Task 1

1. Identify the 30 component stocks after the October 18, 2018 US trading session and record the end of-day prices. Verify that the DJIA index value is indeed equal to the sum of the 30 prices divided by 0.14748071991788. Report your finding.

The value of the DJIA index (“DJIA”) per our calculations is 25,379.45, and the reported value of the index is also 25,379.45 (source: Bloomberg). There is a miniscule difference of $1.183x10^5\%$ which can be considered a rounding error. Therefore, the closing price of the DJIA index reported by Bloomberg on the 18th of October 2018 is not different from the calculated value.

7. Compute and report the annualized average and volatility of log return in percent

Annualized log return of Dow Jones Index : 8.8647\%

Annualized volatility of Dow Jones Index : 17.4493\%

Annualized log return of S\&P Index : 8.1597\%

Annualized volatility of S\&P Index : 17.8570\%

We use the additive feature of log returns to compute the annualized log return of the Dow and S\&P 500 Index (“S\&P500”).

We recognize that the S\&P has a superior breadth and depth of coverage compared to the Dow. Therefore, we conclude firstly, that the annualized log return of US stock market is approximately equal to the S\&P’s (at 8.16%).

The annualized log returns of the Dow is about 0.7050\% higher than the S\&P’s. In contrast, the Dow’s volatility is 0.4077\% less than the S\&P’s. From this, we can draw 2 conclusions. Firstly, the Dow has a comparatively better Sharpe ratio, and therefore, has a better risk-reward profile. Secondly, the Dow is empirically less risky compared to the S\&P index. This in turn means that the Dow would have a lower Beta compared to overall US stock market.

8. Compute and report the sample skewedness and sample kurtosis

Skewedness of Dow Jones Index : -1.6916

Kurtosis of Dow Jones Index : 45.3947

Skewedness of S\&P 500 : -1.2832

Kurtosis of S\&P 500 : 31.4436

The Dow and S\&P indices both have negative skewedness, which means that the distribution of log returns is asymmetric and is left-skewed. There are several conclusions we can draw from this finding. Firstly, we know that a left-skewed distribution has a “drawn out” left tail – meaning, it has a longer and fatter left-tail. Also, we know that the mean of the distribution is smaller than the median, which itself is smaller than the mode. And, Finally, we know that the median is closer to the third quartile that to the first quartile.

Intuitively, we do not find that it is unusual to expect negative skewedness in index returns because we observe more extreme downward movement (i.e. larger negative returns) during periods of recession.

The Dow and S\&P have kurtosis values of 45.3947 and 31.4436 respectively. This is very high, considering that the kurtosis of the standard normal distribution is 3.0. Kurtosis is a measure of the sharpness of the peak of a frequency-distribution curve. The high kurtosis means that the Dow and S\&P index (and by extension from the S\&P 500, the US stock market) is generally *\textit{leptokurtic*}. This means that the tails of the log return frequency-distribution approaches 0 more slowly than a normal distribution, and therefore produces more outliers than the normal distribution.

The implication of this finding is that extreme “events” like stock price crash or booms, which produce abnormally large returns (both positive and negative), are happening more often than what is predicted by the normal distribution.

8.1 Additional text for histogram

As we can see from the frequency-distribution histogram overlaid with a normal distribution curve with the same mean and variance to the underlying data, the histogram for both S\&P and DJI has a sharper peak and fatter tail. This indicates that the distribution of log returns has a higher kurtosis compared to the normal distribution.

9. Report the JB and make an inference on the test of normality at the 5\% significance level.

The JB test’s null hypothesis is JB = 0. If the null hypothesis is not rejected, it indicates that the distribution that we are testing is normal distribution (under a certain level of significance).

The critical Chi-Square Value with 2 Degrees of Freedom and at 5\% significance level is 5.9915. The JB test statistic for both the Dow and S\&P are above that critical value. We therefore reject the null hypothesis for both the S\&P as well as the Dow. And, we conclude that based on the JB test at the 5\% significance level, the log return of these two indices are not normally distributed.

Task 2

1. Compute and report the correlation between the log returns of DJIA and S\&P500

The correlation of the daily log returns from 30th January 1985 to 17th October 2018 is 0.9651. As the correlation is very close to 1, we say that the daily log returns of the S\&P and Dow are highly correlated. We believe that the high correlation can be explained by the fact that Dow’s 30 component stocks are also included in the S\&P index.

When we analyse the correlation on a 252-day rolling basis, we noted that there are periods where the correlation drops sharply to approximately 0.8. This finding is logical because the Dow, unlike the S\&P, is sometimes affected by large price swings in one of its component stocks. The same price swing would not affect the S\&P as much because of the S\&P’s larger coverage.