QF620 Stochastic Modelling in Finance
Assignment 4/4
Due Date: 26-Nov-2018

1. Consider the Black-Scholes stock process

$$dS_t = rS_t dt + \sigma S_t dW_t^*,$$

where W_t^* is a standard Brownian motion under the risk-neutral measure $\mathbb{Q}^*$ associated with the risk-free bond numeraire. A European contract pays

$$V_T = \log\left(\frac{S_T}{K}\right)$$

on maturity date T.

   (a) Derive the valuation formula for this contract.

   (b) Derive the delta (Δ) and (Γ) of this contract.

2. Derive the valuation formula for a cash-or-nothing digital call option struck at K and maturing at T

$$V_T = \mathbb{1}_{S_T > K}$$

under the Black-Scholes model. Proceed to derive its

   (a) delta

   (b) vega

3. Consider a portfolio Π consisting of an option V and Δ amount of stock S, i.e. $\Pi_t = V_t + \Delta \times S_t$. The stock pays no dividend and its price follows the lognormal stochastic differential equation

$$dS_t = \mu S_t dt + \sigma S_t dW_t.$$

   (a) Use Itô's formula to write down the SDE for the process dV_t.

   (b) Write down the process $d\Pi_t$, and determine what choice of Δ will give rise to a risk-free portfolio.

   (c) A risk-free portfolio must earn the continuously compounded risk-free rate by no-arbitrage. Show that this leads to the Black-Scholes partial differential equation.