

Summary of Applications (Python)



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➤ Pandigital Formula for New Year Ver. 1.0.0

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Application 1: HDB Loan Calculator

<https://services2.hdb.gov.sg/webapp/BB29MTHLY/BB29SMTHLY>

The screenshot shows the HDB Loan Calculator web application. The browser address bar displays the URL `2.hdb.gov.sg/webapp/BB29MTHLY/BB29SMTHLY`. The page header includes the HDB logo, the Singapore Government logo, and navigation links for FAQ, Contact Us, Sitemap, and Feedback. A search bar is also present. The main navigation menu includes links for Home, About Us, Residential, Community, Business, Car Parks, e-Services, and a Login button. The page title is "Enquiry on Monthly Instalment". Below the title, a description states: "This service allows you to compute the estimated monthly instalment of the loan for the repayment period that you intend to apply." A section titled "Important Notes:" contains a single note: "1. Please read the [Privacy Statement](#) and [Terms of Use](#) before proceeding further." The main form area contains input fields for "Loan Amount (S\$)" (800,000), "Repayment Period (In Years) (Between 1 - 25 years)" (25), and "Interest Rate Of Loan (%)" (empty). Below these fields, a note states: "(You may EITHER click on one of the buttons below or enter an interest rate that you wish to enquire.)". A table with two columns, "With Effect From" and "Interest Rate", shows a selected option of "01 Jul 2017" and "2.60%". At the bottom, a message states: "Please click the 'Accept' button if you agree and accept the [Terms and Conditions](#) and wish to proceed with this service." Two buttons, "Accept" and "Clear", are provided for user action.

2.hdb.gov.sg/webapp/BB29MTHLY/BB29SMTHLY

HDB InfoWEB e-Services : E... x

Tools Help

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Enquiry on Monthly Instalment

This service allows you to compute the estimated monthly instalment of the loan for the repayment period that you intend to apply.

Important Notes:

1. Please read the [Privacy Statement](#) and [Terms of Use](#) before proceeding further.

Loan Amount (S\$)

Repayment Period (In Years) (Between 1 - 25 years)

Interest Rate Of Loan (%)

(You may EITHER click on one of the buttons below or enter an interest rate that you wish to enquire.)

With Effect From	Interest Rate
<input checked="" type="radio"/> 01 Jul 2017	2.60%

Please click the 'Accept' button if you agree and accept the [Terms and Conditions](#) and wish to proceed with this service.

105%



Application 1: HDB Loan Calculator

<https://services2.hdb.gov.sg/webapp/BB29MTHLY/BB29SMTHLY>

The screenshot shows a web browser window displaying the HDB Loan Calculator. The browser's address bar shows the URL <https://services2.hdb.gov.sg/webapp/BB29MTHLY/BB29SMTHLY>. The page header includes the HDB logo, the Singapore Government logo, and navigation links for FAQ, Contact Us, Sitemap, and Feedback. A search bar is also present. The main navigation menu includes links for Home, About Us, Residential, Community, Business, Car Parks, e-Services, and a Login button. The main content area is titled "Enquiry on Monthly Instalment" and contains a disclaimer about the estimated results. Below the disclaimer, it shows the following information entered: Estimated Loan Amount: \$800,000, Repayment Period (In Years): 25, and Interest Rate of Loan: 2.60%. The estimated monthly instalment is shown as \$3,630. A footnote explains that the actual monthly instalment is dependent on the loan granted, repayment term, and interest rate.

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Enquiry on Monthly Instalment

The results and the loan amount are an indicative estimated computation based on the accuracy of the information (including monthly income, age and available CPF Ordinary Account balance, etc) provided by you. It does not constitute an approval by HDB to buy / take over an HDB flat / an HDB resale flat / a DBSS flat or to grant a housing loan at concessionary interest rates or on any other terms. If you wish to apply for an HDB loan, you must still submit an HDB Loan Eligibility Application to know your eligible maximum loan quantum and furnish the supporting documents. HDB shall not be liable for any information provided by this service. The Financial Plan will be worked out during your HDB appointment.

The following information is entered as follows:

- Estimated Loan Amount : \$800,000
- Repayment Period (In Years) : 25
- Interest Rate of Loan : 2.60%

The estimated monthly instalment[^] for your loan is **\$3,630**

[^]Actual monthly instalment is dependent on the loan granted, repayment term and interest rate as at the instalment commencement date.



HDB Loan Calculator

Loan Amount (S\$) 800000

Repayment Period (In Years) (Between 1-25 years) 25

Interest Rate of Loan (%) 2.6

Calculate

Monthly Installment (S\$, rounded to the next dollar) 3630



Algorithm:

Given PV , r and t , compute P using

$$P = \frac{\frac{r}{12} (PV)}{1 - \left(1 + \frac{r}{12}\right)^{-12t}}$$



Application 1a: HDB Loan Calculator (★☆☆☆☆)



1. ☐ Numeric Literals
2. ☐ Arithmetic Operators
- ❖ ☐ (optional) String Literals
- ❖ ☐ (optional) Variables
- ❖ ☐ (optional) Assignment Statements
- ❖ ☐ (optional) Built-in Function `input`, `float`
- ❖ ☐ (optional) Built-in Function `print`
- ❖ ☐ (optional) Built-in Function `round`
- ❖ ☐ (optional) `math` Function `ceil`

Hint: About 4 lines

Application 1b: HDB Loan Calculator GUI (★ ★ ☆ ☆ ☆)



3. ☐ Qt Designer + PyQt5 Template (Label, Push Button, Line Edit)
 - a) Functions
 - b) Modules
 - c) Classes
4. ☐ String Literals
5. ☐ Variables
6. ☐ Assignment Statements
7. ☐ Built-in Function `float` (convert to number)
8. ☐ Built-in Function `str` (convert to string)
9. ☐ Built-in Function `round` (round numbers)
10. ☐ `math` Function `ceil` (round to the next integer)

Hint: About 30 lines + .ui

Application 1c: Bank Loan Calculator

<https://www.dbs.com.sg/personal/calculators/homeloans-calculators-repayment-schedule.page>



Repayment Schedule Calculator

Your Details

Loan Amount	<input type="text" value="\$"/>
<small>Purchase Price or Valuation of Property must be between \$80,000 and \$99,999,999</small>	
Year of Birth	<input type="text" value="yyyy"/>
<small>Age must be between 21 to 70 years old</small>	
Loan Tenure	<input type="text" value="yy"/>
<small>Loan Tenure must be between 5 to 35 years (Private Property) or 5 to 30 years (HDB Flat)</small>	

Interest Rates

Year 1	<input type="text" value="%"/>
<small>Interest Rate must be between 1 to 100</small>	
Year 2	<input type="text" value="%"/>
<small>Interest Rate must be between 1 to 100</small>	
Year 3	<input type="text" value="%"/>
<small>Interest Rate must be between 1 to 100</small>	
Thereafter	<input type="text" value="%"/>
<small>Interest Rate must be between 1 to 100</small>	

Clear

Calculate

Repayment Schedule Calculator

Your Details

Loan Amount	<input type="text" value="800000"/>	✓
<small>Purchase Price or Valuation of Property must be between \$80,000 and \$99,999,999</small>		
Year of Birth	<input type="text" value="1990"/>	✓
<small>Age must be between 21 to 70 years old</small>		
Loan Tenure	<input type="text" value="30"/>	✓
<small>Loan Tenure must be between 5 to 35 years (Private Property) or 5 to 30 years (HDB Flat)</small>		

Interest Rates

Year 1	<input type="text" value="1.75"/>	✓
<small>Interest Rate must be between 1 to 100</small>		
Year 2	<input type="text" value="2"/>	✓
<small>Interest Rate must be between 1 to 100</small>		
Year 3	<input type="text" value="2.2"/>	✓
<small>Interest Rate must be between 1 to 100</small>		
Thereafter	<input type="text" value="2.6"/>	✓
<small>Interest Rate must be between 1 to 100</small>		

Clear

Calculate



Result

Your monthly repayment schedule is as follow

Month	Interest Rate	Beginning Principal	Monthly Installment	Interest Paid	Principal Paid	Ending Principal
+ 1-12 (Yr 1)	1.75%	\$800,000.00	\$2,857.95	\$13,836.42	\$20,458.95	\$779,541.05
+ 13-24 (Yr 2)	2.00%	\$779,541.05	\$2,953.94	\$15,407.79	\$20,039.49	\$759,501.56
+ 25-36 (Yr 3)	2.20%	\$759,501.56	\$3,029.67	\$16,509.71	\$19,846.30	\$739,655.26
+ 37-48 (Yr 4)	2.60%	\$739,655.26	\$3,179.54	\$19,003.90	\$19,150.62	\$720,504.63
+ 49-60 (Yr 5)	2.60%	\$720,504.63	\$3,179.54	\$18,500.00	\$19,654.52	\$700,850.12
+ 61-72 (Yr 6)	2.60%	\$700,850.12	\$3,179.54	\$17,982.85	\$20,171.67	\$680,678.45
+ 73-84 (Yr 7)	2.60%	\$680,678.45	\$3,179.54	\$17,452.09	\$20,702.43	\$659,976.02
+ 85-96 (Yr 8)	2.60%	\$659,976.02	\$3,179.54	\$16,907.37	\$21,247.15	\$638,728.87
+ 97-108 (Yr 9)	2.60%	\$638,728.87	\$3,179.54	\$16,348.31	\$21,806.21	\$616,922.66
+ 109-120 (Yr 10)	2.60%	\$616,922.66	\$3,179.54	\$15,774.55	\$22,379.97	\$594,542.69
+ 121-132 (Yr 11)	2.60%	\$594,542.69	\$3,179.54	\$15,185.68	\$22,968.84	\$571,573.85
+ 133-144 (Yr 12)	2.60%	\$571,573.85	\$3,179.54	\$14,581.32	\$23,573.20	\$548,000.66
+ 145-156 (Yr 13)	2.60%	\$548,000.66	\$3,179.54	\$13,961.06	\$24,193.46	\$523,807.20
+ 157-168 (Yr 14)	2.60%	\$523,807.20	\$3,179.54	\$13,324.48	\$24,830.04	\$498,977.16

+ 157-168 (Yr 14)	2.60%	\$523,807.20	\$3,179.54	\$13,324.48	\$24,830.04	\$498,977.16
+ 169-180 (Yr 15)	2.60%	\$498,977.16	\$3,179.54	\$12,671.15	\$25,483.37	\$473,493.80
+ 181-192 (Yr 16)	2.60%	\$473,493.80	\$3,179.54	\$12,000.63	\$26,153.89	\$447,339.91
+ 193-204 (Yr 17)	2.60%	\$447,339.91	\$3,179.54	\$11,312.47	\$26,842.05	\$420,497.86
+ 205-216 (Yr 18)	2.60%	\$420,497.86	\$3,179.54	\$10,606.20	\$27,548.32	\$392,949.54
+ 217-228 (Yr 19)	2.60%	\$392,949.54	\$3,179.54	\$9,881.35	\$28,273.17	\$364,676.37
+ 229-240 (Yr 20)	2.60%	\$364,676.37	\$3,179.54	\$9,137.42	\$29,017.10	\$335,659.27
+ 241-252 (Yr 21)	2.60%	\$335,659.27	\$3,179.54	\$8,373.92	\$29,780.60	\$305,878.67
+ 253-264 (Yr 22)	2.60%	\$305,878.67	\$3,179.54	\$7,590.33	\$30,564.19	\$275,314.48
+ 265-276 (Yr 23)	2.60%	\$275,314.48	\$3,179.54	\$6,786.12	\$31,368.40	\$243,946.09
+ 277-288 (Yr 24)	2.60%	\$243,946.09	\$3,179.54	\$5,960.75	\$32,193.76	\$211,752.32
+ 289-300 (Yr 25)	2.60%	\$211,752.32	\$3,179.54	\$5,113.67	\$33,040.85	\$178,711.47
+ 301-312 (Yr 26)	2.60%	\$178,711.47	\$3,179.54	\$4,244.30	\$33,910.22	\$144,801.25
+ 313-324 (Yr 27)	2.60%	\$144,801.25	\$3,179.54	\$3,352.05	\$34,802.47	\$109,998.78
+ 325-336 (Yr 28)	2.60%	\$109,998.78	\$3,179.54	\$2,436.32	\$35,718.20	\$74,280.58
+ 337-348 (Yr 29)	2.60%	\$74,280.58	\$3,179.54	\$1,496.50	\$36,658.02	\$37,622.57
+ 349-360 (Yr 30)	2.60%	\$37,622.57	\$3,179.54	\$531.95	\$37,622.57	\$0.00

Print Results

Result

Your monthly repayment schedule is as follow

Month	Interest Rate	Beginning Principal	Monthly Instalment	Interest Paid	Principal Paid	Ending Principal
+ 1-12 (Yr 1)	1.75%	\$800,000.00	\$2,857.95	\$13,836.42	\$20,458.95	\$779,541.05
+ 13-24 (Yr 2)	2.00%	\$779,541.05	\$2,953.94	\$15,407.79	\$20,039.49	\$759,501.56

Diagram illustrating the calculation flow:

- Given:** Interest Rate and Beginning Principal for Year 1.
- To compute:** Monthly Instalment, Interest Paid, Principal Paid, and Ending Principal for Year 1.
- Given:** Interest Rate and Beginning Principal for Year 2 (where Beginning Principal is the Ending Principal of Year 1).
- To compute:** Monthly Instalment, Interest Paid, Principal Paid, and Ending Principal for Year 2.

1. Given “Interest Rate”, “Beginning Principal”, we compute “Monthly Instalment”, “Interest Paid”, “Principal Paid” and “Ending Principal”.
2. Repeat 1 for the next year with a new “Interest Rate” and use the “Ending Principal” of the previous year as the “Beginning Principal” of the current year, till the end of the repayment period.

Formulas:

(yearly)

r: Interest Rate,

bp: Beginning Principal,

n: remaining periods (in years)

m: Monthly Instalment,

ip: Interest Paid,

pp: Principal Paid,

ep: Ending Principal.

$$m = \frac{\frac{r}{12} (bp)}{1 - \left(1 + \frac{r}{12}\right)^{-12n}}$$

$$ep = bp \left(1 + \frac{r}{12}\right)^{12} + \frac{m \left[1 - \left(1 + \frac{r}{12}\right)^{12}\right]}{\frac{r}{12}}$$

$$pp = bp - ep$$

$$ip = 12m - pp$$

Formulas:

(monthly)

r: Interest Rate,

bp: Beginning Principal,

n: remaining periods (in months)

m: Monthly Instalment,

ip: Interest Paid,

pp: Principal Paid,

ep: Ending Principal.

$$m = \frac{\frac{r}{12} (bp)}{1 - \left(1 + \frac{r}{12}\right)^{-n}}$$

$$ep = bp \left(1 + \frac{r}{12}\right) - m$$

$$pp = bp - ep$$

$$ip = m - pp$$

Loan Amount

800000

Loan Tenue

30

Calculate

Interest Rates

Year 1

1.75

Year 2

2

Year 3

2.2

Thereafter

2.6

Month	Interest Rate	Beginning Principal	Monthly Installment	Interest Paid	Principal Paid	Ending Principal
1-12 (Yr 1)	1.75%	\$800,000.00	\$2,857.95	\$13,836.42	\$20,458.95	\$779,541.05
1	1.75%	\$800,000.00	\$2,857.95	\$1,166.67	\$1,691.28	\$798,308.72
2	1.75%	\$798,308.72	\$2,857.95	\$1,164.20	\$1,693.75	\$796,614.97
3	1.75%	\$796,614.97	\$2,857.95	\$1,161.73	\$1,696.22	\$794,918.75
4	1.75%	\$794,918.75	\$2,857.95	\$1,159.26	\$1,698.69	\$793,220.06
5	1.75%	\$793,220.06	\$2,857.95	\$1,156.78	\$1,701.17	\$791,518.89
6	1.75%	\$791,518.89	\$2,857.95	\$1,154.30	\$1,703.65	\$789,815.25
7	1.75%	\$789,815.25	\$2,857.95	\$1,151.81	\$1,706.13	\$788,109.11
8	1.75%	\$788,109.11	\$2,857.95	\$1,149.33	\$1,708.62	\$786,400.49
9	1.75%	\$786,400.49	\$2,857.95	\$1,146.83	\$1,711.11	\$784,689.38
10	1.75%	\$784,689.38	\$2,857.95	\$1,144.34	\$1,713.61	\$782,975.77
11	1.75%	\$782,975.77	\$2,857.95	\$1,141.84	\$1,716.11	\$781,259.66
12	1.75%	\$781,259.66	\$2,857.95	\$1,139.34	\$1,718.61	\$779,541.05
13-24 (Yr 2)	2.00%	\$779,541.05	\$2,953.94	\$15,407.79	\$20,039.49	\$759,501.56
25-36 (Yr 3)	2.20%	\$759,501.56	\$3,029.67	\$16,509.71	\$19,846.30	\$739,655.26
37-48 (Yr 4)	2.60%	\$739,655.26	\$3,179.54	\$19,003.90	\$19,150.62	\$720,504.63
49-60 (Yr 5)	2.60%	\$720,504.63	\$3,179.54	\$18,500.00	\$19,654.52	\$700,850.12
61-72 (Yr 6)	2.60%	\$700,850.12	\$3,179.54	\$17,982.85	\$20,171.67	\$680,678.45

Application 2: Income Tax Calculator

https://www.iras.gov.sg/irashome/uploadedFiles/IRASHome/Individuals/Tax%20Calculator%20-%20Residents_YA17.xls

The screenshot shows the Excel spreadsheet 'Tax Calculator - Residents_YA17.xls' in Compatibility Mode. The spreadsheet is divided into two main sections. The top section, starting from row 67, explains the concept of chargeable income and provides a table of tax rates. The bottom section, starting from row 49, shows the calculation of tax payable for YA 2017.

Chargeable Income Table:

	Chargeable Income	Rate	Gross Tax Payable
	\$	(%)	\$
On the first	20,000	0	0
On the next	10,000	2.0	200
On the first	30,000		200
On the next	10,000	3.5	350
On the first	40,000		550
On the next	40,000	7.0	2,800
On the first	80,000		3,350
On the next	40,000	11.5	4,600
On the first	120,000		7,950
On the next	40,000	18	7,200
On the first	160,000		13,950
On the next	40,000	19	7,600
On the first	200,000		21,150
On the next	40,000	19.5	7,800
On the first	240,000		28,750
On the next	40,000	20	8,000
On the first	280,000		36,550
On the next	40,000		8,000
On the first	320,000		44,550
Above	320,000		

Tax Calculation for YA 2017:

CHARGEABLE INCOME	SS	100,000 .00
Tax Payable on Chargeable Income	SS	5,650.00
Less: Personal Income Tax Rebate (20%)	SS	500.00
Tax Payable after Personal Income Tax Rebate	SS	5,150.00
Less: Parenthood Tax Rebate	SS	0.00
NET TAX PAYABLE	SS	5,150.00

<https://www.iras.gov.sg/irashome/Individuals/Locals/Working-Out-Your-Taxes/Income-Tax-Rates/>

The screenshot shows the IRAS website's 'Income Tax Rates' page for Residents. The page displays a table of tax rates for various chargeable income brackets. The table is titled 'Resident Tax Rates' and 'From YA 2017'.

Chargeable Income	Income Tax Rate (%)	Gross Tax Payable (\$)
First \$20,000	0	0
Next \$10,000	2	200
First \$30,000	-	200
Next \$10,000	3.50	350
First \$40,000	-	550
Next \$40,000	7	2,800
First \$80,000	-	3,350
Next \$40,000	11.5	4,600
First \$120,000	-	7,950
Next \$40,000	15	6,000
First \$160,000	-	13,950
Next \$40,000	18	7,200
First \$200,000	-	21,150
Next \$40,000	19	7,600
First \$240,000	-	28,750
Next \$40,000	19.5	7,800
First \$280,000	-	36,550
Next \$40,000	20	8,000
First \$320,000	-	44,550
In excess of \$320,000	22	

The page also includes a 'RELATED' section with links to calculators and pages, and a 'For YA 2013 to YA 2016' section at the bottom. A chatbot icon is visible in the bottom right corner.



IRAS Income Tax (Simple) Calculator

IRAS Website

<https://www.iras.gov.sg/irashome/Individuals/Locals/Working-Out-Your-Taxes/Income-Tax-Rates/>

tableID: 0

Reload Rates

	1	2	3
1	Chargeable Income	Income Tax Rate (%)	Gross Tax Payable (\$)
2	First \$20,000 Next \$10,000	0 - 2	0 - 200
3	First \$30,000 Next \$10,000	- 3.50	200 - 350
4	First \$40,000 Next \$40,000	- 7	550 - 2,800
5	First \$80,000 Next \$40,000	- 11.5	3,350 - 4,600
6	First \$120,000 Next \$40,000	- 15	7,950 - 6,000
7	First \$160,000 Next \$40,000	- 18	13,950 - 7,200
8	First \$200,000 Next \$40,000	- 19	21,150 - 7,600
9	First \$240,000 Next \$40,000	- 19.5	28,750 - 7,800
10	First \$280,000 Next \$40,000	- 20	36,550 - 8,000
11	First \$320,000 In excess of \$320,000	- 22	44,550

Chargeable Income (net)

100000

☒ Tax Rebate

Rate (%) 20

Cap (\$) 500

Calculate

Tax Amount:

5150.0

Resident Tax Rates

From YA 2017

Chargeable Income	Income Tax Rate (%)	Gross Tax Payable (\$)
First \$20,000 Next \$10,000	0 2	0 200
First \$30,000 Next \$10,000	- 3.50	200 350
First \$40,000 Next \$40,000	- 7	550 2,800
First \$80,000 Next \$40,000	- 11.5	3,350 4,600
First \$120,000 Next \$40,000	- 15	7,950 6,000
First \$160,000 Next \$40,000	- 18	13,950 7,200
First \$200,000 Next \$40,000	- 19	21,150 7,600
First \$240,000 Next \$40,000	- 19.5	28,750 7,800
First \$280,000 Next \$40,000	- 20	36,550 8,000
First \$320,000 In excess of \$320,000	- 22	44,550

```
xi=[int(x.split()[1].replace('$','').replace(',','')) for x in tables[0][0][1:]]
xi
```

```
[20000, 30000, 40000, 80000, 120000, 160000, 200000, 240000, 280000, 320000]
```

```
mi=[float(x.split()[1]) for x in tables[0][1][1:]]
mi
```

```
[2.0, 3.5, 7.0, 11.5, 15.0, 18.0, 19.0, 19.5, 20.0, 22.0]
```

```
bi=[int(x.split()[0].replace(',','')) for x in tables[0][2][1:]]
bi
```

```
[0, 200, 550, 3350, 7950, 13950, 21150, 28750, 36550, 44550]
```

$$y(x) = \begin{cases} 0, & 0 \leq x < 20,000 \\ 0 + 0.02(x - 20000), & 20,000 \leq x < 30,000 \\ 200 + 0.035(x - 30000), & 30,000 \leq x < 40,000 \\ 550 + 0.07(x - 40000), & 40,000 \leq x < 80,000 \\ 3350 + 0.115(x - 80000), & 80,000 \leq x < 120,000 \\ 7950 + 0.15(x - 120000), & 120,000 \leq x < 160,000 \\ 13950 + 0.18(x - 160000), & 160,000 \leq x < 200,000 \\ 21150 + 0.19(x - 200000), & 200,000 \leq x < 240,000 \\ 28750 + 0.195(x - 240000), & 240,000 \leq x < 280,000 \\ 36550 + 0.20(x - 280000), & 280,000 \leq x < 320,000 \\ 44550 + 0.22(x - 320000), & 320,000 \leq x \end{cases}$$



Application 2a: Income Tax Calculator (★ ★ ☆ ☆ ☆)



1. ☐ Numeric Literals
2. ☐ Arithmetic Operators
3. ☐ Comparison (Relational) Operators
4. ☐ Variables
5. ☐ Assignment Statements
6. ☐ Lists
7. ☐ Lists Comprehension (or `for` Statements)
8. ☐ `if` Statements
9. ☐ Built-in Functions `float`, `int`, `sum`, `max`, `print`
- ❖ ☐ (optional) Built-in Function `input`

Hint: About 15 lines

Application 2b: Income Tax Calculator GUI (★★★★☆)



10. ☐ Qt Designer + PyQt5 Template (Label, Push Button, Line Edit, Check Box, Table)
- a) Functions, Modules and Classes
 - b) `pandas` Functions `read_html`
 - c) `pandas.DataFrame`
 - d) String Functions `split`, `replace`

Hint: About 100 lines + .ui

Application 3: Stock Chart & Moving Average (MA) Crossover



Yahoo Finance (CC3.SI) (*created on 10/Jul/2017)
 Summary > Click Chart "Full screen"
 > Add Indicator > Simple Moving Average (SMA)
 (window size: 1 year)

Stock Chart & Moving Average Crossover

Load Another CSV File

C:/Users/ybzhao/myPython/CC3.SI.csv

2015-07-01 to 2017-07-07

Start Date 2016-07-11

End Date 2017-07-11

Update Window (after changing "Start Date" or "End Date")

☒ SMA-1 (<=252) 15

☒ SMA-2 (<=252) 50



Algorithm:

Given historical daily “Close Price”, $data_i, i = 0, 1, 2, \dots, N$

1. Compute $SMA15$ and $SMA50$.
2. Compare $SMA15$ to $SMA50$ by computing $SMA15 - SMA50$. If the difference is positive, it indicates that $SMA15$ is above $SMA50$, and vice versa. We denote the earlier situation by 1, and the latter by 0. It results a sequence of 0's and 1's.
3. Compute the difference of each pair of consecutive numbers in the above mentioned sequence of 0's and 1's. It results a new sequence of values of 0 (when $0 - 0$ or $1 - 1$), 1 (when $1 - 0$) and -1 (when $0 - 1$).
4. Find the locations of 1's and -1's in the new sequence. Those will be the locations of crossovers, after shifting by 1 to the right.

Challenge: There might be missing data.



Application 3a: Stock Chart & MA Crossover (★★★★★)

1. ☐ `pandas` Functions `read_csv`
2. ☐ `pandas DataFrame` Functions `drop`, `plot`
3. ☐ `pandas Series` Functions `rolling().mean()`
4. ☐ `numpy` Functions `round`, `nan`, `diff`
5. ☐ `matplotlib.pyplot` Functions `subplots`, `tight_layout`
6. ☐ Comparison (Relational) Operators
7. ☐ Variables
8. ☐ Literals (numeric and strings)
9. ☐ Assignment Statements
10. ☐ Arithmetic Operators on `pandas Series`
11. ☐ Logical Indexing



Hint: About 20 lines

Application 3b: Stock Chart & MA Crossover GUI (★★★★☆)

- 12. ☐ Qt Designer + PyQt5 Template (Label, Push Button, Line Edit, Check Box)
 - a) Functions, Modules and Classes
 - b) `datetime` Functions `datetime`, `strptime`, `date`, `date.today`, `date.year`, `date.month`, `date.day`, `strftime`,
 - c) `matplotlib.dates` Function `DateFormatter`
- 13. ☐ Built-in Functions `enumerate`, `next`,
- 14. ☐ Generator Expression
- 15. ☐ List Comprehension
- 16. ☐ `pandas` `DataFrame` Function `copy`
- 17. ☐ `assert` Statement



Hint: About 200 lines + .ui

Application 4: Option Price (and Greeks) Calculator



Option pricing refers to the amount per share that an option is traded. Options are derivative contracts that give the holder (the "buyer") the right, but not the obligation, to buy or sell the underlying instrument at an agreed-upon price on or before a specified future date. Although the holder of the option is not obligated to exercise the option, the option writer (the "seller") has an obligation to buy or sell the underlying instrument if the option is exercised.

Price ↔ Implied Volatility

Read more:

Options Pricing <http://www.investopedia.com/university/options-pricing/#ixzz4mVw9p0nB>

Application 4: Option Price (and Greeks) Calculator



Many option traders rely on the "Greeks" to evaluate option positions and to determine option sensitivity. The Greeks are a collection of statistical values that measure the risk involved in an options contract in relation to certain underlying variables. Popular Greeks include Delta, Vega, Gamma and Theta.

For example, let $V(S, K, r, q, \tau, \sigma)$ denote the option price function,

$$\text{Delta: } \Delta = \frac{\partial V}{\partial S} \approx \frac{V(S+\Delta S, K, r, q, \tau, \sigma) - V(S, K, r, q, \tau, \sigma)}{\Delta S}$$

$$\tau = T - t$$

Read more: Options Pricing:

The Greeks <http://www.investopedia.com/university/options-pricing/greeks.asp#ixzz4mVxbx200>

Application 4: Option Price (and Greeks) Calculator

<http://www.fintools.com/resources/online-calculators/options-calcs/options-calculator/>



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OptionsCalc

Model ☐ Black-Scholes

Stock Price

Exercise Price

Value Date

Early-Exercise Date

Expiration Date 183 days

Volatility (%)

Interest Rate (%)

Dividend Method

Yield Rate (%)

	Call	Put
Theoretical Value <input type="checkbox"/>	5.9399	5.1973
Delta <input type="checkbox"/>	0.5743	-0.4207
Delta 100's <input type="checkbox"/>	57.4307	-42.0691
Lambda (%) <input type="checkbox"/>	4.8343	-4.0472
Gamma <input type="checkbox"/>	0.0275	0.0275
Gamma (1%) <input type="checkbox"/>	0.0138	0.0138
Theta <input type="checkbox"/>	-0.0168	-0.0128
Theta (7 days) <input type="checkbox"/>	-0.1186	-0.0905
Vega <input type="checkbox"/>	0.1379	0.1379
Rho <input type="checkbox"/>	0.1142	-0.1315
Psi <input type="checkbox"/>	-0.1440	0.1055
Strike Sensitivity <input type="checkbox"/>	-0.4555	0.5246
Intrinsic Value <input type="checkbox"/>	0.0000	0.0000
Time Value <input type="checkbox"/>	5.9399	5.1973
Zero Volatility <input type="checkbox"/>	0.7427	0.0000
Market Option Price <input type="checkbox"/>	20.04	8.51
Implied Volatility (%) <input type="checkbox"/>	147.42	64.19

Calculate | Default | Reset | Print | Close

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Gamma (1%)

The rate of change in an option's delta with respect to 1% change in the price of the underlying assets or instrument.

$$[\text{Delta}(1.01S) - \text{Delta}(0.99S)]/2$$

Theta

The rate of change in an option's price as time (1 day) passes with all else remaining the same.

$$[\text{Price}(\tau-1 \text{ days}) - \text{Price}(\tau \text{ days})]$$

Theta (7 days)

The rate of change in an option's price as time (7 day) passes with all else remaining the same.

$$[\text{Price}(\tau-1 \text{ days}) - \text{Price}(\tau \text{ days})]$$



OptionsCalc

Model ?	Black-Scholes ▼		
Stock Price ?	50.00		
Exercise Price ?	50.00		
Value Date ?	01/01/2011		
Early-Exercise Date ?	07/03/2011		
Expiration Date ?	07/03/2011		183 days
Volatility (%) ?	40.00		
Interest Rate (%) ?	4.00		
Dividend Method ?	Continuous ▼		
Yield Rate (%) ?	1.00		

	Call	Put
Theoretical Value ?	5.9399	5.1973
Delta ?	0.5743	-0.4207
Delta 100's ?	57.4307	-42.0691
Lambda (%) ?	4.8343	-4.0472
Gamma ?	0.0275	0.0275
Gamma (1%) ?	0.0138	0.0138
Theta ?	-0.0168	-0.0128
Theta (7 days) ?	-0.1186	-0.0905
Vega ?	0.1379	0.1379
Rho ?	0.1142	-0.1315
Psi ?	-0.1440	0.1055
Strike Sensitivity ?	-0.4555	0.5246
Intrinsic Value ?	0.0000	0.0000
Time Value ?	5.9399	5.1973
Zero Volatility ?	0.7427	0.0000
Market Option Price ?	20.04	8.51
Implied Volatility (%) ?	147.42	64.19

Calculate

Default

Reset

Print

Close

OptionsCalc

Model

Binomial

100.5

Stock Price

50.00

Exercise Price

50.00

Value Date

01/01/2011

Early-Exercise Date

07/03/2011

Expiration Date

07/03/2011

183 days

Volatility (%)

40.00

Interest Rate (%)

4.00

Dividend Method

Continuous

Yield Rate (%)

1.00

Repo Rate (%)

4.00

Theoretical Value

Call

Put

Delta

5.9398

5.1971

Delta 100's

57.5293

-41.9706

Lambda (%)

4.8430

-4.0381

Gamma

0.0274

0.0274

Gamma (1%)

0.0137

0.0137

Theta

-0.0169

-0.0128

Theta (7 days)

-0.1138

-0.0905

Vega

0.1379

0.1379

Rho

0.1439

-0.1055

Psi

-0.1439

0.1055

Strike Sensitivity

-0.1555

0.5246

Intrinsic Value

0.0000

0.0000

Time Value

5.9398

5.1971

Zero Volatility

0.7609

0.0182

Market Option Price

20.04

8.51

Implied Volatility (%)

147.46

64.19

Calculate

Default

Reset

Print

Close

Numerical Approximations To Derivatives

Delta: $\Delta = \frac{\partial V}{\partial S} \approx \frac{V(S+\Delta S) - V(S)}{\Delta S}$ or $\frac{V(S+\Delta S) - V(S-\Delta S)}{2\Delta S}$

Delta 100's: $\Delta_{100} = 100\Delta$

Lambda: $\lambda = \Delta \times \frac{S}{V}$

Gamma: $\Gamma = \frac{\partial^2 V}{\partial S^2} \approx \frac{V(S+\Delta S) - 2V(S) + V(S-\Delta S)}{(\Delta S)^2}$

Gamma 1%: $= \frac{\Delta(1.01S) - \Delta(0.99S)}{2}$

Theta 1 day: $\theta_1 = V(\tau - 1/365) - V(\tau)$

Theta 7 days: $\theta_7 = V\left(\tau - \frac{7}{365}\right) - V(\tau)$

Vega: $v = \frac{\partial V}{\partial \sigma} \approx \frac{V(\sigma+\Delta\sigma) - V(\sigma)}{\Delta\sigma}$ or $\frac{V(\sigma+\Delta\sigma) - V(\sigma-\Delta\sigma)}{2\Delta\sigma}$

Rho: $\rho = \frac{\partial V}{\partial r} \approx \frac{V(r+\Delta r) - V(r)}{\Delta r}$ or $\frac{V(r+\Delta r) - V(r-\Delta r)}{2\Delta r}$

Psi: $\Psi = \frac{\partial V}{\partial q} \approx \frac{V(q+\Delta q) - V(q)}{\Delta q}$ or $\frac{V(q+\Delta q) - V(q-\Delta q)}{2\Delta q}$

Strike Sensitivity (SS): $\frac{V-S\Delta}{K}$

Intrinsic Value (IV):
 $\max(S - K, 0),$
 $\max(K - S, 0)$

Time Value: $V - IV$

Zero Volatility:
 $e^{-r\tau} \max(S e^{(r-q)\tau} - K, 0)$
 $e^{-r\tau} \max(K - S e^{(r-q)\tau}, 0)$



option_BS - DataFrame			
Index	Attributes	Call	Put
S	Stock Price	50.0000	50.0000
K	Exercise Price	50.0000	50.0000
t	Time to Maturity	0.5014	0.5014
sigma	Volatility (%)	40.0000	40.0000
r	Interest Rate (%)	4.0000	4.0000
q	Yield Rate (%)	1.0000	1.0000
V	Theoretical Value	5.9399	5.1973
delta	Delta	0.5743	-0.4207
delta100	Delta 100's	57.4308	-42.0691
lambda	Lambda (%)	4.8343	-4.0472
gamma	Gamma	0.0275	0.0275
gamma1pct	Gamma (1%)	0.0138	0.0138
theta	Theta	-0.0168	-0.0128
theta7day	Theta (7 days)	-0.1186	-0.0905
vega	Vega	0.1379	0.1379
rho	Rho	0.1142	-0.1315
psi	Psi	-0.1440	0.1055
ss	Strike Sensitivity	-0.4555	0.5246
iv	Intrinsic Value	0.0000	0.0000
tv	Time Value	5.9399	5.1973
zv	Zero Volatility	0.7427	0.0000

Format Resize ☐ Background co ☒ Column min/max OK Cancel

MainWindow

Option Price (and Greeks) Calculator

Model

Black-Scholes

Stock Price

50.00

Exercise Price

50.00

Value Date

01/01/2011

Expiration Date

07/03/2011

Volatility (%)

40.00

Interest Rate (%)

4.00

Dividend Method

Continuous

Yield Rate (%)

1.00

Calculate

Call

Put

Theoretical Value

5.9399

5.1973

Delta

0.5743

-0.4207

Delta 100's

57.4308

-42.0691

Lambda (%)

4.8343

-4.0472

Gamma

0.0275

0.0275

Gamma (1%)

0.0138

0.0138

Theta

-0.0168

-0.0128

Theta (7 days)

-0.1186

-0.0905

Vega

0.1379

0.1379

Rho

0.1142

-0.1315

Psi

-0.1440

0.1055

Strike Sensitivity

-0.4555

0.5246

Intrinsic Value

0.0000

0.0000

Time Value

5.9399

5.1973

Zero Volatility

0.7427

0.0000

option_Binomial - DataFrame			
Index	Attributes	Call	Put
S	Stock Price	50.0000	50.0000
K	Exercise Price	50.0000	50.0000
t	Time to Maturity	0.5014	0.5014
sigma	Volatility (%)	40.0000	40.0000
r	Interest Rate (%)	4.0000	4.0000
q	Yield Rate (%)	1.0000	1.0000
V	Theoretical Value	5.9398	5.1971
delta	Delta	0.5742	-0.4208
delta100	Delta 100's	57.4232	-42.0817
lambda	Lambda (%)	4.8338	-4.0486
gamma	Gamma	0.0277	0.0277
gamma1pct	Gamma (1%)	0.0138	0.0138
theta	Theta	-0.0168	-0.0128
theta7day	Theta (7 days)	-0.1186	-0.0905
vega	Vega	0.1379	0.1379
rho	Rho	0.1142	-0.1315
psi	Psi	-0.1439	0.1055
ss	Strike Sensitivity	-0.4554	0.5248
iv	Intrinsic Value	0.0000	0.0000
tv	Time Value	5.9398	5.1971
zv	Zero Volatility	0.7427	0.0000

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MainWindow

Option Price (and Greeks) Calculator

Model

Binomial (N=100.5)

Stock Price

50.00

Exercise Price

50.00

Value Date

01/01/2011

Expiration Date

07/03/2011

Volatility (%)

40.00

Interest Rate (%)

4.00

Dividend Method

Continuous

Yield Rate (%)

1.00

Calculate

Call

Put

Theoretical Value

5.9398

5.1971

Delta

0.5742

-0.4208

Delta 100's

57.4232

-42.0817

Lambda (%)

4.8338

-4.0486

Gamma

0.0277

0.0277

Gamma (1%)

0.0138

0.0138

Theta

-0.0168

-0.0128

Theta (7 days)

-0.1186

-0.0905

Vega

0.1379

0.1379

Rho

0.1142

-0.1315

Psi

-0.1439

0.1055

Strike Sensitivity

-0.4554

0.5248

Intrinsic Value

0.0000

0.0000

Time Value

5.9398

5.1971

Zero Volatility

0.7427

0.0000

$N=10,000,000+10,000,000$ (antithetic), seed(0)

option_MonteCarlo - DataFrame			
Index	Attributes	Call	Put
S	Stock Price	50.0000	50.0000
K	Exercise Price	50.0000	50.0000
t	Time to Maturity	0.5014	0.5014
sigma	Volatility (%)	40.0000	40.0000
r	Interest Rate (%)	4.0000	4.0000
q	Yield Rate (%)	1.0000	1.0000
V	Theoretical Value	5.9383	5.1966
delta	Delta	0.5743	-0.4207
delta100	Delta 100's	57.4298	-42.0682
lambda	Lambda (%)	4.8355	-4.0477
gamma	Gamma	0.0277	0.0277
gamma1pct	Gamma (1%)	0.2871	-0.2104
theta	Theta	-0.0168	-0.0128
theta7day	Theta (7 days)	-0.1185	-0.0905
vega	Vega	0.1378	0.1379
rho	Rho	0.1142	-0.1315
psi	Psi	-0.1440	0.1055
ss	Strike Sensitivity	-0.4555	0.5246
iv	Intrinsic Value	0.0000	0.0000
tv	Time Value	5.9383	5.1966
zv	Zero Volatility	0.7427	0.0000

Format

Resize

☐ Background color

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OK

Cancel

MainWindow

Option Price (and Greeks) Calculator

Model

Monte Carlo Simulation

Stock Price

50.00

Exercise Price

50.00

Value Date

01/01/2011

Expiration Date

07/03/2011

Volatility (%)

40.00

Interest Rate (%)

4.00

Dividend Method

Continuous

Yield Rate (%)

1.00

Calculate

Theoretical Value

Call

5.9383

Delta

0.5743

Delta 100's

57.4298

Lambda (%)

4.8355

Gamma

0.0277

Gamma (1%)

0.2871

Theta

-0.0168

Theta (7 days)

-0.1185

Vega

0.1378

Rho

0.1142

Psi

-0.1440

Strike Sensitivity

-0.4555

Intrinsic Value

0.0000

Time Value

5.9383

Zero Volatility

0.7427

Put

5.1966

-0.4207

-42.0682

-4.0477

0.0277

-0.2104

-0.0128

-0.0905

0.1379

-0.1315

0.1055

0.5246

0.0000

5.1966

0.0000

Algorithm (Black-Scholes):

Given S, K, r, q, t, T and σ , (1 day=1/365 years)

- Compute Option Prices/Values c and p using

$$c = S \cdot e^{-q(T-t)} \cdot \Phi(d_1) - K \cdot e^{-r(T-t)} \cdot \Phi(d_2)$$

and

$$p = K \cdot e^{-r(T-t)} \cdot \Phi(-d_2) - S \cdot e^{-q(T-t)} \cdot \Phi(-d_1)$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}, d_2 = d_1 - \sigma\sqrt{T-t}$$

$$c = V_c(S, K, r, q, t, T, \sigma), p = V_p(S, K, r, q, t, T, \sigma)$$

- Compute Greeks using :

- Delta

$$\Delta_c = e^{-q(T-t)} \cdot \Phi(d_1), \Delta_p = -e^{-q(T-t)} \cdot \Phi(-d_1)$$

$$\Delta_c = \Delta_c(S, K, r, q, t, T, \sigma), \Delta_p = \Delta_p(S, K, r, q, t, T, \sigma)$$

- Delta 100's

$$100\Delta_c, 100\Delta_p$$

- Lambda

$$\lambda_c = \Delta_c \times \frac{S}{c}, \lambda_p = \Delta_p \times \frac{S}{p}$$

- Compute Greeks (continued)

- Gamma

$$\Gamma_c = \Gamma_p = e^{-q(T-t)} \frac{\phi(d_1)}{S\sigma\sqrt{T-t}}$$

- Gamma (1%)

$$\frac{\Delta_c(1.01S, K, r, q, t, T, \sigma) - \Delta_c(0.99S, K, r, q, t, T, \sigma)}{2},$$

$$\frac{\Delta_p(1.01S, K, r, q, t, T, \sigma) - \Delta_p(0.99S, K, r, q, t, T, \sigma)}{2}$$

- Theta (1 day)

$$V_c\left(S, K, r, q, t - \frac{1}{365}, T, \sigma\right) - V_c(S, K, r, q, t, T, \sigma),$$

$$V_p\left(S, K, r, q, t - \frac{1}{365}, T, \sigma\right) - V_p(S, K, r, q, t, T, \sigma)$$

- Theta (7 days)

$$V_c\left(S, K, r, q, t - \frac{7}{365}, T, \sigma\right) - V_c(S, K, r, q, t, T, \sigma),$$

$$V_p\left(S, K, r, q, t - \frac{7}{365}, T, \sigma\right) - V_p(S, K, r, q, t, T, \sigma)$$

- Vega

$$v_c = v_p = S e^{-q(T-t)} \phi(d_1) \sqrt{T-t} \times \frac{1}{100}$$



Algorithm (continued):

- Compute Greeks (continued)

- Rho

$$\rho_c = K(T-t)e^{-r(T-t)}\Phi(d_2) \times \frac{1}{100}, \quad \rho_p = -K(T-t)e^{-r(T-t)}\Phi(-d_2) \times \frac{1}{100}$$

- Psi

$$\Psi_c = \left[-S(T-t)e^{-q(T-t)}\Phi(d_1) - \frac{S\sqrt{T-t}}{\sigma}e^{-q(T-t)}\phi(d_1) + \frac{K\sqrt{T-t}}{\sigma}e^{-r(T-t)}\phi(d_2) \right] \times \frac{1}{100},$$

$$\Psi_p = \left[S(T-t)e^{-q(T-t)}\Phi(-d_1) - \frac{S\sqrt{T-t}}{\sigma}e^{-q(T-t)}\phi(-d_1) + \frac{K\sqrt{T-t}}{\sigma}e^{-r(T-t)}\phi(-d_2) \right] \times \frac{1}{100}$$

- Strike Sensitivity

$$SS_c = \frac{c - S\Delta_c}{K}, SS_p = \frac{p - S\Delta_p}{K}$$

- Intrinsic Value

$$IV_c = \max(S - K, 0), IV_p = \max(K - S, 0)$$

- Time Value

$$TV_c = c - IV_c, TV_p = p - IV_p$$

- Zero Volatility

$$ZV_c = \max(S e^{(r-q)(T-t)} - K, 0) e^{-r(T-t)}, ZV_p = \max(K - S e^{(r-q)(T-t)}, 0) e^{-r(T-t)}$$



Appendix:



- Theta (instantaneous rate of change)

$$\theta_c = -e^{-q(T-t)} \frac{S\phi(d_1)\sigma}{2\sqrt{T-t}} - rKe^{-r(T-t)}\Phi(d_2) + qSe^{-q(T-t)}\Phi(d_1)$$

$$\theta_p = -e^{-q(T-t)} \frac{S\phi(d_1)\sigma}{2\sqrt{T-t}} + rKe^{-r(T-t)}\Phi(-d_2) - qSe^{-q(T-t)}\Phi(-d_1)$$

$$\tau = T - t$$

Algorithm (Binomial Tree):

Given S, K, r, q, t, T, σ and N (1 day=1/365 years)

1. $\Delta t = \frac{T-t}{N}, u = e^{\sigma\sqrt{\Delta t}}, d = \frac{1}{u}, p = \frac{e^{(r-q)\Delta t} - d}{u - d}$
2. $f_{N,j} = \max(0, Su^j d^{N-j} - K)$ (Call), $f_{N,j} = \max(0, K - Su^j d^{N-j})$ (Put),
for $j = 0, 1, \dots, N$
3. $f_{i,j} = e^{-r\Delta t} [p \cdot f_{i+1,j+1} + (1 - p) \cdot f_{i+1,j}]$ for $i = N - 1, N - 2, \dots, 1, 0; j = 0, 1, \dots, i$
4. Option (call/put) price: $V = f_{0,0}$
5. Option Greeks (*Delta and *Gamma):

$$\Delta = \frac{f_{1,1} - f_{1,0}}{S \cdot u - S \cdot d}, \Gamma = \frac{d(f_{2,2} - f_{2,1}) - u(f_{2,1} - f_{2,0})}{S^2(u - d)^2}$$

~~$$\text{Delta: } \Delta = \frac{\partial V}{\partial S} \approx \frac{V(S+\Delta S) - V(S)}{\Delta S} \text{ or } \frac{V(S+\Delta S) - V(S-\Delta S)}{2\Delta S}$$~~

Delta 100's: $\Delta_{100} = 100\Delta$

$$\text{Lambda: } \lambda = \Delta \times \frac{S}{V}$$

~~$$\text{Gamma: } \Gamma = \frac{\partial^2 V}{\partial S^2} \approx \frac{V(S+\Delta S) - 2V(S) + V(S-\Delta S)}{(\Delta S)^2}$$~~

$$\text{Gamma 1\%: } = \frac{\Delta(1.01S) - \Delta(0.99S)}{2}$$

$$\text{Theta 1 day: } \theta_1 = V\left(\tau - \frac{1}{365}\right) - V(\tau)$$

$$\text{Theta 7 days: } \theta_7 = V\left(\tau - \frac{7}{365}\right) - V(\tau)$$

$$\text{Vega: } v = \frac{\partial V}{\partial \sigma} \approx \frac{V(\sigma+\Delta\sigma) - V(\sigma)}{\Delta\sigma} \text{ or } \frac{V(\sigma+\Delta\sigma) - V(\sigma-\Delta\sigma)}{2\Delta\sigma}$$

$$\text{Rho: } \rho = \frac{\partial V}{\partial r} \approx \frac{V(r+\Delta r) - V(r)}{\Delta r} \text{ or } \frac{V(r+\Delta r) - V(r-\Delta r)}{2\Delta r}$$

$$\text{Psi: } \Psi = \frac{\partial V}{\partial q} \approx \frac{V(q+\Delta q) - V(q)}{\Delta q} \text{ or } \frac{V(q+\Delta q) - V(q-\Delta q)}{2\Delta q}$$

$$\text{Strike Sensitivity (SS): } \frac{V - S\Delta}{K}$$

Intrinsic Value (IV):

$$\max(S - K, 0), \\ \max(K - S, 0)$$

Time Value: $V - IV$

Zero Volatility:

$$e^{-r\tau} \max(S e^{(r-q)\tau} - K, 0) \\ e^{-r\tau} \max(K - S e^{(r-q)\tau}, 0)$$

$$\tau = T - t$$



Algorithm (Monte Carlo Simulation):

Given S, K, r, q, t, T, σ and N (1 day=1/365 years)

$$S_T = S e^{\left(r - q - \frac{1}{2}\sigma^2\right)(T - t) + \sigma\sqrt{(T - t)}z}$$

where z is a random variable following the standard normal distribution.

1. Draw N random numbers $z(i), i = 1, 2, \dots, N$, from the standard normal distribution.
2. Calculate $S_T(i)$ using the above equation for each $z = z(i)$.
3. Calculate the option's value at maturity as

$$\text{Call option: } h_T(i) = \max(S_T(i) - K, 0)$$

$$\text{or Put option: } h_T(i) = \max(K - S_T(i), 0)$$

4. Estimate the option's price/value using:

$$V = e^{-r(T-t)} \frac{1}{N} \sum_{i=1}^N h_T(i)$$

$$\tau = T - t$$

$$\text{Delta: } \Delta = \frac{\partial V}{\partial S} \approx \frac{V(S+\Delta S) - V(S)}{\Delta S} \text{ or } \frac{V(S+\Delta S) - V(S-\Delta S)}{2\Delta S}$$

$$\text{Delta 100's: } \Delta_{100} = 100\Delta$$

$$\text{Lambda: } \lambda = \Delta \times \frac{S}{V}$$

$$\text{Gamma: } \Gamma = \frac{\partial^2 V}{\partial S^2} \approx \frac{V(S+\Delta S) - 2V(S) + V(S-\Delta S)}{(\Delta S)^2}$$

$$\text{Gamma 1\%: } = \frac{\Delta(1.01S) - \Delta(0.99S)}{2}$$

$$\text{Theta 1 day: } \theta_1 = V\left(\tau - \frac{1}{365}\right) - V(\tau)$$

$$\text{Theta 7 days: } \theta_7 = V\left(\tau - \frac{7}{365}\right) - V(\tau)$$

$$\text{Vega: } v = \frac{\partial V}{\partial \sigma} \approx \frac{V(\sigma+\Delta\sigma) - V(\sigma)}{\Delta\sigma} \text{ or } \frac{V(\sigma+\Delta\sigma) - V(\sigma-\Delta\sigma)}{2\Delta\sigma}$$

$$\text{Rho: } \rho = \frac{\partial V}{\partial r} \approx \frac{V(r+\Delta r) - V(r)}{\Delta r} \text{ or } \frac{V(r+\Delta r) - V(r-\Delta r)}{2\Delta r}$$

$$\text{Psi: } \Psi = \frac{\partial V}{\partial q} \approx \frac{V(q+\Delta q) - V(q)}{\Delta q} \text{ or } \frac{V(q+\Delta q) - V(q-\Delta q)}{2\Delta q}$$

$$\text{Strike Sensitivity (SS): } \frac{V - S\Delta}{K}$$

Intrinsic Value (IV):

$$\max(S - K, 0), \\ \max(K - S, 0)$$

Time Value: $V - IV$

Zero Volatility:

$$e^{-r\tau} \max(Se^{(r-q)\tau} - K, 0) \\ e^{-r\tau} \max(K - Se^{(r-q)\tau}, 0)$$



Application 4a: Option Price (and Greeks) Calculator (★ ★ ☆ ☆ ☆)

(Using Black-Scholes Formula)

1. ☐ Literals (Numbers and Strings)
2. ☐ Arithmetic Operators
3. ☐ Variables
4. ☐ Assignment Statements
5. ☐ Function Definitions
6. ☐ `return` Statement
7. ☐ `scipy.stats.norm` Functions `cdf`, `pdf`
8. ☐ `numpy` Functions `sqrt`, `exp`, `log`
- ❖ ☐ List or Tuple Indexing and Slicing
- ❖ ☐ (optional) `pandas.DataFrame`
- ❖ ☐ (optional) Built-in Function `print`

Hint: About 15+60 lines



Application 4b: Option Price (and Greeks) Calculator (★★★★☆)

(Using Binomial Tree)

1. ☐ Literals (numbers and strings)
2. ☐ Arithmetic Operators
3. ☐ Variables
4. ☐ Assignment Statements
5. ☐ (Nested) List Comprehension
6. ☐ List or Tuple Indexing and Slicing
7. ☐ (Nested) `for` Statements
8. ☐ (Recursive) Functions
9. ☐ Built-in Functions `isinstance`, `range`, `int`, `max`
10. ☐ `return` Statement
11. ☐ `if` Statements
12. ☐ `numpy` Functions `sqrt`, `exp`
- ❖ ☐ (optional) `pandas.DataFrame`

Hint: About 40+70 lines



Application 4c: Option Price (and Greeks) Calculator (★★★★☆)

(Using Monte Carlo Simulation)

1. ☐ Literals (Numbers and Strings)
 2. ☐ Arithmetic Operators
 3. ☐ Variables
 4. ☐ Assignment Statements
 5. ☐ Functions
 6. ☐ `if` Statements
 7. ☐ `return` Statement
 8. ☐ `numpy` Functions `sqrt`, `exp`, `maximum`, `mean`,
`random.seed`, `random.standard_normal`, `append`
 - ❖ ☐ (optional) `pandas.DataFrame`
- Hint: About 10+70 lines



Application 4d: Option Price (and Greeks) Calculator GUI (★★★★☆)

- ✓ ☐ Qt Designer + PyQt5 Template (Label, Push Button, Line Edit, Date Edit, Combo Box)
 - a) Functions
 - b) Modules
 - c) Classes
- ✓ ☐ `datetime` Attribute `date.days`
- ✓ ☐ Built-in Function `float`, `format`
- ✓ ☐ `pandas.DataFrame` Indexing



Hint: About 150 lines

Application 4e: Option Price (and Greeks) Calculator (★★★★☆)

(Using 3 Numerical Solutions to the Black-Scholes PDE with Continuous Dividend)

$$V_t + \frac{1}{2}\sigma^2 S^2 V_{SS} + (r - q)SV_S - rV = 0$$

Explicit Method:

Given $S, K, r, q, t = 0, T, \sigma, M$ and N . (Assume $S < 2K$.)

1. Compute $\Delta t = \frac{T}{N}$, $\Delta S = \frac{S_{\max}}{M}$, where $S_{\max} = 2K$.
2. Compute (Call option) $f_{N,j} = \max(j(\Delta S) - K, 0)$, (Put option) $f_{N,j} = \max(K - j(\Delta S), 0)$, for $j = 0, 1, \dots, M$.
3. For $i = N - 1, N - 2, \dots, 1, 0$, repeat 3.1 and 3.2.

3.1. Compute vector $\hat{F}_i = A \cdot F_{i+1}$, where

$$\hat{F}_i = \begin{pmatrix} \hat{f}_{i,0} \\ \hat{f}_{i,1} \\ \vdots \\ \hat{f}_{i,M-1} \\ \hat{f}_{i,M} \end{pmatrix}, F_{i+1} = \begin{pmatrix} f_{i+1,0} \\ f_{i+1,1} \\ \vdots \\ f_{i+1,M-1} \\ f_{i+1,M} \end{pmatrix}, A = \begin{pmatrix} 1 & 0 & 0 & 0 & \dots & & & \\ a_1^* & b_1^* & c_1^* & 0 & & & & \\ 0 & a_2^* & b_2^* & c_2^* & & & & \\ \vdots & & \ddots & \ddots & \ddots & & & \\ \vdots & & & \ddots & \ddots & \ddots & & \\ & & & & a_{M-2}^* & b_{M-2}^* & c_{M-2}^* & 0 \\ & & & & 0 & a_{M-1}^* & b_{M-1}^* & c_{M-1}^* \\ \dots & & & & 0 & 0 & 0 & 1 \end{pmatrix} \text{ with } \begin{cases} a_j^* = \frac{1}{2} \cdot (\Delta t) \cdot [\sigma^2 \cdot j^2 - (r - q) \cdot j] \\ b_j^* = 1 - (\Delta t) \cdot [\sigma^2 \cdot j^2 + r] \\ c_j^* = \frac{1}{2} \cdot (\Delta t) \cdot [\sigma^2 \cdot j^2 + (r - q) \cdot j] \end{cases}, \text{ for } j = 1, \dots, M - 1.$$

3.2. Compute vector F_i ,

$$F_i = \begin{pmatrix} f_{i,0} \\ f_{i,1} \\ \vdots \\ f_{i,M-1} \\ f_{i,M} \end{pmatrix} \text{ with } f_{i,j} = \begin{cases} 0 & , \text{ for } j = 0 \\ \hat{f}_{i,j} & , \text{ for } j = 1, 2, \dots, M - 1 \text{ (Call)}; \\ S_{\max} - K e^{-r(N-i)(\Delta t)} & , \text{ for } j = M \end{cases} \quad f_{i,j} = \begin{cases} K e^{-r(N-i)(\Delta t)} & , \text{ for } j = 0 \\ \hat{f}_{i,j} & , \text{ for } j = 1, 2, \dots, M - 1 \text{ (Put)}. \\ 0 & , \text{ for } j = M \end{cases}$$

4. Find k , such that $k(\Delta S) \leq S < (k + 1)(\Delta S)$, i.e. $k = \left\lfloor \frac{S}{\Delta S} \right\rfloor$. ($\lfloor \cdot \rfloor$ represents the floor of \cdot .)
5. Option price: $V = f_{0,k} + \frac{f_{0,k+1} - f_{0,k}}{\Delta S} [S - k(\Delta S)]$.

Implicit Method:

Given $S, K, r, q, t = 0, T, \sigma, M$ and N . (Assume $S < 2K$.)

1. Compute $\Delta t = \frac{T}{N}$, $\Delta S = \frac{S_{\max}}{M}$, where $S_{\max} = 2K$.
2. Compute (Call option) $f_{N,j} = \max(j(\Delta S) - K, 0)$, (Put option) $f_{N,j} = \max(K - j(\Delta S), 0)$, for $j = 0, 1, \dots, M$.
3. For $i = N - 1, N - 2, \dots, 1, 0$, repeat 3.1 and 3.2.

3.1. Compute vector $\hat{\mathbf{F}}_{i+1}$,

$$\hat{\mathbf{F}}_{i+1} = \begin{pmatrix} \hat{f}_{i+1,0} \\ \hat{f}_{i+1,1} \\ \vdots \\ \hat{f}_{i+1,M-1} \\ \hat{f}_{i+1,M} \end{pmatrix} \text{ with } \hat{f}_{i+1,j} = \begin{cases} 0 & , \text{ for } j = 0 \\ f_{i+1,j} & , \text{ for } j = 1, 2, \dots, M-1 \text{ (Call)}; \\ S_{\max} - K e^{-r(N-i)(\Delta t)} & , \text{ for } j = M \end{cases} \quad \hat{f}_{i+1,j} = \begin{cases} K e^{-r(N-i)(\Delta t)} & , \text{ for } j = 0 \\ f_{i+1,j} & , \text{ for } j = 1, 2, \dots, M-1 \text{ (Put)}. \\ 0 & , \text{ for } j = M \end{cases}$$

3.2. Compute vector $\mathbf{F}_i = \mathbf{A}^{-1} \cdot \hat{\mathbf{F}}_{i+1}$, where

$$\mathbf{F}_i = \begin{pmatrix} f_{i,0} \\ f_{i,1} \\ \vdots \\ f_{i,M-1} \\ f_{i,M} \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 1 & 0 & 0 & 0 & \dots & \dots & \dots & \dots \\ a_1 & b_1 & c_1 & 0 & & & & \\ 0 & a_2 & b_2 & c_2 & & & & \\ \vdots & & \ddots & \ddots & \ddots & & & \\ \vdots & & & \ddots & \ddots & \ddots & & \\ & & & & a_{M-2} & b_{M-2} & c_{M-2} & 0 \\ & & & & 0 & a_{M-1} & b_{M-1} & c_{M-1} \\ & & & & & 0 & 0 & 1 \end{pmatrix} \text{ with } \begin{cases} a_j = \frac{1}{2} \cdot (\Delta t) \cdot [(r - q) \cdot j - \sigma^2 \cdot j^2] \\ b_j = 1 + (\Delta t) \cdot (\sigma^2 \cdot j^2 + r) \\ c_j = -\frac{1}{2} \cdot (\Delta t) \cdot [\sigma^2 \cdot j^2 + (r - q) \cdot j] \end{cases} , \text{ for } j = 1, \dots, M-1.$$

4. Find k , such that $k(\Delta S) \leq S < (k+1)(\Delta S)$, i.e. $k = \left\lfloor \frac{S}{\Delta S} \right\rfloor$. ($\lfloor \cdot \rfloor$ represents the floor of \cdot .)
5. Option price: $V = f_{0,k} + \frac{f_{0,k+1} - f_{0,k}}{\Delta S} [S - k(\Delta S)]$.

Crank-Nicolson Method:

Given $S, K, r, q, t = 0, T, \sigma, M$ and N . (Assume $S < 2K$.)

1. Compute $\Delta t = \frac{T}{N}$, $\Delta S = \frac{S_{\max}}{M}$, where $S_{\max} = 2K$.
2. Compute (Call option) $f_{N,j} = \max(j(\Delta S) - K, 0)$, (Put option) $f_{N,j} = \max(K - j(\Delta S), 0)$, for $j = 0, 1, \dots, M$.
3. For $i = N - 1, N - 2, \dots, 1, 0$, repeat 3.1, 3.2 and 3.3.
 - 3.1. Compute vector $\hat{\mathbf{b}} = \mathbf{M}_2 \cdot \mathbf{F}_{i+1}$, where

$$\hat{\mathbf{b}} = \begin{pmatrix} \hat{b}_0 \\ \hat{b}_1 \\ \vdots \\ \hat{b}_{M-1} \\ \hat{b}_M \end{pmatrix}, \mathbf{F}_{i+1} = \begin{pmatrix} f_{i+1,0} \\ f_{i+1,1} \\ \vdots \\ f_{i+1,M-1} \\ f_{i+1,M} \end{pmatrix}, \mathbf{M}_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & \dots & & & \\ \alpha_1 & 1 + \beta_1 & \gamma_1 & 0 & & & & \\ 0 & \alpha_2 & 1 + \beta_2 & \gamma_2 & & & & \\ \vdots & & \ddots & \ddots & \ddots & & & \\ \vdots & & & \ddots & \ddots & \ddots & & \\ & & & & \alpha_{M-2} & 1 + \beta_{M-2} & \gamma_{M-2} & 0 \\ & & & & 0 & \alpha_{M-1} & 1 + \beta_{M-1} & \gamma_{M-1} \\ & & & & & 0 & 0 & 0 & 1 \end{pmatrix} \text{ with } \begin{cases} \alpha_j = \frac{1}{4} \cdot (\Delta t) \cdot [\sigma^2 \cdot j^2 - (r - q) \cdot j] \\ \beta_j = -\frac{1}{2} (\Delta t) \cdot (\sigma^2 \cdot j^2 + r) \\ \gamma_j = \frac{1}{4} \cdot (\Delta t) \cdot [\sigma^2 \cdot j^2 + (r - q) \cdot j] \end{cases}, \text{ for } j = 1, \dots, M - 1.$$

- 3.2. Compute vector \mathbf{b} ,

$$\mathbf{b} = \begin{pmatrix} b_0 \\ b_1 \\ \vdots \\ b_{M-1} \\ b_M \end{pmatrix} \text{ with } b_j = \begin{cases} 0 & , \text{ for } j = 0 \\ \hat{b}_j & , \text{ for } j = 1, 2, \dots, M - 1 \text{ (Call)}; \\ S_{\max} - Ke^{-r(N-i)(\Delta t)} & , \text{ for } j = M \end{cases} \quad b_j = \begin{cases} Ke^{-r(N-i)(\Delta t)} & , \text{ for } j = 0 \\ \hat{b}_j & , \text{ for } j = 1, 2, \dots, M - 1 \text{ (Put)} \\ 0 & , \text{ for } j = M \end{cases}$$

- 3.3. Compute vector $\mathbf{F}_i = (\mathbf{M}_1)^{-1} \cdot \mathbf{b}$, where

$$\mathbf{F}_i = \begin{pmatrix} f_{i,0} \\ f_{i,1} \\ \vdots \\ f_{i,M-1} \\ f_{i,M} \end{pmatrix}, \mathbf{M}_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & \dots & & & \\ -\alpha_1 & 1 - \beta_1 & -\gamma_1 & 0 & & & & \\ 0 & -\alpha_2 & 1 - \beta_2 & -\gamma_2 & & & & \\ \vdots & & \ddots & \ddots & \ddots & & & \\ \vdots & & & \ddots & \ddots & \ddots & & \\ & & & & -\alpha_{M-2} & 1 - \beta_{M-2} & -\gamma_{M-2} & 0 \\ & & & & 0 & -\alpha_{M-1} & 1 - \beta_{M-1} & -\gamma_{M-1} \\ & & & & & 0 & 0 & 0 & 1 \end{pmatrix}.$$

4. Find k , such that $k(\Delta S) \leq S < (k + 1)(\Delta S)$, i.e. $k = \left\lfloor \frac{S}{\Delta S} \right\rfloor$.
5. Option price: $f_{0,k} + \frac{f_{0,k+1} - f_{0,k}}{\Delta S} [S - k(\Delta S)]$.

Fun Application 1a: Pandigital Formula For New Year – 1

Insert $+$, $-$, $*$ or $/$ in “123456789” to obtain 2016.

$$98 * 76 - 5432 * 1 = 2016$$

$$1 \times 2 \times 3 \times 4 \times 5 \times 6 - 7 - 8 - 9 = 2016$$

$$9 \times 8 \times 7 + 6 \times 5 \times 4 + 3 \times 2 + 1 = 2016$$

$$98 + 7 \times 6 + 5 \times 4 \times 3 + 2 - 1 = 2016$$



Fun Application 1a: Pandigital Formula For New Year – 1 (★ ★ ☆ ☆ ☆)

1. ☐ Numeric Literals
2. ☐ Lists
3. ☐ Arithmetic Operators
4. ☐ Comparison (Relational) Operators
5. ☐ Variables
6. ☐ String Concatenation
7. ☐ Assignment Statements
8. ☐ `if` Statements
9. ☐ `for` Statements
10. ☐ Built-in Functions `eval`, `print`
11. ☐ (optional) Error and Exceptions



(brute force solution)

Hint: About 30 lines

Fun Application 1b: Pandigital Formula For New Year – 2

Insert $+$, $-$, $*$, $/$ or $^$ in “123456789” to obtain 2016. (Note that Python does not have $^$ operator.)

Additional Requirement:

$$2^3^4: (2^3)^4$$

$$98 * 76 - 5432 * 1 = 2016$$

$$1 \times 2 \times 3 \times 4 \times 5 \times 6 - 7 - 8 - 9 = 2016$$

$$9 \times 8 - 7 + 6 - 5 + 4 + 3 \times 2 + 1 = 2016$$

$$98 + 7 \times 6 + 5 - 4 \times 3 + 2 - 1 = 2016$$



Fun Application 1b: Pandigital Formula For New Year – 2 (★★★★☆)

1. ☐ Numeric Literals
2. ☐ Lists
3. ☐ Arithmetic Operators
4. ☐ Comparison (Relational) Operators
5. ☐ Variables
6. ☐ String Concatenation
7. ☐ String Function `rfind`
8. ☐ Assignment Statements
9. ☐ `if` Statements
10. ☐ `for` Statements
11. ☐ Built-in Functions `power`, `eval`, `print`
12. ☐ Recursive Functions
13. ☐ (optional) Error and Exceptions



(brute force solution)

Hint: About 50 lines

Fun Application 2: Sudoku Solving



Fun Application 2: Sudoku Solving (★★★★☆)

1. ☐ Variables
2. ☐ Assignment Statements
3. ☐ Numeric and String Literals
4. ☐ Arithmetic Operators
5. ☐ Comparison (Relational) Operators
6. ☐ Logical Operators
7. ☐ Strings (Indexing and Slicing, Function `find`)
8. ☐ Recursive Function
9. ☐ Set Comprehension (or `for` loop)
10. ☐ `for` Statements
11. ☐ `if` Statements
12. ☐ Built-in Functions `range`, `print`
13. ☐ Errors and Exceptions
14. ☐ Class
15. ☐ `pass` statement



(brute force solution)

Hint: About 25 lines