Date No.

Assignment 3.

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1. $dS_t = rS_t dt + \sigma S_t dW_t^*$

$$S_T = S_0 \cdot e^{(r-\frac{1}{2}\sigma^2)T + \sigma W_T^*}$$

$$S_T^n = S_0^n \cdot e^{n(r-\frac{1}{2}\sigma^2)T + n\sigma W_T^*}$$

Let V_t denote the value of the financial contract at time t.

Under the Q^* measure, we have

$$\frac{V_0}{B_0} = E^{Q^*}\left[\frac{V_T}{B_T}\right]$$

$$V_0 = e^{-rT} E^{Q^*}\left[S_0^n \cdot e^{n(r-\frac{1}{2}\sigma^2)T + n\sigma\sqrt{T}\lambda}\right] \quad \lambda \sim N(0,1)$$

$$= e^{-rT} \cdot S_0^n \cdot e^{n(r-\frac{1}{2}\sigma^2)T} E^{Q^*}\left[e^{n\sigma\sqrt{T}\lambda}\right]$$

$$= e^{-rT} S_0^n \cdot e^{n(r-\frac{1}{2}\sigma^2)T} e^{\frac{1}{2}\sigma^2 n^2 T}$$

$$= S_0^n \cdot e^{(\frac{1}{2}\sigma^2 n + r)(n-1)T}$$

2. $\int_0^F h(k) \frac{\partial^2 p(k)}{\partial k^2} dk + \int_F^\infty h(k) \cdot \frac{\partial^2 C(k)}{\partial k^2} dk$

Using integration-by-parts twice, we obtain

$$V_0 = e^{-rT} h(F) + h'(F)\left(\cancel{C(F) - p(F)}\right) + \int_0^F h''(k) \cdot p(k) dk + \int_F^\infty h''(k) C(k) dk$$

$$= e^{-rT} S_T^n + \int_0^F n(n-1) k^{n-2} \cdot p(k) \cdot dk + \int_F^\infty n(n-1) \cdot k^{n-2} \cdot C(k) dk.$$

where $h(S_T) = S_T^n \quad h'(S_T) = n \cdot S_T^{n-1}, \quad h''(S_T) = n(n-1) S_T^{n-2}$