Session 4
$\mathbb{Q}$uantitative Analysis of Financial Markets
Capital Asset Pricing Model & Stock Picking

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Broad Lesson Plan

Learning Objectives

- Recall the familiar concept of CAPM and appreciate how it is further developed in the context of $\mathbb{Q}$F.
- Describe the market model and connect it with simple (univariate) OLS regression model.
- Gain a deep understanding of conditional mean and conditional variance in CAPM.
- Develop a working knowledge and a deeper understanding of how alpha, beta, systematic, and unsystematic risks are estimated.
- Describe and discuss security market line, capital market line, market risk premium, Sharpe's ratio, and other risk-adjusted performance measures.
- Obtain a familiarization with the notion of market timing.

Risk–Return Duality

- Expected return: $\mu = \mathbb{E}(r_t)$
- Expected variance $\sigma^2 = \mathbb{V}(r_t)$
- Volatility σ is a proxy for the potential of a risk.
- Risk factor $=$ Risk premium
- Risk adjusted excess return, e.g. **Sortino ratio**

$$\frac{\mathbb{E}\left(r_t - r_{ft}\right)}{\sigma_d},$$

where r_{ft} is the risk-free rate, and σ_d is asset's or portfolio's downside standard deviation.

Market Model

- Market portfolio's log return is r_{mt}.
- The market model assumes that any stock's log return r_{it} is **bivariate normally distributed** with r_{mt}. The covariance is denoted by σ_{im}.
- The conditional distribution of r_{it} is normal with conditional mean and conditional variance given by

$$
\begin{aligned}
\mathbb{E}\big(r_{it}\big|r_{mt}\big) &= \mathbb{E}\big(r_{it}\big) + \frac{\sigma_{im}}{\sigma_m^2}\Big(r_{mt} - \mathbb{E}\big(r_{mt}\big)\Big) \\
&= \left(\mathbb{E}\big(r_{it}\big) - \frac{\sigma_{im}}{\sigma_m^2}\mathbb{E}\big(r_{mt}\big)\right) + \frac{\sigma_{im}}{\sigma_m^2} r_{mt} \\
\mathbb{V}\big(r_{it}\big|r_{mt}\big) &= \sigma_i^2 - \frac{\sigma_{im}^2}{\sigma_m^2}
\end{aligned}
$$

OLS Approach to Market Model

- A linear regression model of r_{it} on r_{mt} is

$$r_{it} = a + b\, r_{mt} + e_{it}$$

where

$$\begin{aligned} a &= \mathbb{E}\big(r_{it}\big) - \frac{\sigma_{im}}{\sigma_m^2}\mathbb{E}\big(r_{mt}\big) \\ b &= \frac{\sigma_{im}}{\sigma_m^2} \end{aligned}$$

- Properties of residuals e_{it}:
  - Uncorrelated with r_{mt}: $\mathbb{E}\big(e_{it}\big|r_{mt}\big) = 0$
  - By the law of iterated expectations, $\mathbb{E}(e_{it}) = 0$.

Proof for Conditional Variance

- Unconditional variance of r_{it} has two parts

$$\sigma_i^2 = b^2\,\sigma_m^2 + \sigma_e^2$$

  - Systematic risk: $b\,\sigma_m$
  - Unsystematic risk, idiosyncratic, or diversifiable risk: σ_e

- Now, the variance of r_{it} given r_{mt} is simply

$$\mathbb{V}\big(r_{it}\big|r_{mt}\big) = \mathbb{V}\big(a + b\,r_{mt} + e_{it}\big|r_{mt}\big) = \sigma_e^2$$

because r_{mt} is known and thus zero variance.

- Hence

$$\mathbb{V}\big(r_{it}\big|r_{mt}\big) = \sigma_i^2 - \frac{\sigma_{im}^2}{\sigma_m^2}$$

Capital Asset Pricing Model

- So far, bivariate normal distribution is assumed.
- When economic equilibrium is added to the market model, then a is restricted to

$$a = r_f(1 - b),$$

where r_f is the riskfree rate.
- Imposing this theoretical CAPM restriction, the OLS regression of market model becomes

$$r_{it} = r_{ft} + b\big(r_{mt} - r_{ft}\big) + e_{it}.$$

- Capital Asset Pricing Model

$$\mathbb{E}\big(r_{it} - r_{ft}\big) = b_i\,\mathbb{E}\big(r_{mt} - r_{ft}\big).$$

- Regression specification

$$r_{it} - r_{ft} = a_i + b_i\big(r_{mt} - r_{ft}\big) + e_{it}.$$

Estimation of Alpha and Beta

- Time series of returns $\{r_{it}\,,\, r_{mt}\}_{t=1,2,\ldots,T}$
- OLS estimation of beta

$$\widehat{b}_i = \frac{\sum_{t=1}^{T} \big(r_{mt} - \overline{r}_m\big)\big(r_{it} - \overline{r}_i\big)}{\sum_{t=1}^{T} \big(r_{mt} - \overline{r}_m\big)^2}$$

- Theoretically alpha a_i is zero in equilibrium. But
  - if $\widehat{a}_i > 0$, then positive abnormal return
  - if $\widehat{a}_i < 0$, then negative abnormal return
- Alpha is also called the **Jensen measure** in the context of portfolio theory.
- Practical issues
  - What is the ideal sampling frequency?
  - What is the ideal sampling size?

Example

- Linear regression model: $r_{it} - r_{ft} = a_i + b_i \left(r_{mt} - r_{ft} \right) + e_{it}$

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.003119	0.002653	1.175459	0.2426
MKT EXC RET	0.624225	0.080491	7.755199	0.0000
R-squared	0.375559	Mean dependent var.		0.000904
Adjusted R-squared	0.369314	S.D. dependent var.		0.033544
S.E. of regression	0.026639	Akaike info criterion		-4.393447
Sum squared resid	0.070965	Schwarz criterion		-4.341977
Log likelihood	226.0658	F-statistic		60.14311
Durbin-Watson stat	2.513910	Prob(F-statistic)		0.000000

Terms Used in the Example

- The standard error of $\widehat{a}_i$ is “Std Error of C”:

$$\widehat{\sigma}_e \sqrt{\frac{1}{T} + \frac{\overline{X}^2}{\sum_{t=1}^{T}(X_t - \overline{X})^2}}$$

- The standard error of $\widehat{b}_i$ is “Std Error of Coefficient of MKT EXC RET” is

$$\widehat{\sigma}_e \sqrt{\frac{1}{\sum_{t=1}^{T}(X_t - \overline{X})^2}}$$

- The SSR, “Sum squared resid" is SSR = $\displaystyle\sum_{t=1}^{T} \widehat{e}_t^2$ (aka RSS)

- The standard error of e_t, “S.E. of regression” is $\sigma_e = \sqrt{\dfrac{1}{T-2}\mathrm{SSR}}$.

Security Market Line

- A line in a graph of expected return versus beta:

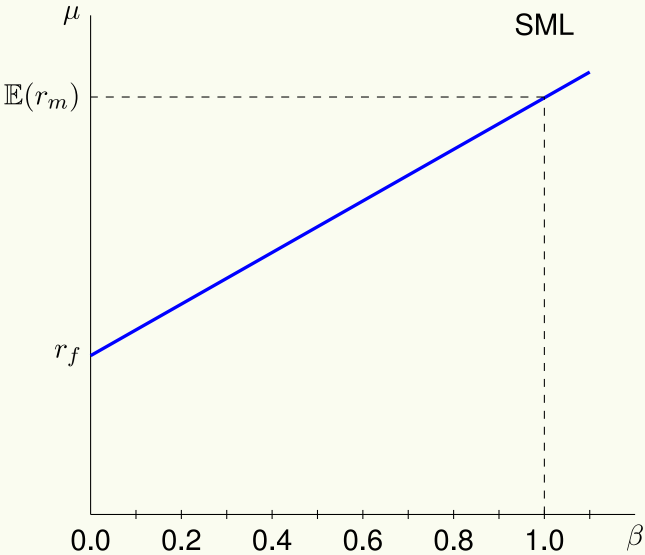
$$\mu := \mathbb{E}(r_j) = r_f + \beta_j\big(\mathbb{E}(r_m) - r_f\big)$$

or

$$\mu - r_f = \frac{\mathbb{C}(r_j\,,\, r_m)}{\mathbb{V}(r_m)}\big(\mathbb{E}(r_m) - r_f\big)$$

- The market portfolio has a beta of 1.

Illustration of SML



Capital Market Line

- CML is the line containing all possible portfolios of investors. Each portfolio is a linear combination of 2 assets — the market portfolio and the risk-free asset.

- Any portfolio with return r_p and volatility σ_p has the same **Sharpe ratio** as the market portfolio has

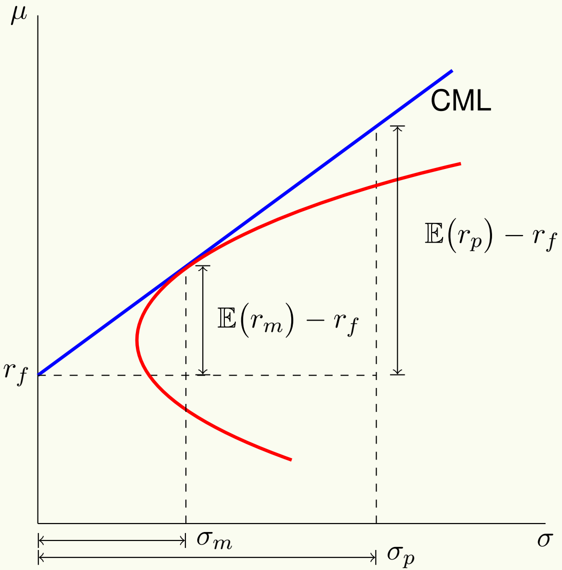
$$\frac{\mathbb{E}\big(r_p - r_f\big)}{\sigma_p} = \frac{\mathbb{E}\big(r_m - r_f\big)}{\sigma_m} =: \lambda \quad \Longrightarrow \quad \mathbb{E}\big(r_m - r_f\big) = \lambda \sigma_m .$$

- Estimation of market risk premium λ:

$$r_{mt} - r_{ft} = \lambda \sigma_{mt} + u_t$$

- Merton's proposal: $\mathbb{E}\big(r_{mt} - r_{ft}\big) = \lambda \sigma_m^2$, where λ is interpreted as the **relative risk aversion**.

Illustration of CML



Treynor and Jensen Measures

- Treynor measure: expected excess portfolio return r_{pt} per unit of portfolio beta b_p

$$\frac{\mathbb{E}(r_{pt} - r_{ft})}{b_p}$$

- If Jensen measure indicates superior performance, so does Treynor measure:

$$a = \mathbb{E}(r_{pt} - r_{ft}) - b_p \mathbb{E}(r_{mt} - r_{ft}) > 0 \quad \Longleftrightarrow \quad \frac{\mathbb{E}(r_{pt} - r_{ft})}{b_p} > \mathbb{E}(r_{mt} - r_{ft})$$

Sharpe's Ratio and M^2

- Sharpe's ratio

$$\frac{\mathbb{E}\big(r_{pt} - r_{ft}\big)}{\sigma_p}$$

shows how well the portfolio is performing relative to CML with slope $\dfrac{\mathbb{E}(r_{mt} - r_{ft})}{\sigma_m}$

$$\frac{\mathbb{E}\big(r_{pt} - r_{ft}\big)}{\sigma_p} > \frac{\mathbb{E}\big(r_{mt} - r_{ft}\big)}{\sigma_m} \quad \Longleftrightarrow \quad a > 0$$

- M^2 measure in percent

$$M^2 := \mathbb{E}\big(r_{pt} - r_{ft}\big)\frac{\sigma_m}{\sigma_p} - \mathbb{E}\big(r_{mt} - r_{ft}\big).$$

Information Ratio

- A generalized version of the Sharpe ratio with a generic benchmark r_{bt}

- Definition

$$\frac{\mathbb{E}\left(r_{pt} - r_{bt}\right)}{\sigma_{p-b}},$$

  where σ_{p-b} is the standard deviation of the difference in returns between the portfolio and its benchmark.

- The portfolio's excess return $r_{pt} - r_{bt}$ is also known as its **active return**.

- The variability σ_{p-b} of the excess return is also referred to as **active risk**, tracking risk, or **tracking error**.

Application: Stock Picking

- Ideal stock to hold:
  - positive alpha
  - large beta during bull market
  - small beta during bear market
  - large Sharpe ratio
- Ideal stock to short is the reverse.
- Long the “good” stocks, short the “bad” stocks. Will this quant strategy work?
- So exactly how could one search for those good and bad stocks?

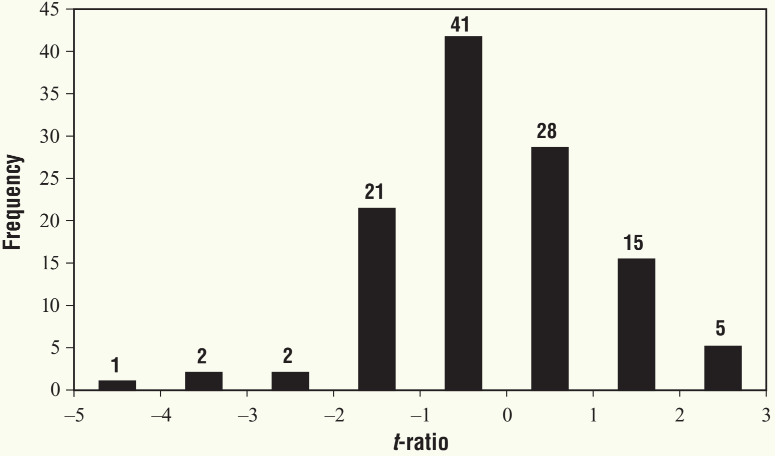
Market Timing

- Shift investment funds into the market portfolio when market is rising, and to shift out of the stock market into money market when market is falling, especially if the market falls below risk-free return.
- The goal is to avoid being invested in e.g. mutual funds during a market decline.
- Typically, trend-following indicators are used to determine the direction and identify buy and sell signals.
- In an up move "buy signal," money is transferred from a money market fund into a mutual fund in an attempt to capture a capital gain.
- In a down move "sell signal," the assets in the mutual fund are sold and moved back into the money market for safe keeping until the next up move.

Jensen's Empirical Tests

- Testing for the presence and significance of abnormal returns ("Jensen's alpha" - Jensen, 1968).
- Data: Annual Returns on the portfolios of 115 mutual funds from 1945–1964.
- The model: $R_{jt} - R_{ft} = \alpha_j + \beta_j(R_{mt} - R_{ft}) + u_{jt}$ for $j = 1, 2, \ldots, 115$.
- Are α_j significant?
- The null hypothesis is H_0: α_j=0 .

Distribution of Mutual Fund Alphas' t-Ratios



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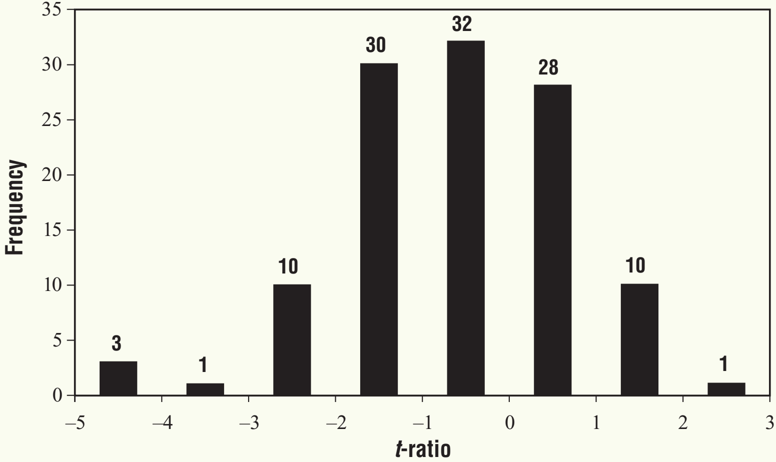
Net of Transactions Costs



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Takeaways

- The flip side of risk is risk premium, e.g., $\mathbb{E}\left(r_{pt} - r_{ft}\right)$ for market risk, which is systematic
- Market model is based on simple linear regression of log returns.
- Unsystematic risk can be diversified away by the portfolio approach.
- Security market line is a pictorial description of CAPM.
- Capital market line allows you to perform allocation of fund between risk-free and risky assets
- Treynor's ratio, Jensen's alpha, Sharpe's ratio, M^2 measure, information ratio
- Most active fund managers could not beat the market.