Portfolio Mean-Variance Analysis

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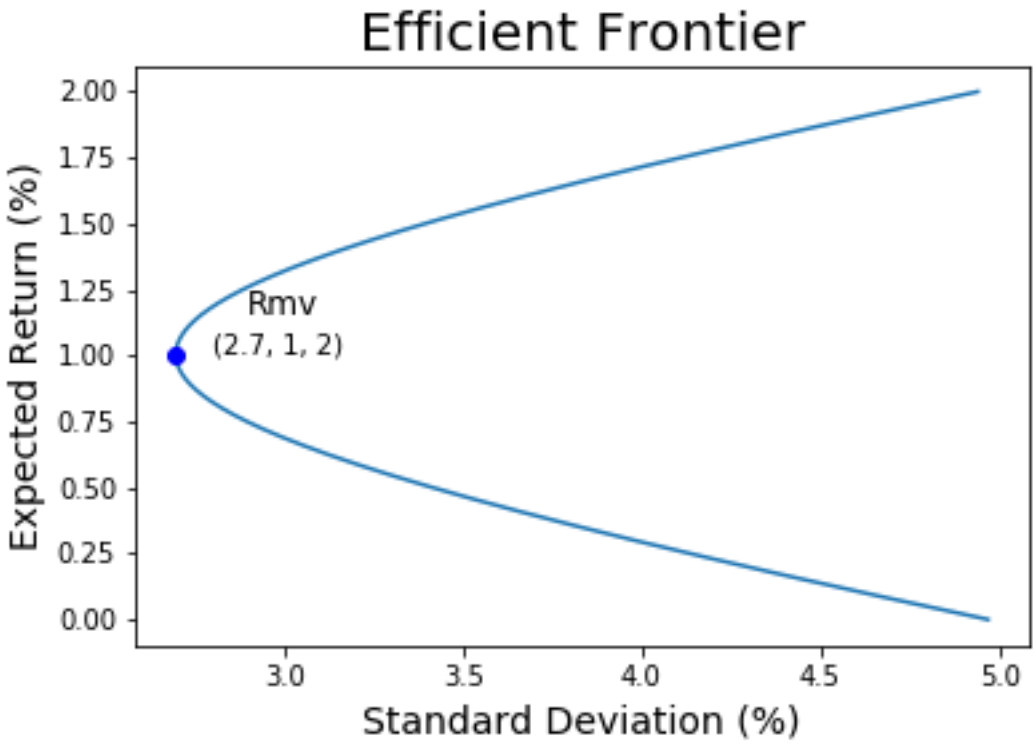
1. Table of the mean returns and standard deviation of returns for the ten industry portfolios

Industry	Expected Return	Standard Deviation
NoDur	0.902833	3.345657
Durbl	0.733333	8.361852
Manuf	1.012833	5.31027
Enrgy	1.231167	6.081524
HiTec	0.76625	5.381191
Telcm	0.881417	4.448284
Shops	0.916333	4.093786
Hlth	0.783833	3.787172
Utils	0.907167	3.701763
Other	0.489083	5.582452

2. Covariance Matrix

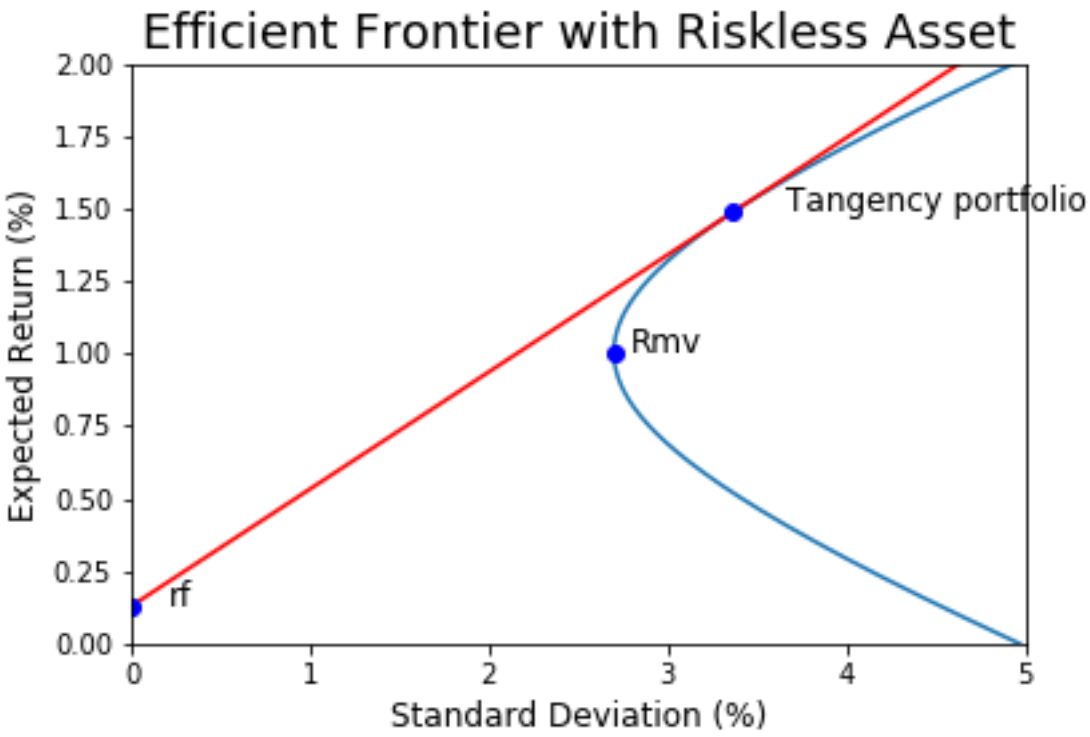
	NoDur	Durbl	Manuf	Enrgy	HiTec	Telcm	Shops	Hlth	Utils	Other
NoDur	11.193422	18.449666	14.104907	10.531341	12.922949	11.968078	10.170832	9.953112	7.866653	14.438409
Durbl	18.449666	69.920577	39.178097	27.019794	35.466652	27.490543	27.444731	16.824003	12.746136	39.361987
Manuf	14.104907	39.178097	28.19897	23.14538	24.618739	19.55015	17.622867	13.596447	11.440612	26.313423
Enrgy	10.531341	27.019794	23.14538	36.984933	19.267276	15.366817	11.2978	9.630327	14.027168	18.320469
HiTec	12.922949	35.466652	24.618739	19.267276	28.95722	18.708273	17.837115	13.254064	10.304187	23.85547
Telcm	11.968078	27.490543	19.55015	15.366817	18.708273	19.787227	14.169356	11.506599	10.991596	19.610836
Shops	10.170832	27.444731	17.622867	11.2978	17.837115	14.169356	16.759084	10.178849	6.69435	19.226524
Hlth	9.953112	16.824003	13.596447	9.630327	13.254064	11.506599	10.178849	14.342669	7.475036	14.864553
Utils	7.866653	12.746136	11.440612	14.027168	10.304187	10.991596	6.69435	7.475036	13.703052	9.99296
Other	14.438409	39.361987	26.313423	18.320469	23.85547	19.610836	19.226524	14.864553	9.99296	31.163771

3. Minimum-variance frontier and its economic significance



Efficient frontier is the set of the possible minimum variances for given target return. For an investor, investing in the tangent point where his/her utility curve intersects the upper-half of efficient frontier maximizes his/her utility. The more risk-tolerant an investor is, the higher point tangent point he/she will have. The lower-half of the curve can be ignored when making investment decisions, because for any given standard deviation, the upper-half of the curve will always yield higher return than the lower-half.

4. Efficient frontier with riskless asset, tangency portfolio and its economic significance



If riskless asset is included in the portfolio, the efficient frontier, a set of minimum variance point for the given return is linear. Y-intercept of the efficient frontier denotes the return from the risk-free asset. At y-intercept, 100% of the his/her wealth is invested in risk-free asset. Likewise in the previous chart, the more risk-tolerant an investor is, the higher point the investor is likely to pick in the curve. At the tangent point, the portfolio consists of only risky assets, and it is called tangency portfolio. Above tangent point, the proportion of risk-free assets goes negative, and this implies that the investor takes a short position on risk-free asset.

5. Weight of ten industry portfolio at tangency portfolio

Industry	Weight
NoDur	0.5679722
Durbl	-0.2140726
Manuf	0.7141051
Enrgy	0.1040872
HiTec	-0.3634382
Telcm	-0.0954633
Shops	0.9916468
Hlth	0.0755702
Utils	0.1326433
Other	-0.9130508
Sum	1

The weight of each industry in tangency portfolio is as above. As mentioned previously, tangency portfolio does not contain risk-free asset making the sum of ten industry weight one. By definition, correlation between tangency portfolio and risk-free portfolio is zero. That is, we can say that risk-free portfolio is the orthogonal portfolio of the tangency portfolio.