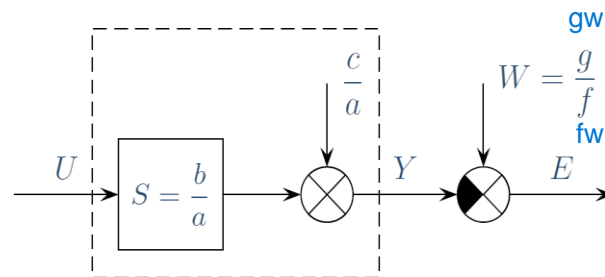# RAL

Exercise 05

# Ex. 1 Time-optimal feedforward control

▷ Design the Time-optimal feedforward control for the system

$$S = \frac{b}{a} = \frac{0.2d + 0.1d^2}{1 - 0.5d + 0.1d^2}$$

▷ Required value

$$W = \frac{g_W}{f_W} = \frac{d + d^2}{1}$$

▷ Z transform $E(d)$ must be finite polynomial

E = W - Y

$$E = \frac{g_W}{f_W} - \frac{b}{a}U$$

$$E f_W + \frac{b f_W}{a} U = g_W$$

$$f_W x + by = g_W$$

$$\begin{bmatrix} f_W & b \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \sim \begin{bmatrix} g_{NSD} & 0 \\ p & r \\ q & s \end{bmatrix}$$

$$x = p\frac{c}{g_{NSD}} + rh$$

$$y = q\frac{c}{g_{NSD}} + sh$$

$$x = E \qquad\qquad y = \frac{f_W}{a} U$$

$$k_e = 1 + degE$$

gw

$$\frac{c}{a}$$

$$W = \frac{g}{f}$$

fw

$U$  $S = \frac{b}{a}$  $Y$  $E$

# Ex. 2 Time-optimal feedforward control

➤ Design the Time-optimal feedforward control for the system

$$S = \frac{0.2d + 0.1d^2}{1 - 1.4d + 0.4d^2} = \frac{b}{a}$$

➤ Required value

$$W = \frac{g_W}{f_W} = \frac{1}{1 - d}$$

➤ Use the function *axminbyc()*;

➤ Result:    U = (10/3 – 4/3*d)
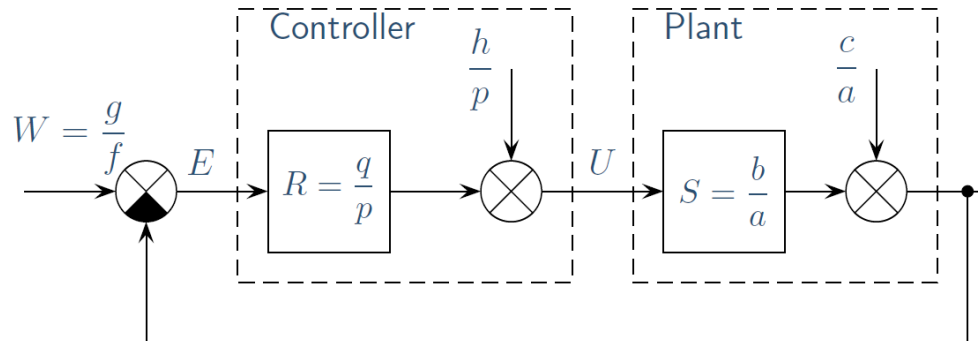
# Ex. 2 Time-optimal feedforward control

➢ Use the function *axminbyc()*;

▸ syms d
▸ a = 1-1.4*d + 0.4*d^2;
▸ b = 0.2*d + 0.1*d^2;
▸ fw = 1-d;
▸ gw = 1;
▸
▸ [x,y] = axminbyc(fw,b,gw,d);
▸
▸ U = simplify(y*a/fw);
▸ E = simplify(x);

➢ Result:     U = (10/3 - 4/3*d)

# Ex. 3 Time–optimal feedback control

➤ Design the Time-optimal feedback control for the system

$$S = \frac{b}{a} = \frac{0.2d + 0.1d^2}{1 - 0.5d + 0.1d^2}$$

➤ Required value:

$$W = \frac{g_W}{f_W} = \frac{1}{1 - d}$$



➤ Solvability condition:

$$E = \frac{a^0 p_r g_w}{\Delta f_W^0} - \frac{c p_r}{\Delta} - \frac{hb}{\Delta}$$

$$U = \frac{a^0 q_r g_w}{\Delta f_W^0} - \frac{c q_r}{\Delta} + \frac{ha}{\Delta}$$

# Ex. 3

➤ Design the Time-optimal feedback control for the system

$$S = \frac{b}{a} = \frac{0.2d + 0.1d^2}{1 - 0.5d + 0.1d^2}$$

➤ Required value:

$$W = \frac{g_W}{f_W} = \frac{1}{1 - d}$$

➤ To have error e and action u finite it is necessary to obtain their Z transforms E(d) and U(d) as finite polynomials

➤ => $f_W^0 = 1$, $\Delta = 1$ (the minimum degree of polynomials $p_r$ and $q_r$).

$$\frac{a}{f_W} = \frac{a^0 * NSD}{f_W^0 * NSD} = \frac{a^0}{f_W^0} = \frac{1 - 0.5d + 0.1d^2}{1 - d}$$

$$f_W^0 \neq 1 \Rightarrow no\ solution$$

# Ex. 4 Time-optimal feedback control - Matlab

➤ Design the Time-optimal feedback control for the system

$$S = \frac{b}{a} = \frac{0.2d + 0.1d^2}{1 - 1.4d + 0.4d^2}$$

➤ Required value

$$W = \frac{g_W}{f_W} = \frac{1}{1 - d}$$

➤ Use the function *axminbyc( )*;

➤ Result:     R = −(44*d − 134)/(11*d + 27)

# Ex. 4

➢ Design the Time-optimal feedback control for the system

$$S = \frac{0.2d + 0.1d^2}{1 - 1.4d + 0.4d^2} = \frac{b}{a}$$

➢ Required value: $\qquad W = \frac{g_W}{f_W} = \frac{1}{1-d}$

➢ Solvability condition: $f_W^0 = 1$

$$\frac{a}{f_W} = \frac{a^0 * NSD}{f_W^0 * NSD} = \frac{1 - 1.4d + 0.4d^2}{1-d} = \frac{(1-d)(1-0.4d)}{1-d} = \frac{1-0.4d}{1} = \frac{a^0}{f_W^0}$$

➢ Use the function *axminbyc()*;

➢ Result: $\qquad$ R = –(44*d – 134)/(11*d + 27)

# Ex. 4

> Use the function *axminbyc()*;

```
syms d
a = 1−1.4*d+0.4*d^2;
b = 0.2*d+0.1*d^2;
c = 1;

[p,q] = axminbyc(a,b,c,d);
R = simplify(q/p)
```

> Result:  R = −(44*d − 134)/(11*d + 27)

# Ex. 5 Stable 1DOF Time-optimal feedback control

➢ Design the Stable Time-optimal feedback control for the system

$$S = \frac{b}{a} = \frac{0.2d + 0.1d^2}{1 - 0.5d + 0.1d^2}$$

➢ Required value:

$$W = \frac{g_W}{f_W} = \frac{1}{2 - d}$$

$$ap + bq = b^+$$

$$af_w^0 x + b^- q = 1$$

$$R = \frac{q_r}{p_r} = \frac{q}{xb^+ f_w^0}$$

$$E = \frac{a^0 p_r g_w}{\Delta f_W^0} - \frac{cp_r}{\Delta} - \frac{hb}{\Delta}$$

$$\frac{a}{f_W} = \frac{a^0 * NSD}{f_W^0 * NSD}$$

$$b = 0.2d + 0.1d^2 = b^+ b^-$$

➢ R = -(d^2 - 7*d + 20)/(d^2 - 4)

# Ex. 5

- syms d
- a = 1 – 0.5*d + 0.1*d^2;
- b = 0.1*d^2 + 0.2*d;
- bminus = d;
- bplus = 0.1*(d + 2);
- c = 1;
- fw = 2–d;
-
- gnsd = gcd(a,fw);
- fw0 = simplify(fw/gnsd);
- [x,q] = axminbyc(a*fw0,bminus,c,d);
- R = simplify(q/(fw0*x*bplus));

- Result: R = –(d^2 – 7*d + 20)/(d^2 – 4)

# Ex. 6 Matlab/Simulink

➢ Create the model of Time-optimal feedforward control in MATLAB/Simulink from Ex. 1 (h = 1, h = 10).

➢ System $\quad S = \dfrac{b}{a} = \dfrac{0.2d + 0.1d^2}{1 - 0.5d + 0.1d^2}$

➢ Required value: $\quad W = \dfrac{g_W}{f_W} = \dfrac{d + d^2}{1}$

➢ Feedforward control $\quad U = y\dfrac{a}{f_w} = y\dfrac{a_0}{f_{w0}} = h\,\dfrac{1 - 0.5d + 0.1d^2}{1}$

$$y = h$$