1 Chapter 1

2 Chapter 2

3 Chapter 3

3.1 Derivatives of Polynomials and Exponential Functions

3.1.1 Power Rule

General Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$

3.1.2 Constant Multiple Rule

If c is a constant and f is a differentiable function:

$$\frac{d}{dx}cf(x) = c\frac{d}{dx}f(x)$$

3.1.3 Sum Rule

If f and g are both differentiable:

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

3.1.4 Difference Rule

If f and g are both differentiable:

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

3.1.5 Product Rule

If f and g are both differentiable:

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

3.1.6 Quotient Rule

If f and g are differentiable:

$$\frac{d}{dx}[\frac{f(x)}{g(x)}] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

3.2 The Product and Quotient Rules

3.3 Derivatives of Trigonometric Functions

$\frac{d}{dx}(\sin x) = \cos x$
$\frac{d}{dx}(\cos x) = -\sin x$
$\frac{d}{dx}(\tan x) = \sec^2 x$
$\frac{d}{dx}(\csc x) = \csc x \cot x$
$\frac{d}{dx}(\sec x) = \sec x \tan x$
$\frac{d}{dx}(\cot x) = -\csc^2 x$
$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$
$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$
$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$
$\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$

3.4 Chain Rule

Given the function $\frac{d}{dx}f(g(x))$, $\frac{d}{dx} = f'(g(x)) * g'(x)$

3.5 Chain Rule

If g is differentiable at x and f is differentiable at $g(x)$ then the composite function $F = f * g$ defined by $F(x) = f(g(x))$ is differentiable at x and that $F'$is given by the product:

$$F'(x) = f'(g(X)) * g'(x)$$

In Leibniz notation, if $y = f(u)$ and $u = g(x)$ are both differentiable functions then:

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

Theorem 1. ***Power Rule Combined With Chain Rule*** *If n is any real number and $u = g(x)$ is differentiable then,*

$$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$$

As such, $\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1} * g'(x)$

3.6 Implicit Differentiation

Definition 1. ***Implicit Differentiation:***
The process of differentiating both sides of the equation with respect to x and then solving the resulting equation for y'

Essentially, if the function says to take the derivative with respect to x, and there is a variable other than x present in the equation, derive the equation using normal differentiation rules, and at a $\frac{dn}{dx}$ where n is the variable letter present in the equation

3.7 Derivatives of Logarithmic Functions

General Form: $\frac{d}{dx}(log_b x) = \frac{1}{x \ln b}$ Proof: Let $y = \log_b x$ Then

$$b^y = x$$

Using the general form for taking the derivative, you get:

$$b^y(\ln b)\frac{dy}{dx} = 1$$
so
$$\frac{dy}{dx} = \frac{1}{b^y \ln b} = \frac{1}{x \ln b}$$

Taking the Derivative of $y = \ln x$:
Using the above General Form, you can solve for the derivative of $y = \ln x$:

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

IMPORTANT:
When taking the derivative of a logarithmic function, such as the derivative of $y = \ln x^3 + 1$ you will have to use chain rule.
$\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$ and $\frac{d}{dx}[\ln g(x)] = \frac{g'(x)}{g(x)}$
Another Derivative to remember is the derivative of $\ln|x|$ which derives to $\frac{1}{x}$

3.7.1 Using Logarithmic Differentiation

1. Take natural logarithms of both sides of an equation $y = f(x)$ and use the Laws of Logarithms to simplify
2. Differentiate implicitly with respect to x.
3. Splve the resulting equation for y' For example, let $y = x^n$

$$\ln|y| = \ln|x|^n = n\ln x \text{ when } x \neq 0$$

For Logarithmic differentiation to be used, both the **base** and the **exponent** must not be constants

Definition 2. $e = \lim_{x\to 0}(1+x)^{\frac{1}{x}}$
which when evaluated, defines e as $e \approx 2.7182818$
Another form to define e is: $e = \lim_{n\to\infty}(1+\frac{1}{n})^n$

3.8 Rates of Change in the Natural and Social Sciences

Rates of change:
If x changes from $\Delta x = x_2 - x_1$
Which means the change in y is $\Delta y = f(x_2) - f(x_1)$
The difference quotient, which will give us the average rate of change in y with respect to x is defined as

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2)-f(x_1)}{x_2-x_1}$$

Finding the instantaneous rate of change of y with respect to x is defined as:

$$\frac{dy}{dx} = \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}$$