

QF620 Stochastic Modelling in Finance
Assignment 2/4
Due Date: 5-Nov-2018

1. Assume that a stock price S_t follows the Black-Scholes lognormal process:

$$dS_t = rS_t dt + \sigma S_t dW_t.$$

- (a) Use Itô's formula to derive the stochastic differential equation for the process S_t^2 .
- (b) Solve the stochastic differential equations of S_t and S_t^2 .
- (c) Use your solutions to verify the following expectations:

$$\begin{aligned}\mathbb{E}[S_t] &= Se^{rt} \\ \mathbb{E}[S_t^2] &= S^2 e^{(2r+\sigma^2)t},\end{aligned}$$

2. If we write the following stochastic differential equation for a stock price process S_t

$$dS_t = rS_t dt + \sigma S_t dW_t,$$

what is the numeraire of this world? Verify your answer by deriving the stochastic differential equation for the process of the stock price divided by this numeraire.

3. Show that the solution to the inverted stock price process

$$d\frac{1}{S_t} = -r\frac{1}{S_t}dt - \sigma\frac{1}{S_t}dW_t^S$$

is given by

$$\frac{1}{S_t} = \frac{1}{S_0} \exp \left[\left(-r - \frac{1}{2}\sigma^2 \right) t - \sigma W_t^S \right].$$

Given this solution, proceed to evaluate the expectation

$$V_0 = S_0 \mathbb{E}^S \left[\left(1 - K \frac{1}{S_T} \right)^+ \right].$$