

```

2. 3. gcc -Wall -Werror -std=c11 -c asd.c. char 1,int 4,*pointer 4,float 4,double 8, %c,%d,%f,%lf%-double. Pointer: p = &a atoi(*str -> int) atof(*str->float/double)
insertionSort(A);
// Input array A[0..n-1] of n elements
for all i=1..n-1 do
  element=A[i], j=i-1
  while j>0 and A[j]>element do
    A[j+1]=A[j]
    j=j-1
  end while
  A[j+1]=element
end for

```

address of first element
address of last element + 1
access current element
pointer arithmetic (move to next element)

4. 5. code 是像固定的函数 f0, main0. global 是全局变量, heap 是动态分配的内存, stack 是函数局部变量, 双指针 **P 一般矩阵, 传参数

- code ... fixed-size, read-only region
 - contains the machine code instructions for the program
- global data ... fixed-size, read-write region
 - contains global variables and constant strings
- heap ... very large, read-write region
 - contains dynamic data structures created by malloc() (see later)
- stack ... dynamically-allocated data (function local vars)
 - consists of frames, one for each currently active function
 - each frame contains local variables and house-keeping info

```

Matrix:
float **matrix = malloc(m * sizeof(float *));
assert(matrix != NULL);
int i;
for (i = 0; i < m; i++) {
  matrix[i] = malloc(n * sizeof(float));
  assert(matrix[i] != NULL);
}
4m + 4 * mn bytes allocated

```

```

MatrixProduct(A,B)
Input: n×n matrices A, B
Output: n×n matrix A·B
for all i=1..n do
  for all j=1..n do
    C[i,j]=0
    for all k=1..n do
      C[i,j]=C[i,j]+A[i,k]·B[k,j]
    end for
  end for
end for
return C

```

6. 7. adjacency matrix 用双指针, DFS array 0(V+E2) matrix 0(V2) list 0(V+E), BFS 0(V+E2). Ham path 是点不重复 0(V-1!), Euler path 是边不重复 0(n2). Components. 可达到.

Comparison of Graph Representations

	array of edges	adjacency matrix	adjacency list
space usage	E	V^2	$\sum V_i E_i$
initialise	T	V^2	V
insert edge	T	T	T
find/delete edge	E	T	V

Other operations:

	array of edges	adjacency matrix	adjacency list
disconnected?	E	V	T
isPath(x,y)?	$E \cdot \log V$	V^2	$\sum V_i E_i$
copy graph	E	V^2	E
destroy graph	T	V	E

BFS algorithm (records visiting order, marks vertices as visited when put on q)

```

visited[] // array of visiting orders, indexed by vertex 0..V-1
findPathBFS(G,src,dest):
  Input graph G, vertices src,dest
  for all vertices v∈G do
    Visited[v]=1
  end for
  enqueue src into new queue q
  visited[src]=true
  found=false
  while not found and q is not empty do
    dequeue v from q
    if v=dest then
      found=true
    else
      for each (v,w)∈Edges(G) such that visited[w]==1 do
        enqueue w into q
        visited[w]=true
      end for
    end if
  end while
  if found then
    display path in dest..src order
  end if

```

... Graph ADT (Adjacency Matrix)

Implementation of graph initialisation (adjacency-matrix representation)

```

Graph newGraph(int V) {
  assert(V >= 0);
  int i;
  Graph g = malloc(sizeof(GraphRep));
  g->nV = V; g->nE = 0;
  // allocate memory for each row
  g->edges = malloc(V * sizeof(int *));
  assert(g->edges != NULL);
  // allocate memory for each column and initialise with 0
  for (i = 0; i < V; i++) {
    g->edges[i] = calloc(V, sizeof(int));
    assert(g->edges[i] != NULL);
  }
  return g;
}

```

```

components(G):
  Input graph G
  for all vertices v∈G do
    componentOf[v]=-1
  end for
  compID=0
  for all vertices v∈G do
    if componentOf[v]==-1 then
      dfsComponents(G,v,compID)
      compID=compID+1
    end if
  end for
  return compID
dfsComponents(G,v,id):
  componentOf[v]=id
  for all vertices w adjacent to v do
    if componentOf[w]==-1 then
      dfsComponents(G,w,id)
    end if
  end for

```

```

hasPath(G,src,dest):
  Input graph G, vertices src,dest
  Output true if there is a path from src to dest in G, false otherwise
  return dfsPathCheck(G,src,dest)
dfsPathCheck(G,v,dest):
  mark v as visited
  if v=dest then
    return true
  else
    for all (v,w)∈Edges(G) do
      if w has not been visited then
        return dfsPathCheck(G,w,dest) // found path via w to dest
      end if
    end for
  end if
  return false // no path from v to dest
hasEulerPath(G,src,dest):
  Input graph G, vertices src,dest
  Output true if G has Euler path from src to dest false otherwise
  if src=dest then
    if degree(G,src) or degree(G,dest) is even then
      return false
    end if
  else if degree(G,src) is odd then
    return false
  end if
  for all vertices v∈G do
    if v=src and v=dest and degree(G,v) is odd then
      return false
    end if
  end for
  return true

```

Theorem. A graph has an Euler circuit if and only if it is connected and all vertices have even degree

... Transitive Closure

Cost analysis:

- storage: additional V^2 items (each item may be 1 bit)
- computation of transitive closure: V^3
- computation of reachable(): $O(T)$ after having generated tc[][]

visited[] // array [0..N-1] to keep track of visited

```

hasHamiltonianPath(G,src,dest):
  for all vertices v∈G do
    visited[v]=false
  end for
  return hamilton(G,src,dest,#vertices(G)-1)
hamilton(G,v,dest,d):
  Input G graph
  v current vertex considered
  dest destination vertex
  d distance "remaining" until path found
  if v=dest then
    if d=0 then return true else return false
  else
    visited[v]=true
    for each (v,w)∈Edges(G) ^ ~visited[w] do
      if hamilton(G,w,dest,d-1) then
        return true
      end if
    end for
  end if
  visited[v]=false // reset visited mark
  return false

```

... Transitive Closure

If we implement the above as:

```

make tc[][] a copy of edges[][]
for all i∈vertices(G) do
  for all s∈vertices(G) do
    for all t∈vertices(G) do
      if tc[s][i]=1 and tc[i][t]=1 then
        tc[s][t]=1
      end if
    end for
  end for
end for

```

then we get an algorithm to convert edges into a tc

This is known as *Warshall's algorithm*

8. MST (Kruskal) 是先找最小边, prim 是从一个顶点找最小边条件是一个点在已找到的一个不在, Djs 最小路径, 欧拉最小路径和 transitive closure 很像 0(v3), Maxflow

Relax along() 是 dist(v)+(v,w)(dist(w)) 那么就替换

```

dist[] // array of cost of shortest path from s to t
pend[] // array of predecessor in shortest path from s
dijkstraSSSP(G,source):
  Input graph G, source node
  initialise dist[] to all ∞, except dist[source]=0
  initialise pend[] to all -1
  vSet=all vertices of G
  while vSet≠∅ do
    find s∈vSet with minimum dist[s]
    for each (s,t,w)∈Edges(G) do
      relax along (s,t,w)
    end for
    vSet=vSet-{s}
  end while

```

```

Data: G, dist[][] path[][] Algorithm
dist[][] // array of cost of shortest path from s to t
path[][] // array of next node after s on shortest path from s to t
floydDPSP(G):
  Input graph G
  initialise dist[s][t]=0 for each s=t
  == for each (s,t,w)∈Edges(G)
  == otherwise
  initialise path[s][t]=s for each (s,t,w)∈Edges(G), otherwise to -1
  for all i∈vertices(G) do
    for all j∈vertices(G) do
      for all k∈vertices(G) do
        if dist[s][i]+dist[i][j] < dist[s][j] then
          dist[s][j]=dist[s][i]+dist[i][j]
          path[s][j]=path[s][i]
        end if
      end for
    end for
  end for

```

```

maxflow(G):
  Input flow network G with source s and sink t
  Output maximum flow value
  initialise flow[v][w]=0 for all vertices v, w
  maxflow=0
  while !shortest augmenting path visited[] from s to t do
    df = maximum additional flow via visited[]
    // adjust flow so as to represent residual graph
    v=t
    while v≠s do
      flow[visited[v]][v] = flow[visited[v]][v] + df;
      flow[v][visited[v]] = flow[v][visited[v]] - df;
      v=visited[v]
    end while
    maxflow=maxflow+df
  end while
  return maxflow

```

Shortest augmenting path can be found by standard BFS

Maxflow: Augmenting path 是一条可以获得最多 flow 的路径 (找最小的限制) eg, flow 是 2. 用当前的容量 c-f(v,w), 添加另一个方向边容量是 f(v,w). 之后在找新的 Augmenting path 重复这个过程. 最后把箭头反转就是 flow 的流程图. Edmonds 有两个表, 一个是 flow 一个是剩余流量

```

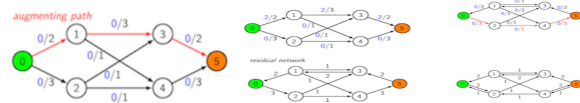
KruskalMST(G):
  Input: Graph G with n nodes
  Output: a minimum spanning tree of G
  sort edges(E) by weights
  for each e∈sortEdges(G) do
    MST = MST ∪ {e}
    if MST has a cycle then
      MST = MST - {e}
    end if
  end for
  if MST has n-1 edges then
    return MST
  end if

```

same vertex set V
for each edge $v-w \in E \dots$

- $c(f(v,w)) < c$ ⇒ add edge $(v-w, f(v,w))$ to E'
- $c(f(v,w)) > 0$ ⇒ add edge $(v-w, f(v,w))$ to E'

9. Tree 一般搜索复杂度是 $O(\log n)$, design $O(\log n)$ n 是节点数量, insert0(n). splay tree 类似 insert 就是插到根但是会出现 double 旋转, 每条两次一次性旋转两次, 记住标记



Edmonds-Karp Algorithm

One approach to solving maximum flow problem ...

maxflow(G):

- Find a shortest augmenting path
- Update flow[] so as to represent residual graph
- Repeat until no augmenting path can be found

$O(V^2E)$

Cost of searching:

	Array	List	File
Unsorted	$O(n)$ (linear scan)	$O(n)$ (linear scan)	$O(n)$ (linear scan)
Sorted	$O(\log n)$ (binary search)	$O(n)$ (linear scan)	$O(\log n)$ (seek, seek, ...)

For binary trees, several well-defined visiting orders exist:

- **preorder (NLR)** ... visit root, then left subtree, then right subtree
- **inorder (LNR)** ... visit left subtree, then root, then right subtree
- **postorder (LRN)** ... visit left subtree, then right subtree, then root
- **level-order** ... visit root, then all its children, then all their children

```
showBSTreePreorder(t):
Input tree t
push t onto new stack S
while stack is not empty do
  t=pop(S)
  print data(t)
  if right(t) is not empty then
    push right(t) onto S
  end if
  if left(t) is not empty then
    push left(t) onto S
  end if
end while
```

```
rotateLeft(N2):
Input tree N2
Output N2 rotated to the left
if N2 is empty or right(N2) is empty then
  return N2
end if
N1=right(N2)
right(N2)=left(N1)
left(N1)=N2
return N1
```

... Rebalancing Trees

Implementation of partition operation:

```
partition(tree,i):
Input tree with n nodes, index i
Output tree with ith item moved to the root
m=#nodes(left(tree))
if i < m then
  left(tree)=partition(left(tree),i)
  tree=rotateRight(tree)
else if i > m then
  right(tree)=partition(right(tree),i-m-1)
  tree=rotateLeft(tree)
end if
return tree
```

```
TreeTraversal(tree,style):
Input tree, style of traversal
if tree is not empty then
  if style="NLR" then
    visit(data(tree))
  end if
  TreeTraversal(left(tree),style)
  if style="LNR" then
    visit(data(tree))
  end if
  TreeTraversal(right(tree),style)
  if style="LRN" then
    visit(data(tree))
  end if
end if
```

```
int TreeHeight(Tree t) {
  if (t == NULL) {
    return -1;
  } else {
    int lheight = 1 + TreeHeight(left(t));
    int rheight = 1 + TreeHeight(right(t));
    if (lheight > rheight)
      return lheight;
    else
      return rheight;
  }
}
```

```
NewTreeInsert(tree,item):
Input tree, item
Output tree with item randomly inserted
t=insertAtLeaf(tree,item)
if #nodes(t) mod k = 0 then
  rebalance(t)
end if
return t
```

E.g. rebalance after every 20 insertions \Rightarrow choose $k=20$

... Rebalancing Trees

Implementation of rebalance:

```
rebalance(t):
Input tree t with n nodes
Output t rebalanced
if n%3 then
  t=partition(t,n/2)
  left(t)=rebalance(left(t))
  right(t)=rebalance(right(t))
end if
return t
```

10. random insert $O(\log n)$ worst is $O(n)$, splay tree search $O(n)$ 找到移到根, 否则移最接近的, AVL search $O(\log n)$, 234 search $O(\text{height} = \log n)$, red-black search $O(\log n)$ 最多 2h 旋转

```
insertSplay(tree,item):
Input tree, item
Output tree with item splay-inserted
if tree is empty then return new node containing item
else if item=data(tree) then return tree
else if item<data(tree) then
  if left(tree) is empty then
    left(tree)=new node containing item
  else if item<data(left(tree)) then
    // Case 1: left-child of left-child "zig-zig"
    left(left(tree))=insertSplay(left(left(tree)),item)
    tree=rotateRight(tree)
  else if item>data(left(tree)) then
    // Case 2: right-child of left-child "zig-zag"
    right(left(tree))=insertSplay(right(left(tree)),item)
    left(tree)=rotateLeft(left(tree))
  end if
  return rotateRight(tree)
else // item>data(tree)
  if right(tree) is empty then
    right(tree)=new node containing item
  else if item>data(right(tree)) then
    // Case 3: left-child of right-child "zag-zig"
    left(right(tree))=insertSplay(left(right(tree)),item)
    right(tree)=rotateLeft(right(tree))
  else if item<data(right(tree)) then
    // Case 4: right-child of right-child "zag-zag"
    right(right(tree))=insertSplay(right(right(tree)),item)
    tree=rotateLeft(tree)
  end if
  return rotateLeft(tree)
end if
```

```
insertRB(tree,item):
Input 2-3-4 tree, item
Output tree with item inserted
node=root(tree), parent=NULL
repeat
  if node.order=4 then
    promote = node.data[1] // middle value
    nodeL = new node containing node.data[0]
    nodeR = new node containing node.data[2]
    if parent=NULL then
      make new 2-node root with promote,nodeL,nodeR
    else
      insert promote,nodeL,nodeR into parent
      increment parent.order
    end if
    node=parent
  end if
  if node is a leaf then
    insert item into node
    increment node.order
  else
    parent=node
    if item<node.data[0] then
      node=node.child[0]
    else if item<node.data[1] then
      node=node.child[1]
    else
      node=node.child[2]
    end if
  end if
until item inserted
```

```
insertAVL(tree,item):
Input tree, item
Output tree with item AVL-inserted
if tree is empty then
  return new node containing item
else if item=data(tree) then
  return tree
else
  if item<data(tree) then
    left(tree)=insertAVL(left(tree),item)
  else if item>data(tree) then
    right(tree)=insertAVL(right(tree),item)
  end if
  if height(left(tree))-height(right(tree)) > 1 then
    if item<data(left(tree)) then
      left(tree)=rotateLeft(left(tree))
    end if
    tree=rotateRight(tree)
  else if height(right(tree))-height(left(tree)) > 1 then
    if item>data(right(tree)) then
      right(tree)=rotateRight(right(tree))
    end if
    tree=rotateLeft(tree)
  end if
  return tree
end if
```

... Red-Black Tree Insertion

High-level description of insertion algorithm:

```
insertRB(tree,item,inRight):
Input tree, item, inRight indicating direction of last branch
Output tree with item inserted
if tree is empty then
  return newNode(item)
end if
if left(tree) and right(tree) both are RED then
  split 4-node
end if
recursive insert cases (cf. regular BST)
re-arrange links/colours after insert
return modified tree
insertRedBlack(tree,item):
Input red-black tree, item
Output tree with item inserted
tree=insertRB(tree,item,false)
colour(tree)=BLACK
return tree
```

split-4: 红黑红 变黑黑黑 re-arrange: 右红左红用旋转变右红右红, 黑右红右红变红黑黑

11. string: naïve matching $O(n^2)$, Boyer-Moore $O(mn+s)$, KMP $O(m+n)$, Trie: $O(d \cdot m)$ $d=26$ 一般, 红色框是 finish 一个单词, 后缀压缩, suffix $O(n \cdot m)$, Huffman $O(n \cdot \log n)$

... Boyer-Moore Algorithm

```
BoyerMooreMatch(T,P,i):
Input text T of length n, pattern P of length m, alphabet  $\Sigma$ 
Output starting index of a substring of T equal to P
-1 if no such substring exists
[=lastOccurrenceFunction(P,i)] // start at end of pattern
repeat
  if T[i]=P[1] then
    if j=0 then
      return i // match found at i
    else
      j=i-1, j=j-1
    end if
  else
    i=i+min(j, lastOccurrence(P[i],T[i]))
    j=j-1
  end if
until i=n
return -1 // no match
```

• Biggest jump (in characters ahead) occurs when $L[T[i]] = -1$

```
Traverse a path, using char-by-char from Key:
find(trie,key):
Input trie, key
Output pointer to element in trie if key found
NULL otherwise
node=trie
for each char in key do
  if node.child[char] exists then
    node=node.child[char] // move down one level
  else
    return NULL
  end if
end for
if node.finish then // "finishing" node reached?
  return node
else
  return NULL
end if
```

... Trie Operations

```
insert(trie,item,key):
Input trie, item with key of length m
Output trie with item inserted
if trie is empty then
  t=new trie node
end if
if m=0 then
  t.finish=true, t.data=item
else
  t.child[key[0]]=insert(t.child[key[0]],item,key[1..m-1])
end if
return t
```

12. random and approximation

找 0 值就是 $x1 \cdot mid \cdot 0$, 让 $x1=mid$ 否则 $x2=mid$, 曲线长度用 $a=(end-start)/step$ 为一个段, 之后每段曲线长度是在一段里两点间距离公式, 定点覆盖, 最少点集合 c 覆盖全部边 e . $O(r \cdot v)$

$Xn+1=(a \cdot Xn+c) \bmod m$

Quicksort 是选第一个为 $low=pivot$, $left$ = 最左边 $low+1$ 开始找比 low 大的, $right$ 是最右边比 low 等于小的, 如果 $left > right$, 那么 swap 他们, 再循环知道 $left > right$. 第一个等于 $right$, $right = pivot$. 再 sorted 左半边右半边 $O(n \log n)$. 随机 quick 是从 low 到 $high$ 随机选取一个 low . 之后交换第一个与 low , 其他一样与之前一样. $O(n \log n)$.

MinMax cut 压缩是随机选两个点, 变成一个点, 把所有链接两个点的边连接到新点直到剩下两个点为两组 $O(B)$. Karger 算法是重复压缩 d 次 $d=(\frac{1}{\epsilon} \cdot \frac{1}{n} \cdot \log \frac{1}{\epsilon})$, 在这之中找最小的 cut. $O(\epsilon \cdot v^2 \cdot \log v)$.