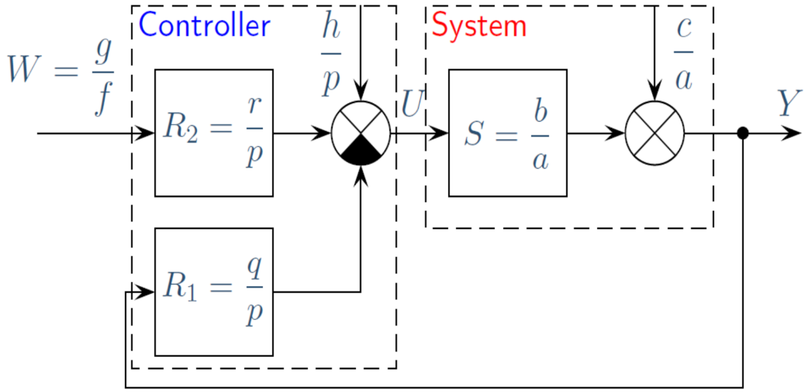
RAL

Ex 06

Ex. 1Time optimal control with 2DOF

- Design the Time-optimal feedback controllers $R_1 = \frac{q_r}{p_r}$ and $R_2 = \frac{r_r}{p_r}$ for system

$$S = \frac{b}{a} = \frac{0.2z + 0.1}{z^2 - 0.5z + 0.1} = \frac{0.2d + 0.1d^2}{1 - 0.5d + 0.1d^2}$$



- Required value:

$$W = \frac{g_W}{f_W} = 1 + d + d^2 + d^3 + d^4$$

$$F_w(d) = \frac{r_r b}{a p_r + b q_r}$$

- Pseudochar. Polynomial $\Delta = a p_r + b q_r$

$$E = \left(1 - \frac{b r_r}{\Delta}\right)\frac{g_w}{f_w} - \frac{c p_r}{\Delta} - \frac{b h}{\Delta}$$

$$\text{U} = \frac{a r_r}{\Delta}\frac{g_w}{f_w} - \frac{c q_r}{\Delta} - \frac{a h}{\Delta}$$

$$ax + by = 1$$

$$a p_r + b q_r = 1$$

$$f_W s_r + b r_r = 1$$

Solvability condition: $f_W^0 = 1$

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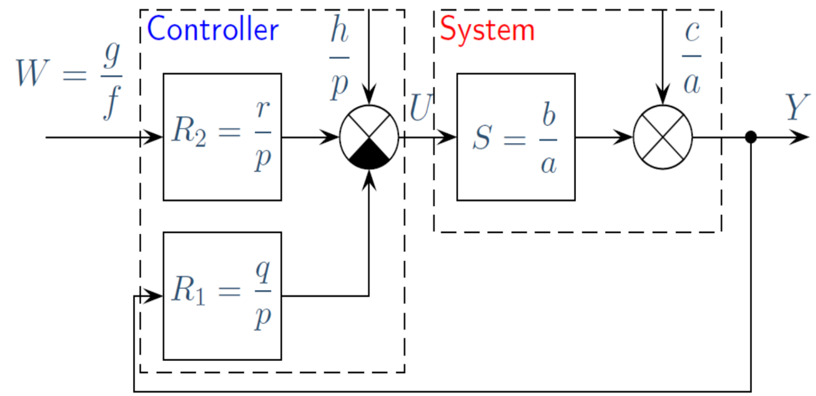


$$S = \frac{b}{a} = \frac{0.2z + 0.1}{z^2 - 0.5z + 0.1} = \frac{0.2d + 0.1d^2}{1 - 0.5d + 0.1d^2}$$

- Solvability condition: $f_W^0 = 1$

$$\frac{a}{f_W} = \frac{a^0 * GCD}{f_W^0 * GCD} \rightarrow f_W^0 = \frac{f_W}{GCD}$$

$$ax + by = 1$$
$$ap_r + bq_r = 1$$
$$f_W s_r + br_r = 1$$

$$\begin{bmatrix} f_W & b \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \sim \begin{bmatrix} g_{NSD} & 0 \\ p & \mathrm{r} \\ q & s \end{bmatrix}$$

$$p_r = x = p_1 \frac{c}{g_{NSD}} + r_1 h_1$$

$$q_r = y = q_1 \frac{c}{g_{NSD}} + s_1 h_1$$

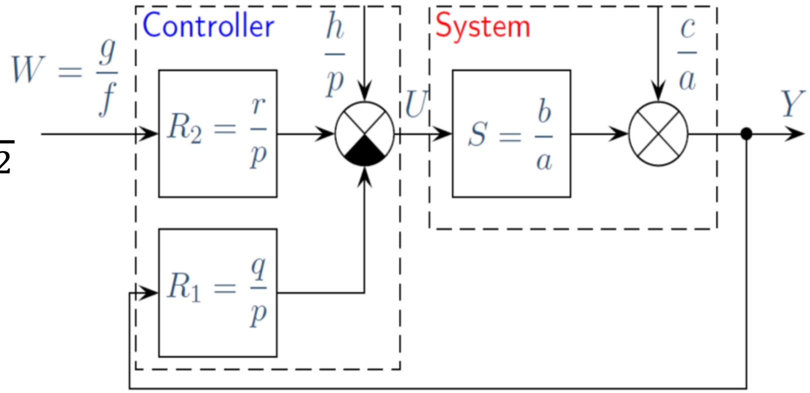
$$s_r = x = p_2 \frac{c}{g_{NSD}} + r_2 h_2$$

$$r_r = y = q_2 \frac{c}{g_{NSD}} + s_2 h_2$$

Ex. 2 Stable time optimal control with 2DOF

- Design the stable time-optimal feedback controllers $R_1 = \frac{q_r}{p_r}$ and $R_2 = \frac{r_r}{p_r}$ for system

$$S = \frac{b}{a} = \frac{0.2z + 0.1}{z^2 - 0.5z + 0.1} = \frac{0.2d + 0.1d^2}{1 - 0.5d + 0.1d^2}$$



- Required value:

$$W = \frac{g_W}{f_W} = 1 + d + d^2 + d^3 + d^4$$

$$F_w(d) = \frac{r_r b}{a p_r + b q_r}$$

- Pseudochar. Polynomial $\Delta = a p_r + b q_r$

$$E = \left(1 - \frac{b r_r}{\Delta}\right)\frac{g_w}{f_w} - \frac{c p_r}{\Delta} - \frac{b h}{\Delta}$$

$$\mathrm{U} = \frac{a r_r}{\Delta}\frac{g_w}{f_w} - \frac{c q_r}{\Delta} - \frac{a h}{\Delta}$$

$$a p_r + b q_r = b^+$$

$$a\frac{p_r}{b^+} + b^- q_r = 1$$

$$p_r = x b^+ \qquad q_r = q_r$$

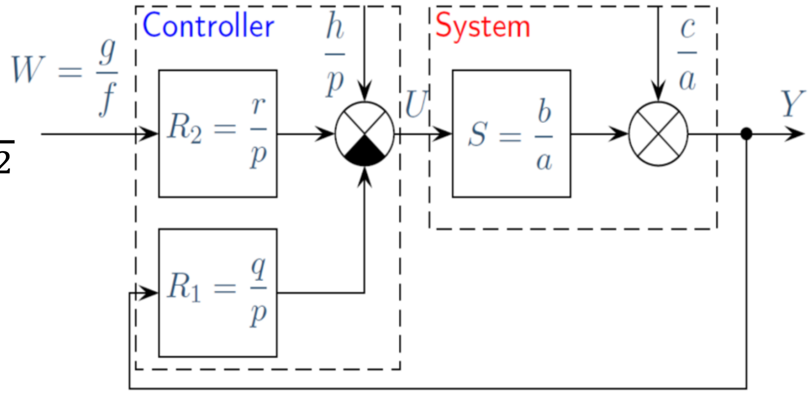
$$a x + b^- q_r = 1$$

$$f_W s_r + b^- r_r = 1$$

Ex. 2 Stable time optimal control with 2DOF

- Design the stable time-optimal feedback controllers $R_1 = \frac{q_r}{p_r}$ and $R_2 = \frac{r_r}{p_r}$ for system

$$S = \frac{b}{a} = \frac{0.2\text{z} + 0.1}{z^2 - 0.5\text{z} + 0.1} = \frac{0.2d + 0.1d^2}{1 - 0.5d + 0.1d^2}$$



$$x = p_1 \frac{c}{g_{NSD}} + r_1 h_1$$

$$q_r = y = q_1 \frac{c}{g_{NSD}} + s_1 h_1$$

$$s_r = x = p_2 \frac{c}{g_{NSD}} + r_2 h_2$$

$$r_r = y = q_2 \frac{c}{g_{NSD}} + s_2 h_2$$

$$\begin{bmatrix} f_w & b \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \sim \begin{bmatrix} g_{NSD} & 0 \\ p & \text{r} \\ q & s \end{bmatrix}$$

Ex. 3 Function recip()

- Create a recip() function that creates a reciprocal polynomial from a vector with polynomial coefficients.

```
>> syms x
>> sym2poly(x^5+3*x^3-2*x-5)
ans =1 0 3 0 -2 -5
```

Ex. 4 RST controller

- Verify the feedback system from ex.1 in Matlab/Simulink.

$$S = \frac{0.2d + 0.1d^2}{1 - 0.5d + 0.1d^2} = \frac{b}{a}$$

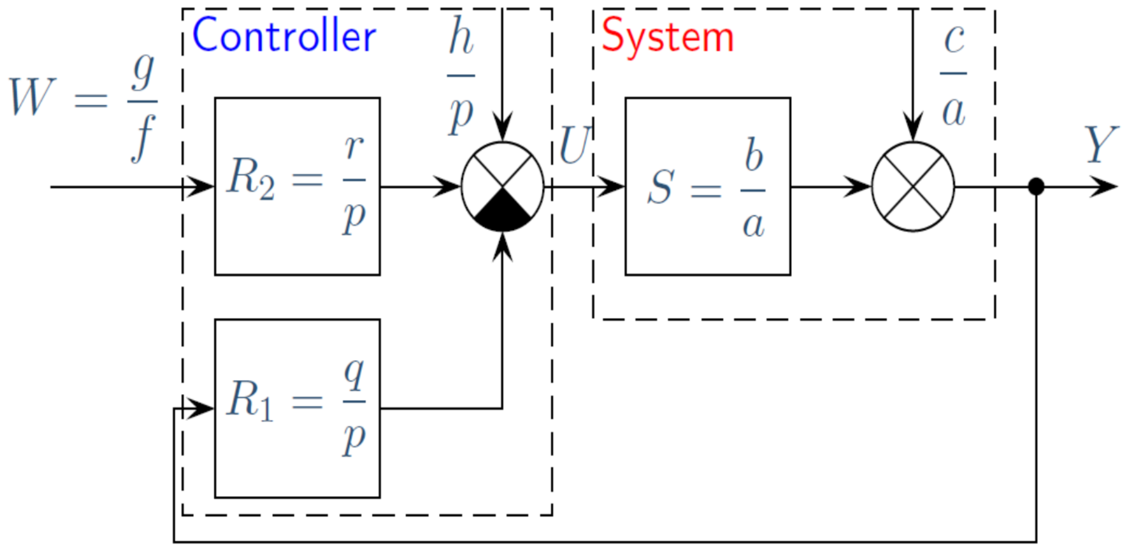
$$W = \frac{g_W}{f_W} = 1 + d + d^2 + d^3 + d^4$$

$$R_1 = \frac{q_r}{p_r} = \frac{\frac{25}{24} - \frac{7}{24}d}{\frac{7}{24}d + 1} \qquad R_2 = \frac{r_r}{p_r} = \frac{\frac{10}{3}}{\frac{7}{24}d + 1}$$

- for two cases :
  - EMMP controller with two degrees of freedom
  - RST controller

Ex. 4 EMMP and RST controllers

- EMMP controller with two degrees of freedom



- RST controller