5.3 INDUCTION-MOTOR EQUIVALENT CIRCUIT

The foregoing considerations of flux and mmf waves can readily be translated to a steady-state equivalent circuit for a polyphase induction machine. In this derivation, only machines with symmetric polyphase windings excited by balanced polyphase voltages are considered. As in many other discussions of polyphase devices, it is helpful to think of three phase machines as being Y-connected, so that currents are line values and voltages always line-to-neutral values. In this case, we can derive the equivalent circuit for one phase, with the understanding that the voltages and currents in the remaining phases can be found simply by an appropriate phase of the phase under study (±120° in the case of a three -phase machine).

First, consider conditions in the stator. The synchronously rotating air-gap flux wave generate balanced polyphase counter emfs in the phases of the stator. The stator terminal voltage differs from the counter emf by the voltage drop in the stator leakage impedance Z_1=R_1+ j$X_{1\sigma}$. Thus

$$\dot{U}_1 = \dot{I}_1(R_1 + \mathrm{j}X_{1\sigma}) - \dot{E}_1 \tag{1}$$

where

$\dot{U}_1$ = Stator line-to- neutral terminal voltage

$\dot{E}_1$ = Counter emf (line-to-neutral) generated by the resultant air-gap flux

$\dot{I}_1$ = Stator current

R_1 = Stator effective resistance

$X_{1\sigma}$ = Stator leakage reactance

The polarity of the voltages and currents are shown in the equivalent circuit of Fig. 1.

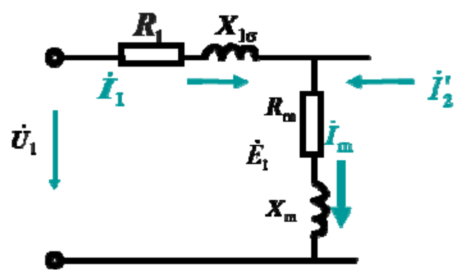

Figure 1 Stator equivalent circuit for a polyphase induction motor.

The resultant air gap flux is created by the combined mmfs of the stator and rotor currents. Just as in the case of a transformer, the stator current can be resolved into two components: a load component and an exciting (magnetizing) component. The load component $\dot{I}_2'$ produces an mmf that corresponds to the mmf of the rotor current. The exciting component $\dot{I}_{\mathrm{m}}$ is the additional stator current required to create the resultant air-gap flux and is a function of the emf $\dot{E}_1$.

The equivalent circuit representing stator phenomena of Fig.1 is exactly like that used to represent the primary of a transformer. To complete our model, the effects of the rotor must be incorporated. From the point of view of the stator equivalent circuit of Fig.1 the rotor can be represented by an equivalent impedance $Z_2' = \dot{E}_2' / \dot{I}_2'$ corresponding to the leakage impedance of an equivalent stationary secondary. To complete the equivalent circuit, we must determine Z_2' by representing the stator and rotor voltages and currents in terms of rotor quantities as referred to the stator. The final result is shown in the load terminals of the stator equivalent circuit of figure 2. The combined effect of shaft load and rotor resistance appears as a reflected resistance R'_2/s, a function of slip and therefore of the mechanical load. The current $\dot{I}'_2$ in the reflected rotor impedance equals 180° out of phase with the load component $\dot{I}_{1L}$ of stator current; the voltage across this impedance equals the stator voltage $\dot{E}_1$. Note that when rotor currents and voltages are reflected into the stator, their frequency is also changed to stator frequency. All rotor electrical phenomena, when viewed from the stator, become stator-frequency phenomena, because the stator winding simply sees mmf and flux waves traveling at synchronous speed.

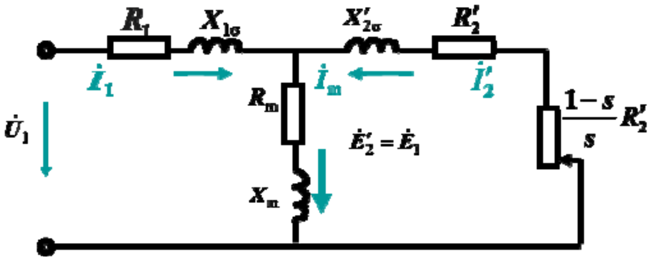

Figure 2 Alternative form of the polyphase induction motor equivalent circuit.

5.4 ANALYSIS OF THE EQUIVALENT CIRCUIT

The single-phase equivalent circuit of figure 2 can be used to determine a wide variety of steady-state performance characteristics of polyphase induction machines. These include variations of current, speed, and losses as the load-torque requirements change, as well as the starting torque, and the maximum torque. Note however that, in practice, the equivalent-circuit parameters may be a function of operating conditions. Specifically, temperature will affect the values of the resistances and the rotor parameters of a squirrel-cage motor may change with slip.

The equivalent circuit shows that the total power P_e (电磁功率) transferred across the air gap from the stator is

$$P_e=mI_2'^2(R_2'/s) \tag{2}$$

Where m is the number of stator phases.

The total rotor I^2R copper loss, P_{cu2}, can be calculated from the I^2R loss in the equivalent rotor as

$$P_{cu2}=mI_2'^2R_2' \tag{3}$$

The total mechanical power P_Ω(总机械功率) developed by the motor can now be determined by subtracting the rotor power dissipation of Eq.3 from the air-gap power of Eq.2.

$$P_\Omega= P_e - P_{cu2}= mI_2'^2(R_2'/s)- mI_2'^2R_2'= mI_2'^2R_2'(1-s)/s \tag{4}$$

Comparing Eq.2 with Eq.4 gives

$$P_\Omega= P_e\,(1-s) \tag{5}$$

and

$$P_{cu2}= sP_e \tag{6}$$

We see then that, of the total power P_e delivered across their air gap to the rotor, the fraction 1-s is converted to total mechanical power P_Ω and the fraction s is dissipated as I^2R loss in the rotor conductors. Similarly, the power dissipated in the rotor can be expressed in terms of the total mechanical power P_Ω as

$$P_{cu2}= sP_{\Omega}/(1-s) \tag{7}$$

From Eqs.6 and 7, it is evident that an induction motor operating at high slip is an inefficient device. The equivalent of Fig 2 emphasizes the relationship between rotor loss and electromechanical power. The rotor power dissipation per stator phase corresponds to the power dissipated in the resistance R_2 while the electromechanical power per stator phase is equal to the power delivered to the resistance $R_2(1-s)/s$.

The electromechanical torque T_e corresponding to the power P_{mech} can be obtained by recalling mechanical power equals torque times angular velocity. Thus,

$$P_{\Omega}=T_e\Omega= T_e\Omega_s(1-s) \tag{8}$$

For P_{Ω} in watts and Ω_s in rad/sec. T_e will be in newton-meters.

Use of Eqs.4 and 5 leads to

$$T_e = P_{\Omega}/\Omega= P_e/\Omega_s \tag{9}$$

With the synchronous mechanical angular velocity Ω_s being given by

$$\Omega_s=2\pi n_s/60 \tag{10}$$

The mechanical torque T_e and power P_{Ω} are not the output values available at the shaft because friction, windage, and stray-load losses remain to be accounted for. It is obviously correct to subtract friction, windage, and other rotational losses from T_e or P_{Ω} and it is generally assumed that stray load effects can be subtracted in the same manner. The remainder is available as output power from the shaft for useful work. Thus

$$P_2= P_{\Omega} - p_{\Omega} - p_{\Delta} \tag{11}$$

Analysis of the transformer equivalent circuit is often simplified by either neglecting the magnetizing branch entirely or adopting the approximation of moving it out directly to the primary terminals. Such approximations are not used in the case of induction machines under normal running conditions because the presence of the air gap results in a relatively lower magnetizing impedance and correspondingly a relatively higher exciting current—30 to 50 percent of full-load current—and because the leakage reactances are also higher.

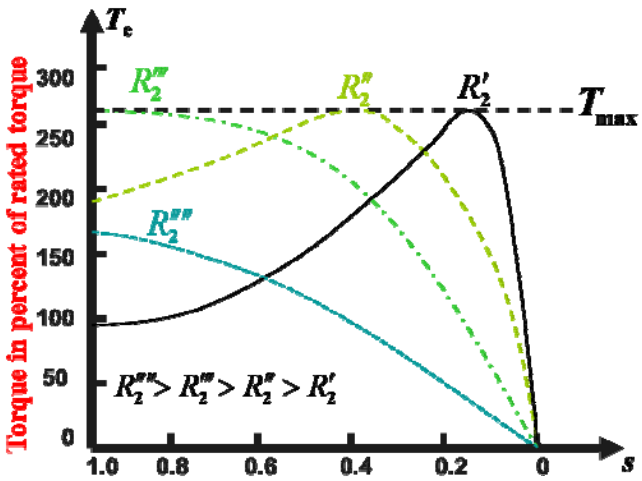

Figure 3 Induction-motor torque-slip curves showing effect of changing rotor circuit resistance.

Under the conditions of constant- frequency operation, a typical conventional induction motor with a squirrel-cage rotor is substantially a constant-speed motor having about 10 percent or less drop in speed from no load to full load. In the case of a wound-rotor induction motor, speed variation can be obtained by inserting external resistance in the rotor circuit; the influence of increased rotor resistance on the torque-speed characteristic is shown by the dashed curves in Fig. 3. For such a motor, significant speed variations can be achieved as the rotor resistance is varied. Similarly, the zero-speed torque variations seen in Fig 3 illustrate how the staring torque of a wound-rotor induction motor can be varied by varying the rotor resistance.

Notice that the slip at maximum torque is directly proportional to rotor resistance R_2 but the value of the maximum torque is independent of R_2. When R_2 is increased by inserting external resistance in the rotor of a wound-rotor motor, the maximum electromechanical torque is unaffected but the speed at which it occurs can be directly controlled. This result can also be seen by observing that the electromechanical torque is a function of the ratio R_2/s. Thus, the torque is unchanged as long as the ratio R_2/s remains constant.