

RAL

Exercise 04

Ex. 1 Modal control (pole placement)

- Design the controller $\frac{q_R}{p_R}$ for the system $S = \frac{b}{a} = \frac{0.2z+0.1}{z^2-0.5z+0.1}$ such that the characteristic polynomial $\Delta = z^i$

$\deg(a) = 2$
 $\deg(\Delta) = 2 \deg(a) - 1 = 3$

$$F_w = (R^*S)/(1+R^*S)$$

$$F_w = \frac{bq_R}{ap_R + bq_R}$$

delta

$$\Delta = ap_R + bq_R$$

$$\Delta = ax + by$$

delta = z³ (minimum degree)

Causality requirement

$$\deg(q_r) \leq \deg(p_r)$$

$$\deg(b) \leq \deg(a)$$

- The degree i of the characteristic polynomial must be sufficiently high to find the causal controller

$$(z^2 - 0.5z + 0.1)pr + (0.2z + 0.1)qr = z^3$$

- Solvability condition: $\deg(\Delta) \geq 2 \deg(a) - 1$

$\deg(\Delta) = \deg(a) + \deg(pr)$
 $\deg(\Delta) \geq 2 \deg(a) - 1$

$$\begin{bmatrix} a & b \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} g_{NSD} & 0 \\ p & r \\ q & s \end{bmatrix}$$

$$p_R = x = p \frac{c}{g_{NSD}} + rh$$

$$q_R = y = q \frac{c}{g_{NSD}} + sh$$

Ex. 2 EMMP – 1 DOF (One Degree Of Freedom)

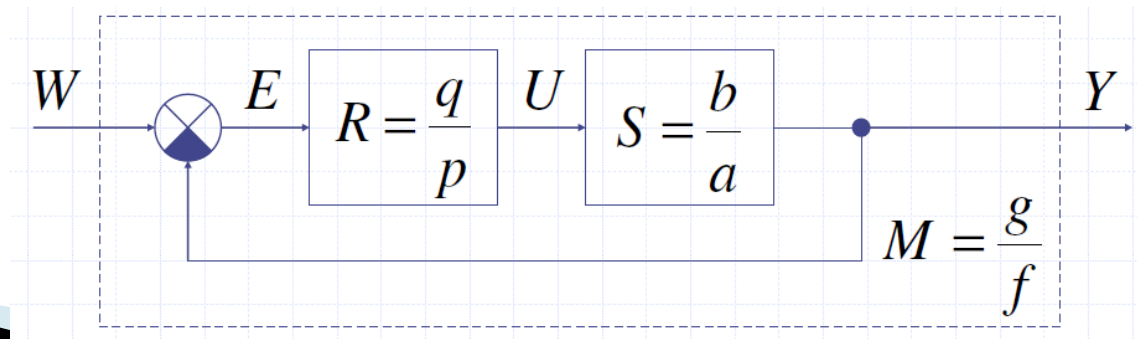
- Design the controller $R = \frac{q_R}{p_R}$ for the system

$$S = \frac{b}{a} = \frac{0.2z^{-1} + 0.1z^{-2}}{1 - 0.5z^{-1} + 0.1z^{-2}}$$

- such that the closed-loop system behaves like a model:

$$M = \frac{0.5z^{-1} + 0.5z^{-2}}{1} = \frac{g_M}{f_M}$$

$$\frac{Y(z)}{W(z)} = \frac{SR}{1 + SR} = \frac{bq_R}{ap_R + bq_R} = \frac{g_M}{f_M} \rightarrow g_M^0(ap_R + bq_R) = f_M b^0 q_R$$



Ex. 3 Matlab – Modal controller (pole placement)

- Design the controller $\frac{q_R}{p_R}$ for the system $S = \frac{b}{a} = \frac{0.2z+0.1}{z^2-0.5z+0.1}$
- Find the solution in positive and negative powers of z .

$$\Delta = z$$

$$\Delta = z^2$$

$$\Delta = z^3$$

$$\Delta = ap_R + bq_R = h$$

Ex. 4 Matlab – 1 DOF EMMP

- In the MATLAB/Simulink simulate the feedback system from Example 3
- Sampling period is $T_s = 1$ s.
- Compare the step response of the feedback system with the response of the model M.

$$S = \frac{b}{a} = \frac{0.2z^{-1} + 0.1z^{-2}}{1 - 0.5z^{-1} + 0.1z^{-2}}$$

$$M = \frac{g_M}{f_M} = \frac{0.5z^{-1} + 0.5z^{-2}}{1}$$

Ex. 5 Matlab – modal controller

- Design the controller $\frac{q_R}{p_R}$ for the system $S = \frac{b}{a} = \frac{0.6d+0.2d^2}{1+0.2d+0.01d^2}$
- Find the solution in negative powers of z i.e. in d .
- Sampling period is $T_s = 1$ s.
- Plot the step response of the closed-loop system.

A) Pseudochar. polynomial $\Delta = 1$

$$\Delta = ap_R + bq_R$$

B) Pseudochar. polynomial $\Delta = 0.1d + 0.2d^2$

Solution: A) $R = -(17*d+240)/(340*d-980)$

Solution: B) $R = -(5*d+183)/(100*d)$