Transformer

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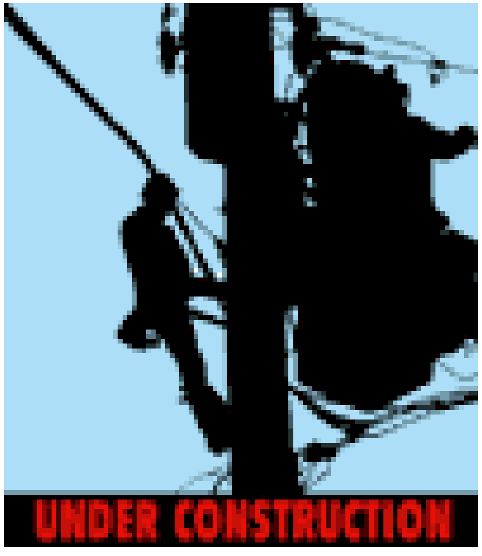
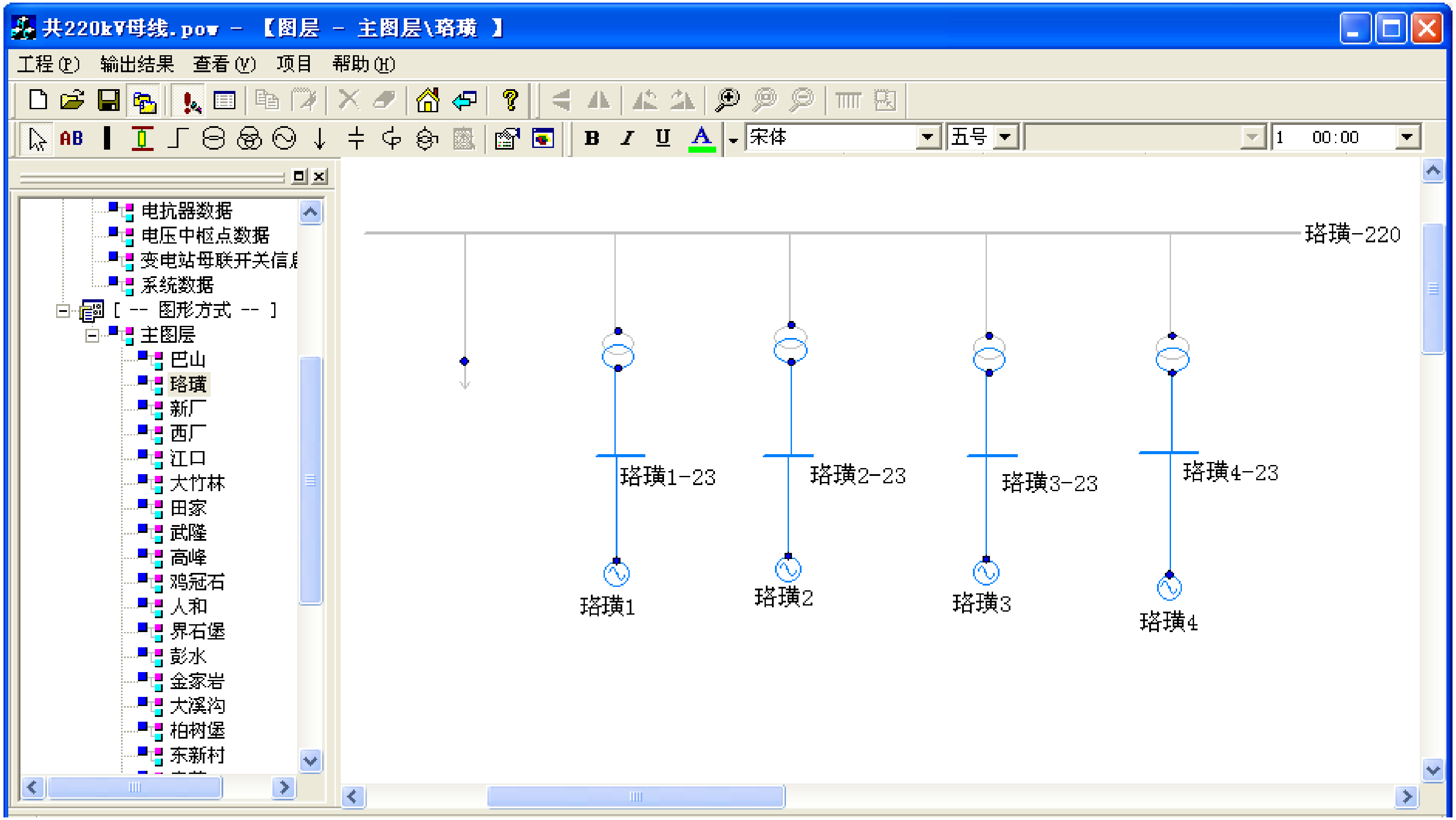




Transformer:

An electromagnetic device that changes ac electric power from one voltage level to another voltage level by the action of magnetic coupling.

SG(H)B10- Transformer

Structures

- Transformer
  - Body
    - Core
      - Pole
      - Yoke
    - Winding
      - Primary
      - Secondary
  - Tank
    - Oil
    - Radiator
    - Insulator
    - Tap switch

Rated values

Rated values : under these operation parameters , the apparatus is safety, reliable and has good performances, such as high efficiency, et al.

f_N, S_N, U_{1N}, U_{2N}, I_{1N}, I_{2N}

U_{2N}: **: When the rated voltage U_{1N} is applied to the primary and the secondary is open, the voltage between any two phase of the secondary is the secondary rated voltage.** $U_{2N}=U_{20}$

Single Phase:

$$\boxed{S_N = U_{1N} \times I_{1N} = U_{2N} \times I_{2N}}$$

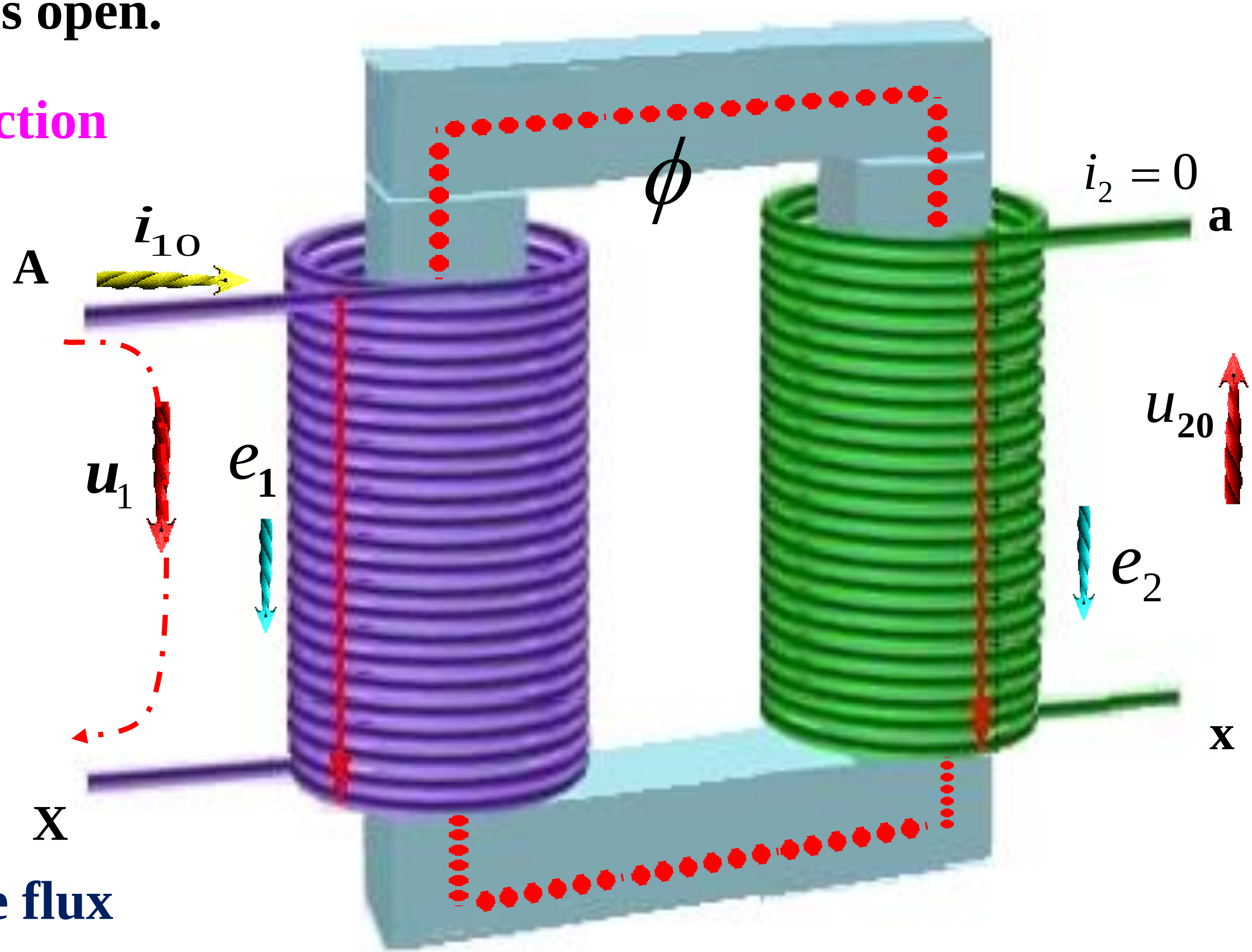
Three Phase:

$$\boxed{S_N = \sqrt{3} \times U_{1N} \times I_{1N} = \sqrt{3} \times U_{2N} \times I_{2N}}$$

No-load Operation

The primary is connected to ac voltage source，and the secondary is open.

Positive direction



Ignore leakage flux

No-load Operation

$$e_1 = -N_1 \frac{\mathrm{d}\phi}{\mathrm{d}t}$$

$$e_2 = -N_2 \frac{\mathrm{d}\phi}{\mathrm{d}t}$$

$$\frac{\boldsymbol{e}_1}{\boldsymbol{e}_2} = \frac{\boldsymbol{N}_1}{\boldsymbol{N}_2}$$

$$u_1 = i_{10}R_1 + (-e_1) = i_{10}R_1 + N_1 \frac{\mathrm{d}\phi}{\mathrm{d}t}$$

$$u_{20} = e_2 = -N_2 \frac{\mathrm{d}\phi}{\mathrm{d}t}$$

$$\left|\frac{u_1}{u_{20}}\right| \approx \frac{e_1}{e_2} = \frac{N_1}{N_2} = k$$

No-load Operation

$$\dot{U}_1 \Rightarrow \dot{I}_0 \Rightarrow \dot{F}_0 = N_1\dot{I}_0 \Rightarrow \dot{\Phi}_0 \Rightarrow \left(\begin{array}{l} \dot{E}_1 \\ \\ \dot{E}_2 \end{array} \right.$$

$$\dot{U}_1 = \dot{I}_{10}R_1 - \dot{E}_1 \approx -\dot{E}_1$$

$$\dot{E}_2 = \dot{U}_{20}$$

No-load Operation

Assume: The Voltage source u_1 is sinusoidal wave.

$$u_1 \approx -e_1 \qquad e_1 = \sqrt{2}E_1 \sin \omega t$$

$$\phi = -\frac{1}{N_1}\int e_1 \mathrm{d}t = -\frac{1}{N_1}\int \sqrt{2}E_1 \sin \omega t \mathrm{d}t$$

$$= \frac{\sqrt{2}E_1}{\omega N_1}\cos \omega t = \frac{\sqrt{2}E_1}{2\pi f N_1}\cos \omega t$$

$$= \frac{E_1}{4.44 f N_1}\cos \omega t = \Phi_{\mathrm{m}} \cos \omega t \qquad \boldsymbol{U_1 \approx E_1 = 4.44 f N_1 \Phi_{\mathrm{m}}}$$

Conclusion:

The waveform and the amplitude of the main flux are determined by those of the voltage source.

No-load Operation

$$U_1 \approx E_1 = 4.44\, f N_1 \Phi_\mathrm{m}$$

When $U_1 = C$: $f\boxed{\nearrow} \rightarrow \Phi_\mathrm{m}\boxed{\searrow} \rightarrow$ Saturation $\boxed{\searrow} \rightarrow \mu_\mathrm{Fe}\boxed{\nearrow}$

When $f = C$: $U_1\boxed{\nearrow} \rightarrow \Phi_\mathrm{m}\boxed{\nearrow} \rightarrow$ Saturation $\boxed{\nearrow} \rightarrow \mu_\mathrm{Fe}\boxed{\searrow}$

Discussion: There are two identical transformers which are all same except their frequencies are 50Hz and 60Hz respectively, represent A and B. In the case of consideration of saturation, please compare the no-load loss and no-load current.

Analysis: The two transformers are all same except their frequencies are 50Hz and 60Hz respectively.

$f \boxed{\nearrow} \rightarrow \Phi_m \boxed{\searrow} \rightarrow B_m \boxed{\searrow}$ when $U_1=4.44fN_1\Phi_m=C$.

Because $p_{Fe}= C_{Fe}\, f^{\,1.3}B_m{}^2G$, this leads to $p_{Fe}\boxed{\searrow}$.

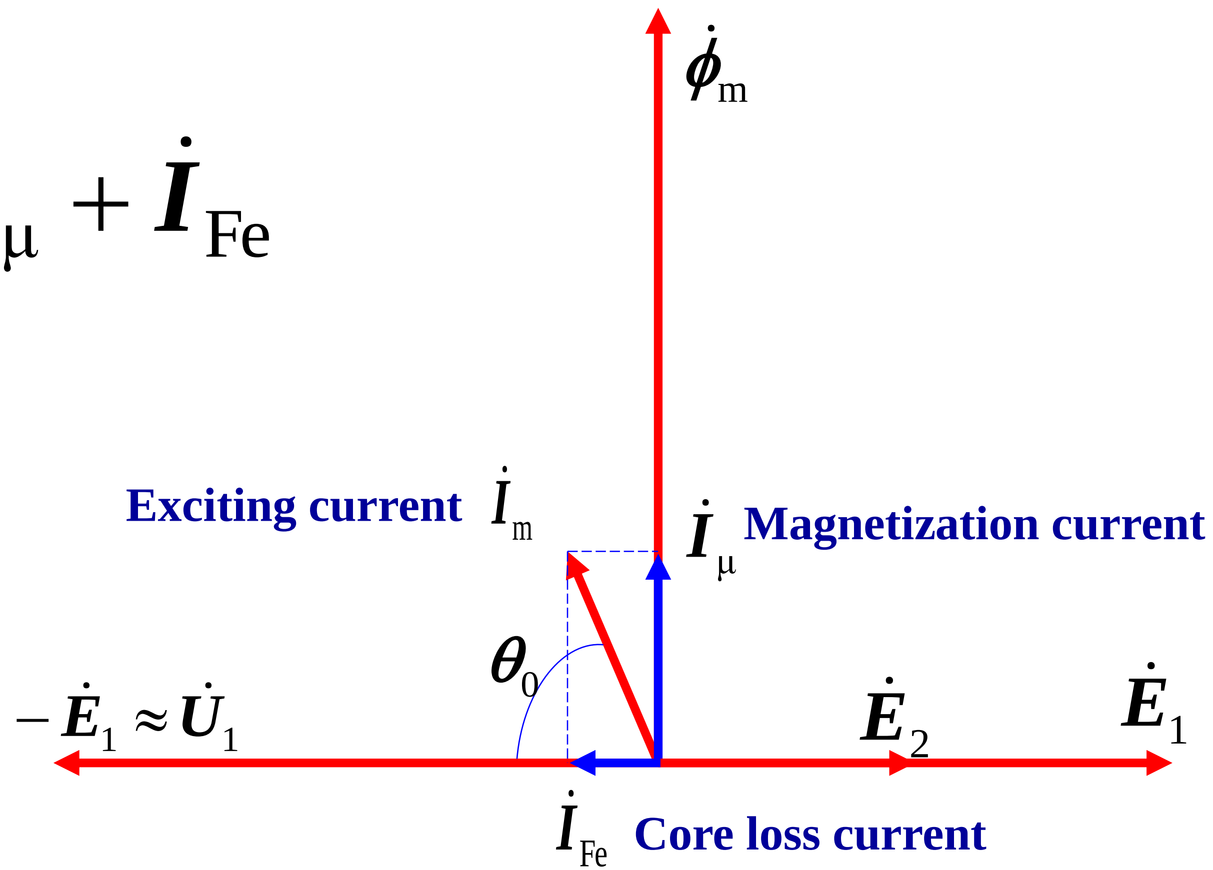
On the other hand, with $p_{Fe}\boxed{\searrow}$, the exciting resistance $R_m\boxed{\searrow}$(R_m represents core loss). With $B_m\boxed{\searrow}$, core saturation $\boxed{\searrow}$ and $\mu_{Fe}\boxed{\nearrow}$.

Because $X_m=2\pi fL_m=2\pi fN^2\Lambda_m= 2\pi fN^2(\mu_{Fe}A/L)$, when $f\boxed{\nearrow}$ and $\mu_{Fe}\boxed{\nearrow}$,$X_m\boxed{\nearrow}$. This leads to $Z_m\boxed{\nearrow}$ and $I_m\boxed{\searrow}$.

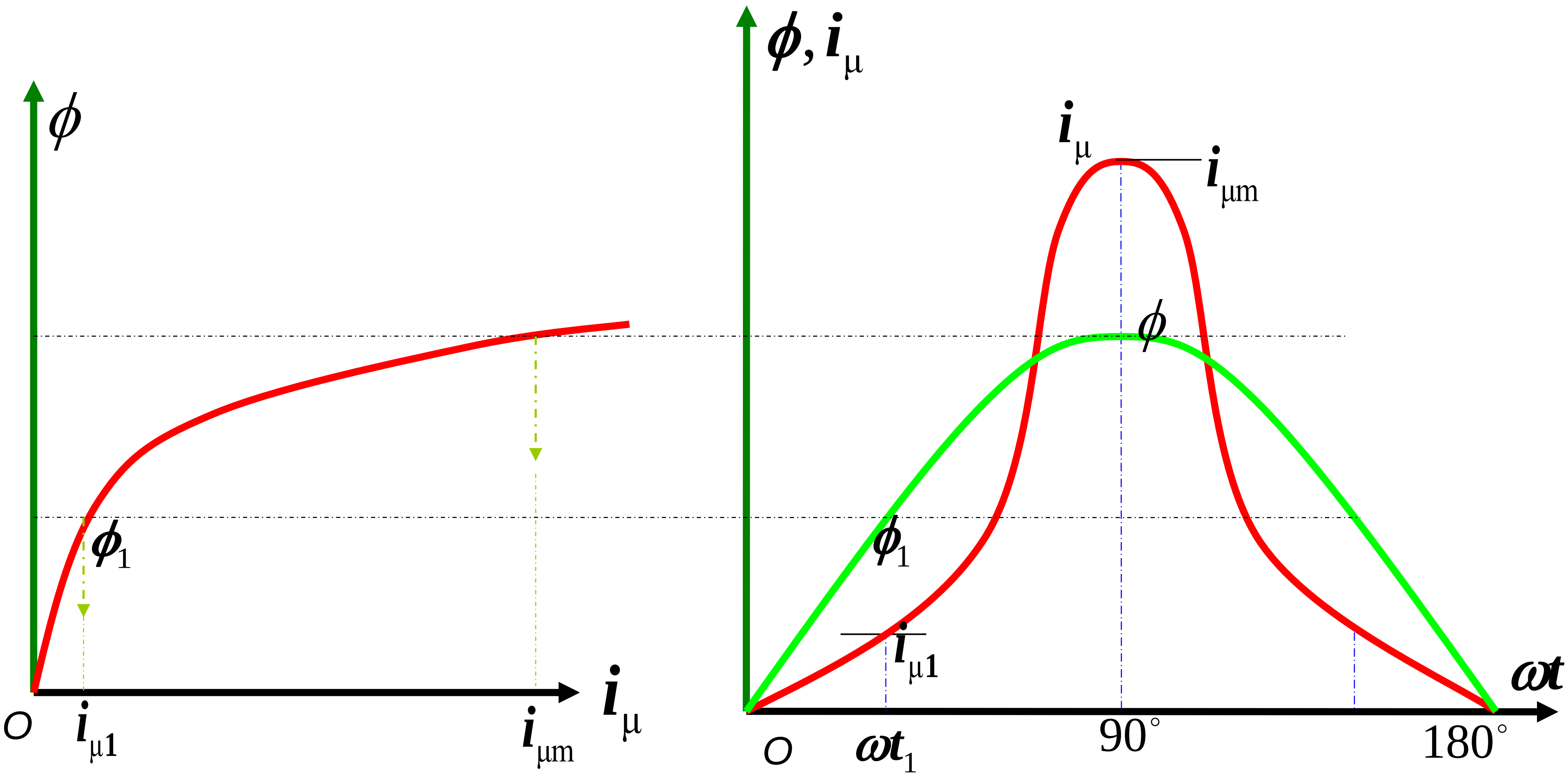
Exciting Current

$$\dot{I}_{10} = \dot{I}_{\mathrm{m}}$$

$$\dot{I}_{\mathrm{m}} = \dot{I}_{\mu} + \dot{I}_{\mathrm{Fe}}$$



Saturation



ϕ
ϕ_1
O
$i_{\mu 1}$
$i_{\mu m}$
i_μ
ϕ, i_μ
i_μ
$i_{\mu m}$
ϕ
ϕ_1
$i_{\mu 1}$
ωt
O
ωt_1
90°
180°

Exciting impedance

Mathematical modeling

$$\phi = N_1 i_\mu \cdot \Lambda_m$$

$$e_1 = -N_1 \frac{d\phi}{dt} = -N_1 \frac{d(N_1 i_\mu \cdot \Lambda_m)}{dt}$$

$$= -N_1^2 \Lambda_m \frac{di_\mu}{dt} = -L_{1\mu} \frac{di_\mu}{dt}$$

$$\dot{E}_1 = -j\omega L_{1\mu} \dot{I}_\mu = -jX_\mu \dot{I}_\mu$$

$$\dot{I}_\mu = -\frac{\dot{E}_1}{jX_\mu}$$

$$X_\mu = \omega L_{1\mu} = \omega N_1^2 \Lambda_m$$

Magnetizing reactance

Exciting impedance

Mathematical modeling

$$p_{\mathrm{Fe}} \propto E_1^2 \Rightarrow p_{\mathrm{Fe}} = \frac{E_1^2}{R_{\mathrm{Fe}}}$$

$$\boxed{\dot{E}_1 = -\dot{I}_{\mathrm{Fe}} R_{\mathrm{Fe}}}$$

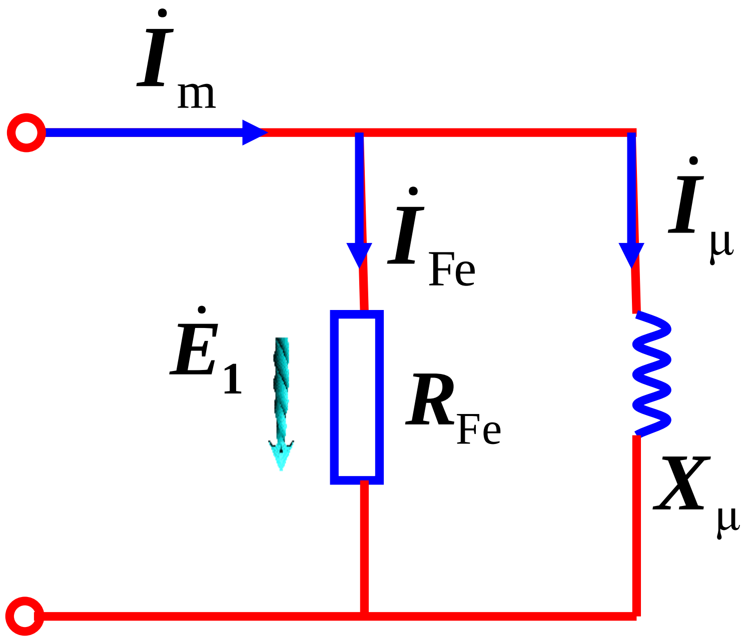
$$\boxed{\dot{I}_{\mathrm{Fe}} = -\frac{\dot{E}_1}{R_{\mathrm{Fe}}}}$$

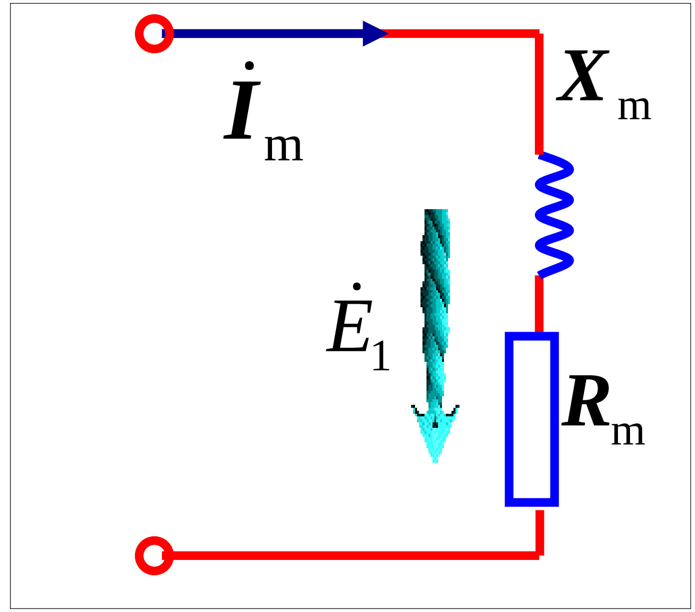
R_{Fe} **Core loss resistance**

$$\dot{I}_\mathrm{m} = \dot{I}_\mu + \dot{I}_\mathrm{Fe}$$

$$\dot{E}_1 = -\dot{I}_\mathrm{Fe} R_\mathrm{Fe}$$

$$\dot{E}_1 = -\mathrm{j} X_\mu \dot{I}_\mu$$

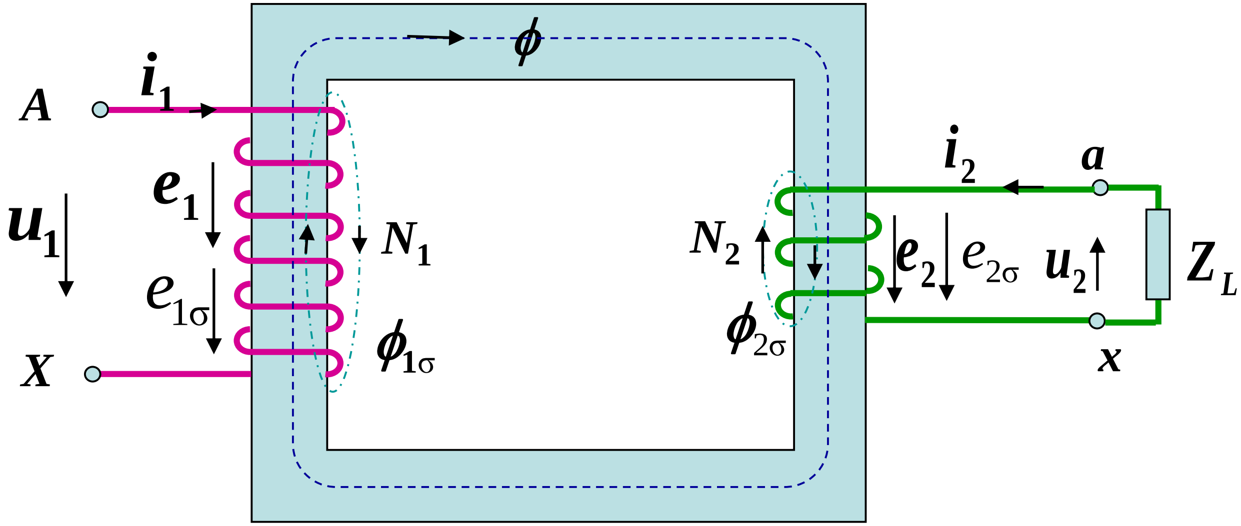




$$\dot{E}_1 = -\dot{I}_\mathrm{m} Z_\mathrm{m}$$

Load Operation

The primary is connected to ac voltage source， and the secondary is closed by a load impedance.



Load Operation

$U_1 \approx E_1 = 4.44\, f N_1 \Phi_{\mathrm{m}}$ Φ_{m} changes almost nothing.

No-load: $\dot{i}_{10} = \dot{i}_{\mathrm{m}}$ $F_{10} = F_{\mathrm{m}} = N_1 \dot{i}_{\mathrm{m}}$

Load: $\dot{i}_1 = \dot{i}_{\mathrm{m}} + \dot{i}_{1L}$

MMF of primary: $F_1 = N_1 \dot{i}_{\mathrm{m}} + N_1 \dot{i}_{1L}$.

MMF of Secondary: $F_2 = N_2 \dot{i}_{12}$

$\boxed{F_1 + F_2 = F_{\mathrm{m}}}$ $N_1 \dot{i}_{1L} + N_2 \dot{i}_{12} = 0$ $\boxed{\dot{i}_{1L} = -\frac{N_2}{N_1} \dot{i}_2}$

$$\dot{F}_1 + \dot{F}_2 = \dot{F}_{\mathrm{m}} \qquad N_1 \dot{I}_1 + N_2 \dot{I}_2 = N_1 \dot{I}_{\mathrm{m}}$$

MMF Equation

Leakage flux and Leakage reactance

$$e_{1\sigma} = -N_1 \frac{\mathrm{d}\phi_{1\sigma}}{\mathrm{d}t} = -N_1 \frac{\mathrm{d}(N_1 i_1 \cdot \Lambda_{1\sigma})}{\mathrm{d}t} = -N_1^2 \Lambda_{1\sigma} \frac{\mathrm{d}i_1}{\mathrm{d}t}$$

$$L_{1\sigma} = N_1^2 \Lambda_{1\sigma} \quad \boxed{e_{1\sigma} = -L_{1\sigma} \frac{\mathrm{d}i_1}{\mathrm{d}t}} \quad X_{1\sigma} = \omega L_{1\sigma} \quad \boxed{\dot{E}_{1\sigma} = -\mathrm{j} X_{1\sigma} \dot{I}_1}$$

$$e_{2\sigma} = -N_2 \frac{\mathrm{d}\phi_{2\sigma}}{\mathrm{d}t} = -N_2 \frac{\mathrm{d}(N_2 i_2 \cdot \Lambda_{2\sigma})}{\mathrm{d}t} = -N_2^2 \Lambda_{2\sigma} \frac{\mathrm{d}i_2}{\mathrm{d}t}$$

$$L_{2\sigma} = N_2^2 \Lambda_{2\sigma} \quad \boxed{e_{2\sigma} = -L_{2\sigma} \frac{\mathrm{d}i_2}{\mathrm{d}t}} \quad X_{2\sigma} = \omega L_{2\sigma} \quad \boxed{\dot{E}_{2\sigma} = -\mathrm{j} X_{2\sigma} \dot{I}_2}$$

Voltage Equation

$$\boldsymbol{u}_1 + \boldsymbol{e}_1 + \boldsymbol{e}_{1\sigma} = \boldsymbol{i}_1 \boldsymbol{R}_1$$

$$\boldsymbol{u}_1 = \boldsymbol{i}_1 \boldsymbol{R}_1 - \boldsymbol{e}_{1\sigma} - \boldsymbol{e}_1$$

$$u_1 = i_1 R_1 + L_{1\sigma} \frac{\mathrm{d}i_1}{\mathrm{d}t} - e_1$$

$$\dot{\boldsymbol{U}}_1 = \dot{\boldsymbol{I}}_1 \boldsymbol{R}_1 + \mathrm{j}\underline{\dot{\boldsymbol{I}}_1 \boldsymbol{X}_{1\sigma}} - \dot{\boldsymbol{E}}_1$$

$$\dot{\boldsymbol{U}}_1 = \dot{\boldsymbol{I}}_1 (\boldsymbol{R}_1 + \mathrm{j}\boldsymbol{X}_{1\sigma}) - \dot{\boldsymbol{E}}_1$$

$$\boldsymbol{Z}_{1\sigma} = \boldsymbol{R}_1 + \mathrm{j}\boldsymbol{X}_{1\sigma}$$

$$\boldsymbol{e}_2 + \boldsymbol{e}_{2\sigma} = \boldsymbol{i}_2 \boldsymbol{R}_2 + \boldsymbol{u}_2$$

$$\boldsymbol{e}_2 = \boldsymbol{i}_2 \boldsymbol{R}_2 - \boldsymbol{e}_{2\sigma} + \boldsymbol{u}_2$$

$$e_2 = i_2 R_2 + L_{2\sigma} \frac{\mathrm{d}i_2}{\mathrm{d}t} + u_2$$

$$\dot{\boldsymbol{E}}_2 = \dot{\boldsymbol{I}}_2 \boldsymbol{R}_2 + \mathrm{j}\boldsymbol{X}_{2\sigma} \dot{\boldsymbol{I}}_2 + \dot{\boldsymbol{U}}_2$$

$$\dot{\boldsymbol{E}}_2 = \dot{\boldsymbol{I}}_2 (\boldsymbol{R}_2 + \mathrm{j}\boldsymbol{X}_{2\sigma}) + \dot{\boldsymbol{U}}_2$$

$$\boldsymbol{Z}_{2\sigma} = \boldsymbol{R}_2 + \mathrm{j}\boldsymbol{X}_{2\sigma}$$

Basic Equation of Transformer

$$\dot{\boldsymbol{U}}_1 = \dot{\boldsymbol{I}}_1 \boldsymbol{Z}_{1\sigma} - \dot{\boldsymbol{E}}_1$$

$$\dot{\boldsymbol{E}}_2 = \dot{\boldsymbol{I}}_2 \boldsymbol{Z}_{2\sigma} + \dot{\boldsymbol{U}}_2$$

$$\boxed{\frac{\dot{\boldsymbol{E}}_1}{\dot{\boldsymbol{E}}_2} = \boldsymbol{k}}$$

$$N_1 \dot{I}_1 + N_2 \dot{I}_2 = N_1 \dot{I}_{\mathrm{m}}$$

$$\boxed{\dot{\boldsymbol{E}}_1 = -\dot{\boldsymbol{I}}_{\mathrm{m}} \boldsymbol{Z}_{\mathrm{m}}}$$

Winding Referring

Original $\Rightarrow$ **Referred**

$$N_2 \qquad \qquad N_1$$

$$N_1\dot{I}_2' = N_2\dot{I}_2 \Rightarrow \dot{I}_2' = \frac{N_2}{N_1}\dot{I}_2 = \frac{1}{k}\dot{I}_2$$

$$\dot{E}_2' = -N_1\frac{\mathrm{d}\phi}{\mathrm{d}t} = \dot{E}_1 = k\dot{E}_2$$

Impedance Referring

$$I_2'^2 R_2' = I_2^2 R_2 \Rightarrow \boldsymbol{R}_2' = \left(\frac{\boldsymbol{I}_2}{\boldsymbol{I}_2/\boldsymbol{k}}\right)^2 \boldsymbol{R}_2 = \boldsymbol{k}^2 \boldsymbol{R}_2$$

$$\boldsymbol{I}_2'^2 \boldsymbol{X}_{2\sigma}' = \boldsymbol{I}_2^2 \boldsymbol{X}_{2\sigma} \Rightarrow \boldsymbol{X}_{2\sigma}' = \left(\frac{\boldsymbol{I}_2}{\boldsymbol{I}_2/\boldsymbol{k}}\right)^2 \boldsymbol{X}_{2\sigma} = \boldsymbol{k}^2 \boldsymbol{X}_{2\sigma}$$

Original

$$\dot{\boldsymbol{U}}_1 = \dot{\boldsymbol{I}}_1 \boldsymbol{Z}_{1\sigma} - \dot{\boldsymbol{E}}_1$$

$$\dot{\boldsymbol{E}}_2 = \dot{\boldsymbol{I}}_2 \boldsymbol{Z}_{2\sigma} + \dot{\boldsymbol{U}}_2$$

$$\boxed{\frac{\dot{\boldsymbol{E}}_1}{\dot{\boldsymbol{E}}_2} = \boldsymbol{k}}$$

$$N_1 \dot{I}_1 + N_2 \dot{I}_2 = N_1 \dot{I}_\mathrm{m}$$

$$\boxed{\dot{\boldsymbol{E}}_1 = -\dot{\boldsymbol{I}}_\mathrm{m} \boldsymbol{Z}_\mathrm{m}}$$

Referred

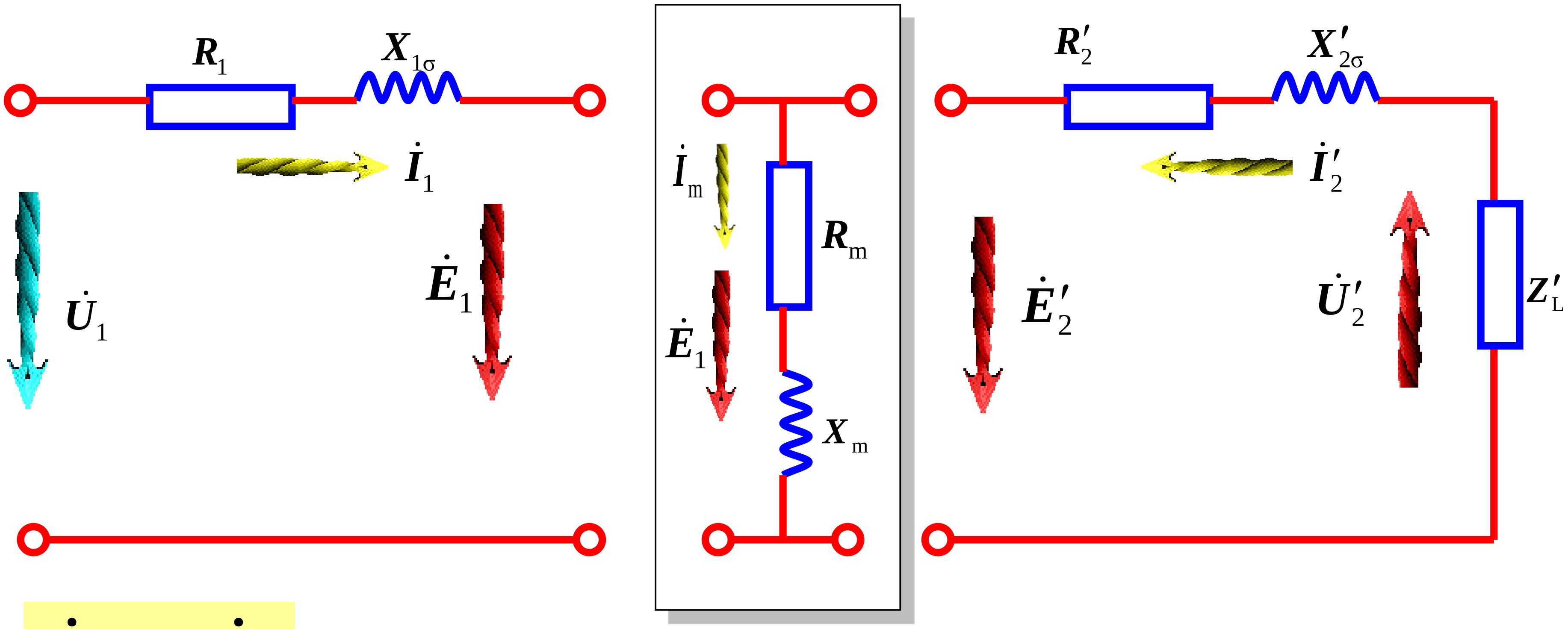
$$\dot{\boldsymbol{U}}_1 = \dot{\boldsymbol{I}}_1 \boldsymbol{Z}_{1\sigma} - \dot{\boldsymbol{E}}_1$$

$$\dot{\boldsymbol{E}}_2' = \dot{\boldsymbol{I}}_2' \boldsymbol{Z}_{2\sigma}' + \dot{\boldsymbol{U}}_2'$$

$$\dot{\boldsymbol{I}}_1 + \dot{\boldsymbol{I}}_2' = \dot{\boldsymbol{I}}_\mathrm{m}$$

$$\dot{\boldsymbol{E}}_1 = \dot{\boldsymbol{E}}_2' = -\dot{\boldsymbol{I}}_\mathrm{m} \boldsymbol{Z}_\mathrm{m}$$

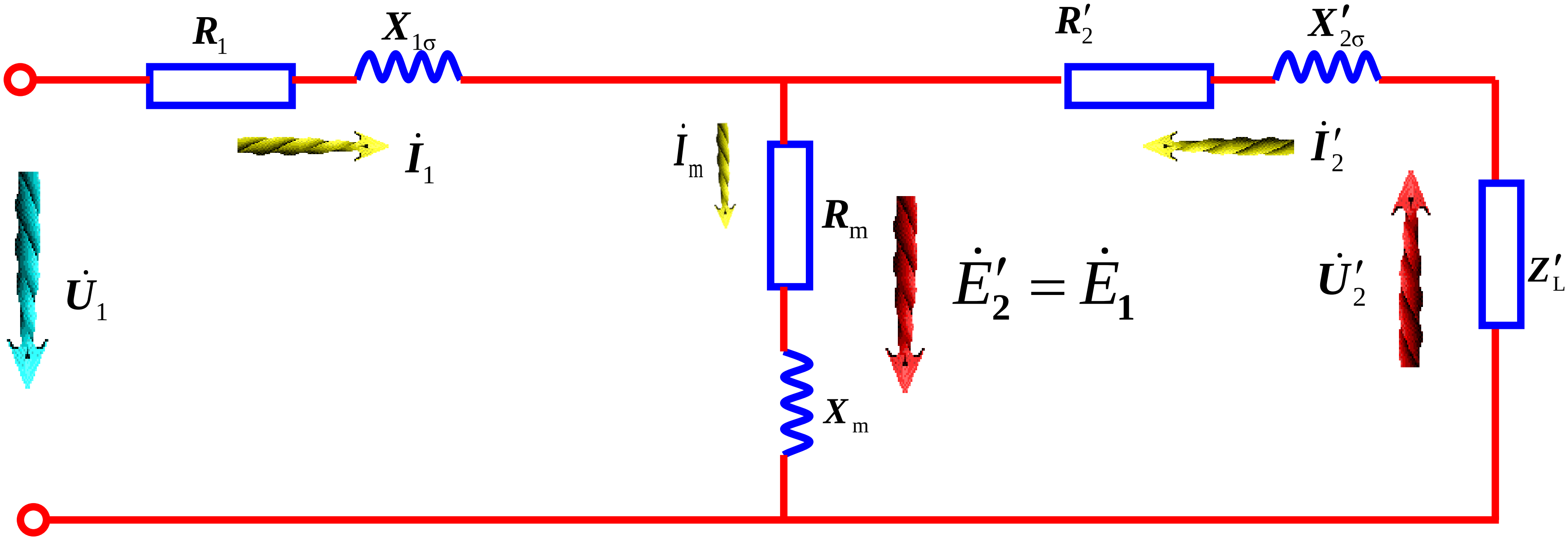
Equivalent Circuits



$$\dot{E}_1 = \dot{E}_2'$$

$$\dot{I}_1 + \dot{I}_2' = \dot{I}_m$$

Equivalent Circuits

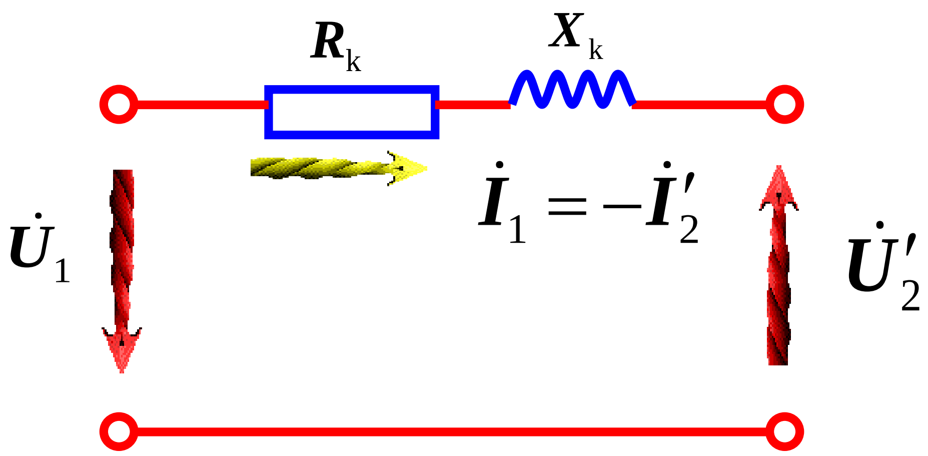


Simplified Equivalent Circuits

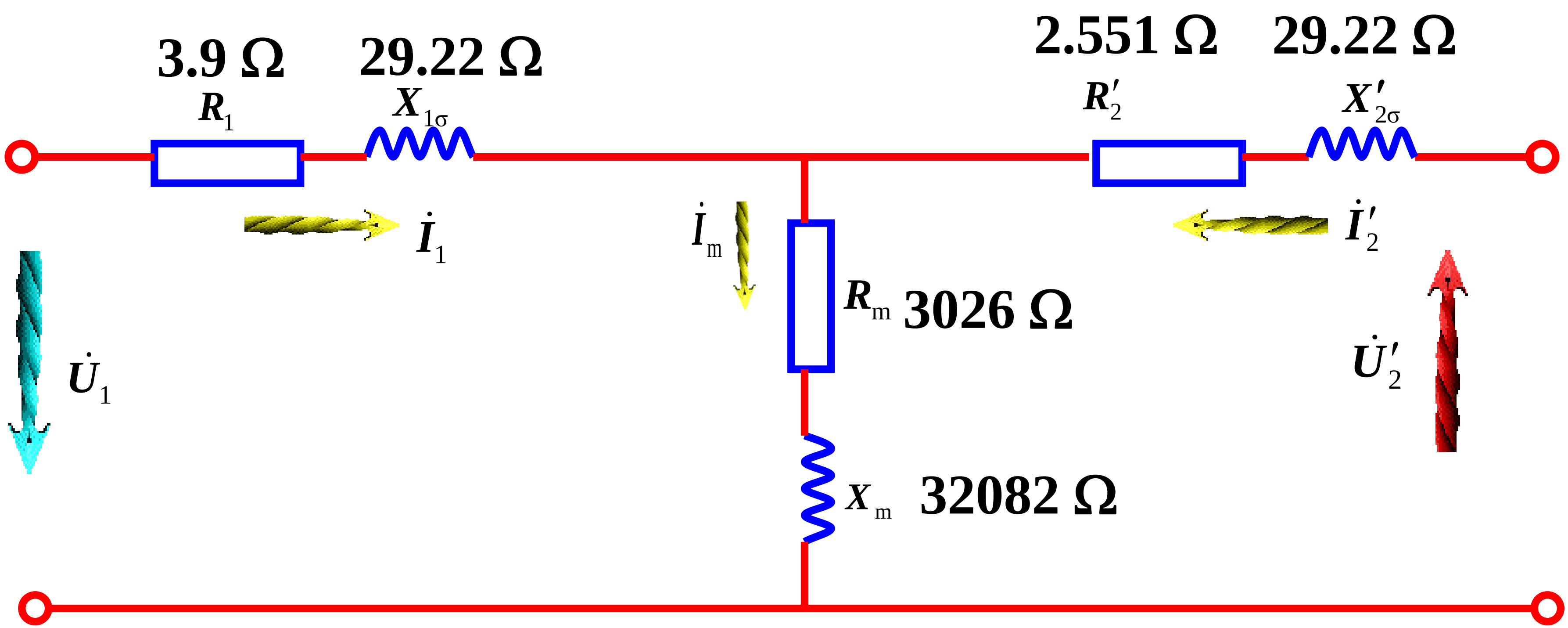
$$R_k = R_1 + R_2'$$

$$X_k = X_{1\sigma} + X_{2\sigma}'$$

$$Z_k = Z_{1\sigma} + Z_{2\sigma}' = R_k + jX_k$$

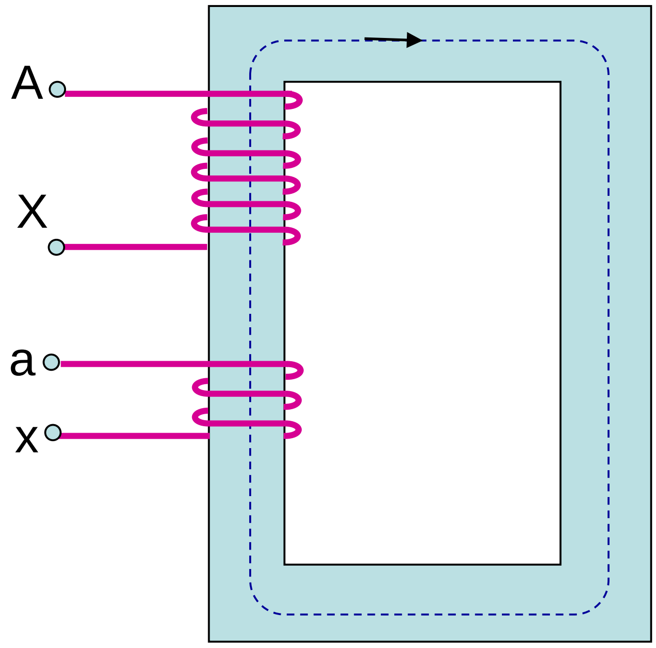


Equivalent Circuits



$S_N = 20000\text{kVA}$

$U_{1N} / U_{2N} = 127\text{kV} / 11\text{kV}$ $\quad$ $I_{1N} = 157.5\text{A}$ $\quad$ $I_{2N} = 1818\text{A}$



If 220V is applied to the high-voltage side of a U_{1N}/U_{2N}=220V/110V single-phase transformer, the exciting current is 0.3A and the no-load loss is 4W. Calculate the exciting current and the no-load loss if

(a) the terminal X is connected to the terminal a and 330V is applied to the high-voltage side (terminal A and x) of the transformer ;

(b) the terminal X is connected to the terminal x and 110V is applied to the high-voltage side (terminal A and a) of the transformer.

Per-Unit Value

$$\text{Per-Unit Value} = \frac{\text{Practice Value}}{\text{Base Value}}$$

Base Value	Primary	Secondary
Voltage	$U_{1b}=U_{1N}$	$U_{2b}=U_{2N}$
Current	$I_{1b}=I_{1N}$	$I_{2b}=I_{2N}$
Impedance	$Z_{1b}=U_{1N}/I_{1N}$	$Z_{2b}=U_{2N}/I_{2N}$
Power	$S_{1b}=S_{1N}$	$S_{2b}=S_{2N}$

Advantages of Per-Unit Value

$$Z_k^* \approx 0.03 \sim 0.1$$

$$I_{1N}^* = U_{1N}^* = I_{2N}^* = U_{2N}^* = 1$$

$$I_0^* \approx 0.02 \sim 0.05$$

$$P_N^* = \frac{P_N}{S_N} = \frac{S_N \cos\varphi}{S_N} = \cos\varphi$$

$$Q_N^* = \frac{Q_N}{S_N} = \frac{S_N \sin\varphi}{S_N} = \sin\varphi$$

$$Z_k^* = \frac{Z_k}{Z_{1b}} = \frac{(I_{1N} Z_k)}{U_{1N}} = u_k$$

$$U_2'^* = \frac{U_2'}{U_{1b}} = \frac{kU_2}{U_{1N}} = \frac{U_2}{U_{1N}/k} = \frac{U_2}{U_{2N}} = \frac{U_2}{U_{2b}} = U_2^*$$

Advantages of Per-Unit Value

$$I_2^* = 0: \quad \text{(No Load)}$$

$$I_2^* = 1: \quad \text{(Full Load)}$$

$$I_2^* = 0.5: \quad \text{(Half Load)}$$

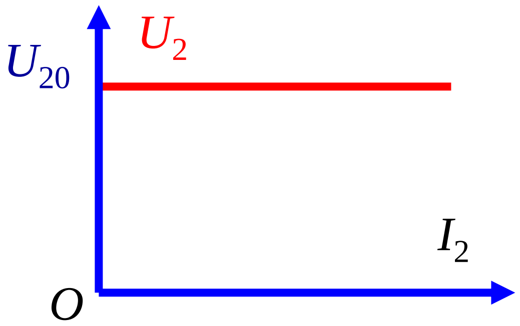
$$I_2^* = 0.7: \quad \text{(Under Load)}$$

$$I_2^* = 1.2: \quad \text{(Over Load)}$$

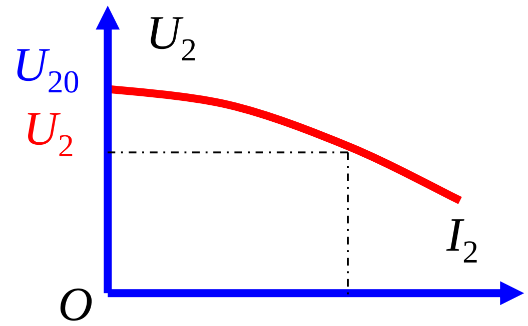
Operation Performances

External Characteristic

What we desire:



What the practice:

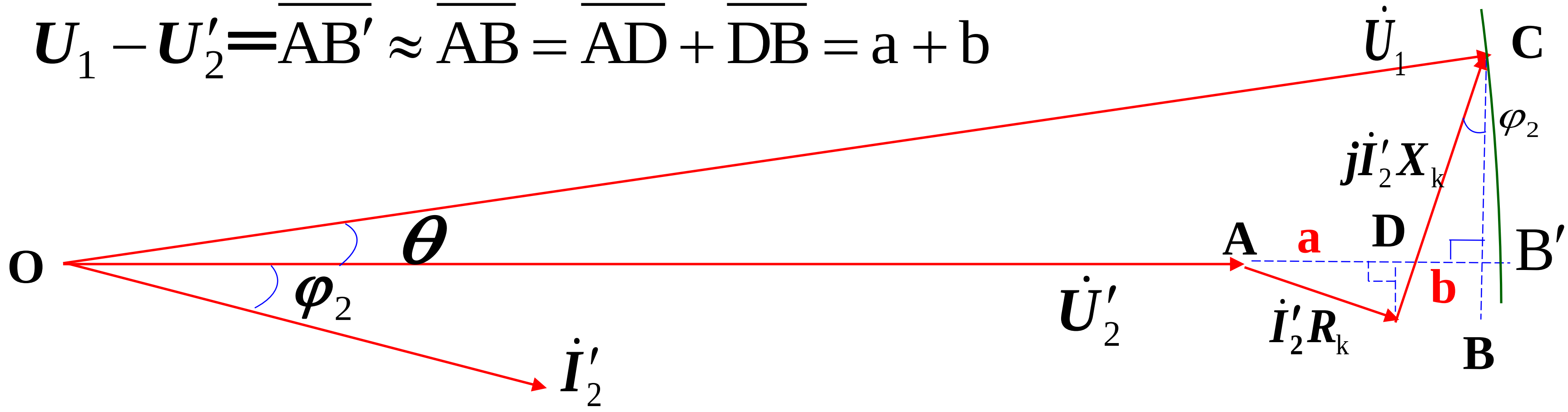


When U_1=U_{1N} and cosφ=C, find $U_2 = f(I_2)$

Voltage Regulation

$$\Delta u = \frac{U_{20} - U_2}{U_{2N\phi}} \times 100\% = \frac{U_{1N\phi} - U_2'}{U_{1N\phi}} \times 100\%$$

$$U_1 - U_2' = \overline{AB'} \approx \overline{AB} = \overline{AD} + \overline{DB} = a + b$$



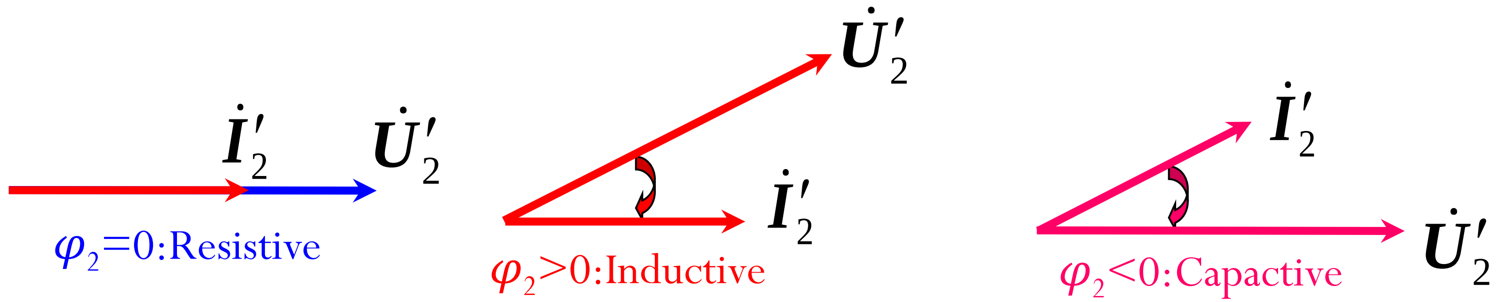
$$a = I_2' R_k \cos\varphi_2 \qquad b = I_2' X_k \sin\varphi_2$$

$$\Delta u = \frac{U_{1N\phi} - U_2'}{U_{1N\phi}} \times 100\% \approx \frac{I_2' R_k \cos\varphi_2 + I_2' X_k \sin\varphi_2}{U_{1N\phi}} \times 100\%$$

$$= \frac{I_2' R_k \cos\varphi_2 + I_2' X_k \sin\varphi_2}{I_{1N\phi}(U_{1N\phi} / I_{1N\phi})} \times 100\%$$

$$\boxed{\Delta u = I^*(R_k^* \cos\varphi_2 + X_k^* \sin\varphi_2) \times 100\%}$$

$$\Delta u = I^{*}(R_{\mathrm{k}}^{*}\cos\varphi_2 + X_{\mathrm{k}}^{*}\sin\varphi_2)\times 100\%$$



Voltage regulation is

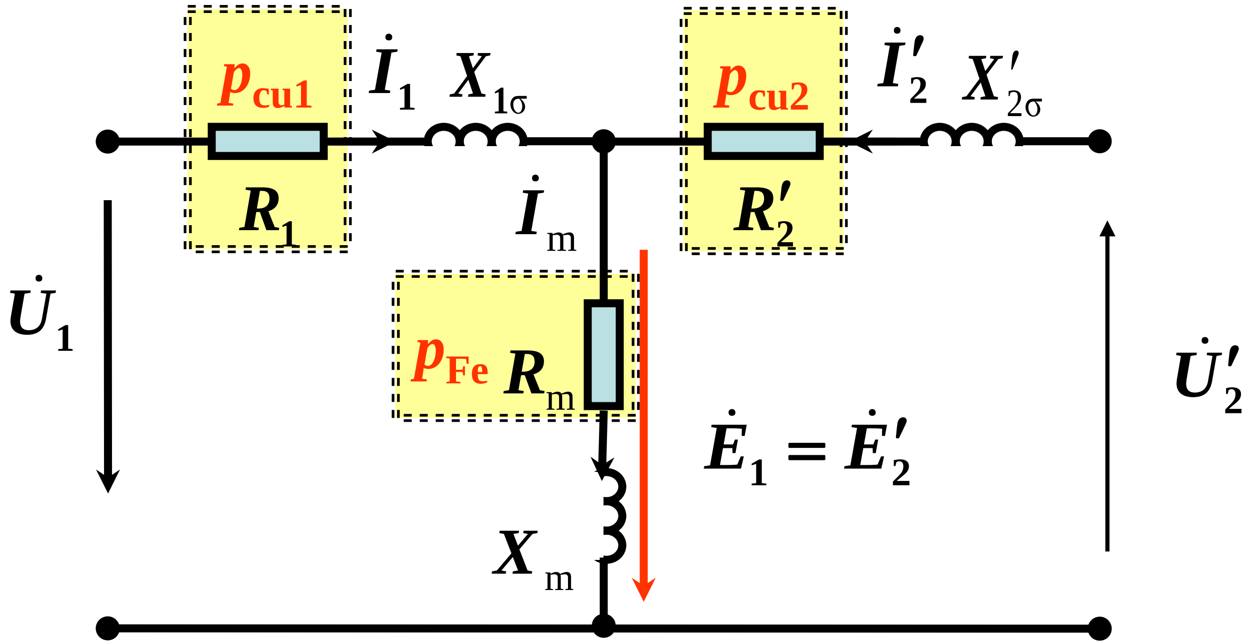
1.related to the load current I^{*}_{2},

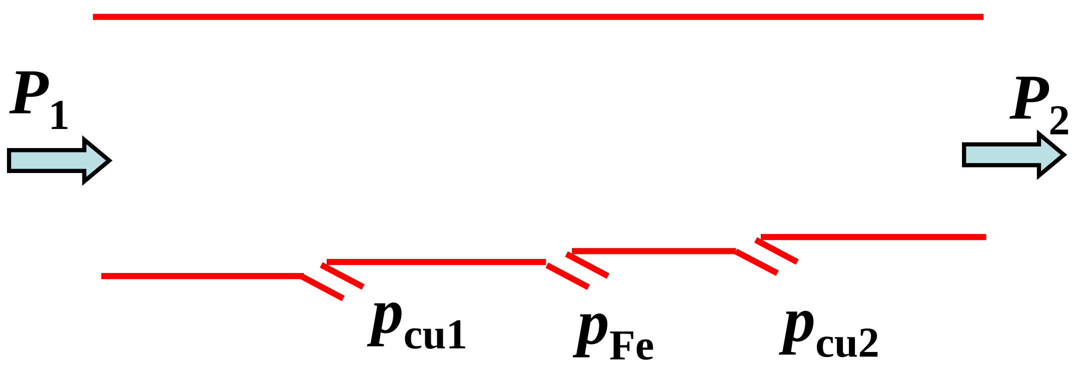
2.related to the short-circuited impedance Z^{*}_{k}, and

3.related to the load property $\cos\varphi_2$.

Tap switch

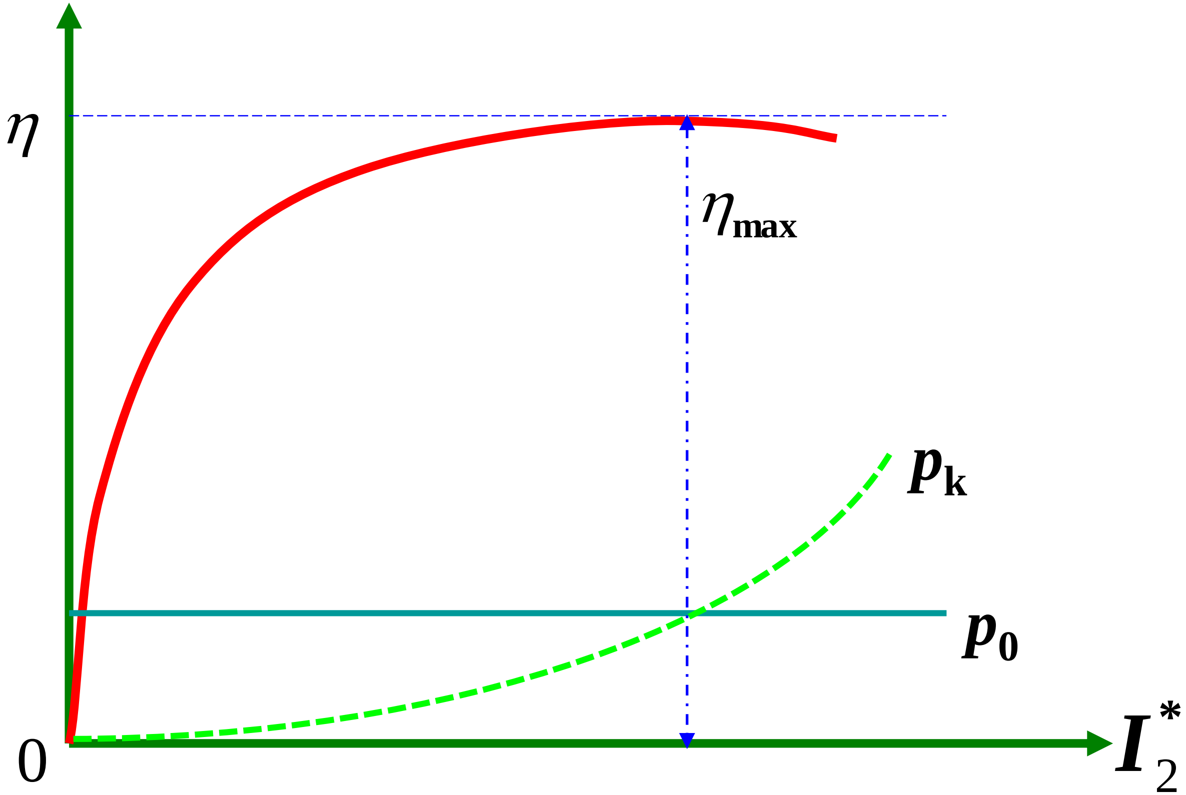
Efficiency Characteristic





$$\eta = 1 - \frac{\sum p}{P_2 + \sum p} = 1 - \frac{p_0 + p_\mathrm{k}}{P_2 + p_0 + p_\mathrm{k}}$$

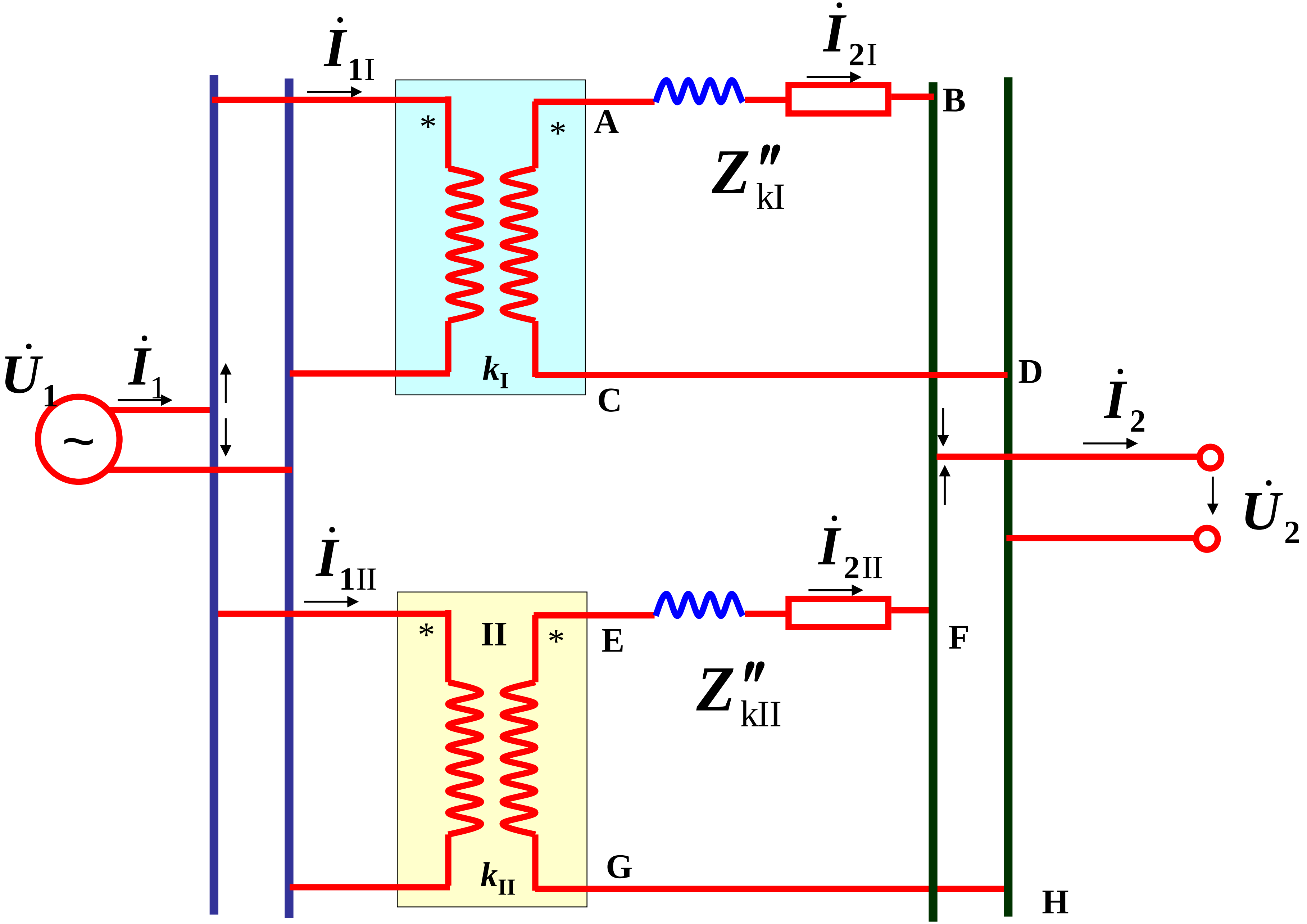
$$\eta = 1 - \frac{p_0 + I_2^{*2} p_\mathrm{kN}}{S_\mathrm{N} I_2^{*} \cos\varphi_2 + p_0 + I_2^{*2} p_\mathrm{kN}}$$



When $\dfrac{d\eta}{dI_2^*} = 0$ **, that is** $I_2^* = \sqrt{\dfrac{p_0}{p_{kN}}}$ **:**

$$p_k = I_2^{*2} p_{kN} = \left(\sqrt{\frac{p_0}{p_{kN}}}\right)^2 p_{kN} = p_0$$

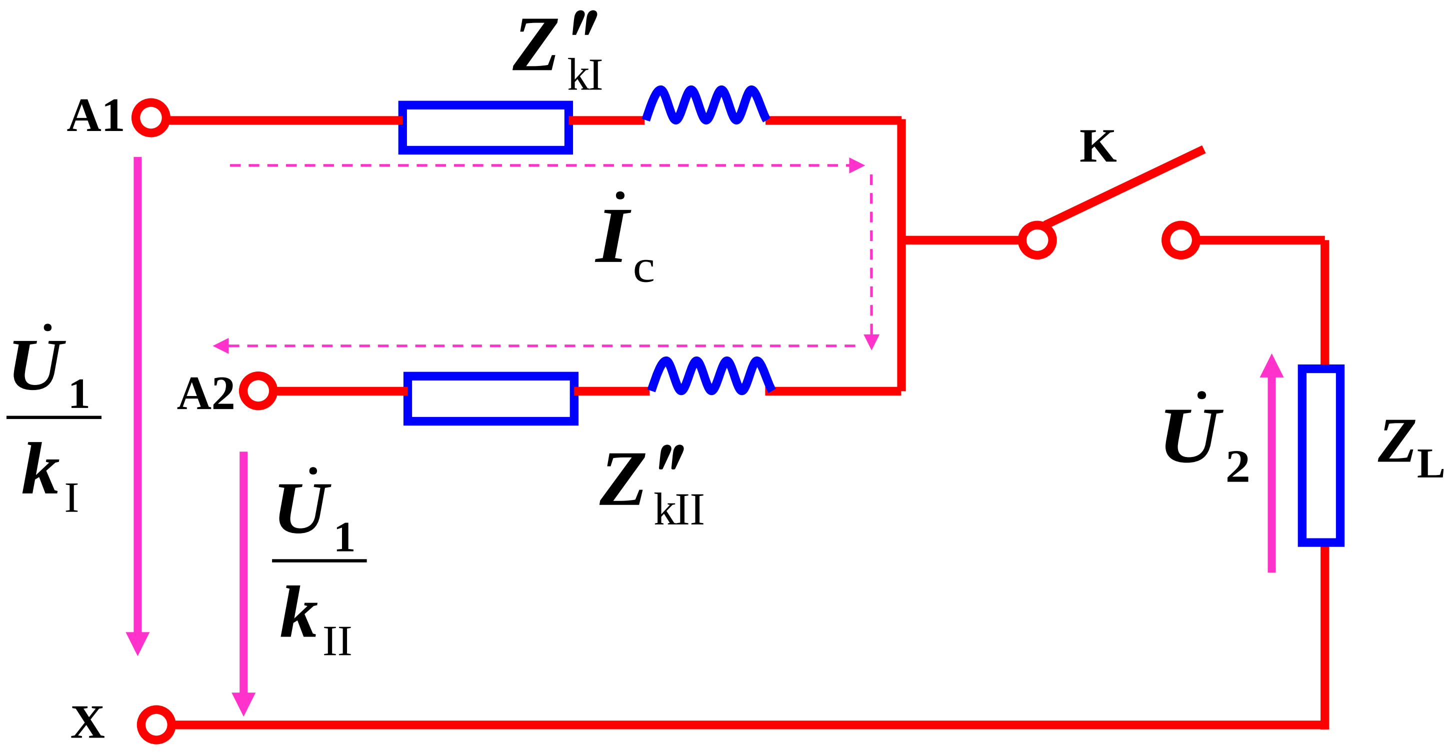
Parallel operation



$\dot{I}_{1\mathrm{I}}$
$\dot{I}_{2\mathrm{I}}$
B
A
$Z''_{k\mathrm{I}}$
$\dot{U}_1$
$\dot{I}_1$
k_{I}
D
C
$\dot{I}_2$
$\dot{U}_2$
$\dot{I}_{1\mathrm{II}}$
$\dot{I}_{2\mathrm{II}}$
II
E
F
$Z''_{k\mathrm{II}}$
k_{II}
G
H

Parallel operation

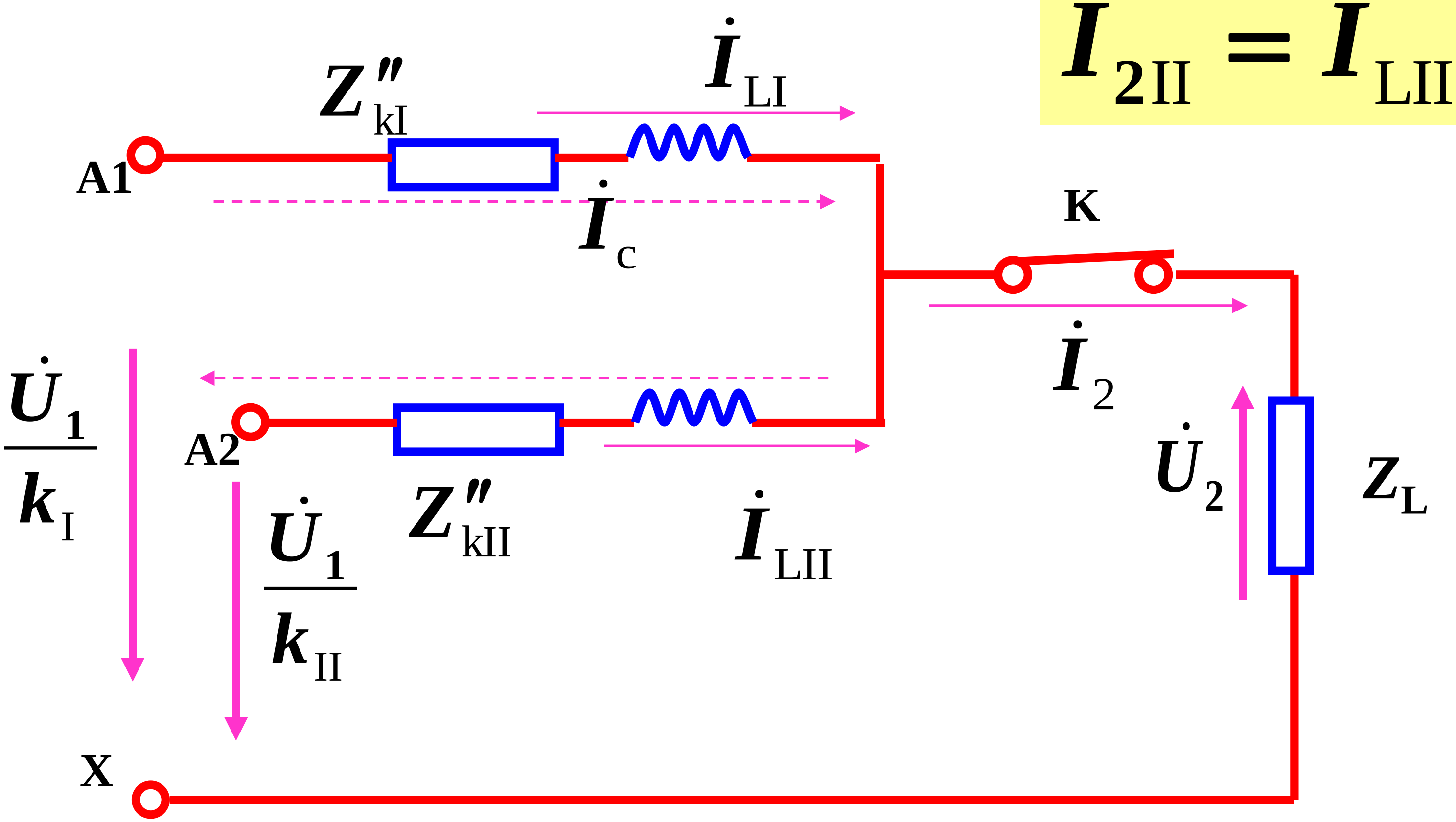
$$\dot{I}_{\mathrm{c}} = \frac{\dfrac{\dot{U}_1}{k_{\mathrm{I}}} - \dfrac{\dot{U}_1}{k_{\mathrm{II}}}}{Z''_{\mathrm{kI}} + Z''_{\mathrm{kII}}}$$



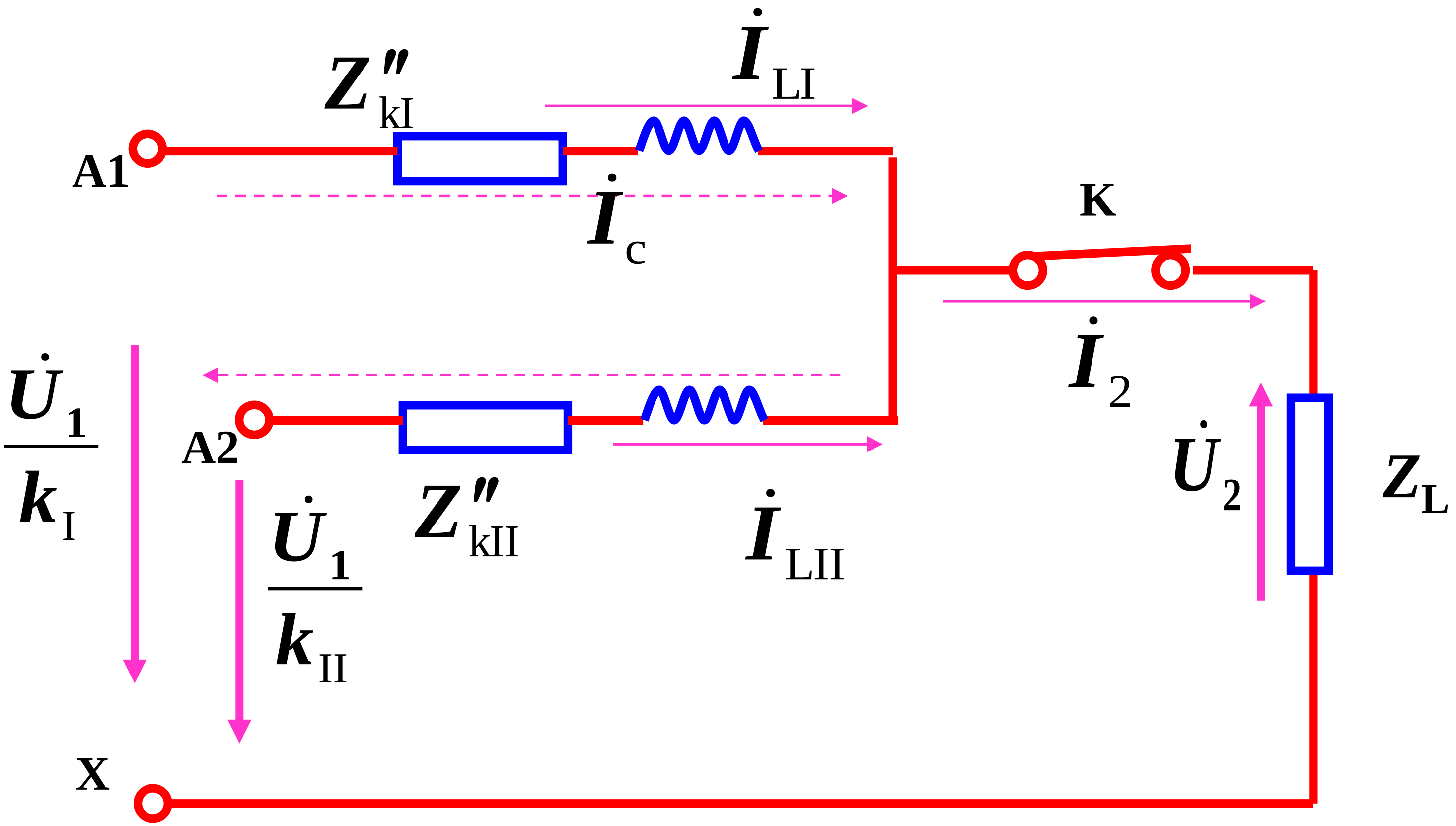
Parallel operation

$$\dot{I}_{2\mathrm{I}} = \dot{I}_{\mathrm{LI}} + \dot{I}_{\mathrm{c}}$$

$$\dot{I}_{2\mathrm{II}} = \dot{I}_{\mathrm{LII}} - \dot{I}_{\mathrm{c}}$$



Parallel operation



$$\dot{I}_{LI} + \dot{I}_{LII} = \dot{I}_2$$

$$\dot{I}_{LI} Z''_{KI} = \dot{I}_{LII} Z''_{LII}$$

For two same single-phase transformers with the voltage ratio of U_{IN}/U_{2N}=220V/110V, any one of them has a no-load current of 0.6A and a no-load loss of 10W if 220V is applied to the primary side at no-load operation. Calculate the total no-load current and the no-load loss if

(a) 440V is applied to the primary side with the primary terminals of two transformers are connected in series;

(b) 220V is applied to the primary side with the primary terminals of two transformers are connected in parallel.