In all the Python programs, we assume students will import Matplotlib, Pandas, Numpy and Scipy as the following:

```
import matplotlib.pyplot as plt
import pandas as pd
import numpy as np
import scipy.stats as ss
import scipy.optimize as so
```

Part 1: [Question 1 – Question 4, 40 marks] Programming in ☐ Python / ☐ MATLAB**

(International-Track Students must use MATLAB.)**

Question 1 [10 marks] (☐ Python / ☐ MATLAB**)

(1) [2 marks] Load data from **`E001.csv`**, which contains the coordinates of **7 points** (see Figure 1) **from row 2 to row 8. Note that it is not accepted if you type these values manually in your program.**
(2) [2 marks] Plot these **7 points**, y (vertical) vs x (horizontal), using **black circles** as the marker.
(3) [2 marks] Draw **the straight line of best fit** of these points in the same axes using a **black solid l**
(4) [2 marks] Use **30 equally spaced points** to draw **the quadratic polynomial of best fit** of these points in the same axes using a **black dotted line**.
(5) [2 marks] **Label the x axis and the y axis**, respectively, using "x" and "y". **Add title** "Seven Points" to the graph.

	A	B
1	x	y
2	1	1
3	2	3
4	3	4
5	4	5
6	5	2
7	6	3
8	7	4

Figure 1: **`E001.csv`** opened in Excel.

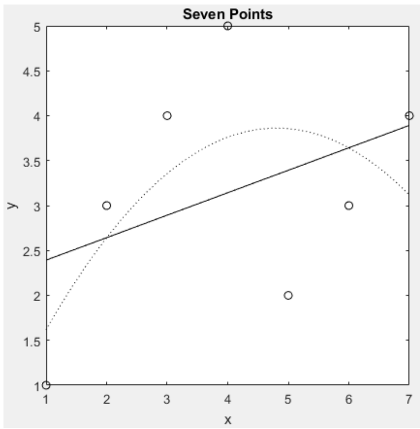


Figure 2: The expected output.

Question 2 [10 marks] (☐ Python / ☐ MATLAB**)

(1) [2 marks] Specify the seed number for the random number generator as **`123456`**.
(2) [2 marks] **Use one command** to generate a 4-by-8 matrix by appending a 4-by-4 matrix of uniformly distributed random numbers to a 4-by-4 matrix of random numbers of standard normal distribution. Name the 4-by-8 matrix as **`X`**.
(3) [2 marks] **Use one command** to update the second row of **`X`** by adding 2 times the elements in its first row to the square of the elements in its third row, element by element.
(4) [2 marks] **Use one command** to calculate the sum of each column and sort the sums from highest to lowest. Save and name the sorted sums as **`S`**.
(5) [2 marks] **Use one command** to find the second smallest sum in **`S`** and name the scalar as **`Q02`**.

Questions 3 [10 marks] (☐ Python / ☐ MATLAB)**

(1) [5 marks] Write a function to compute the European call option price using the following Black-Scholes formula:

$$V = S \cdot \Phi(d_1) - K \cdot e^{-rT} \cdot \Phi(d_2)$$

where $\Phi(x)$ represents the standard normal cumulative distribution function, and

$$d_1 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T\right] \text{ and } d_2 = d_1 - \sigma\sqrt{T}.$$

Inputs to the function should be S, K, r, T and σ and the output should be the value of V. Name this function as **`Q03fun`**.

(2) [5 marks] Given $V = 13.7, S = 100, K = 95, r = 0.1, T = 0.25$, use the function defined in (1) and a library function for equation solving to find the implied volatility, i.e. the value of σ in the Black-Scholes formula. Name the value of σ as **`Q03sigma`**.

Question 4 [10 marks] (☐ Python / ☐ MATLAB)**

[10 marks] Solve the following optimization problem with constraints.

$$\text{Miminize } 3x^2 + 4xy + y^2 + x + 6y - 2$$

Subject to

$$2x + 3y \geq 4$$
$$x \geq 0$$
$$y \geq 0$$

- In the MATLAB program, use the library function **`quadprog`**,

```
[x, fval]=quadprog(H, f, A, b, Aeq, beq, lb, ub)
```

for a problem specified by:

$$\min_x \frac{1}{2}x^T H x + f^T x \text{ such that } \begin{cases} A \cdot x \leq b, \\ Aeq \cdot x = beq, \\ lb \leq x \leq ub \end{cases}$$

Name the minimum value as **`Q04.min`**, and the corresponding values of x and y, respectively, as **`Q04.x`** and **`Q04.y`**.

- In the Python program, use the library function **`scipy.optimize.minimize`**. Name the minimum value as **`Q04min`**, and the corresponding values of x and y, respectively, as **`Q04x`** and **`Q04y`**.

Part 2: [Question 5 – Question 7, 60 marks] Programming in Python-Only

Question 5: [20 marks] Write a Python function, `myLSMput`, to implement the following algorithm.

Input [1 mark]: $\boldsymbol{K}$ (a positive number), $\boldsymbol{r}$ (a positive number), $\boldsymbol{\Delta t}$ (a positive number) and $\boldsymbol{S}$ (an M-by-N matrix) denoted as the following.

$$S = \begin{pmatrix} S_{1,1} & S_{1,2} & \cdots & S_{1,N} \\ S_{2,1} & S_{2,2} & \cdots & S_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ S_{M,1} & S_{M,2} & \cdots & S_{M,N} \end{pmatrix}$$

Output [1 marks]: $\boldsymbol{V}$ (a number)

<u>Algorithm:</u>

(1) [3 marks] Use one command to create an M-element vector, $x = (x_1, x_2, \dots, x_M)$, which is initialized as a constant vector with every element equal to N and set as an integer type.

(2) [3 marks] Use one command to create an M-by-N matrix, IV, as the following:

$$IV = \begin{pmatrix} IV_{1,1} & IV_{1,2} & \cdots & IV_{1,N} \\ IV_{2,1} & IV_{2,2} & \cdots & IV_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ IV_{M,1} & IV_{M,2} & \cdots & IV_{M,N} \end{pmatrix},$$

where $IV_{i,j} = \max(K - S_{i,j}, 0)$ for $i = 1, 2, \dots, M; j = 1, 2, \dots, N$.

(3) [2 marks] Repeat (3.1)-(3.4) for $j = N - 1, N - 1, \dots, 2$.

(3.1) [2 marks] Let idx denote the set of row numbers in the j-th column of S:

$$idx = \{i: IV_{i,j} > 0, i \in \{1, 2, \dots, M\}\}.$$

We denote the corresponding set of values in the j-th column of matrix IV as $IV_{idx,j}$.

(3.2) [2 marks] Let X be the set of values in the j-th column of S: $X = \{S_{i,j}: i \in idx\}$. Let Y be the corresponding values in IV: $Y = \{IV_{i,x_i} e^{-(r)(x_i - j)(\Delta t)}: i \in idx\}$.

(3.3) [2 marks] Regress Y on a constant, X and X^2, (i.e. a degree-2 polynomial regression), we denote the result as: $P_2(x) = a_2 x^2 + a_1 x + a_0$.

(3.4) [2 marks] For $i \in idx$, if $IV_{i,j} > P_2(S_{i,j})$, let $x_i = j$.

(4) [2 marks] Use one command to calculate and return the value V:

$$V = \frac{1}{M} \sum_{i=1}^{M} IV_{i,x_i} e^{-(r)(x_i - 1)(\Delta t)}$$

Question 6: [20 marks] Write a Python function, `my_MCScall_stat`, to implement the following algorithm.

Input [1 mark]: $\boldsymbol{S}$ (a positive number), $\boldsymbol{K}$ (a positive number), $\boldsymbol{r}$ (a positive number), $\boldsymbol{T}$ (a positive number), $\boldsymbol{\sigma}$ (a positive number) and $\boldsymbol{N}$ (a positive even integer).

Output [1 mark]: $\boldsymbol{V}$ (a number)

<u>Algorithm:</u>

(1) [2 marks] Use one command to create a vector of $\frac{N}{2}$ random numbers with standard normal distribution, denoted as d:

$$d = \left(d_1, d_2, \dots, d_{\frac{N}{2}}\right)$$

(2) [4 marks] Use one command and the library function **`numpy.concatenate`** to create a vector of N elements by using d as the first half, and $-d$ as the second half. Name this vector as p.

$$p = (p_1, p_2, \dots, p_N) = \left(d_1, d_2, \dots, d_{\frac{N}{2}}, -d_1, -d_2, \dots, -d_{\frac{N}{2}}\right)$$

(3) [2 marks] Use one command to compute the mean and standard deviation of the N numbers in the vector p, and denote them as μ_p and σ_p, respectively.

(4) [2 marks] Use one command to update every element in the vector p as follows:

$$p_i \leftarrow \frac{p_i - \mu_p}{\sigma_p} \quad \text{for } i = 1, 2, \dots, N.$$

(5) [4 marks] Use one command to obtain the vector $Y = (Y_1, Y_2, \dots, Y_N)$ computed as follows:

$$Y_i = S \cdot e^{\left(r - \frac{\sigma^2}{2}\right)T + \sigma \cdot \sqrt{T} \cdot p_i} \quad \text{for } i = 1, 2, \dots, N.$$

(6) [2 marks] Use one command to obtain the vector $h = (h_1, h_2, \dots, h_N)$ computed as follows:

$$h_i = \max(Y_i - K, 0) \quad \text{for } i = 1, 2, \dots, N.$$

(7) [2 marks] Use one command to calculate and return V as follows:

$$V = \frac{1}{N} e^{-r \cdot T} \sum_{i=1}^{N} h_i$$

Question 7: [20 marks] Write a Python program according to the following instructions.

(1) [2 marks] **Use one command with `pandas.read_csv`** to load data from the CSV file **`Q7.csv`** to a DataFrame, using the **first column as the row labels** and the **first row as column names**. Name this DataFrame as **`df1`**.

Q7.csv - Notepad

```
A,B,C,D,E,F
1,-3.2,3.9,-2.6,-8.1,1.4
2,7.4,1.8,-1.1,2.3,-7
3,8.3,8.7,7.3,8.4,7
4,-7.5,-9.1,7.8,-8.5,4.2
5,-10,5.2,-6,-2.2,2
6,6.5,-1.5,-7.7,0.2,5.6
7,-6.2,-8.7,9,-9.5,-7.2
8,-4.3,-6.5,4.9,-0.8,-7.3
9,2.2,-6.2,-4.8,-6.1,-7.5
10,8.9,-4.8,-0.6,-5.7,-4.8
```

(2) [2 marks] **Use one command** to find and **`print`** the number of rows and number of columns of the DataFrame **`df1`** as **`10 rows, 5 columns`**, if there are 10 rows and 5 columns.

(3) [2 marks] **Use one command with `pandas.DataFrame.drop`** to drop those rows in **`df1`** which have a smaller value in column **C** than the value in column **D**.

(4) [2 marks] **Use one command** to add a column to the DataFrame **`df1`** and initialize the column's values as consecutive integers starting from **`-5`** (i.e., **`-5`**, **`-4`**, **`-3`**, …). Name this column as **G**.

(5) [2 marks] **Use one command** to replace the column **D** of **`df1`** by its rolling mean of 3 using **`pandas.DataFrame.rolling`** (not **`pandas.Series.rolling`**).

(6) [2 marks] **Use one command with `pandas.DataFrame.eval`** to update the **odd rows** of column **E** using **(column E)=(column C)-(column F)*(column B)**.

(7) [2 marks] **Use one command with `pandas.DataFrame.apply`** to update the **even rows** of column **E** using elements in the same row: **(column E)=(column C)+(column F)-(column B)**.

(8) [2 marks] **Use one command with `pandas.DataFrame.where`** to update **`df1`** by changing all negative numbers in the DataFrame to twice of their original values.

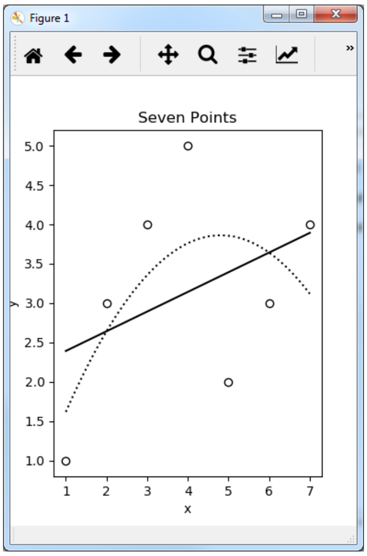
(9) [2 marks] **Use one command with `pandas.DataFrame.plot`** to plot columns **B**, **E**, **F** and **G** in **`df1`**, respectively using **black solid line**, **blue dashed line**, **red dotted line** and **green dash-dot lines**.

(10) [2 marks] Write the DataFrame **`df1`** to a CSV file, **`Q7result.csv`**.

Questions 1-10, Python Sample Solution:

Question 1

```
#(0)
import matplotlib.pyplot as plt
import pandas as pd
import numpy as np
#(1)
df=pd.read_csv('E001.csv')
#(2)
plt.plot(df.x,df.y,'ko',mfc='none')
#(3)
p01=np.polyfit(df.x, df.y, 1)
x0=df.x.sort_values(ascending=True)
plt.plot(x0, np.polyval(p01, x0), 'k-')
#(4)
p02=np.polyfit(df.x, df.y, 2)
x1=np.linspace(df.x.min(), df.x.max(), 30)
plt.plot(x1, np.polyval(p02, x1), 'k:')
#(5)
plt.xlabel('x')
plt.ylabel('y')
plt.title('Seven Points')
plt.show()
```



Question 2

```
#(0)
import numpy as np
#(1)
np.random.seed(123456)
#(2)
x=np.append(np.random.rand(4,4), np.random.randn(4,4), axis=1)
#(3)
x[1,:]=2*x[0,:]+x[2,:]**2
#(4)
s=np.sort(np.sum(x, axis=0))[::-1]
#(5)
Q8=s[-2]
```

Question 3

```
#(0)
import numpy as np
import scipy.stats as ss
import scipy.optimize as so
#(1)
def Q03fun(S, K, r, sigma, T):
    d1 = (np.log(S/K) + (r + sigma**2 / 2) * T)/(sigma * np.sqrt(T))
    d2 = d1 - sigma* np.sqrt(T)
    return S * ss.norm.cdf(d1) - K * np.exp(-r * T) * ss.norm.cdf(d2)
#(2)
V=13.7
S=100
K=95
r=0.1
T=0.25
Q03sigma=so.fsolve(lambda x: Q03fun(S, K, r, x, T)-V, 0.5)[0]
```

Question 4

```
#(0)
import numpy as np
import scipy.optimize as so
fun=lambda x: 3*x[0]**2+4*x[0]*x[1]+x[1]**2+x[0]+6*x[1]-2
cons=({'type': 'ineq', 'fun': lambda x: 2*x[0]+3*x[1]-4},)
bnds=((0, None), (0, None))
res=so.minimize(fun, (1, 1), bounds=bnds, constraints=cons)
Q04min=res.fun
Q04x, Q04y=res.x
```

Question 5

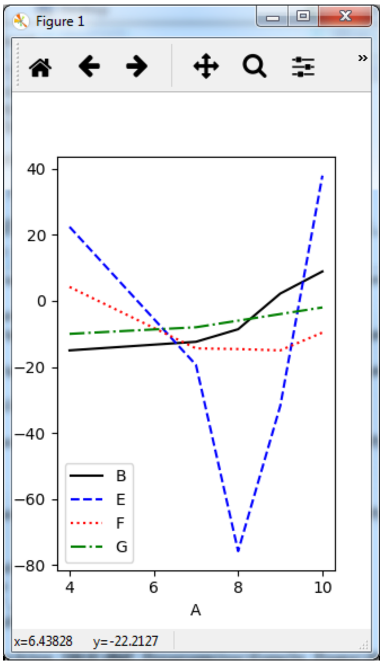
```
#(0)
import numpy as np
def myLSMput(K, r, dt, S):
    M,N=S.shape
    #(1)
    x=(np.ones(M)*N).astype(int)
    #(2)
    IV=np.maximum(K-S,0)
    #(3)
    for j in range(N-1, 2-1, -1):
        #(3.1)
        idx=np.arange(1,M+1)[IV[:,j-1]>0]
        #(3.2)
        X=S[idx-1, j-1]
        Y=IV[idx-1,x[idx-1]-1]*np.exp(-r*(x[idx-1]-j)*dt)
        #(3.3)
        P=np.polyfit(X,Y,2)
        #(3.4)
        for i in idx:
            if IV[i-1,j-1]>np.polyval(P,S[i-1,j-1]):
                x[i-1]=j
    #(4)
    V=np.mean(IV[range(M),x-1]*np.exp(-r*(x-1)*dt))
    return V
```

Question 6

```
#(0)
import numpy as np
def my_MCScall_stat(S, K, r, T, sigma, N):
    #(1)
    d=np.random.standard_normal(N//2)
    #(2)
    p=np.concatenate((d, -d))
    #(3)
    mp, sp=p.mean(), p.std()
    #(4)
    p=(p-mp)/sp
    #(5)
    Y = S * np.exp((r - 0.5 * sigma ** 2) * T + sigma * np.sqrt(T) * p)
    #(6)
    h = np.maximum(Y - K, 0)
    #(7)
    V = np.exp(-r * T) * np.sum(h) / N
    return V
```

Question 7

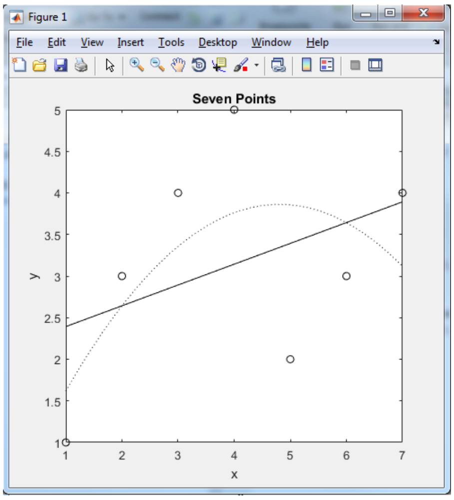
```
#(0)
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
#(1)
df1=pd.read_csv('Q7.csv', index_col=0)
#(2)
print("%d rows, %d columns" % df1.shape)
#(3)
df1.drop(df1.index[df1.C>df1.D], inplace=True)
#(4)
df1['G']=range(-5,-5+df1.shape[0])
#(5)
df1['D']=df1[['D']].rolling(3).mean()
#(6)
df1.iloc[::2,:].eval('E=C-F*B', inplace=True)
#(7)
df1.loc[df1.index[1::2],'E']=df1.apply(
    lambda row: row['C']+row['F']-row['B'],
    axis=1)
#(8)
df1.where(df1>0, 2*df1, inplace=True)
#(9)
df1[['B','E','F','G']].plot(
    style=['k-','b--','r:','g-.'])
plt.show()
#(10)
df1.to_csv('Q7result.csv')
```



Questions 1-4, MATLAB Sample Solution:

Question 1

```
%(1)
t=readtable('E001.csv');
%(2)
plot(t.x,t.y,'o');
%(3)
hold on;
p01=polyfit(t.x,t.y,1);
x0=sort(t.x);
plot(x0, polyval(p01,x0));
%(4)
p02=polyfit(t.x,t.y,2);
x1=linspace(min(t.x), max(t.x), 30);
plot(x1, polyval(p02,x1));
%(5)
xlabel('x');
ylabel('y');
title('Seven Points');
```



Question 2

```
%%(1)
rng(123456);
%%(2)
x=[randn(4,4) rand(4,4)];
%%(3)
x(2,:)=x(3,:).^2+2*x(1,:);
%%(4)
s=sort(sum(x, 1), 'descend');
%%(5)
Q02=s(end-1);
```

Question 3

```
%(1) Q03fun.m
function V=Q03fun(S,K,r,T,sigma)
   d1=(log(S/K)+(r+sigma^2/2)*T)/(sigma*sqrt(T));
   d2=d1-sigma*sqrt(T);
   V=S*normcdf(d1)-K*exp(-r*T)*normcdf(d2);
end
```

```
%(2)
V=13.7;
S=100;
K=95;
r=0.1;
T=0.25;
f=@(x) (Q03fun(S,K,r,T,x)-V)^2;
Q03sigma=fsolve(f,0.8);
```

Question 4

```
H=[6 4;4 2];
f=[1; 6];
Aeq=[];
beq=[];
A=[-2 -3];
b=[-4];
lb=[0 0];
ub=[];
[w, fval]=quadprog(H, f, A, b, Aeq, beq, lb, ub);
Q04.min = fval-2;
Q04.x=w(1);
Q04.y=w(2);
```