JACKAL's Model Predictive Control Model

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1 State Parameters and Control Vectors

1.1 State Parameter Vector

$$\mathbf{x}_t = \begin{bmatrix} x_t \\ y_t \\ \theta_t \\ v_t \\ cte_t \\ e\theta_t \end{bmatrix}$$

where:

- x_t and y_t represent the vehicle's position coordinates.
- θ_t is the vehicle's orientation or heading angle.
- v_t is the vehicle's velocity.
- cte_t (Cross Track Error) measures the lateral distance between the vehicle and the reference trajectory.
- $e\theta_t$ (Error in Theta) represents the orientation error relative to the trajectory's tangent at the vehicle's current position.

1.2 Control Vector

$$\mathbf{u}_t = \begin{bmatrix} \omega_t \\ a_t \end{bmatrix}$$

where:

- ω_t is the angular velocity, which affects the vehicle's heading.
- a_t is the acceleration, controlling the speed increase or decrease.

These control inputs are used to predict and optimize the vehicle's trajectory over the prediction horizon, ensuring adherence to the desired path while maintaining dynamic stability and safety.

2 Total Cost Function

The total cost function J is defined as:

$$J = \sum_{i=0}^{N-1} \left(w_{cte}(cte_i - ref_{cte})^2 + w_\theta(e\theta_i - ref_\theta)^2 + w_v(v_i - ref_v)^2 \right)$$

$$+ \sum_{i=0}^{N-2} \left(w_{angvel} angvel_i^2 + w_{accel} accel_i^2 \right)$$

$$+ \sum_{i=0}^{N-3} \left(w_{angvel\_d}(angvel_{i+1} - angvel_i)^2 + w_{accel\_d}(accel_{i+1} - accel_i)^2 \right)$$

where:

- cte_i, $e\theta_i$, and v_i represent the cross-track error, heading error, and velocity at step i.
- ref_{cte}, ref_θ, and ref_v are the target values for these states.
- w_{cte}, w_θ, w_v, w_{angvel}, w_{accel}, w_{angvel_d}, and w_{accel_d} are weights indicating the importance of each component.
- $angvel_i$ and $accel_i$ are the control inputs at each step for angular velocity and acceleration.

3 Constraint Formulations

3.1 Initial State Constraints

$$\begin{aligned}
x[1] &= x_{start}, \\
y[1] &= y_{start}, \\
\theta[1] &= \theta_{start}, \\
v[1] &= v_{start}, \\
cte[1] &= cte_{start}, \\
e\theta[1] &= e\theta_{start}.
\end{aligned}$$

3.2 Vehicle Dynamics and Control Constraints

$$\begin{aligned}
x_{t+1} &= x_t + v_t \cos(\theta_t) \cdot dt, \\
y_{t+1} &= y_t + v_t \sin(\theta_t) \cdot dt, \\
\theta_{t+1} &= \theta_t + \omega_t \cdot dt, \\
v_{t+1} &= v_t + a_t \cdot dt, \\
cte_{t+1} &= cte_t + (v_t \sin(e\theta_t) \cdot dt), \\
e\theta_{t+1} &= e\theta_t - (\theta_t - \mathrm{atan}(f'(x_t))) + \omega_t \cdot dt.
\end{aligned}$$

where $f'(x_t)$ represents the derivative of the polynomial trajectory fit evaluated at x_t.

3.3 Actuator Limitations

To ensure smooth control actions and avoid abrupt vehicle behavior, actuator limitations are imposed:

$$|\omega_t| \leq \omega_{max},$$
$$|a_t| \leq a_{max}.$$

3.4 Actuator Change Constraints

Minimizing the change in control inputs to promote smoother transitions:

$$|\Delta\omega_t| \leq \Delta\omega_{max},$$
$$|\Delta a_t| \leq \Delta a_{max}.$$