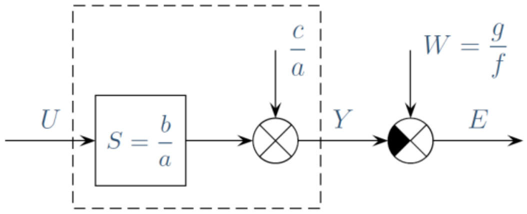
RAL Ex 05

Ex. 1: Time-optimal feedforward control

Design the Time-optimal feedforward control for the system $S = \frac{b}{a} = \frac{0.2d+0.1d^2}{1-0.5d+0.1d^2}$ and following required value $W = \frac{g_W}{f_W} = \frac{d+d^2}{1}$.



Polynomial c_0 represents initial conditions of the plant. Since feedforward control cannot act against it therefore it is assumed equal to zero ($c_0 = 0$). So as to have error e finite, $E(d)$ must be finite polynomial. We assumed that expressions Ef_W and g_W are polynomials, for this reason, the expression $f_W U/a$ must be also polynomial.

$$E = \frac{g_W}{f_W} - \frac{b}{a}U$$

$$Ef_W + \frac{bf_W}{a}U = g_W$$

$$f_W x + by = g_W$$

$$E = x \qquad\qquad y = \frac{f_W}{a}U$$

$$1x + (0.2d + 0.1d^2)y = d + d^2$$

prvni krok gcd

$$\begin{bmatrix} f_W & b \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \sim \begin{array}{c} \begin{array}{cc} \text{fw} & \text{b} \end{array} \\ \begin{bmatrix} 1 & 0.2\mathrm{d} + 0.1\mathrm{d}^2 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \end{array} \sim (2.\mathrm{s} - 1.\mathrm{s} * 0.1\mathrm{d}^2) \sim \begin{bmatrix} 1 & 0.2\mathrm{d} \\ 1 & -0.1\mathrm{d}^2 \\ 0 & 1 \end{bmatrix} \sim (2.\mathrm{s} - 1.\mathrm{s} * 0.2\mathrm{d}) \sim$$

$$\sim \begin{bmatrix} 1 & 0 \\ 1 & -0.1\mathrm{d}^2 - 0.2\mathrm{d} \\ 0 & 1 \end{bmatrix} \sim \begin{bmatrix} g_{NSD} & 0 \\ p & \mathrm{r} \\ q & s \end{bmatrix}$$

$$p = 1 \qquad r = -0.1d^2 - 0.2d$$
$$q = 0 \qquad s = 1$$

$$x = p\frac{g_W}{g_{NSD}} + rh \qquad\qquad y = q\frac{g_W}{g_{NSD}} + sh$$

$$\begin{matrix} x = d + d^2 + (-0.1d^2 - 0.2d)h \\ y = 0 + h \end{matrix} \quad \begin{cases} h = 1 \quad x = 0.8d + 0.9d^2 & E = 0.8d + 0.9d^2 \\ \qquad\qquad y = 1 & U = y\dfrac{a}{f_W} = 1\,\dfrac{1 - 0.5\mathrm{d} + 0.1\mathrm{d}^2}{1} \end{cases}$$

Number of error steps: $k_e = 1 + degE = 3$

$$\begin{matrix} x = d + d^2 + (-0.1d^2 - 0.2d)h \\ y = 0 + h \end{matrix} \quad \begin{cases} h = 10 \quad x = -d & E = -d \\ \qquad\qquad y = 10 & U = y\dfrac{a}{f_W} = y\dfrac{a^0}{f_W^0} = 10\,\dfrac{1 - 0.5\mathrm{d} + 0.1\mathrm{d}^2}{1} \end{cases}$$

$$U = y\frac{a}{f_w} = y\frac{a^0}{f_w^0} \text{ is polynomial only for } f_w^0 = 1$$

Number of error steps: $k_e = 1 + degE = 2$

Ex. 2:

Design the Time-optimal feedforward control for the system $S = \frac{b}{a} = \frac{0.2d+0.1d^2}{1-1.4d+0.4d^2}$ and following required value $W = \frac{g_W}{f_W} = \frac{1}{1-d}$.

$$E = \frac{g_W}{f_W} - \frac{b}{a}U$$

$$Ef_W + \frac{bf_W}{a}U = g_W$$

$$f_W x + by = g_W$$

$$E = x \qquad y = \frac{f_W}{a}U$$

$$(1-d)x + (0.2d + 0.1d^2)y = 1$$

$$\begin{bmatrix} f_W & b \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1-d & 0.2\mathrm{d} + 0.1\mathrm{d}^2 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \sim (2.\mathrm{s} * 10) \sim \begin{bmatrix} 1-d & 2\mathrm{d} + \mathrm{d}^2 \\ 1 & 0 \\ 0 & 10 \end{bmatrix} \sim (2.\mathrm{s} + 1.\mathrm{s} * \mathrm{d}) \sim$$

$$\sim \begin{bmatrix} 1-d & 3\mathrm{d} \\ 1 & \mathrm{d} \\ 0 & 10 \end{bmatrix} \sim (1.\mathrm{s} * 3 + 2.\mathrm{s}) \sim \begin{bmatrix} 3 & 3\mathrm{d} \\ 3+d & \mathrm{d} \\ 10 & 10 \end{bmatrix} \sim (2.\mathrm{s} - 1.\mathrm{s} * \mathrm{d}) \sim \begin{bmatrix} 3 & 0 \\ 3+d & -2\mathrm{d} - \mathrm{d}^2 \\ 10 & 10 - 10d \end{bmatrix} \sim$$

$$\sim \begin{bmatrix} g_{NSD} & 0 \\ p & \mathrm{r} \\ q & s \end{bmatrix}$$

$$p = 3 + d \qquad r = -2\mathrm{d} - \mathrm{d}^2$$
$$q = 10 \qquad s = 10(1-d)$$

$$x = p\frac{g_W}{g_{NSD}} + rh \qquad y = q\frac{g_W}{g_{NSD}} + sh$$

$$x = \frac{3+d}{3} + (-2\mathrm{d} - \mathrm{d}^2)h \qquad y = \frac{10}{3} + 10(1-d)h$$

For $h = 0$

$x = \frac{3+d}{3}$ $\qquad y = \frac{10}{3}$

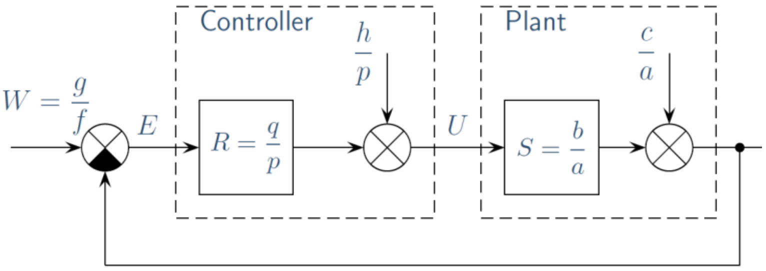
$E = x = \frac{3+d}{3}$ $\qquad U = y\frac{a}{f_w} = y\frac{a^0}{f_W^0} = \frac{10}{3}\frac{1-1.4\mathrm{d}+0.4\mathrm{d}^2}{1-\mathrm{d}} = \frac{10}{3}(1 - 0.4d)$

Number of error steps: $k_e = 1 + degE = 2$

Note. The only way to reduce the order of the polynomial x is to choose $h = 0$, because in other cases h could not be a polynomial.

Ex. 3: Time-optimal feedback control

Design the Time-optimal feedback control for the system $S = \frac{b}{a} = \frac{0.2d+0.1d^2}{1-0.5d+0.1d^2}$



Required value: $W = \frac{g_W}{f_W} = \frac{1}{1-d}$

Solvability condition:

$$E = \frac{a^0 p_r g_W}{\Delta f_W^0} - \frac{c p_r}{\Delta} - \frac{hb}{\Delta}$$

$$U = \frac{a^0 q_r g_W}{\Delta f_W^0} - \frac{c q_r}{\Delta} + \frac{ha}{\Delta}$$

To have error e and action u finite it is necessary to obtain their Z transforms $E(d)$ and $U(d)$ as finite polynomials. Polynomials h and c are unknown and nonzero. => $f_W^0 = 1$, $\Delta = 1$ (the minimum degree of polynomials p_r amd q_r).

$$\frac{a}{f_W} = \frac{a^0 * NSD}{f_W^0 * NSD} = \frac{a^0}{f_W^0} = \frac{1-0.5d+0.1d^2}{1-d}$$

$$f_W^0 \neq 1 \Rightarrow no\ solution$$

Ex.4: Time-optimal feedback control

Design the Time-optimal feedback control $R = \frac{q_r}{p_r}$ for the system $S = \frac{b}{a} = \frac{0.2d+0.1d^2}{1-1.4d+0.4d^2}$

Required value: $W = \frac{g_W}{f_W} = \frac{1}{1-d}$

Solvability condition: $f_W^0 = 1$

$$\frac{a}{f_W} = \frac{a^0 * NSD}{f_W^0 * NSD} = \frac{1-1.4d+0.4d^2}{1-d} = \frac{(1-d)(1-0.4d)}{1-d} = \frac{1-0.4d}{1} = \frac{a^0}{f_W^0}$$

$$a p_r + b q_r = \Delta$$

Generally: $ax + by = \Delta$

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syms d
a = 1-1.4*d + 0.4*d^2;
b = 0.2*d + 0.1*d^2;
c = 1;
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[p,q] = axminbyc(a,b,c,d);
R = simplify(q/p);
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Result: R = -(44*d - 134)/(11*d + 27)

Ex. 5: Stable 1DOF Time-optimal feedback control

Design the Stable Time-optimal feedback control $R = \frac{q_r}{p_r}$ for the system $S = \frac{b}{a} = \frac{0.2d+0.1d^2}{1-0.5d+0.1d^2}$

Required value: $W = \frac{g_W}{f_W} = \frac{1}{2-d}$

To have error e finite it is necessary to obtain their Z transforms $E(d)$ as finite polynomials. Z-transform of control action $U(d)$ must be stable. Polynomials h and c are unknown and nonzero.

The characteristic polynomial is solved for the minimum degree of the polynomial x and q

$$ap + bq = b^+$$

$$af_W^0 x + b^- q = 1$$

$$R = \frac{q_r}{p_r} = \frac{q}{xb^+ f_W^0}$$

$$b = b^+ b^- = 0.2d + 0.1d^2 = 0.1d(2 + d)$$

or $b_{12} = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a} = \frac{-0.2 \pm 0.2}{0.2}$

Roots: $b_1 = 0;\ b_2 = -2$

$$b^+ = 0.1d + 0.2$$

$$b^- = d$$

```
syms d
a = 1 - 0.5*d + 0.1*d^2;
b = 0.1*d^2 + 0.2*d;
bminus = d;
bplus = 0.1*(d + 2);
c = 1;
fw = 2-d;

gnsd = gcd(a,fw);                   % Greatest Common Divisor GCD
fw0 = simplify(fw/gnsd);            % obtaining fw0; fw is dividing by GCD

[x,q] = axminbyc(a*fw0,bminus,c,d); % solve equation a*fw0*x + bminus*q = 1
R = simplify(q/(fw0*x*bplus));      % R = q/p = q/(x*bplus*fw0)
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Result: R = -(d^2 - 7*d + 20)/(d^2 - 4)