Λ	No.
Assignment 4.	CHANJUNG KIM.
$dS_{t} = rS_{t}dt + \sigma S_{t}dWt^{*}$ $S_{t} = S_{0} \cdot e^{(r - \frac{1}{2}\sigma^{2})t + \sigma Wt^{*}}$ $V_{t} = \log \left(\frac{S_{0} \cdot e^{(r - \frac{1}{2}\sigma^{2})t + \sigma Wt^{*}}}{K}\right).$	
Therefore, the value of the European contribution of the European contrib	
$= e^{-tt} \left[\log \left(\frac{S_0}{K} \right) + \left(r - \frac{1}{2} \delta^2 \right) t + \delta \mathcal{A} \right]$ $= e^{-tt} \left[\log \left(\frac{S_0}{K} \right) + \left(r - \frac{1}{2} \delta^2 \right) t \right]$	
b) delta If we take porfial defivortive of above ex we obtain Out x 1/2	quation with respect to So.
gamma. Taking portial derivative of delta with resonant to the second of the second o	pect to So, we obtain

		Date	No.
2.	V Cash Orginal = ett [Stork. e-2 dt.		
	= ert = 500 e-2 bl.		
	$A^* = \frac{\log \frac{k}{5} - (r - \frac{6^2}{2})T}{6\sqrt{T}}$		
	Therefore, V^{c}_{cash} Digital = e^{-rt} $\overline{P}\left(\frac{\log s}{k} + (r - s)\right)$	- 6 ²)T	
	a) Delta Let $lcg \stackrel{S_0}{\leftarrow} + (r - \frac{5^2}{2})T = d_2$.		
	$\frac{\partial V^{c}}{\partial S_{0}} = e^{-\nu t} \cdot \frac{\partial \overline{\phi}(d_{2})}{\partial d_{2}} \cdot \frac{\partial d_{2}}{\partial S_{0}}$		
	$= e^{+k} \cdot \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{dz^2}{2}} \cdot \frac{1}{6S_0f_7}$		
	$= e^{-rt} \cdot \frac{1}{6SoJT} \cdot \sqrt{2\pi} \cdot e^{-\frac{d^2}{2}}$		
	b). Vega.		
	Let $\frac{\log S_0}{k} + \left(k - \frac{6^2}{2}\right)T = \sqrt{2}$.		
	$\frac{\partial V}{\partial o} = e^{-\gamma t} \frac{\Phi(d_1)}{\partial d_2} \frac{\partial d_2}{\partial o}$		
	$= \underbrace{Q^{+1} \cdot \underbrace{1}_{\sqrt{2}L} \cdot \underbrace{Q^{-\frac{d^{2}}{2}}_{\sqrt{2}L} \cdot \left(-\int T - \frac{dz}{6}\right)}_{\sqrt{2}L}$		

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3. a) $V_t = f(t, S_t)$		
$dVt = \frac{dV}{dt} dt + \frac{dV}{dS} dSt + \frac{1}{2} \frac{\partial^2 V}{\partial S^2} (dSt)^2$		
= Jt dt + JV (MSedt + oSedWe) +	12 0° Stat.	
$= \left(\frac{\partial V}{\partial t} + \frac{\partial V}{\partial S} + \frac{1}{2} \frac{\partial^2 V}{\partial S^2} + \frac{1}{2} $	25 o SedWe	
b) Tt = Vt + Dx St dTt = dVt + Dx dSt.		
$= \left(\frac{\partial V}{\partial t} + \frac{\partial V}{\partial S} \mathcal{U} S t + \frac{1}{2} \frac{\partial^2 V}{\partial S^2} \sigma^2 S t^2\right) dt +$	dV o SedWt	
+ DMStdt+ DOSEdWE.		
For the portfotio to be a risk-free portfo	otio, dWt tenr)
should disappear from the equation.		
Therefore, JV o StdWt + Do StdWt = 0		
$\nabla = -\frac{92}{9\Lambda}$		
	- 	

Then we obtain $ \frac{d\pi_t}{dt} = \left(\frac{dV}{dt} + \frac{1}{2}\frac{\partial^2 V}{\partial s^2}\right) dt. $ Since the return of this portfolio is deterministic, the return from this portfolio must be equal to risk-free vate r. to get rid of arbitrage $ \frac{d\pi_t}{dt} = r\pi_t dt = \left(\frac{\partial V}{\partial t} + \frac{1}{2}\frac{\partial^2 V}{\partial s^2}\right) dt. $ Substituting π_t with $V_t - \frac{\partial V}{\partial s}S_t$, (where $S_t = -\frac{\partial V}{\partial s}$) we obtain $ \frac{dV_t}{dt} = \frac{\partial V}{\partial t} + \frac{1}{2}\frac{\partial^2 V}{\partial s^2} + \frac{\partial^2 V}{\partial s^2} + \partial^2 $	$ \frac{d\Pi_t}{dt} = \left(\frac{dV}{dt} + \frac{1}{2}\frac{dV}{ds^2}\right)^2 St^2 ds^2 $ Since the return of this portfolio must the return from this portfolio must risk-free vate r . to get r id of d	is deterministic, be equal to
Since the return of this partiolization is deterministic, the return from this partiolization must be equal to risk-free rate r . to get rid of arbitrage $ \frac{\partial V}{\partial t} = r T t t dt = \left(\frac{\partial V}{\partial t} + 1 \frac{\partial^2 V}{\partial s^2} \sigma^2 S \epsilon^2 \right) dt. $ Substituting $T t = with V t - \frac{\partial V}{\partial s} S t = \frac{\partial V}{\partial s} $ we obtain $ \frac{\partial V}{\partial t} + r \frac{\partial V}{\partial s} S t = \frac{\partial V}{\partial t} + \frac{1}{2} \frac{\partial^2 V}{\partial s^2} \sigma^2 S t^2 $ $ \frac{\partial V}{\partial t} + r \frac{\partial V}{\partial s} S t = \frac{\partial V}{\partial s} + \frac{1}{2} \frac{\partial^2 V}{\partial s^2} \sigma^2 S t^2 $ $ \frac{\partial V}{\partial t} + r \frac{\partial^2 V}{\partial s} + r \frac{\partial^2 V}{\partial s} + \frac{1}{2} \frac{\partial^2 V}{\partial s^2} \sigma^2 S t^2 $	Since the return of this portfolio the return from this portfolio must risk-free vate r. to get rid of a	is deterministic, be equal to
the return from this partialization must be equal to risk-free vate r . to get rid of arbitrage $ \frac{\partial V}{\partial t} + \frac{1}{2}\frac{\partial^2 V}{\partial s^2} + \frac{\partial^2 V}{\partial s^2$	the return from this portfolio must risk-free vate r. to get rid of a	be equal to
Fisk-free vate r to get rid of arbitrage $ \frac{\partial V}{\partial t} = r T t dt = \left(\frac{\partial V}{\partial t} + \frac{1}{2} \frac{\partial^2 V}{\partial s^2} \right) dt. $ Substituting $T t = with V t - \frac{\partial V}{\partial s} S t$, (where $\Delta = -\frac{\partial V}{\partial s}$) We obtain $ \frac{\partial V}{\partial t} + \frac{\partial V}{\partial s} S t = \frac{\partial V}{\partial t} + \frac{1}{2} \frac{\partial^2 V}{\partial s^2} O^2 S t^2 $ $ \frac{\partial V}{\partial t} + \frac{\partial V}{\partial s} + \frac{\partial V}{\partial s} + \frac{1}{2} \frac{\partial^2 V}{\partial s^2} O^2 S t^2 $	risk-free vate r. to get rid of a	be equal to
Substituting .Tt with $V_{\pm} - \frac{\partial V}{\partial S}S_{\pm}$, (where $\Delta = -\frac{\partial V}{\partial S}$) We obtain $VV_{\pm} - VV_{\pm} = \frac{\partial V}{\partial S} + \frac{1}{2}\frac{\partial^{2}V}{\partial S^{2}}O^{2}S_{\pm}^{2}$ $VV_{\pm} = \frac{\partial V}{\partial t} + VS_{\pm}\frac{\partial V}{\partial S} + \frac{1}{2}\frac{\partial^{2}V}{\partial S^{2}}O^{2}S_{\pm}^{2}$.,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,
We obtain $VVt - V \frac{dV}{dS}St = \frac{dV}{dt} + \frac{1}{2} \frac{d^2V}{dS^2} O^2 St^2$ $VVt = \frac{dV}{dt} + VSt \frac{dV}{dS} + \frac{1}{2} \frac{d^2V}{dS^2} O^2 St^2$	$\frac{\partial T}{\partial t} = r \int dt = \left(\frac{\partial v}{\partial t} + \frac{\partial v}{\partial t} \right)$	$\frac{\partial^2 V}{2 \partial S^2} O^2 S \epsilon^2 dt$
$VVt - r\frac{\partial V}{\partial S}St = \frac{\partial V}{\partial t} + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}0^2St^2$ $VVt = \frac{\partial V}{\partial t} + rSc\frac{\partial V}{\partial S} + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}0^2St^2$		St, (where $\Delta = -\frac{\partial V}{\partial S}$)
$rVt = \frac{\partial V}{\partial t} + rSe \frac{\partial V}{\partial S} + \frac{1}{2} \frac{\partial^2 V}{\partial S^2} O^2 Se^2$	We obtain	
	$VVt - V\frac{dV}{dS}St = \frac{dV}{dt} + \frac{1}{2}\frac{d^2y}{dS}$	202St2
Therefore, Ut satisfies the Black-Scholes PDE	$VVt = \frac{\partial V}{\partial t} + FSt \frac{\partial V}{\partial S} + \frac{1}{2} \frac{\partial^2 V}{\partial S^2} C$	2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
	Therefore, Vt sofisfies the black	-Scholes PDE.