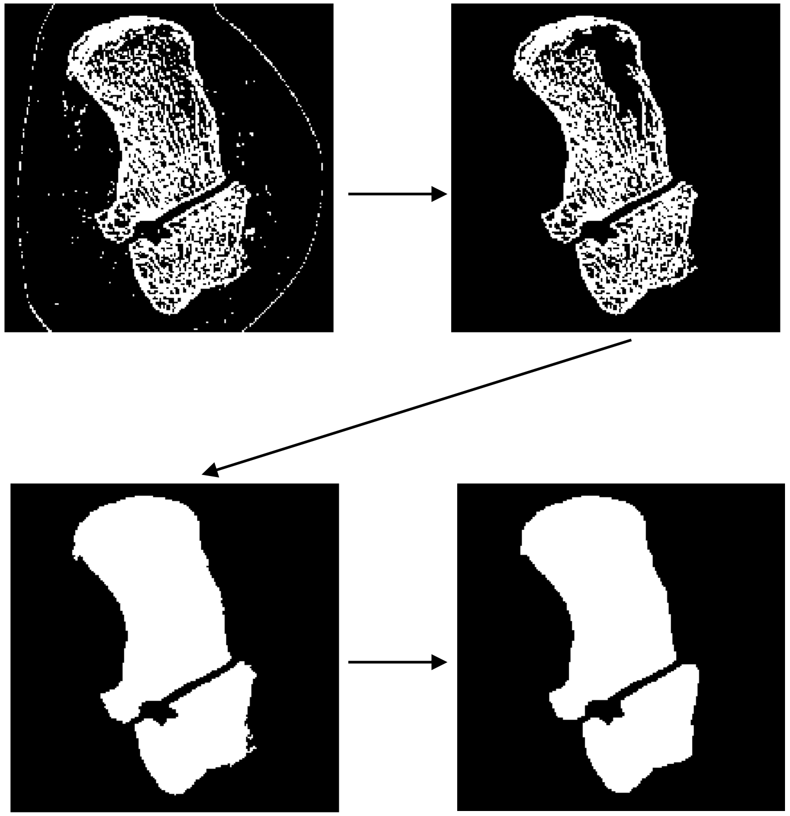
# CSE 554

# Lecture 2: Shape Analysis (Part I)

Fall 2016

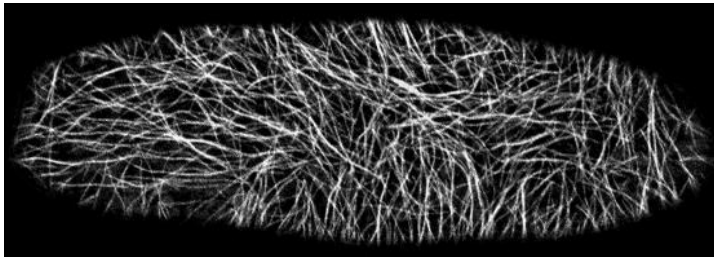
# Review

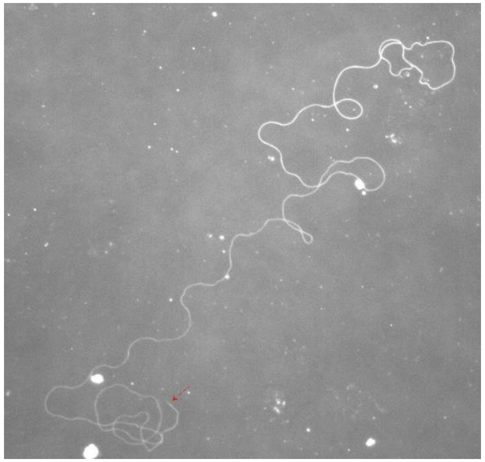
- Binary pictures
  - Tresholding grayscale images
  - Basic operations
    - Connected component labeling
    - Morphological operators



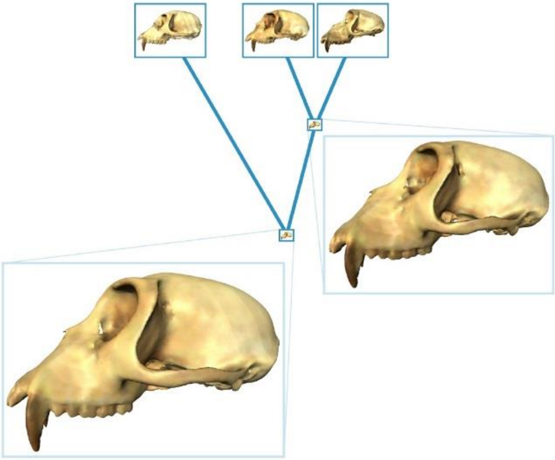
# Shape analysis

- Questions about shapes:
  - Metrics: length? Width? orientation?
  - What are the parts?
  - How similar are two shapes?


Microtubules on the cell surface


Sperms of fruit flies

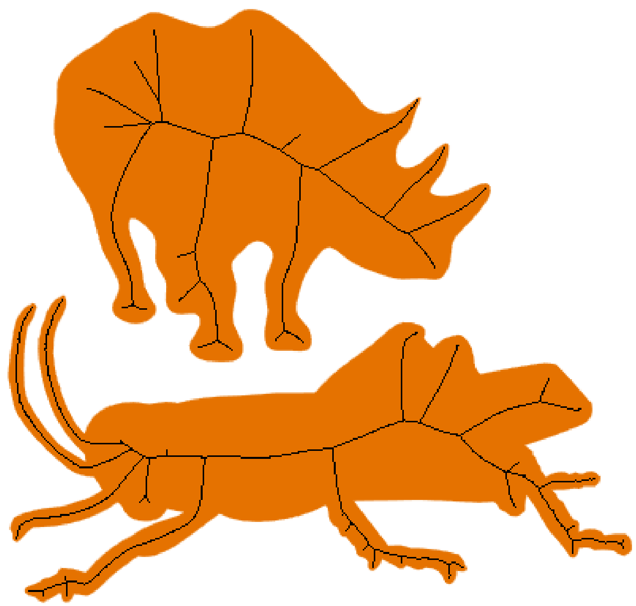

Cerebral artery aneurysms


Monkey skulls

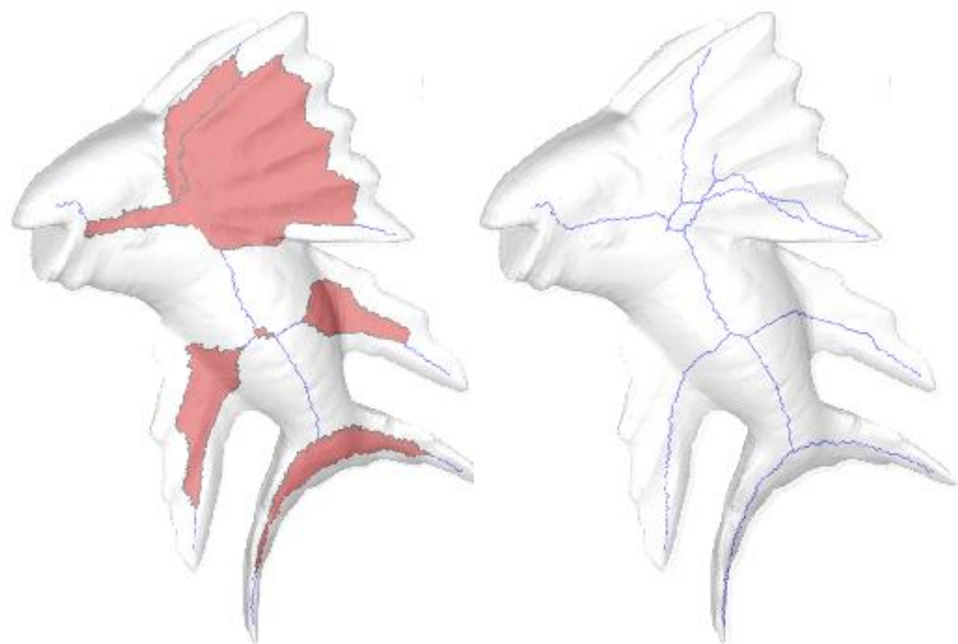
# Skeletons



- Geometry at the *center* of the object
  - Compact, and capturing protruding shape parts



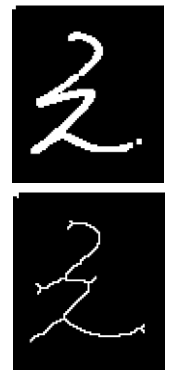
Skeleton of 2D shapes: 1D curves



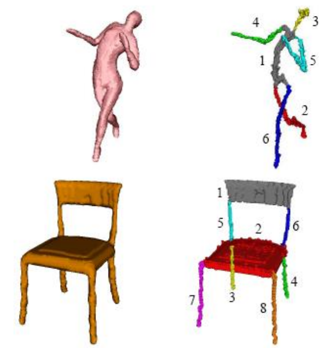
Skeleton of 3D shapes: 1D curves and 2D surfaces

# Applications

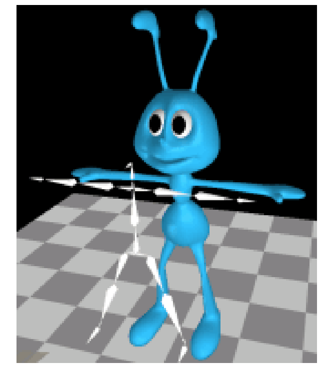
- Computer graphics and vision
  - Optical character recognition (a)
  - Shape retrieval (b)
  - Animating articulated shapes (c)



(a)

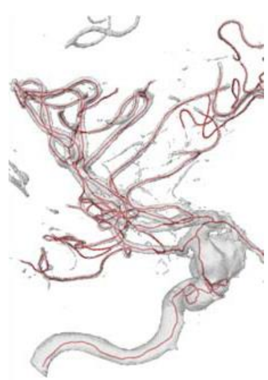


(b)

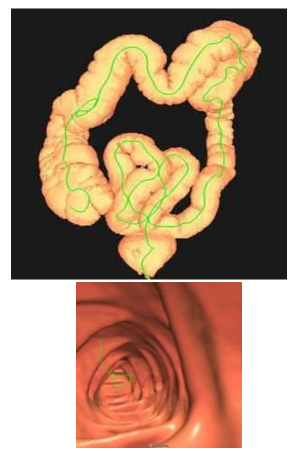


(c)

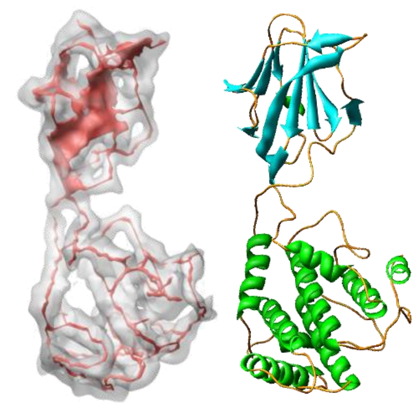
- Bio-medical image analysis
  - Vessel network analysis (d)
  - Virtual colonoscopy (e)
  - Protein modeling (f)



(d)



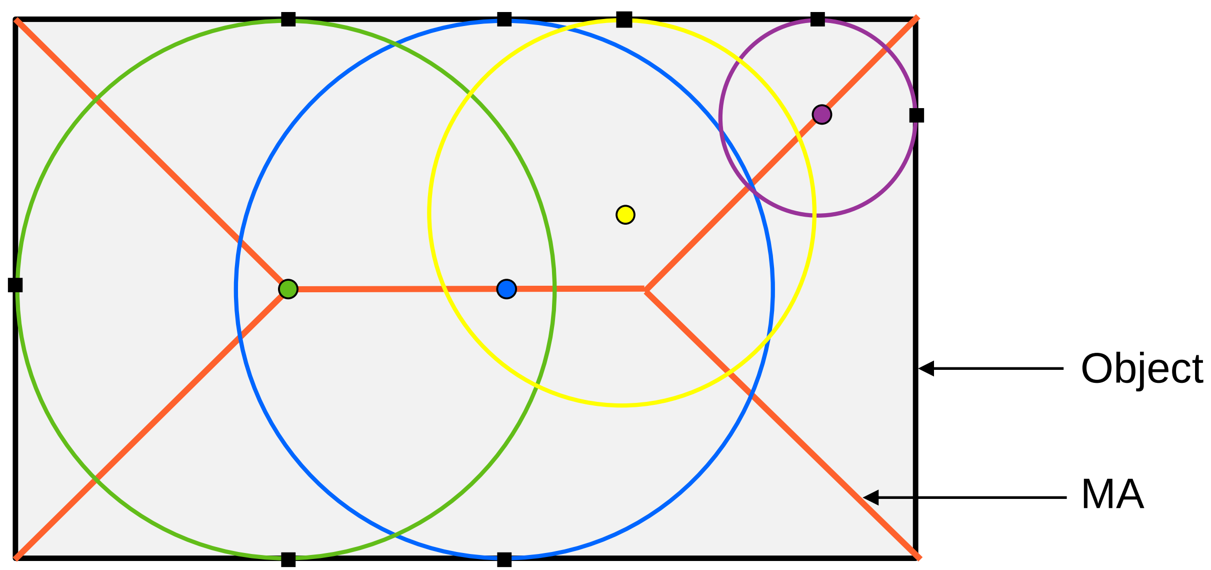
(e)



(f)

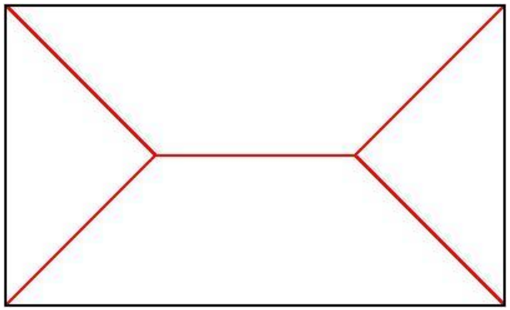
# Medial Axes (MA)

- *Interior points with multiple closest points on the boundary*

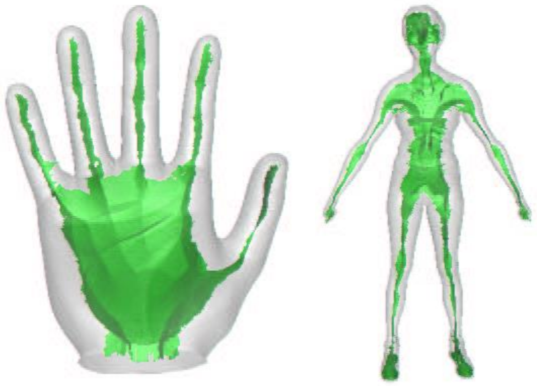


# Medial Axes (MA)

- Properties
  - ✓ Thin
    - MA are curves (1D) in a 2D object, and surfaces (2D) in a 3D object.



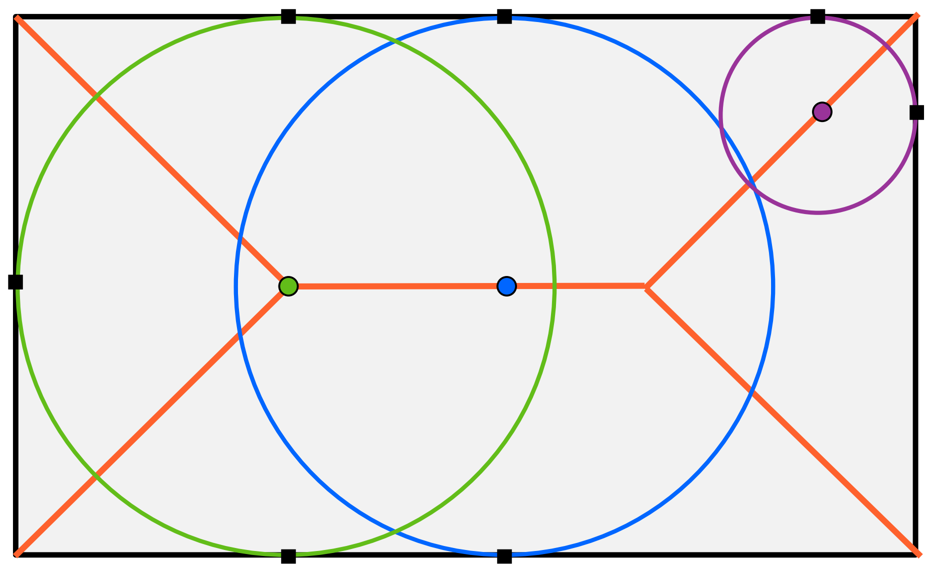
2D MA



3D MA

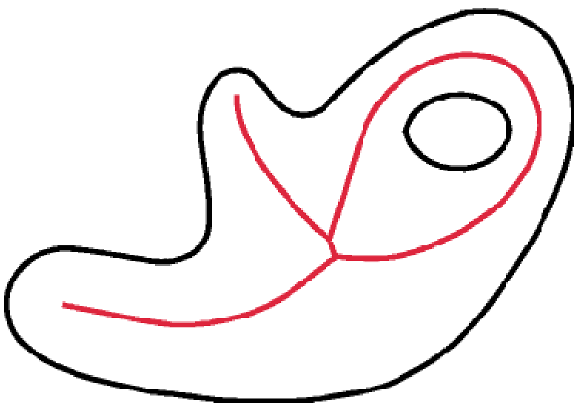
# Medial Axes (MA)

- Properties
  - ✓ Preserves object's shape
    - The object can be reconstructed from MA and its distances to the boundary

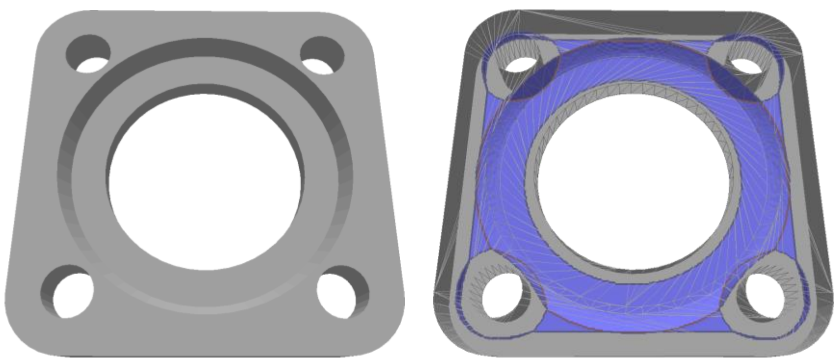


# Medial Axes (MA)

- Properties
  - ✓ Preserves object's topology
    - 2D: # of connected components of object and background
    - 3D: # of connected components of object and background, and # of tunnels



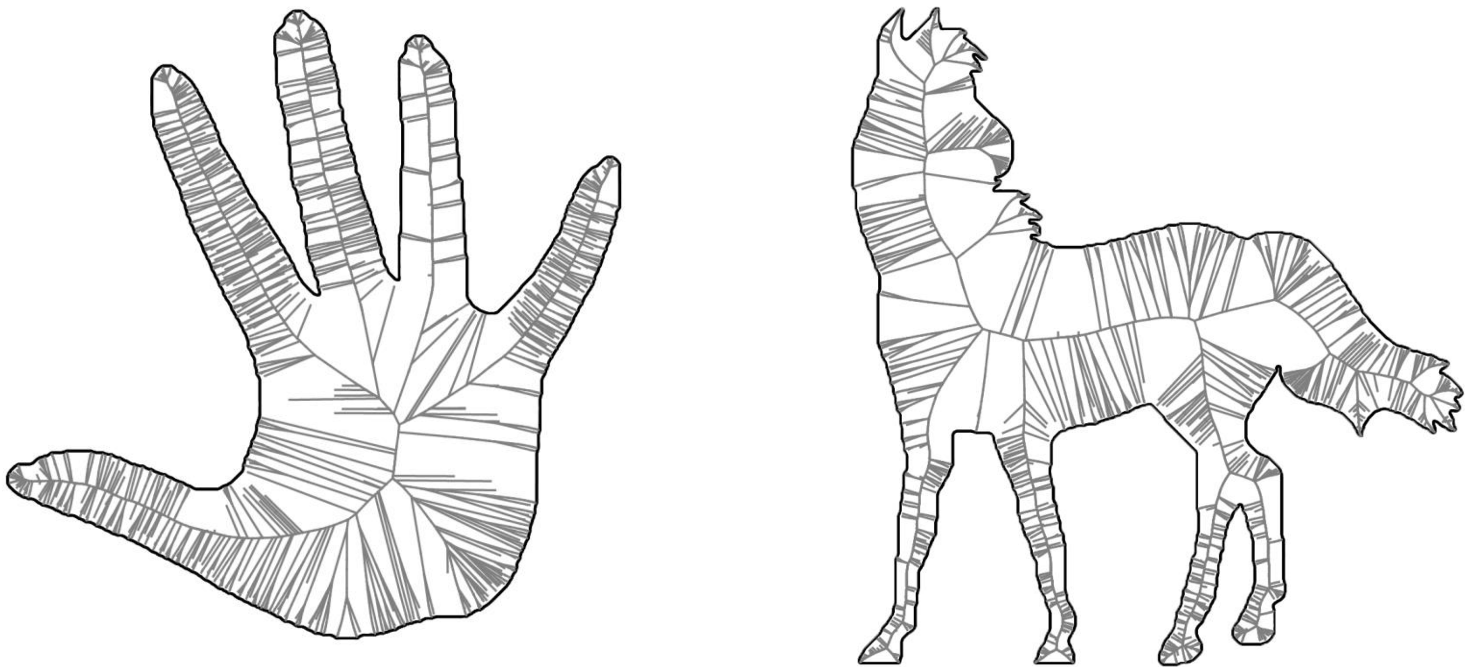
A 2D shape with 1 object component and 2 background components



A 3D shape with 5 tunnels

# Medial Axes (MA)

- Properties
  - ✘ Not stable under boundary perturbation



# Skeletons

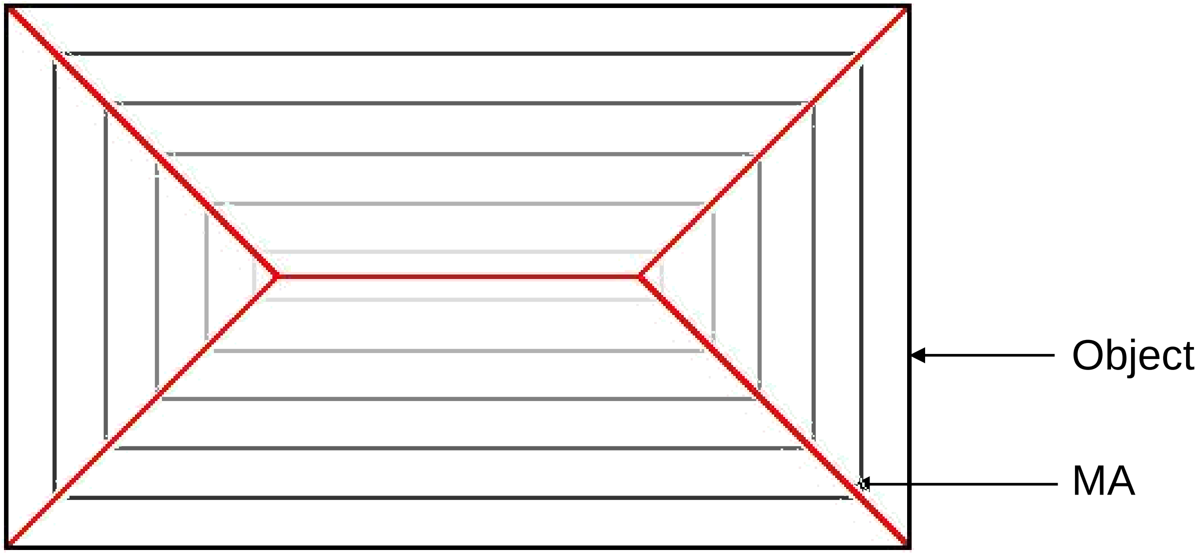
- Approximation of medial axes
  - Roughly corresponds to the stable parts of the medial axes
  - No unique or precise definition (e.g., application dependent)

# Computing Skeletons

- A classical method: thinning
  - Relatively simple to implement
  - Can create curve skeletons in 2D and curve or surface skeletons in 3D
- What we will cover:
  - Thinning on binary pictures (*this lecture*)
  - Thinning on cell complexes (*next lecture*)

# Medial Axes (MA)

- Grassfire analogy:
  - Let the object represent a field of grass. A fire starts at the field boundary, and burns across the field at uniform speed.
  - MA are where the fire fronts meet.



# Medial Axes (MA)

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School of Engineering & Applied Science

- Grassfire analogy:
  - Let the object represent a field of grass. A fire starts at the field boundary, and burns across the field at uniform speed.
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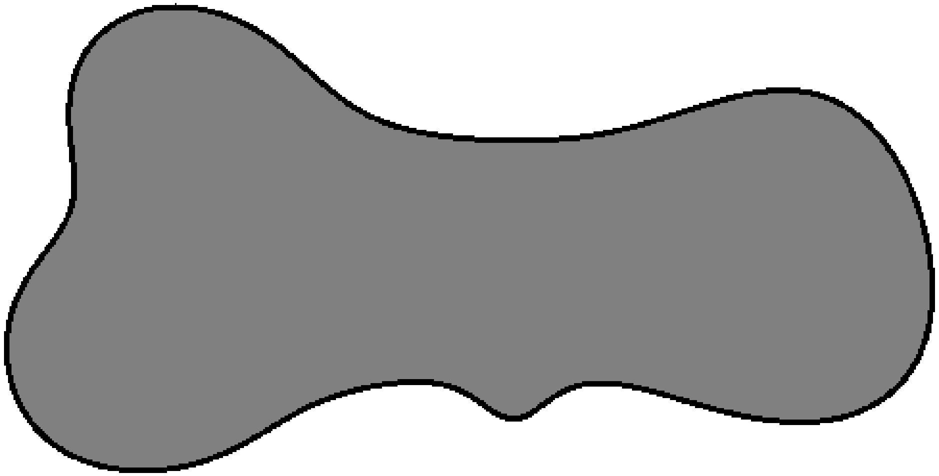
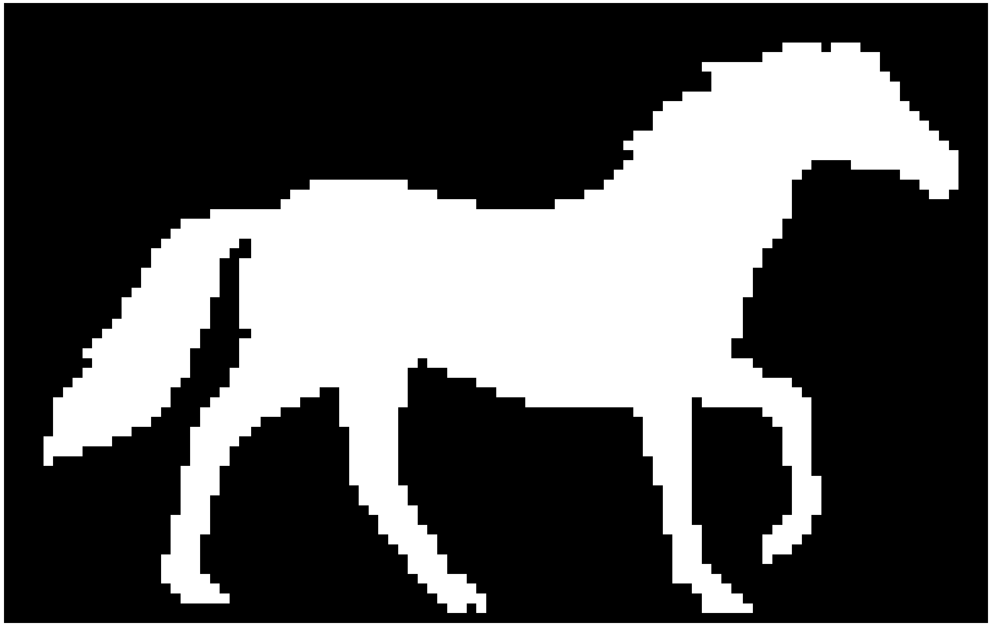


# 2D Thinning

- Iterative process that shrinks a binary picture to a skeleton
  - Simulating the “grassfire burning” that defines MA

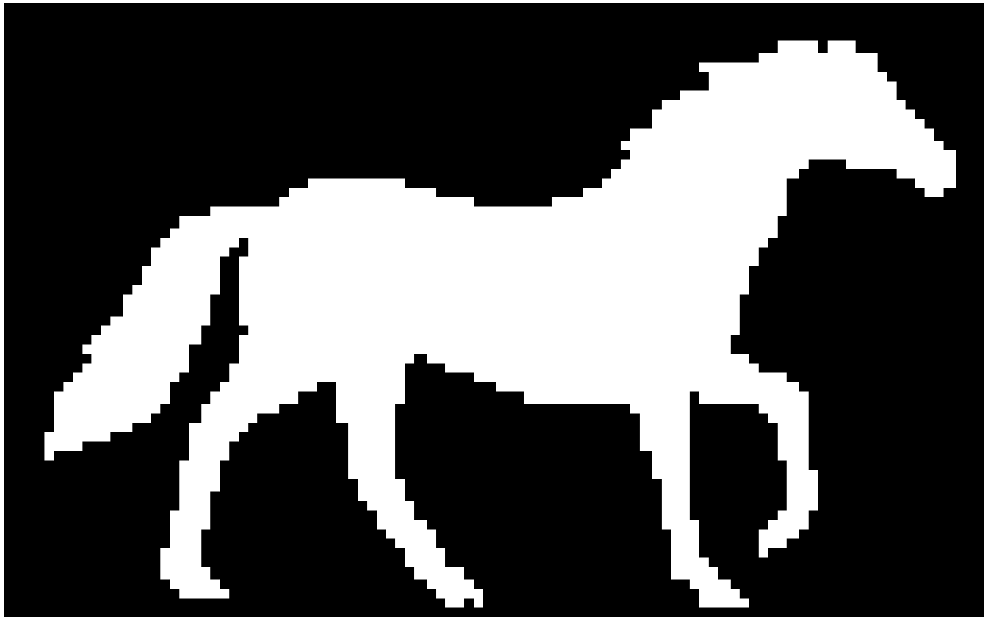
# 2D Thinning

- Shrink a binary picture by iterative erosion



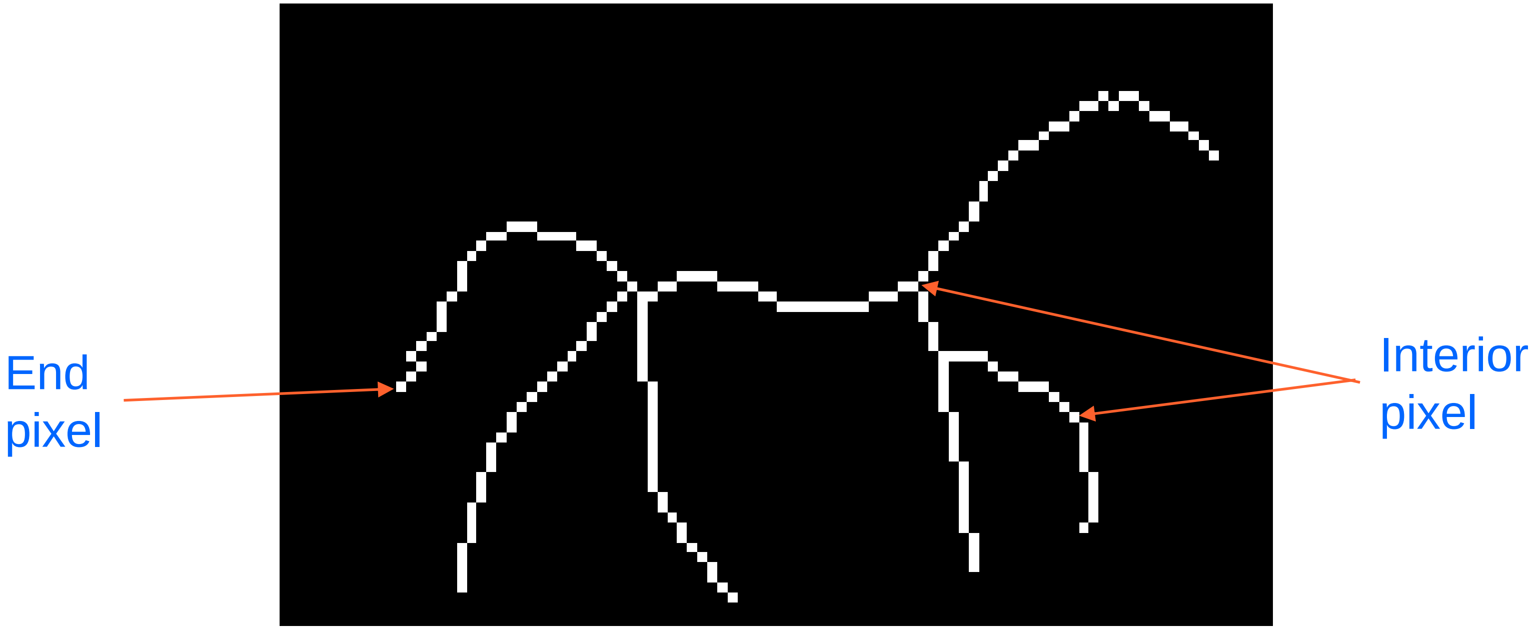
# 2D Thinning

- Shrink a binary picture by iterative erosion
  - Identify pixels where the (digital) fire fronts quench



# 2D Thinning

- Shrink a binary picture by iterative erosion
  - Identify pixels where the (digital) fire fronts quench
    - Two types of pixels



# 2D Thinning

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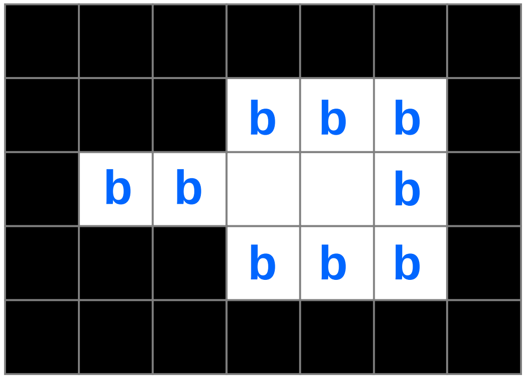
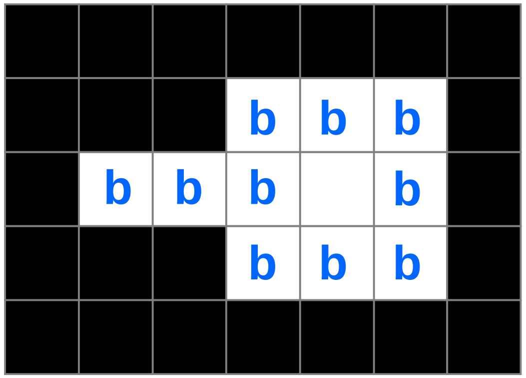
- Border pixels (to be removed by erosion)
  - Object pixel **p** is on the border if and only if **p** is connected to some background pixel
    - 4 or 8 connectivity: erosion by a cross or a square



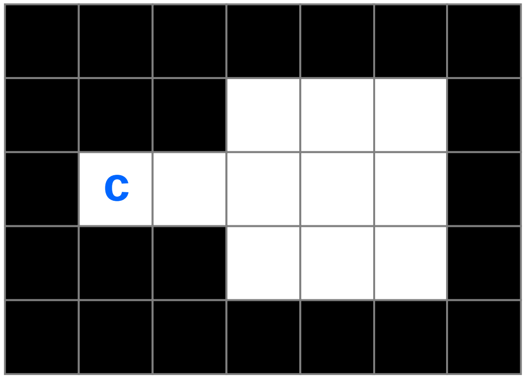

Border pixels for 4-connectivity




Border pixels for 8-connectivity

# 2D Thinning

- Curve-end pixels
  - Object pixels lying at the ends of curves, whose removal would shrink the skeleton (and hence losing shape information).




Curve-end pixel

# 2D Thinning

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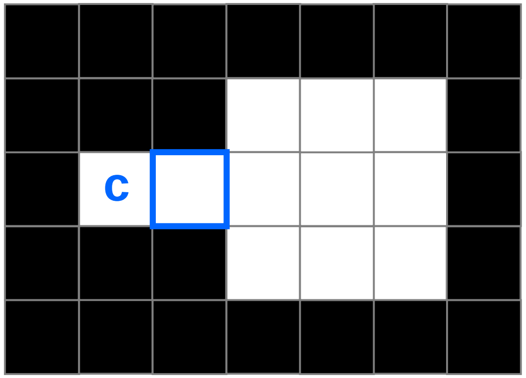
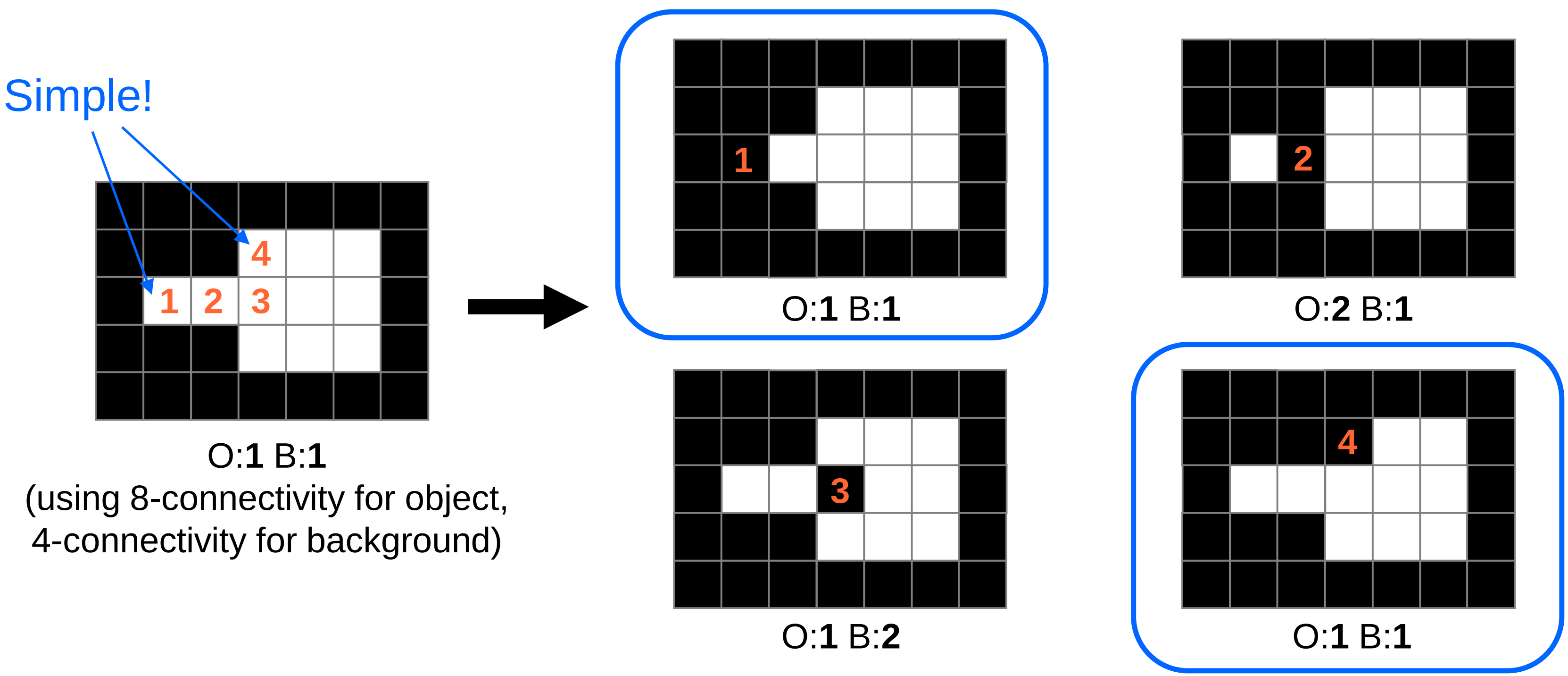
- Curve-end pixels criteria
  - Object pixel **c** is a curve-end pixel if and only if **c** is connected to exactly one object pixel.




Curve-end pixel and its connected pixel

# 2D Thinning

- Simple pixels
  - Object pixels whose removal from the object does not change topology (i.e., # of components of object and background)

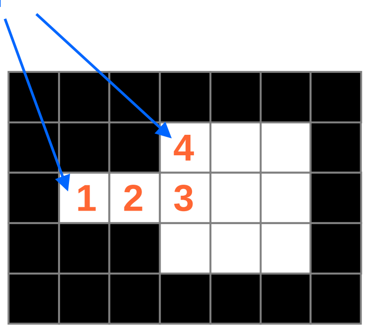


# 2D Thinning

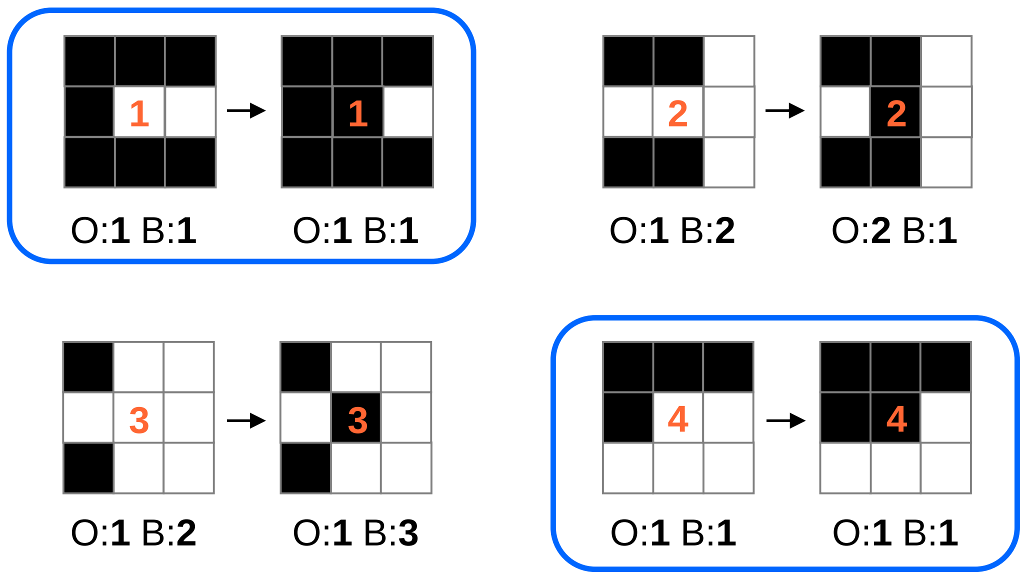


- Simple pixels criteria
  - Object pixel **p** is simple if and only if setting **p** to background does not change the # of components of either the object or background in the 3x3 neighborhood of **p**.

Simple!




(using 8-connectivity for object, 4-connectivity for background)



# 2D Thinning

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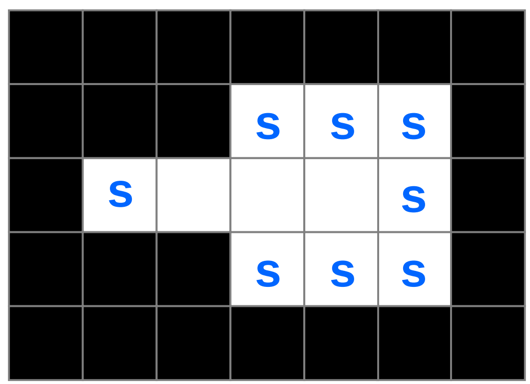
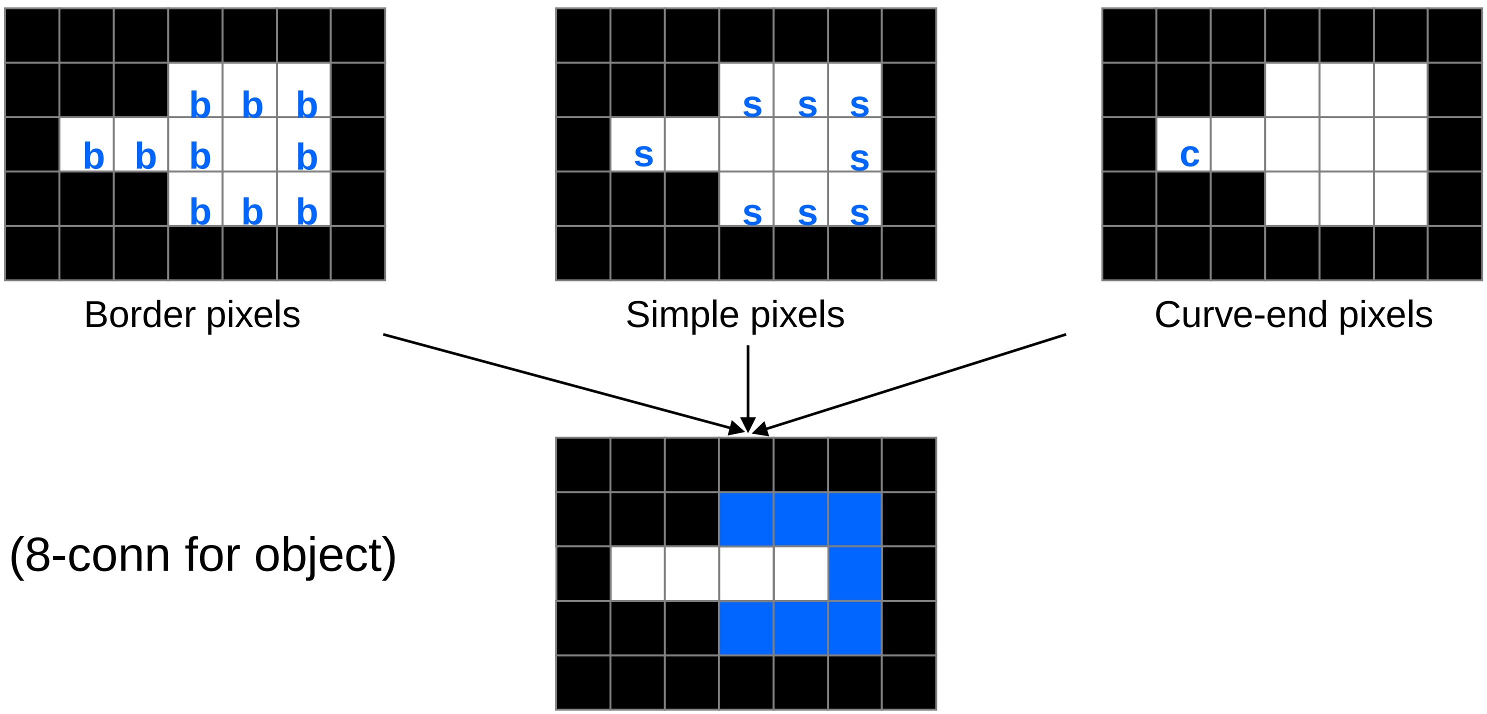
- Simple pixels criteria
  - Object pixel **p** is simple if and only if setting **p** to background does not change the # of components of either the object or background in the 3x3 neighborhood of **p**.




All simple pixels

# 2D Thinning

- Putting together: Removable pixels
  - Border pixels that are simple and not curve-end




Removal pixels

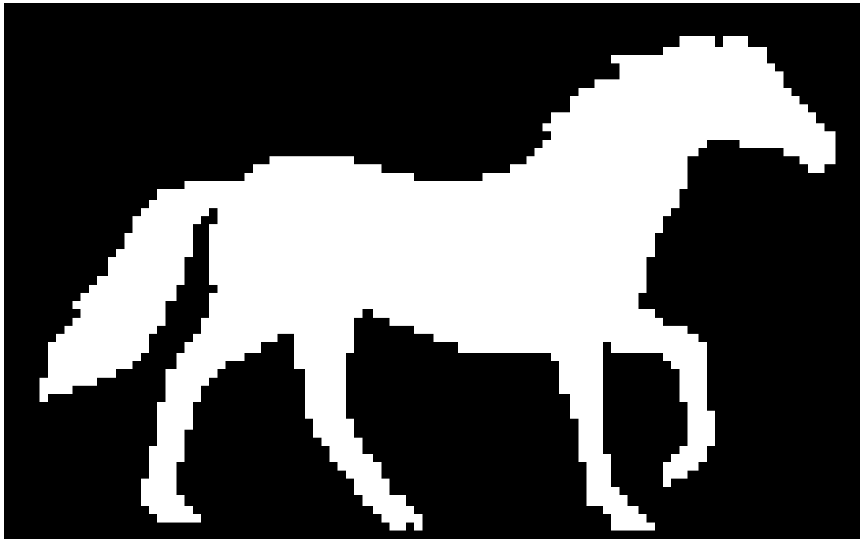
# 2D Thinning

- Algorithm (attempt) 1
  - Simultaneous removal of all removable points (“Parallel thinning”)

```
// Parallel thinning on a binary image I
1. Repeat:
    1. Collect all removable pixels as S
    2. If S is empty, Break.
    3. Set all pixels in S to be background in I
2. Output I
```

# 2D Thinning

- Algorithm (attempt) 1
  - Simultaneous removal of all removable points ("Parallel thinning")



# 2D Thinning

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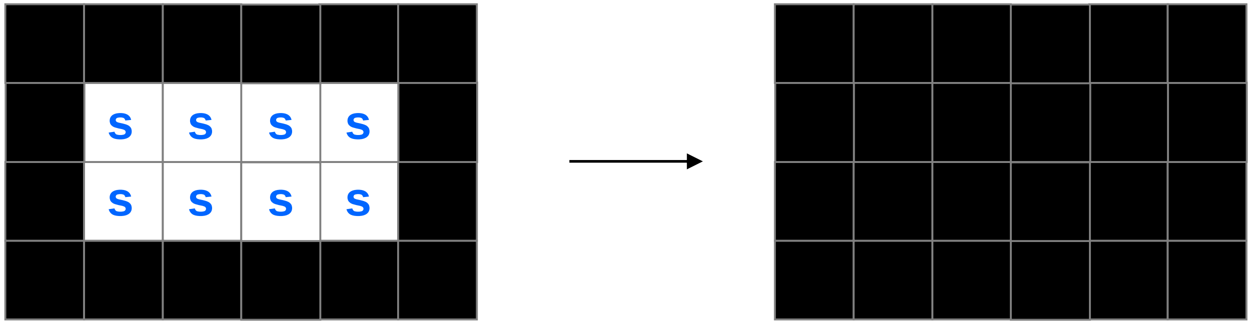
- Why does parallel thinning breaks topology?
  - Simple pixels, when removed together, may change topology



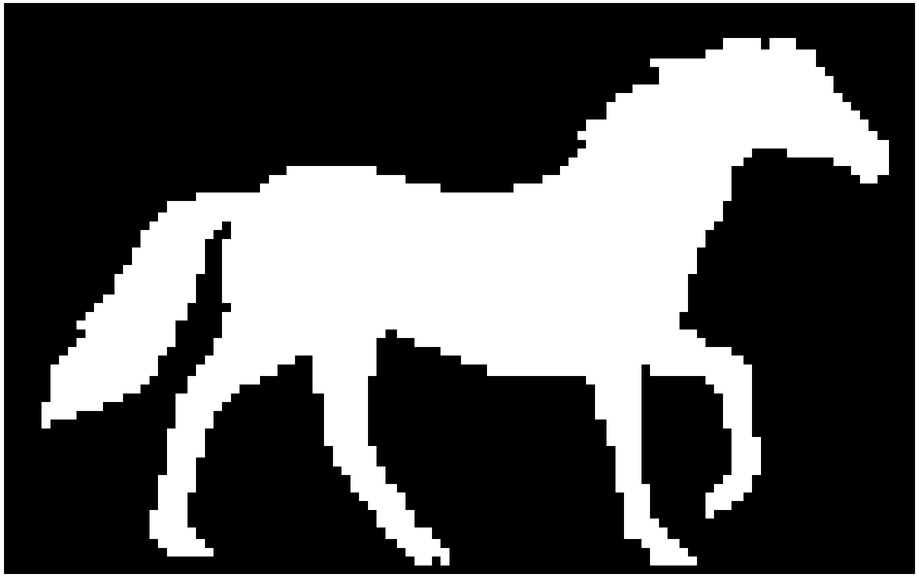
# 2D Thinning

- Algorithm 2
  - Sequentially visit each removable pixel and check its simple-ness before removing the pixel. (“Serial Thinning”)

```
// Serial thinning on a binary image I
1. Repeat:
     1. Collect all border pixels as S
     2. If S is empty, Break.
     3. Repeat for each pixel x in S (in certain order):
          1. If x is currently simple and not curve-end, set x to
             be background in I
2. Output I
```

# 2D Thinning

- Algorithm 2
  - Sequentially visit each removable pixel and check its simple-ness before removing the pixel. (“Serial Thinning”)



Serial thinning

# 2D Thinning

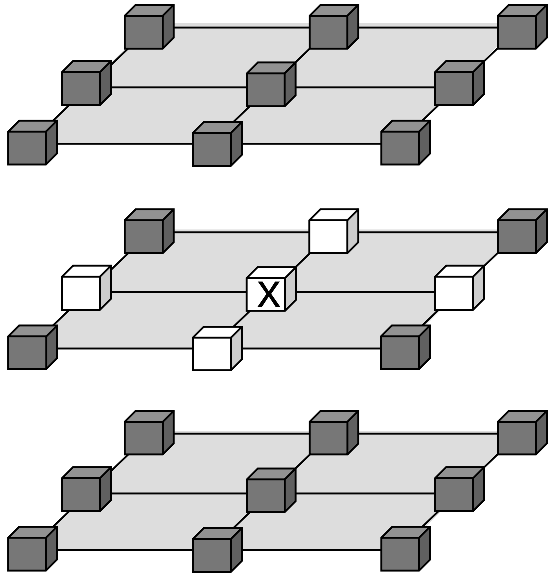
- Algorithm 2
  - Sequentially visit each removable pixel and check its simple-ness before removing the pixel. ("Serial Thinning")
  - Result is affected by the visiting "sequence"

Serial thinning with two different visiting sequences of removable pixels

# 3D Thinning



- Identifying removable voxels
  - Border voxels
    - Similar to 2D: object voxels connected to at least one background voxel
  - Simple voxels
    - Harder to characterize than 2D: Maintaining # of connected components is not sufficient (need to consider # of tunnels too)
  - Curve-end and surface-end voxels
    - Curve-end criteria same as in 2D
    - Surface-end criteria are much harder to describe (e.g., requires a table look-up)

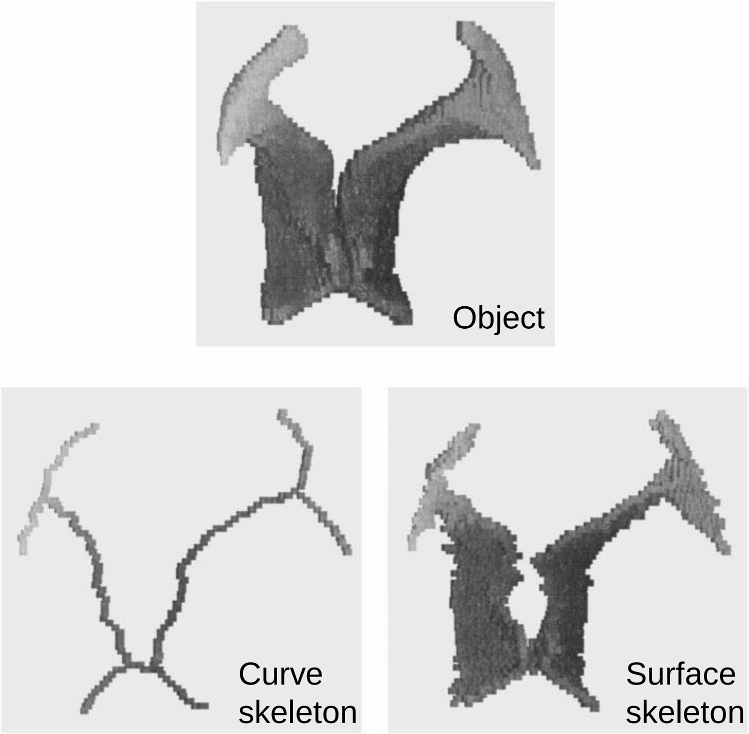



Setting voxel x to background creates a "tunnel" in the object (using 26-conn for object)

# 3D Thinning

- Two kinds of skeletons
  - Curve skeletons: only curve-end voxels are preserved during thinning
  - Surface skeletons: both curve-end and surface-end voxels are preserved

  *(see further readings)*




Method of [Palagyi and Kuba, 1999]

# Skeleton Pruning

- Thinning is sensitive to boundary noise
  - Due to the instability of medial axes
- Skeleton pruning
  - During thinning
    - E.g., using more selective criteria for end pixels (voxels)
  - After thinning
    - E.g., based on branch length
  - See *Further Readings*

Object with boundary noise

Resulting skeleton

# Further Readings on: Binary Pictures, MA and Thinning

- Books
  - "*Digital Geometry: geometric methods for digital picture analysis*", by Klette and Rosenfeld (2004)
  - "*Medial representations: mathematics, algorithms and applications*", by Siddiqi and Pizer (2008)
- Papers
  - "*Digital topology: introduction and survey*", by Kong and Rosenfeld (1989)
    - Theories of binary pictures
  - "*Thinning methodologies - a comprehensive survey*", by Lam et al. (1992)
    - A survey of 2D methods
  - "*A Parallel 3D 12-Subiteration Thinning Algorithm*", by Palagyi and Kuba (1999)
    - Includes a good survey of 3D thinning methods
  - "*Pruning medial axes*", by Shaked and Bruckstein (1998)
    - A survey of MA and skeleton pruning methods