pg 9.

$$S_{t_i} = S_{t_{i-1}} e^{\left(r - \frac{\sigma_i^2}{2}\right)(t_i - t_{i-1}) + \sigma_i\left(W_{t_i} - W_{t_{i-1}}\right)}$$

$$\log S_{t_i} - \log S_{t_{i-1}} = \left(r - \frac{\sigma_i^2}{2}\right)\left(t_i - t_{i-1}\right) + \sigma_i\left(W_{t_i} - W_{t_{i-1}}\right)$$

$$\log \frac{S_{t_i}}{S_{t_{i-1}}} = \left(r - \frac{\sigma_i^2}{2}\right)\left(t_i - t_{i-1}\right) + \sigma_i\left(W_{t_i} - W_{t_{i-1}}\right)$$

$$\left(\log \frac{S_{t_i}}{S_{t_{i-1}}}\right)^2 \simeq \sigma_i^2\left(t_i - t_{i-1}\right) = \sigma_i^2 \, \Delta t$$

$$\sum_{i=1}^{N}\left(\log \frac{S_{t_i}}{S_{t_{i-1}}}\right)^2 = \sum_{i=1}^{N} \sigma_i^2 \, \Delta t = \int_0^T \sigma_t^2 \, dt$$

$$dS_t = rS_t\,dt + \sigma_t S_t\,dW_t^*$$ (Not Black-Scholes)

$$X_t = \log S_t = f(S_t)$$

$$dX_t = d\log S_t = \frac{1}{S_t}\,dS_t - \frac{1}{2}\frac{1}{S_t^2}(dS_t)^2$$

$$= \frac{dS_t}{S_t} - \frac{1}{2}\sigma_t^2\,dt$$

$$dS_t = r S_t \, dt + \sigma_t S_t \, dW_t$$

$$\frac{dS_t}{S_t} = r \, dt + \sigma_t \, dW_t$$

$$\int_0^T \frac{dS_t}{S_t} = rT + \int_0^T \sigma_t \, dW_t$$

$$\mathbb{E}\left[\int_0^T \frac{dS_t}{S_t} \right] = rT$$

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$$\mathbb{E}\left[\int_0^T \sigma_t^2 \, dt\right] = 2\,\mathbb{E}\left[\int_0^T \frac{dS_t}{S_t}\right] - 2\,\mathbb{E}\left[\log \frac{S_T}{S_0}\right]$$

$$= 2rT - 2\cdot\left[\log\frac{F}{S_0} - e^{rT}\int_0^F \frac{P(K)}{K^2}\,dK - e^{rT}\int_F^\infty \frac{C(K)}{K^2}\,dK\right]$$

$$= 2rT - 2\log\frac{S_0 e^{rT}}{S_0} + 2e^{rT}\int_0^F \frac{P(K)}{K^2}\,dK + 2e^{rT}\int_F^\infty \frac{C(K)}{K^2}\,dK$$

$$= 2rT - 2rT + \text{''} + \ldots$$

CBOE:

$$2e^{rT}\left[\sum_{K_i<F} \frac{P(K_i)}{K_i^2}\cdot \Delta K + \sum_{K_i>F} \frac{C(K_i)}{K_i^2}\cdot \Delta K\right]$$

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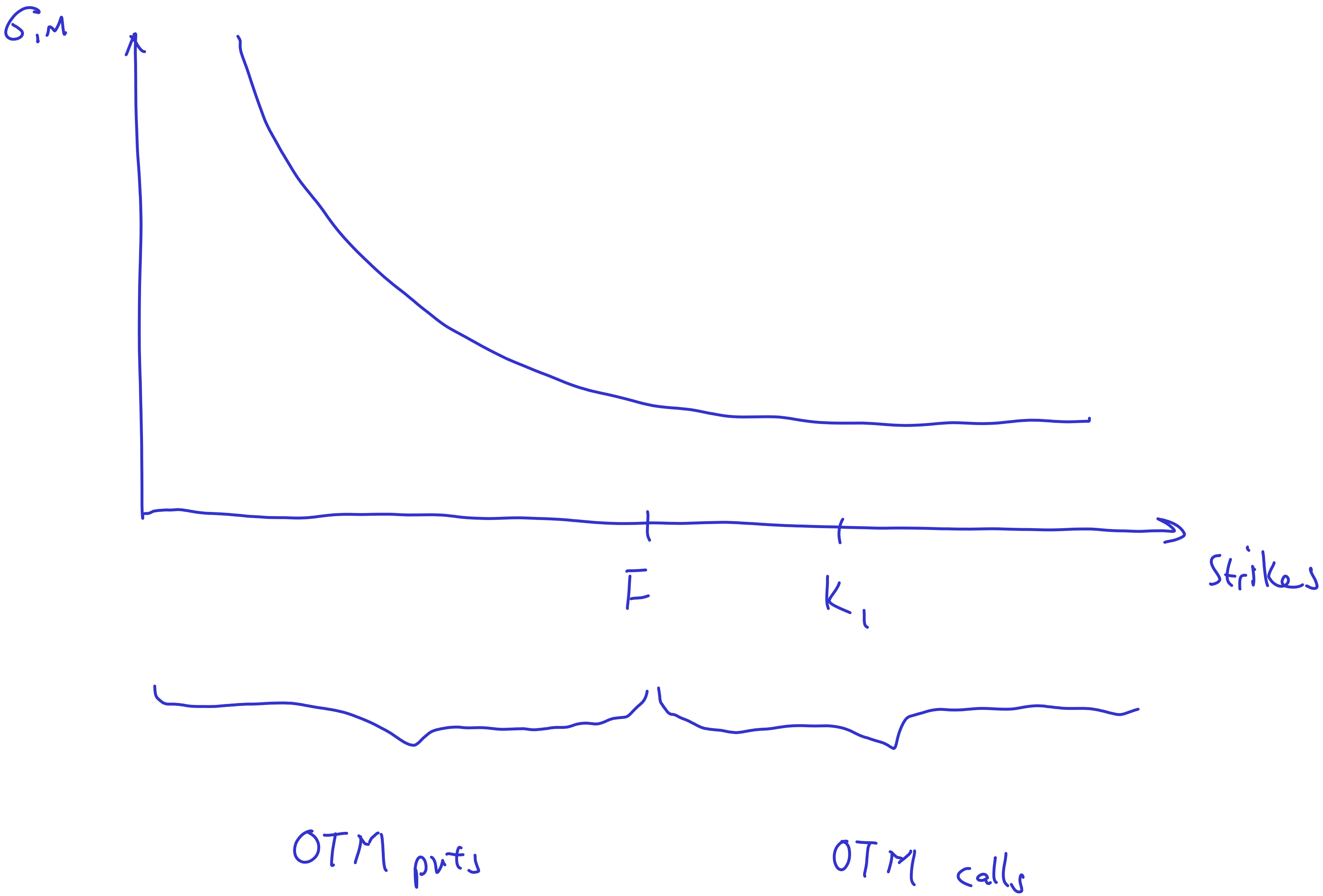

$$-\frac{\partial C(K_1)}{\partial K} \simeq -\frac{C(K_1) - C(K_1 - \Delta K)}{\Delta K}$$

finite difference

$$= \frac{C(K_1 - \Delta K) - C(K_1)}{\Delta K}$$

$$\sigma_{SABR}(F_0, K, \alpha, \beta, \rho, \nu, T)$$

$$Call = Black76(F_0, \sigma_{SABR}, K, T)$$

$$\Delta = \frac{\partial C}{\partial F_0} = \frac{\partial Black76(F_0, \sigma_{SABR}, K, T)}{\partial S_0}$$

$$+ \frac{\partial Black76(F_0, \sigma_{SABR}, K, T)}{\partial \sigma} \cdot \frac{\partial \sigma_{SABR}}{\partial F_0}$$

delta hedging: sell option, buy $\Delta \times$ stock to hedge

buy option, sell $\Delta \times$ stock to hedge.

Ex. sell call, buy $\Delta \times$ stock to hedge

sell put, buy $\Delta \times$ stock to hedge

Pg12.

$$d(X_t Y_t) = Y_t dX_t + X_t dY_t + dX_t dY_t$$

$$V_t = \phi_t S_t + \psi_t B_t$$

$$dV_t = d(\phi_t S_t) + d(\psi_t B_t)$$

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$$X_t = C_t - V_t = C_t - \phi_t S_t - \psi_t B_t$$

$$dX_t = dC_t - dV_t = dC_t - \phi_t dS_t - \psi_t dB_t$$

Itô formula:

$$dC_t = \frac{\partial C}{\partial t} dt + \frac{\partial C}{\partial S} dS_t + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} (dS_t)^2$$

$$= \frac{\partial C}{\partial t} dt + \frac{\partial C}{\partial S} dS_t + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S^2 dt .$$

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$$dS_t = rS_t\,dt + \sigma S_t\,dW_t^*$$, B_t is numeraire

$$dC_t = \frac{\partial C}{\partial t}dt + \frac{\partial C}{\partial S}dS_t + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}(dS_t)^2$$

$$= \frac{\partial C}{\partial t}dt + \frac{\partial C}{\partial S}\left(rS_t\,dt + \sigma S_t\,dW_t^*\right) + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S_t^2\,dt$$

$$= \left(\frac{\partial C}{\partial t} + rS_t\frac{\partial C}{\partial S} + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S_t^2\right)dt + \sigma S_t\frac{\partial C}{\partial S}dW_t^*$$

$$= rC_t\,dt + \sigma S_t\frac{\partial C}{\partial S}dW_t^*$$