4.5 ROTATING MMF WAVES IN AC MACHINES

To understand the theory and operation of polyphase ac machines, it is necessary to study the nature of the mmf wave produced by a polyphase winding. Attention will be focused on a two-pole machine (or equivalently one pole pair of a multi-pole winding; $-\pi \leq \theta_s \leq \pi$). We will begin with an analysis of a single phase winding. This will help us to develop insight into the nature of a polyphase winding.

4.5.1 MMF Wave of a Single-Phase Winding

The space-fundamental mmf distribution of a single phase winding is given by

$$f_{\phi 1} = \frac{4}{\pi} \frac{N k_{w1}}{2p} i_{\phi} \cos\theta_s \tag{4.17}$$

where θ_s is given by Eq. 4.1. When this winding is excited by a current varying sinusoidally in time at electrical frequency ω

$$i_{\phi} = \sqrt{2} I_{\phi} \cos\omega t \tag{4.18}$$

the mmf distribution is given by

$$f_{\phi 1}(\theta_s, t) = \frac{4}{\pi} \frac{N k_{w1}}{2p} \sqrt{2} I_{\phi} \cos\theta_s \cos\omega t = F_{\phi 1} \cos\theta_s \cos\omega t \tag{4.19}$$

Equation 4.19 has been written in a form to emphasize the fact that the result is an mmf distribution of maximum amplitude.

$$F_{\phi 1} = \frac{4}{\pi} \frac{N k_{w1}}{2p} \sqrt{2} I_{\phi} = 0.9 \frac{N k_{w1}}{p} I_{\phi} \tag{4.20}$$

This mmf distribution remains fixed in space with an amplitude that varies sinusoidally in time at frequency ω, as shown in Fig. 4.25a. Note that, to simplify the notation. Eq.4.1 has been used to express the mmf distribution of Eq. 4.19 in terms of the electrical angle θ_s.

Use of a common trigonometric identity permits Eq. 4.19 to be rewritten in the form

$$f_{\phi 1}(\theta_s, t) = \frac{1}{2} F_{\phi 1} \cos(\omega t - \theta_s) + \frac{1}{2} F_{\phi 1} \cos(\omega t + \theta_s) \tag{4.21}$$

which shows that the mmf of a single-phase winding can be resolved into two rotating mmf waves, each of amplitude equal to one-half the maximum amplitude of $F_{\phi 1}$ with one, $f^{+}_{\phi 1}(\theta_s, t)$, rotating in the $+\theta_s$ direction and the other, $f^{-}_{\phi 1}(\theta_s, t)$, rotating in the $-\theta_s$ direction

$$f^{+}_{\phi 1}(\theta_s, t) = \frac{1}{2} F_{\phi 1} \cos(\omega t - \theta_s) \tag{4.22}$$

$$f^{-}_{\phi 1}(\theta_s, t) = \frac{1}{2} F_{\phi 1} \cos(\omega t + \theta_s) \tag{4.23}$$

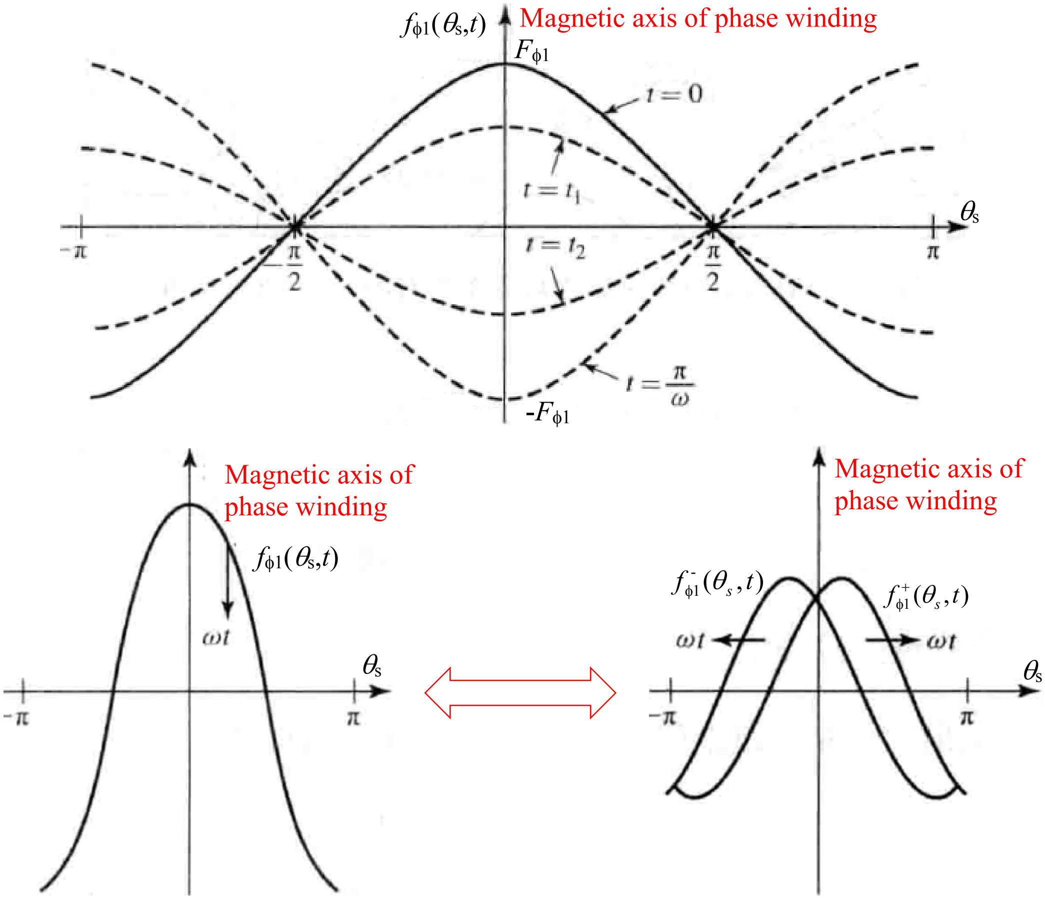

Figure 4.25 Single phase-winding space-fundamental air gap mmf (a) mmf distribution of a single phase winding at various times; (b) total mmf $f_{\phi1}(\theta_s,t)$ decomposed into two traveling waves $f^+_{\phi1}(\theta_s,t)$ and $f^-_{\phi1}(\theta_s,t)$.

Both flux waves rotate in their respective directions with electrical angular velocity ω, corresponding to a mechanical angular velocity Ω where

$$\omega = p\Omega = p(2\pi n)/60 \tag{4.24}$$

where n is the rotational speed in r/min. This decomposition is shown graphically in Figure 4.25b.

The fact that the air-gap mmf of a single-phase winding excited by a source of alternating current can be resolved into rotating traveling waves is an important conceptual step in understanding ac machinery. In single phase ac machinery, the positive-traveling flux wave produces useful torque while the negative-traveling flux wave produces both negative and pulsating torque as well as losses. Although single-phase machines are designed so as to minimize the effects of the negative- traveling flux wave, they cannot be totally eliminated. On the other hand, as shown in Section 4.5.2, in polyphase ac machinery the windings are equally displaced in space phase, and the winding currents are similarly displaced in time phase, with the result that the negative-traveling flux waves of the various windings sum to zero while the positive traveling flux waves reinforce, giving a single positive-traveling flux wave.

4.5.2 MMF Wave of a Polyphase Winding

In this section we study the mmf distributions of three-phase windings such as those found on the stator of three phase induction and synchronous machines. The analyses presented can be readily extended to a polyphase winding with any number of phase. Once again attention is focused on a two-pole machine or one pair of poles of a multi-pole winding.

In a three-phase machine, the windings of the individual phases are displaced from each other by 120 electrical degrees in space around the air-gap circumference, as shown by coils (a, $-a$), (b, $-b$), and(c, $-c$) in Fig.4.26. The concentrated full-pitch coils shown here may be considered to represent distributed windings producing sinusoidal mmf waves centered on the magnetic axes of the respective phases. The space-fundamental sinusoidal mmf waves of the three phases are accordingly displaced 120 electrical degrees in space. Each phase is excited by an alternating current which varies in magnitude sinusoidally with time. Under balanced three-phase conditions, the instantaneous currents are

$$i_a = \sqrt{2} I_\phi \cos \omega t \tag{4.25}$$

$$i_b = \sqrt{2} I_\phi \cos(\omega t - 120°) \tag{4.26}$$

$$i_c = \sqrt{2} I_\phi \cos(\omega t - 240°) \tag{4.27}$$

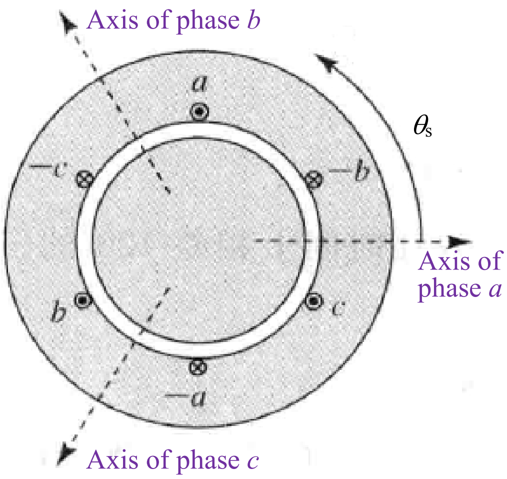

Figure 4.26 Simplified two-pole three-phase stator winding

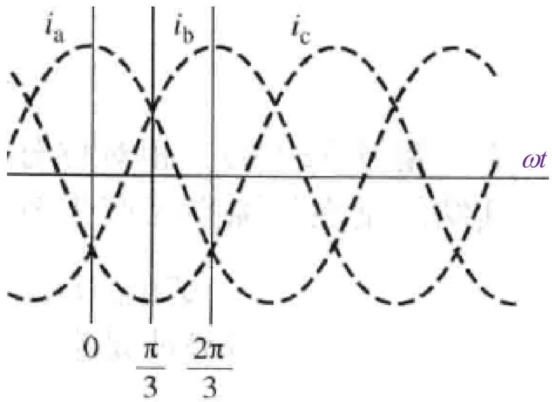

Figure 4.27 Instantaneous phase currents under balanced three-phase conditions.

where the time origin is arbitrarily taken as the instant when the phase-a current is at its positive maximum. The phase sequence is assumed to be abc. The instantaneous currents are shown in Fig. 4.27.The dots and crosses in the coil sides (Fig. 4.26) indicate the reference directions for positive phase currents.

The mmf of phase a has been shown to be

$$f_{a1} = F_{\phi 1} \cos\theta_s \cos\omega t = F^+_{a1} + F^-_{a1} \tag{4.28}$$

where

$$F^+_{a1} = 1/2 \cdot F_{\phi 1} \cos(\omega t - \theta_s) \tag{4.29}$$

$$F^-_{a1} = 1/2 \cdot F_{\phi 1} \cos(\omega t + \theta_s) \tag{4.30}$$

and

$$F_{\phi 1}=0.9\frac{Nk_{w1}}{p}I_{\phi} \tag{4.31}$$

Note that to avoid excessive notational complexity, the subscript ag has been dropped; here the subscript "a1" indicates the space-fundamental component of the phase-a air-gap mmf.

Similarly, for phases b and c, whose axes are at θ_s=120° and θ_s=240° respectively, their air-gap mmfs can be shown to be

$$f_{b1}=F_{\phi 1}\cos(\theta_s-120°)\cos(\omega t-120°) = F^{+}_{b1}+ F^{-}_{b1} \tag{4.32}$$

$$F^{+}_{b1}=1/2\cdot F_{\phi 1}\cos(\omega t-\theta_s) \tag{4.33}$$

$$F^{-}_{b1}=1/2\cdot F_{\phi 1}\cos(\omega t+\theta_s-240°) \tag{4.34}$$

and

$$f_{c1}=F_{\phi 1}\cos(\theta_s-240°)\cos(\omega t-240°) = F^{+}_{c1}+ F^{-}_{c1} \tag{4.35}$$

$$F^{+}_{c1}=1/2\cdot F_{\phi 1}\cos(\omega t-\theta_s) \tag{4.36}$$

$$F^{-}_{c1}=1/2\cdot F_{\phi 1}\cos(\omega t+\theta_s-120°) \tag{4.37}$$

The total mmf is the sum of the contributions from each of the three phases

$$f_1(\theta_s,\ t)=f_{a1}+ f_{b1}+ f_{c1} \tag{4.38}$$

This summation can be performed quite easily in terms of the positive- and negative-traveling waves. The negative-traveling waves sum to zero

$$F^{-}_{a1}+ F^{-}_{b1}+ F^{-}_{c1}$$
$$=1/2\cdot F_{\phi 1}\cos(\omega t+\theta_s)+ 1/2\cdot F_{\phi 1}\cos(\omega t+\theta_s-240°)+1/2\cdot F_{\phi 1}\cos(\omega t+\theta_s-120°)=0 \tag{4.39}$$

while the positive-traveling waves reinforce

$$F^{+}_{a1}+ F^{+}_{b1}+ F^{+}_{c1}=3/2\cdot F_{\phi 1}\cos(\omega t-\theta_s) \tag{4.40}$$

Thus, the result of displacing the three windings by 120° in space phase and displacing the winding currents by 120° in time phase is a single positive-traveling mmf wave

$$f_1(\theta_s, t)= 3/2\cdot F_{\phi 1}\cos(\omega t-\theta_s) \tag{4.41}$$

The air-gap mmf wave described by Eq. 4.41 is a space-fundamental sinusoidal function of the electrical space angle θ_s (and hence of the space angle $\theta=\theta_s/p$). It has a constant amplitude of (3/2) $F_{\phi 1}$, i.e., 1.5 times the peak amplitude of the air-gap mmf wave produced by the individual phases alone. It has a positive peak at angle $\theta = (\omega t/p)$. Thus, under balanced three phase conditions, the three-phase winding produces an air-gap mmf wave which rotates at *synchronous angular velocity* Ω_s

$$\Omega_s=\omega/p \tag{4.42}$$

where

ω = angular frequency of the applied electrical excitation [rad/sec]

Ω_s= synchronous spatial angular velocity of the air-gap mmf wave [rad/sec]

The corresponding *synchronous speed* n_s in r/min can be expressed in terms of the applied electrical frequency

$$\omega=2\pi f,\quad f=\omega/(2\pi) \tag{4.43}$$

as

$$f=\omega/(2\pi) =p\Omega_s/(2\pi)=p(2\pi n_s/60)/(2\pi)=pn_s/60,\quad n_s=60f/p \tag{4.44}$$

In general, a rotating field of constant amplitude will be produced by a m-phase winding ($m\geq 3$) excited by balanced m-phase currents of frequency f when the respective phase axes are located $2\pi/m$ electrical radians apart in space. The amplitude of this flux wave will be $m/2$ times the maximum contribution of any one phase, and the synchronous angular velocity will remain $\Omega_s = \omega/p$ radians per second. For a two phase machine, the phase axes are located $\pi/2$ electrical radians apart in space and the amplitude of the rotating flux wave will be equal to that of the individual phases.

In this section, we have seen that a polyphase winding excited by balanced polyphaser currents produces a rotating mmf wave. Production of a rotating mmf wave and the corresponding rotating magnetic flux wave is key to the operation of polyphase rotating electrical machinery. It is the interaction of this magnetic flux wave with that of the rotor which produces torque. Constant torque is produced when rotor-produced magnetic flux wave rotates in synchronism with that of the stator.