Session 1
ℚuantitative Analysis of Financial Markets
In-Class Exercises

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Question 1

Suppose the probability density function is given by $f(x) = cx^2$ for $-1 \leq x \leq 1$. What is the value of c (in irreducible fraction, e.g. 54/55)?

Answer: Probabilities sum to 1. Hence

$$1 = \int_{-1}^{1} f(x)dx = c\int_{-1}^{1} x^2 dx = \frac{c}{3}x^3\Big|_{-1}^{1} = c\big(1^3 - (-1)^3\big) = \frac{2c}{3}.$$

It follows that $c = \mathbf{\frac{3}{2}}$.

Question 2

The past returns of a stock are chronologically arranged as

$$-2.1\%, 3.4\%, 1.7\%, -0.5\%, -3.2\%, 0.8\%, 0.3\%, -2.8\%, -0.6\%, -1.9\%.$$

What is the 95-th percentile (2 decimals in %, e.g., 1.23%)?

Answer: Sort the past returns from the smallest to the largest:

Percentile	$\frac{0}{9}$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{4}{9}$	$\frac{5}{9}$	$\frac{6}{9}$	$\frac{7}{9}$	$\frac{8}{9}$	$\frac{9}{9}$
Return (%)	-3.2	-2.8	-2.1	-1.9	-0.6	-0.5	0.3	0.8	1.7	3.4

95-th percentile is between 8/9-th percentile and 100-th percentile. Perform linear interpolation, which gives

$$\frac{(0.95 - 8/9) \times 3.4 + (1 - 0.95) \times 1.7}{1/9} = 2.635.$$

Thus, at 2 decimal places, the 95-th percentile is **2.64%**.

Question 3

Fund managers buy either value stocks or growth stocks. It is known that 20% (51.5%) of value (growth) stocks are liquidated after 2 years On average, 30% fund managers liquidate their stocks after 2 years. What is the probability that fund managers will invest in a value stock?

Answer: Let p be the probability that fund managers will invest in a value stock.

$$
\begin{aligned}
& 0.2p + 0.515(1-p) = 0.3 \\
\implies \quad & 0.315p = 0.215 \\
\implies \quad & p = \frac{0.215}{0.315} = \mathbf{\frac{43}{65}}
\end{aligned}
$$

Question 4

Let T with $t \in \{0, 1\}$ be the random variable indicating whether a trader is professional (t = 1) or not (t = 0), and A with $a \in \{0, 1\}$ be the variable indicating the accuracy of a trader's trading algorithm. A professional trader has accurate trading algorithm with probability $\mathbb{P}(A = 1|T = 1) = 0.98$, a non-professional trader with probability $\mathbb{P}(A = 1|T = 0) = 0.001$. One in hundred thousand traders is a professional, i.e., $\mathbb{P}(T = 1) = 0.00001$. What is the probability that a trader having accurate trading algorithm turns out to be professional (2 decimals in %, e.g., 12.34%)?

Answer

$$\begin{aligned}\mathbb{P}(T = 1|A = 1) &= \frac{\mathbb{P}(A = 1|T = 1)\,\mathbb{P}(T = 1)}{\mathbb{P}(A = 1|T = 1)\,\mathbb{P}(T = 1) + \mathbb{P}(A = 1|T = 0)\,\mathbb{P}(T = 0)}\\ &= \frac{0.98 \times 0.00001}{0.98 \times 0.00001 + 0.001 \times (1 - 0.00001)}\\ &= 0.0097 = \mathbf{0.97\%}.\end{aligned}$$

Question 5

In a portfolio, 30% of the stock have good (G) analysts' rating, 50% are blue chips (B), and the remaining are considered ordinary (O). In a ranking exercise, 65% of the good ones, 82% of the blue chips, and 50% of the ordinary ones were selected. Now, a stock is picked randomly from the portfolio, which you know is not ranked (NR) in the exercise, what is the probability that this stock you pick is a blue chip (2 decimals in %, e.g., 12.34%)?

Answer:

$$\begin{aligned}\mathbb{P}(\mathsf{B} \mid \mathsf{NR}) &= \frac{\mathbb{P}(\mathsf{NR} \mid \mathsf{B})\,\mathbb{P}(\mathsf{B})}{\mathbb{P}(\mathsf{G})\,\mathbb{P}(\mathsf{NR} \mid \mathsf{G}) + \mathbb{P}(\mathsf{B})\,\mathbb{P}(\mathsf{NR} \mid \mathsf{B}) + \mathbb{P}(\mathsf{O})\,\mathbb{P}(\mathsf{NR} \mid \mathsf{O})} \\ &= \frac{0.5 \cdot (1 - 0.82)}{(0.3 \cdot (1 - 0.65)) + (0.5 \cdot (1 - 0.82)) + (0.2 \cdot (1 - 0.5))} \\ &= 0.3051 = \mathbf{30.51\%}.\end{aligned}$$