1 Chapter 1

Interval notation: Using the example function $f(x) = \frac{1}{x(x-1)}$ the domain for this function written in interval notation is $(-\infty, 0)U(0, 1)U(1, \infty)$
Set notation:$x|x \neq 0, x \neq 1$

1.1 Even vs Odd Functions

If a function f(x) satisfies the condition $f(-x) = f(x)$ then the function is considered to be an even function
Example: $f(x) = x^2$ Proof: Given the function $f(x) = x^2$ we can prove that $f(-x)$ is equal to f(x) since any values of x no matter whether negative or non-negative it will give the same value as the parent function
Whereas if the function f(x) satisfies the condition $f(-x) = -f(x)$ then the function is considered to be odd
Example: Given the function $f(x) = x^3$ we can prove that the function $f(x) = (-x)^3$ will give the same value as the parent function

Quadratic Function: $x = \frac{-b \pm (\sqrt{b^2 - 4ac})}{2a}$

1.2 Trig Functions

$sin^{-1}x = y$ is equal to $sin(y) = x$ and $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$ Same applies for cosine and tangent

11 $y = \csc^{-1}x \; (|x| \geqslant 1) \iff \csc y = x$ and $y \in (0, \pi/2] \cup (\pi, 3\pi/2]$

$y = \sec^{-1}x \; (|x| \geqslant 1) \iff \sec y = x$ and $y \in [0, \pi/2) \cup [\pi, 3\pi/2)$

$y = \cot^{-1}x \; (x \in \mathbb{R}) \iff \cot y = x$ and $y \in (0, \pi)$

1.3 Domain and Range of Trig Functions

Domain of $f(x) = \sin x$ and $f(x) = \cos x$ is $(-\infty, \infty)$ and the range is $[-1, 1]$
The period of these functions is 2π
$\tan x = \frac{\sin x}{\cos x}$

1.4 Vertical and Horizontal Stretching and reflecting

$y = cf(x)$, stretch the graph of $y = f(x)$ vertically by a factor of c whereas $y = (\frac{1}{c})(x)$ shrinks the graph vertically by a factor of c
$y = f(cx)$ shrinks the graph horizontally by a factor of c, while $y = f(\frac{x}{c})$ stretches the graph horizontally by a factor of c
$y = -f(x)$ reflects the parent function around the x-axis while $y = f(-x)$ reflects the function around the y-axis

1.5 Combinations of functions

$(f + g)(x) = f(x) + g(x)$ and $(f - g)(x) = f(x) - g(x)$
Similarly $(fg)(x) = f(x)g(x)$ and $\frac{f}{g}(x) = \frac{f(x)}{g(x)}$
Definition: A composite function is defined as $(f * g)(x) = f(g(x))$

1.6 Laws of exponents

1. $b^{x+y} = b^x b^y$
2. $b^{x-y} = \frac{b^x}{b^y}$
3. $(b^x)^y = b^{xy}$
4. $(ab)^x = a^x b^x$

1.7 Converting Log to Exponent

$\log_b x = y$ is the same as $b^y = x$

1.8 Laws of Logs

1. $\log_b(xy) = \log_b x + \log_b y$
2. $log_b \frac{x}{y} = log_b x - log_b y$
3. $\log_b(x^r) = r \log_b(x)$
4. $\ln x = y$ which is equal to $e^y = x$
5. $\ln e = 1$
6. Change of base formula: $\log_b(x) = \frac{\ln x}{\ln y}$
7. $b^{\log_b k} = k$

1.9 Limits

Limits are not defined at (a) A jump discontinuity, (b) Asymptotes which are in different directions/values, (b) Oscillates

1.9.1 One Sided Limits

2 Definition of One-Sided Limits We write

$$\lim_{x \to a^-} f(x) = L$$

and say the **left-hand limit of $f(x)$ as x approaches a** [or the **limit of $f(x)$ as x approaches a from the left**] is equal to L if we can make the values of $f(x)$ arbitrarily close to L by taking x to be sufficiently close to a with x *less than* a.

3 $$\lim_{x \to a} f(x) = L \quad \text{if and only if} \quad \lim_{x \to a^-} f(x) = L \quad \text{and} \quad \lim_{x \to a^+} f(x) = L$$

6 Definition The vertical line $x = a$ is called a **vertical asymptote** of the curve $y = f(x)$ if at least one of the following statements is true:

$$\lim_{x \to a} f(x) = \infty \qquad \lim_{x \to a^-} f(x) = \infty \qquad \lim_{x \to a^+} f(x) = \infty$$

$$\lim_{x \to a} f(x) = -\infty \qquad \lim_{x \to a^-} f(x) = -\infty \qquad \lim_{x \to a^+} f(x) = -\infty$$

1.10 Limit Laws

Limit Laws Suppose that c is a constant and the limits

$$\lim_{x \to a} f(x) \quad \text{and} \quad \lim_{x \to a} g(x)$$

exist. Then

1. $\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$
2. $\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$
3. $\lim_{x \to a} [cf(x)] = c \lim_{x \to a} f(x)$
4. $\lim_{x \to a} [f(x)g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$
5. $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ if $\lim_{x \to a} g(x) \neq 0$