QF605
Fixed Income Securities
Group Project

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Part I: Bootstrapping Swap Curves

1 OIS Discount Factors

With the provided OIS rates data, we proceeded to use the following methodology to bootstrap the OIS discount factor curve.

$$
\begin{aligned}
PV_{fix} &= PV_{float} \\
D(0,1y) * OIS_{1y} &= D(0,1y) * [(1+\frac{f_0}{360})^{360} - 1] \\
[D(0,1y) + D(0,2y)] * OIS_{2y} &= D(0,1y) * [(1+\frac{f_0}{360})^{360} - 1] + D(0,2y) * [(1+\frac{f_1}{360})^{360} - 1] \\
&\vdots \\
[D(0,1y) + \cdots + D(0,20y)] * OIS_{20y} &= D(0,1y) * [(1+\frac{f_0}{360})^{360} - 1] + \cdots + D(0,20y) * [(1+\frac{f_{19}}{360})^{360} - 1]
\end{aligned}
$$

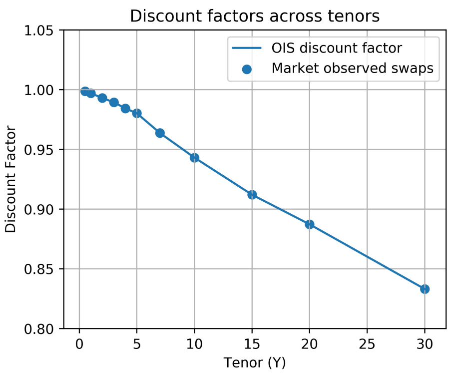
Due to only a handful of OIS swaps observable in the market, we can only then use the OIS swaps of varying tenor [6m,1y,2y,3y,5y,7y,10y,20y] to bootstrap the whole OIS discount curve while linearly interpolate for the rest of the "gap" discount factors. In order to solve for all discount factors, we will have to adopt the following:

$$
\begin{aligned}
PV_{fix} &= PV_{float} \\
[D(0,1y) + \cdots + D(0,7y)] * OIS_{7y} &= D(0,1y) * [(1+\frac{f_0}{360})^{360} - 1] + \cdots + D(0,7) * [(1+\frac{f_6}{360})^{360} - 1]
\end{aligned}
$$

* Assuming all prior discount factors have been bootstrapped, we can then subtitute the following into above equation to help isolate D(0,7y), and then derive D(0,7y):

$$
\begin{aligned}
f_6 &= 360 * [D(0,7y)^{-\frac{1}{360*7}} - 1] \\
D(0,6y) &= \frac{[D(0,7y) - D(0,5y)]}{2} * 1 + D(0,5y) \\
f_5 &= 360 * [[\frac{[D(0,7y) - D(0,5y)]}{2} * 1 + D(0,5y)]^{-\frac{1}{360*6}} - 1]
\end{aligned}
$$

Proceeding to do the similar for all OIS swaps, we derive the following OIS discount factors results and graph.



(a) OIS Discount Curve

Tenor	OIS rate	Overnight rate	OIS DF
0.5	0.00250	0.002497	0.998752
1.0	0.00300	0.002996	0.997009
2.0	0.00325	0.003495	0.993035
3.0	0.00335	0.003545	0.989422
4.0	0.00350	0.003946	0.984339
5.0	0.00360	0.003996	0.980216
7.0	0.00400	0.005281	0.963709
10.0	0.00450	0.005858	0.943106
15.0	0.00500	0.006135	0.912079
20.0	0.00525	0.005980	0.887281
30.0	0.00550	0.006085	0.833138

(b) OIS Discount Factors

Figure 1: OIS Results

2 LIBOR Discount Factors

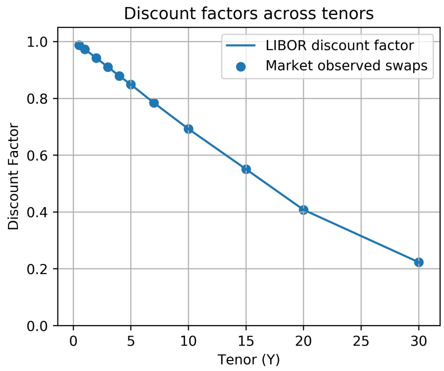
Similarly for LIBOR discount factors, we will adopt the same approach using the OIS discount factors to derive with the forward libor rates and LIBOR discount factors.

$$
\begin{aligned}
PV_{fix} &= PV_{float} \\
0.5 * [D_{OIS}(0,0.5y) + D_{OIS}(0,1y)] * IRS_{1y} &= 0.5 * [D_{OIS}(0,6m) * L(0,6m) + D_{OIS}(0,1y) * L(6m,1y)] \\
&\vdots \\
0.5 * [D_{OIS}(0,0.5y) + \cdots + D_{OIS}(0,20y)] * IRS_{20y} &= 0.5 * [D_{OIS}(0,0.5y) * L(0,6m) + \cdots + D_{OIS}(0,20y) * L(19.5y,20y)]
\end{aligned}
$$

Likewise, we will also substitute the following equations to solve for one unknown (ie. LIBOR discount factor) for each of the above equation starting from 0.5y...20y.In this example, we will use 7 years IRS to be consistent with our OIS approach

$$\begin{aligned}
D(0,5.5y) &= [\frac{[D(0,7y)-D(0,5y)]}{4}*1+D(0,5y)]*\frac{1}{\frac{1}{2}} \\
D(0,6y) &= [\frac{[D(0,7y)-D(0,5y)]}{4}*2+D(0,5y)]*\frac{1}{\frac{1}{2}} \\
D(0,6.5y) &= [\frac{[D(0,7y)-D(0,5y)]}{4}*3+D(0,5y)]*\frac{1}{\frac{1}{2}} \\
L(5y,5.5y) &= \frac{D(0,5y)-D(0,5.5y)}{D(0,5.5y)}*\frac{1}{\frac{1}{2}} \\
L(5.5y,6y) &= \frac{D(0,5.5y)-D(0,6y)}{D(0,6y)}*\frac{1}{\frac{1}{2}} \\
L(6y,6.5y) &= \frac{D(0,6y)-D(0,6.5y)}{D(0,6.5y)}*\frac{1}{\frac{1}{2}} \\
L(6.5y,7y) &= \frac{D(0,6.5y)-D(0,7y)}{D(0,7y)}*\frac{1}{\frac{1}{2}}
\end{aligned}$$

Proceeding to execute the same approach for all IRS , we derive the following LIBOR discount factors results and graph.



(a) LIBOR Discount Curve

Tenor	LIBOR IRS	Forward Libor	LIBOR DF
0.5	0.0250	0.025000	0.987654
1.0	0.0280	0.031005	0.972577
2.0	0.0300	0.032264	0.942178
3.0	0.0315	0.034815	0.910480
4.0	0.0325	0.035842	0.878976
5.0	0.0330	0.035329	0.848982
7.0	0.0350	0.041329	0.784164
10.0	0.0370	0.044084	0.692570
15.0	0.0400	0.051483	0.550788
20.0	0.0450	0.070136	0.407786
30.0	0.0500	0.082638	0.223275

(b) LIBOR Discount Factors

Figure 2: LIBOR Results

3 Forward Swap Rates

With all the necessary OIS discount factors and Forward LIBOR rates, we can go on to derive the Forward Swap rates:

Tenor	Forward Swap Rates
1x1	0.032008
1x2	0.033260
1x3	0.034012
1x5	0.035259
1x10	0.038436

Tenor	Forward Swap Rates
5x1	0.039302
5x2	0.040103
5x3	0.040105
5x5	0.041129
5x10	0.043676

Tenor	Forward Swap Rates
10x1	0.042245
10x2	0.043174
10x3	0.044157
10x5	0.046313
10x10	0.053545

Figure 3: Forward Swap rates