Supply equation for landing model

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1 Model A

The optimization problem that a fisher that harvest both species i and j,

$$\begin{aligned} \underset{E_i, E_j}{\text{maximize}} \quad & p_i H_i - w_i E_i + p_j H_j - w_j E_j, \\ \text{subject to} \quad & E_i \geq 0 \\ & E_j \geq 0 \\ & \bar{E} \geq E_i + E_j. \end{aligned}$$

Assuming $H_k = q S_k^{\alpha} E_k^{\beta}$ and $E_j = \bar{E} - E_i$, is the following

$$\begin{aligned} \underset{E_i}{\text{maximize}} \quad & p_i q S_i^{\alpha} E_i^{\beta} - w_i E_i + p_j q S_j^{\alpha} \left(\bar{E} - E_i\right)^{\beta} - w_j \left(\bar{E} - E_i\right), \\ \text{subject to} \quad & E_i \geq 0 \\ & \bar{E} \geq E_i. \end{aligned}$$

Assuming that $E_i > 0$ and $\bar{E} > E_i$, then the first order condition is

$$\beta p_i q S_i^{\alpha} E_i^{\beta-1} - w_i - \beta p_j q S_j^{\alpha} \left(\bar{E} - E_i\right)^{\beta-1} + w_j = 0$$

Assuming both marginal effort cost are the same, solving for E^*

$$\beta p_i q S_i^{\alpha} E_i^{\beta-1} = \beta p_j q S_j^{\alpha} \left(\bar{E} - E_i\right)^{\beta-1}$$

$$E_i^{\beta-1} = \frac{\beta p_j q S_j^{\alpha}}{\beta p_i q S_i^{\alpha}} \left(\bar{E} - E_i\right)^{\beta-1}$$

$$E_i = \left(\frac{p_j q S_j^{\alpha}}{p_i q S_i^{\alpha}}\right)^{\frac{1}{\beta-1}} \left(\bar{E} - E_i\right)$$

$$E_i^* \left[1 + \left(\frac{p_j q S_j^{\alpha}}{p_i q S_i^{\alpha}}\right)^{\frac{1}{\beta-1}}\right] = \left(\frac{p_j q S_j^{\alpha}}{p_i q S_i^{\alpha}}\right)^{\frac{1}{\beta-1}} \bar{E}$$

$$E_i^* = \frac{\left(\frac{p_j q S_j^\alpha}{p_i q S_i^\alpha}\right)^{\frac{1}{\beta-1}}}{1+\left(\frac{p_j q S_j^\alpha}{p_i q S_i^\alpha}\right)^{\frac{1}{\beta-1}}}\bar{E}$$

and the supply of harvest is:

$$H_i^* = q S_i^\alpha \left[\frac{\left(\frac{p_j q S_j^\alpha}{p_i q S_i^\alpha}\right)^{\frac{1}{\beta-1}}}{1+\left(\frac{p_j q S_j^\alpha}{p_i q S_i^\alpha}\right)^{\frac{1}{\beta-1}}}\bar{E}\right]^\beta$$

if $\beta < 1$

$$H_i^* = q S_i^\alpha \left[\frac{\left(\frac{p_i q S_i^\alpha}{p_j q S_j^\alpha}\right)^{\frac{1}{1-\beta}}}{1+\left(\frac{p_i q S_i^\alpha}{p_j q S_j^\alpha}\right)^{\frac{1}{1-\beta}}}\bar{E}\right]^\beta$$

Let us now assume that marginal cost of effort are different. Then...

$$\beta q \left[p_i S_i^\alpha E_i^{\beta-1} - p_j S_j^\alpha \left(\bar{E} - E_i\right)^{\beta-1}\right] = w_i - w_j$$

2 Model B

Other option is that the marginal cost of effort depend on the scale that produce output? The optimization problem that a fisher that harvest both species i and j, assuming $H_k = q S_k^\alpha E_k^\beta$, $E_j = \bar{E} - E_i$ and $C_k = w_k E_k^\beta$, is the following

$$\begin{aligned} \underset{e_i}{\text{maximize}} \quad & p_i q S_i^\alpha E_i^\beta - w_i E_i^\beta + p_j q S_j^\alpha \left(\bar{E} - E_i\right)^\beta - w_j \left(\bar{E} - E_i\right)^\beta, \\ \text{subject to} \quad & e_i \geq 0 \\ & \bar{E} \geq E_i. \end{aligned}$$

Assuming that $E_i > 0$ and $\bar{E} > E_i$, then the first order condition is

$$\beta p_i q S_i^\alpha E_i^{\beta-1} - \beta w_i E_i^{\beta-1} - \beta p_j q S_j^\alpha \left(\bar{E} - E_i\right)^{\beta-1} + \beta w_j \left(\bar{E} - E_i\right)^{\beta-1} = 0$$

$$\beta \left(p_i q S_i^\alpha - w_i\right) E_i^{\beta-1} - \beta \left(p_j q S_j^\alpha - w_j\right) \left(\bar{E} - E_i\right)^{\beta-1} = 0$$

$$\left(p_i q S_i^\alpha - w_i\right) E_i^{\beta-1} = \left(p_j q S_j^\alpha - w_j\right) \left(\bar{E} - E_i\right)^{\beta-1}$$

$$\left(p_i q S_i^\alpha - w_i\right) \frac{E_i^{\beta-1}}{\left(\bar{E} - E_i\right)^{\beta-1}} = \left(p_j q S_j^\alpha - w_j\right)$$

$$\frac{E_i^{\beta-1}}{\left(\bar{E}-E_i\right)^{\beta-1}} = \frac{\left(p_j q S_j^{\alpha} - w_j\right)}{\left(p_i q S_i^{\alpha} - w_i\right)}$$

$$\frac{E_i}{\bar{E}-E_i} = \left(\frac{p_j q S_j^{\alpha} - w_j}{p_i q S_i^{\alpha} - w_i}\right)^{\frac{1}{\beta-1}}$$

$$E_i^* = \left(\bar{E}-E_i\right)\left(\frac{p_j q S_j^{\alpha} - w_j}{p_i q S_i^{\alpha} - w_i}\right)^{\frac{1}{\beta-1}}$$

$$E_i^*\left[1+\left(\frac{p_j q S_j^{\alpha} - w_j}{p_i q S_i^{\alpha} - w_i}\right)^{\frac{1}{\beta-1}}\right] = \bar{E}\left(\frac{p_j q S_j^{\alpha} - w_j}{p_i q S_i^{\alpha} - w_i}\right)^{\frac{1}{\beta-1}}$$

$$E_i^* = \frac{\left(\frac{p_j q S_j^{\alpha} - w_j}{p_i q S_i^{\alpha} - w_i}\right)^{\frac{1}{\beta-1}}}{1+\left(\frac{p_j q S_j^{\alpha} - w_j}{p_i q S_i^{\alpha} - w_i}\right)^{\frac{1}{\beta-1}}}\bar{E}$$

Assume that $\beta < 1$

$$E_i^* = \frac{\left(\frac{p_i q S_i^{\alpha} - w_i}{p_j q S_j^{\alpha} - w_j}\right)^{\frac{1}{1-\beta}}}{1+\left(\frac{p_i q S_i^{\alpha} - w_i}{p_j q S_j^{\alpha} - w_j}\right)^{\frac{1}{1-\beta}}}\bar{E}$$

and the harvest supply for species i is:

$$H_i^* = q S_i^{\alpha}\left[\frac{\left(\frac{p_i q S_i^{\alpha} - w_i}{p_j q S_j^{\alpha} - w_j}\right)^{\frac{1}{1-\beta}}}{1+\left(\frac{p_i q S_i^{\alpha} - w_i}{p_j q S_j^{\alpha} - w_j}\right)^{\frac{1}{1-\beta}}}\bar{E}\right]^{\beta}$$

Therefore, harvest of species i would depend on

- Catchability (gear?)
- Stock of the species, and substitutes
- Price of the species, and substitutes
- Marginal cost of effort for the species and substitute (We can assume is the same)
- Potential effort capacity

We might include the effect directly, as interaction or as relative values.

3 Model C

$$\begin{aligned} \underset{e_i}{\text{maximize}} \quad & p_i q S_i E_i - w_i E_i^2 + p_j q S_j \left(\bar{E} - E_i\right) - w_j \left(\bar{E} - E_i\right)^2, \\ \text{subject to} \quad & e_i \geq 0 \\ & \bar{E} \geq E_i. \end{aligned}$$

Assuming that $E_i > 0$ and $\bar{E} > E_i$, then the first order condition is

$$p_i q S_i - 2 w_i E_i - p_j q S_j + 2 w_j \left(\bar{E} - E_i\right) = 0$$

Solving for E_i^*

$$p_i q S_i - p_j q S_j + 2 w_j \bar{E} = E_i^* \left(2 w_i + 2 w_j\right)$$

$$E_i^* = \frac{p_i q S_i - p_j q S_j + 2 w_j \bar{E}}{2 w_i + 2 w_j}$$

and the harvest supply for species i is:

$$H_i^* = q S_i \frac{p_i q S_i - p_j q S_j + 2 w_j \bar{E}}{2 w_i + 2 w_j}$$

if $w_i = w_j = w$

$$H_i^* = q S_i \frac{p_i q S_i - p_j q S_j}{4w} + \frac{q S_i \bar{E}}{2}$$

$$H_i^* = \frac{p_i q^2 S_i^2}{4w} - \frac{p_j q^2 S_i S_j}{4w} + \frac{q S_i \bar{E}}{2}$$