

Real numbers and complex numbers are examples of fields. That is, they are each a set closed under addition and multiplication that satisfy

Field axioms

Addition

(A1) $x + y = y + x$.

(A2) $(x + y) + z = x + (y + z)$.

(A3) There exists an element 0 such that for all x , $x + 0 = x$.

(A4) For all x , there exists an element $-x$ such that $x + (-x) = 0$.

Multiplication

(M1) $xy = yx$.

(M2) $(xy)z = x(yz)$.

(M3) There exists an element $1 \neq 0$ such that for all x , $1 \cdot x = x$.

(M4) If $x \neq 0$, then there exists an element x^{-1} such that $x \cdot x^{-1} = 1$.

Distributive law

(D) $x(y + z) = xy + xz$.

Example: \mathbb{F}_2

Here is an example of a field \mathbb{F}_2 with exactly two elements (which must be 0 and 1, of course!), and addition and multiplication mod 2 (The addition and multiplication tables are as follows).

Addition:

+	0	1
0	0	1
1	1	0

Multiplication:

·	0	1
0	0	0
1	0	1