Transformer

The departures of an actual transformer from those of an ideal transformer must be included to a greater or lesser degree in most analyses of transformer performance. A more complete model must take into account the effects of winding resistance, leakage fluxes, and finite exciting current due to the finite(and indeed nonlinear) permeability of the core. In some case, the capacitances of the windings also have important effects, notably in problems involving transformer behavior at frequencies encountered in power system transformers as a result of voltage surges caused by lightning or switching transients. The analysis of these high-frequency problems is beyond the scope of the present treatment however, and accordingly capacitances of the winding will be neglected.

Two methods of analysis by which departures from the ideal can be taken into account are (1) an equivalent-circuit technique based on physical reasoning and (2) mathematical approach based on the classical theory to magnetically coupled circuits. Both methods are in everyday use, and both have very close parallels in the theories of rotating machines. Because it offers an excellent example of the thought process involved in translating physical concepts to a quantitative theory, the equivalent-circuit technique is presented here.

To begin the development of a transformer equivalent circuit, we first consider the primary winding. The total flux linking the primary winding can be divided into two components: the main flux, confined essentially to the iron core and produced by the combined mmfs of the primary and secondary currents, and the primary leakage flux, which links only the primary. These components are identified in the schematic transformer shown in Fig. 1, where for simplicity the primary and secondary windings are shown on opposite legs of the core. In an actual transformer with interleaved windings, the details of the flux distribution are more complicated, but he essential features remain the same.

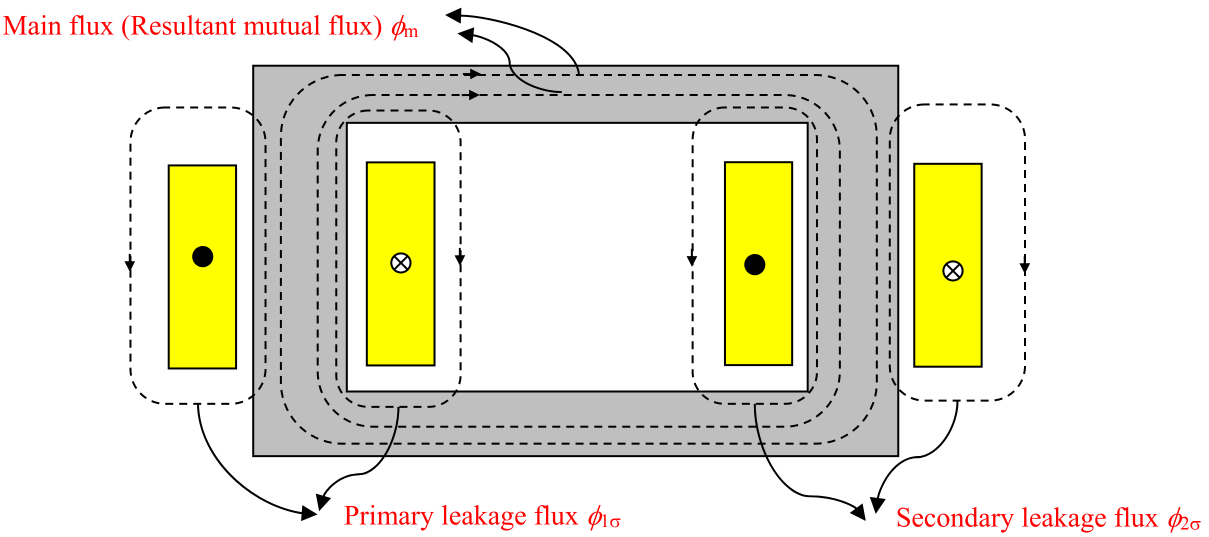


Figure 1 Schematic view of main and leakage flux in a transformer. The X and the dot indicated current directions in the various coils.

The leakage flux induces voltage in the primary windings which adds to that produced by the mutual flux. Because the leakage path is largely in air, this flux and the voltage induced by it vary linearly with primary current $\dot{I}_1$. It can therefore be represented by a primary leakage inductance $L_{1\sigma}$ (equal to the leakage-flux linkages with the primary per unit of primary current). The corresponding primary leakage reactance $X_{1\sigma}$ is found as

$$X_{1\sigma}=2\pi f\ L_{1\sigma} \tag{1}$$

In addition, there will be a voltage drop in the primary resistance R_1 (not shown in Fig 1).

We now see that the primary terminal voltage $\dot{U}_1$ consists of three components: the $\dot{I}_1 R_1$ drop in the primary resistance, the $\mathrm{j}\dot{I}_1 X_{1\sigma}$ drop arising from primary leakage flux, and the emf $\dot{E}_1$ induced in the primary by the main flux (resultant mutual flux). Fig.2 shows an equivalent circuit for the primary winding which includes each of these voltages.

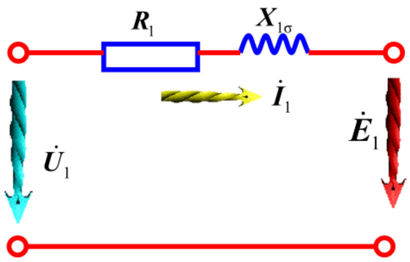


Figure 2 Equivalent circuit for the primary winding

The main flux (resultant mutual flux) links both the primary and secondary windings and is created by their combined mmfs. It is convenient to treat these mmfs by considering that the primary current must meet two requirements of the magnetic circuit: It must not only produce the mmf required to produce the main flux (resultant mutual flux), but it must also counteract the effect of the secondary mmf which acts to demagnetize the core. An alternative viewpoint is that the primary current must not magnetize the core, it must also supply current to the load connected to the secondary. According to this picture (Fig.3), it is convenient to resolve the primary current into two components: an exciting component and a load component. The exciting component $\dot{I}_{\mathrm{m}}$ is defined as the additional primary current required to produce the main flux (resultant mutual flux). The load component $\dot{I}_{1L}$ is defined as the component current in the primary which would exactly counteract the mmf of secondary current $\dot{I}_2$.

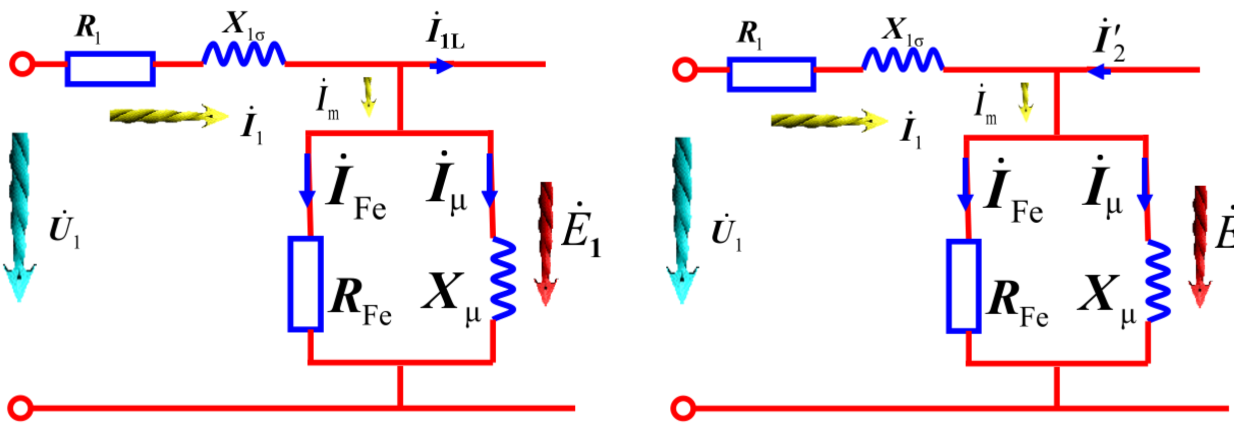


Figure 3 the components of primary current.

Since it is the exciting component which produces the main flux of core, the net mmf must equal $N_1\dot{I}_{\mathrm{m}}$ and thus we see that:

$$N_1\dot{I}_{\mathrm{m}} = N_1\dot{I}_1 + N_2\dot{I}_2 = N_1(\dot{I}_{\mathrm{m}} + \dot{I}_{1\mathrm{L}}) + N_2\dot{I}_2 \quad \Rightarrow \quad \dot{I}_{1\mathrm{L}} = -\frac{N_2}{N_1}\dot{I}_2 = -\dot{I}_2' \tag{2}$$

We see that the load component of the primary current equals the negative secondary current referred to the primary.

The exciting current $\dot{I}_{\mathrm{m}}$ can be resolved into a core-loss component $\dot{I}_{\mathrm{Fe}}$ in opposite phase with emf $\dot{E}_1$ and magnetizing component $\dot{I}_{\mu}$ leading $\dot{E}_1$ by 90°. In the equivalent circuit of Figure 3 the equivalent sinusoidal exciting current is accounted for by means of a shunt branch connected across $\dot{E}_1$, comparing a core-loss resistance R_{Fe} in parallel with a magnetizing reactance X_{μ}.

In the equivalent circuit of figure 3 the power E_1^2/R_{Fe} accounts for the core-loss due to the main flux (resultant mutual flux). R_{Fe}, also referred to as the magnetizing resistance, together with X_{μ} forms the excitation branch of the equivalent circuit, and we will refer to the parallel combination of R_{Fe} and X_{μ} as the magnetizing impedance Z_{m}. When R_{Fe} is assumed constant, the core loss is thereby assumed to vary as E_1^2. Strictly speaking, the magnetizing reactance X_{μ} varies with the saturation of the iron. However, X_{μ} is often assumed constant and the magnetizing current is thereby assumed to be independent of frequency and directly proportional to the main flux. Both R_{Fe} and X_{μ} are usually determined at rated voltage and frequency; they are then assumed to remain constant for the small departure from rated values associated with normal operation.

We will next add to our equivalent circuit a representation of the secondary winding. We begin by recognizing that the main flux $\dot{\Phi}_{\mathrm{m}}$ induced an emf $\dot{E}_2$ in the secondary, and since the flux links both windings, the induced-emf ratio must equal the winding turns ratio, i.e,.

$$\dot{E}_1 / \dot{E}_2 = N_1 / N_2 \tag{3}$$

Just as is the case for the primary winding, the emf $\dot{E}_2$ is not the secondary terminal voltage because of the secondary resistance R2 and because the secondary current $\dot{I}_2$ creates secondary leakage flux (See Figure 1). The secondary terminal voltage $\dot{U}_2$ differs from the induced voltage $\dot{E}_2$ by the voltage drops due to secondary resistance R_2 and secondary leakage reactance $X_{2\sigma}$ (corresponding to the secondary leakage inductance $L_{2\sigma}$) as in the portion of the complete transformer equivalent circuit (Figure 4) to the right of $\dot{E}_2$.

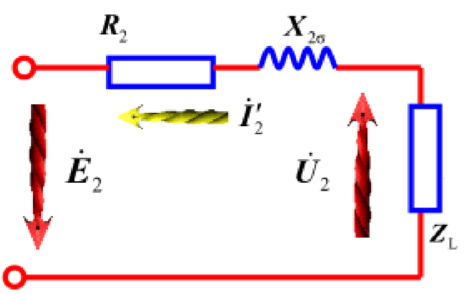
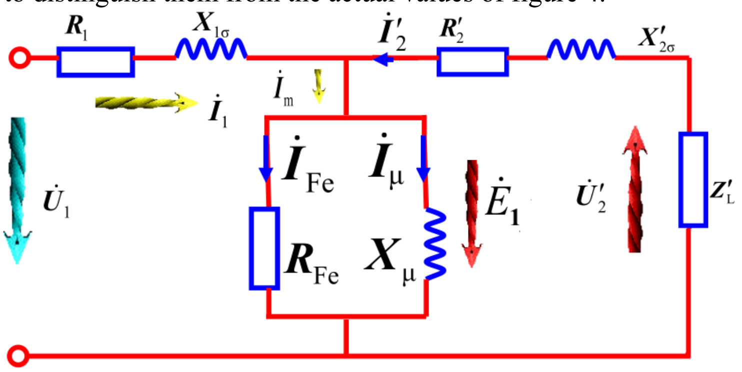


Figure 4. Equivalent circuit for the secondary winding

The final equivalent circuit is usually drawn as in figure 5, all voltages, currents, and impedances referred to the primary winding. Specifically, for figure 5,

$$k = N_1 / N_2 \quad X_{2\sigma}' = k^2 X_{2\sigma} \quad R_2' = k^2 R_2 \quad U_2' = kU_2 \tag{4}$$

The circuit of figure 5 is called the equivalent-T circuit for a transformer. In the figure 5, in which the secondary quantities are referred to the primary, the referred secondary values are indicated with primes (符号撇), for example, $X'_{2\sigma}$ and R'_2 , to distinguish them from the actual values of figure 4.



THE PER-UNIT SYSTEM

Electric power systems typically consist of the interconnection of a large number of generators, transformers, transmissions lines and loads (a large fraction of which include electric motors). The characteristics of these components vary over a large range; with voltages ranging from hundreds of volts to hundreds of kilovolts and power ratings ranging from kilowatts to hundreds of megawatts. Power-system analyses, and indeed analyses of individual power-system components are often carried out in per-unit form, i.e., with all pertinent quantities expressed as decimal fractions of appropriately chosen base values. All the usual computations are then carried out in these per-unit values instead of the familiar volts, amperes, ohms, and so on.

There are a number of advantages to the use of the per-unit system. One is that, when expressed in per-unit based upon their rating, the parameter values of machines and transformers typically fall in a reasonably narrow numerical range. This both permits a quick “sanity check” of parameter values as well enables “back-of-the envelope” estimates of parameter values which are otherwise not available. A second advantage is the when transformer equivalent-circuit parameters are converted to their per-unit values, the transformer turns ratio becomes 1:1. This greatly simplifies analyses since it eliminates the need to refer impedances to one side or the other of transformers.

Quantities such as voltage *V*, current *I*, power *P*, reactive power *Q*, voltamperes *VA*, resistance *R*, reactance *X*, impedance *Z*, conductance *G*, and susceptance *B* can be translated to and from per-unit form as follows.

$$\text{Quantity in per unit} = \frac{\text{Actual quantity}}{\text{Base value of quantity}}$$