QF603 Group Mini-Project 2

Group F

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Abstract

In this report, we implemented a simple linear regression of the Dow Jones Industrial Average ("DJIA") index over the S&P500 ("S&P") using daily and annual log returns. Our key finding is that the return characteristics of the DJIA and S&P are not different from each other at the 5% level of significance.

Task 3: Regression of Daily Log Returns

Estimate of key statistics $\hat{a}, \hat{b}$ and $\hat{\sigma}_{u_t}$

Results

- Alpha = $\hat{a} = -0.000046$, Beta = $\hat{b} = 0.943110$
- Standard Distribution of Residual = $\sigma_{u_t} = 0.002877$

The regression can be expressed as $r_{(DJIA,t)} = -0.000046 + 0.943110 r_{(S\&P500,t)}$.

The positive intercept indicates that the DJIA has a small positive daily excess returns on average as compared to the S&P.

The slope of 0.9431 indicates that the DJIA is slightly less volatile than the S&P. In this context, the slope is interpreted as Beta – the sensitivity of DJIA's returns to the S&P's returns. Assuming investors are risk-averse, a lower Beta is preferred for the same level of return because a lower Beta asset will have more consistency in its returns.

Combining these two measures together, it suggests that the DJIA has a superior risk-adjusted return as compared to the S&P.

T-test for Null Hypothesis a=b=0 at 5% significance

Results

- T-statistics for $\hat{a} = 1.486364$, and for $\hat{b} = 339.973655$
- Degree of Freedom = $8500 - 2 = 8498$
- Null Hypothesis (H_0) are that $a = b = 0$.
- Alternate hypothesis (H_1) are that $a \neq 0 \quad \text{and} \quad b \neq 0$
- Critical value at 5% significance level = ± 1.960243

The test statistic for $\hat{a}$ falls within the critical values, and thus we cannot reject the null hypothesis that $\hat{a} = 0$. The indication of DJIA having higher daily returns than the S&P is therefore not significant at the 5% level.

However, the test statistic for $\hat{b}$ falls outside the critical values, and thus we reject the null hypothesis that $\hat{b} = 0$ and conclude that there is a linear relationship between DJIA and S&P at the 5% significance level.

Goodness of Fit: R^2 and Adjusted R^2 values

Results

- R-Squared (R^2) = 0.931512
- Adjusted R-Squared (adj-R^2) = 0.931504

The R^2 value reports the degree to which our independent variable (the SP500 daily log returns) explains the variation of the dependent variable (the DJIA daily log returns). Since R-square is 0.931512, it means that over 90% of the variation in the dependent variable is explained by the independent variable.

Adj-R^2 also measures the goodness of model fitting, but takes into consideration the number of independent variables in the model. R^2 will only increase or stay the same when we add more independent variables, even if they do not have any relationship with the dependent variable. Adj-R^2 on the other hand, will "penalize" the model for having excessive dependent variables that do not significantly improve the model.

The adj-R^2 is always lower than the R^2 value. And, in our case, because there is only 1 independent variable, the adj-R^2 and the R^2 values are relatively equal.

Jarque-Bera test statistic for the residuals

Results

- Jarque-Bera test stats = 25434.27
- Degrees of Freedom = lalala
- Null Hypothesis $H_0 = 0$
- Alternate Hypothesis $H_1 \neq 0$
- Critical Chi-Square Value at ??% significance level = 5.99146

The JB test statistic exceeds the critical value by a huge margin, strongly indicating that the residuals are not normally distributed. This is due to regression outliers that were a result of extreme market conditions, for example the huge one-day drop on 19-Oct-1987.

Task 4: Regression of Yearly Log Returns

Estimate of key statistics $\hat{a}$,$\hat{b}$ and $\hat{\sigma_u}$

Results

- Alpha = $\hat{a} = 0.019784$, Beta = $\hat{b} = 0.842545$
- Standard Distribution of Residual = $\sigma_{u_t} = 0.037969$

The regression can be expressed as $r_{(DJIA,t)} = 0.019784 + 0.842545 r_{(S\&P500,t)}$.

The positive intercept indicates that the DJIA has a small positive daily excess returns on average as compared to the S&P.

The slope of 0.8425 indicates that the DJIA is slightly less volatile than the S&P.

Combining these two measures together, it suggests that the DJIA has a superior risk-adjusted return as compared to the S&P.

T-test for Null Hypothesis a=b=0 at 5% significance

Results

- T-statistics for $\hat{a} = 2.649890$, and for $\hat{b} = 20.436300$
- Degree of Freedom = $32 - 2 = 30$
- Null Hypothesis (H_0) are that $a = b = 0$.

- Alternate hypothesis (H_1) are that $a \neq 0 \quad$ and $\quad b \neq 0$
- Critical value at 5% significance level = ± 2.042272

The test statistic for $\hat{a}$ falls outside the critical values, and thus we reject the null hypothesis that $\hat{a}$=0. The t-test at 5% significance concludes that the DJIA has higher annual returns as compared to the S&P.

The test statistic for $\hat{b}$ also falls outside the critical values, and thus we reject the null hypothesis that $\hat{b}$=0 and conclude that there is a linear relationship between the annual returns of the DJIA and the S&P at the 5% significance level.

Goodness of Fit: R^2 and Adjusted R^2 values

Results

- R-Squared (R^2) = 0.932983
- Adjusted R-Squared (adj-R^2) = 0.930749

The R^2 value is very high, further supporting our previous point that there exists a linear relationship between the annual returns of the DJIA and the S&P.

Jarque-Bera test statistic for the residuals

Results

- Jarque-Bera test stats = 25434.27
- Degrees of Freedom = lalala
- Null Hypothesis $H_0 = 0$
- Alternate Hypothesis $H_1 \neq 0$
- Critical Chi-Square Value at ??% significance level = 5.99146

The JB test statistic falls within the critical value, indicating that the regression residuals are normally distributed.