Electric Machinery

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Requirement

- **More sinusoidal waveform of EMF and MMF, less harmonics;**
- **Symmetry of each phase；**
- **More utilization ratio of the conductors;**
- **Reliable in structure, insulation, heat, and so on.**

Classification

Winding Arrangement

Lap winding, Wave winding

Layer

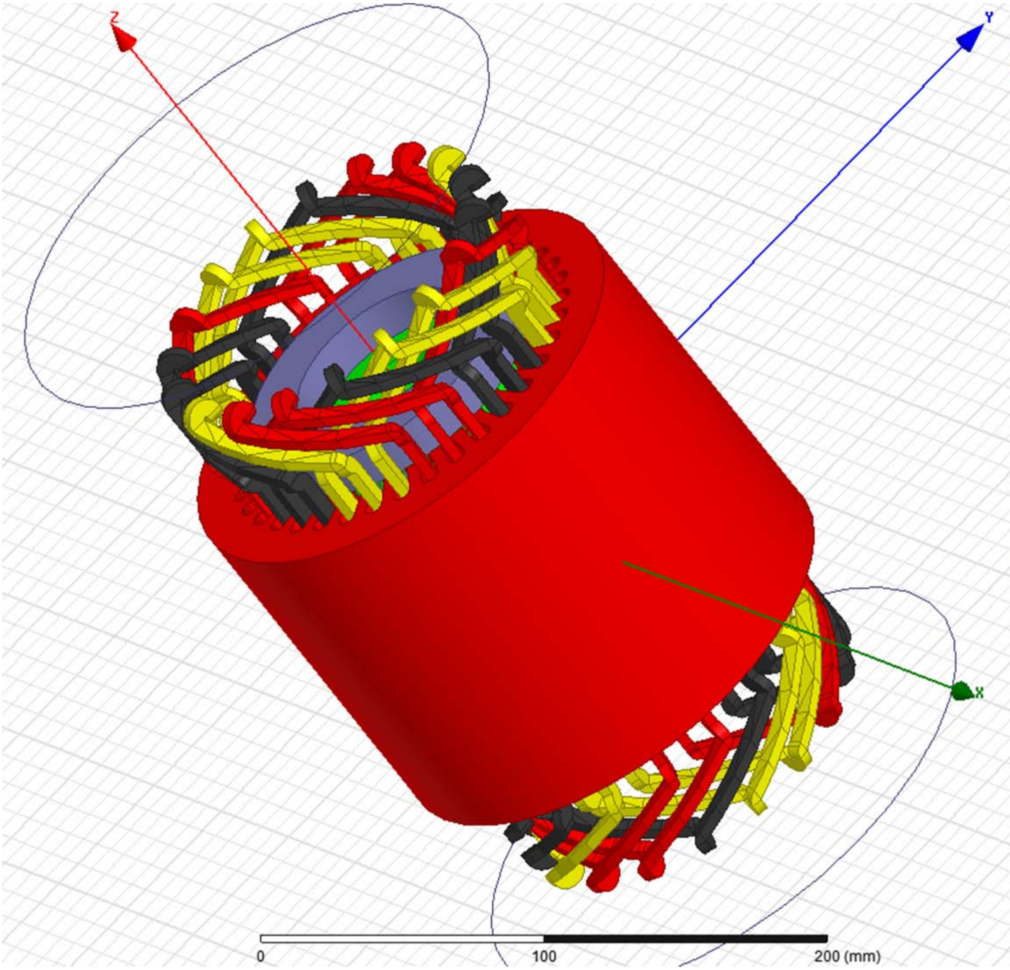
Single-layer, Double-layer, Multi-layer

Phase

Single-phase, Three-phase, Polyphase

Slots per pole per phase

Integral, fractional


Z
Y
X
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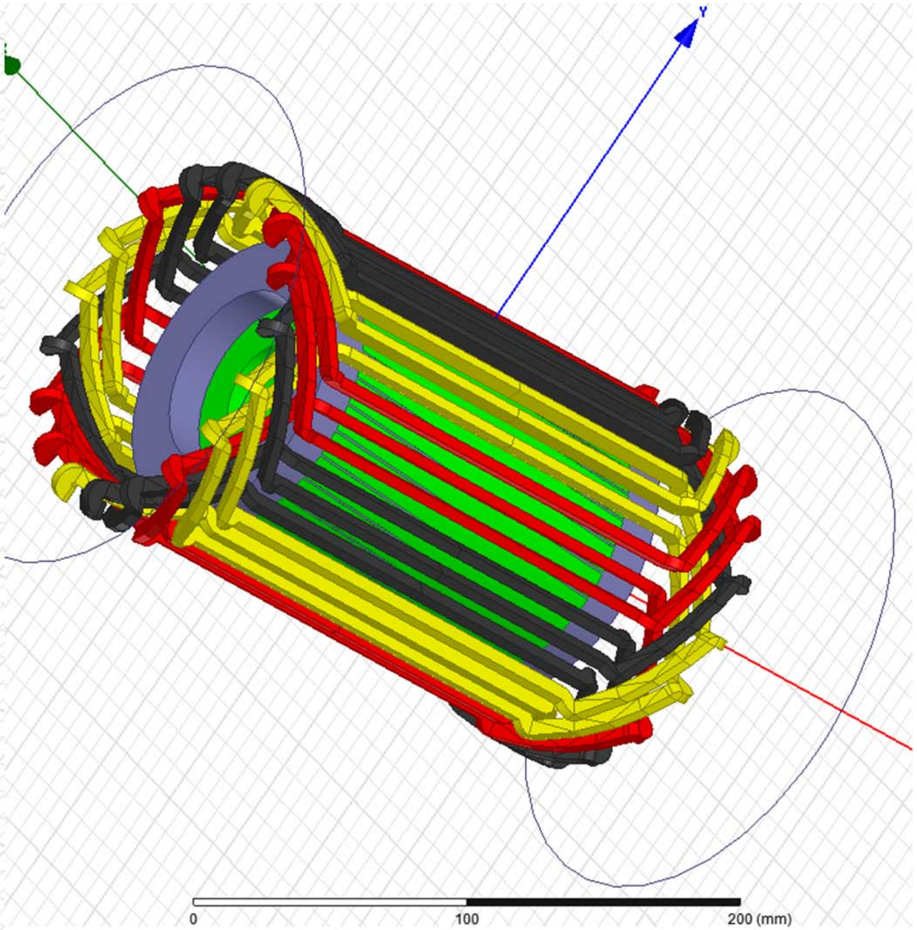
200 (mm)

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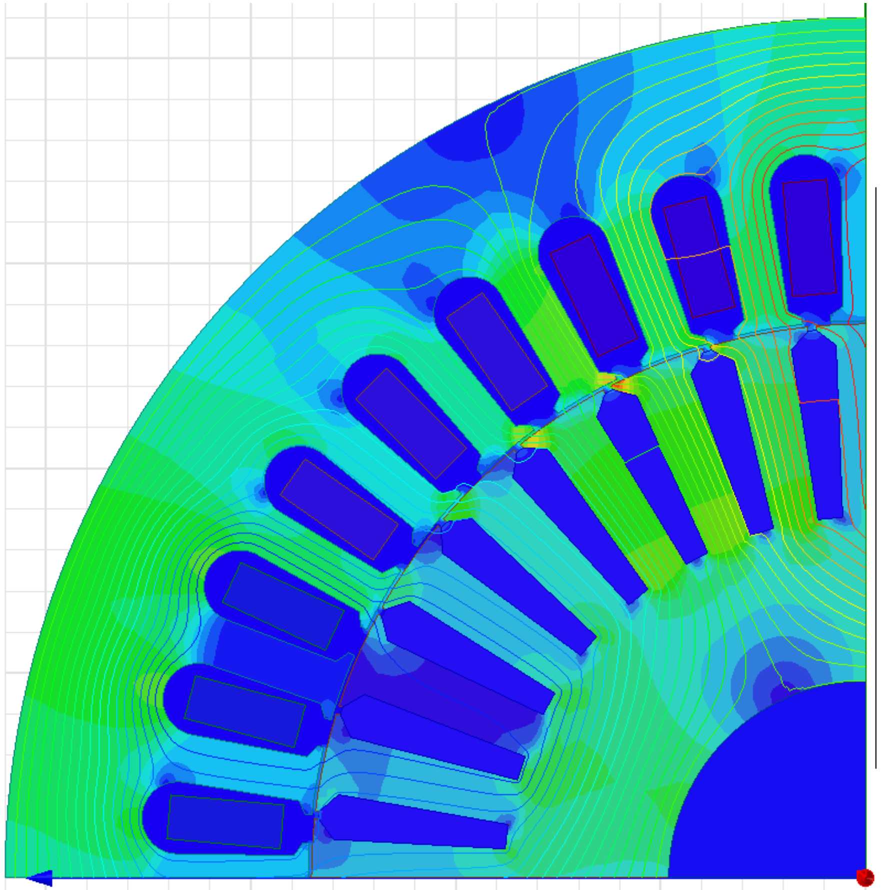

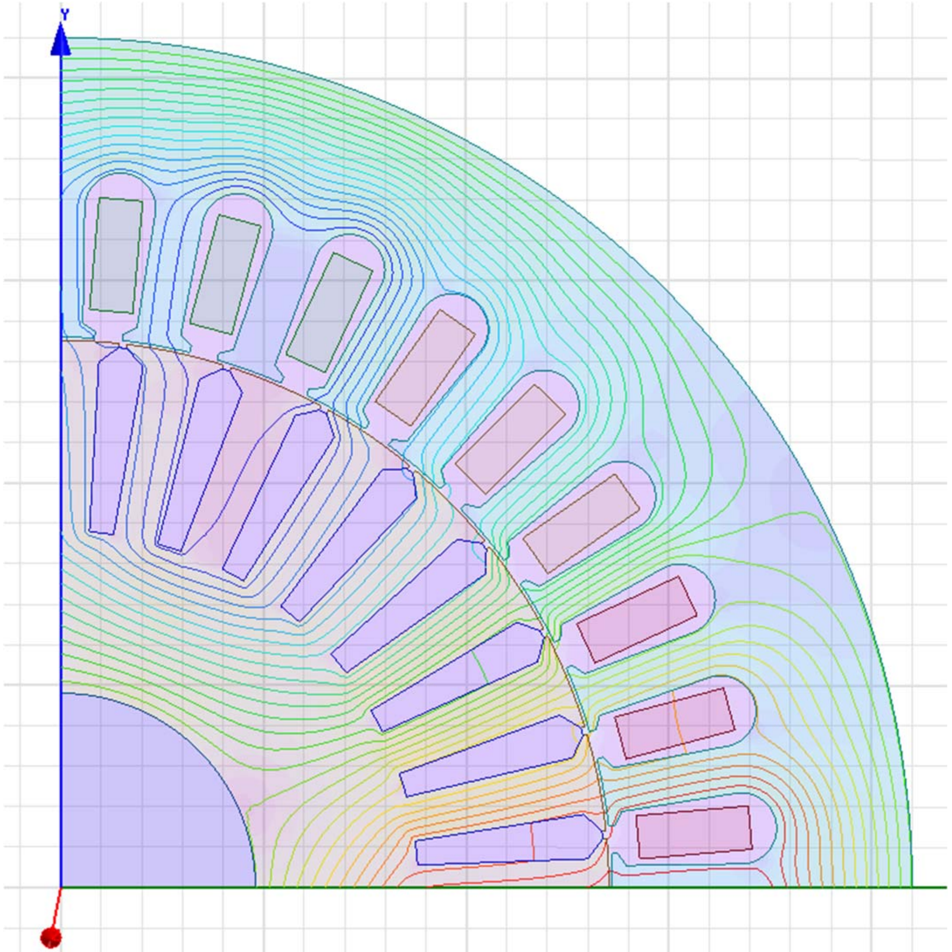
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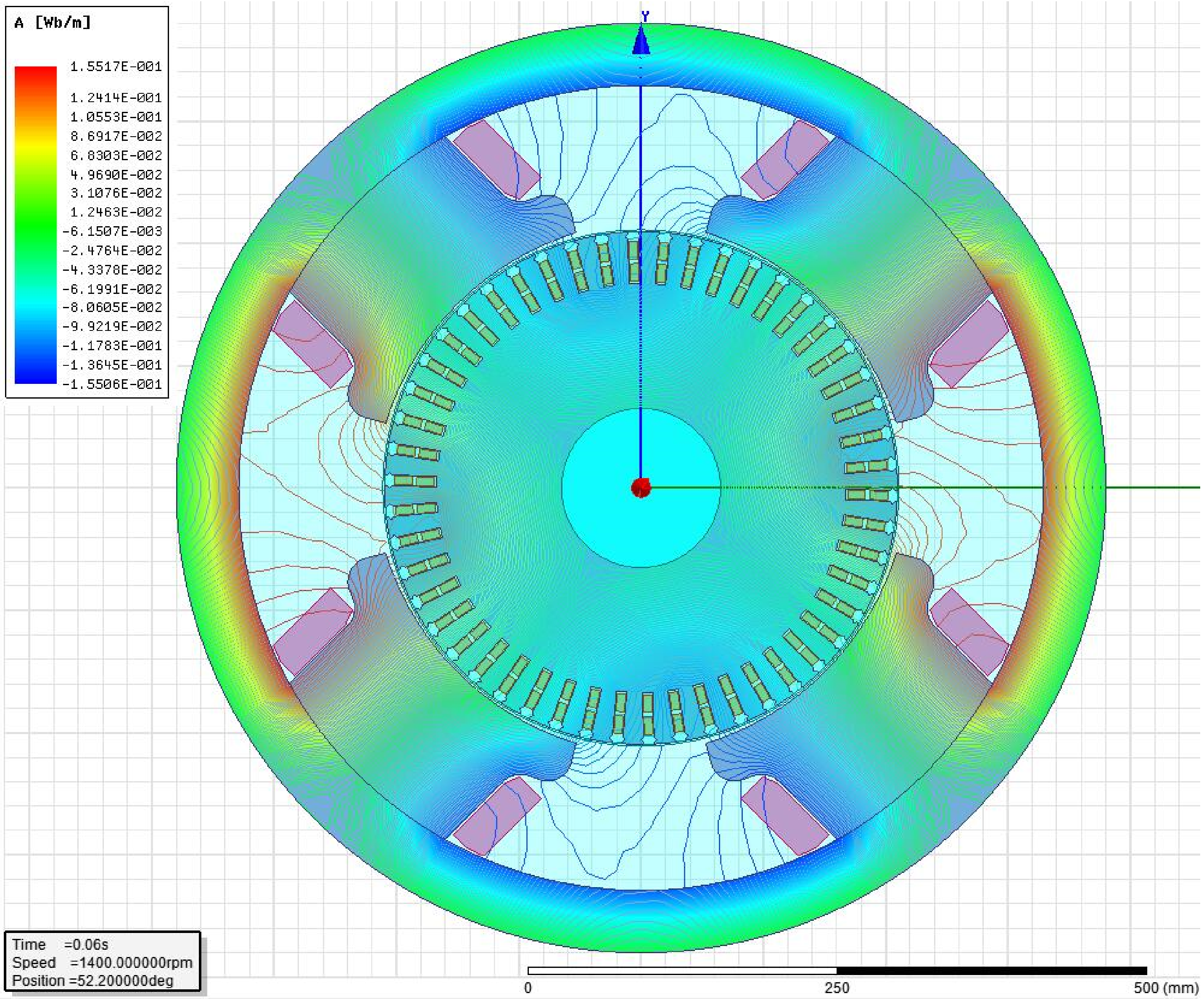
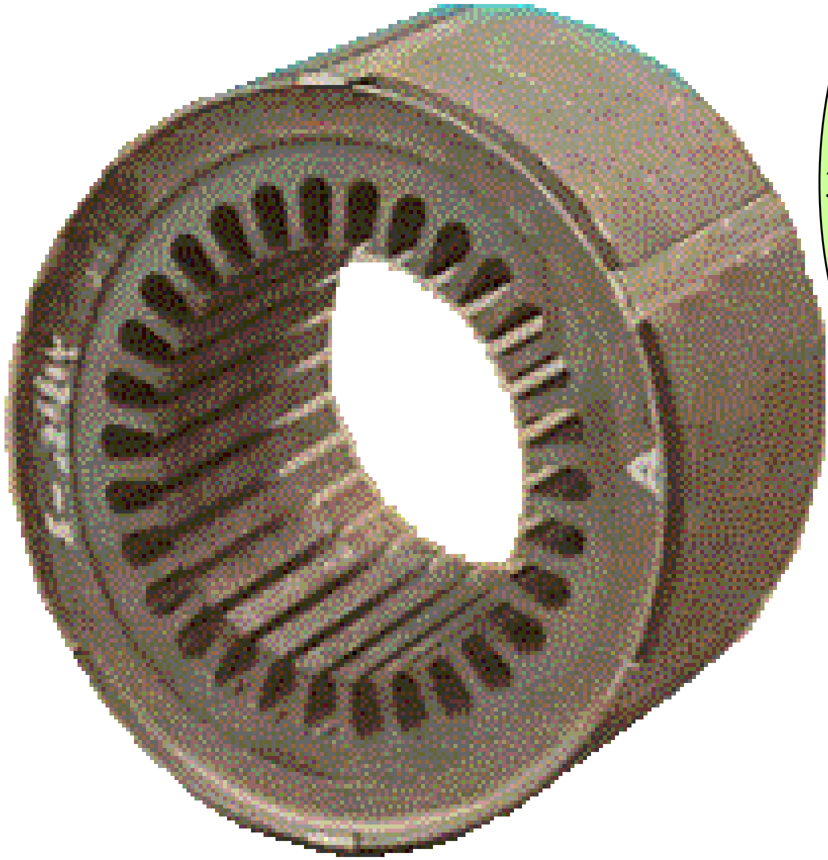
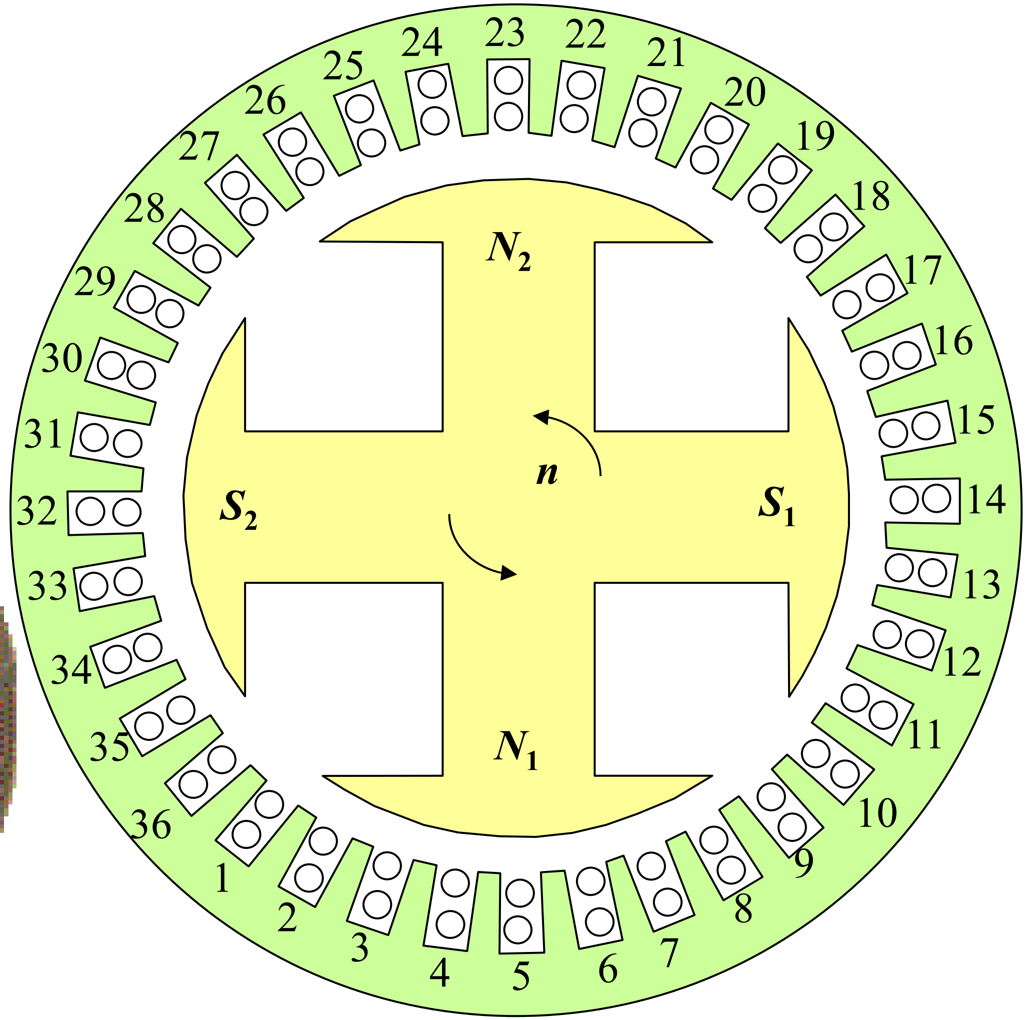


Electrical angle = *p* × Mechanical angle

Model of one synchronous machine





Technical Terms

Phase number: m=3

Slot number: Q=36

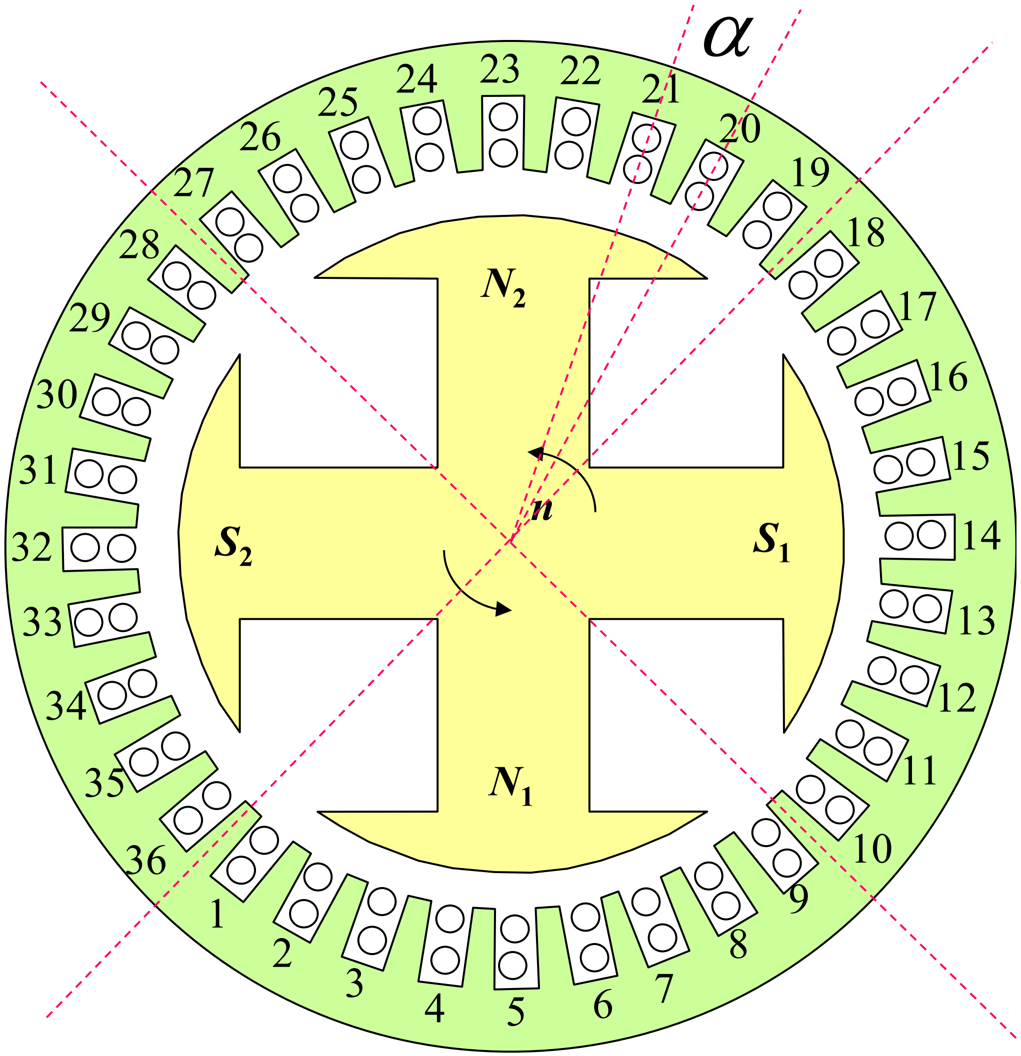
Pole number: 2p=4

Pole pitch: τ=Q/2p=9

Coil pitch: y_1=8 ($y_1<\tau$)

Slots per pole per phase:

$$q = \frac{Q}{2pm} = \frac{\tau}{m} = 3$$



Mechanical angle:

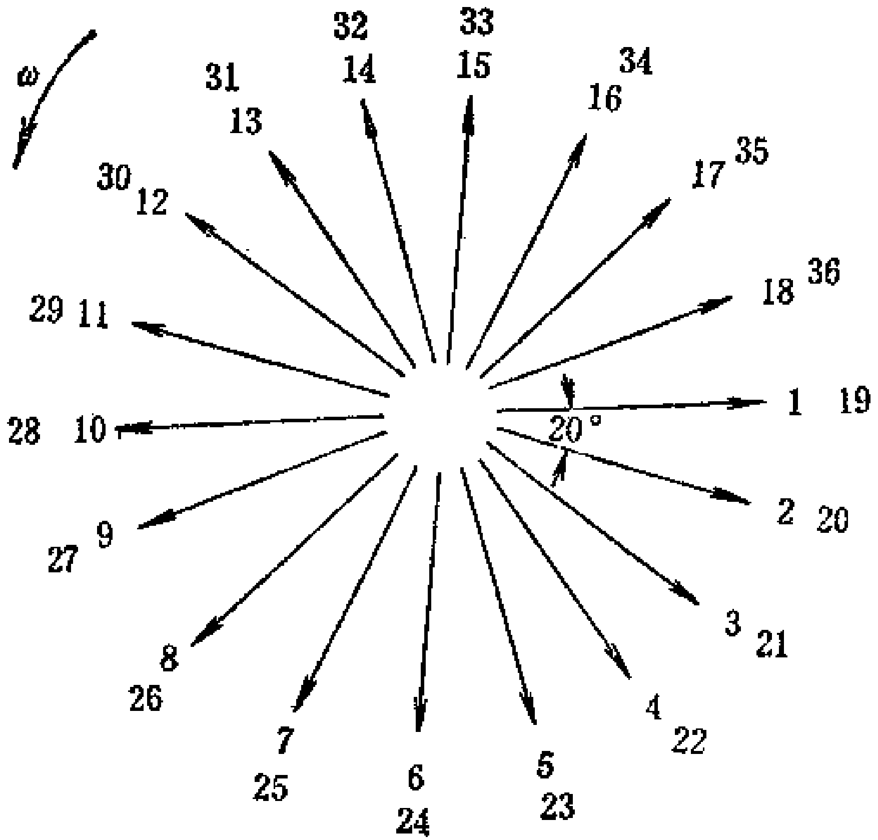
Geometrical angle

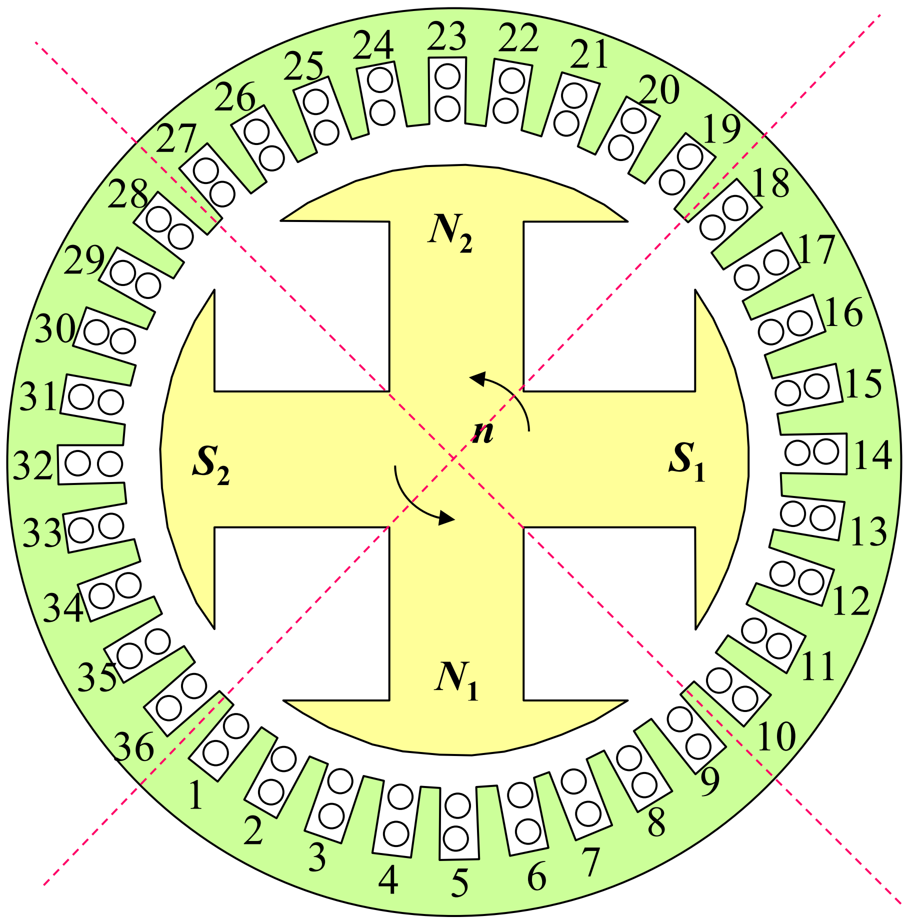
Electrical angle = p × Mechanical angle

Slot pitch in mechanical angle: α_0=360°/Q=10°

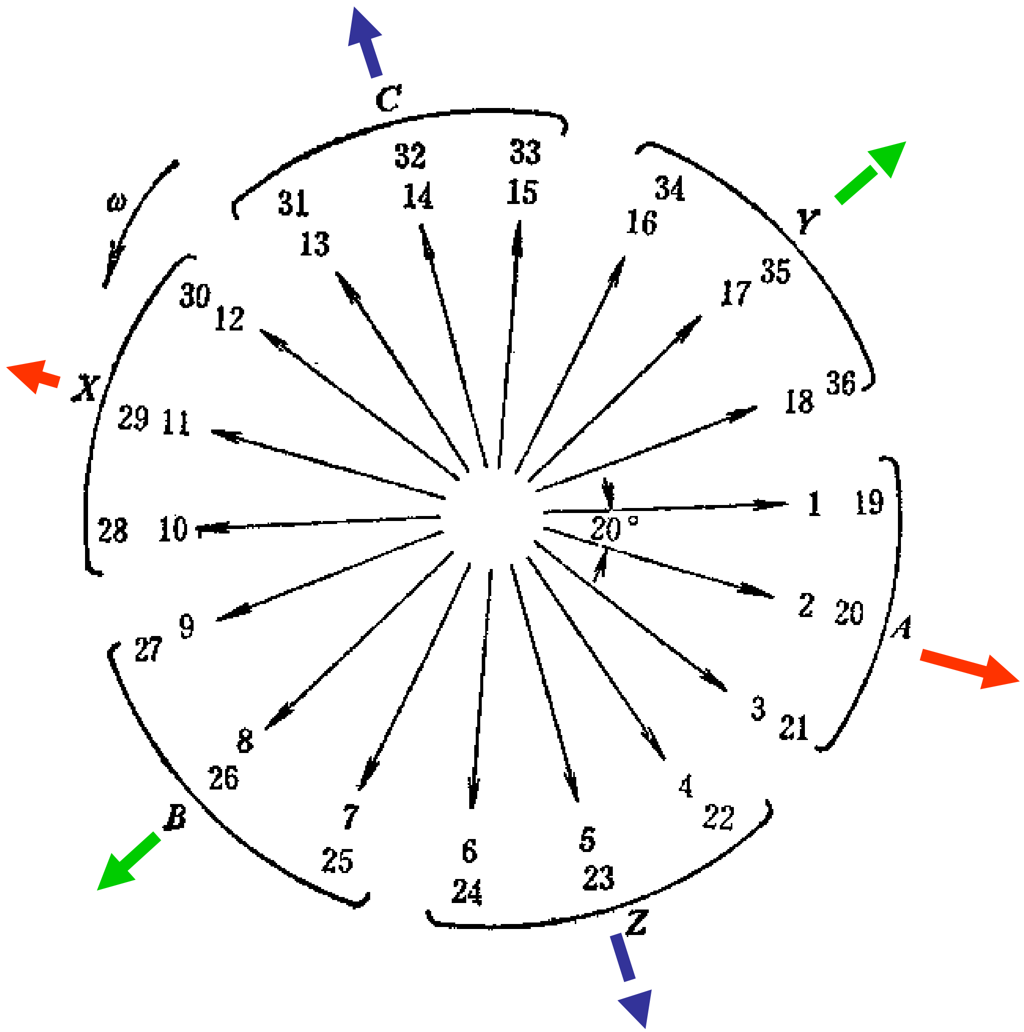
Slot pitch in electrical angle: α=p×360°/Q=20°

Slot EMF Star Vector Graph





The first pair of poles: 1-18 slot, the fist loop.
The second pair of poles: 19-36 slot, the second loop.



C
Y
A
Z
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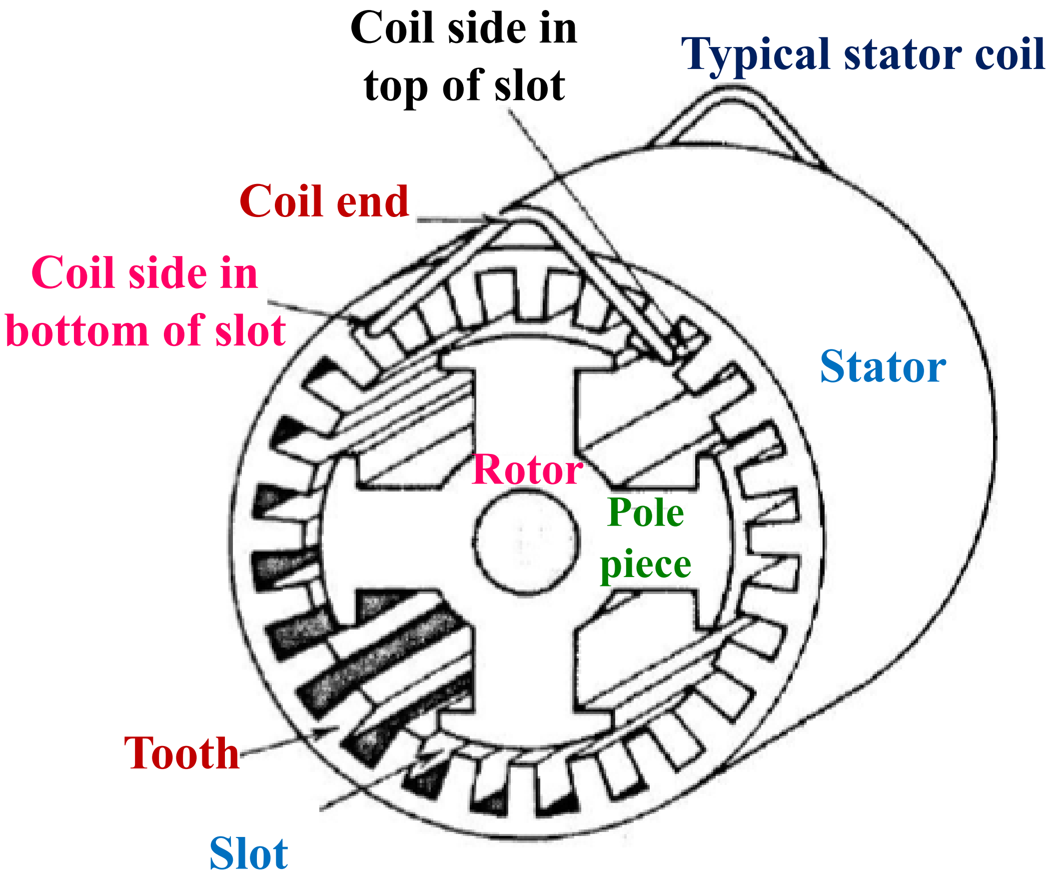
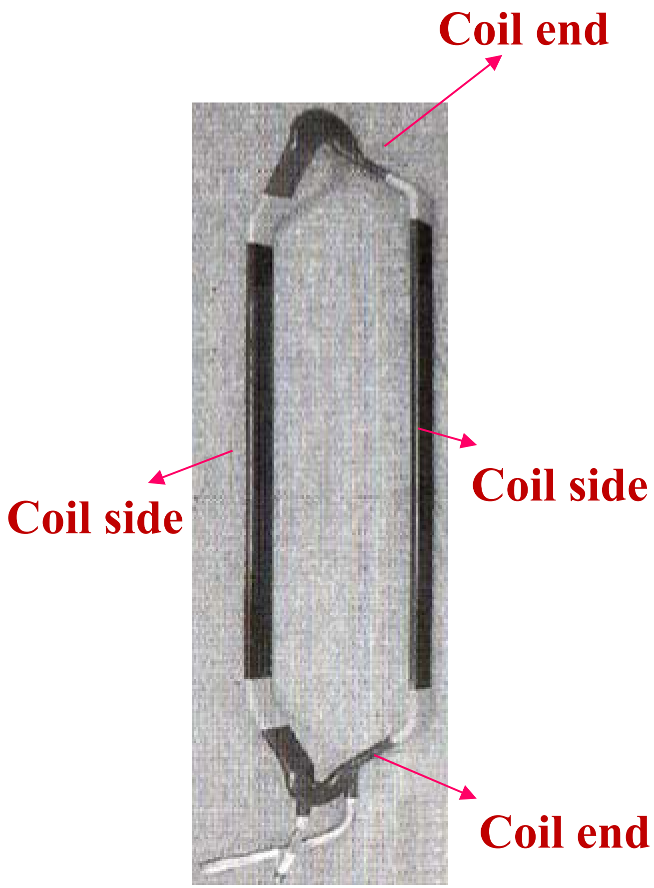
X
ω
20°
1 19
2 20
3 21
4 22
5 23
6 24
7 25
8 26
9 27
10 28
11 29
12 30
13 31
14 32
15 33
16 34
17 35
18 36

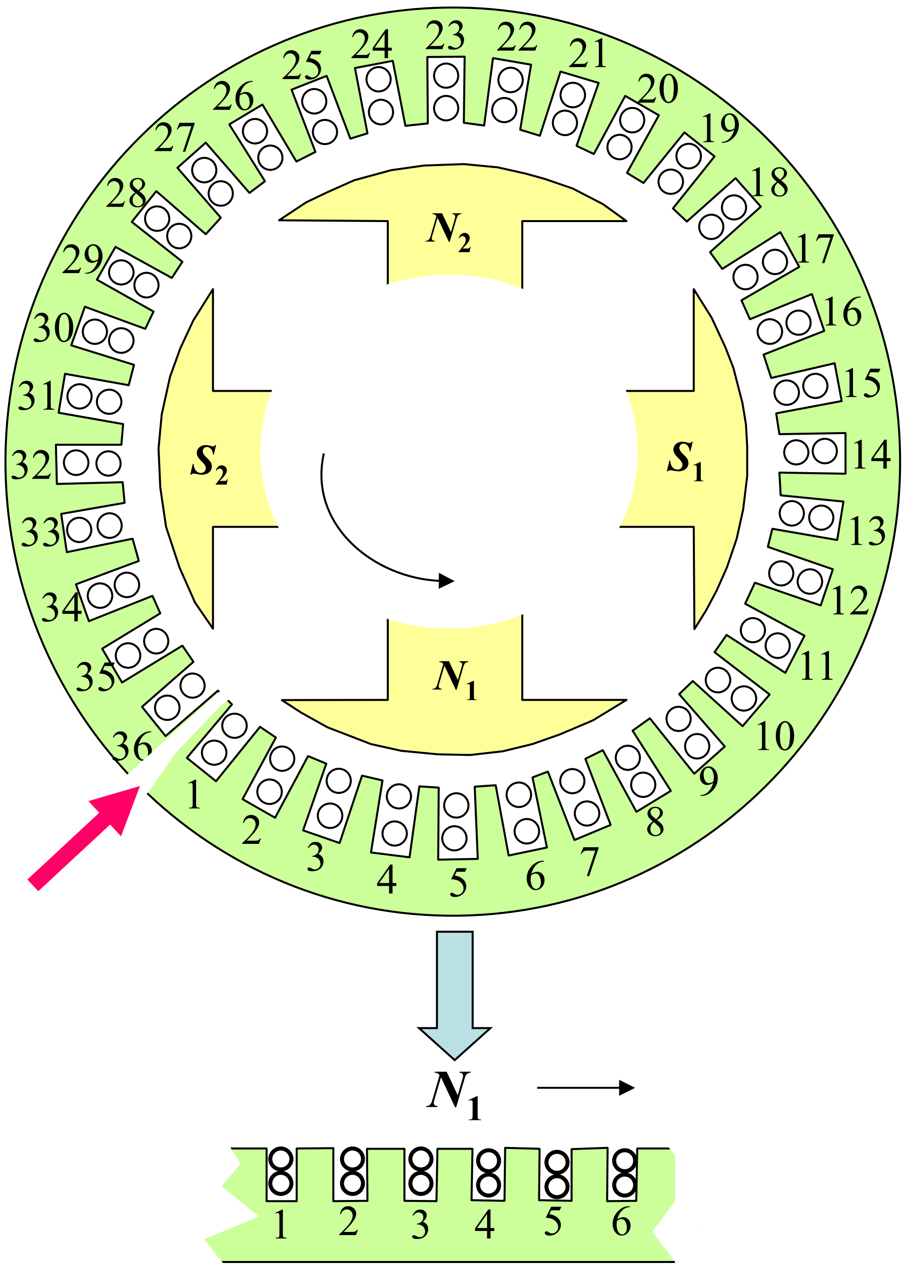
3-Phase double-layer lap winding



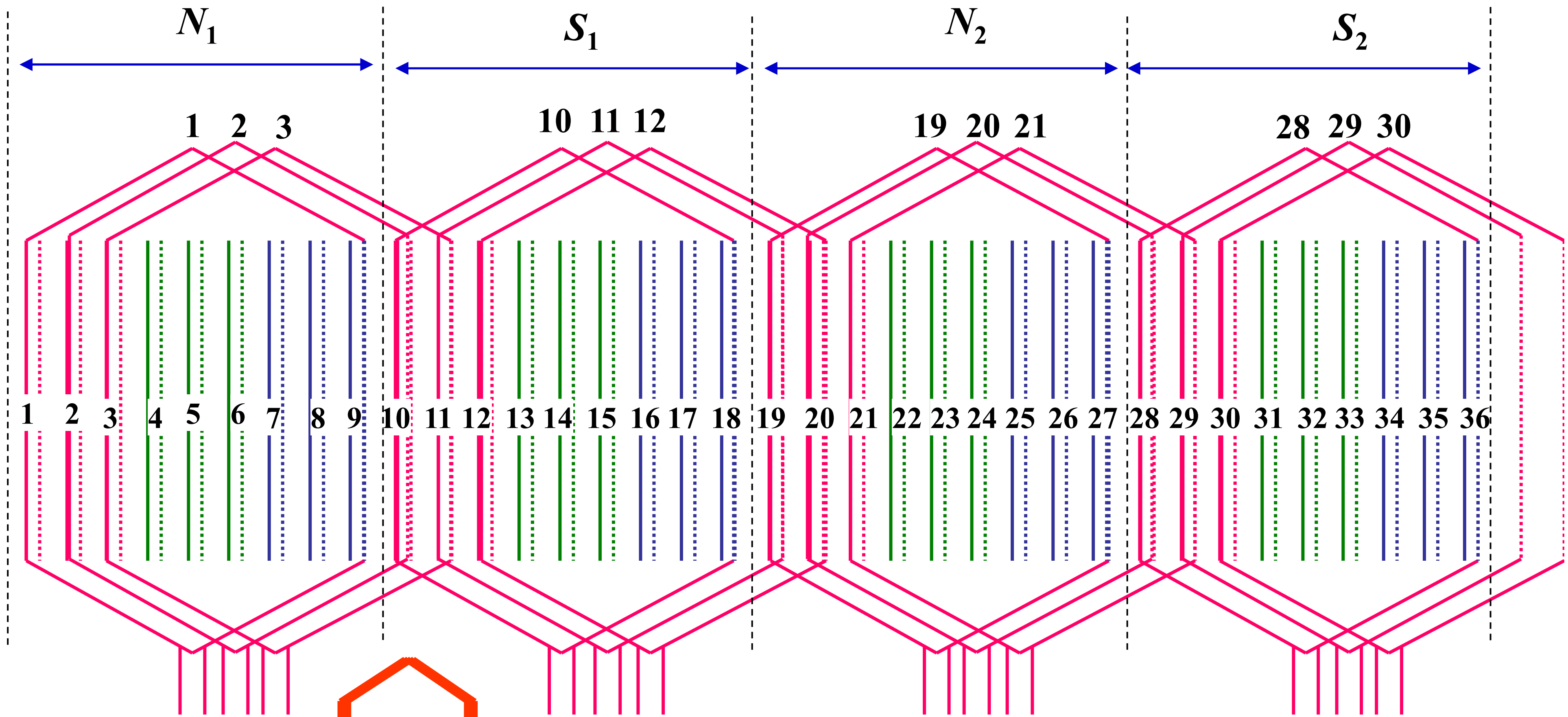


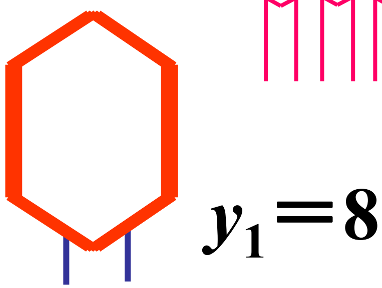
N_c turns, $2N_c$ Conductors

Developed Diagram

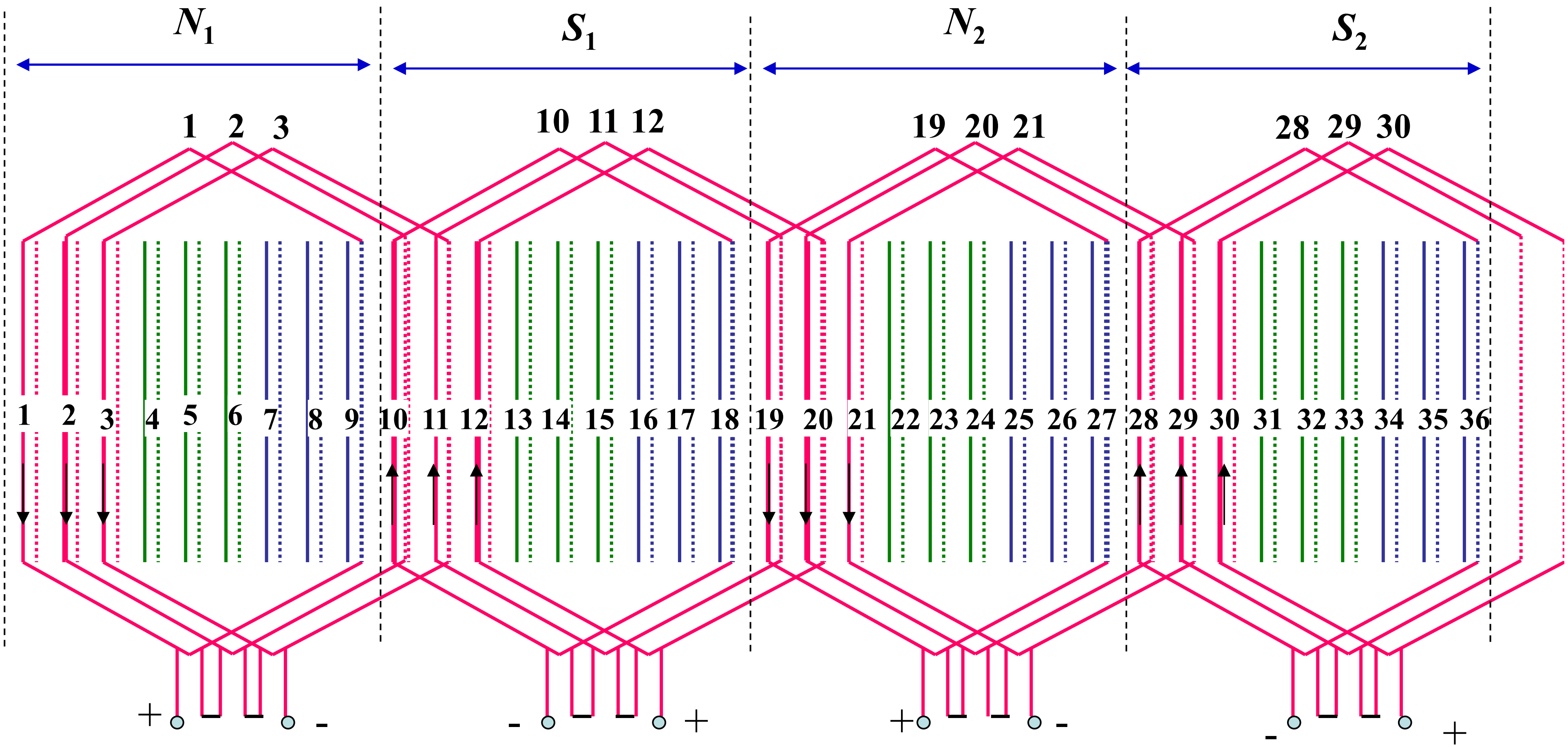


Developed Diagram



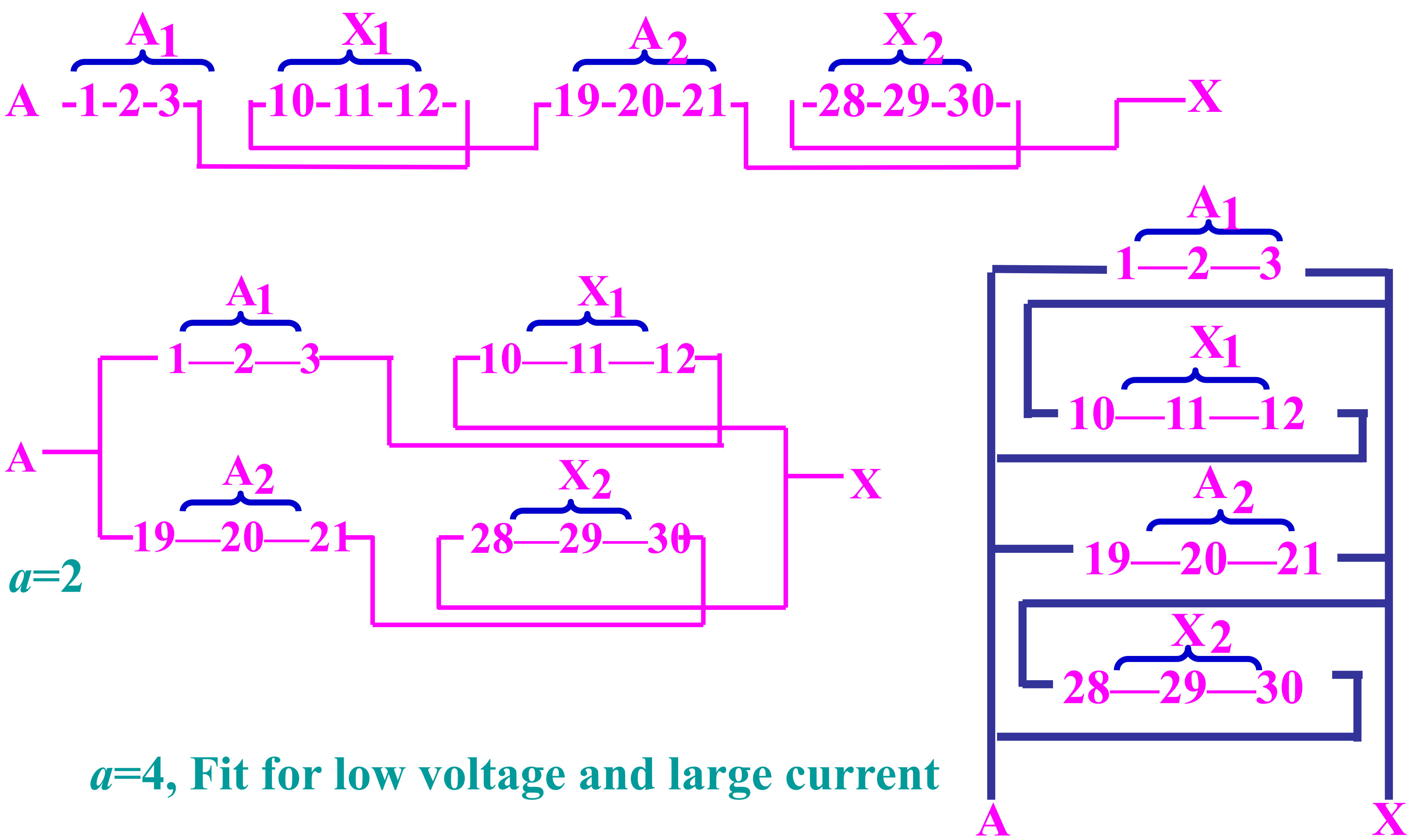


Developed Diagram



Parallel Branches

a=1, Fit for high voltage and small current



a=2

a=4, Fit for low voltage and large current

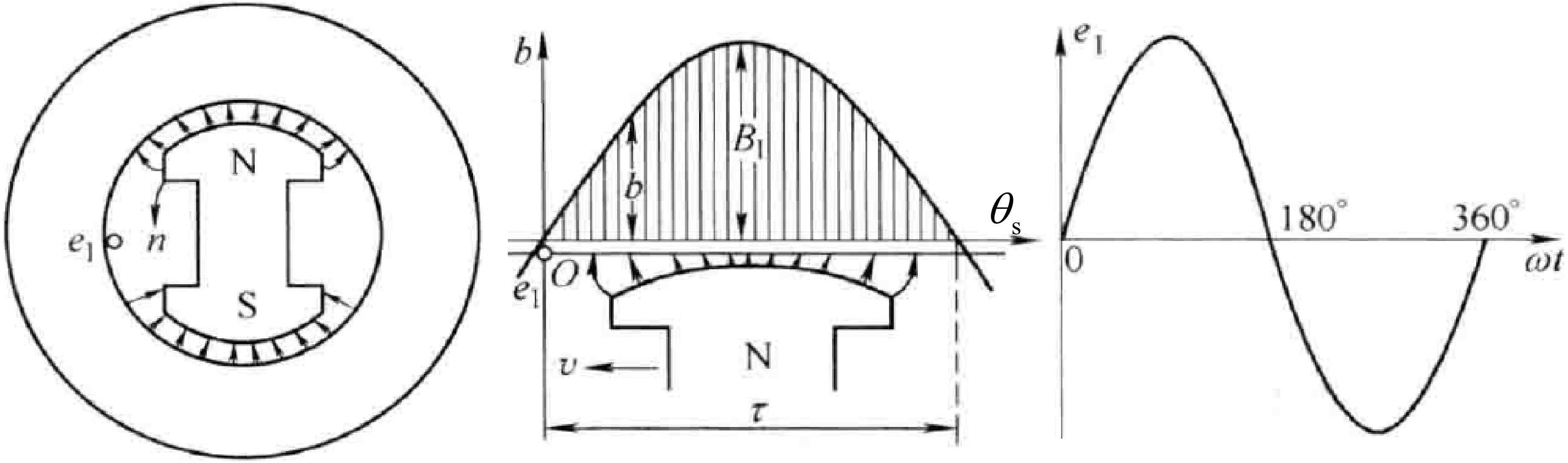
Turns of AC Winding

Total turns per phase: $N_t = 2p \times q \times N_c$
Total turns per branch per phase: $N = 2p \times q \times N_c / a$

Total turns per branch per phase N is basic and important to calculate EMF and MMF of a AC winding.

Electromotive Force of AC Windings

Sinusoidal air-gap magnetic field



$$b = B_1 \sin \theta_s$$

$$\theta_s = \omega t$$

$$b = B_1 \sin \omega t$$

$$e = blv = B_1 lv \sin \omega t$$

$$= \sqrt{2} E_1 \sin \omega t$$

$$f = p \frac{n_s}{60}$$

Electromotive Force of AC Windings

Synchronous speed

Angular frequency: $\omega = 2\pi f$

$$f = p\frac{n_s}{60} \qquad\qquad n_s = \frac{60f}{p}$$

Where *p* is pole pairs, *n* is speed (r/min).

Pole Paris	1	2	3	4	…
Synchronous Speed (rpm)	3000	1500	1000	750	…

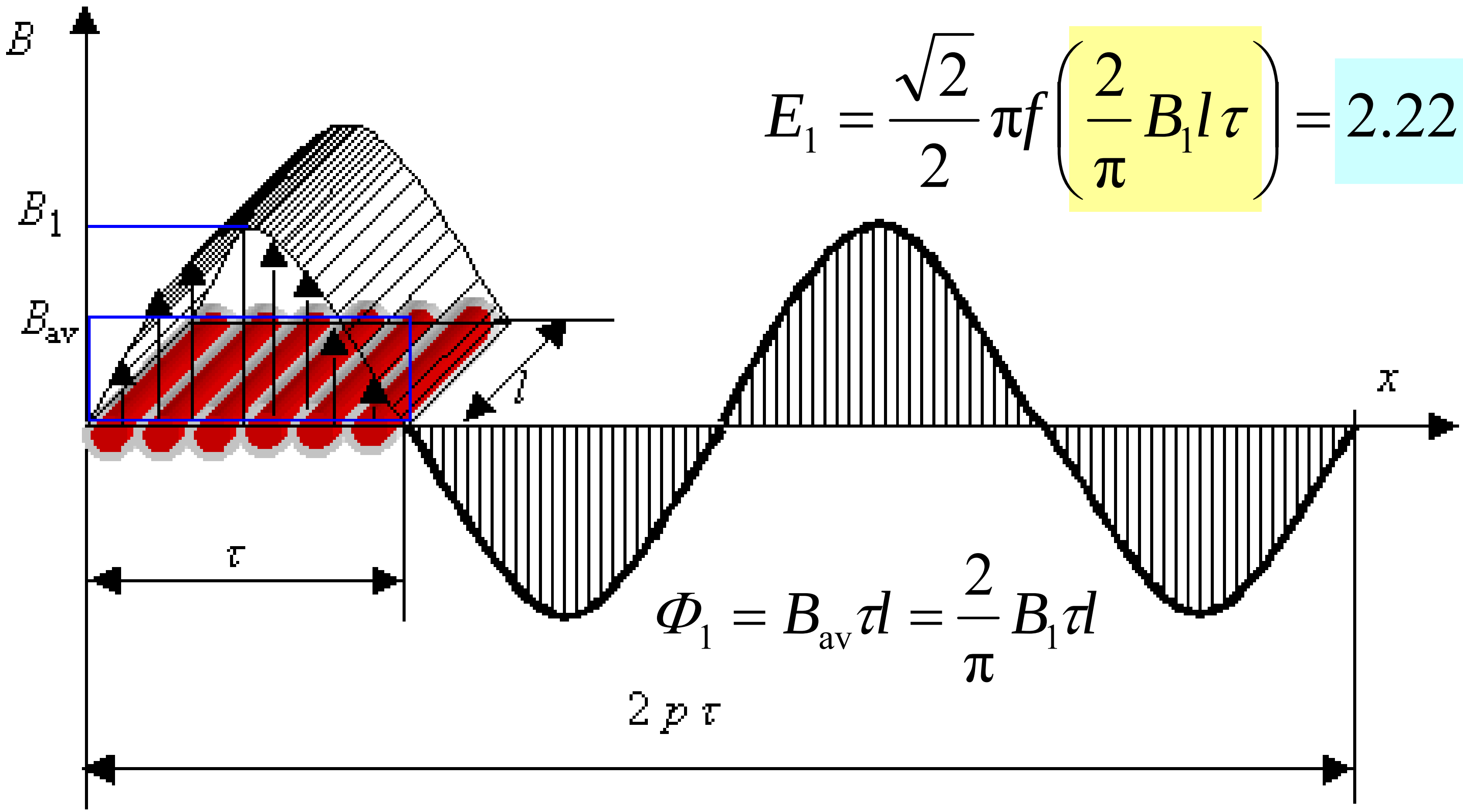
EMF of a conductor

$$e = blv = B_1 lv \sin \omega t$$

$$= \sqrt{2} E_1 \sin \omega t$$

$$E_1 = \frac{B_1 l}{\sqrt{2}} v = \frac{B_1 l}{\sqrt{2}} \cdot 2\tau f = \sqrt{2}\, f B_1 \tau l$$

$$E_1 = \frac{\sqrt{2}}{2} \pi f \left(\frac{2}{\pi} B_1 l \tau \right) = 2.22\, f \Phi_1$$



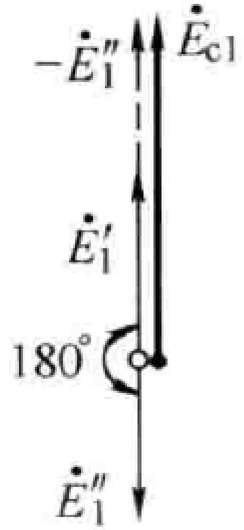
$$\Phi_1 = B_{av} \tau l = \frac{2}{\pi} B_1 \tau l$$

EMF of a full-pitch coil

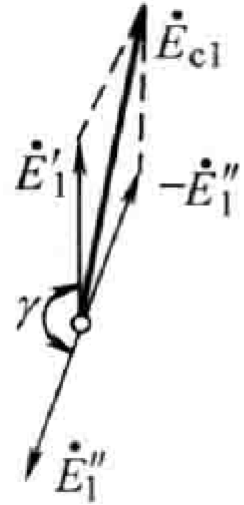
$$\dot{E}_{c1} = \dot{E}_1' - \dot{E}_1'' = 2\dot{E}_1'$$

$$E_{c1(N_c=1)} = 2E_1' = 4.44 f\Phi_1$$

$$E_{c1} = 4.44 fN_c\Phi_1$$



EMF of a fractional-pitch coil



$$\gamma = \frac{y_1}{\tau} \times 180^\circ$$

$$\dot{\boldsymbol{E}}_{c1} = \dot{\boldsymbol{E}}_1' - \dot{\boldsymbol{E}}_1'' = \boldsymbol{E}_1\angle 0^\circ - \boldsymbol{E}_1\angle\gamma$$

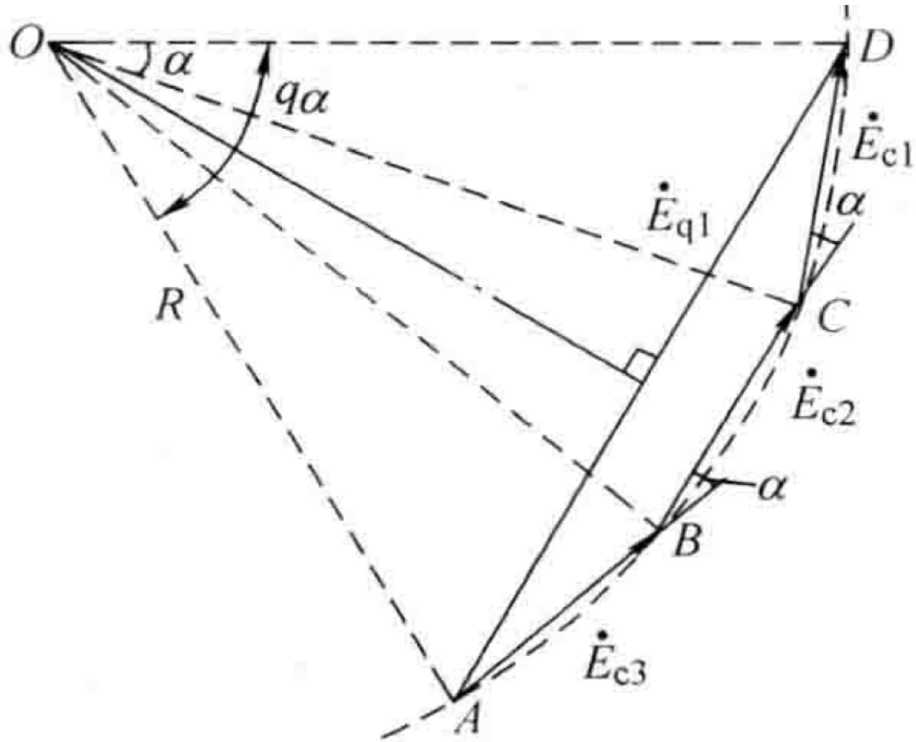
Pitch Factor

$$2\boldsymbol{E}_1 \cos\frac{180^\circ - \gamma}{2} = 2\boldsymbol{E}_1 \sin\frac{y_1}{\tau}90^\circ = 4.44\, f\boldsymbol{k}_{p1}\boldsymbol{\Phi}_1$$

$$\boldsymbol{k}_{p1} = \sin\frac{y_1}{\tau}90^\circ$$

EMF of a fractional-pitch coil group

$$2R\sin\frac{\alpha}{2} = E_{c1} \qquad 2R\sin\frac{q\alpha}{2} = E_{q1}$$



$$E_{q1}\sin\frac{\alpha}{2} = E_{c1}\sin\frac{q\alpha}{2}$$

$$E_{q1} = qE_{c1}\frac{\sin\frac{q\alpha}{2}}{q\sin\frac{\alpha}{2}} = qE_{c1}k_{d1}$$

Distribution Factor $$k_{d1} = \frac{E_{q1}}{qE_{c1}} = \frac{\sin\frac{q\alpha}{2}}{q\sin\frac{\alpha}{2}}$$

$$\boldsymbol{k}_{d1} < 1$$

EMF of a fractional-pitch coil group

$$E_{q1} = q \times 4.44 f N_c k_{p1} \Phi_1 k_{d1} = 4.44 f (q N_c) k_{w1} \Phi_1$$

$k_{w1} = k_{p1} k_{d1}$ **Winding Factor**

EMF of one phase

$$E_{\phi 1} = \frac{2p}{a} E_{q1} = \frac{2p}{a} 4.44 f (q N_c) k_{w1} \Phi_1$$

$$= 4.44 f (2 p q N_c / a) k_{w1} \Phi_1$$

$$= 4.44 f N k_{w1} \Phi_1$$

$$N = \frac{2p}{a} q N_c$$

MMF of Single-phase AC Winding

Assumption

1.The coil current is cosinoidal wave:

$$i_{\mathrm{c}} = \sqrt{2} I_{\mathrm{c}} \cos \omega t$$

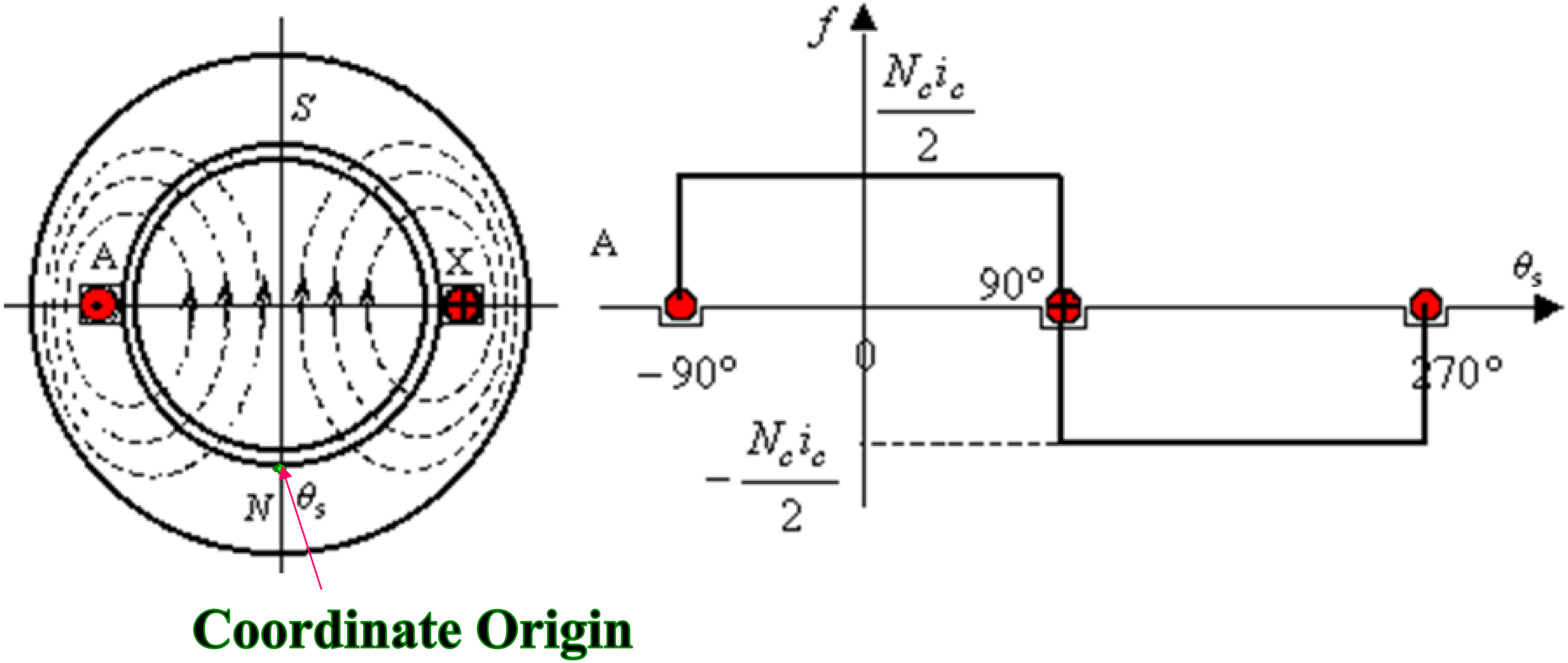
2. The core permeability $\mu_{\mathrm{Fe}}=\infty$

3. The air-gap is uniform

4. The coil current is located in slot center

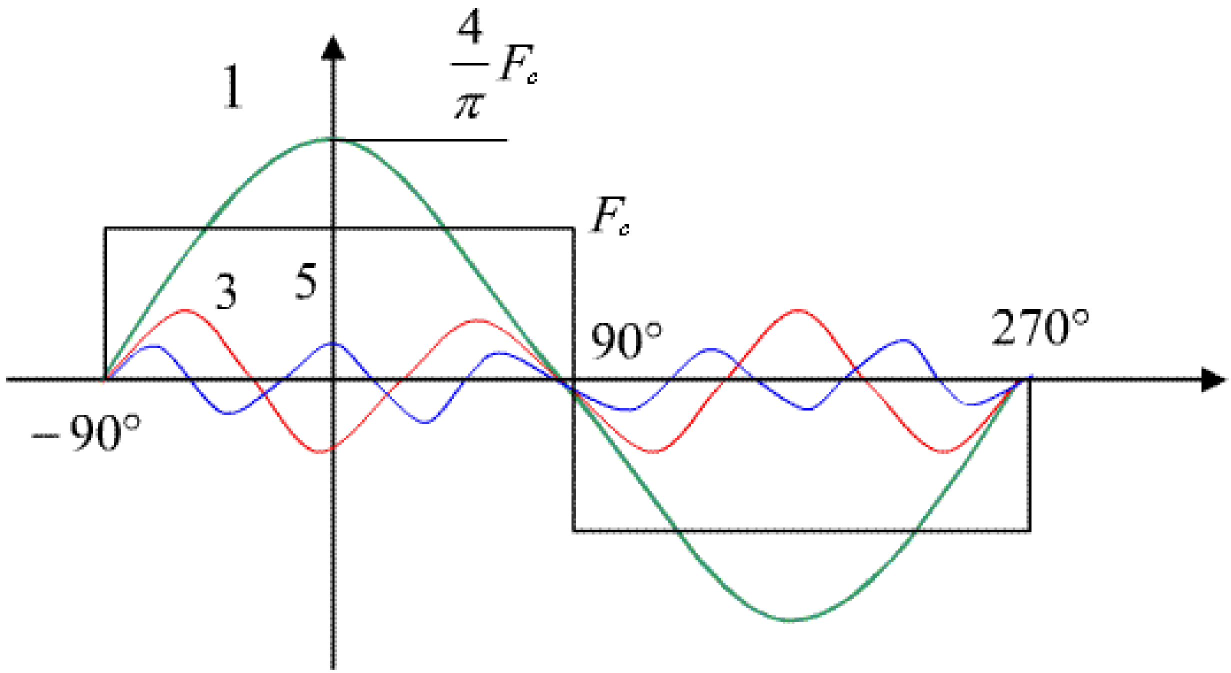
MMF of a full-pitch coil

$$i_c = \sqrt{2} I_c \cos \omega t$$



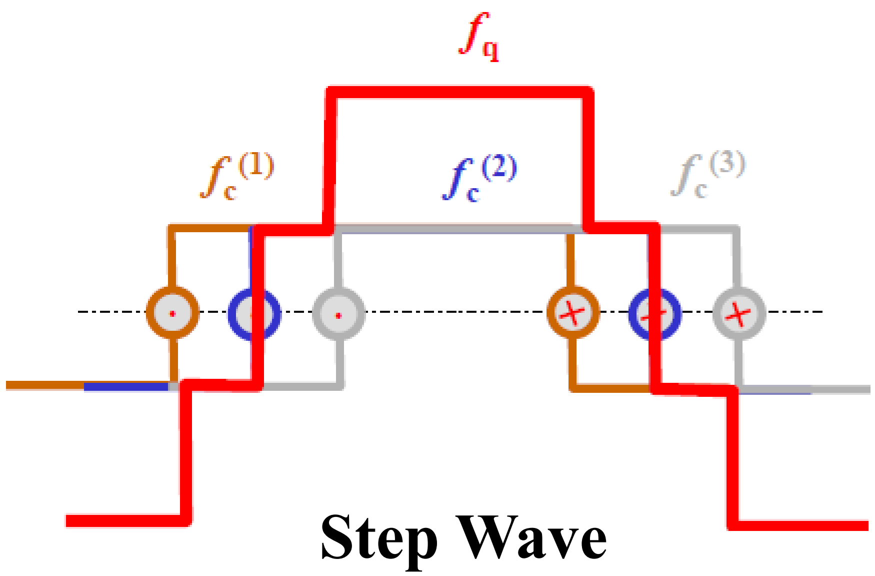
1. **Waveform**: is rectangular.
2. **Position**: is fixed at the coil axis.
3. **Amplitude**: is proportional to i_c, and pulses with the frequency of i_c.

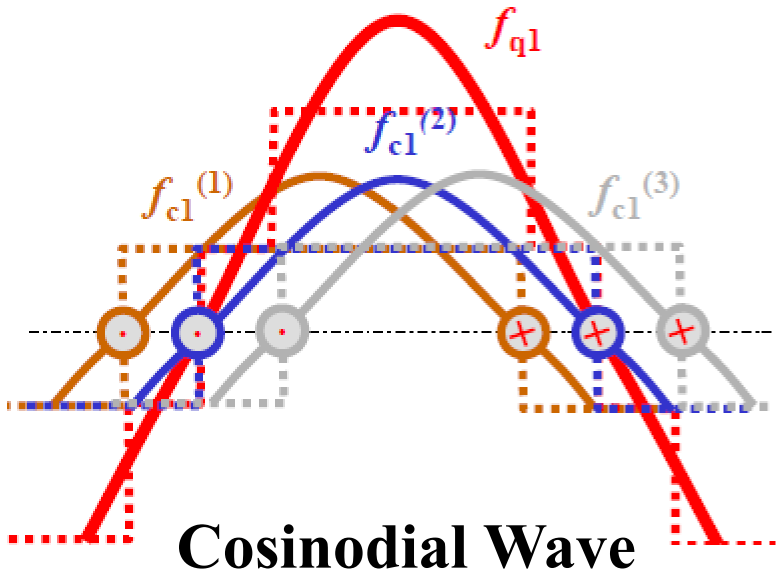
Analysis of the rectangular waveform



$$f_{c1} = \frac{4}{\pi} \times \frac{N_c i_c}{2} \cos\theta_s = \frac{2\sqrt{2}}{\pi} N_c I_c \cos\theta_s \cos\omega t$$

MMF of a full-pitch coil group

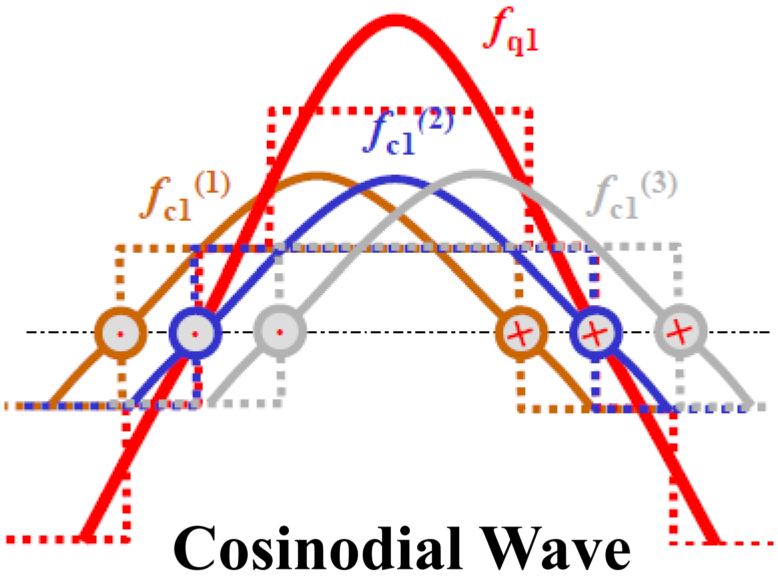


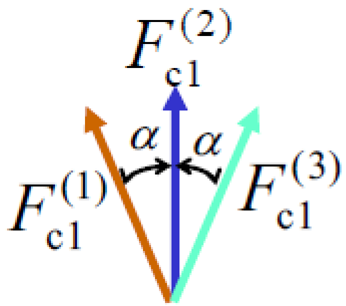


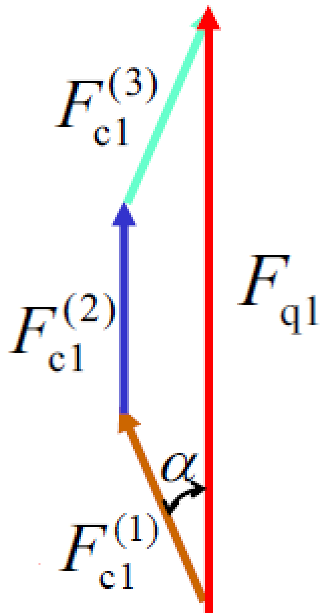
$$f_q = f_c^{(1)} + f_c^{(2)} + f_c^{(3)}$$

$$f_{q1} = f_{c1}^{(1)} + f_{c1}^{(2)} + f_{c1}^{(3)}$$

MMF of a full-pitch coil group







$$f_{q1}=f_{c1}^{(1)}+f_{c1}^{(2)}+f_{c1}^{(3)}$$

For single-layer winding

$$f_{q1}=(qf_{c1})k_{d1}=\frac{4}{\pi}\times\frac{qN_c i_c}{2}k_{d1}\cos\theta_s=\frac{2\sqrt{2}}{\pi}qN_c k_{d1}I_c\cos\theta_s\cos\omega t$$

MMF of a full-pitch coil group

For double-layer winding

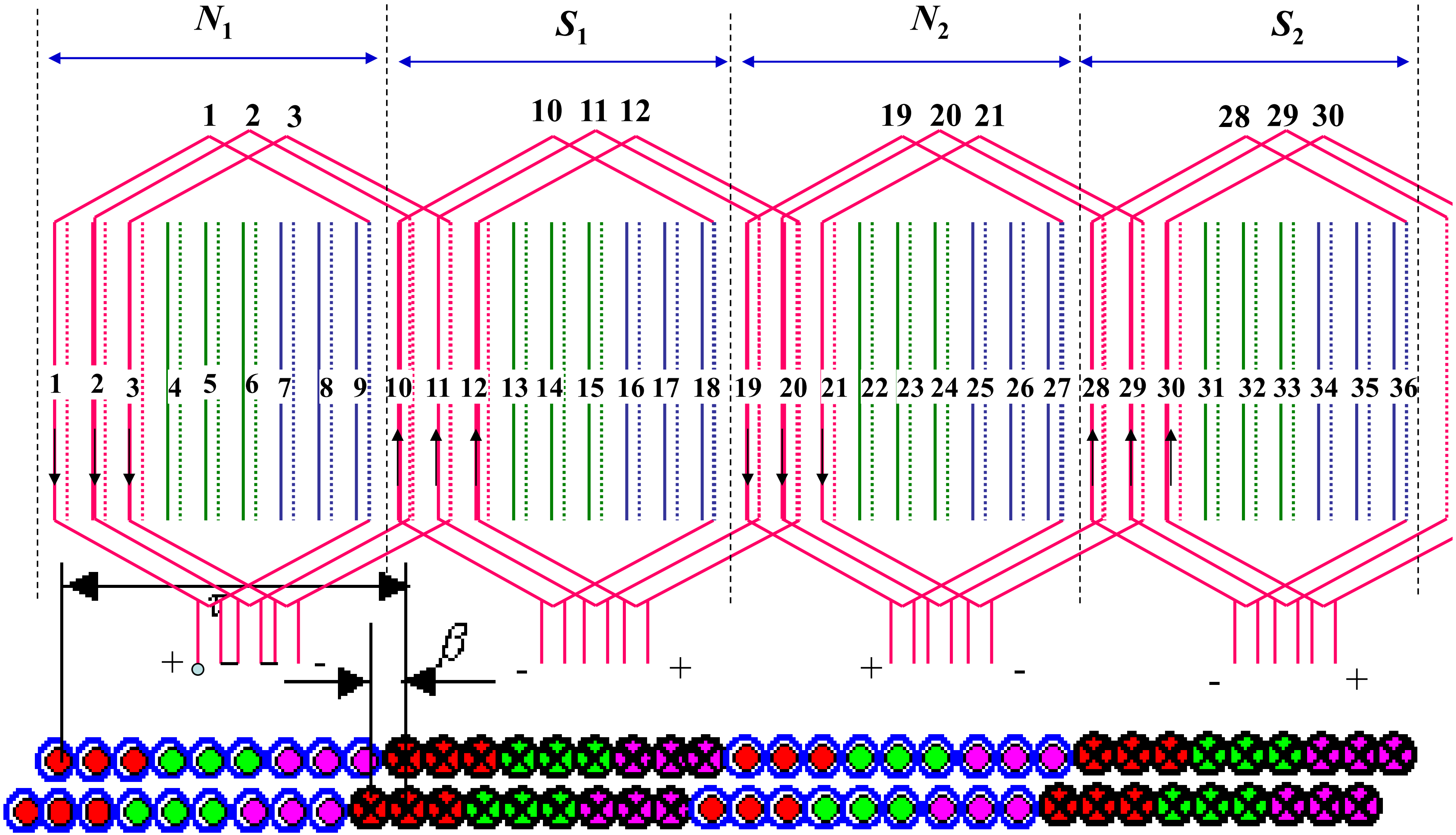
$$f_{q1} = 2(qf_{c1})k_{d1} = \frac{4}{\pi} \times qN_c i_c k_{d1} \cos\theta_s$$

$$= \frac{4}{\pi}\frac{2pa}{2pa} \times qN_c i_c k_{d1} \cos\theta_s$$

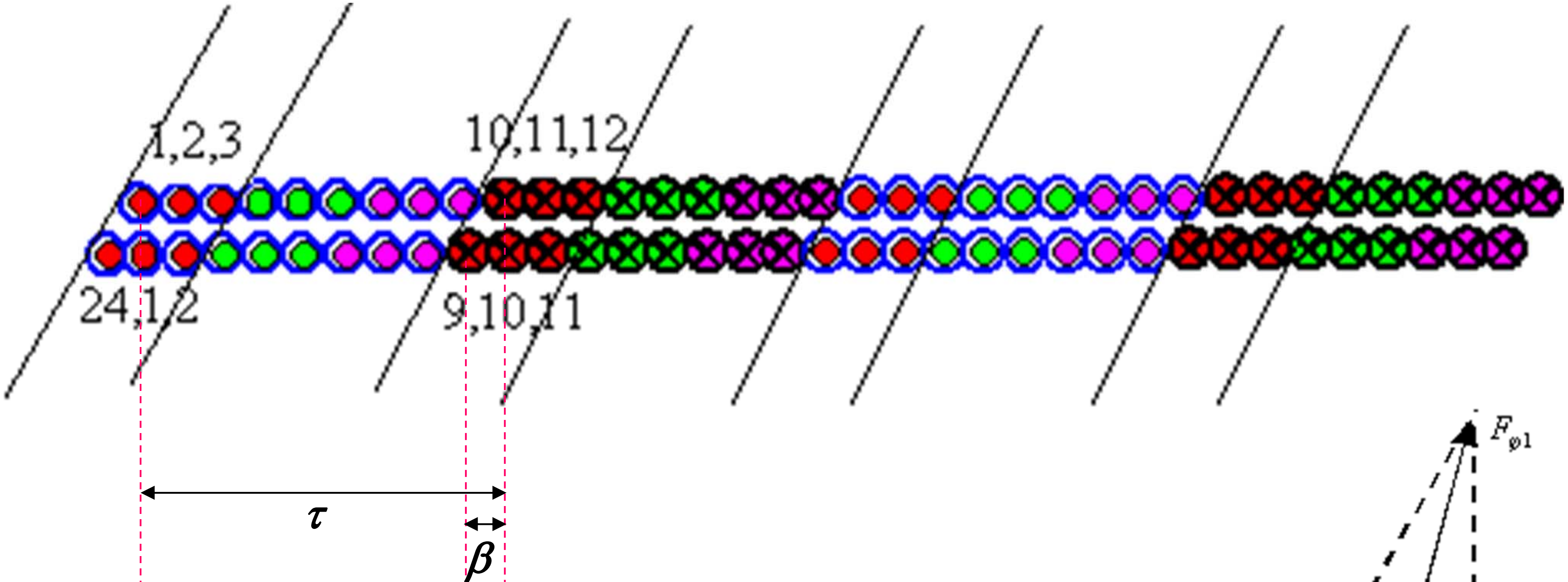
$$= \frac{4}{\pi} \times \frac{1}{2p} \times \underbrace{\frac{2pqN_c}{a}}_{N} \overbrace{(ai_c)}^{i_\phi} k_{d1} \cos\theta_s$$

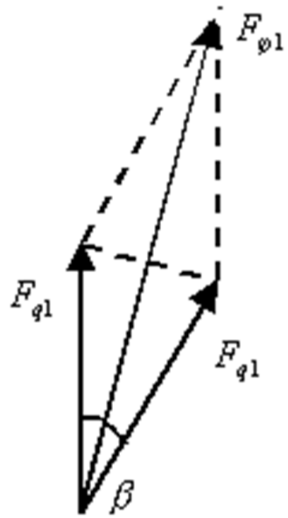
$$= \frac{4}{\pi}\frac{Nk_{d1}}{2p} i_\phi \cos\theta_s$$

MMF of a fractional-pitch coil group



MMF of a fractional-pitch coil group





$$f_{\mathrm{q1}} = \frac{4}{\pi}\frac{Nk_{\mathrm{d1}}}{2p}k_{\mathrm{p1}}i_{\phi}\cos\theta_{\mathrm{s}} = \frac{4}{\pi}\frac{Nk_{\mathrm{w1}}}{2p}i_{\phi}\cos\theta_{\mathrm{s}}$$

$$\boldsymbol{k}_{\mathrm{w}\mathbf{1}} = \boldsymbol{k}_{\mathrm{d}\mathbf{1}}\boldsymbol{k}_{\mathrm{p}\mathbf{1}}$$

MMF of one phase winding

$$f_{\phi 1} = f_{q1} = \frac{4}{\pi}\frac{Nk_{w1}}{2p} i_{\phi} \cos\theta_{s}$$

$$f_{\phi 1}(\theta_{s}, t) = \frac{4}{\pi}\frac{\sqrt{2}Nk_{w1}}{2p} I_{\phi} \cos\theta_{s} \cos\omega t = F_{\phi 1} \cos\theta_{s} \cos\omega t$$

$$F_{\phi 1} = \frac{4}{\pi}\frac{\sqrt{2}Nk_{w1}}{2p} I_{\phi} = 0.9\frac{Nk_{w1}}{p} I_{\phi}$$

MMF of single-phase AC winding is a pulsing mmf.

A function of both time and space.

Distributes cosinoidally on space.

Varies cosinoidally with time.

The maximum position is fixed on the axis of the phase winding.

The pulsing frequency is the frequency of the AC current.

MMF of Three-phase AC Winding

Space phase difference of each axis is 120°

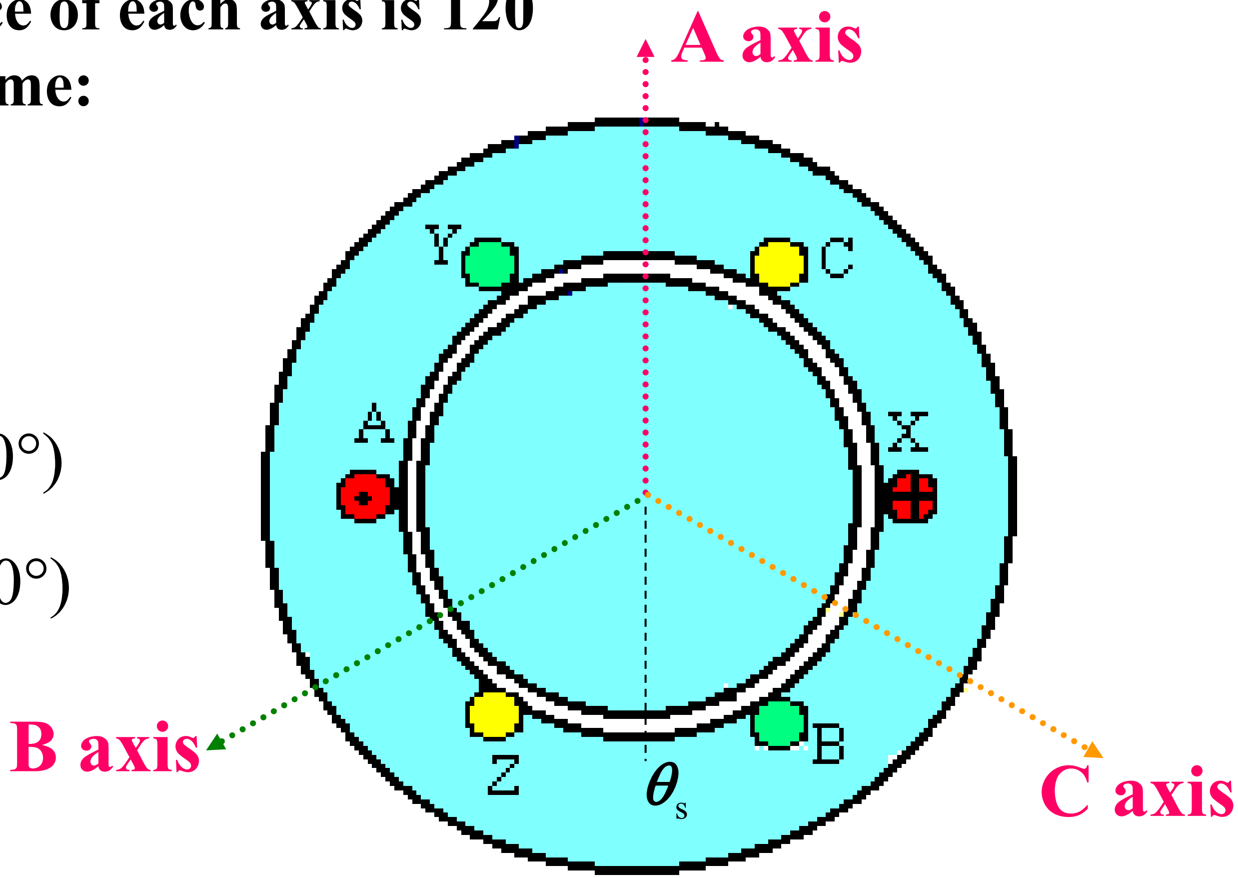
Effective turns are same:

$\boldsymbol{k_{wA}N_A=k_{wB}N_B=k_{wC}N_C}$

$$i_A = \sqrt{2}I_\phi \cos\omega t$$

$$i_B = \sqrt{2}I_\phi \cos(\omega t - 120°)$$

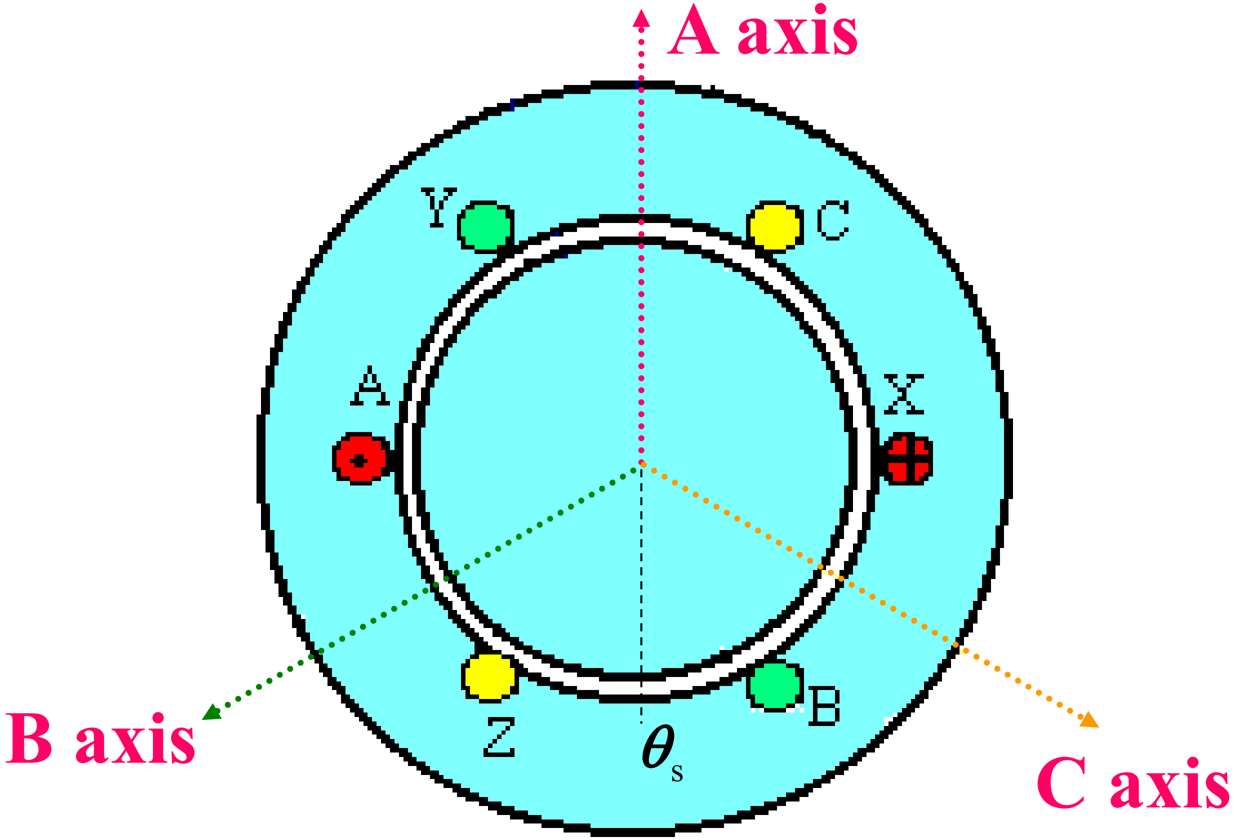
$$i_C = \sqrt{2}I_\phi \cos(\omega t - 240°)$$



Amplitudes are same: $\boldsymbol{I_A=I_B=I_C}$

Time phase difference of each current is 120°

MMF of Three-phase AC Winding



$$f_{A1} = F_{\phi 1} \cos\theta_s \cos\omega t$$

$$f_{B1} = F_{\phi 1} \cos(\theta_s - 120°) \cos(\omega t - 120°)$$

$$f_{C1} = F_{\phi 1} \cos(\theta_s - 240°) \cos(\omega t - 240°)$$

MMF of Three-phase AC Winding

$$f_1(\theta_s, t) = f_{A1} + f_{B1} + f_{C1} = F_{\phi 1}\cos\theta_s \cos\omega t$$
$$+ F_{\phi 1}\cos(\theta_s - 120°)\cos(\omega t - 120°)$$
$$+ F_{\phi 1}\cos(\theta_s - 240°)\cos(\omega t - 240°)$$

$$f_1(\theta_s, t) = \frac{1}{2}F_{\phi 1}\cos(\omega t - \theta_s) + \frac{1}{2}F_{\phi 1}\cos(\omega t + \theta_s) +$$
$$\frac{1}{2}F_{\phi 1}\cos(\omega t - \theta_s) + \frac{1}{2}F_{\phi 1}\cos(\omega t + \theta_s - 240°) +$$
$$\frac{1}{2}F_{\phi 1}\cos(\omega t - \theta_s) + \frac{1}{2}F_{\phi 1}\cos(\omega t + \theta_s - 120°)$$
$$= \frac{3}{2}F_{\phi 1}\cos(\omega t - \theta_s) = F_1\cos(\omega t - \theta_s)$$

Traveling Wave

a) Traveling speed

$$n_s = \frac{2\pi f}{p \cdot 2\pi} 60 = \frac{60 f}{p}$$

b) Traveling direction

Forward Traveling Wave ABC

Backward Traveling Wave ACB

$$\boxed{f_1(\theta_s, t) = F_1 \cos(\omega t - \theta_s)}$$

Property: A circular rotating MMF is produced when 3-phase symmetrical currents flow in 3-phase symmetrical windings.

Amplitude : The amplitude of the MMF keeps constant, does not change with time.

Speed: The speed of the MMF is same with the synchronous speed.

Direction: The rotating direction of the MMF is from the winding axis with current leading to the winding axis with current lagging.

Position: The position of the MMF axis is overlapped with the winding axis in which the current is maximum. The space electric angle of the MMF rotating is equal to the time phase angle of the current changing.

The relationship between rotating and pulsing MMFs

Pulsing

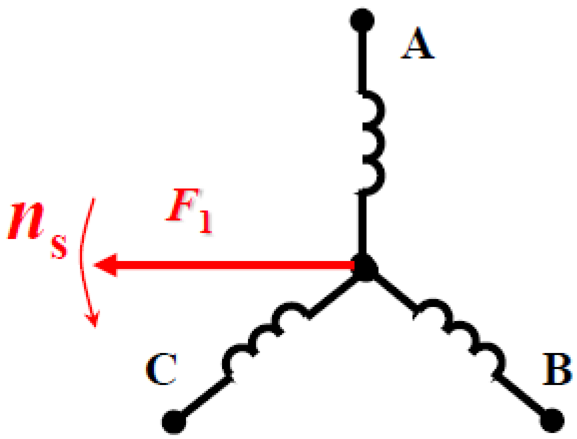
$$f_{\phi 1}(\theta_s, t) = F_{\phi 1} \cos\theta_s \cos\omega t$$

$$= \frac{1}{2} F_{\phi 1} \cos(\omega t - \theta_s) + \frac{1}{2} F_{\phi 1} \cos(\omega t + \theta_s)$$

Forward Rotating   Backward Rotating

A pulsing MMF can be decomposed to two rotating MMFs which have same speeds, same amplitudes but opposite directions.

Example: Assume the 3-phase symmetrical windings flow in 3-phase symmetrical currents: $i_A=\cos(\omega t)$, $i_B=\cos(\omega t+120°)$ and $i_C=\cos(\omega t+240°)$. When $\omega t=90°$, determine the rotating direction and the axis position of the rotating MMF.



Phase sequence: A→C→B, F_1 is backward.
When $\omega t=0°$, $i_A=1$, F_1 overlaps with A axis.
When $\omega t=90°$, F_1 rotates anticlockwise over 90° from A axis.