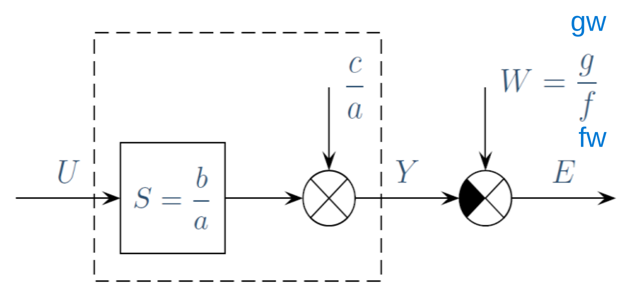
RAL

Exercise 05

Ex. 1 Time-optimal feedforward control

- Design the Time-optimal feedforward control for the system

$$S = \frac{b}{a} = \frac{0.2d + 0.1d^2}{1 - 0.5d + 0.1d^2}$$



- Required value $\quad W = \dfrac{g_W}{f_W} = \dfrac{d + d^2}{1}$
- Z transform $E(d)$ must be finite polynomial

E = W - Y

$$E = \frac{g_W}{f_W} - \frac{b}{a}U$$

$$E f_W + \frac{b f_W}{a} U = g_W$$

$$f_W x + b y = g_W$$

$$x = E \qquad\qquad y = \frac{f_W}{a}\,U$$

$$\begin{bmatrix} f_W & b \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \sim \begin{bmatrix} g_{NSD} & 0 \\ p & \mathrm{r} \\ q & s \end{bmatrix}$$

$$x = p\frac{c}{g_{NSD}} + rh$$

$$y = q\frac{c}{g_{NSD}} + sh$$

$$k_e = 1 + degE$$

Ex. 2 Time-optimal feedforward control

- Design the Time-optimal feedforward control for the system

$$S = \frac{0.2d + 0.1d^2}{1 - 1.4d + 0.4d^2} = \frac{b}{a}$$

- Required value $W = \frac{g_W}{f_W} = \frac{1}{1 - d}$

- Use the function *axminbyc()*;

- Result: U = (10/3 - 4/3*d)

Ex. 2 Time-optimal feedforward control

- Use the function *axminbyc()*;

```
syms d
a = 1-1.4*d + 0.4*d^2;
b = 0.2*d + 0.1*d^2;
fw = 1-d;
gw = 1;

[x,y] = axminbyc(fw,b,gw,d);

U = simplify(y*a/fw);
E = simplify(x);
```

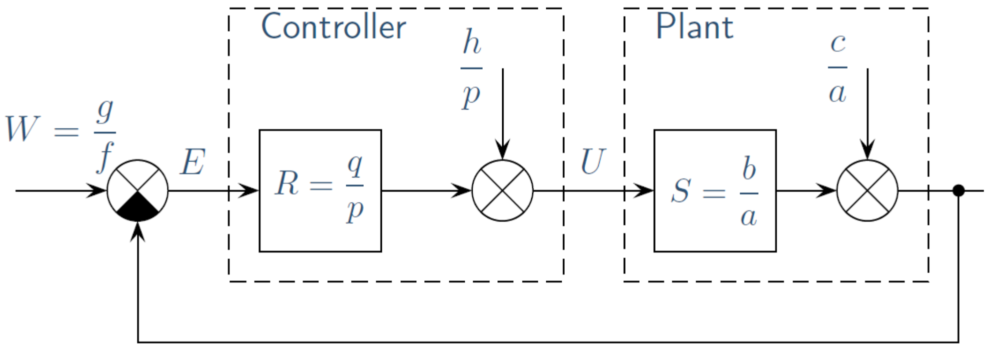
- Result: U = (10/3 - 4/3*d)

Ex. 3 Time-optimal feedback control

- Design the Time-optimal feedback control for the system

$$S = \frac{b}{a} = \frac{0.2d + 0.1d^2}{1 - 0.5d + 0.1d^2}$$

- Required value: $W = \frac{g_W}{f_W} = \frac{1}{1-d}$



- Solvability condition: $E = \frac{a^0 p_r g_W}{\Delta f_W^0} - \frac{c p_r}{\Delta} - \frac{hb}{\Delta}$

$$U = \frac{a^0 q_r g_W}{\Delta f_W^0} - \frac{c q_r}{\Delta} + \frac{ha}{\Delta}$$

Ex. 3

- Design the Time-optimal feedback control for the system

$$S = \frac{b}{a} = \frac{0.2d + 0.1d^2}{1 - 0.5d + 0.1d^2}$$

- Required value: $$W = \frac{g_W}{f_W} = \frac{1}{1 - d}$$

- To have error e and action u finite it is necessary to obtain their Z transforms E(d) and U(d) as finite polynomials
- => $f_W^0 = 1,\ \Delta = 1$ (the minimum degree of polynomials p_r and q_r).

$$\frac{a}{f_W} = \frac{a^0 * NSD}{f_W^0 * NSD} = \frac{a^0}{f_W^0} = \frac{1 - 0.5d + 0.1d^2}{1 - d}$$

$$f_W^0 \neq 1 \Rightarrow no\ solution$$

Ex. 4 Time-optimal feedback control – Matlab

- Design the Time-optimal feedback control for the system

$$S = \frac{b}{a} = \frac{0.2d + 0.1d^2}{1 - 1.4d + 0.4d^2}$$

- Required value $$W = \frac{g_W}{f_W} = \frac{1}{1 - d}$$

- Use the function *axminbyc()*;

- Result: R = –(44*d – 134)/(11*d + 27)

Ex. 4

- Design the Time-optimal feedback control for the system

$$S = \frac{0.2d + 0.1d^2}{1 - 1.4d + 0.4d^2} = \frac{b}{a}$$

- Required value: $W = \frac{g_W}{f_W} = \frac{1}{1-d}$

- Solvability condition: $f_W^0 = 1$

$$\frac{a}{f_W} = \frac{a^0 * NSD}{f_W^0 * NSD} = \frac{1 - 1.4d + 0.4d^2}{1-d} = \frac{(1-d)(1-0.4d)}{1-d} = \frac{1-0.4d}{1} = \frac{a^0}{f_W^0}$$

- Use the function *axminbyc()*;

- Result: R = –(44*d – 134)/(11*d + 27)

Ex. 4

- Use the function *axminbyc()*;

```
syms d
a = 1-1.4*d+0.4*d^2;
b = 0.2*d+0.1*d^2;
c = 1;

[p,q] = axminbyc(a,b,c,d);
R = simplify(q/p)
```

- Result: R = -(44*d - 134)/(11*d + 27)

Ex. 5 Stable 1DOF Time-optimal feedback control

- Design the Stable Time-optimal feedback control for the system $S = \frac{b}{a} = \frac{0.2d + 0.1d^2}{1 - 0.5d + 0.1d^2}$

b = b_minus * b_plus
a*p + b*q = b_plus
a/b_plus * p + b_minus * q = 1

p = x * b_plus * f0

- Required value: $W = \frac{g_W}{f_W} = \frac{1}{2-d}$

a/b_plus * x*b_plus*f0 + b_minus*q = 1
a*f0*x + b_minus*q = 1

b_minus jsou nestabilni pro z osu => mimo jednotkovou kruznici

$$ap + bq = b^+$$

$$af_W^0 x + b^- q = 1$$

$$R = \frac{q_r}{p_r} = \frac{q}{xb^+ f_W^0}$$

is b_plus not + operation

$$E = \frac{a^0 p_r g_W}{\Delta f_W^0} - \frac{cp_r}{\Delta} - \frac{hb}{\Delta}$$

$$\frac{a}{f_W} = \frac{a^0 * NSD}{f_W^0 * NSD}$$

$$b = 0.2d + 0.1d^2 = b^+ b^-$$

- R = -(d^2 - 7*d + 20)/(d^2 - 4)

Ex. 5

```
syms d
a = 1 - 0.5*d + 0.1*d^2;
b = 0.1*d^2 + 0.2*d;
bminus = d;
bplus = 0.1*(d + 2);
c = 1;
fw = 2-d;

gnsd = gcd(a,fw);
fw0 = simplify(fw/gnsd);
[x,q] = axminbyc(a*fw0,bminus,c,d);
R = simplify(q/(fw0*x*bplus));
```

- Result: R = -(d^2 - 7*d + 20)/(d^2 - 4)

Ex. 6 Matlab/Simulink

- Create the model of Time-optimal feedforward control in MATLAB/Simulink from Ex. 1 (h = 1, h = 10).

- System $$S = \frac{b}{a} = \frac{0.2d + 0.1d^2}{1 - 0.5d + 0.1d^2}$$

- Required value: $$W = \frac{g_W}{f_W} = \frac{\mathrm{d} + d^2}{1}$$

- Feedforward control $$U = y\frac{a}{f_w} = y\frac{a_0}{f_{w0}} = h\,\frac{1 - 0.5\mathrm{d} + 0.1\mathrm{d}^2}{1}$$
$$y = h$$