1 Chapter 4

1.1 Minimum and Maximum Values

Definition 1. *Let c be a number in the domain D of a function f. Then $f(c)$ is the:*

Absolute maximum *value of f on D if $f(c) \geq f(x)$ for all x in D.*
Absolute minimum *value of f on D if $f(c) \leq f(x)$ for all x in D.*

Definition 2. *The number $f(c)$ is a*

Local Maximum *value of f if $f(c) \geq f(x)$ when x is near c.*
Local Minimum *values of f if $f(c) \leq f(x)$ when x is near c.*

Theorem 1. ***Extreme Value Theorem***
If f is continuous on a closed interval $[a, b]$ then f attains an absolute maximum values $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

Theorem 2. ***Fermat's Theorem***
If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = o$.
If f has a local maximum or minimum at c, then c is a critical number of f.

Definition 3. *A **critical number** of a function f is a number c c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.*

1.1.1 Finding the Absolute Max and Min Values using the Closed Interval Method

To find the *absolute* maximum and minimum values of a continuous function f on a closed interval $[a, b]$:
1. Find the values of f at the critical numbers of f in (a, b).
2. Find the values of f at the endpoints of the interval.
3. The largest of the values from Step 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

Theorem 3. ***Role's Theorem:***
Let f be a function that satisfies the following three hypotheses:
1. f is continuous on the closed interval $[a, b]$
2. f is differentiable on the open interval (a, b)
3. $f(a) = f(b)$
Then there is a number c in (a, b) such that $f'(c) = 0$

Theorem 4. ***Mean Value Theorem:***
Let f be a function that satisfies the following hypotheses:
1. f is continuous on the closed interval $[a, b]$
2. f is differentiable on the open interval (a, b)
Then there is a number c in (a, b) such that:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

or equivalently

$$f(b) - f(a) = f'(c)(b - a)$$

The slope of the secant line AB is $m_{ab} = \frac{f(b)-f(a)}{b-a}$ $\frac{d}{dx}(\sin x) = \cos x$
$\frac{d}{dx}(\cos x) = -\sin x$
$\frac{d}{dx}(\tan x) = \sec^2 x$
$\frac{d}{dx}(\csc x) = \csc x \cot x$
$\frac{d}{dx}(\sec x) = \sec x \tan x$
$\frac{d}{dx}(\cot x) = -\csc^2 x$
$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$
$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$
$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$
$\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$ General Form: $\frac{d}{dx}(log_b x) = \frac{1}{x \ln b}$ Proof: Let $y = \log_b x$
Then

$$b^y = x$$

Using the general form for taking the derivative, you get:

$$b^y(\ln b)\frac{dy}{dx} = 1$$
so
$$\frac{dy}{dx} = \frac{1}{b^y \ln b} = \frac{1}{x \ln b}$$

Taking the Derivative of $y = \ln x$:
Using the above General Form, you can solve for the derivative of $y = \ln x$:

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

IMPORTANT:
When taking the derivative of a logarithmic function, such as the derivative of $y = \ln x^3 + 1$ you will have to use chain rule.
$\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$ and $\frac{d}{dx}[\ln g(x)] = \frac{g'(x)}{g(x)}$
Another Derivative to remember is the derivative of $\ln |x|$ which derives to $\frac{1}{x}$ 1. Take natural logarithms of both sides of an equation $y = f(x)$ and use the Laws of Logarithms to simplify
2. Differentiate implicitly with respect to x.
3. Splve the resulting equation for y' For example, let $y = x^n$

$$\ln |y| = \ln |x|^n = n \ln x \text{ when } x \neq 0$$

For Logarithmic differentiation to be used, both the **base** and the **exponent** must not be constants

Definition 4. $e = \lim_{x\to 0}(1+x)^{\frac{1}{x}}$
which when evaluated, defines e as $e \approx 2.7182818$
Another form to define e is: $e = \lim_{n\to\infty}(1+\frac{1}{n})^n$