$\mathcal{H}_\infty$ loopshaping

RAL - Robust Control

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This exercise is about H-infinity loopshaping controller design method. First few steps are based on the classical method of shaping the open-loop frequency response. Brief summary of this method follows. (1)

Classical loopshaping method

Characteristic polynomial of the closed loop transfer function $\frac{GK}{1+GK}$ is decisive for the overall stability of the closed loop system. Characteristic polynomial itself consists only of the open loop transfer function L and +1 =¿ 1+GK (in most cases). Goals of the shaping procedure is to take advantage of this property (that overall stability and performance of the closed loop is based on the open loop properties). Open loop frequency response should have following properties:

- High gain at low frequencies (to suppress the process noise)
- Crossover frequency should be as high as possible, and should cross the 0dB axis with declination of -20dB/dek (for high performance/speed), gain plot should cross the 0dB axis before the phase plot crosses angle -180° (for stability margin)
- Small gain at high frequencies (to supress the measurement noise)

All these requirements are summed by the following picture:

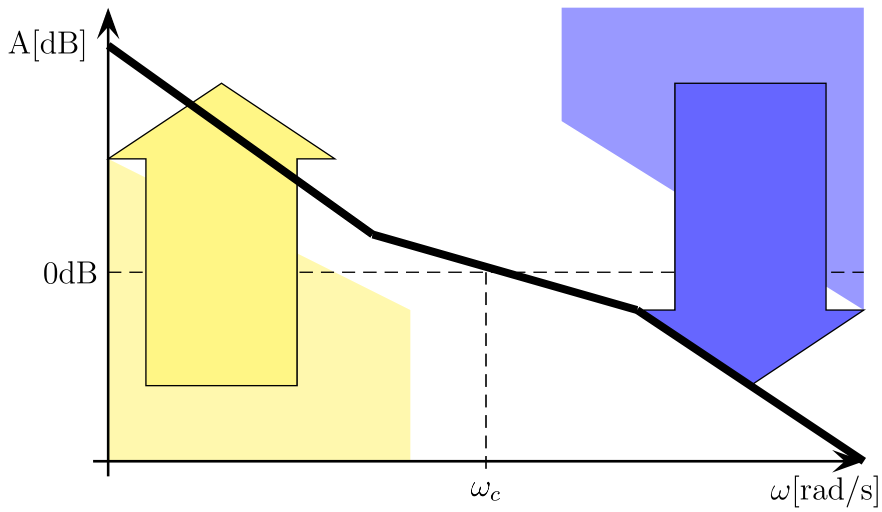

Figure 1: Open loop frequency response template

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Details about this method can be found in the article (1).

This method extends the classical loopshaping method with additional robust controller. You can use ncfsyn command to compute the stabilizing controller.

Computer exercises

Task 1

Use MATLAB to compute the stabilizing controller using H-infinity loopshaping method. Use the following system:

$$G = \frac{1}{0.0025p^2 + 0.06p + 1} \tag{1}$$

This system has damping coeficient $\zeta = 0.6$ of and time constant $T = 0.05s$.

1. Design the "pre-compensator" W1 (its basically a classical PI controller) using the sisotool(G) command, modify the controller - place pole at origin (an integrator) and zero at some initial (stable) location $W_1 = \frac{K(1+T_k p)}{p}$.
2. Move the zero and the gain of your controller to achieve open loop frequency response compliant with the classical loopshaping method
3. Robustify the plant $G_S = W_1 G W_2$, where $W_2 = 1$ using the ncfsyn command. Function will return the controller $K = W_1 K_S W_2$, to get the controller alone use INFO.Ks structure.
4. Implement the controller according to the following diagram:

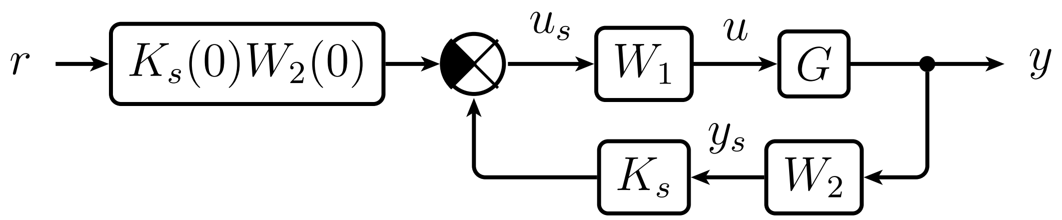

Figure 2: Controller implementation

Ks is the gain of the controller, you can use function "freqresp" with zero frequency to get the gain of the controller.

Follow all the signs in the diagram!!! (positive feedback with negative setpoint)

$$K_s(0)W_2(0) = \lim_{s \to 0} K_s(s)W_2(s) \tag{2}$$

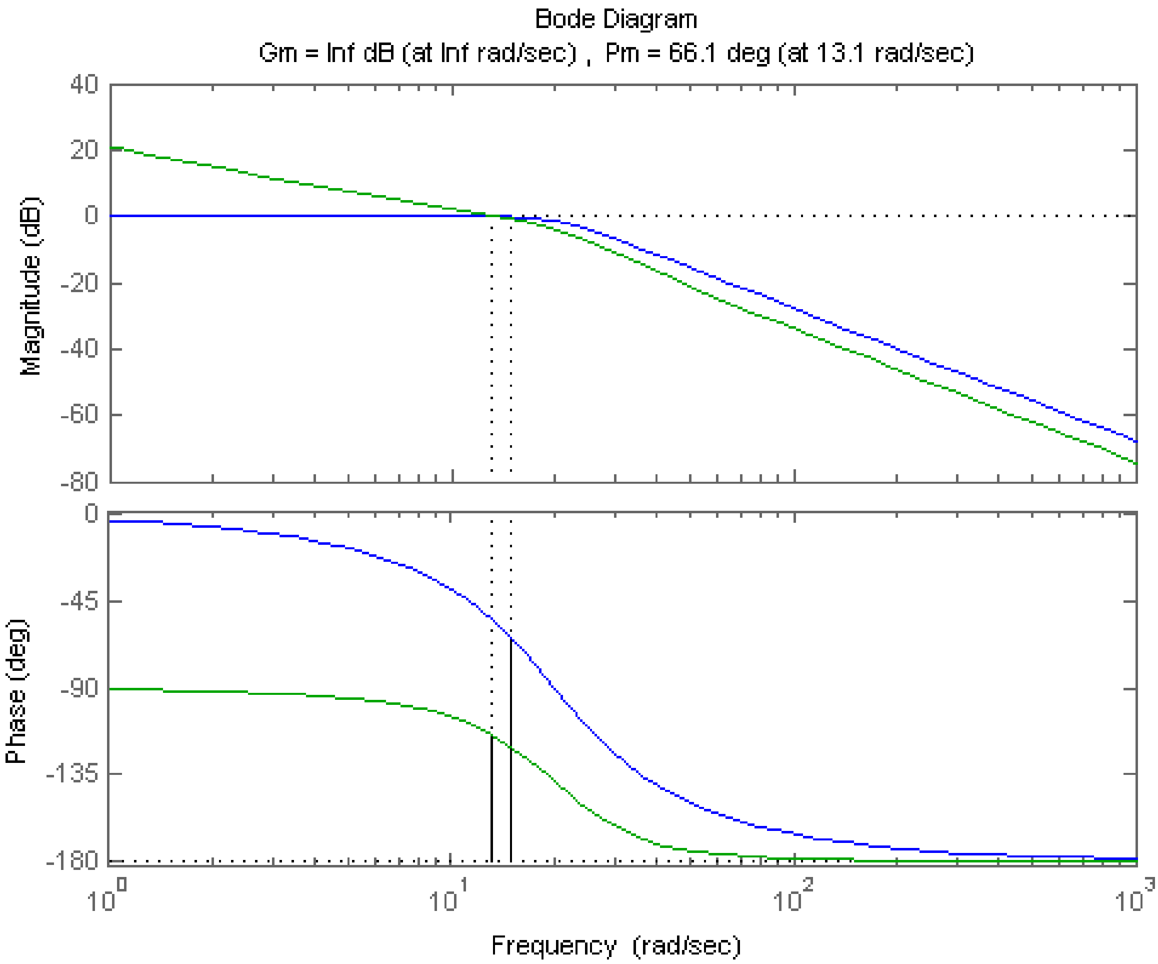

Figure 3: Frequency response of the plant (blue) and open loop frequency response (green)

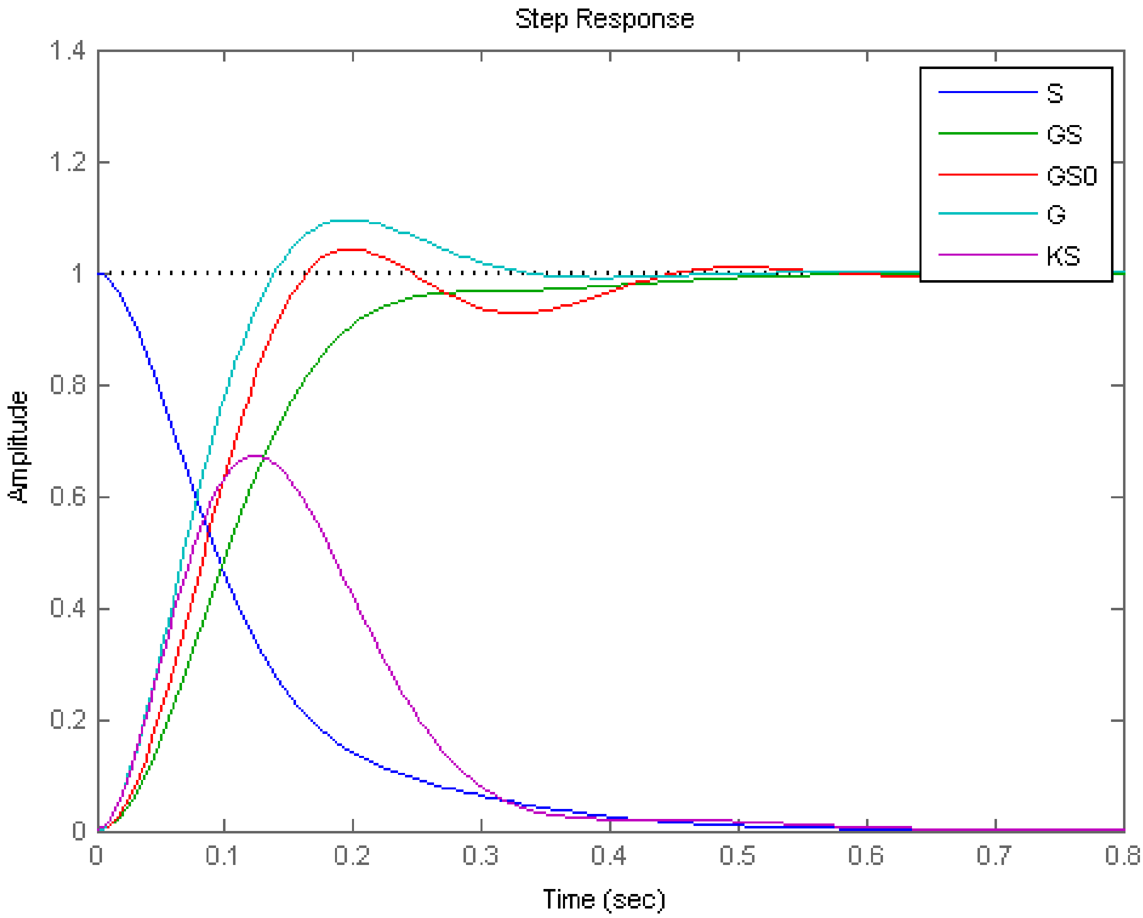

Figure 4: Step response

Task 2

Apply the same steps for the mass-dampenner-spring system from previous exercise. System definition together with the uncertainty definition is in the template.

References

[1] McFarlane, D.; Glover, K.; , "A loop-shaping design procedure using $\mathcal{H}_\infty$ synthesis ," Automatic Control, IEEE Transactions on , vol.37, no.6, pp.759-769, Jun 1992

[2] BOTURA, C. P., NETO, M. F. S., FILHO, S. A. A.: Robust Speed Control of an Induction Motor: An $\mathcal{H}_\infty$ Control Theory Approach with Field Orientation and μ-Analysis, IEEE Transactions on Power Electronics, Vol. 15, No. 5, September 2000.

[3] Gu, Da-Wei, Petkov, Petko Hr., Konstantinov, Mihail M., Robust Control Design with MATLAB. 2005.

[4] K. Glover. All optimal Hankel-norm approximations of linear multivariable systems and their $\mathcal{L}_\infty$ error bounds. International Journal of Control, 39:1115– 1193, 1984.