1. You are invested in two hedge funds. The probability that hedge fund Alpha generates positive returns in any given year is 60%. For hedge fund Omega, it is 70%. Assume the returns are independent. What is the probability that both funds lose money in a given year (no decimal in %, e.g., 12%)?

2. Your firm forecasts that there is a 50% probability that the market will be up significantly next year, a 20% probability that the market will be down significantly next year, and a 30% probability that the market will be flat, neither up or down significantly. You are asked to evaluate the prospects of a new portfolio manager. The manager has a long bias and is likely to perform better in an up market. Based on past data, you believe that the probability that the manager will be up if the market is up significantly is 80%, and that the probability that the manager will be up if the market is down significantly is only 10%. If the market is flat, the manager is just as likely to be up as to be down. What is the unconditional probability that the manager is up next year (no decimal in %, e.g., 12%)?

3. 1% of a population of fund managers definitely are skillful; the rest have bad years. 90% of definitely skillful managers are tested positive by a $\mathbb{Q}$F method. Also, 90% of those having bad years are tested negative by the same method. If you randomly select a manager who is tested positive, what is the probability that he or she definitely are skillful (irreducible fraction, e.g., 4/37)?

4. A \$100 notional, zero coupon bond has one year to expiry. The probability of default is 10%. In the event of default, assume that the recovery rate is 40%. The continuously compounded discount rate is 5%. What is the present value of this bond (2 decimals in \$, e.g., \$10.22)?

5. Consider the indicator variable $1_A(\omega)$, which is defined as

$$1_A(\omega) = \begin{cases} 1, & \text{if } \omega \in A; \\ 0, & \text{if } \omega \in A^c. \end{cases}$$

Let $\Omega := A \cup A^c$, which is the full set. Suppose $f(\omega)$ is the probability density function.

$$\mathbb{E}\left(1_A\right) = \int_\Omega 1_A f(\omega) d\omega = \int_A f(\omega) d\omega = \mathbb{P}(A) =: p.$$

What is the variance of 1_A in terms of p?

Requirements:

(A) All answers are to be presented strictly in A4-size paper.

(B) You are free to either typeset in latex or write. If you write your answers, write nicely and legibly. The probability of inadvertently grading a sloppily written submission wrongly is quite high.

(C) Deadline: Upload your submission to elearn's dropbox by 2359 hours, October 17, 2018.