Pg 7.

$$dX_t = \mu\, dt + \sigma\, dW_t \quad , \quad W_t \to \mathbb{P}-\text{Brownian}$$

$$X_t = \mu t + \sigma W_t$$

$$\mathbb{E}^P[X_t] = \mu t \quad , \quad V[X_t] = \sigma^2 t = \mathbb{E}^P[X_t^2] - \mathbb{E}^P[X_t]^2$$

$$\Rightarrow \mathbb{E}^P[X_t^2] = (\mu t)^2 + \sigma^2 t$$

$$dX_t = \sigma\, d\tilde{W}_t \quad , \quad \tilde{W}_t \to \mathbb{Q}-\text{Brownian}$$

$$X_t = \sigma \tilde{W}_t$$

$$\mathbb{E}^Q[X_t] = 0 \quad , \quad V[X_t] = \sigma^2 t = \mathbb{E}^Q[X_t^2] - \cancel{\mathbb{E}^Q[X_t]^2}$$

pg 8.

$$dX_t = \mu X_t dt + \sigma X_t dW_t \quad , \quad W_t \to \mathbb{P}\text{-Brownian}$$

$$= \nu X_t dt - \nu X_t dt + \mu X_t dt + \sigma X_t dW_t$$

$$= \nu X_t dt + \sigma X_t \left(dW_t + \frac{\mu - \nu}{\sigma} dt \right)$$

$$= \nu X_t dt + \sigma X_t d\tilde{W}_t \quad , \quad \tilde{W}_t \to \mathbb{Q}\text{-Brownian}$$

pg15.

$$C_t = C(t, S_t)$$

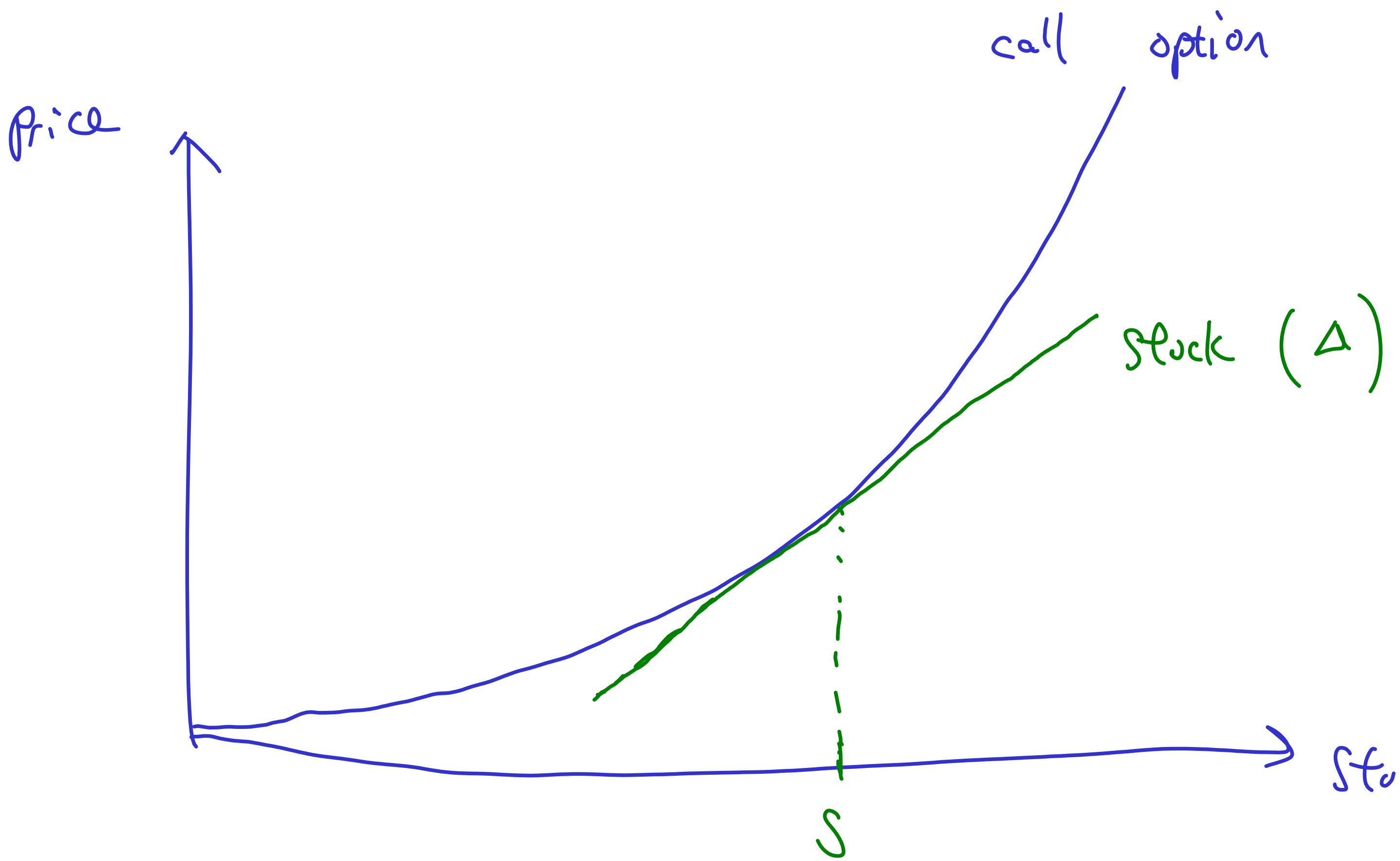
$$dC_t = \frac{\partial C}{\partial t} dt + \frac{\partial C}{\partial S} dS_t + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} (dS_t)^2$$

Risk-free portfolio $\qquad d\Pi_t = r\,\Pi_t\,dt$

$$dB_t = r\,B_t\,dt$$

$$\Pi_t = \frac{\partial C}{\partial S} S_t - C_t$$

$$d\Pi_t = \frac{\partial C}{\partial S} dS_t - dC_t$$



pg18.

$$\frac{S_t}{B_t} = \mathbb{E}\left[\frac{S_{t+\Delta t}}{B_{t+\Delta t}}\right]$$

$$= \mathbb{E}\left[\frac{S_{t+\Delta t}}{B_t e^{r\Delta t}}\right]$$

$$S_t = e^{-r\Delta t}\,\mathbb{E}\left[S_{t+\Delta t}\right]$$

$$\frac{S_t}{N_t} = \mathbb{E}^{\mathbb{Q}^N}\left[\frac{S_{t+\Delta t}}{N_{t+\Delta t}}\right]$$

numeraire

risk-neutral measure $\mathbb{Q}^N$, numeraire is N_t

Pg 21.

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

$$dB_t = r B_t dt$$

$$X_t = f(S_t, B_t) = \frac{S_t}{B_t}$$

$$f(s,b) = \frac{s}{b}$$

$$\frac{\partial f}{\partial b} = -\frac{s}{b^2}, \quad \frac{\partial f}{\partial s} = \frac{1}{b}, \quad \frac{\partial^2 f}{\partial s^2} = 0$$

Itô's formula:

$$dX_t = \frac{\partial f(S_t, B_t)}{\partial b} dB_t + \frac{\partial f(S_t, B_t)}{\partial s} dS_t + \frac{1}{2} \frac{\partial^2 f(S_t, B_t)}{\partial s^2} (dS_t)^2$$

$$= -\frac{S_t}{B_t^2} \cdot r B_t dt + \frac{1}{B_t}\left(\mu S_t dt + \sigma S_t dW_t\right) + 0$$

$$dX_t = -r X_t dt + \mu X_t dt + \sigma X_t dW_t$$

$$dX_t = (\mu - r) X_t \, dt + \sigma X_t \, dW_t$$

$$= \sigma X_t \left(dW_t + \frac{\mu - r}{\sigma} dt \right)$$

$$= \sigma X_t \, dW_t^B \quad ; \quad W_t^B : \longrightarrow \mathbb{Q}\text{-Brownian}$$

(where B_t is numeraire)

$$\therefore \quad dW_t^B = dW_t + \frac{\mu - r}{\sigma} dt$$

$$\therefore \quad dW_t = dW_t^B - \frac{\mu - r}{\sigma} dt$$

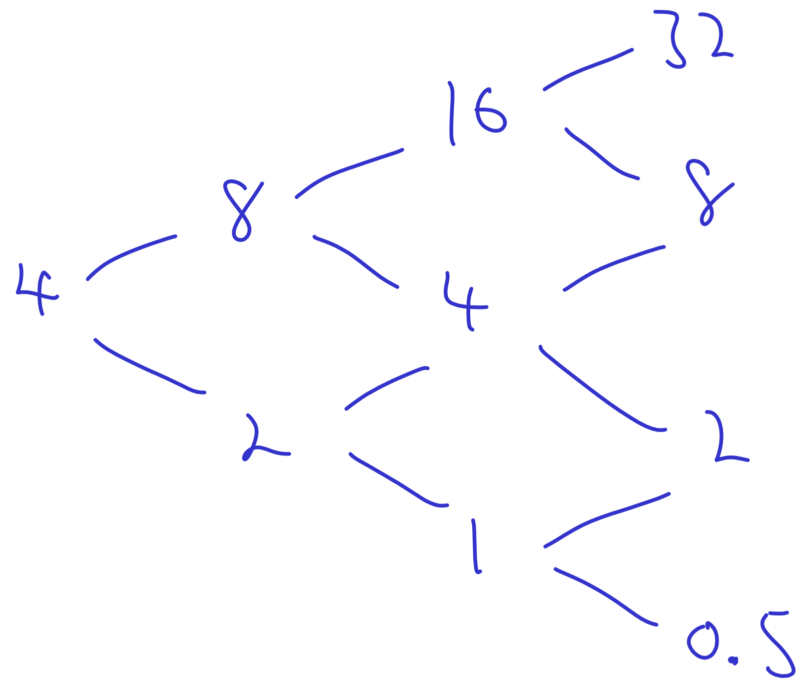
pg23.

$$\frac{S_t}{B_t} = \mathbb{E}^{\mathbb{Q}^B}\left[\frac{S_T}{B_T}\right]$$

$$X_t = \mathbb{E}^{\mathbb{Q}^B}\left[X_T\right]$$

$$\frac{C_t}{B_t} = \mathbb{E}^{\mathbb{Q}^B}\left[\frac{C_T}{B_T}\right]$$

pg24.



$(32-10)^+ = 22$

$(8-10)^+ = 0$

$(2-10)^+ = 0$

$(0.5-10)^+ = 0$

$$\frac{S_0}{B_0} = \mathbb{E}^{Q^B}\left[\frac{S_1}{B_1}\right]$$

$$\frac{4}{1} = \frac{p^B \times 8 + (1-p^B) \times 2}{1.25} \Rightarrow p^B = 0.5$$

$$\frac{C_0}{B_0} = \mathbb{E}^{Q^B}\left[\frac{C_3}{B_3}\right] = \frac{\frac{1}{2^3} \times 22}{1.25^3} \Rightarrow C_0 = 1.408$$

stock as numeraire :

$$\frac{B_0}{S_0} = \mathbb{E}^{Q^S}\left[\frac{B_1}{S_1}\right]$$

$$\frac{1}{4} = 1.25 \times \left[p^S \times \frac{1}{8} + (1-p^S) \times \frac{1}{2}\right]$$

$$\Rightarrow p^S = 0.8$$

$$\frac{C_0}{S_0} = \mathbb{E}^{Q^S}\left[\frac{C_3}{S_3}\right]$$

$$\frac{C_0}{4} = 0.8^3 \times \frac{22}{32} \Rightarrow C_0 = 1.408$$

. .

pg2.

$$V_t = N_t \, \mathbb{E}^{\mathbb{Q}^N}\left[\frac{V_T}{N_T}\right]$$

$$V_t = M_t \, \mathbb{E}^{\mathbb{Q}^M}\left[\frac{V_T}{M_T}\right]$$

$$N_t \, \mathbb{E}^{\mathbb{Q}^N}\left[\frac{V_T}{N_T}\right] = M_t \, \mathbb{E}^{\mathbb{Q}^M}\left[\frac{V_T}{M_T}\right]$$

$G_t = \frac{V_t}{N_t}$:

$$N_t \, \mathbb{E}^{\mathbb{Q}^N}\left[G_T\right] = M_t \, \mathbb{E}^{\mathbb{Q}^M}\left[\frac{V_T}{N_T} \cdot \frac{N_T}{M_T}\right]$$

$$= M_t \, \mathbb{E}^{\mathbb{Q}^M}\left[G_T \cdot \frac{N_T}{M_T}\right]$$

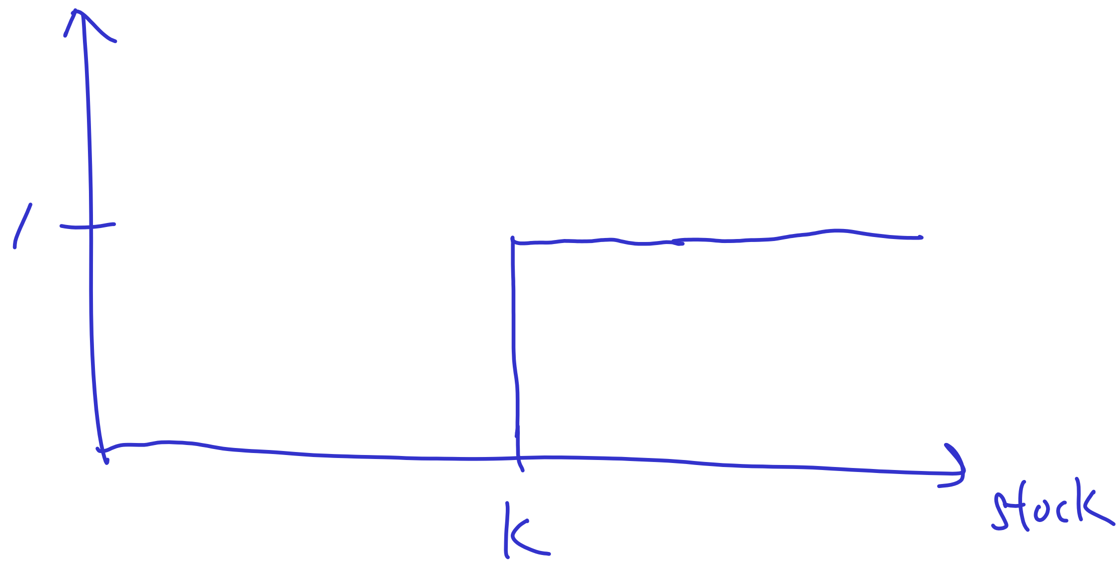
$$\mathbb{E}^{Q^N}\left[G_T \right] = \mathbb{E}^{Q^M}\left[G_T \cdot \frac{N_T/N_t}{M_T/M_t} \right]$$

$$= \mathbb{E}^{Q^M}\left[G_T \cdot \frac{dQ^N}{dQ^M} \right]$$

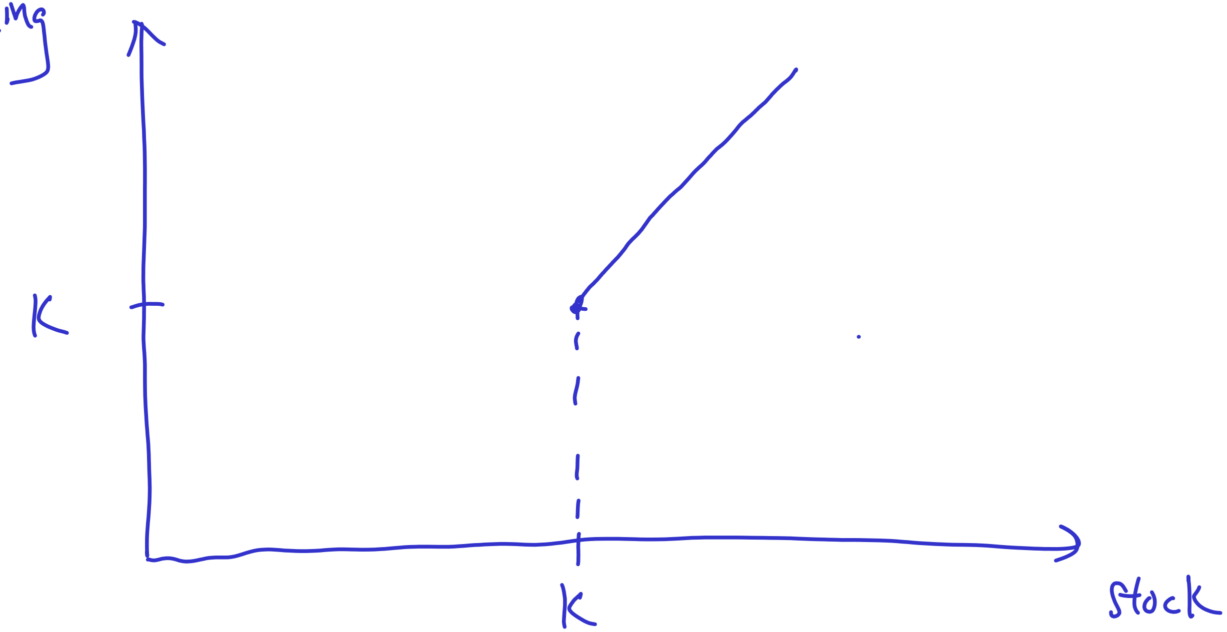
$$\frac{dQ}{dP} = \exp\left[-\frac{\kappa^2 t}{2} - \kappa W_t \right]$$

pg4.

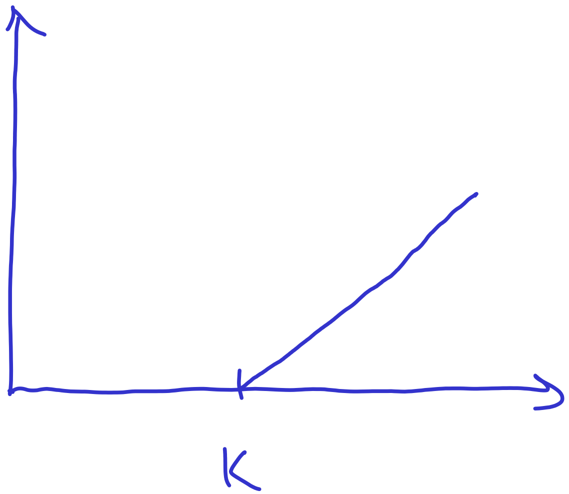
Cash or nothing call



asset-or-nothing call



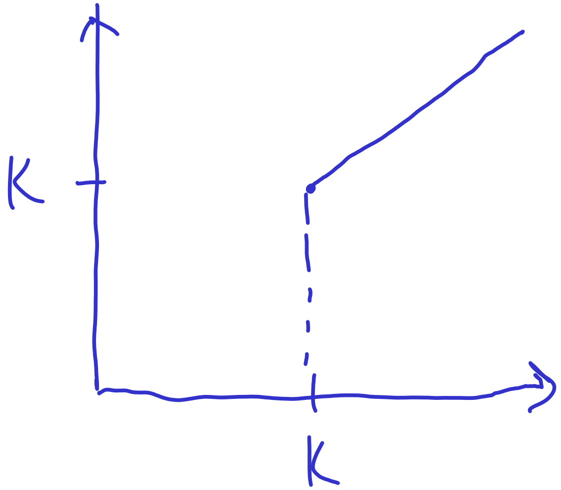
vanilla call



$$S_0 \Phi(d_1) - Ke^{-rT} \Phi(d_2)$$

=

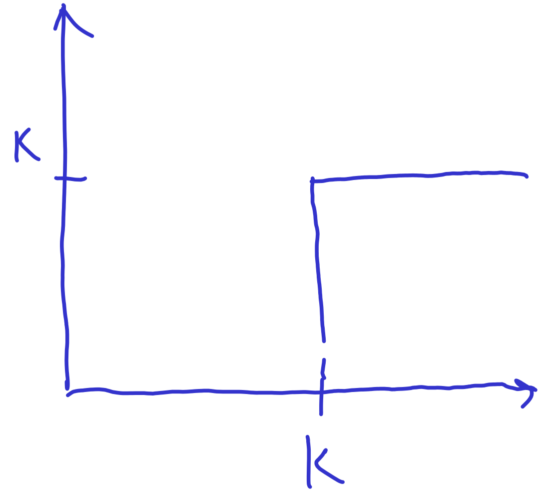
asset digital call



$$S_0 \Phi(d_1)$$

−

cash digital call



$$Ke^{-rT} \Phi(d_2)$$

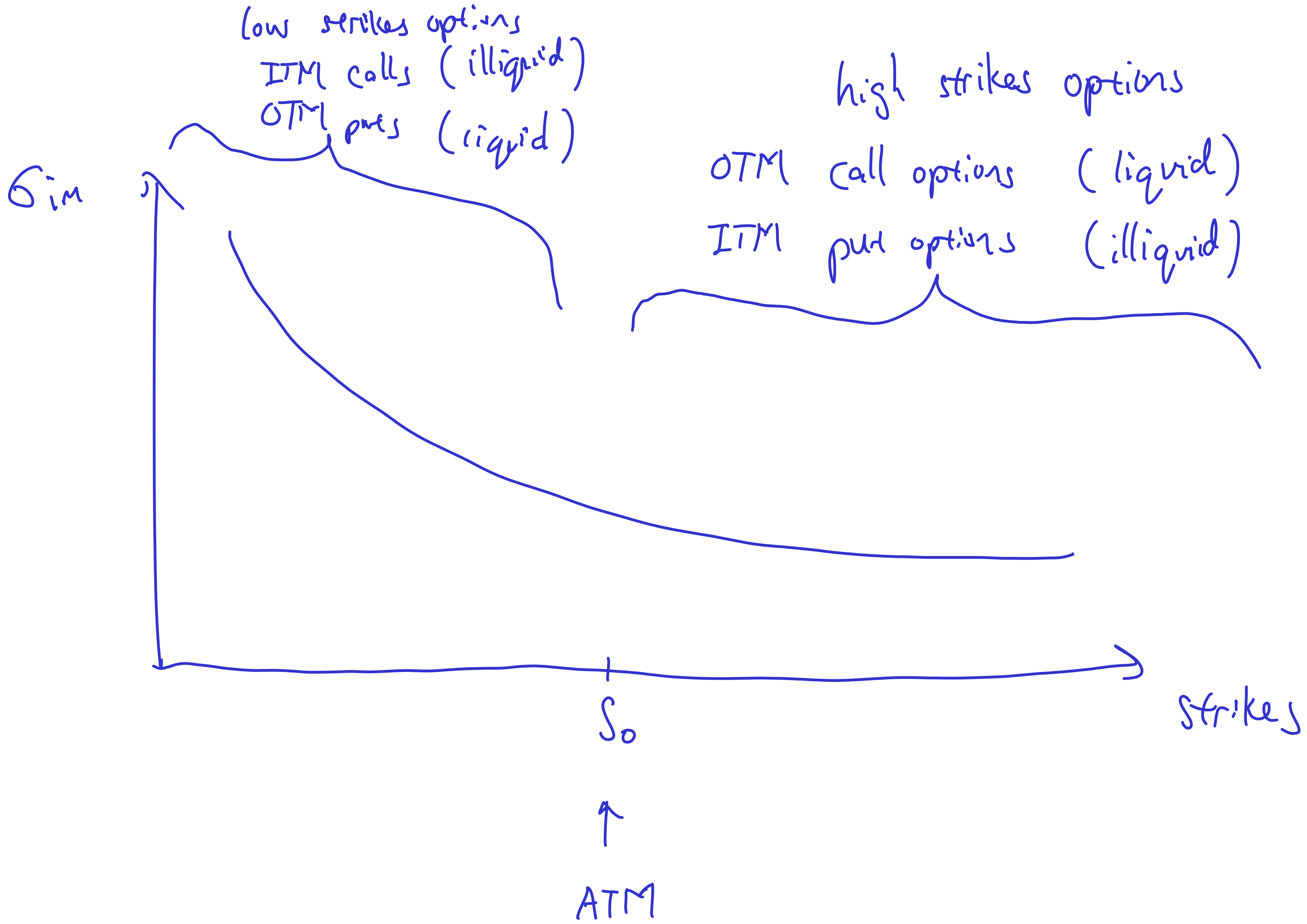
pg7.

$$e^{-rT}\,\mathbb{E}^{\mathbb{Q}^*}\left[S_T\,\mathbb{1}_{S_T>K}\right] = e^{-rT}\,\mathbb{E}^{\mathbb{Q}^S}\left[S_T\,\mathbb{1}_{S_T>K}\cdot\frac{d\mathbb{Q}^*}{d\mathbb{Q}^S}\right]$$

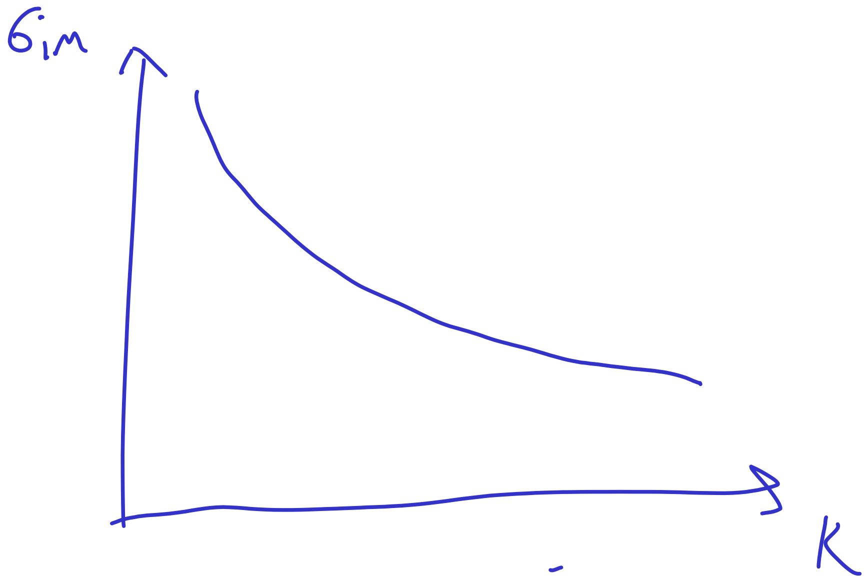
$$= \cancel{e^{-rT}}\,\mathbb{E}^{\mathbb{Q}^S}\left[S_T\,\mathbb{1}_{S_T>K}\cdot\frac{B_T/B_0}{S_T/S_0}\right]$$

$$= S_0\,\mathbb{E}^{\mathbb{Q}^S}\left[\mathbb{1}_{S_T>K}\right]$$

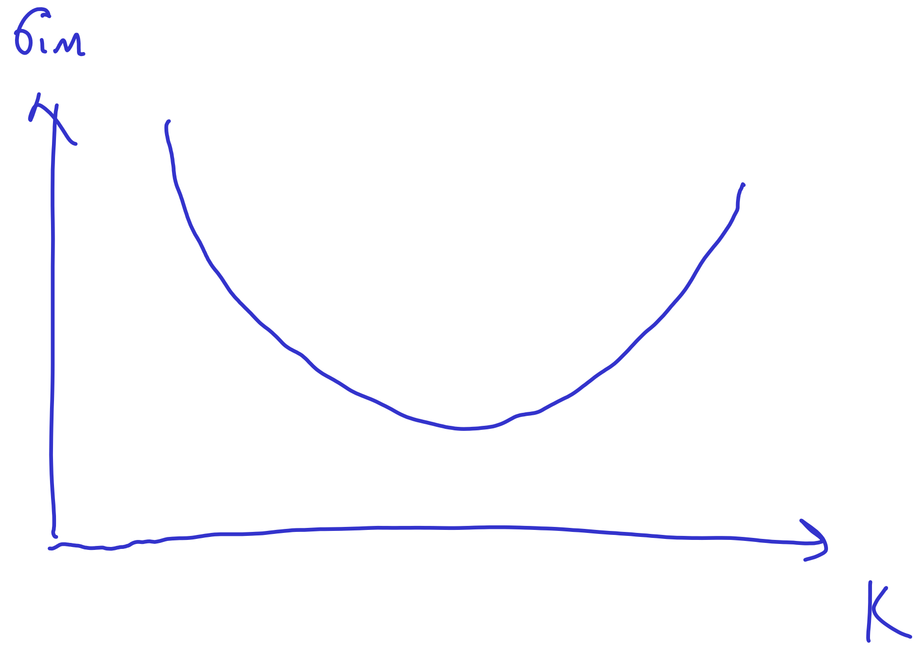
pg9.


low strikes options
ITM calls (illiquid)
OTM puts (liquid)
high strikes options
OTM call options (liquid)
ITM put options (illiquid)
σ_{im}
strikes
S_0
ATM

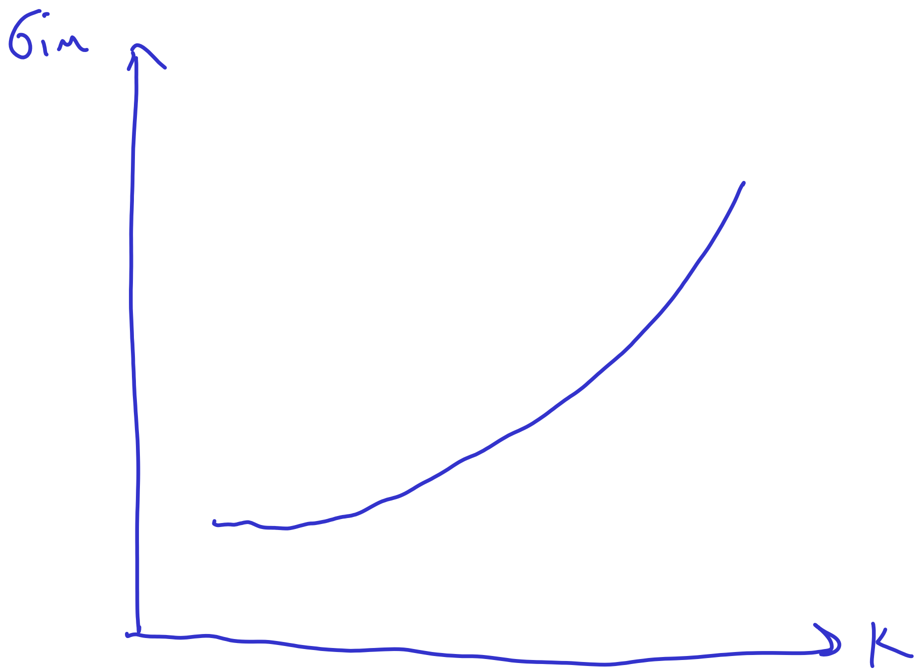
Equity option mkt



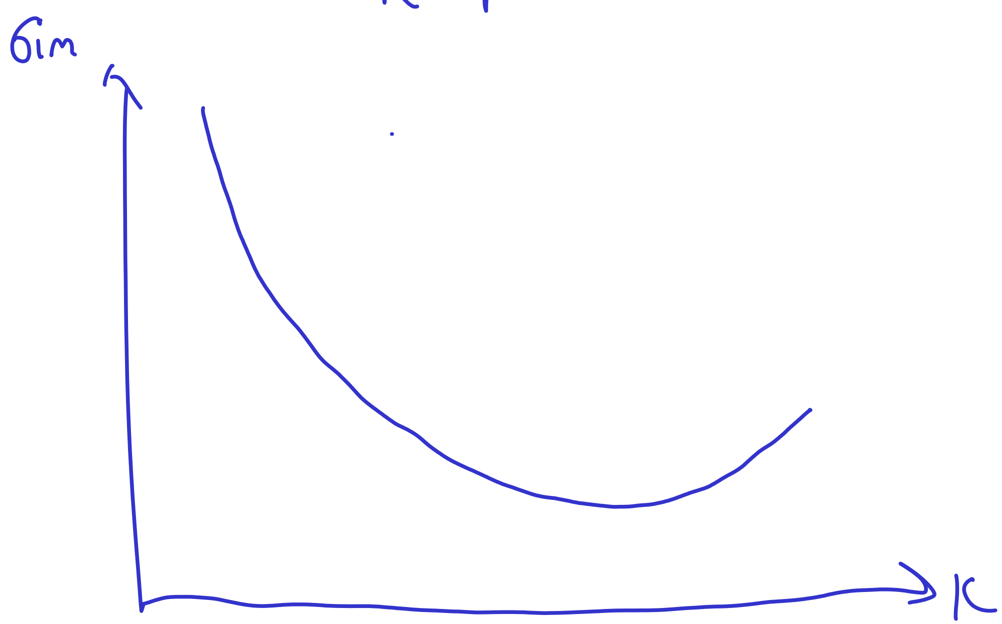
FX option mkt



Commodity option mkt

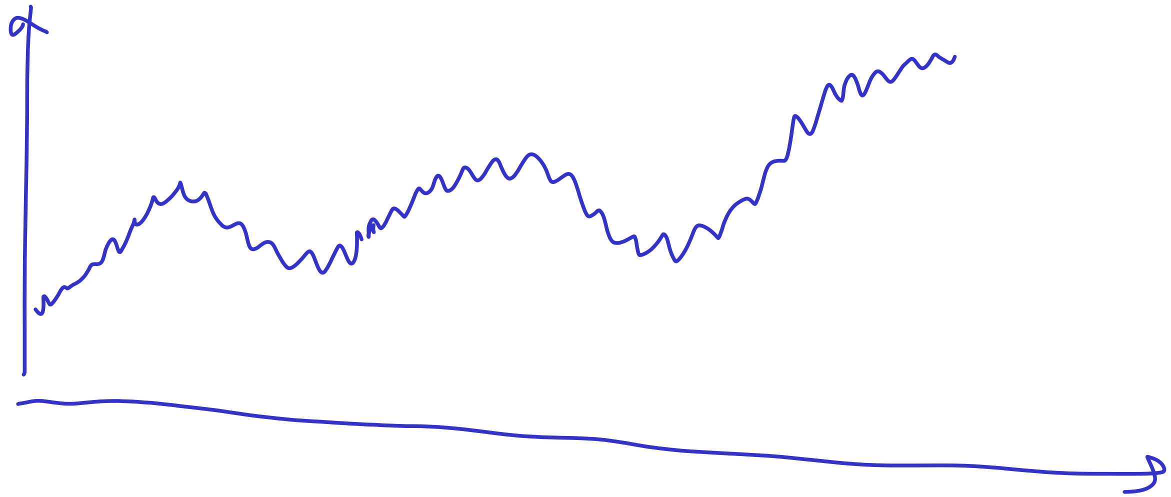


IR option mkt

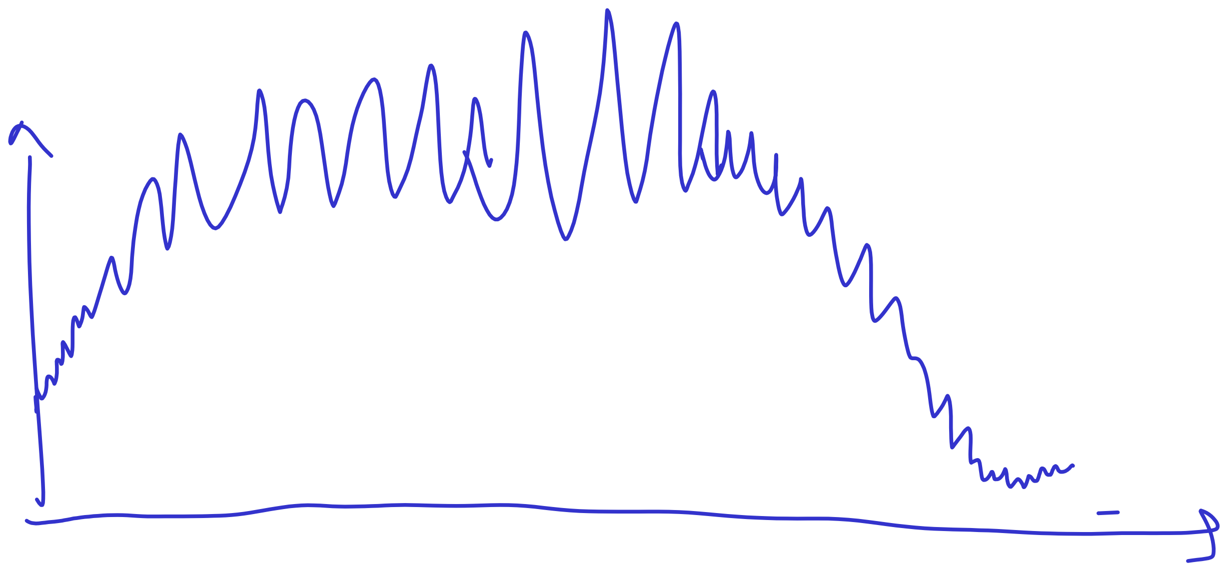


pg16.

Normal process



Lognormal process



$$dF_t = \sigma F_t \, dW_t^* \quad \text{vs} \quad dF_t = \sigma F_0 \, dW_t^*$$