

$$\begin{aligned}\frac{d}{dx}(\sin x) &= \cos x & \frac{d}{dx}(\cos x) &= -\sin x \\ \frac{d}{dx}(\tan x) &= \sec^2 x & \frac{d}{dx}(\csc x) &= \csc x \cot x \\ \frac{d}{dx}(\sec x) &= \sec x \tan x & \frac{d}{dx}(\cot x) &= -\csc^2 x \\ \frac{d}{dx}(\sin^{-1} x) &= \frac{1}{\sqrt{1-x^2}} & \frac{d}{dx}(\cos^{-1} x) &= -\frac{1}{\sqrt{1-x^2}} \\ \frac{d}{dx}(\tan^{-1} x) &= \frac{1}{1+x^2} & \frac{d}{dx}(\cot^{-1} x) &= -\frac{1}{1+x^2} \\ \frac{d}{dx}(\sec^{-1} x) &= \frac{1}{x\sqrt{x^2-1}} & \frac{d}{dx}(\csc^{-1} x) &= -\frac{1}{x\sqrt{x^2-1}}\end{aligned}$$

Anti-Derivatives to Remember:

$$cf(x) \rightarrow cF(x) \quad / \quad f(x) + g(x) \rightarrow F(x) + G(x)$$

$$x^n (n \neq -1) \rightarrow \frac{x^{n+1}}{n+1} \quad / \quad \frac{1}{x} \rightarrow \ln |x|$$

$$e^x \rightarrow e^x \quad / \quad b^x \rightarrow \frac{b^x}{\ln b}$$

$$\cos x \rightarrow \sin x \quad / \quad \sin x \rightarrow -\cos x$$

$$\sec^2 x \rightarrow \tan x \quad / \quad \sec x \tan x \rightarrow \sec x$$

$$\frac{1}{\sqrt{1-x^2}} \rightarrow \sin^{-1} x \quad / \quad \frac{1}{1+x^2} \rightarrow \tan^{-1} x$$

$$\cosh x \rightarrow \sinh x \quad / \quad \sinh x \rightarrow \cosh x$$

Indeterminate Forms:

$$\begin{aligned}\frac{0}{0} &/ \frac{\infty}{\infty} \\ \frac{-\infty}{-\infty} &/ 0^0\end{aligned}$$

$\infty^0$  /  $1^\infty$  Use logs to convert to a workable form for L'Hospital's Rule

**Definition 1.** The area  $A$  of the region  $S$  that lies under the graph of the continuous function  $f$  is the limit of the sum or the areas of approximating rectangles:

$$A = \lim_{n \rightarrow \infty} [f(x_1)\Delta x + f(x_2)\Delta x + \dots + f(x_n)\Delta x]$$

Sigma notation or a Riemann sum is read as  $\sum_{i=1}^n f(x_i)\Delta x = f(x_1)\Delta x + f(x_2)\Delta x + \dots + f(x_n)\Delta x$

$n$  is where the ending x-value of the last rectangle

$i = 1$  is where the starting x-value that the rectangle will be

## 0.1 The Definite Integral

**Definition 2.** If  $f$  is a function defined for  $a \leq x \leq b$ , we divide the interval  $[a, b]$  into  $n$  sub-intervals of equal width  $\Delta x = \frac{b-a}{n}$ . Letting  $x_0 (= a), x_1, x_2, \dots, x_n (= b)$  be the endpoints for the sub-intervals, while also letting  $x_1^*, x_2^*, \dots, x_n^*$  be sample points in each of the subintervals, which means that  $x_i^*$  is in the subinterval  $[x_{i-1}, x_i]$

This leads to the definite integral from  $a$  to  $b$  being  $\int_a^b f(x)dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*)\Delta x$

**THE LIMIT OF THIS FUNCTION MUST EXIST FOR THE ABOVE EQUATION TO WORK**

### 0.1.1 Evaluating Integrals

$$1. \sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$2. \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$3. \sum_{i=1}^n i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Rules for working with Sigma Notation

1.  $\sum_{i=1}^n c = nc$
2.  $\sum_{i=1}^n ca_i = c \sum_{i=1}^n a_i$
3.  $\sum_{i=1}^n (a_i + b_i) = \sum_{i=1}^n na_i + \sum_{i=1}^n nb_i$
4.  $\sum_{i=1}^n (a_i - b_i) = \sum_{i=1}^n a_i - \sum_{i=1}^n b_i$

Since the limit of a Riemann sum works no matter if  $a < b$  or  $a > b$  we can assume that  $\int_b^a f(x)dx = -\int_a^b f(x)dx$

Similar to the derivative rule, if  $a = b$  then the change in  $x$  (or  $\Delta x$ ) = 0 so the integral evaluates to zero

Here are some of the basic properties of the integral:

1.  $\int_b^a cdx = c(b - a)$  when  $c$  is a constant
2.  $\int_b^a f(x) + g(x)dx = \int_b^a f(x)dx + \int_b^a g(x)dx$
3.  $\int_b^a cf(x)dx = c \int_b^a f(x)dx$  when  $c$  is a constant
4.  $\int_b^a [f(x) - g(x)]dx = \int_b^a f(x)dx - \int_b^a g(x)dx$

### 0.1.2 The Fundamental Theorem of Calculus

Part 1: If  $f$  is continuous on  $[a, b]$  then the function  $g$  defined by

$$g(x) = \int_a^x f(t)dt \text{ when } a \leq x \leq b$$

As long as the function is continuous along its interval and differentiable along that same interval

Part 2: If  $f$  is continuous on  $[a, b]$  then:

$$\int_a^b f(x)dx = F(b) - F(a)$$

Meaning that as long as  $F$  is any anti derivative of  $f$ ,  $F'(x) = f(x)$