QF620 Stochastic Modelling in Finance Assignment 3/4

Due Date: 19-Nov-2018

1. Suppose the stock price process follows the stochastic differential equation

$$dS_t = rS_t dt + \sigma S_t dW_t^*,$$

where W_t^* is a standard Brownian motion under the risk-neutral measure \mathbb{Q}^* associated with the risk-free bond B_t as numeraire, where $dB_t = rB_t dt$. We would like to value a contract which pays

$$S_T^n$$

on maturity date T, where $n \in \mathbb{N}$ is a natural number. Derive a valuation formula for this contract.

2. Consider the same contract as above, determine how we can use static replication to value this contract payoff $h(S_T) = S_T^n$. We have access to the vanilla European call and put options market:

$$\int_0^F h(K) \frac{\partial^2 P(K)}{\partial K^2} \ dK + \int_F^\infty h(K) \frac{\partial^2 C(K)}{\partial K^2} \ dK$$