Behavioral Finance

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Barberis, Huang and Santos (2001)

- Quasi-behavioral model with fully rational investors and non-standard preferences
- Preferences incorporate concepts from **prospect theory** of Kahneman and Tversky (1979):
  1. Gains and losses are measured relative to **reference level**
  2. Investors exhibit **loss aversion**: more sensitive to losses than gains (of same magnitude)
- Preferences can also incorporate **house money** effect: less risk averse after prior gains and more risk averse after prior losses
- Embed preferences into endowment economy, and solve numerically for equilibrium prices and expected returns

Economic Environment

- Riskless bond is in zero net supply and provides (constant) risk-free return of R_f over next time period
- Risky stock represents equity claim on perishable output, and provides (random) return of R_{t+1} over next time period
- In equilibrium, aggregate consumption and dividend growth rates both evolve as i.i.d. lognormal processes:

$$\ln\left(\frac{D_{t+1}}{D_t}\right) = \ln\left(\frac{\overline{C}_{t+1}}{\overline{C}_t}\right) = \mu + \sigma\epsilon_{t+1}$$

- Here $\epsilon_t \sim N(0,1)$ represent normal economic fluctuations

Investor Preferences

- Infinitely-lived investor maximises time-separable lifetime utility, from consumption and recent financial gains or losses:

$$E\left[\sum_{t=0}^{\infty}\left(\delta^t \frac{C_t^{1-\gamma}}{1-\gamma} + \delta^{t+1} b_t \nu(X_{t+1})\right)\right]$$

- Here $\delta = e^{-\rho} \in (0, 1)$ is subjective discount factor and $\gamma > 0$ is coefficient of relative risk aversion for consumption shocks
- Also X_t represents recent financial gains or losses, and $\nu(X_t)$ represents utility of recent financial gains or losses
- Then b_t is scale factor so that utility of consumption remains comparable in magnitude to utility of recent gains or losses

Prospect Theory – Part 1

- Let w_t be (dollar) value of investment in stock at time t, and assume that investor keeps track of year-to-year fluctuations
- Recent financial gain or loss is measured relative to reference level based on risk-free rate:

$$X_{t+1} = w_t \left(R_{t+1} - R_f\right)$$

- Loss aversion makes investor more sensitive to shortfall in financial gain (or outright financial loss), so $\lambda > 1$:

$$\nu(X_{t+1}) = \begin{cases} X_{t+1}, & X_{t+1} \geq 0 \\ \lambda X_{t+1}, & X_{t+1} < 0 \end{cases}$$

Prospect Theory – Part 2

- Since utility from recent gain or loss is (piecewise) linear, can define scale-invariant utility function for recent gain or loss:

$$\nu(X_{t+1}) = \nu(w_t\,(R_{t+1} - R_f)) = w_t\widehat{\nu}(R_{t+1})\,,$$
$$\widehat{\nu}(R_{t+1}) = \begin{cases} R_{t+1} - R_f\,, & R_{t+1} \geq R_f \\ \lambda\,(R_{t+1} - R_f)\,, & R_{t+1} < R_f \end{cases}$$

- Use marginal utility of aggregate consumption as scale factor:

$$b_t = b_0 \overline{C}_t^{-\gamma}$$

- Here $b_0 \geq 0$ is parameter that determines impact of recent gain or loss on investor's utility (relative to consumption)

Optimisation Problem

- Investor's consumption and asset allocation problem:

$$\max_{\{C_t, w_t\}} E\left[\sum_{t=0}^{\infty}\left(\delta^t \frac{C_t^{1-\gamma}}{1-\gamma} + \delta^{t+1} b_0 \overline{C}_t^{-\gamma} w_t \widehat{\nu}(R_{t+1})\right)\right]$$

- Subject to investor's intertemporal budget constraint:

$$W_{t+1} = (W_t - C_t)\, R_f + w_t\,(R_{t+1} - R_f)$$

- Assume complete market, so there exists representative investor who consumes (per capita) aggregate consumption

Optimal Choices

- First-order condition for optimal consumption:

$$\delta R_f E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right] = \delta R_f E_t\left[e^{-\gamma(\mu+\sigma\epsilon_{t+1})}\right] = 1$$

$$\Rightarrow \quad R_f = e^{\rho+\gamma\mu-\frac{1}{2}\gamma^2\sigma^2}$$

- First-order condition for optimal asset allocation:

$$\delta b_0 E_t[\widehat{\nu}(R_{t+1})] + \delta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} R_{t+1}\right] = 1$$

Price-Dividend Ratio

- Let $f_t = P_t / D_t$ be price-dividend ratio for stock:

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{1 + f_{t+1}}{f_t} \frac{D_{t+1}}{D_t}$$

- Assume that price-dividend ratio is constant: $f_t = f$, so that stock return is i.i.d. (since dividend growth is i.i.d.)
- Solve numerically for equilibrium price-dividend ratio:

$$\delta b_0 E_t \left[\widehat{\nu} \left(\frac{1 + f}{f} e^{\mu + \sigma \epsilon_{t+1}} \right) \right] + \frac{1 + f}{f} \delta E_t \left[e^{(1-\gamma)(\mu + \sigma \epsilon_{t+1})} \right] = 1$$

Empirical Results

- Set $\lambda = 2.25$, based on Tversky and Kahneman (1992)
- Set $\mu = 1.84\%$ and $\sigma = 3.79\%$ based on data from 1889–1995
- Set $\gamma = 0.9$ and $\delta = 0.98$, so $R_f \approx 3.5\%$ per year
- Equity premium should increase with b_0, as investor puts more emphasis on utility from recent financial gain or loss
- Annual equity premium is 0.06% for $b_0 = 0$ and 0.91% for $b_0 = 2$, and converges to 1.2% in limit as $b_0 \to \infty$
- Even for $b_0 \to \infty$, model-based equity premium is much smaller than historical equity premium for U.S. stock market

House Money Effect – Part 1

- Thaler and Johnson (1990) found evidence of "house money" effect, where prior outcomes affect risk aversion
- Loss (of given magnitude) is less painful after prior gains and more painful after prior losses
- Hence subjects are more willing to gamble after prior gains and less willing to gamble after prior losses
- Prior financial gains provide "buffer" or "cushion" that reduces sensitivity to subsequent financial losses
- Conversely, prior financial losses make investors even more sensitive to subsequent financial losses

House Money Effect – Part 2

- Barberis, Huang and Santos (2001) incorporate house money effect by assuming that investors keep track of **benchmark level** for value of investment in risky stock
- Investor's benchmark level evolves over time, based on expected return and realised return for risky stock
- Investor becomes more risk averse with prior losses (i.e., if value of investment in risky stock falls short of benchmark) and less risk averse with prior gains
- Price-dividend ratio of risky stock will no longer be constant, but will become function of investor's benchmark level

References

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