1. Basic Math

1. Working knowledge of integration such as integration by parts, area of simple geometry such as triangle and rectangle
2. Solutions $x_\pm$ of quadratic equation $ax^2 + bx + c = 0$: $\quad x_\pm = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
3. Working knowledge of matrix multiplication, which is essentially inner product of a row vector and a column vector
4. Determinant of a 2× 2 matrix $\boldsymbol{M} := \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $\quad \det \boldsymbol{M} \equiv |\boldsymbol{M}| := ad - bc$
5. Inverse of a 2× 2 matrix $\boldsymbol{M}$: $\quad \boldsymbol{M}^{-1} = \dfrac{1}{\det \boldsymbol{M}} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

2. Probability and Statistics

1. Working knowledge of sample mean, unbiased sample variance, covariance, correlation, percentile by linear interpolation
2. Bayes theorem $\mathbb{P}\left(A \middle| B\right) = \dfrac{\mathbb{P}\left(B \middle| A\right)\mathbb{P}\left(A\right)}{\mathbb{P}\left(B\right)}$.
3. Probabilities add up to one.
4. Working knowledge to apply $\mathbb{E}(\ )$, $\mathbb{V}(\ )$, and $\mathbb{C}(\ )$ on both sides of the equation.

3. OLS

1. Slope estimate of simple linear regression: $\quad \widehat{b} = \dfrac{\sum_{i=1}^{n}\left(X_i - \overline{X}\right)\left(Y_i - \overline{Y}\right)}{\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2}$
2. y intercept of simple linear regression: $\quad \widehat{a} = \overline{y} - \widehat{b}\,\overline{x}$
3. Normal distribution

$$\begin{pmatrix} \widehat{a} \\ \widehat{b} \end{pmatrix} \overset{d}{\sim} N\left(\begin{pmatrix} a \\ b \end{pmatrix}, \begin{pmatrix} \sigma_e^2\left(\frac{1}{n} + \frac{\overline{X}^2}{\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2}\right) & -\sigma_e^2\left(\frac{\overline{X}}{\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2}\right) \\ -\sigma_e^2\left(\frac{\overline{X}}{\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2}\right) & \sigma_e^2\left(\frac{1}{\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2}\right) \end{pmatrix}\right)$$

4. Working knowledge of unbiased variance of the residuals and t statistic
   - When the sample size is more than 30, t statistic is close to the standard normal distribution.
   - The critical value at 2-sided 95% confidence level is 1.96.
   - For simple OLS, square of the t statistic is the F statistic.

5. Multiple linear regression estimates in general: $\hat{\boldsymbol{\beta}} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{y}$.

6. Let $\boldsymbol{X} = \begin{pmatrix} \boldsymbol{\iota} & \boldsymbol{x} \end{pmatrix}$ and $\boldsymbol{\beta} = \begin{pmatrix} \alpha & \beta \end{pmatrix}'$.

$$\boldsymbol{X}'\boldsymbol{X} = \begin{pmatrix} \boldsymbol{\iota}' \\ \boldsymbol{x}' \end{pmatrix} \begin{pmatrix} \boldsymbol{\iota} & \boldsymbol{x} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\iota}'\boldsymbol{\iota} & \boldsymbol{\iota}'\boldsymbol{x} \\ \boldsymbol{x}'\boldsymbol{\iota} & \boldsymbol{x}'\boldsymbol{x} \end{pmatrix} = \begin{pmatrix} n & \sum x_t \\ \sum x_t & \sum x_t^2 \end{pmatrix}$$

$$\left(\boldsymbol{X}'\boldsymbol{X}\right)^{-1} = \frac{1}{n\sum x_t^2 - (\sum x_t)^2} \begin{pmatrix} \sum x_t^2 & -\sum x_t \\ -\sum x_t & n \end{pmatrix}$$

$$\boldsymbol{X}'\boldsymbol{y} = \begin{pmatrix} \boldsymbol{\iota}'\boldsymbol{y} \\ \boldsymbol{x}'\boldsymbol{y} \end{pmatrix} = \begin{pmatrix} \sum y_t \\ \sum x_t y_t \end{pmatrix}$$

7. Therefore $(\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{y}$ gives $\hat{\boldsymbol{\beta}} = \begin{bmatrix} \hat{\alpha} & \hat{\beta} \end{bmatrix}'$:

$$\hat{\alpha} = \frac{\sum x_t^2 \sum y_t - \sum x_t \sum x_t y_t}{n\sum x_t^2 - (\sum x_t)^2} \qquad \hat{\beta} = \frac{n\sum x_t y_t - \sum x_t \sum y_t}{n\sum x_t^2 - (\sum x_t)^2}$$

8. The affine form is $y = a + bx + e$. The mean square error is the variance of the error e, i.e.,

$$\sigma_e{}^2 := \mathbb{V}\left[e\right] = \text{MSE}\,.$$

9. OLS Algorithm for Model M

   a) Estimate the model by OLS: $\hat{\boldsymbol{\beta}} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{y}$
   b) Compute the fitted values of $\boldsymbol{y}$: $\widehat{\boldsymbol{y}}$ $\widehat{\boldsymbol{y}} = \boldsymbol{X}\widehat{\boldsymbol{\beta}}$
   c) Compute the residuals or "surprise": $\widehat{\boldsymbol{u}} = \boldsymbol{y} - \widehat{\boldsymbol{y}}$
   d) Compute the residual sum of squares (RSS)

$$\text{SSE} \equiv \text{RSS} = \widehat{\boldsymbol{u}}'\widehat{\boldsymbol{u}} = \sum_{i=1}^{n} \widehat{u}_i{}^2$$

   e) The variance of the residuals is

$$\widehat{\sigma}_u^2 = \frac{1}{n-K}\widehat{\boldsymbol{u}}'\widehat{\boldsymbol{u}}$$

   f) Let $\boldsymbol{\Omega} := \left(\boldsymbol{X}'\boldsymbol{X}\right)^{-1}$. The variance of $\widehat{\beta}_i$ is

$$\mathbb{V}(\widehat{\beta}_i) = \widehat{\sigma}_u^2 \Omega_{ii}.$$

10. Statistical Inference

- For all $j = 1, 2, \ldots, K$, the t test statistic for $\widehat{\beta}_j$ is, given the null-hypothesized value β_j:

$$\frac{\widehat{\beta}_j - \beta_j}{\widehat{\sigma}_u\sqrt{\Omega_{jj}}} \sim t_{n-K}$$

  Here, $\boldsymbol{\Omega} := \left(\boldsymbol{X}'\boldsymbol{X}\right)^{-1}$, and Ω_{jj} is the j-th diagonal element.

- The α% significance level for β_j is, assuming two-tail test,

$$\widehat{\beta}_j - q\widehat{\sigma}_u\sqrt{\Omega_{jj}} \le \beta_j \le \widehat{\beta}_j + q\widehat{\sigma}_u\sqrt{\Omega_{jj}},$$

  where q is the $(1-\alpha/2)$-th quantile of the t_{n-K} distribution.

4. Asset Pricing

1. Sharpe's ratio, M^2, information ratio, Treynor's measuere, and Jensen's alpha
2. SML, CML

5. Introductory Time Series and GARCH

1. Stationarity determination by characteristic equation
2. Working knowledge of VaR
3. $\omega = \gamma V_L$, where V_L is the long run variance
4. ARCH model for a process on the variance of u_t

   $$\mathbb{V}(u_t) = \alpha_0 + \alpha_1 u_{t-1}^2$$

   More generally, with $e_t \overset{d}{\sim} N(0,1)$,

   $$u_t = e_t \sqrt{\alpha_0 + \alpha_1 u_{t-1}^2}$$

5. GARCH(1,1)
6. Long-term variance in GARCH(1,1)

   $$V_L = \frac{\omega}{\gamma} = \frac{\omega}{1 - \alpha - \beta}.$$