Digital Signal Processing Sheet 1

Question 1: **Find The Following Summations:**

1. $\sum_{n=-\infty}^{\infty} n^2 \delta(n+4)$

2. $\sum_{n=0}^{\infty} \delta(n+1)\, 4^n$

3. $\sum_{n=-\infty}^{\infty} \delta(n-2)\, e^{n^2}$

(c) Given $\sum_{n=-\infty}^{\infty} n^2 \delta(n+4)$

We know that $\delta(n+4) = \begin{cases} 1 & \text{for } n = -4 \\ 0 & \text{elsewhere} \end{cases}$

$\therefore \quad \sum_{n=-\infty}^{\infty} n^2 \delta(n+4) = [n^2]_{n=-4} = 16$

(d) Given $\sum_{n=-\infty}^{\infty} \delta(n-2)\, e^{n^2}$

We know that $\delta(n-2) = \begin{cases} 1 & \text{for } n = 2 \\ 0 & \text{elsewhere} \end{cases}$

$\therefore \quad \sum_{n=-\infty}^{\infty} \delta(n-2)\, e^{n^2} = [e^{n^2}]_{n=2} = e^{2^2} = e^4$

(e) Given $\sum_{n=0}^{\infty} \delta(n+1)\, 4^n$

We know that $\delta(n+1) = \begin{cases} 1 & \text{for } n = -1 \\ 0 & \text{for } n \neq -1 \end{cases}$

$\therefore \quad \sum_{n=0}^{\infty} \delta(n+1)\, 4^n = 0$

Digital Signal Processing Sheet 1

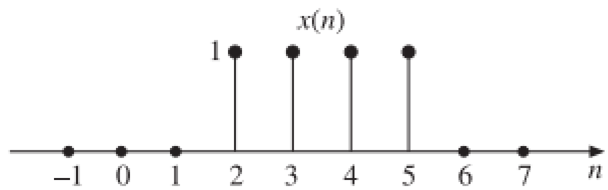
Question 2: **Sketch The Following Signals:**

① $u(n+1)\,u(-n+4)$

② $x(n) = u(n+3) - u(n-1)$

Digital Signal Processing Sheet 1

Question 3: **Express The Signal as the sum of singular functions:**



(b) The signal shown in Figure 1.17(b) is:

$$x(n) = \delta(n-2) + \delta(n-3) + \delta(n-4) + \delta(n-5)$$

$$x(n) = \begin{cases} 0 & \text{for } n \leq 1 \\ 1 & \text{for } 2 \leq n \leq 5 \\ 0 & \text{for } n \geq 6 \end{cases}$$

$$\therefore \quad x(n) = u(n-2) - u(n-6)$$

Digital Signal Processing Sheet 1

Question 4:

Let $x(t)$ be the complex exponential signal, $x(t) = e^{j\omega_0 t}$ with radian frequency ω_0 and fundamental period $T = 2\pi/\omega_0$. Consider the discrete-time sequence $x(n)$ obtained by the uniform sampling of $x(t)$ with sampling interval T_s, i.e.,

$$x(n) = x(nT_s) = e^{jn\omega_0 T_s}$$

Show that $x(n)$ is periodic if the ratio of the sampling interval T_s to the fundamental period T of $x(t)$, i.e., T_s/T is a rational number.

Solution: Given

$$x(t) = e^{j\omega_0 t}$$

$$x(n) = x(nT_s) = e^{jn\omega_0 T_s}$$

where T_s is the sampling interval. Then,

$$\text{Fundamental period } T = \frac{2\pi}{\omega_0}$$

$$\omega_0 = \frac{2\pi}{T}$$

If $x(n)$ is periodic with the fundamental period N, then

$$x(n + N) = x(n)$$

i.e. $$e^{j(n+N)\omega_0 T_s} = e^{jn\omega_0 T_s}$$

i.e. $$e^{jn\omega_0 T_s} \; e^{jN\omega_0 T_s} = e^{jn\omega_0 T_s}$$

This is true only if $$e^{jN\omega_0 T_s} = 1$$

i.e. $$N\omega_0 T_s = 2\pi m$$

where m is a positive integer.

i.e. $$N\frac{2\pi}{T}T_s = 2\pi m$$

$\therefore$ $$\frac{T_s}{T} = \frac{m}{N} = \text{Rational number}$$

This shows that $x(n)$ is periodic if the ratio of the sampling interval to the fundamental period of $x(t)$, T_s/T is a rational number.

Digital Signal Processing Sheet 1

Question 5: **Determine whether the following discrete-time signals are periodic or not. If periodic, determine the fundamental period:**

1. $\sin(5\pi n)$
2. $\cos 4n$
3. $\cos\left(\frac{n}{6}\right)\cos\left(\frac{n\pi}{6}\right)$
4. $\cos\left(\frac{\pi}{2}+0.3n\right)$
5. $1+e^{j2\pi n/3}-e^{j4\pi n/7}$

(b) Given $\quad x(n) = \sin(5\pi n)$

Comparing it with $\quad x(n) = \sin(2\pi f n)$

we have $\quad 2\pi f = 5\pi \quad$ or $\quad f = \frac{5}{2} = \frac{k}{N}$

Here f is a ratio of two integers with $k = 5$ and $N = 2$. Hence it is rational. Hence the given signal is periodic with fundamental period $N = 2$.

(c) Given $\quad x(n) = \cos 4n$

Comparing it with $\quad x(n) = \cos 2\pi f n$

we have $\quad 2\pi f = 4 \quad$ or $\quad f = \frac{2}{\pi}$

Since $f = (2/\pi)$ is not a rational number, $x(n)$ is not periodic.

(e) Given $\quad x(n) = \cos\left(\frac{n}{6}\right)\cos\left(\frac{n\pi}{6}\right)$

Comparing it with $\quad x(n) = \cos(2\pi f_1 n)\cos(2\pi f_2 n)$

we have $\quad 2\pi f_1 n = \frac{n}{6} \quad$ or $\quad f_1 = \frac{1}{12\pi}$

which is not rational.

And $\quad 2\pi f_2 n = \frac{n\pi}{6} \quad$ or $\quad f_2 = \frac{1}{12}$

which is rational.

Thus, $\cos(n/6)$ is non-periodic and $\cos(n\pi/6)$ is periodic. $x(n)$ is non-periodic because it is the product of periodic and non-periodic signals.

Digital Signal Processing Sheet 1

(f) Given $$x(n) = \cos\left(\frac{\pi}{2} + 0.3n\right)$$

Comparing it with $$x(n) = \cos(2\pi f n + \theta)$$

we have $2\pi f n = 0.3n$ and phase shift $\theta = \frac{\pi}{2}$

$$\therefore \quad f = \frac{0.3}{2\pi} = \frac{3}{20\pi}$$

which is not rational.
Hence, the signal $x(n)$ is non-periodic.

(h) Given $$x(n) = 1 + e^{j2\pi n/3} - e^{j4\pi n/7}$$

Let $$x(n) = 1 + e^{j2\pi n/3} - e^{j4\pi n/7} = x_1(n) + x_2(n) + x_3(n)$$

where $x_1(n) = 1$, $x_2(n) = e^{j2\pi n/3}$ and $x_3(n) = e^{j4\pi n/7}$

$x_1(n) = 1$ is a d.c. signal with an arbitrary period $N_1 = 1$

$$x_2(n) = e^{j2\pi n/3} = e^{j2\pi f_2 n}$$

$$\therefore \quad \frac{2\pi n}{3} = 2\pi f_2 n \quad \text{or} \quad f_2 = \frac{1}{3} = \frac{k_2}{N_2} \quad \text{where } N_2 = 3$$

Hence $x_2(n)$ is periodic with period $N_2 = 3$.

$$x_3(n) = e^{j4\pi n/7} = e^{j2\pi f_3 n}$$

$$\therefore \quad \frac{4\pi n}{7} = 2\pi f_3 n \quad \text{or} \quad f_3 = \frac{2}{7} = \frac{k_3}{N_3} \quad \text{where } N_3 = \frac{7}{2}$$

Now, $$\frac{N_1}{N_2} = \frac{1}{3} = \text{Rational number}$$

$$\frac{N_1}{N_3} = \frac{1}{7/2} = \frac{2}{7} = \text{Rational number}$$

The LCM of $$N_1, N_2, N_3 = \frac{7}{2} \times 3 = \frac{21}{2}$$

$\therefore$ The given signal $x(n)$ is periodic with fundamental period $N = 10.5$.