

BOURNEMOUTH UNIVERSITY

TRANSFER DOCUMENT

**From Brush Painting to
Bas-relief**

Author:
Yunfei FU

First Supervisor:
Dr. Hongchuan YU
Second Supervisor:
Pro. Jian Jun ZHANG

October 28, 2017

ABSTRACT

Among all forms of sculpture, bas-relief is arguably the closest to painting. We present a new approach for generating a bas-relief from a single 2d painting. We do not aim to recover exact depth of a painting, which is a tricky computer vision problem, requiring assumptions that are rarely satisfied. Instead, our approach exploits the concept of brush strokes, making each brush stroke possible to generate a correspond bas-relief proxies (depth map of strokes), used for bas-relief editing. To segment brush strokes in 2D paintings, we reformulate layer decomposition and stroke segmentation by imposing boundary constraints, which can make strokes smooth and complete. The resulting brush strokes are sufficient to evoke the impression of the consistent 3D shapes on bas-relief, so that they may be further edited in 3D space. This fulfills the request of recomposition in bas-relief design. Currently, our research focus on brush paintings with relatively sparse and clear strokes.

This accomplished part of the research leads to a few future works as well, such as,

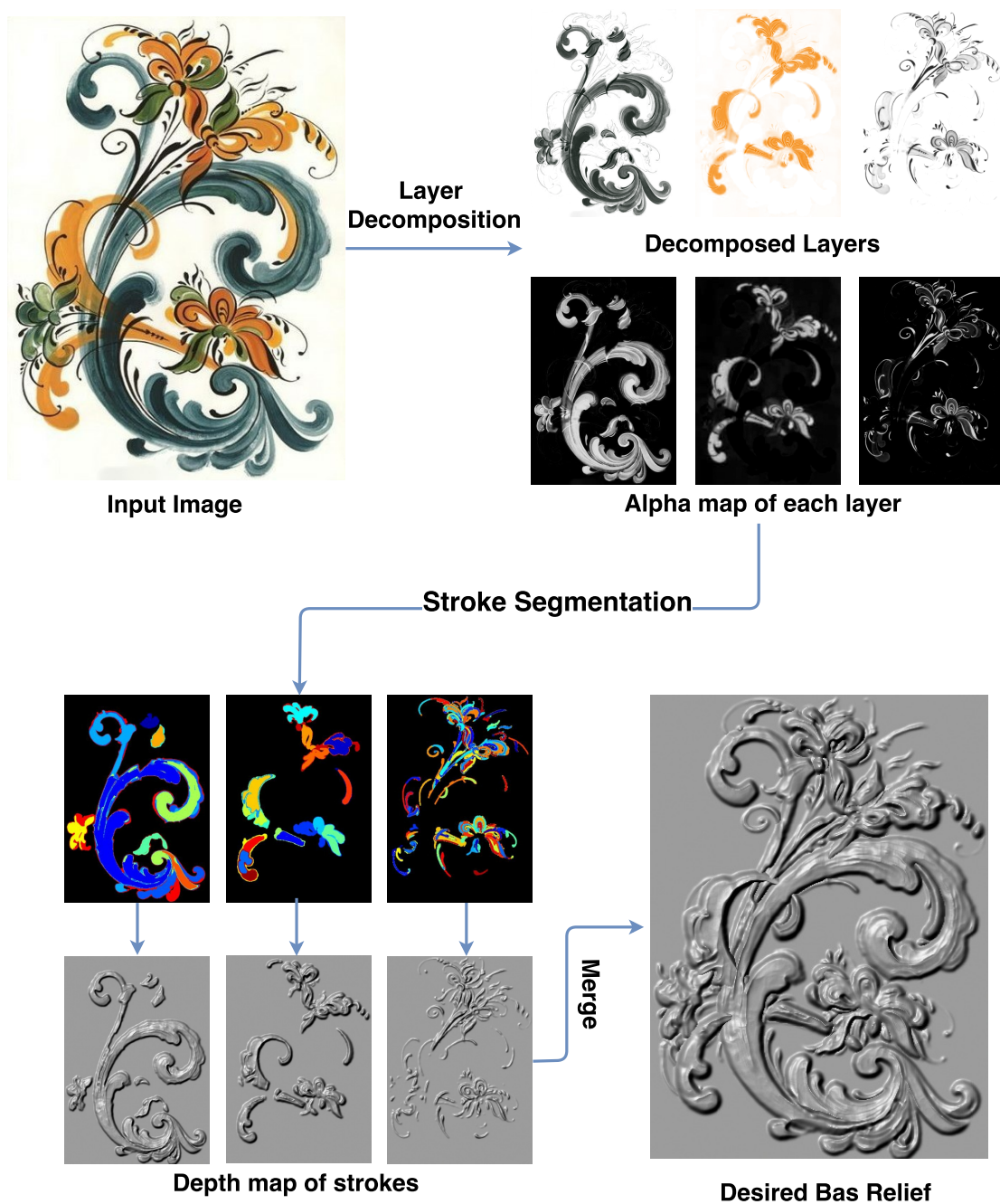


Figure 1: Pipeline Overview

LIST OF BASIC TERMINOLOGY

There are other terms to consider, but many of the important ones are explained in the report where relevant. These are terms not explicitly explained elsewhere.

Height/Depth map: A depth map is an image or image channel that contains information relating to the distance of the surfaces of scene objects from a viewpoint.

Intensity: The measure of brightness of images. In most applications, measured between 0 (dark) and 255 (bright).

Brushstroke: Brushstrokes are the marks made on a surface by a painter's brush.

Contents

1	INTRODUCTION	6
1.1	Background	6
1.2	Motivation	8
1.3	Contribution of Current Work	8
1.4	Document Structure	9
2	INTRODUCTION	10
2.1	Thesis Structure	12
3	LITERATURE REVIEW	14
3.1	Stroke Segmentation	14
3.2	Bas-Relief generation from 2D images	15
3.2.1	Gradient and intensity based methods	15
3.2.2	Shape from shading(SFS) based methods	16
3.2.3	Line drawing based methods	16

List of Figures

1	Pipeline Overview	2
2.1	Bas-relief	10

Chapter 1

INTRODUCTION

1.1 Background

Relief is a kind of sculpture in which 3D models are carved into a relatively flat surface. In essence, it creates a bridge between a full 3D sculpture and a 2D painting. As an artistic form, relief spans the continuum between a 2D painting and a full 3D sculpture[1]. On this spectrum, alto-relievo (high relief) is closest to full 3D, whereas flatter artworks are described as basso-relievo (low relief, and also called bas-relief). Among all the sculpture forms, bas-relief is arguably the closest to 2D paintings, as claimed by[3] and[9].

Bas-relief sculpting has been practiced for thousands of years. Since antiquity, artisans from many ancient cultures (including Greek, Persian, Egyptian, Mayan, and Indian art) have created bas-reliefs.

Today bas-reliefs are commonly found in a variety of media, commemorative medals, coins, souvenirs, 2.5D animation, and artistic sculptures for blind people. However, crafting bas-reliefs is a laborious, challenging and time consuming process. There are many ways to reach the same goals more easily with the help of computers. As mentioned above, relief spans the continuum between a 2D painting and a full 3D sculpture [1]. In general, the bas-relief generation methods can be classified in two different categories:

- 3D model based : using a 3D model (sculpture) as input
- 2D image based : using a 2D image as input

3D model based: Most existing bas-relief production methods have demonstrate how to compress 3D sculpture(model) into bas-relief [1]and[3]. This approach requires a 3D model as a starting point.

2D image based: There have been some bas-relief production approaches available based on 2D image in [4][5] and [6]. These approaches almost follow the “bas-relief ambiguity” [7], that is, roughly speaking, from a frontal view the sculpture looks like a full 3D object while a side-view reveals the disproportional depth. However, these image based methods are focusing on general photograph, which is unsuited for specific problems for paintings, such as stroke overlapping and its transparency.

Little is done in the area of bas-relief generation from artistic paintings, as maintaining the information of the brush paintings proves much trickier than simply manipulating the height of the contour lines. In the case of bas-reliefs, although there is no 3D model available, pseudo 3D effect reflecting the style and subtlety is crucial in preserving the artistic essence.

The aim of our research is to provide a new tool allowing user to convert existing brush paintings to bas-reliefs. We also argue that because traditional paintings are produced with individual strokes, ‘3D bas-relief strokes’ will enable them to ‘paint/sculpt’ a bas-relief naturally, especially if they wanted to quickly convert an existing painting into a relief. With the commonly and cheaply available 3D printing facilities, there is a growing trend in the need of bas-relief art products.

A brush painting can be regarded as the union of a set of hypothetical strokes by a brush [8]. Differing from the other bas-relief generation methods, our method will honor this very feature by constructing the brush strokes individually as 3D geometric entities. This however demands to conquer several challenges. First, each brush stroke covers a region on the canvas and they may overlap each other, some quite heavily in a painting. To make sure the information is retained, every stroke has to be faithfully extracted. Second, spatial occlusion has to be dealt with, since artists are used to depicting it through controlling the transparency of strokes as one of the art elements. Third, as an artistic tool, the generated bas-relief should be further editable allowing the artist to rearrange, tweaking and reshaping the extracted strokes.

The shape, color and opacity of a stroke vary due to the shape and firmness of the brush as well as the forces the artist imposes. Although these variables add the complexity to stroke extraction, stylized strokes often follow distinct patterns. For example, Rosemaling paintings, a typical example of brush painting popular in North Europe, is a traditional form of decorative folk art that originated in the rural valleys of Norway. The Rosemaling designs use C and S strokes, feature scroll, flowing lines, floral designs, and both subtle and vibrant colors. The brush strokes may further be viewed as graphical objects which are meaningful with respect to the objects the painting portrays. Moreover, each stroke is clearly visible due to both subtle

and vibrant colors. The similar properties may be found in some Chinese brush paints.

To extract the strokes from a brush painting, we need to identify and segment the overlapped strokes. We will then generate the depth map for every stroke separately using the shape from shading (SFS) technique on the opacity. All the strokes are finally merged together to yield the resulting bas-relief with the original 2D painting preserved.

1.2 Motivation

Relief belong to a category of art that bridges the gap between 2D painting and 3D sculpting[2].

Bas-relief generation has wide applications in making objects such as commemorative medals, coins, souvenirs, and artistic sculptures for blind people, while current methods are not suitable for bas-relief generation based on Chinese brush paintings.

Previous researches Bas-relief is an art form part way between sculpture and drawing. [4]

1.3 Contribution of Current Work

Our contributions include,

- (1) Extraction of brush strokes. We develop a novel method to extract brush strokes from input paintings with palette analysis and decomposed layers.
- (2) 3D modeling of brush strokes. We develop a novel method which may entirely construct every stroke to bas-relief based on the opacity of paintings.
- (3) Recomposition in bas-relief design. Artists may redefine the brush strokes' order and shapes by sketches, which enable recomposition in bas-relief design, making it a useful tool for sculpture artists.

Overview As showed in Figure 1, the approach performs in three steps:

First, based on the point cloud of the input image in RGB space (Figure ??), we select the palette colors of the input brush painting, and based on the palette colors we decompose a input image into different layers with transparency.

Second, the original painting is decomposed into element brush strokes by applying modified MSERs segmentation. Based on our observation, alpha map is more suitable for our MSERs segmentation, so the segmentation is based on the transparency. Third, based on the segmented brush strokes, Shape from Shading algorithm has been applied to generate the depth map

of each stroke.

Finally, based on those depth maps of strokes , we can edit the generated bas relief. For editing,we have already generated the 3D proxies for each MSER region, namely, the extracted 2D brush region, so, we can change the depth of specific stroke on bas relief,we can also stitch them on each other based on the user input and change the shape of a certain stroke with given indicated skeleton. By doing so, we fulfill the request of recomposition in bas-relief design.The resulting 3D proxies of brush strokes are sufficient to evoke the impression of the consistent 3D shapes,so that they may be further edited in 3D space.

Currently, our research focus on brush paintings with relatively sparse strokes. Experiments show that our method can effectively generate digital bas-reliefs for a range of input images, including some Chinese paintings and rose-mailing paintings. We also demonstrate the utility of the resulting decompositions for image recoloring and image object insertion and animation.

1.4 Document Structure

Chapter 2

INTRODUCTION

Relief sculpting has been practiced for thousands of years. Since antiquity, artisans from many ancient cultures (including Greek, Persian, Egyptian, Mayan, and Indian art) have created bas-reliefs (Figure 2.1). Today bas-reliefs are commonly found in a variety of media in architecture, industrial design and coins. However, the production of bas-reliefs is currently a costly and time-consuming process, requiring skilled sculptors and engravers. Even with the advent of computer-driven techniques providing a foundation for automation in bas-relief, making the design of bas-relief sculptures remains largely in the hands of artists.

As an artistic form, relief spans the continuum between a 2D painting and a full 3D sculpture[1]. On this spectrum, alto-relievo (high relief) is closest to full 3D, whereas flatter artworks are described as basso-relievo (low relief, and also called bas-relief), as showed in Figure 2.1. Among all the sculpture forms, bas-relief is arguably the closest to 2D paintings, as claimed by[3] and[9].



Figure 2.1: Bas-relief

Currently, most existing bas-relief production methods focus on com-

pressing 3D scenes/models into surfaces with a small depth[1]and[3]. This approach requires a 3D model as a starting point.

Another option is to generate bas-reliefs directly from 2D images. There have been some image based bas-relief production approaches available in [4][5] and [6]. These approaches almost follow the “bas-relief ambiguity” [7], that is, roughly speaking, from a frontal view the sculpture looks like a full 3D object while a side-view reveals the disproportional depth.

While most image based methods are focusing on general photographs, which are unsuited for specific problems for brush paintings, such as spatial occlusion (i.e. one stroke occludes other strokes in the painting to demonstrate the depth perception) and stroke transparency. Another clear shortcoming is current image based methods can’t take the artistic intent into account, as all what they do is to infer the height information from the image. Concerning reproduction or modifying of an artistic painting, it is crucial that the style of the originals is preserved, which is not considered in existing image-based methods. However, little is done in the area of bas-relief generation from artistic paintings, as maintaining the styles of the brush paintings proves much trickier than simply manipulating the height of the contour lines. In the case of bas-reliefs, although there is no 3D model available, pseudo 3D effect reflecting the style and subtlety is crucial in preserving the artistic essence.

The aim of our research is to provide the bas-relief sculptors with a new tool allowing them to convert and recompose existing brush paintings to bas-reliefs. We also argue that because traditional paintings are produced with individual strokes, ‘3D bas-relief strokes’ will enable them to ‘paint/sculpt’ a bas-relief naturally, especially if they wanted to quickly convert an existing painting into a relief. With the commonly and cheaply available 3D printing facilities, there is a growing trend in the need of bas-relief art products. A brush painting can be regarded as the union of a set of hypothetical strokes by a brush [8]. Differing from the other bas-relief generation methods, our method will honor this very feature by constructing the brush strokes individually as 3D geometric entities. This however demands to conquer several challenges. First, each brush stroke covers a region on the canvas and they may overlap each other, some quite heavily in a painting. To make sure the information is retained, every stroke has to be faithfully extracted. Second, spatial occlusion has to be dealt with, since artists are used to depicting it through controlling the transparency of strokes as one of the art elements. Third, as an artistic tool, the generated bas-relief should be further editable allowing the artist to rearrange, tweaking and reshaping the extracted strokes.

The shape, color and opacity of a stroke vary due to the shape and firm-

ness of the brush as well as the forces the artist imposes. Although these variables add the complexity to stroke extraction, stylized strokes often follow distinct patterns. For example, Rosemaling paintings, a typical example of brush painting popular in North Europe, is a traditional form of decorative folk art that originated in the rural valleys of Norway. The Rosemaling designs use C and S strokes, feature scroll, flowing lines, floral designs, and both subtle and vibrant colors. The brush strokes may further be viewed as graphical objects which are meaningful with respect to the objects the painting portrays. Moreover, each stroke is clearly visible due to both subtle and vibrant colors. The similar properties may be found in some Chinese brush paints. To extract the strokes from a brush painting, we need to identify and segment the overlapped strokes. We will then generate the depth map for every stroke separately using the shape from shading (SFS) technique on the opacity. All the strokes are finally merged together to yield the resulting bas-relief with the original 2D painting preserved. Our contributions include,

- (1) Extraction of brush strokes. We develop a novel method to extract brush strokes from input paintings with palette analysis and decomposed layers.
- (2) 3D modeling of brush strokes. We develop a novel method which may entirely construct every stroke in 3D based on the opacity of paintings.
- (3) Recomposition in bas-relief design. Artists may redefine the brush strokes' order and shapes by sketches, which enable recomposition in bas-relief design, making it a useful tool for sculpture artists.

2.1 Thesis Structure

The organization of the document is as follows:

Chapter 1: Introduction. This section provides the background, the motivation and the contribution made in current research.

Chapter 2: Literature Review. This section classifies and reviews related previous works in a systematic way.

Chapter 3: Overview of proposed approach. This section explains the methodology selected and defines some basic concepts used in the research.

Chapter 4: Layer Decomposition. This section introduces concept "Layer Decomposition" in this research, which is the basis of the proposed algorithm.

Chapter 5: Extraction of Brush strokes. This section describes the proposed algorithm of brush stroke extraction.

Chapter 6: Depth map and '3D strokes'. This section shows how to generate the individual depth maps of extracted strokes.

Chapter 7: Results. This section reviews the up-to-date results based on proposed the method.

Chapter 8: Conclusions . This section concludes the progress up-to-date.

Chapter 9: Future work. This section discusses possible future work about high relief and 3D painting.

Chapter 3

LITERATURE REVIEW

3.1 Stroke Segmentation

To the best of our knowledge, there is lack of study on extracting brush strokes from paintings. We give a brief overview of the work related to the relevant topics, i.e. decomposing images into layers and stroke segmentation, which are employed in our implementation. In digital image editing, layers organize images. However, scanned paintings and photographs have no such layers. Without layers, simple edits may become very challenging.

Richardt et al.[10] present an approach to produce editable vector graphics, in which the selected region is decomposed into a linear or radial gradient and the residual, background pixels. [11],[12]present two generalized layer decomposition methods, which allow pixels to have independent layer orders and layers to partially overlap each other.[13] present a layer decomposition method based on RGB-space geometry. They assume that all possible image colors are convex combinations of the paint colors. Computing the convex hull of image colors and per-pixel layer opacities is converted into a convex optimization problem. Thus, their method can work well without prior knowledge of shape and overlap of strokes.

Li et al.[14] describe a method based on seed growing for brush stroke segmentation. Starting from seed coordinates, neighboring pixels are visited by exploiting a region-growing-based approach with a variable threshold, which is initialized at a predefined and automatic updated by a shape validation method. But their brush stroke segmentation is limited to finding brush strokes in Van Gogh's paintings, and it does not support segmenting overlapped strokes.

Xu et al.[8] aims at decomposing Chinese paintings into a collection of layered brush strokes, with an assumption that at most two strokes are over-

lapping. And their approach requires a good amount of prior knowledge of shape and order of strokes. The segmentation of brush strokes is based on using a brush stroke library. While our method needs no brush stroke library, and our segmentation is based on multiple layers which more than two brush strokes could overlap each others (Figure ??).

Most Stable Extremal Regions (MSERs) algorithm [15] has been proved to be a very efficient way in stroke segmentation from scene text images [16][17], and the revised version has been widely used in stroke segmentation of hand-written characters [18].

The Most Stable Extremal Regions (MSERs) algorithm [15] was used for establishing correspondence in wide-baseline stereo. [19] introduced the data structure of the component tree in it and further developed it as an efficient segmentation approach, which prunes the component tree and selects only the regions with a stable shape within a range of level sets, see Section 5.1.

However, it is likely that MSERs may fail in segmentation with the following scenarios,

- (1) two adjacent regions with the similar intensity;
- (2) the region with a high transparency.
- (3) Moreover, like the other existing segmentation approaches, the MSERs algorithm encounters over-segmentation issue as well. To tackle these challenges, the coherent lines [20] is introduced into MSERs in our algorithm, which both enhances the edges of strokes and preserves the completeness of strokes, see Section 5.2.

3.2 Bas-Relief generation from 2D images

Automatic bas-relief generation from 2D image has recently become a significant research topic. Recently, researches have explored recovering depth information from images specifically for the purpose of generating bas-reliefs.

3.2.1 Gradient and intensity based methods

Wang et al. [21] demonstrate a method by constructing bas-reliefs from 2D images based on gradient operations. In their research image gradients was first calculated, then by smoothing gradients to smooth shape changes. Finally, they boost fine features by masks. The height image was constructed modified height map. The pixel heights are compressed, and a triangle mesh representing the bas-relief is determined by placing a vertex at each pixel position. By proposed algorithm most features can be preserve, but no con-

sideration is made for the overlapping relationship between different image regions, which is not suitable for brush paintings.

[22] present a two-level approach for height map estimation from the rubbing images of restoring brick and stone relief. The relief is separated into low and high frequency components. The base relief of the low frequency component is estimated automatically with a partial differential equation (PDE)-based mesh deformation scheme. The high frequency detail is estimated directly from rubbing images automatically or optionally with minimal interactive processing. This method works well for reliefs based on brick or stone, but is unsuited to general photographs or brush paintings.

3.2.2 Shape from shading(SFS) based methods

Some work uses Shape from shading for generating bas-relief from 2D photograph. SFS is a relative classic way for 3D shape recovery; see Zhang’s survey [23]. The computation process normally involved with several concepts : depth $Z(x, y)$, surface normal n_x, n_y, n_z , and surface gradient p, q . The depth can either be considered as distance from viewpoint to query surface or the height from surface to default $x - y$ plane. The normal is perpendicular to the surface gradient, namely $(n_x, n_y, n_z) * (p, q, 1)^T = 0$, the surface gradient is the changing rate of depth in x and y direction.

Shape from shading (SFS) is able to recover the shape of an object from a given single image, assuming illumination and reflectance are known (or assuming reflectance is uniform across the entire image). Many methods have been developed, which may be categorized into four PDEs models[24], (1) orthographic SFS with a far light source [25]; (2) perspective SFS with a far light source[26]; (3) perspective SFS with a point light source at the optical center[24]; (4) a generic Hamiltonian. However, SFS is an ill-posed problem. The notable difficulty in SFS is the bas-relief ambiguity[7], that is, the absolute orientation and scaling of a surface are ambiguous given only shading information. To amend it, many SFS algorithms impose priors on shape, depth cues produced by a stereo system, or assume that the light source, the surface reflectance, and the camera are known[23]; [27]; [28]; [9];

Unfortunately, SFS based methods is assuming illumination and reflectance are known, and the image is formed from lighting and shading, which works well for realistic photographs rather than paintings.

3.2.3 Line drawing based methods

Rather than generating bas-relief from a photograph, some researches start with line drawings. Kolomenkin et al.[29] aims to reconstruct a model from

a complex drawing that consists of many inter-related strokes. At first, they extract the curves from the image of the drawing. Then, junctions are detected and margins are generated. By analyzing the connectivity between boundaries and curves, they reduce the problem to a constrained topological ordering of a graph. From these boundaries and curves with given depth, they use smooth interpolation across regions to generate the bas-relief surface. Similarly, line labeling methods have been applied for shape construction from line drawings [30][31][32]. A labeling process would classify segmented lines into different labels, such as concave, convex and occluding, and these labels can give clues for the shape generation of bas-relief.

But line drawing based methods are limited to using information contained in a line drawing, which is not suited for brush strokes which contain information such as color, transparency and stroke shape.

Bibliography

- [1] Tim Weyrich, Jia Deng, Connelly Barnes, Szymon Rusinkiewicz, and Adam Finkelstein. Digital bas-relief from 3d scenes. In *ACM Transactions on Graphics (TOG)*, volume 26, page 32. ACM, 2007.
- [2] Jens Kerber, Meili Wang, Jian Chang, Jian J Zhang, Alexander Belyaev, and H-P Seidel. Computer assisted relief generation—a survey. In *Computer Graphics Forum*, volume 31, pages 2363–2377. Wiley Online Library, 2012.
- [3] Jens Kerber, Art Tevs, Alexander Belyaev, Rhaleb Zayer, and Hans-Peter Seidel. Feature sensitive bas relief generation. In *Shape Modeling and Applications, 2009. SMI 2009. IEEE International Conference on*, pages 148–154. IEEE, 2009.
- [4] Qiong Zeng, Ralph R Martin, Lu Wang, Jonathan A Quinn, Yuhong Sun, and Changhe Tu. Region-based bas-relief generation from a single image. *Graphical Models*, 76(3):140–151, 2014.
- [5] Jing Wu, Ralph R Martin, Paul L Rosin, X-F Sun, Frank C Langbein, Y-K Lai, A David Marshall, and Y-H Liu. Making bas-reliefs from photographs of human faces. *Computer-Aided Design*, 45(3):671–682, 2013.
- [6] Marc Alexa and Wojciech Matusik. Reliefs as images. *ACM Trans. Graph.*, 29(4):60–1, 2010.
- [7] Peter N Belhumeur, David J Kriegman, and Alan L Yuille. The bas-relief ambiguity. *International journal of computer vision*, 35(1):33–44, 1999.
- [8] Songhua Xu, Yingqing Xu, Sing Bing Kang, David H Salesin, Yunhe Pan, and Heung-Yeung Shum. Animating chinese paintings through stroke-based decomposition. *ACM Transactions on Graphics (TOG)*, 25(2):239–267, 2006.

- [9] Jonathan Barron and Jitendra Malik. Color constancy, intrinsic images, and shape estimation. *Computer Vision–ECCV 2012*, pages 57–70, 2012.
- [10] Christian Richardt, Jorge Lopez-Moreno, Adrien Bousseau, Maneesh Agrawala, and George Drettakis. Vectorising bitmaps into semi-transparent gradient layers. In *Computer Graphics Forum*, volume 33, pages 11–19. Wiley Online Library, 2014.
- [11] James McCann and Nancy Pollard. Local layering. *ACM Transactions on Graphics (TOG)*, 28(3):84, 2009.
- [12] James McCann and Nancy S Pollard. Soft stacking. In *Computer Graphics Forum*, volume 31, pages 469–478. Wiley Online Library, 2012.
- [13] Jianchao Tan, Jyh-Ming Lien, and Yotam Gingold. Decomposing images into layers via rgb-space geometry. *ACM Transactions on Graphics (TOG)*, 36(1):7, 2016.
- [14] Jia Li, Lei Yao, Ella Hendriks, and James Z Wang. Rhythmic brushstrokes distinguish van gogh from his contemporaries: findings via automated brushstroke extraction. *IEEE transactions on pattern analysis and machine intelligence*, 34(6):1159–1176, 2012.
- [15] Jiri Matas, Ondrej Chum, Martin Urban, and Tomás Pajdla. Robust wide-baseline stereo from maximally stable extremal regions. *Image and vision computing*, 22(10):761–767, 2004.
- [16] Lukas Neumann and Jiri Matas. Text localization in real-world images using efficiently pruned exhaustive search. In *Document Analysis and Recognition (ICDAR), 2011 International Conference on*, pages 687–691. IEEE, 2011.
- [17] Lluís Gomez and Dimosthenis Karatzas. Multi-script text extraction from natural scenes. In *Document Analysis and Recognition (ICDAR), 2013 12th International Conference on*, pages 467–471. IEEE, 2013.
- [18] Lluís Gomez and Dimosthenis Karatzas. A fast hierarchical method for multi-script and arbitrary oriented scene text extraction. *International Journal on Document Analysis and Recognition (IJDAR)*, 19(4):335–349, 2016.
- [19] Michael Donoser and Horst Bischof. Efficient maximally stable extremal region (mscr) tracking. In *Computer Vision and Pattern Recognition, 2006 IEEE Computer Society Conference on*, volume 1, pages 553–560. IEEE, 2006.

- [20] Henry Kang, Seungyong Lee, and Charles K Chui. Coherent line drawing. In *Proceedings of the 5th international symposium on Non-photorealistic animation and rendering*, pages 43–50. ACM, 2007.
- [21] Meili Wang, Jian Chang, Junjun Pan, and J Zhang. Image-based bas-relief generation with gradient operation. In *Proceedings of the 11th IASTED International Conference Computer Graphics and Imaging, Innsbruck*, volume 679, page 33, 2010.
- [22] Zhuwen Li, Song Wang, Jinhui Yu, and Kwan-Liu Ma. Restoration of brick and stone relief from single rubbing images. *IEEE Transactions on Visualization and Computer Graphics*, 18(2):177–187, 2012.
- [23] Ruo Zhang, Ping-Sing Tsai, James Edwin Cryer, and Mubarak Shah. Shape-from-shading: a survey. *IEEE transactions on pattern analysis and machine intelligence*, 21(8):690–706, 1999.
- [24] Emmanuel Prados and Olivier D Faugeras. ” perspective shape from shading” and viscosity solutions. In *ICCV*, volume 3, page 826, 2003.
- [25] Pierre-Louis Lions, Elisabeth Rouy, and A Tourin. Shape-from-shading, viscosity solutions and edges. *Numerische Mathematik*, 64(1):323–353, 1993.
- [26] Emmanuel Prados and Olivier Faugeras. Unifying approaches and removing unrealistic assumptions in shape from shading: Mathematics can help. *Computer Vision-ECCV 2004*, pages 141–154, 2004.
- [27] Neil G Aldrin, Satya P Mallick, and David J Kriegman. Resolving the generalized bas-relief ambiguity by entropy minimization. In *Computer Vision and Pattern Recognition, 2007. CVPR’07. IEEE Conference on*, pages 1–7. IEEE, 2007.
- [28] Micah K Johnson and Edward H Adelson. Shape estimation in natural illumination. In *Computer Vision and Pattern Recognition (CVPR), 2011 IEEE Conference on*, pages 2553–2560. IEEE, 2011.
- [29] Michael Kolomenkin, George Leifman, Ilan Shimshoni, and Ayellet Tal. Reconstruction of relief objects from line drawings. In *Computer Vision and Pattern Recognition (CVPR), 2011 IEEE Conference on*, pages 993–1000. IEEE, 2011.
- [30] Peter AC Varley and Ralph R Martin. Estimating depth from line drawing. In *Proceedings of the seventh ACM symposium on Solid modeling and applications*, pages 180–191. ACM, 2002.

- [31] Jitendra Malik. Interpreting line drawings of curved objects. *International Journal of Computer Vision*, 1(1):73–103, 1987.
- [32] Daniel Šykora, Ladislav Kavan, Martin Čadík, Ondřej Jamriška, Alec Jacobson, Brian Whited, Maryann Simmons, and Olga Sorkine-Hornung. Ink-and-ray: Bas-relief meshes for adding global illumination effects to hand-drawn characters. *ACM Transactions on Graphics (TOG)*, 33(2):16, 2014.