QF600 Asset Pricing

Assignment 4 – Woon Tian Yong

Simulated $\tilde{\epsilon}$ values:	10,000
Simulated $\tilde{\nu}$ values:	10,000
Simulated $\tilde{g}$ values:	10,000
Generated γ values:	2,000

Part 1: Hansen-Jagannathan Bound

Smallest γ value which satisfies Hansen-Jagannathan bound: **3.32**

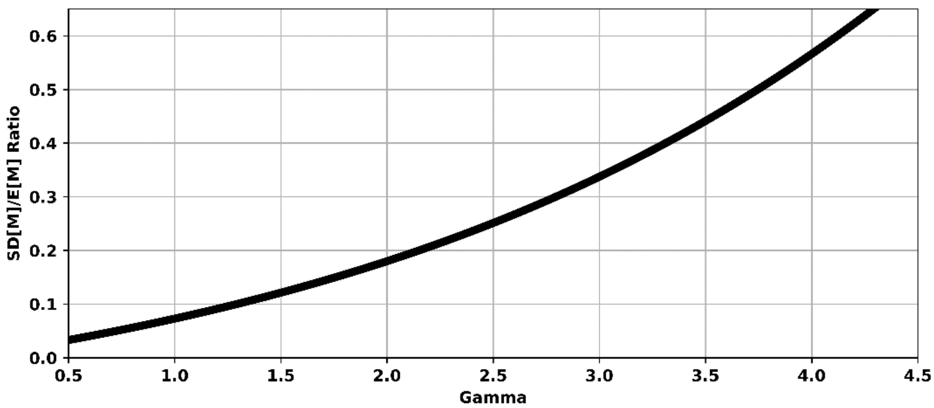
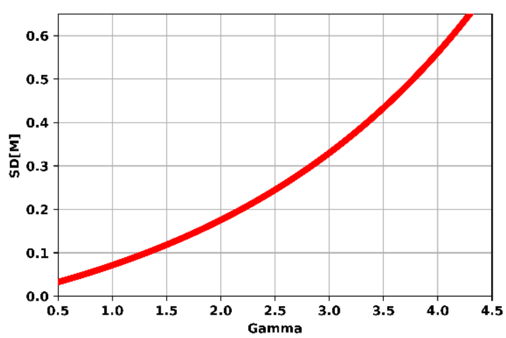
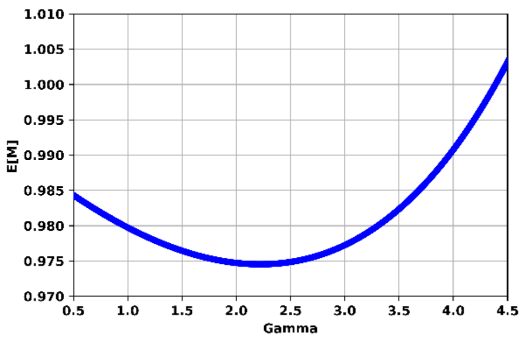


Figure 1.1: SD[M]/E[M] Ratio vs. γ

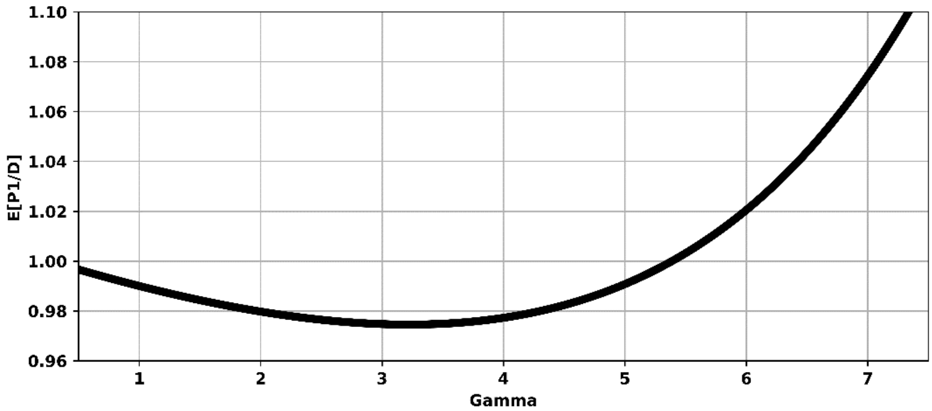
It was observed that the SD[M]/E[M] ratio increased as γ increased, driven by the numerator of the ratio. This makes numerical sense as higher γ values will more greatly exaggerate the ranges of the generated $\tilde{g}$ values and by extension, increase the variances and standard deviations of said $\tilde{g}$ values. On the contrary, the means of the generated $\tilde{g}$ values across different levels of γ were only relatively mildly changed. Since the Hansen-Jagannathan (HJ) bound states that the Sharpe ratio of an asset cannot exceed SD[M]/E[M], where M is the pricing kernel or the stochastic discount factor, for an observed/assumed Sharpe ratio of 0.4 to be within the HJ bound, it was found that a γ parameter of 3.32 and above must be used when investors are assumed to have power utility functions. Further, this also means that investors need to be assumed to have constant relative risk aversion coefficients of at least 3.32, since the γ term also represents the Arrow-Pratt coefficient of constant relative risk aversion for the power utility function.





Figures 1.2, 1.3: SD[M] vs. γ, E[M] vs. γ

Part 2: Price-Dividend Ratio



Figures 2: E[P1/D] vs. γ

It was unsurprising to see that the shape of the E[P1/D] vs. γ curve is of a similar shape to the E[M] vs. γ curve shown in Part 1, given how the mathematical formulas for E[P1/D] and E[M] are very similar. For γ values of between 1 and 7, it was found that the E[P1/D] term is minimised at 0.9745 with a corresponding γ value of 3.2216.

Part 3: Equity Premium

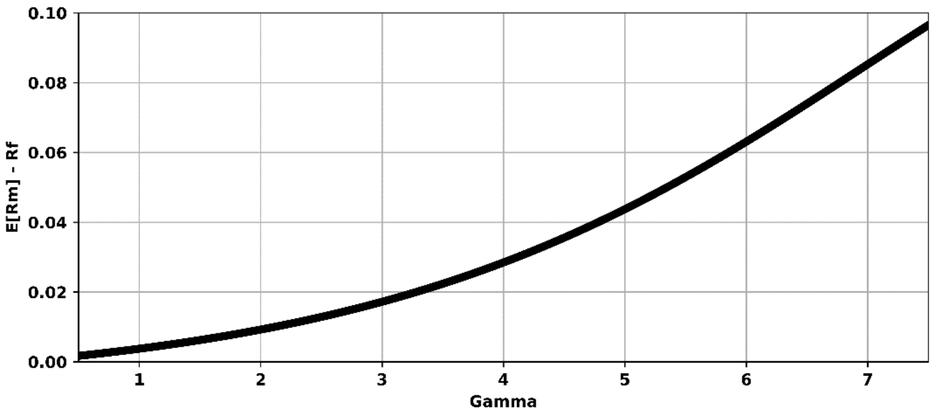


Figure 3.1: E[Rm] - Rf vs. γ

The equity premium was seen to be increasing with γ mainly due to the effect of Rf values decreasing at a greater pace relative to E[Rm] values as γ values increased (see **Figure 3.2** below). Furthermore, it was found that the smallest γ value at 3.32 which satisfies the HJ bound in Part 1 corresponds with an equity premium of about 0.019 or 1.9%, which is unreasonably low by many real-world standards. For instance, the equity premium for the U.S. stock market is around 7% per year, as noted from the 05_Stochastic_Discount_Factor lecture slides. This implies that a larger γ value will likely have to be used so that the corresponding equity premium matches what is measured in the real world, with all else being constant. For the case of the U.S. equity premium of 7% mentioned earlier, a γ value of about 6.32 will have to be used instead. However, the γ value, which is also the Arrow-Pratt coefficient of constant relative risk aversion for the power utility function, should not be too high lest they suggest unrealistically high levels of constant relative risk aversion in investors. In the famous paper published by Mehra and Prescott in 1985, titled "The Equity Premium: A Puzzle", several studies were cited to have estimated the γ-equivalent term to be between about 0 and 2, hinting that an assumption of a γ value of 6.32 may already be too high. These conflicts lie at the heart of the "Equity Premium Puzzle" highlighted by Mehra and Prescott.

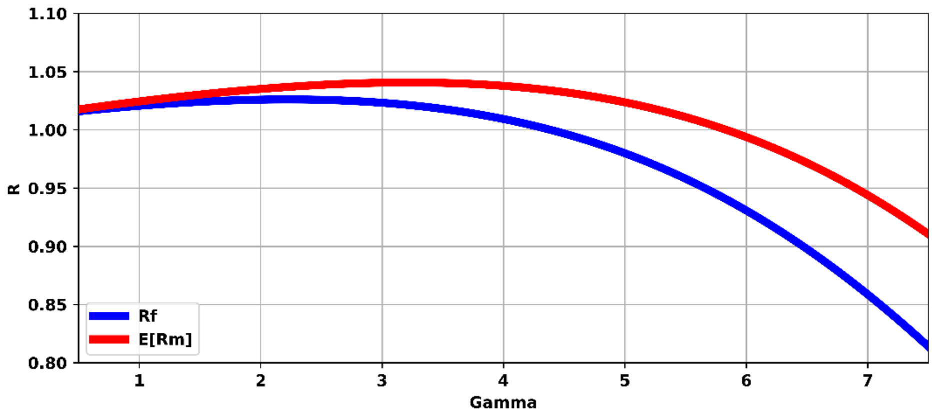


Figure 3.2: E[Rm] and Rf vs. γ