第五次作业

2020010768 无05 付宇辉

1.

判别函数为

$$g_i(x) = -\frac{1}{2}(x-\mu_i)^T\Sigma_i^{-1}(x-\mu_i) - \frac{d}{2}\ln 2\pi - \frac{1}{2}\ln|\Sigma_i| + \ln p(\omega_i)$$

(1) 两类分类判别边界满足$g_1(x) = g_2(x)$

$$\begin{aligned} g_1(x) - g_2(x) &= (\mu_1^T\Sigma_1^{-1} - \mu_2^T\Sigma_2^{-1})x + \left(-\frac{1}{2}\mu_1^T\Sigma_1^{-1}\mu_1 + \frac{1}{2}\mu_2^T\Sigma_2^{-1}\mu_2\right) \\ &= \left(\begin{bmatrix}0 & 0\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} - \begin{bmatrix}2 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\right)\begin{bmatrix}x_1\\x_2\end{bmatrix} \\ &+ \left(-\frac{1}{2}\begin{bmatrix}0 & 0\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix} + \frac{1}{2}\begin{bmatrix}2 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix}\right) = -2x_1 - x_2 + \frac{5}{2} \end{aligned}$$

决策边界为

$$-2x_1 - x_2 + \frac{5}{2} = 0$$

(2) 两类分类判别边界满足$g_1(x) = g_2(x)$

$$\begin{aligned} g_1(x) - g_2(x) &= \left(= -\frac{1}{2}x^T\Sigma_1^{-1}x + \frac{1}{2}x^T\Sigma_2^{-1}x\right) + (\mu_1^T\Sigma_1^{-1} - \mu_2^T\Sigma_2^{-1})x \\ &+ \left(-\frac{1}{2}\mu_1^T\Sigma_1^{-1}\mu_1 + \frac{1}{2}\mu_2^T\Sigma_2^{-1}\mu_2 - \frac{1}{2}\ln|\Sigma_1| + \frac{1}{2}\ln|\Sigma_2|\right) \\ &= \left(-\frac{1}{2}\begin{bmatrix}x_1 & x_2\end{bmatrix}\begin{bmatrix}\frac{1}{4} & 0\\0 & \frac{1}{3}\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} + \frac{1}{2}\begin{bmatrix}x_1 & x_2\end{bmatrix}\begin{bmatrix}\frac{1}{4} & -\frac{1}{8}\\-\frac{1}{8} & \frac{5}{16}\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}\right) \\ &+ \left(\begin{bmatrix}0 & 0\end{bmatrix}\begin{bmatrix}\frac{1}{4} & 0\\0 & \frac{1}{3}\end{bmatrix} - \begin{bmatrix}1 & 1\end{bmatrix}\begin{bmatrix}\frac{1}{4} & -\frac{1}{8}\\-\frac{1}{8} & \frac{5}{16}\end{bmatrix}\right)\begin{bmatrix}x_1\\x_2\end{bmatrix} \\ &+ \left(-\frac{1}{2}\begin{bmatrix}0 & 0\end{bmatrix}\begin{bmatrix}\frac{1}{4} & 0\\0 & \frac{1}{3}\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix} + \frac{1}{2}\begin{bmatrix}1 & 1\end{bmatrix}\begin{bmatrix}\frac{1}{4} & -\frac{1}{8}\\-\frac{1}{8} & \frac{5}{16}\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} - \frac{1}{2}\ln\begin{vmatrix}4 & 0\\0 & 3\end{vmatrix}\right. \\ &\left.+ \frac{1}{2}\ln\begin{vmatrix}5 & 2\\2 & 4\end{vmatrix}\right) = -\frac{1}{96}x_2^2 - \frac{1}{8}x_1x_2 - \frac{1}{8}x_1 - \frac{3}{16}x_2 + 2\ln 2 - \frac{1}{2}\ln 12 + \frac{5}{32} \end{aligned}$$

决策边界为

$$-\frac{1}{96}x_2^2 - \frac{1}{8}x_1x_2 - \frac{1}{8}x_1 - \frac{3}{16}x_2 + 2\ln 2 - \frac{1}{2}\ln 12 + \frac{5}{32} = 0$$

(1)中的结果是线性超平面，(2)的结果是非线性的分类界面。

2.

状态转移概率矩阵为

$$\begin{bmatrix}0.6 & 0.4\\0.3 & 0.7\end{bmatrix}$$

观测概率矩阵为

$$\begin{bmatrix}0.4 & 0.1 & 0.4 & 0.1\\0.1 & 0.4 & 0.1 & 0.4\end{bmatrix}$$

初始概率矩阵为

$$\begin{bmatrix}0.5\\0.5\end{bmatrix}$$

(1)

$t=1$时：

$$\alpha_1:\ \begin{bmatrix}0.5\\0.5\end{bmatrix}*\begin{bmatrix}0.4\\0.1\end{bmatrix}=\begin{bmatrix}0.2\\0.05\end{bmatrix}$$

$t=2$时：

$$\alpha_2:\begin{bmatrix}0.2\\0.05\end{bmatrix}*\begin{bmatrix}0.6 & 0.4\\0.3 & 0.7\end{bmatrix}=\begin{bmatrix}0.12 & 0.08\\0.015 & 0.035\end{bmatrix}\quad\begin{bmatrix}0.135\\0.115\end{bmatrix}*\begin{bmatrix}0.1\\0.4\end{bmatrix}=\begin{bmatrix}0.0135\\0.046\end{bmatrix}$$

$t=3$时：

$$\alpha_3:\begin{bmatrix}0.0135\\0.046\end{bmatrix}*\begin{bmatrix}0.6 & 0.4\\0.3 & 0.7\end{bmatrix}=\begin{bmatrix}0.0081 & 0.0054\\0.0138 & 0.0322\end{bmatrix}\quad\begin{bmatrix}0.0376\\0.1461\end{bmatrix}*\begin{bmatrix}0.1\\0.4\end{bmatrix}=\begin{bmatrix}0.00376\\0.05844\end{bmatrix}$$

所以

$$P(O|\lambda)=0.00376=0.05844=0.0622$$

(2)

$t=1$时：

$$\delta_1:\ \begin{bmatrix}0.5\\0.5\end{bmatrix}*\begin{bmatrix}0.4\\0.1\end{bmatrix}=\begin{bmatrix}0.2\\0.05\end{bmatrix}\ \ \varphi_1=\begin{bmatrix}0\\0\end{bmatrix}$$

$t=2$时：

$$\delta_2:\ \begin{bmatrix}0.2\\0.05\end{bmatrix}*\begin{bmatrix}0.6 & 0.4\\0.3 & 0.7\end{bmatrix}=\begin{bmatrix}0.12 & 0.08\\0.015 & 0.035\end{bmatrix}\quad\begin{bmatrix}0.12\\0.08\end{bmatrix}*\begin{bmatrix}0.1\\0.4\end{bmatrix}=\begin{bmatrix}0.012\\0.032\end{bmatrix}\quad\varphi_2=\begin{bmatrix}1\\1\end{bmatrix}$$

$t=3$时：

$$\delta_3:\begin{bmatrix}0.012\\0.032\end{bmatrix}*\begin{bmatrix}0.6 & 0.4\\0.3 & 0.7\end{bmatrix}=\begin{bmatrix}0.0072 & 0.0048\\0.0096 & 0.0224\end{bmatrix}\quad\begin{bmatrix}0.0096\\0.0224\end{bmatrix}*\begin{bmatrix}0.1\\0.4\end{bmatrix}=\begin{bmatrix}0.00096\\0.00896\end{bmatrix}\quad\varphi_3=\begin{bmatrix}2\\2\end{bmatrix}$$

$$q_3 = S_2, q_2 = S_2, q_1 = S_1$$

最可能的隐含状态序列为：$S_1 S_2 S_2$