Problem 1. Suppose Y_t is the return on an equity portfolio at month t, and X_t is the market return. Their sample means are, respectively, 0.003 and 0.005. Suppose we run an OLS regression

$$Y_t = a + b\,X_t + e_t.$$

(a) Find the estimates for a and b given that

$$\sum_{t=1}^{60} X_t Y_t = 0.005; \qquad \sum_{t=1}^{60} X_t^2 = 0.004.$$

(b) Given that the residual sum of squares (RSS) is 5.8×10^{-5}, compute the t statistic of the a estimate under the hypothesis that $H_0 : a = 0$. What inference can be drawn?

(c) Do likewise under the null hypothesis that $H_0 : b = 1$. What inference can be drawn?

Problem 2. A student runs the following regression of stock i's return r_{it} on market portfolio return r_{mt} based on the market model:

$$r_{it} = a + b r_{mt} + e_{it},$$

where e_{it} is a residual noise that is i.i.d. and independent of r_{mt}.

(a) Is e_{it} independent of r_{it}?

(b) He performs OLS regression and obtains OLS estimates $\widehat{a}$ and $\widehat{b}$. He interprets $\widehat{b}$ as a parameter estimate that is proportionate to CAPM's notion of systematic risk of stock i, and determines $\widehat{a}$ as Jensen's alpha. Comment if his interpretation is sound.

(c) The student selects all stocks with positive $\widehat{a}$ and forms a portfolio. Is this portfolio likely to outperform the market index on average? Provide an explanation for your answer.

Problem 3. Let $\boldsymbol{X}'\boldsymbol{X} = \begin{pmatrix} 6 & 45 \\ 45 & 355 \end{pmatrix}$ and $\boldsymbol{X}'\boldsymbol{y} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$. The covariance between the intercept estimate and the slope estimate is $-\dfrac{3}{28}$.

(a) What is the dimension of matrix $\boldsymbol{X}$?

(b) What is the slope of the simple linear regression (as an irreducible fraction, e.g., 11/21)?

(c) What is the average of the explanatory variable (as an irreducible fraction, e.g., 11/21)?

(d) What is the unbiased variance of the explanatory variable (as an irreducible fraction, e.g., 11/21)?

(e) What is the unbiased variance of the residuals (as an irreducible fraction, e.g., 11/21)?

(f) What is the t statistic of the y-intercept estimate (rounded to 2 decimal places)?

(g) Suppose a new observation of the explanatory variable is obtained and its value is 1.5.

  (i) What is the point forecast for y (rounded to 2 decimal places)?

  (ii) What is the upper bound of the prediction interval at the 5% level of significance (rounded to 2 decimal places)? (Hint: You need to set $\boldsymbol{x} = (1 \quad 1.5)'$ and apply your understanding about Slide 31 of `S4_2_MLR.pdf`)