QF608: Research Methods for Quantitative Professionals

Jin, Kim, Quek, Wang and Woon (2019)

The Fama-French Three-Factor Model

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Research Null Hypothesis

Research Null Hypothesis

The three stock-market factors suggested by Eugene F. Fama and Kenneth R. French in their 1993 paper titled “Common Risk Factors in the Returns on Stocks and Bonds” **do not** significantly explain the returns of stocks listed on the NYSE, NASDAQ and AMEX for the period between Jul 2011 – Dec 2018.

2 Literature Review

Literature Review

- The CAPM, developed by Sharpe (1964), Lintner (1965) and Black (1972) attempts to explain stock returns using excess market returns as an all-encompassing risk factor, an idea which was quickly popularised, but has since been met with much scrutiny and many empirical contradictions.
- Amongst the critics of the CAPM are Fama and French (1993), who have built a case arguing that market excess returns alone do not fully explain cross-sectional variations in equity returns and that the addition of two empirically-backed factors are better explaining said variations. The model is also known as the Fama and French (1993) three-factor model:

$$R(t) - R_f(t) = a + b(R_{mkt} - R_f) + s(SMB) + h(HML) + \epsilon_i$$

3 Data, Research Design & Methodology

Data, Research Design & Methodology

<u>Data Source:</u>

Wharton Research Data Services (WRDS)

- Center for Research in Security Prices (CRSP)
- Compustat – Capital IQ
- CRSP/Compustat Merged (CCM)

CRSP

Available Relevant Data (Monthly)

- Closing Stock Price
- Number of Shares Outstanding
- Returns (Simple, Dividend, Delisting)

Identifiers

- PERMCO
- PERMNO
- CUSIP – Not unique
- Ticker – Not unique

Compustat – Capital IQ

Available Relevant Data (Balance Sheet, Annual)

- Total Stockholders' Equity
- Deferred Taxes
- Investment Tax Credit
- Preferred Stock (Redemption, Liquidation, Carrying Values)

Identifiers

- GVKEY
- IID
- CUSIP – Not unique
- Ticker – Not unique, different from CRSP tickers

CRSP vs Compustat

- Entity Level: **PERMCO (CRSP)** and **GVKEY (Compustat)** do not always agree, difficult to obtain perfect matching between datasets. As such, 1 PERMCO may correspond to multiple GVKEY and vice versa.
- **CRSP/Compustat Merged (CCM)** tries to bridge the gap but some crucial data (e.g. number of shares outstanding) is missing/formatted in an inconvenient manner.
- Solution: Use CCM to obtain entity level matching data, then raw data was obtained from standalone datasets. New identifier **GVKEY PERMCO** was created for purposes of traversing between datasets.
- Example: If Apple Inc's GVKEY is 1690 and its PERMCO is 7, its GV KEY PERMCO is 16907, which is a unique identifier.

Data, Research Design & Methodology

Research Design & Methodology:

- Size
- Book to Market Equity
- SMB & HML
- Rm-Rf
- Quantile Based Portfolios

Size

Formulas

- Size = Market Capitalisation of the Firm at June of time t
- Market Capitalisation = Share Price x Number of Shares Outstanding

Small vs. Big

- Every year, Sizes of firms traded on the NYSE, NASDAQ and Amex (where available) are computed.
- Then, for only NYSE-traded firms, Size is ranked, then the median is used as the distinguisher between Small and Big companies.

Book-to-Market Equity

Formulas

- Book-to-Market Equity= BE/ME
- Book Common Equity (BE) = (Total Stockholders' Equity + Deferred Taxes + Investment Tax Credit – Book Value of Preferred Stock) of the Firm at time t-1.
- Book Value of Preferred Stock = Redemption, Liquidation or Carrying Value of Preferred Stock, in that order, when available.
- Market Common Equity (ME) = Market Capitalisation of the Firm at December of time t-1.

High vs. Low

- Every year, **positive-only** BE-ME for firms traded on the NYSE, NASDAQ and Amex (where available) are computed.
- Then, for only NYSE-traded firms, BE-ME is ranked, then cut off points of 30% and 70% are used as distinguishers of Low, Medium and High firms.
- Low = Bottom 30%, Medium = Next 40%, High = Top 30%

SMB & HML Factors

- Eligible firms are then bundled into 6 value-weighted portfolios ("the factor portfolios") based on Size (S or B) and BE-ME (H, M or L): S/L, S/M, S/H, B/L, B/M, B/H.
- **Monthly returns** for these portfolios are then computed.

- Small Minus Big (SMB) = $\frac{1}{3}(R_{S/L} + R_{S/M} + R_{S/H}) - \frac{1}{3}(R_{B/L} + R_{B/M} + R_{B/H})$
- High Minus Low (HML) = $\frac{1}{2}(R_{S/H} + R_{B/H}) - \frac{1}{2}(R_{S/L} + R_{B/L})$

Excess Market Return Factor

- Excess Market Return = $R_M - R_F$
- R_M = Return of the value-weighted portfolio comprised of all previously eligible firms, and including firms with negative Book Common Equity (BE).
- R_F = 1-Month Treasury Bill Rate

Quintile-Based Portfolios

- The same firms eligible for the 6 factor portfolios are then bundled into 25 portfolios based on their Size and Book-to-Market Equity quintiles.
- Again, NYSE-based breakpoints are used to distinguish the firms into their respective quintiles.
- **Monthly returns** for these portfolios are then computed, and regressed against the three Fama-French factors introduced earlier.

4 Summary of Statistics

Summary of Statistics

The Explanatory Variables:

Table 01: Descriptive Statistics of the Explanatory Variables

	Rm-Rf	SMB	HML
Mean	0.82%	-0.59%	-0.35%
Std	3.41%	2.76%	2.59%
P(T<=t) Two-Tail	2.52%	4.69%	20.90%

Summary of Statistics

The Dependent Variables:

Table 02: Average Number of Firms in Quintile-Based Portfolios

	Book-to-Market Equity (BE/ME) Quintiles				
Size quintile	**Low**	**2**	**3**	**4**	**High**
Small	**237**	**148**	**168**	**179**	**356**
2	**147**	100	96	76	88
3	**112**	75	64	47	50
4	**142**	83	55	44	40
Big	**159**	80	51	38	35

Summary of Statistics

The Dependent Variables:

Table 03: Means of Monthly Excess Returns

	Book-to-Market Equity (BE/ME) Quintiles				
Size quintile	**Low**	**2**	**3**	**4**	**High**
Small	-1.14%	-0.28%	0.03%	-0.14%	-0.40%
2	0.30%	0.33%	0.24%	0.26%	0.11%
3	0.50%	0.62%	0.59%	0.42%	0.10%
4	0.70%	0.65%	0.69%	0.36%	0.34%
Big	1.12%	0.81%	0.78%	0.45%	0.42%

5 Regression Analysis

FF3 (2011-2018) vs FF3(1963-1991)

Table 04: Regression Summary for FF3 Model 2011-2018

	Book-to-Market Equity (BE/ME) Quintiles									
	a (intercept)					t(a)				
Size quintile	**Low**	**2**	**3**	**4**	**Higher**	**Low**	**2**	**3**	**4**	**Higher**
Small	-0.012	-0.002	0.000	-0.002	-0.001	**-4.586**	-1.090	0.191	-1.275	-0.507
2	0.001	0.001	0.000	0.001	0.001	0.513	0.562	0.090	0.438	0.904
3	0.000	0.001	0.003	0.001	-0.001	0.047	0.963	1.520	0.268	-0.440
4	0.000	0.000	0.002	-0.002	0.001	0.298	-0.244	1.004	-1.215	0.623
Big	0.001	0.000	0.000	-0.001	-0.002	0.964	0.137	-0.007	-1.048	-1.079
	b (slope for Rm-Rf)					t(b)				
Small	0.924	0.841	0.878	0.906	0.858	11.729	12.502	15.269	16.929	16.009
2	0.892	0.931	1.018	0.982	1.109	15.635	18.619	23.090	19.409	26.002
3	1.007	1.056	1.036	1.062	1.082	23.131	22.986	20.588	18.617	18.898
4	1.028	1.047	0.975	1.028	1.002	25.102	25.113	21.216	18.034	16.250
Big	1.034	0.930	0.990	0.903	0.909	41.850	27.809	24.789	23.178	17.639
	s (slope for SMB)					t(s)				
Small	1.301	1.268	1.142	0.941	1.254	13.361	15.254	16.071	14.234	18.914
2	1.179	0.953	0.889	0.884	1.002	16.716	15.413	16.317	14.132	19.010
3	0.782	0.615	0.740	0.617	0.579	14.543	10.827	11.893	8.748	8.181
4	0.454	0.313	0.331	0.256	0.410	8.980	6.082	5.823	3.628	5.376
Big	-0.180	-0.090	-0.051	-0.033	-0.133	-5.878	-2.170	**-1.033**	**-0.682**	-2.080
	h (slope for HML)					t(h)				
Small	-0.170	-0.054	0.174	0.310	0.793	**-1.845**	**-0.686**	2.578	4.939	12.628
2	-0.485	-0.089	0.259	0.292	0.979	-7.258	**-1.525**	5.011	4.928	19.603
3	-0.349	0.085	0.238	0.404	1.063	-6.839	**1.583**	4.040	6.039	15.858
4	-0.244	-0.026	0.197	0.299	1.074	-5.093	**-0.539**	3.665	4.479	14.877
Big	-0.253	0.053	0.172	0.521	0.632	-8.747	**1.361**	3.669	11.421	10.470
	R2					AIC				
Small	0.873	0.893	0.914	0.915	0.937	-422.97	-451.36	-479.70	-492.63	-492.17
2	0.923	0.926	0.945	0.926	0.963	-480.99	-504.71	-527.50	-502.64	-533.43
3	0.942	0.928	0.922	0.898	0.917	-529.76	-519.96	-503.56	-480.96	-480.43
4	0.933	0.923	0.899	0.858	0.888	-540.78	-537.55	-520.00	-481.23	-467.08
Big	0.961	0.914	0.898	0.900	0.840	-631.57	-577.17	-545.23	-549.70	-499.32
	Adjusted R2									
Small	0.868	0.889	0.911	0.912	0.935					
2	0.920	0.924	0.943	0.923	0.962					
3	0.940	0.926	0.919	0.895	0.914					
4	0.931	0.920	0.896	0.853	0.884					
Big	0.959	0.911	0.895	0.896	0.834					

	s				
Small	1.46	1.26	1.19	1.17	1.23
2	1.00	0.98	0.88	0.73	0.89
3	0.76	0.65	0.60	0.48	0.66
4	0.37	0.33	0.29	0.24	0.41
Big	– 0.17	– 0.12	– 0.23	– 0.17	– 0.05
			h		
Small	– 0.29	0.08	0.26	0.40	0.62
2	– 0.52	0.01	0.26	0.46	0.70
3	– 0.38	– 0.00	0.32	0.51	0.68
4	– 0.42	0.04	0.30	0.56	0.74
Big	– 0.46	0.00	0.21	0.57	0.76

FF3 (2011-2018) vs CAPM(2011-2018)

Table 04: Regression Summary for FF3 Model 2011-2018

	Book-to-Market Equity (BE/ME) Quintiles									
	a (intercept)					t(a)				
Size quintile	Low	2	3	4	Higher	Low	2	3	4	Higher
Small	-0.012	-0.002	0.000	-0.002	-0.001	**-4.586**	-1.090	0.191	-1.275	-0.507
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Big	-0.180	-0.090	-0.051	-0.033	-0.133	-5.878	-2.170	**-1.033**	**-0.682**	-2.080
	h (slope for HML)					t(h)				
Small	-0.170	-0.054	0.174	0.310	0.793	**-1.845**	**-0.686**	2.578	4.939	12.628
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4	0.933	0.923	0.899	0.858	0.888	-540.78	-537.55	-520.00	-481.23	-467.08
Big	0.961	0.914	0.898	0.900	0.840	-631.57	-577.17	-545.23	-549.70	-499.32
	Adjusted R2									
Small	0.868	0.889	0.911	0.912	0.935					
2	0.920	0.924	0.943	0.923	0.962					
3	0.940	0.926	0.919	0.895	0.914					
4	0.931	0.920	0.896	0.853	0.884					
Big	0.959	0.911	0.895	0.896	0.834					

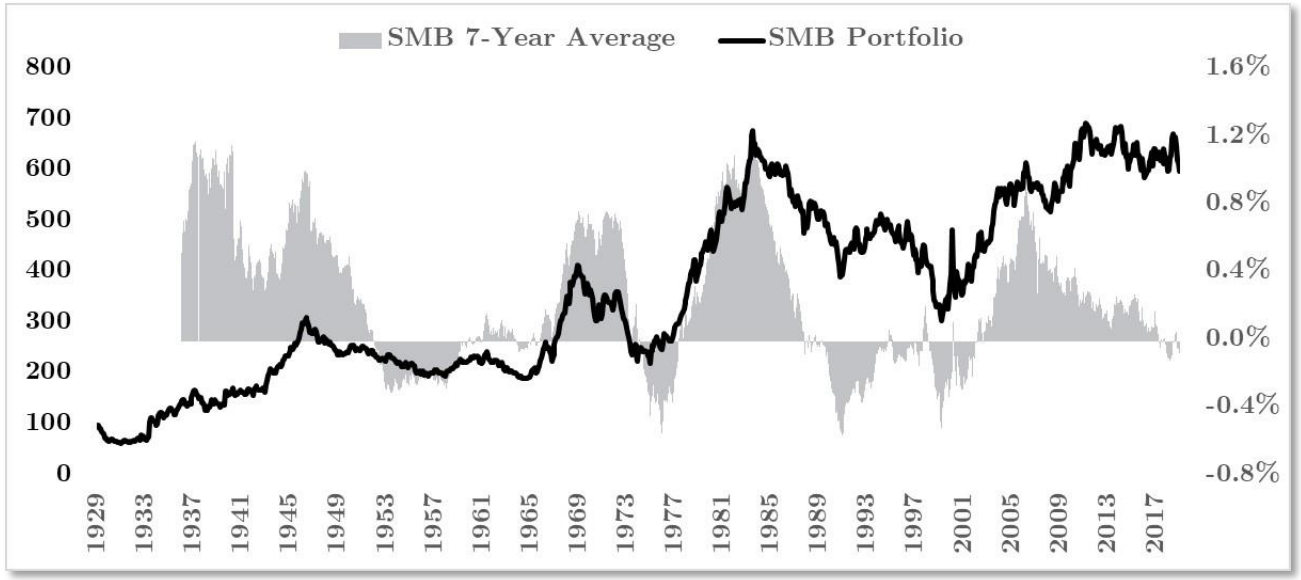
Table 05: Regression Summary for CAPM 2011-2018

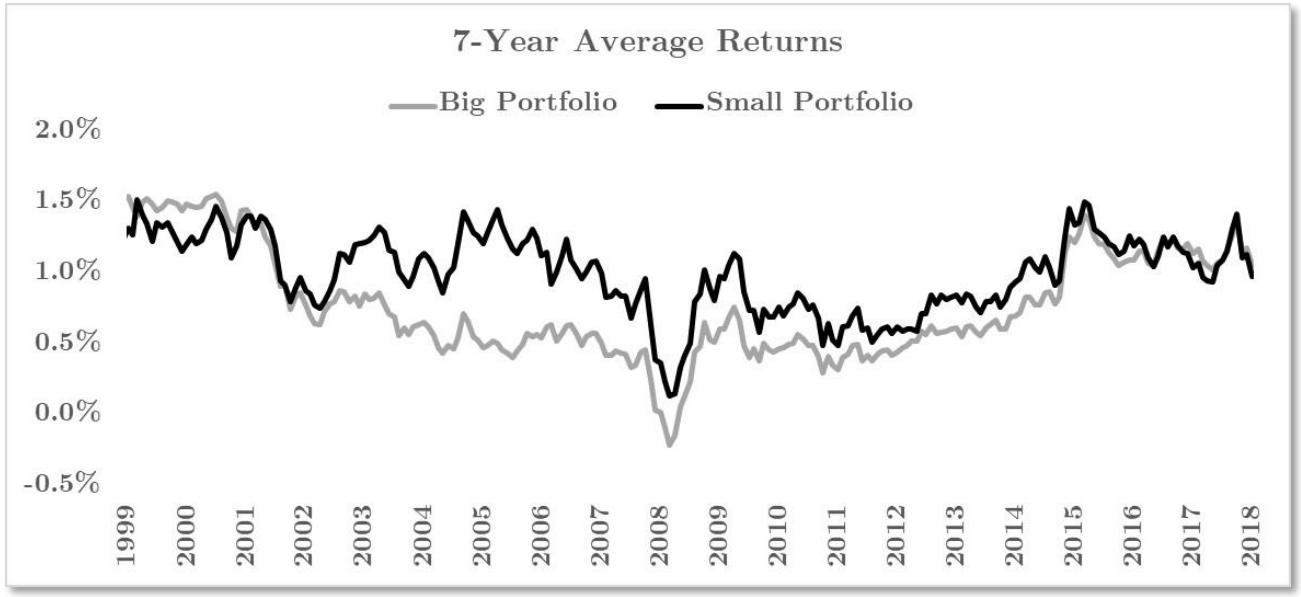
	Book-to-Market Equity (BE/ME) Quintiles									
	a (intercept)					t(a)				
Size quintile	Low	2	3	4	Higher	Low	2	3	4	Higher
Small	-0.023	-0.014	-0.010	-0.012	-0.015	**-5.418**	**-3.417**	**-2.895**	**-3.676**	**-3.290**
2	-0.008	-0.007	-0.009	-0.008	-0.011	**-2.102**	**-2.447**	**-3.012**	**-2.713**	**-2.596**
3	-0.006	-0.004	-0.005	-0.006	-0.009	**-2.201**	-1.962	-1.841	**-2.331**	**-2.429**
4	-0.003	-0.003	-0.002	-0.006	-0.006	-1.544	-1.993	-1.163	**-2.668**	-1.539
Big	0.003	0.001	0.000	-0.003	-0.003	2.753	0.736	-0.078	-1.439	-1.119
	b (slope for Rm-Rf)					t(b)				
Small	1.412	1.314	1.296	1.246	1.300	11.636	11.577	12.653	13.759	10.227
2	1.345	1.288	1.340	1.301	1.451	12.371	15.136	16.365	15.270	12.194
3	1.308	1.282	1.304	1.279	1.264	17.380	20.481	17.367	16.579	11.421
4	1.204	1.164	1.091	1.113	1.121	22.683	26.638	21.584	18.796	10.355
Big	0.976	0.895	0.966	0.874	0.840	29.345	29.477	25.562	16.144	12.256
	R2					AIC				
Small	0.606	0.604	0.645	0.683	0.543	-325.38	-337.48	-355.89	-378.18	-317.14
2	0.635	0.722	0.753	0.726	0.628	-345.16	-389.30	-396.28	-389.06	-329.03
3	0.774	0.827	0.774	0.757	0.597	-411.42	-444.57	-411.88	-406.94	-342.06
4	0.854	0.890	0.841	0.801	0.549	-474.29	-509.31	-483.01	-454.54	-346.06
Big	0.907	0.908	0.881	0.748	0.631	-558.48	-574.83	-535.49	-470.63	-428.25
	Adjusted R2									
Small	0.602	0.599	0.641	0.679	0.538					
2	0.631	0.719	0.750	0.723	0.624					
3	0.772	0.825	0.772	0.755	0.593					
4	0.852	0.888	0.839	0.798	0.544					
Big	0.906	0.907	0.880	0.745	0.626					

6

SMB, HML and UMD: A Deeper Dive

Small Minus Big (SMB) Factor



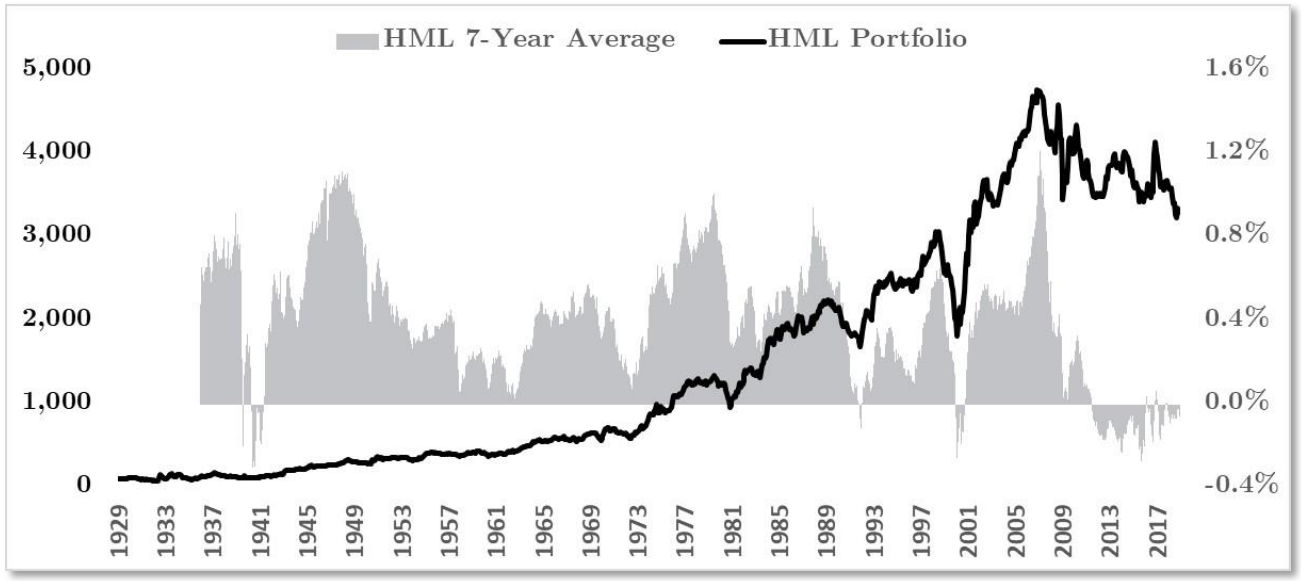


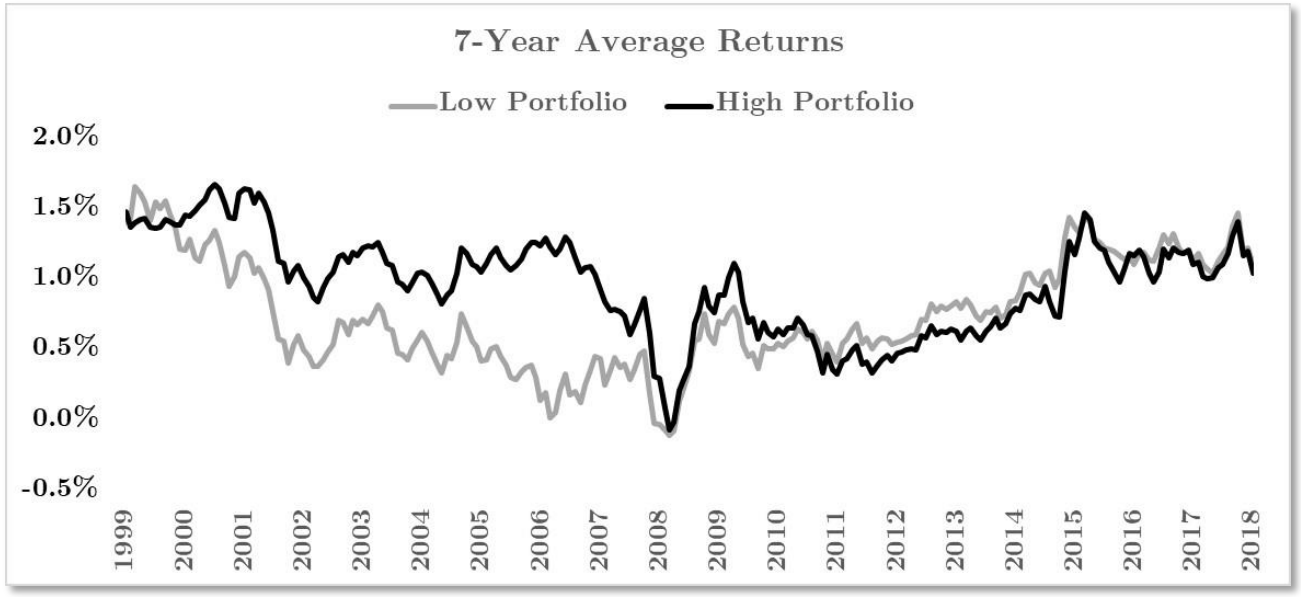
- SMB factor realisations do not exhibit consistently positive readings over our sample period, though said SMB readings are, on average, mildly positive (at 0.21%).
- The rolling 7-year SMB average shows how the SMB factor realisations have trended in and out of positive territories over our extended sample period.
- Net gain the portfolio has clocked over the extended sample period suggests that SMB factor realisations are still on average, positive.
- The sample period used in our main study (July 2011 - December 2018) happens to denote a time period when rolling 7-year average SMB factor realisations are negative, explaining the divergence.
- Delving deeper, we found the main driver behind the recent downtrend in rolling 7-year average SMB factor realisations stem mainly from movements in rolling 7-year Big portfolio returns.

Small Minus Big (SMB) Factor

SMB: WRDS-Provided Factors					
January 1929 to December 2018					
Series	Count	Mean	Std Dev	Skewness	Kurtosis
Full Sample	1080	0.21%	0.0322	1.94	19.35
Positive Values Only	556	2.35%	0.0275	5.36	51.60
Negative Values Only	522	-2.06%	0.0182	-2.08	9.35
July 2011 to December 2018					
Series	Count	Mean	Std Dev	Skewness	Kurtosis
Full Sample	90	-0.12%	0.0231	0.16	-0.37
Positive Values Only	47	1.67%	0.0142	1.11	0.39
Negative Values Only	43	-2.08%	0.0125	-0.36	-0.76
SMB: Computed Factors					
July 2011 to December 2018					
Series	Count	Mean	Std Dev	Skewness	Kurtosis
Full Sample	90	-0.59%	0.0277	-0.12	-0.57
Positive Values Only	42	1.79%	0.0147	0.73	-0.70
Negative Values Only	48	-2.67%	0.0179	-0.50	-0.94
SMB: Constructed Portfolio Returns					
Time Period	Simple Mean Return	Geometric Mean Return	Cumulative Return	Mean 1-Mth T-Bill Rate	Sharpe Ratio
Jan 1929 to Dec 2018	0.21%	0.17%	500.96%	0.27%	-0.0191
Jul 2011 to Dec 2018	-0.12%	-0.15%	-12.67%	0.03%	-0.0675

High Minus Low (HML) Factor



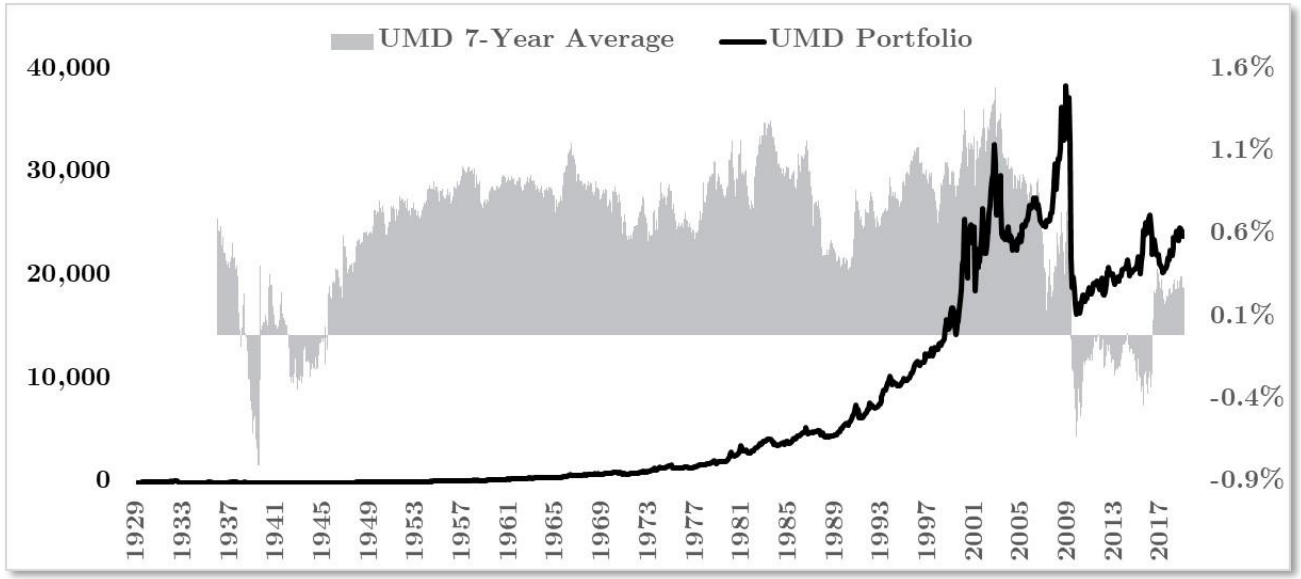


- The HML factor was observed to exhibit mildly negative mean readings for the period between July 2011 and December 2018, though the HML factor's mean reading of -0.10% is deemed to be statistically indifferent from 0 at the 5% level.
- The HML factor was seen to have displayed more consistently and persistently positive readings over the extended sample period of between January 1929 and December 2018 compared to the SMB factor.
- This makes the unravelling of the HML factor's positive readings over the recent decade all the more an unprecedented one, which also explains the divergence between our observed mean HML and that of Fama and French (1993).
- Our constructed HML portfolio has grossly outperformed our constructed SMB portfolio over the extended sample period in terms of nominal returns.
- The HML portfolio clocked 3,200.59% returns while the SMB portfolio only clocked 500.96%, with the latter's Sharpe ratio coming up as negative for returns over the extended sample period.

High Minus Low (HML) Factor

HML: WRDS-Provided Factors					
January 1929 to December 2018					
Series	**Count**	**Mean**	**Std Dev**	**Skewness**	**Kurtosis**
Full Sample	1080	0.38%	0.0351	2.19	19.27
Positive Values Only	573	2.50%	0.0317	4.99	39.75
Negative Values Only	506	-2.01%	0.0202	-2.13	5.97
July 2011 to December 2018					
Series	**Count**	**Mean**	**Std Dev**	**Skewness**	**Kurtosis**
Full Sample	90	-0.10%	0.0215	0.88	1.77
Positive Values Only	36	1.88%	0.0176	1.55	3.48
Negative Values Only	54	-1.42%	0.0113	-0.79	-0.22
HML: Computed Factors					
July 2011 to December 2018					
Series	**Count**	**Mean**	**Std Dev**	**Skewness**	**Kurtosis**
Full Sample	90	-0.35%	0.0260	-0.14	0.80
Positive Values Only	42	1.75%	0.0158	1.37	2.39
Negative Values Only	48	-2.18%	0.0180	-1.22	2.12
HML: Constructed Portfolio Returns					
Time Period	**Simple Mean Return**	**Geometric Mean Return**	**Cumulative Return**	**Mean 1-Mth T-Bill Rate**	**Sharpe Ratio**
Jan 1929 to Dec 2018	0.38%	0.32%	3200.59%	0.27%	0.0307
Jul 2011 to Dec 2018	-0.10%	-0.12%	-10.36%	0.03%	-0.0608

Bonus: Up Minus Down (UMD) Factor



- We also looked at one of the additional stock market factors introduced and popularised by Carhart (1997): the momentum factor (UMD).
- Like the HML factor, it was observed that the UMD factor is consistently and persistently positive.
- Also, like the SMB and HML factors, we observed an unravelling of the long UMD play in the recent decade, though said unravelling took place relatively earlier from April 2009 and has already recovered since April 2016.
- In terms of cumulative returns, the long UMD play has grossly outperformed both the long SMB play and the long HML play, with cumulative nominal returns of 24,252.43% observed over the extended sample period and 24.18% over the original sample period.
- On a risk-adjusted basis and in terms of Sharpe ratios, the long UMD outperforms the long SMB play and the long HML play too.

Bonus: Up Minus Down (UMD) Factor

UMD: WRDS-Provided Factors					
January 1929 to December 2018					
Series	Count	Mean	Std Dev	Skewness	Kurtosis
Full Sample	1080	0.64%	0.0473	-3.05	27.74
Positive Values Only	675	2.90%	0.0262	2.03	6.02
Negative Values Only	403	-3.15%	0.0505	-5.29	38.67
July 2011 to December 2018					
Series	Count	Mean	Std Dev	Skewness	Kurtosis
Full Sample	90	0.29%	0.0303	0.07	1.04
Positive Values Only	51	2.27%	0.0207	1.47	3.20
Negative Values Only	39	-2.31%	0.0192	-1.34	1.67

UMD: Constructed Portfolio Returns					
Time Period	Simple Mean Return	Geometric Mean Return	Cumulative Return	Mean 1-Mth T-Bill Rate	Sharpe Ratio
Jan 1929 to Dec 2018	0.64%	0.51%	24252.43%	0.27%	0.0773
Jul 2011 to Dec 2018	0.29%	0.24%	24.18%	0.03%	0.0839

7 Conclusion

Conclusion

Research Null Hypothesis Revisited:

The three stock-market factors suggested by Eugene F. Fama and Kenneth R. French in their 1993 paper titled “Common Risk Factors in the Returns on Stocks and Bonds” **do not** significantly explain the returns of stocks listed on the NYSE, NASDAQ and AMEX for the period between Jul 2011 – Dec 2018.

Conclusion: We **Reject** Our Research Null Hypothesis.

Additional Finding:

SMB factor turns negative in our testing period due to recent large cap relative outperformance.

8 References

Reference List

- Markowitz, H. (1952). Portfolio selection. The Journal of Finance, 7(1), 77-91.
- Sharpe, W. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. Journal of Finance, 19(3), 425-442.
- Lintner, J. (1965). The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. Review of Economics and Statistics, 47(1), 12-37.
- Black, Fischer. Michael C. Jensen. and Myron Scholes. (1972). The capital asset pricing model: Some empirical tests. in: M. Jensen. ed.. Studies In the theory of capital markets (Praeger, New York, NY).
- Fama, E., & French, K. (1993). Common risk factors in the returns on stocks and bonds.Journal of Financial Economics, 33(1), 3-56..
- Fama. Eugene F. and Kenneth R. French. (1992a). The cross-section of expected stock returns.Journal of Finance 47, 427 - 465.
- Banz. Rolf W.. 1981. The relationship between return and market value of common stocks, Journal of Financial Economics 9. 3-15.
- Daniel, K., & Titman, S. (1997). Evidence on the characteristics of cross sectional variation in stock returns