Answer. Suggested solution for Q1

Let R_α be the random variable for the return of Alpha hedge fund, and R_ω that of Omega hedge fund. Under the assumption of independence between R_α and R_ω, the probability in any given year that both funds make money is

$$\mathbb{P}\left(R_\alpha > 0 \text{ and } R_\omega > 0\right) = \mathbb{P}\left(R_\alpha > 0\right) \times \mathbb{P}\left(R_\omega > 0\right) = 0.6 \times 0.7 = 0.42 = 42\%.$$

Conversely, in any given year that both funds lose money, the probability is

$$\mathbb{P}\left(R_\alpha < 0 \text{ and } R_\omega < 0\right) = \Big(1 - \mathbb{P}\left(R_\alpha > 0\right)\Big) \times \Big(1 - \mathbb{P}\left(R_\omega > 0\right)\Big) = (1 - 0.6) \times (1 - 0.7) = \mathbf{12}\%.$$

□

Answer. Suggested solution for Q2

Let P be the performance direction of the portfolio manager, and M be the market direction.

First, we need to translate English statements into mathematical symbols.

1. 50% probability that the market will be up $\implies \mathbb{P}\left(M = \text{up}\right) = 50\%$.
2. 20% probability that the market will be down $\implies \mathbb{P}\left(M = \text{down}\right) = 20\%$.
3. 30% probability that the market will be flat $\implies \mathbb{P}\left(M = \text{flat}\right) = 30\%$.
4. 80% probability that the manager will be up if the market is up $\implies \mathbb{P}\left(P = \text{up} \middle| M = \text{up}\right) = 80\%$.
5. 10% probability that the manager will be up if the market is down $\implies \mathbb{P}\left(P = \text{up} \middle| M = \text{down}\right) = 10\%$.
6. 50% probability that the manager will be up if the market is flat $\implies \mathbb{P}\left(P = \text{up} \middle| M = \text{flat}\right) = 50\%$.

Given these probabilities, the unconditional probability that the manager will be up is

$$\begin{aligned}
\mathbb{P}\left(P = \text{up}\right) &= \mathbb{P}\left(P = \text{up} \middle| M = \text{up}\right) \times \mathbb{P}\left(M = \text{up}\right) \\
&\quad + \mathbb{P}\left(P = \text{up} \middle| M = \text{down}\right) \times \mathbb{P}\left(M = \text{down}\right) \\
&\quad + \mathbb{P}\left(P = \text{up} \middle| M = \text{flat}\right) \times \mathbb{P}\left(M = \text{flat}\right) \\
&= 0.8 \times 0.5 + 0.1 \times 0.2 + 0.5 \times 0.3 \\
&= \mathbf{57}\%.
\end{aligned}$$

□

Answer. **Suggested solution for Q3**

Let S indicate that the fund manager is skillful, and U unskillful. Let P denote that the test is positive and N negative. Since 1% of fund managers are skillful, we have $\mathbb{P}(S) = 0.01 = \frac{1}{100}$, implying that $\mathbb{P}(U) = 0.99 = \frac{99}{100}$. We also have $\mathbb{P}(P|S) = 0.9 = \frac{9}{10}$.

First we want to compute $\mathbb{P}(P)$, the probability of tested positive. It can be obtained as

$$\mathbb{P}(P) = \mathbb{P}(P|S)\,\mathbb{P}(S) + \mathbb{P}(P|U)\,\mathbb{P}(U) = \frac{9}{10} \times \frac{1}{100} + \frac{1}{10} \times \frac{99}{100} = \frac{108}{1000}.$$

Next, we apply Bayes' theorem as follows

$$\mathbb{P}(S|P) = \frac{\mathbb{P}(P|S)\,\mathbb{P}(S)}{\mathbb{P}(P)} = \frac{\frac{9}{10} \times \frac{1}{100}}{\frac{108}{1000}} = \frac{9}{108} = \boldsymbol{\frac{1}{12}}.$$

□

Answer. **Suggested solution for Q4**

Let PD denote the probability of default. So we have PD = 10%, and no default is $\overline{\text{PD}} = 90\%$. The recovery rate is 40%, i.e., you can recover 40% of the notional amount. The expected payoff or cash flow at maturity of the zero coupon bond is

$$0.9 \times \$100 + 0.1 \times 0.4 \times \$100 = \$94.0$$

The discount future cash flow is the present value. Hence, the present value of the bond is, under continuous compounding,

$$\text{PV} = e^{-0.05\times 1} \times \$94.0 = \boldsymbol{\$89.42}.$$

□

Answer. **Suggested solution for Q5**

Let $\Omega = A \cup A^c$.

$$\begin{aligned}
\mathbb{V}\left(1_A\right) &= \mathbb{E}\left(1_A^2\right) - \mathbb{E}\left(1_A\right)^2 \\
&= \int_\Omega 1_A^2 f(\omega)\, d\omega - p^2 \\
&= \int_\Omega 1_A f(\omega)\, d\omega - p^2 \\
&= p - p^2 = \boldsymbol{p(1-p)}.
\end{aligned}$$

□