PHYS 342 - EM 02

Problem Set 1

This problem set covers the first 15 lectures of the course.

1. Consider a straight quasi-filiform conductor of infinite length, carrying a steady current I (cf. Fig. 1.1).

   1.1. Calculate the magnetostatic field $\vec{B}$ due to the conductor at any point P in space by means of Laplace's elementary law (Eq. (3.18) of the course). [15 points]

   1.2. Show under what conditions the field $\vec{B}(P)$ calculated in 1.1 can be represented by a scalar potential. Is such a potential always a single-valued function? [5 points]

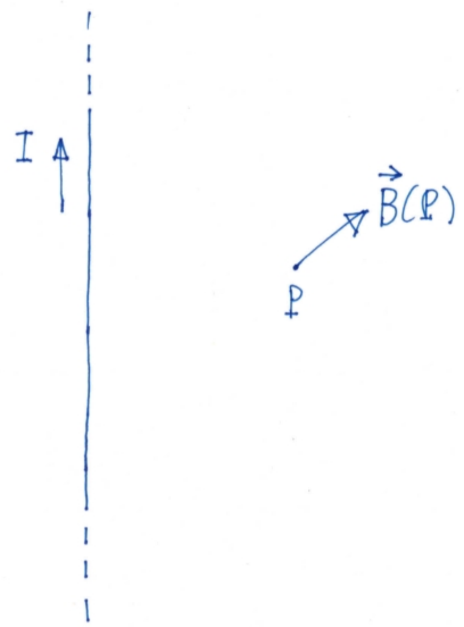

Figure 1.1

2. Consider a semi-circular quasi-filiform conductor starting at point A and ending at point B connected to a straight quasi-filiform conductor between points B and A (cf. Fig. 1.2). The structure carries a steady current I that flows counterclockwise on the circular line γ_1 and from B to A on the straight line γ_2. The radius of the semi-circular conductor is R.

Calculate the magnetostatic field $\vec{B}$ at any point P on the axis of the semi circle by means of Laplace's elementary law. [20 points]

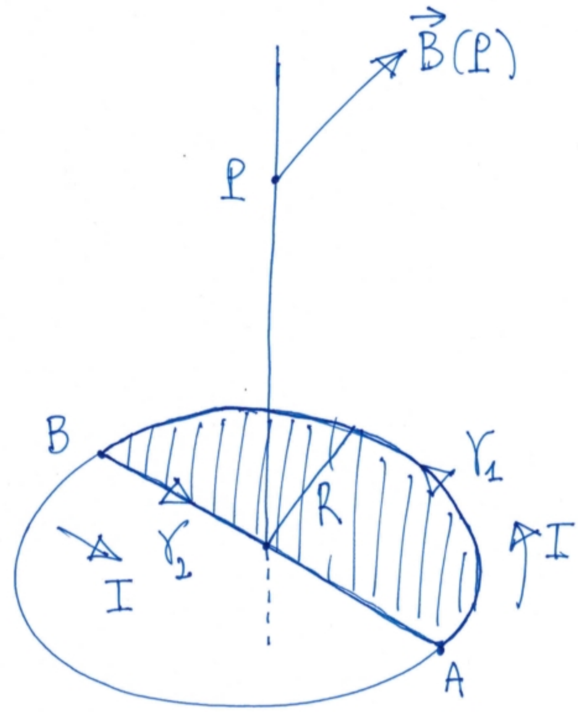

Figure 1.2

The area of the circuit $\gamma_1 \cup \gamma_2$ to be considered has been hashed.

3. Consider an infinite conducting plane of negligible thickness carrying a surface steady current with density $\vec{J}$ uniformly directed on the plane (cf. Fig. 1.3).

   Calculate the magnetostatic field $\vec{B}$ due to $\vec{J}$ at any point P in space by means of the vector potential. [20 points]

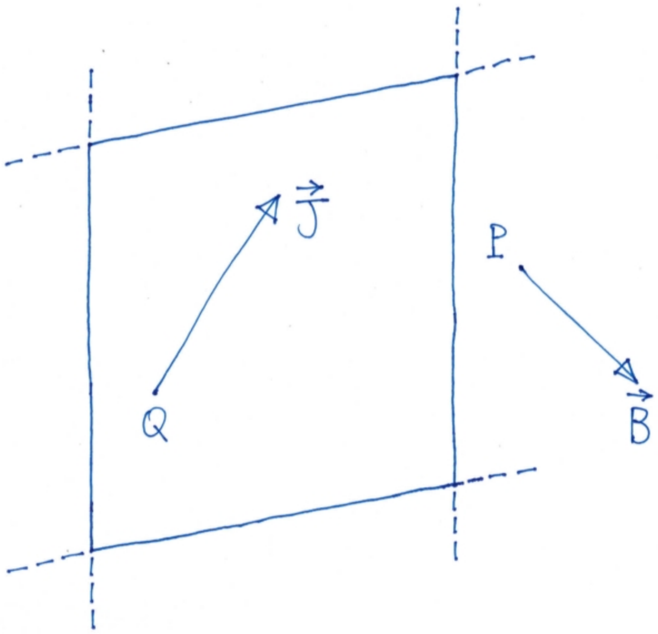

Figure 1.3

4. Consider the same infinite plane with current $\vec{J}$ of problem 3 in this set. Assume a particle with charge $-|q|$ is free to move in front of the plane at a constant velocity $\vec{v}$; the velocity is parallel to the plane, directed as $\vec{J}$, and oriented opposite of $\vec{J}$. Further assume the velocity of the charge carriers (negative) on the surface of the plane is also $\vec{v}$. The thickness of the plane is negligible and, thus, only surface conduction electrons can exist on it (cf. Fig. 1.4).

   Calculate the force acting on the particle $-|q|$ both in the reference frame of the plane and in that of the particle. [20 points]

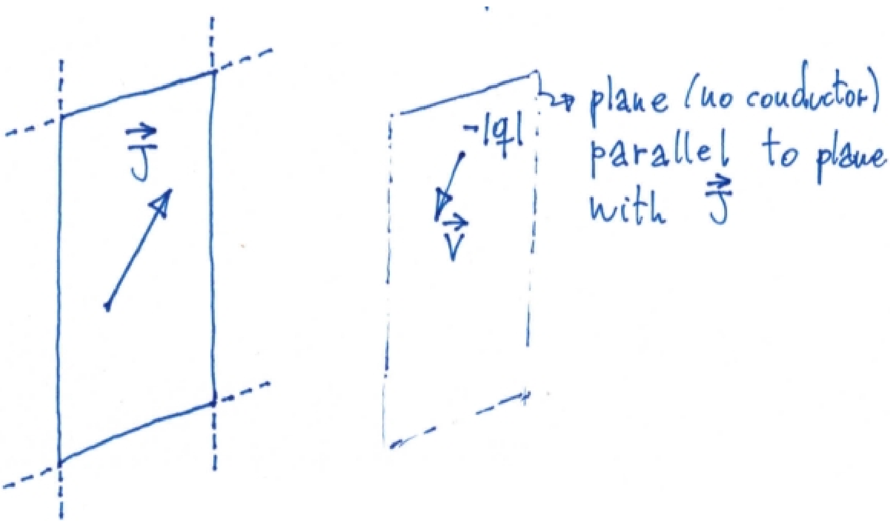

Figure 1.4

5. Consider a solid sphere (hard sphere) of radius a and center O. The sphere is made of a perfect conductor in electrostatic equilibrium in vacuum and is at potential V. A conductive shell of radius b and center O surrounds the

hard sphere. The shell is at zero potential. Figure 1.5 shows the system under consideration.

By means of the weak form of Poisson's problem calculate the approximate capacitance of the system under consideration, in first and second order (as C_1 and C_2 in lecture 7). Compare the approximate solutions to the exact ones obtained from

$$C = 4\pi\epsilon_0 \frac{ab}{b-a} \tag{1.1}$$

In the calculations, assume $a = 1$m. For the first order approximation C_1, consider the ratios $b/a = 2, 4, 10, 100; 1.5, 1.1$. For C_2 only the ratios $b/a = 10$; 1.5. For C_2, we recommend to use WolframAlpha extensively. [20 points]

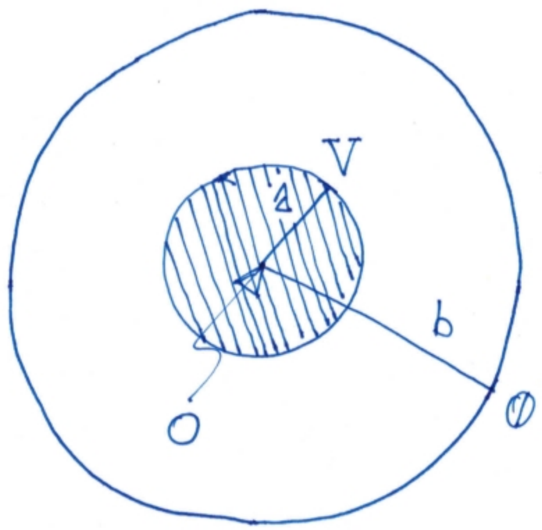

Figure 1.5

6. The Neumann integral (9.7) in the lecture notes represents the mutual inductance M_{21} between two concentric circles of radii A and a, respectively, at a distance δ from each other.

   6.1. Find a similar Neumann integral (no need to solve it) for the case of two concentric ellipses with semi-major axis A and a and semi-minor axis B and b, respectively. Assume the two ellipses belong to two parallel planes at a distance δ from each other, as shown in Fig. 1.6. [15 points]

   6.2. Show that the result found in 6.1 reduces to (9.7) when $A = B$ and $a = b$. Consider then the solution (9.8) of (9.7). Assume $A = a = 1$ m and $\delta = e^{-1/4}r$ with $r = 1$ mm. Under these conditions, δ can be assumed to be small. Using these numerical values and following closely all derivations in lecture 9, compare the approximate result for M_{21} when considering the first two terms (i.e., up to k^2) in the series expansions (9.11a) and (9.11b) for $F(k)$ and $E(k)$ to the approximate result obtained when considering just the first term in the series expansions (9.12a) and (9.12b), i.e., $\ln 4/k_1$ and 1. [5 points]

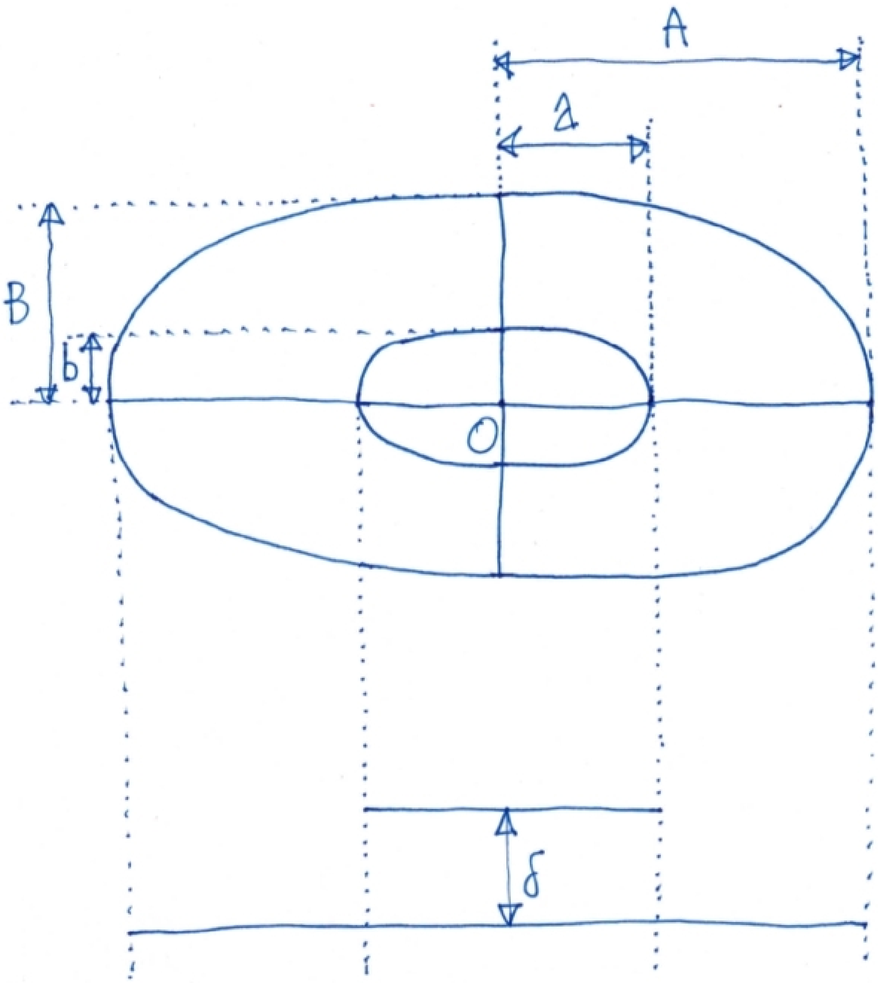

Figure 1.6

7. Consider a parallel-plate condenser with plates of area A and placed at a distance $d(<< \sqrt{A})$ from each other, in vacuum. The condenser gets charged by applying a potential difference $\Delta\phi$ between the two plates by means of a generator (battery). After disconnecting the condenser from the battery, one of the two plates is displaced in space, while remaining parallel to the other plate, until the distance between the plates becomes $d' > d$.

   7.1. Calculate the potential difference $\Delta\phi'$ between the plates at the end of the displacement. [5 points]

   7.2. Calculate the work W required to displace the plate, first, by means of the concept of electrostatic energy and, second, by means of Eq. (10.20) of the lecture notes. [15 points]

8. Consider a straight quasi-filiform conductor of length ℓ attached on one end to a rotating axis, which is normal to the plane containing the conductor (cf. Fig. 1.7). The conductor ℓ rotates about the axis at a frequency ν. The

other end of the conductor moves around a closed conducting quasi-filiform loop of radius ℓ and center on the axis. Suppose the axis to be a quasi-filiform conductor and assume the conductor ℓ to touch the loop at all times. Moreover, suppose there is no mechanical friction between the conductor ℓ and the loop. The entire system is supposed to be in an external uniform magnetostatic field $\vec{B}_0$ directed as the axis and oriented upward. Suppose the electric resistance of the conductor ℓ, the vertical axis, and the loop to be negligible. As shown in the figure, the axis can be connected to the loop by means of a resistance R, which can be opened or closed by means of a switch.

8.1. Assume the switch is open. Calculate the potential difference between the axis and the loop and show which of these two elements is at larger potential if the conductor ℓ rotates in the counterclockwise direction. In addition, calculate the current I carried by the conductor ℓ when the switch is closed. [10 points]

8.2. Calculate the mechanical torque that an engine must apply to the conductor ℓ in order to keep it rotating both when the switch is close and open. In both cases, calculate the mechanical power due to the engine. [10 points]

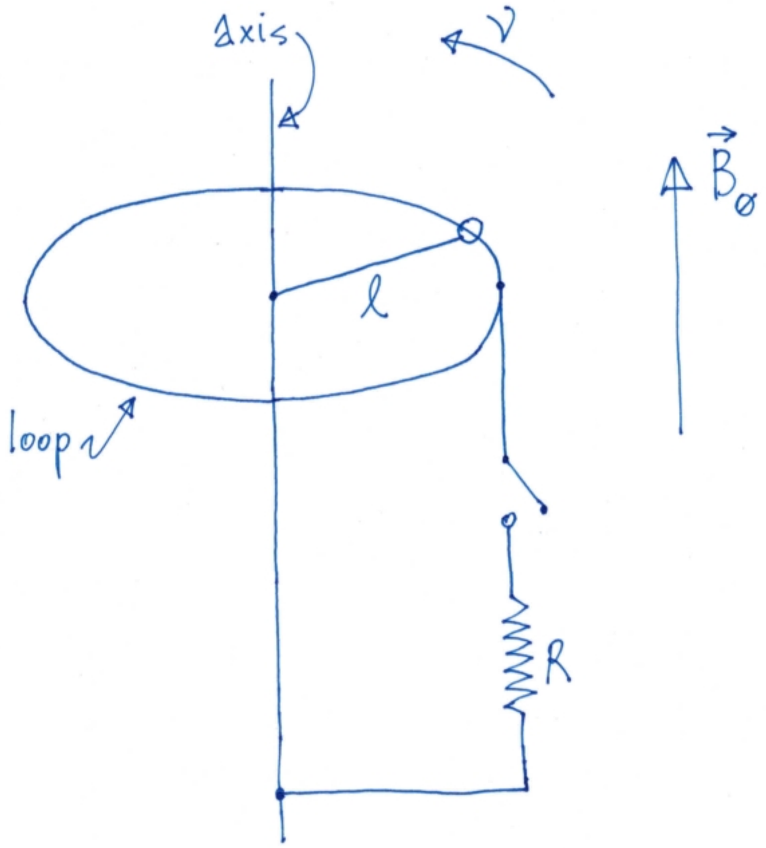

Figure 1.7

9. Consider a small magnetic needle made of iron. The needle can be modelled as a parallelepiped of square section with sides of length a and height $\ell >> a$.

Assume the magnetic dipole moment of the needle is $\vec{\mu}$ and assume the needle is free to oscillate without mechanical friction about an axis passing through its center of mass and normal to it. When the magnetic needle is placed in an external magnetostatic field $\vec{B}_0$, uniform and with field lines on the plane of the needle, the needle tends to align parallel to $\vec{B}_0$ (this could be a compass in the Earth's magnetic field). Figure 1.8 shows the system under consideration.

Calculate the period of the small oscillations when the needle is slightly displaced by rotation from its equilibrium condition (equilibrium condition: $\vec{\mu}$ and $\vec{B}_0$ are parallel with equal sign). Assume the density of iron with respect to water is ρ. [Hint: See last part of lecture 15]. [20 points]

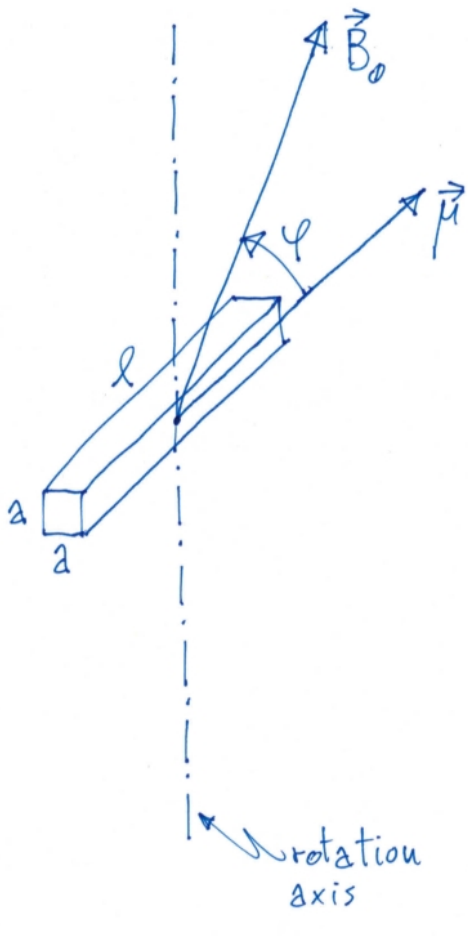

Figure 1.8

10. Consider a thin disk of radius R and uniformly charged with total electric charge q. Assume the thickness of the disk is negligible. Refer to Fig. 1.9.

    10.1. By means of the multipole series expansion, calculate the electrostatic potential at a generic point P on the disk central axis (which is normal to the disk) at a distance h from the center of the disk, with $h >> R$. Stop the series to the quadrupole term. [10 points]

    10.2. Using the same method, calculate the potential at a generic point P' on a line passing through the center of the disk and belonging to the disk plane. As before, indicate with $h(>> R)$ the distance between P' and the disk center. [10 points]

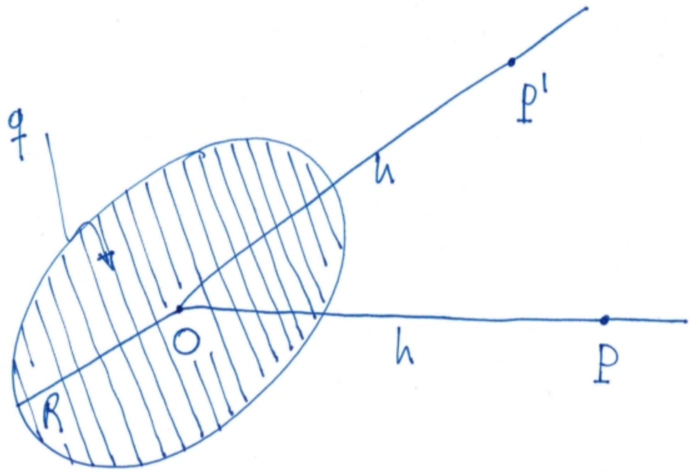

Figure 1.9