

# Supply equation for landing model

May 7, 2022

## 1 Model A

The optimization problem that a fisher that harvest both species  $i$  and  $j$ , assuming  $H_k = qS_k^\alpha E_k^\beta$  and  $E_j = \bar{E} - E_i$ , is the following

$$\begin{aligned} \underset{e_i}{\text{maximize}} \quad & p_i q S_i^\alpha E_i^\beta - w_i E_i + p_j q S_j^\alpha (\bar{E} - E_i)^\beta - w_j (\bar{E} - E_i), \\ \text{subject to} \quad & e_i \geq 0 \\ & \bar{E} \geq E_i. \end{aligned}$$

Assuming that  $E_i > 0$  and  $\bar{E} > E_i$ , then the first order condition is

$$\beta p_i q S_i^\alpha E_i^{\beta-1} - w_i - \beta p_j q S_j^\alpha (\bar{E} - E_i)^{\beta-1} + w_j = 0$$

Assuming both marginal effort cost are the same, solving for  $E^*$

$$\beta p_i q S_i^\alpha E_i^{\beta-1} = \beta p_j q S_j^\alpha (\bar{E} - E_i)^{\beta-1}$$

$$E_i^{\beta-1} = \frac{\beta p_j q S_j^\alpha}{\beta p_i q S_i^\alpha} (\bar{E} - E_i)^{\beta-1}$$

$$E_i = \left( \frac{p_j q S_j^\alpha}{p_i q S_i^\alpha} \right)^{\frac{1}{\beta-1}} (\bar{E} - E_i)$$

$$E_i^* \left[ 1 + \left( \frac{p_j q S_j^\alpha}{p_i q S_i^\alpha} \right)^{\frac{1}{\beta-1}} \right] = \left( \frac{p_j q S_j^\alpha}{p_i q S_i^\alpha} \right)^{\frac{1}{\beta-1}} \bar{E}$$

$$E_i^* = \frac{\left( \frac{p_j q S_j^\alpha}{p_i q S_i^\alpha} \right)^{\frac{1}{\beta-1}} \bar{E}}{1 + \left( \frac{p_j q S_j^\alpha}{p_i q S_i^\alpha} \right)^{\frac{1}{\beta-1}}}$$

and the supply of harvest is:

$$H_i^* = q S_i^\alpha \left[ \frac{\left( \frac{p_j q S_j^\alpha}{p_i q S_i^\alpha} \right)^{\frac{1}{\beta-1}} \bar{E}}{1 + \left( \frac{p_j q S_j^\alpha}{p_i q S_i^\alpha} \right)^{\frac{1}{\beta-1}}} \right]^\beta$$

if  $\beta < 1$

$$H_i^* = qS_i^\alpha \left[ \frac{\left( \frac{p_i q S_i^\alpha}{p_j q S_j^\alpha} \right)^{\frac{1}{1-\beta}}}{1 + \left( \frac{p_i q S_i^\alpha}{p_j q S_j^\alpha} \right)^{\frac{1}{1-\beta}}} \bar{E} \right]^\beta$$

Let us now assume that marginal cost of effort are different. Then...

$$\beta q \left[ p_i S_i^\alpha E_i^{\beta-1} - p_j S_j^\alpha (\bar{E} - E_i)^{\beta-1} \right] = w_i - w_j$$

## 2 Model B

Other option is that the marginal cost of effort depend on the scale that produce output? The optimization problem that a fisher that harvest both species  $i$  and  $j$ , assuming  $H_k = qS_k^\alpha E_k^\beta$ ,  $E_j = \bar{E} - E_i$  and  $C_k = w_k E_k^\beta$ , is the following

$$\begin{aligned} \underset{e_i}{\text{maximize}} \quad & p_i q S_i^\alpha E_i^\beta - w_i E_i^\beta + p_j q S_j^\alpha (\bar{E} - E_i)^\beta - w_j (\bar{E} - E_i)^\beta, \\ \text{subject to} \quad & e_i \geq 0 \\ & \bar{E} \geq E_i. \end{aligned}$$

Assuming that  $E_i > 0$  and  $\bar{E} > E_i$ , then the first order condition is

$$\beta p_i q S_i^\alpha E_i^{\beta-1} - \beta w_i E_i^{\beta-1} - \beta p_j q S_j^\alpha (\bar{E} - E_i)^{\beta-1} + \beta w_j (\bar{E} - E_i)^{\beta-1} = 0$$

$$\beta (p_i q S_i^\alpha - w_i) E_i^{\beta-1} - \beta (p_j q S_j^\alpha - w_j) (\bar{E} - E_i)^{\beta-1} = 0$$

$$(p_i q S_i^\alpha - w_i) E_i^{\beta-1} = (p_j q S_j^\alpha - w_j) (\bar{E} - E_i)^{\beta-1}$$

$$(p_i q S_i^\alpha - w_i) \frac{E_i^{\beta-1}}{(\bar{E} - E_i)^{\beta-1}} = (p_j q S_j^\alpha - w_j)$$

$$\frac{E_i^{\beta-1}}{(\bar{E} - E_i)^{\beta-1}} = \frac{(p_j q S_j^\alpha - w_j)}{(p_i q S_i^\alpha - w_i)}$$

$$\frac{E_i}{\bar{E} - E_i} = \left( \frac{p_j q S_j^\alpha - w_j}{p_i q S_i^\alpha - w_i} \right)^{\frac{1}{\beta-1}}$$

$$E_i^* = (\bar{E} - E_i) \left( \frac{p_j q S_j^\alpha - w_j}{p_i q S_i^\alpha - w_i} \right)^{\frac{1}{\beta-1}}$$

$$E_i^* \left[ 1 + \left( \frac{p_j q S_j^\alpha - w_j}{p_i q S_i^\alpha - w_i} \right)^{\frac{1}{\beta-1}} \right] = \bar{E} \left( \frac{p_j q S_j^\alpha - w_j}{p_i q S_i^\alpha - w_i} \right)^{\frac{1}{\beta-1}}$$

$$E_i^* = \frac{\left(\frac{p_j q S_j^\alpha - w_j}{p_i q S_i^\alpha - w_i}\right)^{\frac{1}{\beta-1}} \bar{E}}{1 + \left(\frac{p_j q S_j^\alpha - w_j}{p_i q S_i^\alpha - w_i}\right)^{\frac{1}{\beta-1}}}$$

Assume that  $\beta < 1$

$$E_i^* = \frac{\left(\frac{p_i q S_i^\alpha - w_i}{p_j q S_j^\alpha - w_j}\right)^{\frac{1}{1-\beta}} \bar{E}}{1 + \left(\frac{p_i q S_i^\alpha - w_i}{p_j q S_j^\alpha - w_j}\right)^{\frac{1}{1-\beta}}}$$

and the harvest supply for species  $i$  is:

$$H_i^* = q S_i^\alpha \left[ \frac{\left(\frac{p_i q S_i^\alpha - w_i}{p_j q S_j^\alpha - w_j}\right)^{\frac{1}{1-\beta}} \bar{E}}{1 + \left(\frac{p_i q S_i^\alpha - w_i}{p_j q S_j^\alpha - w_j}\right)^{\frac{1}{1-\beta}}} \right]^\beta$$

Therefore, harvest of species  $i$  would depend on

- Catchability (gear?)
- Stock of the species, and substitutes
- Price of the species, and substitutes
- Marginal cost of effort for the species and substitute (We can assume is the same)
- Potential effort capacity

We might include the effect directly, as interaction or as relative values.

### 3 Model C

$$\begin{aligned} & \underset{e_i}{\text{maximize}} && p_i q S_i E_i - w_i E_i^2 + p_j q S_j (\bar{E} - E_i) - w_j (\bar{E} - E_i)^2, \\ & \text{subject to} && e_i \geq 0 \\ & && \bar{E} \geq E_i. \end{aligned}$$

Assuming that  $E_i > 0$  and  $\bar{E} > E_i$ , then the first order condition is

$$p_i q S_i - 2w_i E_i - p_j q S_j + 2w_j (\bar{E} - E_i) = 0$$

Solving for  $E_i^*$

$$p_i q S_i - p_j q S_j + 2w_j \bar{E} = E_i^* (2w_i + 2w_j)$$

$$E_i^* = \frac{p_i q S_i - p_j q S_j + 2w_j \bar{E}}{2w_i + 2w_j}$$

and the harvest supply for species  $i$  is:

$$H_i^* = qS_i \frac{p_i q S_i - p_j q S_j + 2w_j \bar{E}}{2w_i + 2w_j}$$

if  $w_i = w_j = w$

$$H_i^* = qS_i \frac{p_i q S_i - p_j q S_j}{4w} + \frac{qS_i \bar{E}}{2}$$

$$H_i^* = \frac{p_i q^2 S_i^2}{4w} - \frac{p_j q^2 S_i S_j}{4w} + \frac{qS_i \bar{E}}{2}$$