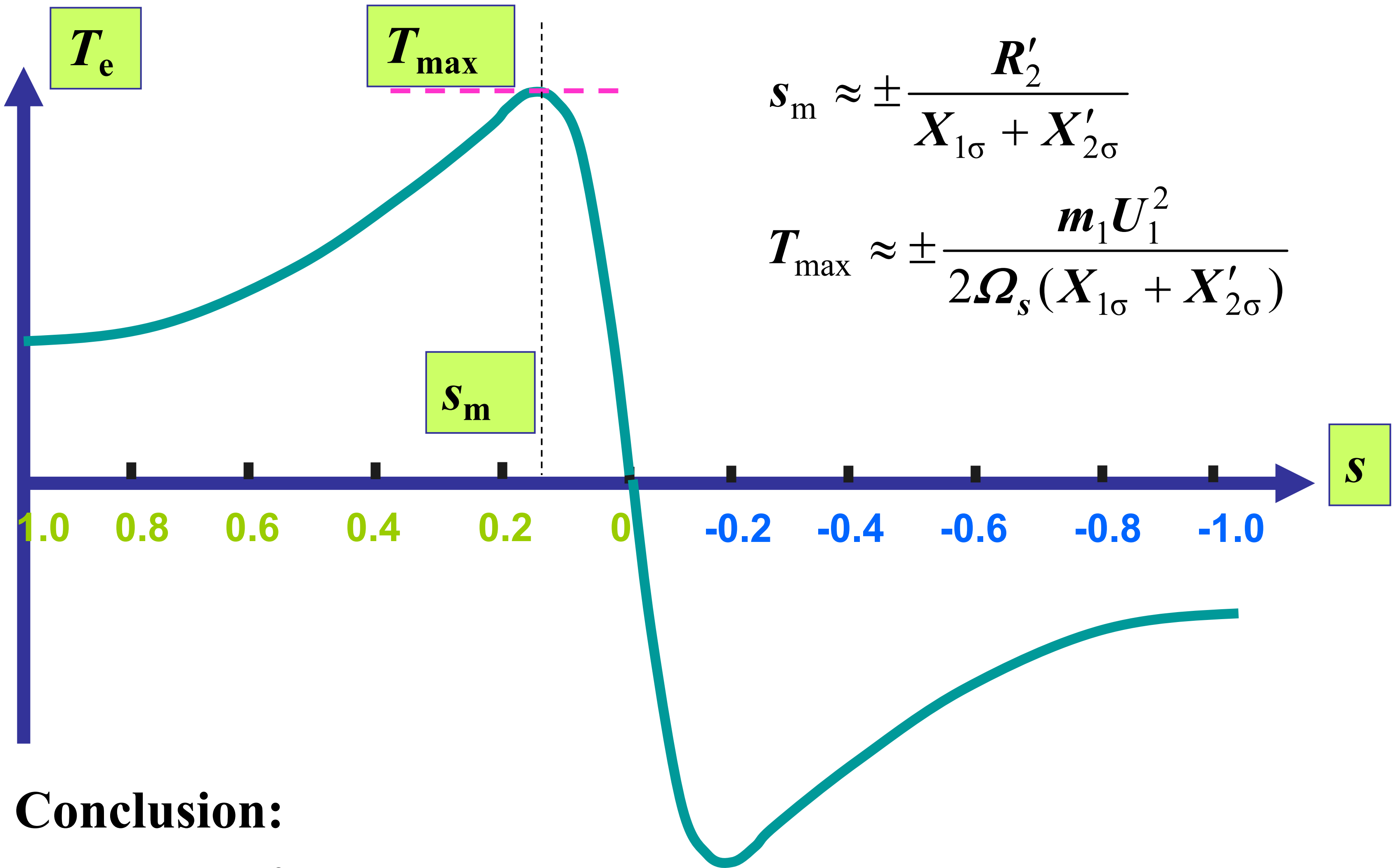
Maximum Electromagnetic Torque

$$s_{\mathrm{m}} = \pm \frac{cR_2'}{\sqrt{R_1^2 + (X_{1\sigma} + cX_{2\sigma}')^2}} \approx \pm \frac{R_2'}{X_{1\sigma} + X_{2\sigma}'}$$

$$T_{\max} = \pm \frac{m_1}{\Omega_{\mathrm{s}}} \frac{U_1^2}{2c[\pm R_1 + \sqrt{R_1^2} + (X_{1\sigma} + cX_{2\sigma}')^2]} \approx \pm \frac{m_1 U_1^2}{2\Omega_s (X_{1\sigma} + X_{2\sigma}')}$$

Starting Torque

$$T_{\mathrm{st}} = \frac{m_1}{\Omega_{\mathrm{s}}} \frac{U_1^2 R_2'}{(R_1 + cR_2')^2 + (X_{1\sigma} + cX_{2\sigma}')^2}$$



$$s_m \approx \pm \frac{R'_2}{X_{1\sigma} + X'_{2\sigma}}$$

$$T_{max} \approx \pm \frac{m_1 U_1^2}{2\Omega_s (X_{1\sigma} + X'_{2\sigma})}$$

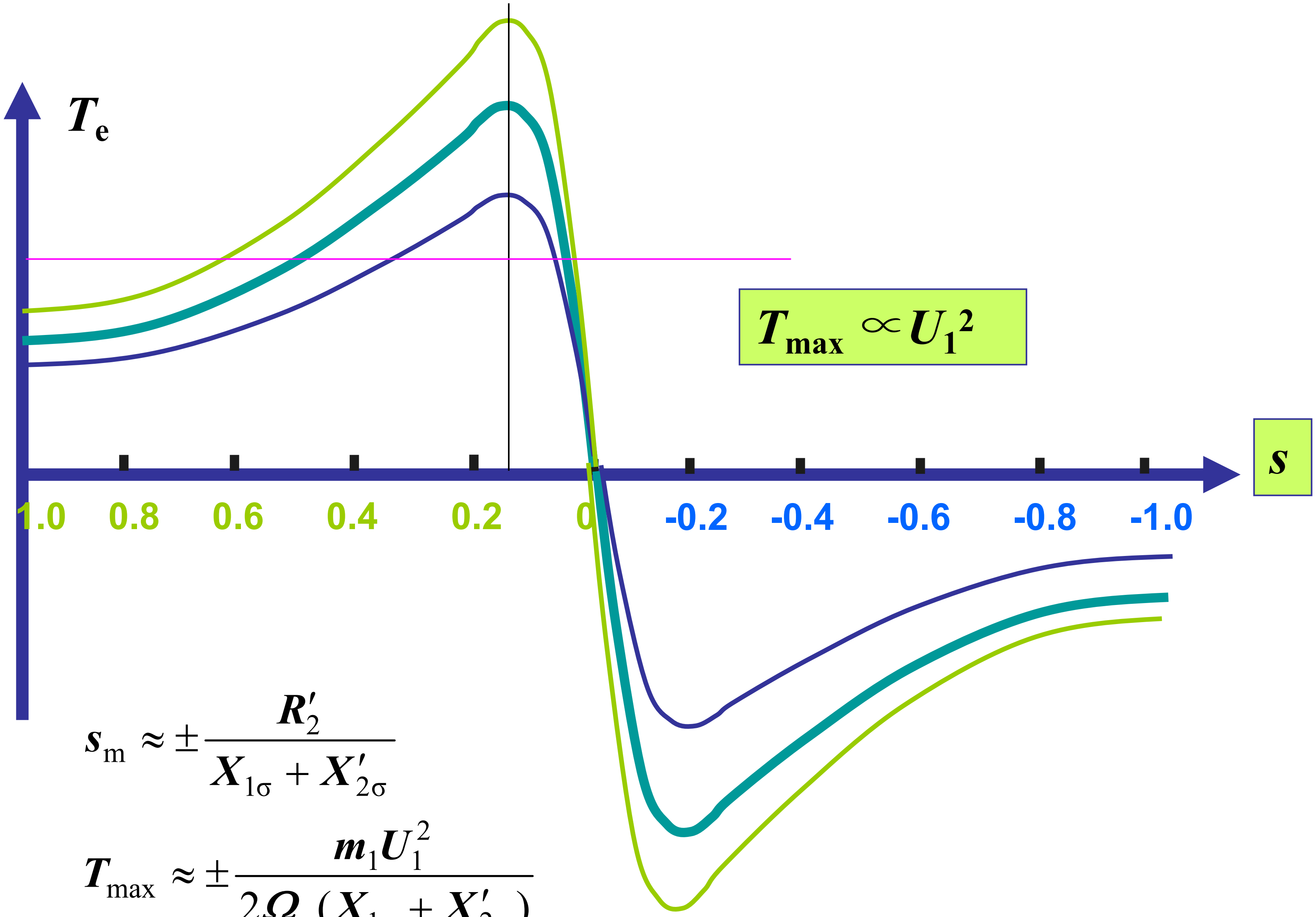
Conclusion:

1. $T_{max} \propto U_1^2$, but s_m is irrelevant with U_1.

2. Both T_{max} and $s_m \propto 1/(X_{1\sigma} + X'_{2\sigma})$.

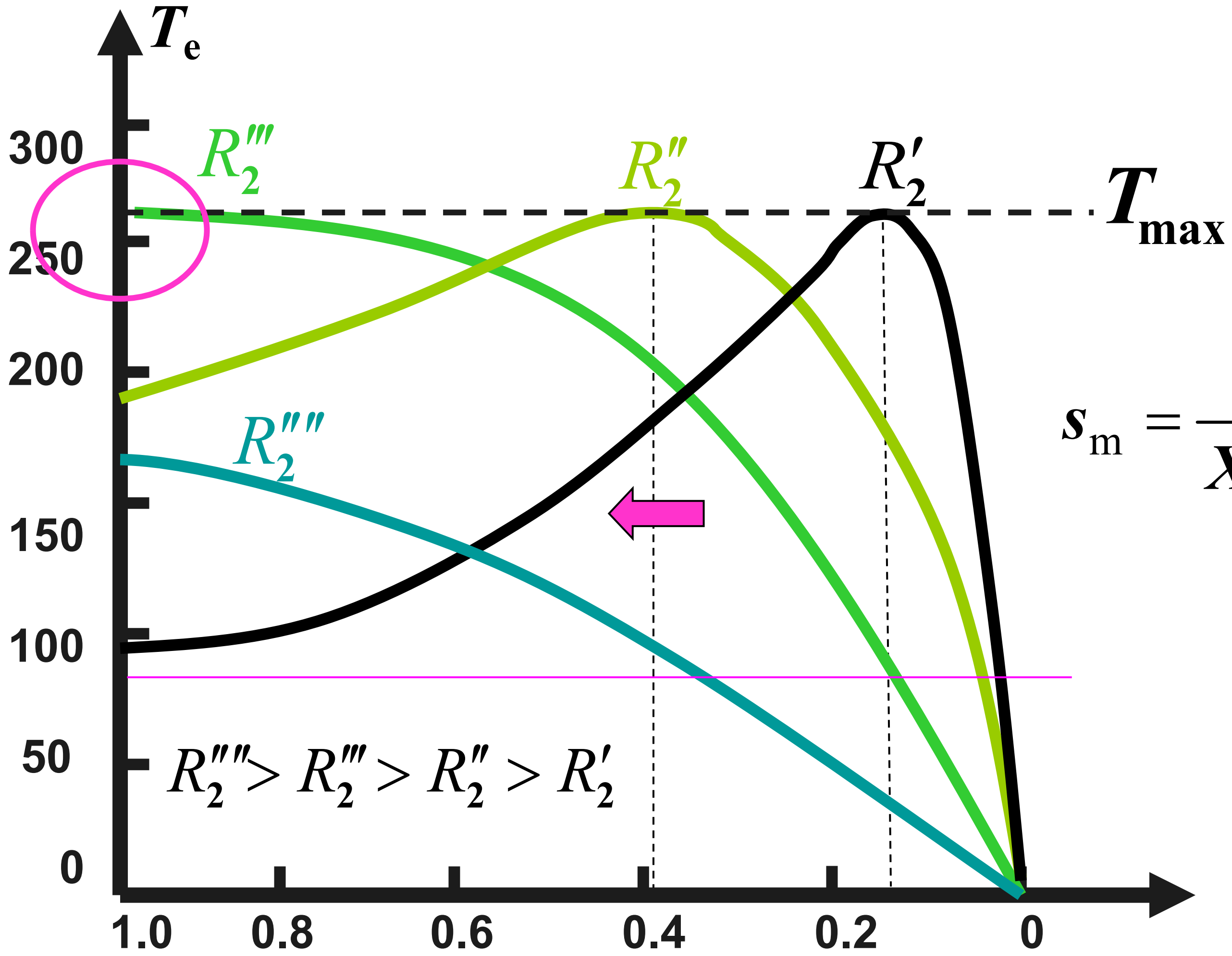
3. $T_{max} \propto 1/f_1^2$, $s_m \propto 1/f_1$.

4. $s_m \propto R'_2$, but T_{max} is irrelevant with R'_2.



$$s_m \approx \pm \frac{R_2'}{X_{1\sigma} + X_{2\sigma}'}$$

$$T_{max} \approx \pm \frac{m_1 U_1^2}{2\Omega_s (X_{1\sigma} + X_{2\sigma}')}$$



$$s_m = \frac{R_2'}{X_{1\sigma} + X_{2\sigma}'}$$

Features for constant torque load :

1.Voltage and current keep constant.

2.Electromagnetic Torque T_e keeps constant.

3.Electromagnetic power P_e keeps constant.

4.R'_2↗ →s↗ , p_{cu2}↗ and P_Ω↘

Mechanical Characteristic

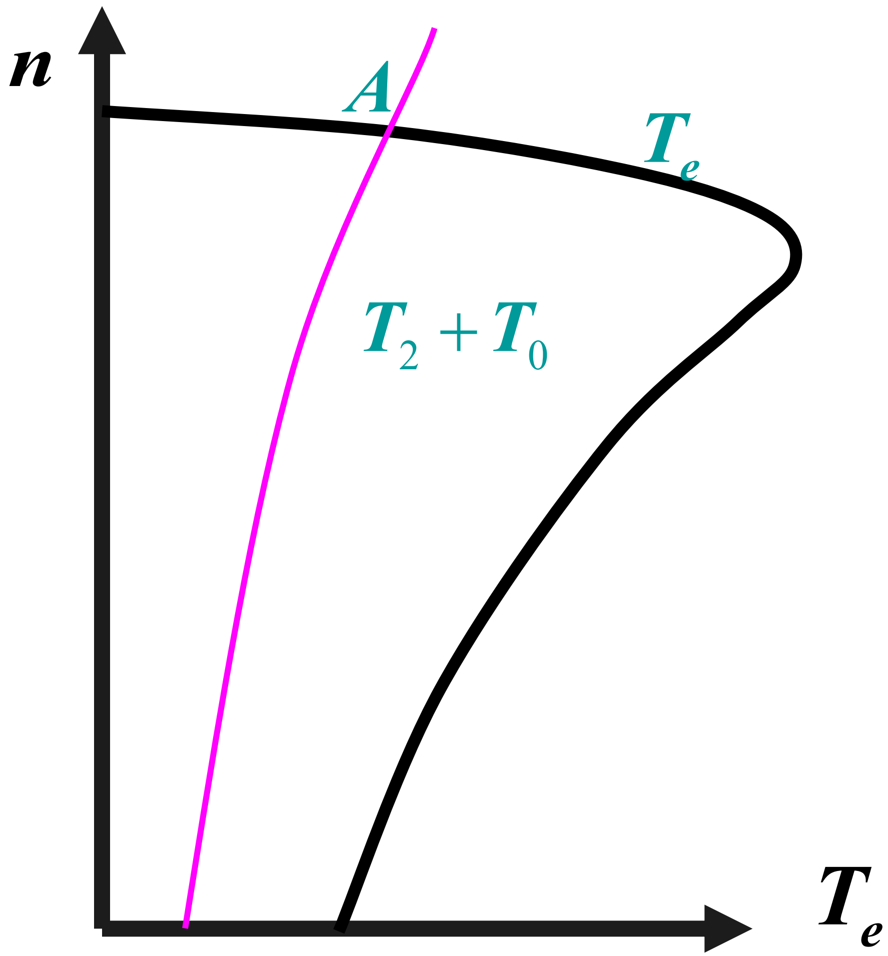
Because $n = n_s(1-s)$

$Te = f(s) \rightarrow$ $n = f(Te)$

Stable condition:

$$\frac{\mathrm{d}T_e}{\mathrm{d}n} = \frac{\mathrm{d}T_L}{\mathrm{d}n}$$

Stable region of induction motor is $0 < s < s_m$



Operation Performances

Keep $U_1=U_{1N}$ and $f_1=f_N$

Operating Characteristics:

Speed Char. :

$$n=f(P_2)$$

Stator Current Char. :

$$I_1=f(P_2)$$

Power Factor Char. :

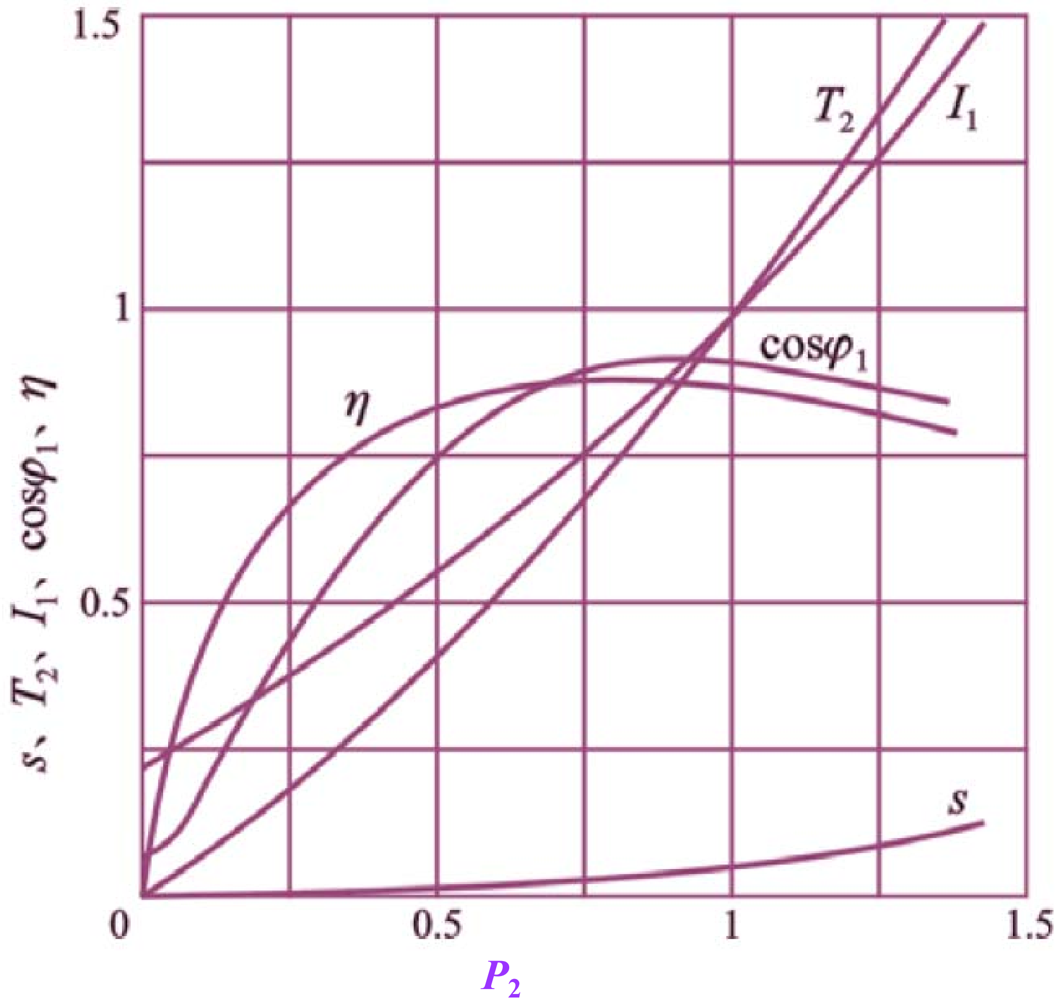
$$\cos\varphi_1=f(P_2)$$

Torque Char. :

$$Te=f(P_2)$$

Efficiency Char. :

$$\eta=f(P_2)$$



Example: The overload ability of an induction motor is k_T=1.6. If the stator voltage decreases 15%, can the motor continue to operate ?

(1) $T_2 \approx Te \propto U_1^2 \qquad T_{max} = k_T T_N$

(2) $U_1 \searrow 15\% \rightarrow U'_1 = 85\% U_N$

$T'_{max} = T_{max} * (0.85)^2$

$T'_{max} = 1.6 * T_N (0.85)^2 = 1.156 T_N > T_N$

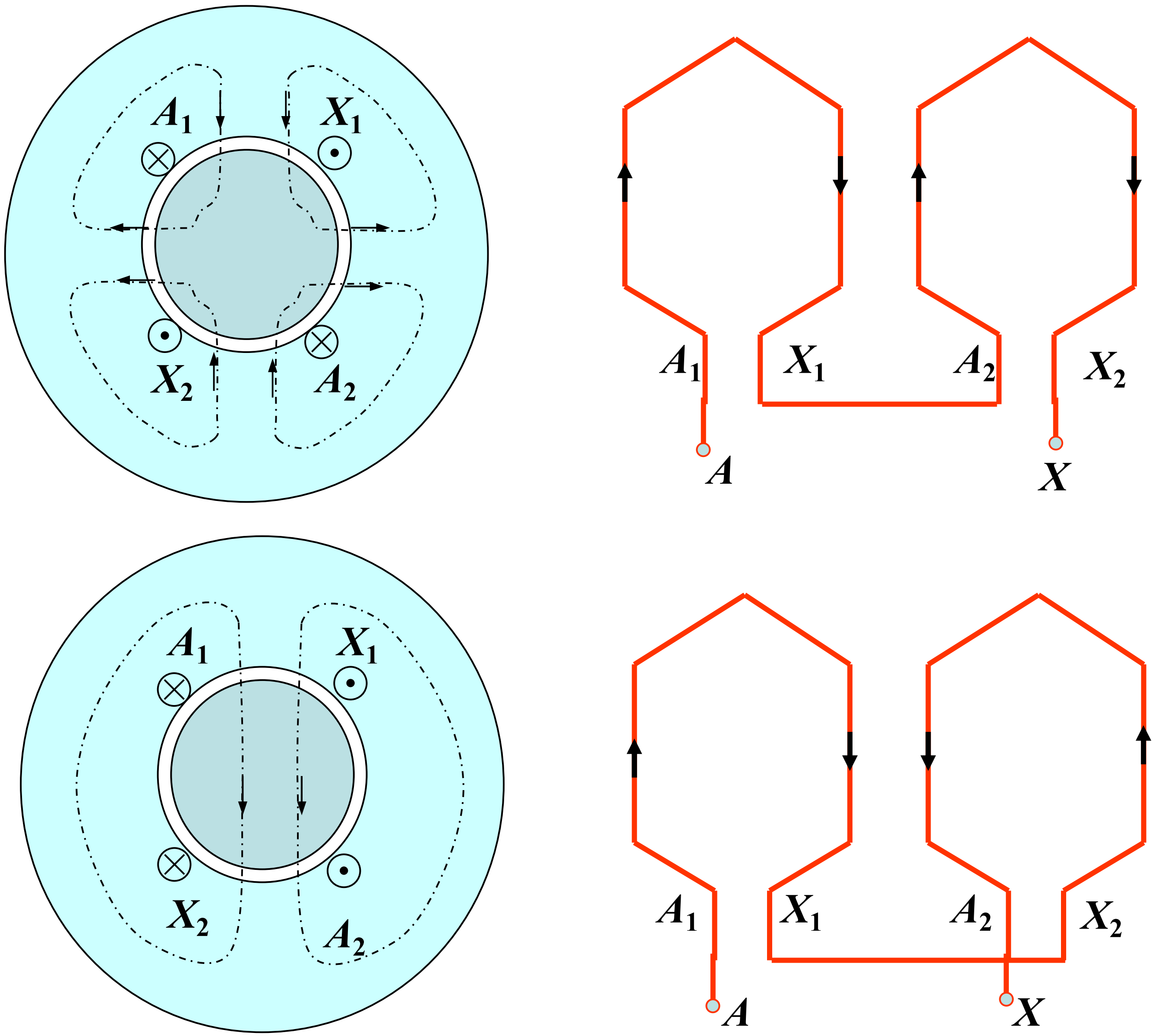
Conclusion: The motor can drive the load to operate.

Speed Control

$$n = \frac{60 f_1}{p}(1 - s)$$

1. Pole Changing
2. Frequency Changing
3. Slip Changing

Pole-Changing Motor



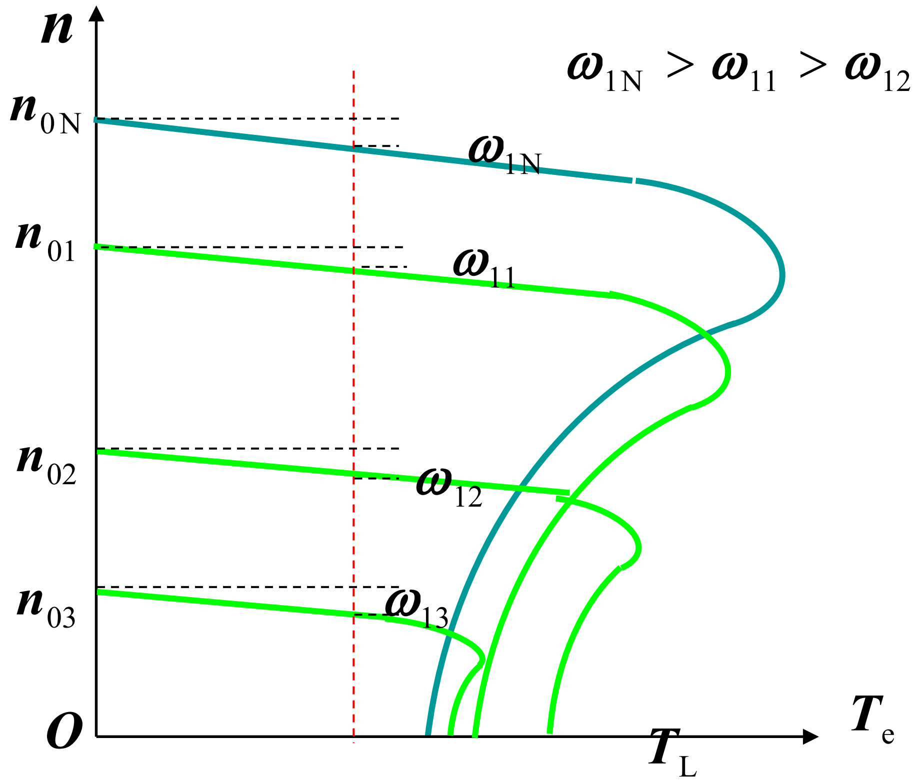
Advantage

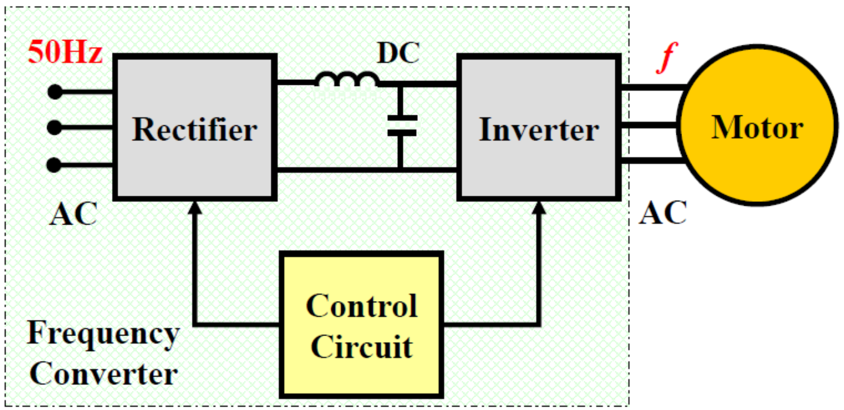
Speed control is easy.

Disadvantage

Speed cannot be changed continuously.

Frequency Control





Require: $\Phi_m \approx C$, maintains the saturation of MC constant.

According to: $U_1 \approx E_1 = 4.44 f_1 N_1 k_{w1} \Phi_m \quad U_1/f_1 \approx C$

Advantage: Speed can be changed continuously.

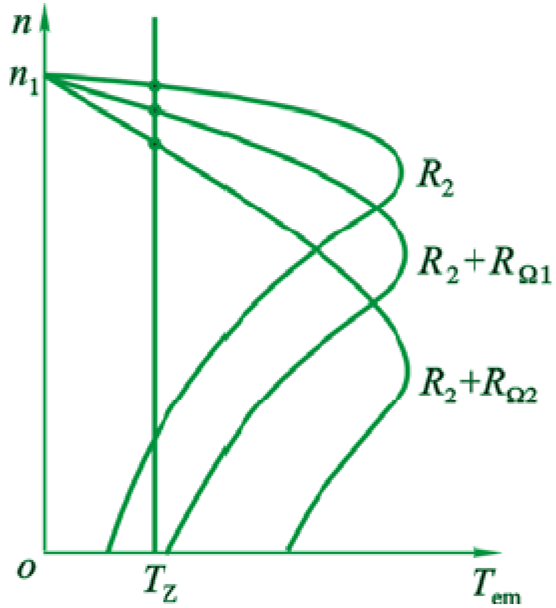
Slip Control

$s = f\,(U_1, R_1, X_{1\sigma}, R'_2, X'_{2\sigma})$

Voltage control

Stator resistor or reactor control

Rotor resistor or reactor control



Rotor-Resistance Control