PHYS 325 - EM 02

Syllabus

- Background required
  - Vector calculus
  - Maxwell's equations for electro and magnetostatics; we will revise the latter
  - Scalar (and vector) potential
  - Perfect conductors in vacuum
  - Electric current
  - Definition of capacitance, resistance, and inductance (for steady currents)
- Topics
  - Electro and magnetostatics in depth
    * Laplace's elementary law and vector potential
    * Lorentz force and relativity
    * Weak form of Poisson's problem and Maxwell's inductance
    * Mechanical effects due to the electro and magnetostatic field
    * Multipole expansion for the scalar (in depth) and vector potential
  - The electrostatic field in presence of dielectric materials (in depth)
    * Polarization and polarization vector
    * Electric displacement and boundary conditions
    * Electric susceptibility
    * Macroscopic electric field inside a dielectric
    * Microscopic polarization phenomena
  - The magnetostatic field in presence of magnetic materials
    * Magnetization and magnetization vector
    * Magnetic field intensity and boundary conditions
    * Diamagnetism, paramagnetism, and ferromagnetism
  - The electromagnetic field
    * Electromagnetic induction
    * Displacement currents
    * Maxwell's equations for the electromagnetic field
    * Electromagnetic potentials
    * Energy of the electromagnetic field

- Grades
  - Attendance: 7%
  - Home assignments: 18%
  - Midterm exam: 27%
  - Final exam: 48%
  - No changes to the above scheme will be allowed.
- Home assignments

  There will be four (4) sets of problems, one for each topic.
  - Set 1: Handed out on 2015-09-21; handed in on 2015-10-19.
  - Set 2: Handed out on 2015-11-02; handed in on 2015-11-12.
  - Set 3: Handed out on 2015-11-16; handed in on 2015-11-26.
  - Set 4: Handed out on 2015-11-23; handed in on 2015-12-04.
- Tutorials

  There will be six (6) tutorials. The first will be a review of magnetostatics (basic phenomena from PHYS 242). The next four will be a detailed review of the home assignments. The last one will be a review of the course before the final exam. The schedule for the tutorials will be decided in class, the first day of lectures. The tutorials will likely be scheduled a few days before the midterm and final exams, in order to prepare for these events.
- Office hours

  Contact me (or the TAs) at matteo.mariantoni@uwaterloo.ca. I prefer to meet with small group of students rather than single students, so that many people can benefit from the same discussion.
- Literature
  - R.P.Feynman, R.B.Leighton, and M.Sands, The Feynman Lectures on Physics - Volume 2, any edition
  - E.M.Purcell and D.J.Morin, Electricity and Magnetism, Cambridge University Press; 3rd edition, 2013
- My notes on Learn

PHYS 342 - EM 02

Lecture 1

1.1 Review of Electrostatics

Consider a volume charge distribution with density $\rho \in C^{\circ}(\Omega)$ in a domain Ω of the 3D Euclidean space.

According to Gauss' theorem

$$\oiint_{\Sigma} \vec{E} \cdot \vec{n} dA = \frac{1}{\epsilon_0} \iiint_{\Omega} \rho dV \tag{1.1}$$

Where Σ is a surface enclosing Ω. This equation can be written in differential form as (if ρ is continuous and limited)

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0} \rho \tag{1.2}$$

which is also valid when $\rho = 0$.

The irrotational property of $\vec{E}$ can be written in integral form as

$$\oint \vec{E} \cdot \vec{t} d\ell = 0 \tag{1.3}$$

and, in differential form, as

$$\vec{\nabla} \times \vec{E} = \vec{0} \tag{1.4}$$

In the case of a surface charge distribution with density ρ in Ω, the local form of Gauss' theorem and the irrotational property is

$$\left(\vec{E}_1 - \vec{E}_2\right) \cdot \vec{n} = \frac{1}{\epsilon_0} \sigma \tag{1.5}$$

and

$$\left(\vec{E}_1 - \vec{E}_2\right) \cdot \vec{t} = 0 \tag{1.6}$$

respectively (cf. conventions in the next figure).

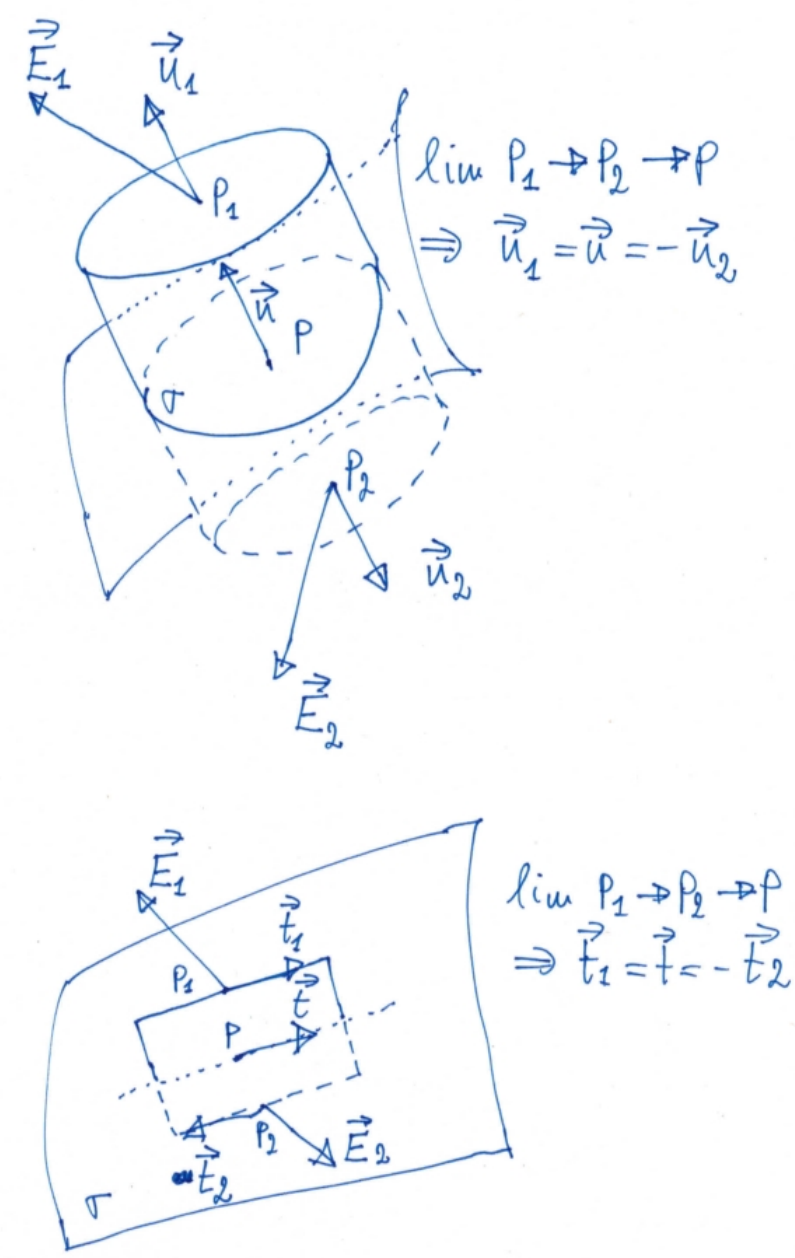

Figure 1.1

In a simply connected (or star) domain, the electrostatic field $\vec{E}$ is not only irrotational, but also conservative. In this case, a scalar potential ϕ can be defined, such that

$$\vec{E} = -\vec{\nabla}\phi \tag{1.7}$$

From this definition and the differential form of Maxwell's equations for $\vec{E}$, it can be shown that

$$\nabla^2\phi = 0 \tag{1.8}$$

when $\rho = 0$ (Laplace equation) and

$$\nabla^2\phi = -\frac{1}{\epsilon_0}\rho \tag{1.9}$$

when $\rho \neq 0$ (Poisson equation).

1.2 Perfect Conductors in Vacuum

Consider a perfect and homogeneous conductor in vacuum.

The conductor is said to be in electrostatic equilibrium when no macroscopic motion of charges exists in the conductor. Under these conditions

$$\vec{E} = \vec{0} \tag{1.10}$$

at each point inside the conductor. Following the notation in the figure below,

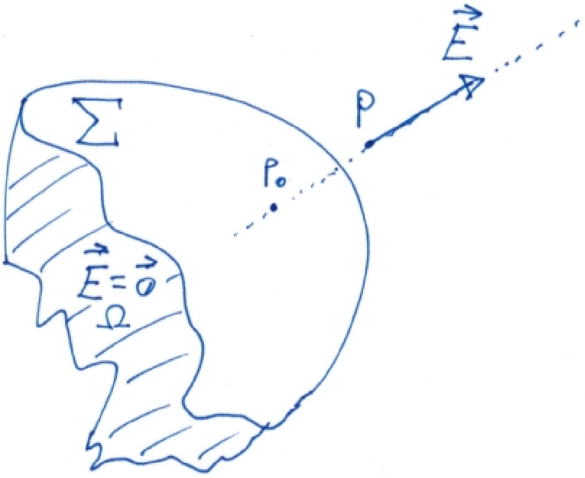

Figure 1.2

when the conductor is in an external field $\vec{E}_{ex}$, a reaction field $\vec{E}_r$ is generated until

$$\vec{E} = \vec{E}_{ex} + \vec{E}_r = \vec{0} \tag{1.11}$$

at each point inside the conductor. Under these conditions, a surface charge density σ is generated on the conductor surface. σ depends on the point P_0 on the surface. According to Coulomb's theorem

$$\lim_{P \to P_0^+} E_n(P) = \frac{\sigma(P_0)}{\epsilon_0} \tag{1.12}$$

where P_0^+ indicates that the point is located on the outer skin of the conductor.

From the definition of ϕ, it follows

$$\phi = \text{const}\Big|_{\Omega \cup \Sigma} \tag{1.13}$$

and

$$E_n = -\frac{\partial}{\partial n}\phi\Big|_{\Sigma^+} \tag{1.14}$$

where Σ^+ indicates the outer skin of the conductor.

If the conductor is charged with Q,

$$Q = \epsilon_0 \oiint_{\Sigma' \supset \Sigma} E_n dA = \oiint_{\Sigma} \sigma dA \tag{1.15}$$

In general,

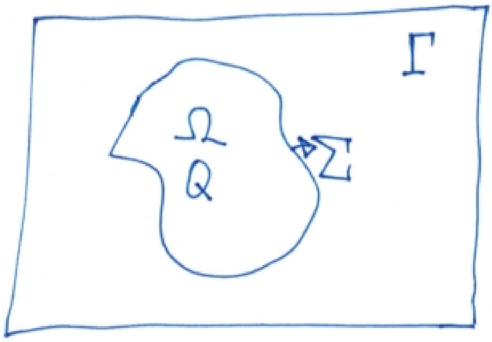

Figure 1.3

i) $\phi \in C^\circ(\Gamma \cup \Sigma)$

ii) $\vec{\nabla}^2 \phi = 0 \qquad \forall P \in \Gamma$

iii) $\phi = \text{const} \qquad \forall P \in \Omega \cup \Sigma$

iv) $-\oiint\limits_{\Sigma^+} \frac{\partial}{\partial n} \phi dA = \frac{1}{\epsilon_0} Q$

v) $\lim\limits_{P \to \infty} \phi(P) = 0$

- Definition of capacitance

Given two conductors, A and B, with charges $+|Q|$ and $-|Q|$, respectively, the capacitance of the system is defined as

$$C \equiv \frac{|Q|}{\Delta\phi} \tag{1.16}$$

where $\Delta\phi = \phi_A - \phi_B$.