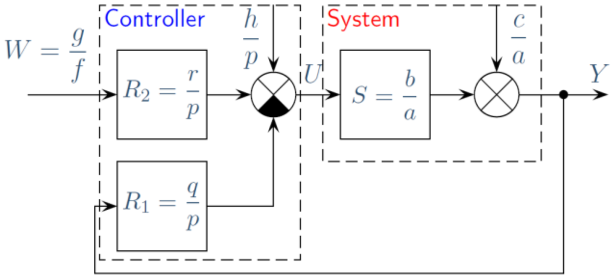
RAL Ex 06

Ex. 1: Time optimal control with 2DOF



Design the Time-optimal feedback control for the system $R_1 = \frac{q_r}{p_r}$ and $R_2 = \frac{r_r}{p_r}$ for system $S = \frac{b}{a} = \frac{0.2d+0.1d^2}{1-0.5d+0.1d^2}$. Required value: $W = \frac{g_W}{f_W} = 1 + d + d^2 + d^3 + d^4$.

Solve two diophantine equations so that p_r, q_r, s_r and r_r are polynomials of the lowest possible degree.

$$\text{Generally:} \quad ax + by = 1$$

$$ap_r + bq_r = 1$$

$$f_W s_r + br_r = 1$$

Solvability condition: $f_W^0 = 1$

$$\frac{a}{f_W} = \frac{a^0 * GCD}{f_W^0 * GCD} \Rightarrow f_W^0 = \frac{f_W}{GCD}$$

$$f_W^0 = 1$$

Controller design: $R_1 = \frac{q_r}{p_r}$

$$\begin{bmatrix} a & b \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 - \frac{1}{2}d + \frac{1}{10}d^2 & \frac{1}{5}\mathrm{d} + \frac{1}{10}\mathrm{d}^2 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \sim \begin{pmatrix} 1.\mathrm{c} * 10; \\ 2.\mathrm{c} * 10; \end{pmatrix} \sim \begin{bmatrix} 10 - 5d + d^2 & 2\mathrm{d} + \mathrm{d}^2 \\ 10 & 0 \\ 0 & 10 \end{bmatrix} \sim$$

$$\sim (1.\mathrm{c} - 2.\mathrm{c}) \sim \begin{bmatrix} 10 - 7d & 2\mathrm{d} + \mathrm{d}^2 \\ 10 & 0 \\ -10 & 10 \end{bmatrix} \sim (2.\mathrm{c} * 7 + 1.\mathrm{s} * \mathrm{d}) \sim \begin{bmatrix} 10 - 7d & 24\mathrm{d} \\ 10 & 10d \\ -10 & 70 - 10d \end{bmatrix} \sim (2.\mathrm{c}/2)$$

$$\sim \begin{bmatrix} 10 - 7d & 12\mathrm{d} \\ 10 & 5d \\ -10 & 35 - 5d \end{bmatrix} \sim (1.\mathrm{c} * 12 + 2.\mathrm{c} * 7) \sim \begin{bmatrix} 120 & 12\mathrm{d} \\ 120 + 35d & 5d \\ 125 - 35d & 35 - 5d \end{bmatrix} \sim (2.\mathrm{c} * 10 - 1.\mathrm{c} * \mathrm{d}) \sim$$

$$\sim \begin{bmatrix} 120 & 0 \\ 120 + 35d & -35\mathrm{d}^2 - 70d \\ 125 - 35d & 35\mathrm{d}^2 - 175d + 350 \end{bmatrix} \sim \begin{pmatrix} 1.\mathrm{c}/120; \\ 2.\mathrm{c}/35; \end{pmatrix} \sim \begin{bmatrix} 1 & 0 \\ 1 + \frac{7}{24}d & -d^2 - 2d \\ \frac{25}{24} - \frac{7}{24}d & d^2 - 5d + 10 \end{bmatrix} \sim \begin{bmatrix} g_{NSD} & 0 \\ p & \mathrm{r} \\ q & s \end{bmatrix}$$

$$p_1 = 1 + \frac{7}{24}d \quad r_1 = -2d - d^2$$

$$q_1 = \frac{25}{24} - \frac{7}{24}d \quad s_1 = 10 - 5d + d^2$$

Substitution: $\quad p_r = x = p_1 \frac{c}{g_{NSD}} + r_1 h_1 \qquad q_r = y = q_1 \frac{c}{g_{NSD}} + s_1 h_1$

$$p_r = 1 + \frac{7}{24}d + (-2d - d^2)h_1$$

$$q_r = \frac{25}{24} - \frac{7}{24}d + (10 - 5d + d^2)h_1$$

for $h_1 = 0$

$$p_r = 1 + \frac{7}{24}d \qquad q_r = \frac{25}{24} - \frac{7}{24}d$$

Controller design: $R_2 = \frac{r_r}{p_r}$

$$f_W s_r + b r_r = 1$$

$$\begin{bmatrix} f_w & b \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & \frac{1}{5}\mathrm{d} + \frac{1}{10}\mathrm{d}^2 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \sim \left(2.\mathrm{c} - 1.\mathrm{c} * (\frac{1}{5}\mathrm{d} + \frac{1}{10}\mathrm{d}^2)\right) \sim \begin{bmatrix} 1 & 0 \\ 1 & -\frac{1}{5}\mathrm{d} - \frac{1}{10}\mathrm{d}^2 \\ 0 & 1 \end{bmatrix} \sim$$

$$\sim (2.\mathrm{c} * 5) \sim \begin{bmatrix} 1 & 0 \\ 1 & -\mathrm{d} - \frac{1}{2}\mathrm{d}^2 \\ 0 & 5 \end{bmatrix} \sim \begin{bmatrix} g_{NSD} & 0 \\ p & \mathrm{r} \\ q & s \end{bmatrix}$$

$$p_2 = 1 \quad r_2 = -\frac{1}{2}d^2 - d$$
$$\mathrm{q}_2 = 0 \qquad \mathrm{s}_2 = 5$$

Substitution: $\quad s_r = x = p_2 \frac{c}{g_{NSD}} + r_2 h_2 \qquad r_r = y = q_2 \frac{c}{g_{NSD}} + s_2 h_2$

$$s_r = 1 + \left(-\frac{1}{2}d^2 - d\right)h_2 \qquad r_r = 0 + 5h_2$$

Polynomial r_r must not be zero.

$$R_1 = \frac{q_r}{p_r} = \frac{\frac{25}{24} - \frac{7}{24}d}{\frac{7}{24}d + 1} \qquad R_2 = \frac{r_r}{p_r} = \frac{5h_2}{\frac{7}{24}d + 1}$$

Closed-loop transfer function:

$$F_w(d) = \frac{r_r b}{a p_r + b q_r} = \frac{5h_2 * \left(0.2d + 0.1d^2\right)}{\left(1 - \frac{1}{2}d + \frac{1}{10}d^2\right)\left(\frac{7}{24}d + 1\right) + \left(\frac{1}{5}d + \frac{1}{10}d^2\right)\left(\frac{25}{24} - \frac{7}{24}d\right)}$$
$$= \frac{5h_2 * \left(0.2d + 0.1d^2\right)}{1}$$

Transfer function in steady-state can be equal to one (can use the final value theorem), we choose $h_2 = \frac{2}{3}$ in numerator $r_r b = h_2 d + \frac{1}{2} h_2 d^2 = \frac{2}{3} d + \frac{1}{3} d^2$

Controller R_2: $R_2 = \frac{r_r}{p_r} = \frac{\frac{10}{3}}{\frac{7}{24}d+1}$

Ex. 2: Stable time optimal control with 2DOF

Design the Stable time-optimal feedback control for the system $R_1 = \frac{q_r}{p_r}$ and $R_2 = \frac{r_r}{p_r}$ for system $S = \frac{b}{a} = \frac{0.2d+0.1d^2}{1-0.5d+0.1d^2}$. Required value: $W = \frac{g_W}{f_W} = 1 + d + d^2 + d^3 + d^4$

$$\text{Generally:} \quad ax + by = 1$$

$$ap_r + bq_r = b^+$$

$$a\frac{p_r}{b^+} + b^- q_r = 1$$

$$p_r = xb^+ \qquad q_r = q_r$$

$$ax + b^- q_r = 1$$

$$f_W s_r + b^- r_r = 1$$

$$b = b^+ b^- = 0.2d + 0.1d^2$$

$$\text{Roots: } 0.2d + 0.1d^2 = d(0.2 + 0.1d)$$

$$b_1 = 0, b_2 - 2$$

$$b^+ = 0.1d + 0.2$$

$$b^- = d$$

Controller design: $R_1 = \frac{q_r}{p_r}$

$$\begin{bmatrix} a & b^- \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 - 0.5d + 0.1d^2 & \mathrm{d} \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \sim (1.\mathrm{c} - 2.\mathrm{c} * 0.1\mathrm{d}) \sim \begin{bmatrix} 1 - 0.5d & \mathrm{d} \\ 1 & 0 \\ -0.1d & 1 \end{bmatrix} \sim (1.\mathrm{c} + 2.\mathrm{c} * 0.5) \sim$$

$$\sim \begin{bmatrix} 1 & \mathrm{d} \\ 1 & 0 \\ -0.1d + 0.5 & 1 \end{bmatrix} \sim (2.\mathrm{c} - 1.\mathrm{c} * \mathrm{d}) \sim \begin{bmatrix} 1 & 0 \\ 1 & -d \\ -0.1d + 0.5 & 1 - 0.5d + 0.1d^2 \end{bmatrix} \sim \begin{bmatrix} g_{NSD} & 0 \\ p & \mathrm{r} \\ q & s \end{bmatrix}$$

$$p_1 = 1 \qquad r_1 = -d$$

$$q_1 = 0.5 - 0.1d \qquad s_1 = 1 - 0.5d + 0.1d^2$$

Substitution: $x = p_1 \frac{c}{g_{NSD}} + r_1 h_1$ $\qquad q_r = y = q_1 \frac{c}{g_{NSD}} + s_1 h_1$

$$x = 1 + (-d)h_1$$

$$q_r = 0.5 - 0.1d + (1 - 0.5d + 0.1d^2)h_1$$

for $h_1 = 0$

$$x = 1$$

$$p_r = xb^+ = 0.1d + 0.2$$

$$q_r = 0.5 - 0.1d$$

Controller design: $R_2 = \frac{r_r}{p_r}$

$$f_W s_r + b^- r_r = 1$$

$$\begin{bmatrix} f_W & b^- \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & \mathrm{d} \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \sim (2.\mathrm{c} - 1.\mathrm{c} * \mathrm{d}) \sim \begin{bmatrix} 1 & 0 \\ 1 & -d \\ 0 & 1 \end{bmatrix} \sim \begin{bmatrix} g_{NSD} & 0 \\ p & \mathrm{r} \\ q & s \end{bmatrix}$$

$$p_2 = 1 \quad r_2 = -d$$

$$q_2 = 0 \quad s_2 = 1$$

$$s_r = x = p_2 \frac{c}{g_{NSD}} + r_2 h_2$$

$$r_r = y = q_2 \frac{c}{g_{NSD}} + s_2 h_2$$

$$s_r = 1 + (-d)h_2$$

$$r_r = 0 + h_2$$

Polynomial r_r must not be zero.

$$R_1 = \frac{q_r}{p_r} = \frac{0.5 - 0.1d}{0.2 + 0.1d} \qquad R_2 = \frac{r_r}{p_r} = \frac{h_2}{0.2 + 0.1d}$$

Closed-loop transfer function:

$$F_w(d) = \frac{r_r b}{a p_r + b q_r} = \frac{h_2 * \left(0.2d + 0.1d^2\right)}{\left(1 - 0.5d + 0.1d^2\right)(0.2 + 0.1d) + \left(0.2d + 0.1d^2\right)(0.5 - 0.1d)}$$
$$= \frac{h_2 * \left(0.2d + 0.1d^2\right)}{(0.2 + 0.1d)}$$

Transfer function in steady-state can be equal to one (can use the final value theorem), we choose $h_2 = 1$ in numerator $r_r b = 0.2h_2 d + 0.1h_2 d^2 = 0.2d + 0.1d^2$

Controller R_2: $\quad R_2 = \frac{r_r}{p_r} = \frac{1}{0.2 + 0.1d}$