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3.1 Derivatives of Polynomials and Exponential Functions

3.1.1 Power Rule

General Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$

3.1.2 Constant Multiple Rule

If c is a constant and f is a differentiable function:

$$\frac{d}{dx}cf(x) = c\frac{d}{dx}f(x)$$

3.1.3 Sum Rule

If f and g are both differentiable:

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

3.1.4 Difference Rule

If f and g are both differentiable:

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

3.1.5 Product Rule

If f and g are both differentiable:

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

3.1.6 Quotient Rule

If f and g are differentiable:

$$\frac{d}{dx}\big[\frac{f(x)}{g(x)}\big] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

3.2 The Product and Quotient Rules

3.3 Derivatives of Trigonometric Functions

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\csc x) = \csc x \cot x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\csc^{-1}x) = -\frac{1}{x\sqrt{x^2-1}}$$

3.4 Chain Rule

Given the function $\frac{d}{dx}f(g(x))$, $\frac{d}{dx}=f'(g(x))*g'(x)$

3.5 Chain Rule

If g is differentiable at x and f is differentiable at g(x) then the composite function F = f * g defined by F(x) = f(g(x)) is differentiable at x and that F' is given by the product:

$$F'(x) = f'(g(X)) * g'(x)$$

In Leibniz notation, if y = f(u) and u = g(x) are both differentiable functions then:

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Theorem 1. Power Rule Combined With Chain Rule If n is any real number and u = g(x) is differentiable then,

$$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$$

As such, $\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1} * g'(x)$

3.6 Implicit Differentiation

Definition 1. Implicit Differentiation: The process of differentiating both sides of the equation with respect to x and then solving the resulting equation for y'

Essentially, if the function says to take the derivative with respect to x, and there is a variable other than x present in the equation, derive the equation using normal differentiation rules, and at a $\frac{dn}{dx}$ where n is the variable letter present in the equation