Digital Signal Processing

SECTION 4

OPERATIONS ON MATRIX

LECTURE 2 EXERCISES

ASMAA FATHY

Unit Impulse Sequence

The discrete-time unit impulse function $\delta(n)$, also called unit sample sequence, is defined as:

$$\delta(n) = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{for } n \neq 0 \end{cases}$$

This means that the unit sample sequence is a signal that is zero everywhere, except at $n = 0$, where its value is unity. It is the most widely used elementary signal used for the analysis of signals and systems.

The shifted unit impulse function $\delta(n - k)$ is defined as:

$$\delta(n - k) = \begin{cases} 1 & \text{for } n = k \\ 0 & \text{for } n \neq k \end{cases}$$

EXAMPLE 1.1

- Find the following summations:

(a) $\sum_{n=-\infty}^{\infty} e^{3n}\,\delta(n-3)$

(b) $\sum_{n=-\infty}^{\infty} \delta(n-2)\cos 3n$

(a) $\sum_{n=-\infty}^{\infty} e^{3n}\,\delta(n-3)$

Solution

Given $$\sum_{n=-\infty}^{\infty} e^{3n}\,\delta(n-3)$$

We know that $$\delta(n-3)=\begin{cases}1 & \text{for } n=3\\ 0 & \text{elsewhere}\end{cases}$$

$\therefore$ $$\sum_{n=-\infty}^{\infty} e^{3n}\delta(n-3)=[e^{3n}]_{n=3}=e^{9}$$

(b) $\sum_{n=-\infty}^{\infty} \delta(n-2)\cos 3n$

Solution

Given $$\sum_{n=-\infty}^{\infty} \delta(n-2)\cos 3n$$

We know that $$\delta(n-2) = \begin{cases} 1 & \text{for } n = 2 \\ 0 & \text{elsewhere} \end{cases}$$

$$\therefore \quad \sum_{n=-\infty}^{\infty} \delta(n-2)\cos 3n = [\cos 3n]_{n=2} = \cos 6$$

Unit Step Sequence

The discrete-time unit step sequence $u(n)$ is defined as:

$$u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

The shifted version of the discrete-time unit step sequence $u(n - k)$ is defined as:

$$u(n-k) = \begin{cases} 1 & \text{for } n \geq k \\ 0 & \text{for } n < k \end{cases}$$

It is zero if the arugment $(n - k) < 0$ and equal to 1 if the arugment $(n - k) \geq 0$.

The graphical representation of $u(n)$ and $u(n - k)$ is shown in Figure 1.3[(a) and (b)].

EXAMPLE 1.2

- Sketch the following signals:

(a) $u(n+2)\,u(-n+3)$

(b) $x(n) = u(n+4) - u(n-2)$

(a) $u(n+2)\,u(-n+3)$

Solution

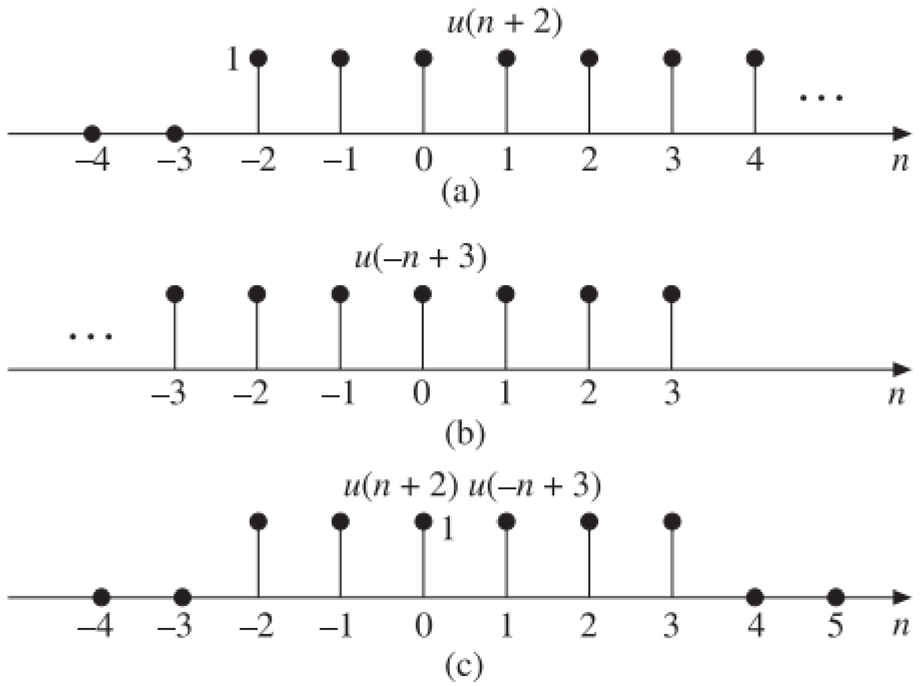


Figure 1.13 Plots of (a) $u(n+2)$ (b) $u(-n+3)$ (c) $u(n+2)\,u(-n+3)$.

(b) $x(n) = u(n+4) - u(n-2)$

Solution

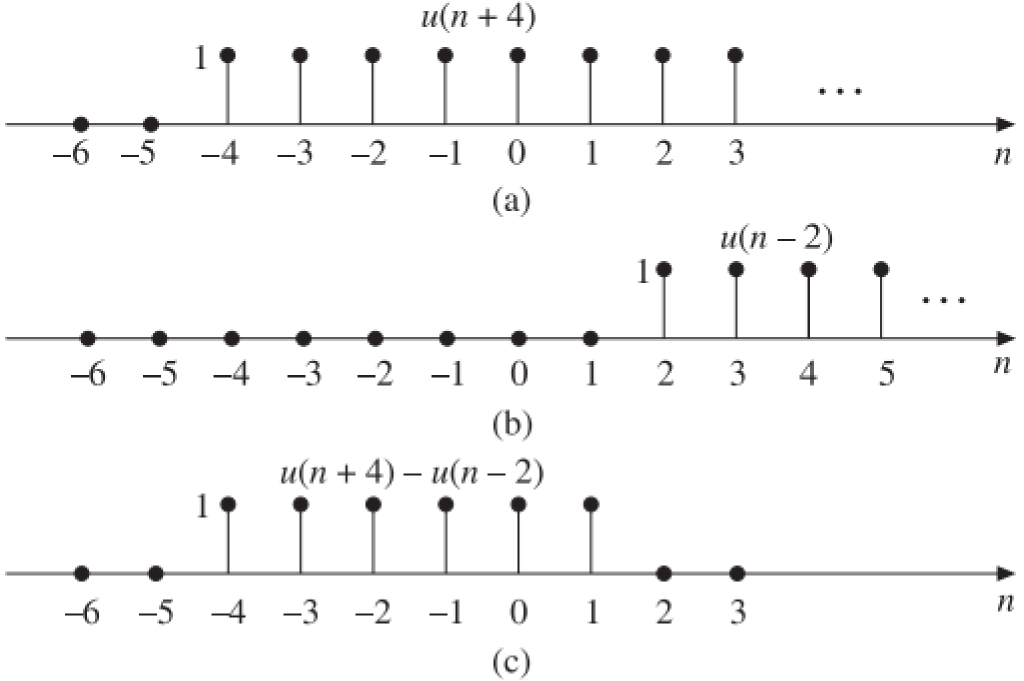
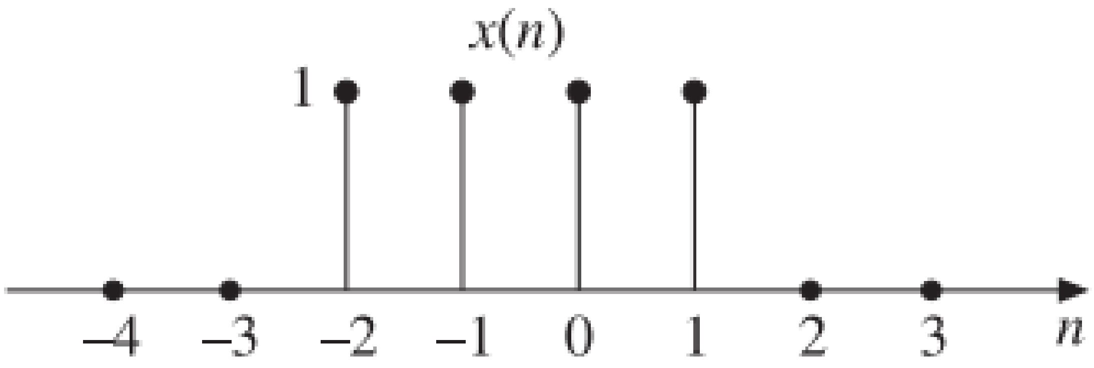


Figure 1.14 Plots of (a) $u(n + 4)$ (b) $u(n - 2)$ (c) $u(n + 4) - u(n - 2)$.

EXAMPLE 1.3

- Express the signal as the sum of singular functions:



Solution

$$x(n) = \delta(n+2) + \delta(n+1) + \delta(n) + \delta(n-1)$$

$$x(n) = \begin{cases} 0 & \text{for } n \le -3 \\ 1 & \text{for } -2 \le n \le 1 \\ 0 & \text{for } n \ge 2 \end{cases}$$

$\therefore$

$$x(n) = u(n+2) - u(n-2)$$

Deterministic and Random Signals

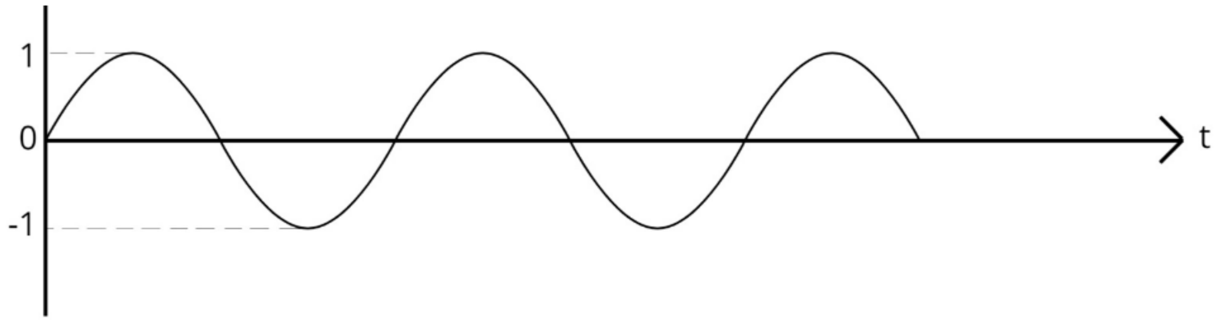
- A deterministic signal can be completely represented by mathematical equation at any time and its nature and amplitude at any time can be predicted.
- *Examples:* Sinusoidal sequence $x(n) = \cos \omega n$, Exponential sequence $x(n) = e^{j\omega n}$, ramp sequence $x(n) = \alpha n$.

- A signal characterized by uncertainty about its occurrence is called a non-deterministic or random signal. A random signal cannot be represented by any mathematical equation. The behavior of such a signal is probabilistic in nature and can be analyzed only stochastically.

Periodic and Non-periodic Sequences

- A signal which has a definite pattern and repeats itself at regular intervals of time is called a periodic signal, and a signal which does not repeat at regular intervals of time is called a non-periodic or aperiodic signal.
- A discrete-time signal $x(n)$ is said to be periodic if it satisfies the condition
  $x(n) = x(n + N)$ for all integers n.
- The smallest value of N which satisfies the above condition is known as fundamental period.
- The angular frequency is given by $\omega = \frac{2\pi}{N}$

$\therefore$ Fundamental period $N = \frac{2\pi}{\omega}$



EXAMPLE 1.4

- Show that the complex exponential sequence $x(n) = e^{j\omega_o n}$ is periodic only if $\omega_o/2\pi$ is a rational number.

Solution

Given	$x(n) = e^{j\omega_0 n}$
$x(n)$ will be periodic if	$x(n+N) = x(n)$
i.e.	$e^{j[\omega_0(n+N)]} = e^{j\omega_0 n}$
i.e.	$e^{j\omega_0 N}\, e^{j\omega_0 n} = e^{j\omega_0 n}$
This is possible only if	$e^{j\omega_0 N} = 1$
This is true only if where k is an integer.	$\omega_0 N = 2\pi k$
$\therefore$	$\dfrac{\omega_0}{2\pi} = \dfrac{k}{N}$ Rational number

This shows that the complex exponential sequence $x(n) = e^{j\omega_0 n}$ is periodic if $\omega_0/2\pi$ is a rational number.

EXAMPLE 1.5

- Determine whether the following discrete-time signals are periodic or not. If periodic, determine the fundamental period:

(a) $\sin(0.02\pi n)$

(d) $\sin\dfrac{2\pi n}{3} + \cos\dfrac{2\pi n}{5}$

(g) $e^{j(\pi/2)n}$

(a) $\sin(0.02\pi n)$

Solution

Given $x(n) = \sin(0.02\pi n)$

Comparing it with $x(n) = \sin(2\pi f n)$

we have $0.02\pi = 2\pi f$ or $f = \dfrac{0.02\pi}{2\pi} = 0.01 = \dfrac{1}{100} = \dfrac{k}{N}$

Here f is expressed as a ratio of two integers with $k = 1$ and $N = 100$. So it is rational. Hence the given signal is periodic with fundamental period $N = 100$.

(d) $\sin \frac{2\pi n}{3} + \cos \frac{2\pi n}{5}$

Solution

Given $$x(n) = \sin \frac{2\pi n}{3} + \cos \frac{2\pi n}{5}$$

Comparing it with $$x(n) = \sin 2\pi f_1 n + \cos 2\pi f_2 n$$

we have $$2\pi f_1 = \frac{2\pi}{3} \quad \text{or} \quad f_1 = \frac{1}{3} = \frac{k_1}{N_1}$$

$\therefore \quad N_1 = 3$

and $$2\pi f_2 n = \frac{2\pi}{5} \quad \text{or} \quad f_2 = \frac{1}{5}$$

$\therefore \quad N_2 = 5$

Since $\frac{N_1}{N_2} = \frac{3}{5}$ is a ratio of two integers, the sequence $x(n)$ is periodic. The period of $x(n)$ is the LCM of N_1 and N_2. Here LCM of $N_1 = 3$ and $N_2 = 5$ is 15. Therefore, the given sequence is periodic with fundamental period $N = 15$.

(g) $e^{j(\pi/2)n}$

Solution

Given $x(n) = e^{j(\pi/2)n}$

Comparing it with $x(n) = e^{j2\pi fn}$

we have $2\pi f = \dfrac{\pi}{2}$ or $f = \dfrac{1}{4} = \dfrac{k}{N}$

which is rational.

Hence, the given signal $x(n)$ is periodic with fundamental period $N = 4$.

Thank You