RAL Ex 04

Ex. 1 Modal control (pole placement)

Design the controller $R = \frac{q_R}{p_R}$ for the system $S = \frac{b}{a} = \frac{0.2z+0.1}{z^2-0.5z+0.1}$ such that the characteristic polynomial of the closed loop system is equal to $\Delta = z^i$. The position of the poles can be set.

$$ap_R + bq_R = \Delta$$

Generally: $ax + by = \Delta$

The degree of the characteristic polynomial must be sufficiently high to find the solution leading to causal controller.

Solvability condition: $\deg(\Delta) \geq 2\deg(a) - 1$ (holds only for positive powers of z)

$$i = 2\deg(a) - 1 = 4 - 1 = 3$$

$$(z^2 - 0.5z + 0.1)p + (0.2z + 0.1)q = z^3$$

reduce higher polynomeal =>

$$\begin{bmatrix} a & b \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \sim \begin{bmatrix} z^2 - \frac{1}{2}z + \frac{1}{10} & \frac{1}{5}z + \frac{1}{10} \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \sim \begin{bmatrix} z^2 - \frac{1}{2}z + \frac{1}{10} & z + \frac{1}{2} \\ 1 & 0 \\ 0 & 5 \end{bmatrix} {}^{*z}_{*z} \sim \begin{bmatrix} -z + \frac{1}{10} & z + \frac{1}{2} \\ 1 & 0 \\ -5z & 5 \end{bmatrix} {}^{+}_{+} \sim \begin{bmatrix} \frac{3}{5} & z + \frac{1}{2} \\ 1 & 0 \\ -5z + 5 & 5 \end{bmatrix} \sim$$

$* 5/3$

$$\sim \begin{bmatrix} 1 & z + \frac{1}{2} \\ \frac{5}{3} & 0 \\ -\frac{25}{3}z + \frac{25}{3} & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{5}{3} & -\frac{5}{3}z \\ -\frac{25}{3}z + \frac{25}{3} & 5 + \frac{25}{3}z^2 - \frac{25}{3}z \end{bmatrix} \sim \begin{bmatrix} 1 & 0 \\ \frac{5}{3} & -\frac{10}{3}z - \frac{5}{3} \\ -\frac{25}{3}z + \frac{25}{3} & \frac{5}{3} + \frac{50}{3}z^2 - \frac{25}{3}z \end{bmatrix} \sim \begin{bmatrix} g_{NSD} & 0 \\ p & r \\ q & s \end{bmatrix}$$

$$p_R = x = p\frac{c}{g_{NSD}} + rh$$

$$q_R = y = q\frac{c}{g_{NSD}} + sh$$

$$p_R = x = \frac{5}{3}z^3 + \left(-\frac{10}{3}z - \frac{5}{3}\right)h$$

$$q_R = y = \left(-\frac{25}{3}z + \frac{25}{3}\right)z^3 + \left(\frac{5}{3} + \frac{50}{3}z^2 - \frac{25}{3}z\right)h$$

In our controller, p_R is x and q_R is y, h is arbitrary polynomial.

Appropriate selection of the polynomial h enables us to reduce the degree of the controller. This selection of h cancelles the high powers in the polynomial q_R.

$$h = h_2z^2 + h_1z + h_0$$

After the substitution h

$$p_R = \left(\frac{5}{3} - \frac{10}{3}h_2\right)z^3 + \left(-\frac{5}{3}h_2 - \frac{10}{3}h_1\right)z^2 + \left(-\frac{5}{3}h_1 - \frac{10}{3}h_0\right)z - \frac{5}{3}h_0$$

$$q_R = \left(-\frac{25}{3} + \frac{50}{3}h_2\right)z^4 + \left(\frac{25}{3} - \frac{25}{3}h_2 + \frac{50}{3}h_1\right)z^3 + \left(\frac{5}{3}h_2 - \frac{25}{3}h_1 + \frac{50}{3}h_0\right)z^2 + \left(\frac{5}{3}h_1 - \frac{25}{3}h_0\right)z + \frac{5}{3}h_0$$

We search for the simplest controller. It is the one with lowest degree, i.e. lowest degree of the polynomial q_R. Successively we choose:

$$q4 = 0 \rightarrow h_2 = 0.5 \rightarrow p3 = 0$$

$$q3 = 0 \rightarrow h_1 = -0.25 \rightarrow p2 = 0$$

$$q2 = 0 \rightarrow h_0 = -\frac{7}{40} \rightarrow p1 = 1$$

$$q1 = \frac{25}{24}$$

Witch gives: $q_R = \frac{25}{24}z - \frac{7}{24}$ $\quad p_R = z + \frac{7}{24}$

The transfer function of the controller is $R = \frac{\frac{25}{24}z - \frac{7}{24}}{z + \frac{7}{24}}$

Test of correctness:

Closed loop transfer function is: $F_w = \frac{bq_R}{ap_R + bq_R} = \frac{\frac{5}{24}z^2 + \frac{11}{240}z - \frac{7}{240}}{z^3}$

Characteristic polynomial is as it was desired.

Ex. 2 EMMP – Exact Model Matching Problem

Design the feedback controller $R = \frac{q_R}{p_R}$ for the system $S = \frac{b}{a}$ such that the closed loop system behaves like a model with the transfer function $M = \frac{g_M}{f_M}$.

$$S = \frac{0.2z^{-1} + 0.1z^{-2}}{1 - 0.5z^{-1} + 0.1z^{-2}} \qquad M = \frac{0.5z^{-1} + 0.5z^{-2}}{1} = \frac{g_M}{f_M}$$

$$\frac{Y(z)}{W(z)} = \frac{SR}{1 + SR} = \frac{bq_R}{ap_R + bq_R} = \frac{g_M}{f_M} \rightarrow g_M^0(ap_R + bq_R) = f_M b^0 q_R$$

$$p_R a g_M^0 + g_M^0 b q_R - f_M b^0 q_R = 0$$ all uknown to left side

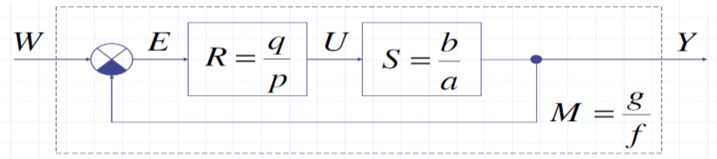
$$p_R a g_M^0 - q_R(f_M - g_M)b^0 = 0$$

$$p_R = b^0(f_M - g_M) * x_{lib}$$

$$q_R = a g_M^0 * x_{lib}$$ h - ve cvikach

x_{lib}-arbitrary polinomial

$$R=\frac{q_R}{p_R}=\frac{ag_M^0}{b^0(f_M-g_M)}=\frac{(1-0.5d+0.1d^2)(0.5d+0.5d^2)}{(0.2d+0.1d^2)(1-0.5d-0.5d^2)}$$



The goal is to find polynomials p, q such that the following equality:

$$\frac{Y(z)}{W(z)}=\frac{SR}{1+SR}=\frac{bq_R}{ap_R+bq_R}\equiv\frac{g_M}{f_M}=M$$

Test of correctness: $\frac{SR}{1+SR}=M$

$$F_W=\frac{\frac{0.2d+0.1d^2}{1-0.5d+0.1d^2}*\frac{(1-0.5d+0.1d^2)(0.5d+0.5d^2)}{(0.2d+0.1d^2)(1-0.5d-0.5d^2)}}{1+\frac{0.2d+0.1d^2}{1-0.5d+0.1d^2}*\frac{(1-0.5d+0.1d^2)(0.5d+0.5d^2)}{(0.2d+0.1d^2)(1-0.5d-0.5d^2)}}=\frac{\frac{0.5d+0.5d^2}{1-0.5d-0.5d^2}}{1+\frac{0.5d+0.5d^2}{1-0.5d-0.5d^2}}$$
$$=\frac{0.5d+0.5d^2}{1-0.5d-0.5d^2+0.5d+0.5d^2}=0.5d+0.5d^2=M$$

Ex. 3 Matlab – Modal control (pole placement)

Design the controller $R=\frac{q_R}{p_R}$ for the system $S=\frac{0.2z+0.1}{z^2-0.5z+0.1}$. Find the solution in both, positive and negative powers of z.

$$\Delta=z$$

$$\Delta=z^2$$

$$\Delta=\mathrm{z}^3$$

$$\Delta=ap_R+bq_R=h$$

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syms d
a = 1-0.5*d+0.1*d^2;
b = 0.2*d+0.1*d^2;
c = 1;
[xmin,y] = axminbyc(a,b,c,d)
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*p = xmin = (7*d)/24 + 1*

*q = y = 25/24 - (7*d)/24*

```
syms z
a = z^2-0.5*z+0.1;
b = 0.2*z+0.1;
c = z^1;
[ymin,x] = axminbyc(b,a,c,z)

c = z^1
q = ymin =(25*z)/6 + 5/6
p = x = -5/6
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- Controller is not causal
- Degree of the numerator is higher than the degree of the denominator

```
c = z^2
q = ymin =(35*z)/12
p = x = 5/12
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- Controller is not causal
- Degree of the numerator is higher than the degree of the denominator

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c = z^3
q = ymin =(25*z)/24 - 7/24
p = x = z + 7/24
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