MSc IN QUANTITATIVE FINANCE

AY 2017-18 Term 1 Final Examination

QF603 Quantitative Analysis of Financial Market

INSTRUCTIONS TO STUDENTS

1 Make sure you have written your name and student ID on the Answer Sheet provided.

2 This **closed-book** exam paper must be completed in **THREE** hours.

3 This exam paper contains a total of 56 questions and comprises 10 pages including this instruction sheet.

4 For multiple choice questions (MCQs), select THE *best* choice.

5 Write all your answers **neatly** on the Answer Sheet only.

6 The exam booklet provided is for your working and you don't need to submit it.

7 You may use your own electronic calculators.

8 You are required to return the **Answer Sheet** and this exam paper at the end of the exam.

1. Which one of the following is the most appropriate as a definition of R^2 for a simple linear regression?
   A. It is the proportion of the total variability of y that is explained by the model.
   B. It is the correlation between the fitted values and the residuals.
   C. It is the correlation between the fitted values and the regressor's values.
   D. It is the proportion of the total variability of y about its mean value that is explained by the model.

2. What is the relationship, if any, between t-distributed and F-distributed random variables?
   A. A t-variate with z degrees of freedom is also an $F(1, z)$.
   B. The square of a t-variate with z degrees of freedom is also an $F(1, z)$.
   C. A t-variate with z degrees of freedom is also an $F(z, 1)$.
   D. There is no relationship between the two distributions.

3. Consider a model for time series: $Y_t = 0.8Y_{t-1} + 0.2Y_{t-2} + \epsilon_t$, where ϵ_t is i.i.d with mean 0 and variance 1. Which of the following statements is most likely the least accurate?
   A. $\mathbb{E}(Y_t|Y_{t-2}) = 0.84Y_{t-2} + 0.16\,\mathbb{E}(Y_{t-3})$
   B. $\mathbb{E}(Y_t|Y_{t-2}, Y_{t-1}) = 0.8Y_{t-1} + 0.2Y_{t-2} + \epsilon_t$
   C. The time series value at $t = 7$ has a significant impact on today's (time T) time series value.
   D. $\mathbb{V}(Y_t|Y_{t-2}, Y_{t-1}, \epsilon_t) = 1$

4. Under which of the following situations would bootstrapping be preferred to pure Monte Carlo method?
   A. The distributional properties of simulation data are the same as those of the actual data.
   B. The distributional properties of simulation data are known.
   C. The distributional properties of the actual data are known.
   D. The sample of actual data available is very small.

5. Which of the following statements are most accurate concerning the use of antithetic variates as part of a Monte Carlo experiment? Antithetic variates involve
   A. increasing the number of replications required to cover the whole probability space.
   B. employing a similar variable to that used in the simulation, but whose properties are known analytically.
   C. taking one over each of the random draws and repeating the experiment using those values as the draws.
   D. using the negative of each of the random draws and repeating the experiment using those values as the draws.

6. If our regression equation is $\boldsymbol{y} = \boldsymbol{X\beta}+\boldsymbol{u}$, where we have T observations, k regressors, an an intercept, what will be the dimension of $\widehat{\boldsymbol{\beta}}$ using the standard matrix notation?
   A. $T \times 1$
   B. $k \times k$
   C. $T \times (k+1)$
   D. $(k+1) \times 1$

7. Consider the linear regression specification as follows:

$$y_t = \beta_1 + \beta_2 X_{2,t} + \beta_3 X_{3,t} + \beta_4 X_{4,t} + u_t.$$

   Which of the following null hypotheses cannot be tested using an F-test?
   A. $\beta_2 = 0$
   B. $\beta_2 = 1$ and $\beta_3 + \beta_4 = 1$
   C. $\beta_3 \beta_4 = 1$
   D. $\beta_2 - \beta_3 - \beta_4 = 1$

8. For the regression specification $y_t = \beta_1 + \beta_2 X_{2,t} + \beta_3 X_{3,t} + u_t$, you are given the following data: the residual sum of squares is 86 and the number of observations is 103. Moreover,

$$\left(\boldsymbol{X'X}\right)^{-1} = \begin{pmatrix} 1.3 & 2.1 & -1.4 \\ 2.1 & 0.8 & 1.9 \\ -1.4 & 1.9 & 3.4 \end{pmatrix} \quad \text{and} \quad \left(\boldsymbol{X'y}\right) = \begin{pmatrix} -1.6 \\ 2.9 \\ 0.8 \end{pmatrix}$$

   What is the value of $\widehat{\beta}_1$ (2 decimals, e.g., 1.23)?

9. For the data in Question 8 what is the standard error of $\widehat{\beta}_2$ (2 decimals, e.g., 1.23)?

10. For the data in Question 8 what is the t statistic of $\widehat{\beta}_3$ (2 decimals, e.g., 1.23)?

11. If the total sum of squares for Question 8 is 373, what is the adjusted R^2 (2 decimals in %, e.g., 12.56%)?

12. The residual from a standard regression model is defined as
   A. the difference between the actual value, y, and the mean, $\overline{y}$
   B. the difference between the actual value, y, and the fitted value, $\widehat{y}$
   C. the difference between the fitted value, $\widehat{y}$, and the mean, $\overline{y}$
   D. the square of the difference between the fitted value, $\widehat{y}$, and the mean, $\overline{y}$

13. Which of the following statements is most accurate concerning the standard regression model?
   A. The dependent variable y has a probability distribution.
   B. The explanatory x has a probability distribution.
   C. The disturbance term is assumed to be correlated with x.
   D. For a well-defined model, the residual will be zero for all sample data points.

14. If an estimator is said to have minimum variance, which one of the following statements is least accurate?
    A. The probability that the estimate is far from its true value is minimized.
    B. The estimator is efficient.
    C. Such an estimator would be termed "best".
    D. Such an estimator will always be unbiased.

15. Consider the following two regressions:

$$y_t = \beta_1 + \beta_2 x_{2,t} + \beta_3 y_{t-1} + u_t;$$
$$\Delta y_t = \gamma_1 + \gamma_2 x_{2,t} + \gamma_3 y_{t-1} + v_t.$$

    Which one of the following statements is most accurate?
    A. The sum of squared residuals will be the same for the two models.
    B. The R^2 will be the same for the two models.
    C. The adjusted R^2 will be the same for the two models.
    D. The regression F-test will be the same for the two models.

16. Let α, β, and γ be parameters, and u_t be the i.i.d. Gaussian noise. Which one of the following models can be estimated using linear OLS, after suitable transformations if necessary?
    (i) $y_t = \alpha + \beta x_t^2 + \gamma t + u_t$ (ii) $y_t = \alpha e^{\beta x_t + \gamma t} + u_t$ (iii) $y_t = \alpha + \beta t x_t + u_t$ (iv) $e^{y_t} = \alpha e^{\beta x_t + u_t}$
    A. (i) only
    B. (i) and (iii) only
    C. (i), (iii) and (iv) only
    D. (ii), (iii), and (iv) only

17. Suppose that a test that the true value of the intercept coefficient is zero results in non-rejection. What would be the most appropriate conclusion?
    A. Drop the intercept and re-run the regression.
    B. Retain the intercept.
    C. Re-compute the test statistic.
    D. The regression line is running exactly through the origin.

18. Which one of the following statements is most accurate?
    A. The restricted residual sum of squares is always larger than the unrestricted residual sum of squares.
    B. The adjusted R^2 always increases whenever a new regressor is added to the regression specification.
    C. The unrestricted residual sum of squares is always larger than the restricted residual sum of squares.
    D. R^2 sometimes increases whenever a new regressor is added to the regression specification.

19. Suppose R^2 for an estimated regression model is exactly zero. Which of the following are true?

(i) All coefficient estimates on the regressors will be zero.

(ii) The fitted line will be horizontal with respect to all of the explanatory variables.

(iii) The regression line has not explained any of the variability of y.

(iv) The intercept coefficient estimate must be zero.

A. (ii) and (iv) only
C. (i), (ii), and (iii) only
B. (i) and (iii) only
D. (i), (ii), (iii), and (iv)

20. Consider the following two regression models:

$$\text{Model 1:} \quad y_t = \beta_1 + \beta_2 X_{2,t} + u_t;$$
$$\text{Model 2:} \quad y_t = \alpha_1 + \alpha_2 X_{2,t} + \alpha_3 X_{3,t} + u_t.$$

Which of the following statements are true?

(i) Model 2 must have an R^2 at least as high as that of Model 1.

(ii) Model 2 must have an adjusted R^2 at least as high as that of Model 1.

(iii) Models 1 and 2 would have identical value of R^2 if the estimated coefficient α_3 is zero.

(iv) Models 1 and 2 would have identical value of adjusted R^2 if the estimated coefficient α_3 is zero.

A. (ii) and (iv) only
C. (i), (ii), and (iii) only
B. (i) and (iii) only
D. (i), (ii), (iii), and (iv)

21. Which of the following assumptions are required to show the consistency, unbiasedness and efficiency of the OLS estimator?

(i) $\mathbb{E}(u_t) = 0$ (ii) $\mathbb{V}(u_t) = \sigma^2$ (iii) $\mathbb{C}(u_t, u_{t-j}) = 0, \ \forall j$ (iv) $u_t \sim N(0, \sigma^2)$

A. (ii) and (iv) only
C. (i), (ii), and (iii) only
B. (i) and (iii) only
D. (i), (ii), (iii), and (iv)

22. Which one of the following is least likely to be the consequence when the classical linear regression assumptions fail to hold?

A. The coefficient estimates are not unbiased.
B. The standard error estimates are not optimal.
C. The distributions assumed for the test statistics are inappropriate.
D. Conclusions regarding the relationships between the dependent and independent variables may be invalid.

23. Earnings per share for 25 blue chips produce the following quantities:

$$\overline{y} = 20, \qquad \overline{x} = 10, \qquad \sum_{i=1}^{n} \left(y_i - \overline{y}\right)^2 = 100, \qquad \sum_{i=1}^{n} \left(x_i - \overline{x}\right)^2 = 60, \qquad \sum_{i=1}^{n} \left(x_i - \overline{x}\right)\left(y_i - \overline{y}\right) = 30,$$

where y_i is the earnings per share and x is the logarithm of revenue. Consider the linear regression model $y_i = \beta_1 + \beta_2 x_i + u_i$, $i = 1, 2, \ldots, 25$. What is the value of β_2 estimate (2 decimals, e.g., 1.23)?

24. What is the β_1 estimate for the data in 23 (2 decimals, e.g., 1.23)?

25. For the data in 23, and suppose the unbiased variance of the residuals is 30, what is the t statistic for the null hypothesis $H_0 : \beta_2 = 1$ (2 decimals, e.g., 1.23)?

26. Which one of the following cannot be a component for a time series plot?

A. Seasonality
C. Noise
B. Trend
D. None of the above

27. Which one of the following is most accurate concerning autocovariance?

A. Linear dependence between multiple points on the different series observed at different times
B. Linear dependence between two points on the same series observed at different times
C. Linear dependence between two points on different series observed at the same time
D. Quadratic dependence between two points on the same series observed at different times

28. Consider the following AR(1) model with the disturbances (u_t) having zero mean and unit variance:

$$y_t = 0.4 + 0.2y_{t-1} + u_t.$$

The unconditional variance of y is closest to

A. 0.04 B. 0.40 C. 1.00 D. 1.04

29. For the following process $y_t = \mu + \epsilon_t + \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2} + \theta_3\epsilon_{t-3}$, where ϵ_t is a zero mean white noise process with variance σ_ϵ^2. Which one of the following autocorrelation functions is likely to be true?

A. ACF(5) = 0 B. ACF(3) = 0 C. ACF(2) =0 D. ACF(1) =1

30. Consider the following set of data:

23.32 32.33 32.88 28.98 33.16 26.33 29.88 32.69 18.98 21.23 26.66 29.89

What is the lag-one sample autocorrelation of the time series (2 decimals, e.g., 1.23)?

31. Which one of the following statements is least accurate?

A. BIC penalizes complex models more strongly than the AIC.
B. Smoothing parameter close to one gives more weight or influence to recent observations over the forecast.
C. Adjacent observations in time series data other than white noise are independent and identically distributed.
D. Stationarity is a desirable property for a time series process.

32. In the paradigm of online machine learning, current probabilities that are updated upon the arrival of new available information are known as

    A. Priors B. Posteriors C. Predictor priors D. Likelihoods

33. You toss two fair coins. Given that one of the coins is head, what is the probability that both coins are head (in irreducible fraction, e.g., 73/373)?

34. For two events A and B, what is the relationship between $\mathbb{P}(A|B)$ and $\mathbb{P}(A)$?

    A. $\mathbb{P}(A|B) > \mathbb{P}(A)$ B. $\mathbb{P}(A|B) < \mathbb{P}(A)$ C. $\mathbb{P}(A|B) = \mathbb{P}(A)$ D. Undetermined

35. Which one of the following is not an element of the formal definition for VaR?

    A. Worst Case Loss
    B. Tolerance Level
    C. Normality of returns
    D. Liquidation period

36. Which one of the following statements is least accurate?

    A. We can learn more about the true risk distribution of the portfolio by looking at historical VaR and the VaR trend line over time.
    B. We can compare recent risk of the portfolio with historical risk of the portfolio by comparing the results of historical VaR and EWMA VaR.
    C. One of the applications of VaR is to use it as a tool for margin projection and margin management purposes.
    D. In the VaR framework, you must use log returns rather than simple returns.

37. On any given day, the probability that General Motors (GM) and General Electric (GE) both go up is 40% and the probability that they both go down is 20%. Otherwise, one goes up and the other goes down. If you know that at least one went up, what is the probability that both went up (in irreducible fraction, e.g., 73/373)?

38. The daily volatility is 0.74%. The number of trading days in a year is 252 days. The confidence level is 95% and the length of the holding period is 10 days. What is the 10-day VaR (2 decimals in %, e.g., 1.23%)?

39. A quant analyzes data and finds that $\overline{X} = 50$, $\overline{Y} = 30$, $\sqrt{\mathbb{V}(X)} = 8$, $\sqrt{\mathbb{V}(Y)} = 16$, and that the correlation between X and Y is 0.75. Which one of the following equations gives the best-fit line?

    A. $Y - 30 = 1.5(X - 50)$
    B. $Y = 1.5X$
    C. $Y = 0.375X + 48.75$
    D. $Y = 0.66X$

40. Estimated from daily data, suppose that the parameters in a GARCH(1,1) model are $\alpha = 0.03$, $\beta = 0.95$, and $\omega = 0.000002$. What is the long-run average volatility (2 decimals in %, e.g., 7.23%) per day?

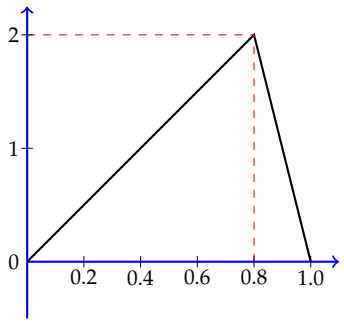
41. Refer to Question 40. If the current volatility is 1.5% per day, what is your estimate of the volatility in 20 days (2 decimals in %, e.g., 7.23%) per day?

42. Refer to Question 40. What volatility per annum (252 days per year) should be used to price a 20-day option (2 decimals in %, e.g., 7.23%)?

43. Which of the following is least accurate?
    A. GARCH models can be used for updating variance rate estimates and forecasting future level of variance rates.
    B. GARCH models can be used for updating covariance rate estimates and forecasting future level of covariance rates.
    C. The tail dependence is higher in bivariate Student's t-distribution than a bivariate normal distribution.
    D. In the context of multivariate normal distributions, the $m(> 1)$ factors in the m-factor model have correlated standard normal distributions.

44. The PDF for a triangular distribution with a minimum of a, a maximum of b, and a mode of c is described by the following two-part function:

$$f(x) = \begin{cases} \dfrac{2(x-a)}{(b-a)(c-a)}, & x \in [a, c] \\ \dfrac{2(b-x)}{(b-a)(b-c)}, & x \in (c, b] \end{cases}$$



What is the value of $a + b + c$ (2 decimals, e.g., 1.23)?

45. Given the PDF in Question 44, what can be said about the skewness γ or/and the excess kurtosis κ?
    A. $\gamma > 0$ B. $\gamma < 0$ C. $\gamma > 0$ and $\kappa = 0$ D. $\gamma < 0$ and $\kappa = 0$

46. The probability that at least one company is going to announce a particular corporate matter in the next hour is 84%. What is the probability that at least one announces in the next 30 minutes, assuming that the distribution of announcement times is independently uniformly distributed?
    A. 60% B. 56% C. 42% D. 40%

47. A population has a known mean of 750. Suppose 4,000 samples are randomly drawn with replacement from this population. The mean of the observed samples is 732.7, and the standard deviation of the observed samples is 60. What is the standard error of the sample mean (2 decimals, e.g., 1.23)?

48. Daixiong, being a quant, divides the stock's performance into three categories of "increase", "constant" and "decrease". He obtains the following conclusion after data analysis:
    - The probability that the state of the economy is GOOD is 20%. If the state of the economy is GOOD, the probability that the stock price increases is 80% and the probability that the stock price decreases is 10%.
    - The probability that the state of the economy is NEUTRAL is 30%. If the state of the economy is NEUTRAL, the probability that the stock price increases is 50% and the probability that the stock price decreases is 30%.
    - If the state of the economy is POOR, the probability that the stock price increases is 15% and the probability that the stock price decreases is 70%.

    What is the probability that the state of the economy is NEUTRAL given that the stock performance is constant (no decimal in %, e.g., 40%)?

49. You built a linear regression model to analyze annual salaries for a developed country. You incorporated two independent variables, age and experience, into your model. Upon reading the regression results, you noticed that the coefficient of "experience" is negative, which appears to be counter-intuitive. In addition, you have discovered that the coefficients have low t-statistics but the regression model has a high R^2. What is the most likely cause of these results?

    A. Incorrect standard errors
    B. Heteroskedasticity
    C. Multicollinearity
    D. Serial correlation

50. A fixed income portfolio manager currently holds a portfolio of bonds of various companies. Assuming all these bonds have the same annualized probability of default and that the defaults are independent, the number of defaults in this portfolio over the next year follows which type of distribution?

    A. Bernoulli
    B. Normal
    C. Binomial
    D. Exponential

51. Which of the following is most likely to happen in a typical financial market?

    A. expected shortfall $>$ VaR(95%)
    B. expected shortfall $<$ VaR(95%)
    C. VaR(95%) = VaR(99%)
    D. VaR(99%) $<$ VaR(95%)

52. The PDF $p(x)$ of daily profit x at Triangle Asset Management can be described by

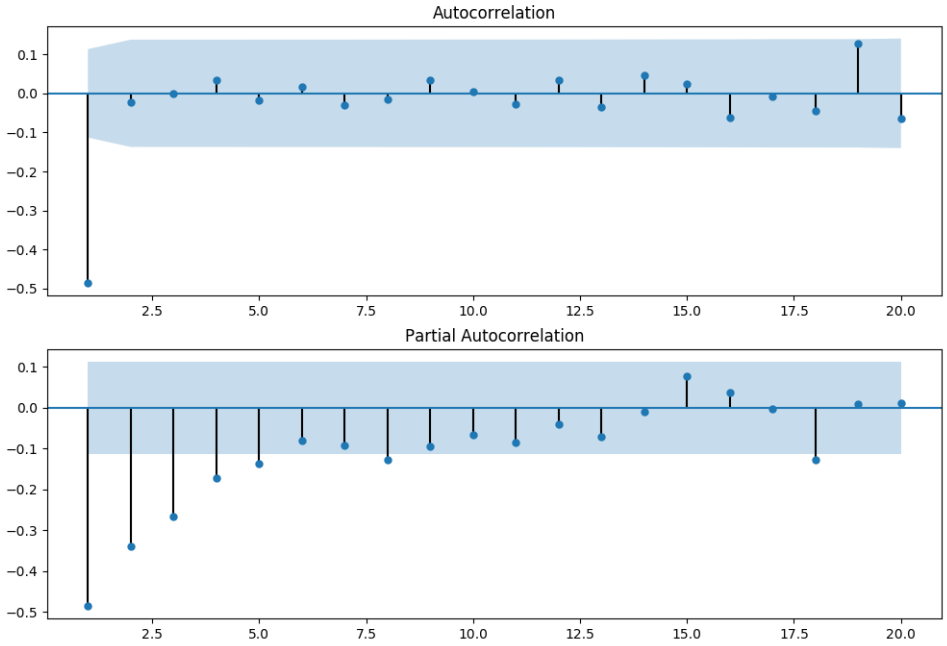
$$p(x) = \begin{cases} \frac{2}{15} + \frac{1}{100}x, & -20 \le x \le 0; \\ \frac{2}{15} - \frac{1}{50}x, & 0 < x \le 10. \end{cases}$$

    What is the 1-day 95% VaR (2 decimals, e.g., 4.73)?

53. For the PDF $p(x)$ in Question 52, what is the expected value of daily profit?

54. What is the time series model that will generate the plotted ACF and PACF?

    A. AR(1) B. AR(2) C. MA(1) D. MA(2)



55. The quarterly earnings per share of a company has a time trend modeled as

$$\text{EPS}_t = 0.20 + 0.05t + 0.0001t^2, \qquad \text{where } t = 1, 2, \ldots, 30.$$

    What is the point forecast of 31-th quarter (2 decimals, e.g., 3.45)?

56. Consider a time series model $Y_t = \alpha Y_{t-2} + \beta Y_{t-4} + u_t$, where $\alpha > 0$ and $\beta > 0$. Which one of the following statements is most accurate?

    A. for the model to be invertible, $\alpha + \beta < 1$.

    B. For the model to be covariance stationary, $\alpha + \beta < 1$.

    C. For the model to be not covariance stationary, $\alpha + \beta < 1$.

    D. for the model to be not invertible, $\alpha + \beta < 1$.