Limit Definition: $\lim_{x \leftarrow a} f(x) = L$ if and only if $\lim_{x \leftarrow a^-} f(x) = L$ and $\lim_{x \leftarrow a^+} f(x) = L$
A vertical asymptote defined by the line $x = a$ is called a **vertical asymptote** of the function $f(x)$ if the both the left and right limit go to either $-\infty$ or ∞

0.1 Limit Laws

Limit Laws Suppose that c is a constant and the limits

$$\lim_{x \to a} f(x) \quad \text{and} \quad \lim_{x \to a} g(x)$$

exist. Then

1. $\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$
2. $\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$
3. $\lim_{x \to a} [cf(x)] = c \lim_{x \to a} f(x)$
4. $\lim_{x \to a} [f(x)g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$
5. $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ if $\lim_{x \to a} g(x) \neq 0$

Power law: $\lim_{x \to a} [f(x)]^n$ is equal to $[\lim_{x \to a} f(x)]^n$
$\lim_{x \to a} x^n = a^n$ where n is a positive integer
Roots: $\lim_{x \to a} \sqrt[n]{x} = \sqrt[n]{a}$
Direct Substitution Property: If f is a polynomial or a rational function and a is in the domain of f then: $\lim x \to af(x) = f(a)$

3 The Squeeze Theorem If $f(x) \leqslant g(x) \leqslant h(x)$ when x is near a (except possibly at a) and

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$$

then

$$\lim_{x \to a} g(x) = L$$

0.2 Precie Definition of a Limit

2 Precise Definition of a Limit Let f be a function defined on some open interval that contains the number a, except possibly at a itself. Then we say that the **limit of $f(x)$ as x approaches a is L**, and we write

$$\lim_{x \to a} f(x) = L$$

if for every number $\varepsilon > 0$ there is a number $\delta > 0$ such that

$$\text{if} \quad 0 < |x - a| < \delta \qquad \text{then} \qquad |f(x) - L| < \varepsilon$$

0.3 Continuity

A function **f** is continuous at a number a if $\lim_{x \to a} f(x) = f(a)$
This means that 1. $f(a)$ is defined 2. $\lim_{x \to a} f(x) exists$ 3. $limx \to af(x) =$

$f(a)$
Definition: A function f is continuous on an interval if it is continuous for every number in the interval

10 The Intermediate Value Theorem Suppose that f is continuous on the closed interval $[a, b]$ and let N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there exists a number c in (a, b) such that $f(c) = N$.

0.4 Limits at infinity/Horizontal Asymptotes

The line $y = L$ is called a horizontal aymptote of the curve $y = f(x)$ if either $\lim_{x\to\infty} f(x) = L$ or $\lim_{x\to-\infty} f(x) = L$
If $r > 0$ is a rational number then : $\lim x \to \infty \frac{1}{x^r} = 0$
If $r > 0$ is a rational number such that x^r is defined for all x then: $\lim_{x\to-\infty} \frac{1}{x^r} = 0$
$\lim_{x\to-\infty} e^x = 0$

0.5 Derivatives and Rates of Change

The tangent line to the curve $y = f(x)$ at the point $P(a, f(a))$ is the line through P with slope $m = \lim x \to a \frac{f(x)-f(a)}{x-a}$ provided the limit exists
LIMIT DEFINITION OF A FUNCTION: $f'(a) = \lim h \to a \frac{f(a+h)-f(a)}{h}$
Instantaneous rate of change: $\lim_{\delta x\to 0} \frac{\delta y}{\delta x} = \lim x_2 \to x_1 \frac{f(x_2)-f(x_1)}{x_2-x_1}$
FUNCTIONS CAN BE CONTINUOUS AND DIFFERENTIABLE BUT NOT DIFFERENTIABLE AND CONTINUOUS

0.6 Derivation Rules

Derivative of a constant: $\frac{d}{dx}(x) = 1$
Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$
Constant Multiple Rule: $\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$
Sum Rule: $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$
Difference Rule: $\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$
Definition of the Number e: e is a number such that $\lim_{h\to 0} \frac{e^h-1}{h} = 1$ as such the derivative of $e^x ise^x$
When taking the derivative of e^{nx} the derivative will become ne^{nx}
Product Rule: If f and g are both differentiable then: $\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$
Quotient Rule: If f and g are both differentiable then: $\frac{d}{dx}\frac{f(x)}{dx} = \frac{g(x)\frac{d}{dx}[f(x)]-f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$

Derivatives of Trigonometric Functions

$\frac{d}{dx}(\sin x) = \cos x$ $\qquad$ $\frac{d}{dx}(\csc x) = -\csc x \cot x$

$\frac{d}{dx}(\cos x) = -\sin x$ $\qquad$ $\frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\tan x) = \sec^2 x$ $\qquad$ $\frac{d}{dx}(\cot x) = -\csc^2 x$