4. Consider an economy with $k = 2$ states of nature, a "good" state and a "bad" state.[1] There are two assets, a risk-free asset with $R_f = 1.05$ and a second risky asset that pays cashflows

$$X_2 = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

The current price of the risky asset is 6.

a. Solve for the prices of the elementary securities p_1 and p_2 and the risk-neutral probabilities of the two states.

Answer: Let

$$P = \begin{bmatrix} 1/1.05 \\ 6 \end{bmatrix}$$

and

$$X = \begin{bmatrix} 1 & 10 \\ 1 & 5 \end{bmatrix}$$

Then

$$[p_1 \quad p_2] = P'X^{-1} = \begin{bmatrix} \frac{1}{1.05} & 6 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 0.2 & -0.2 \end{bmatrix} = [0.2476 \quad 0.7048]$$

Hence, the risk-neutral probabilities are $\hat{\pi}_1 \equiv p_1 R_f = 0.26$ and $\hat{\pi}_2 \equiv p_2 R_f = 0.74$.

b. Suppose that the physical probabilities of the two states are $\pi_1 = \pi_2 = 0.5$. What is the stochastic discount factor for the two states?

Answer: $m_1 = p_1/\pi_1 = 0.495$. $m_2 = p_2/\pi_2 = 1.410$.