QF608 Research Methods for Quantitative Professionals

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1 Summary of Statistics

The explanatory variables
The table below gives mean, standard deviation and test statistics for the three explanatory variables. The mean of market risk premium, 0.82%, which implies 9.84% on the annual basis, is much higher than what Fama and French reported (0.43%) for their test period during 1963-1991. Neither SMB nor HML are as significant as market risk premium given their higher p-values. This suggests returns of small caps minus large cap and returns of high BE/ME minus low BE/ME are indifferent from 0.

Table 1: Stats of Explanatory Variables

	Rm-Rf	SMB	HML
Mean	0.82%	-0.59%	-0.35%
Std	3.41%	2.76%	2.59%
P(T<=t) two-tail	2.52%	4.69%	20.90%

The correlation matrix shows HML has little correlation with market risk premium and SMB. This implies HML brings in additional explanatory power that CAPM is not able to capture.

Table 2: Correlation among Explanatory Variables

Correlation	Rm-Rf	SMB	HML
Rm-Rf	1.00	0.46	-0.04
SMB	0.46	1.00	0.03
HML	-0.04	0.03	1.00

The dependent variables
The table below lists average of annual number of firms in each portfolio. 61.7% of firms lie in the union of smallest cap quintile and lowest BE/ME quintiles (as highlighted).

Table 3: Number of Firms

	Book-to-market equity (BE/ME) quintiles				
Size quintile	Low	2	3	4	High
Small	**237**	**148**	**168**	**179**	**356**
2	**147**	100	96	76	88
3	**112**	75	64	47	50
4	**142**	83	55	44	40
Big	**159**	80	51	38	35

The table below demonstrates average of monthly excess return for all 25 portfolios between 2011-2018. We are able to find a clear trend that excess returns get higher when size increase in each BE/ME quintiles. On the other hand, we can see lower BE/ME portfolios yields higher excess return in size quintiles 3 and Big, but the consistency does not hold for the other size quintiles.

Table 4: Mean of Monthly Eccess Return

	Book-to-market equity (BE/ME) quintiles				
Size quintile	Low	2	3	4	Higher
Small	-0.0114	-0.0028	0.0003	-0.0014	-0.0040
2	0.0030	0.0033	0.0024	0.0026	0.0011
3	0.0050	0.0062	0.0059	0.0042	0.0010
4	0.0070	0.0065	0.0069	0.0036	0.0034
Big	0.0112	0.0081	0.0078	0.0045	0.0042

2 Time-series Regression Results

After performing linear regression for 25 portfolios between 2011-2018, coefficients for all regressors have been summarized in Table 5. First of all, for intercept a, we are unable to reject null hypothesis in 24/25 portfolios, except for the portfolio on the upper left corner as highlighted. This is to convey there is no abnormal return in FF 3-factor model, which implies its capability in explaining stock return variation. Secondly, t-test results for slope b suggest rejection of null hypothesis. As anticipated, values of slope b are close to 1. Thirdly, t-statistics are strong enough to reject null hypothesis for s (slope for SMB) in four smaller size quintiles. The exception happens for Big size quintiles where slopes turn negative as highlighted. Similarly, for HML, t-tests are not able to reject null hypothesis when h turns into positive from lower BE/ME quintile to higher BE/ME quintile.

Table 5: Regression Summary for FF3 Model 2011-2018

	Book-to-market equity (BE/ME) quintiles									
	a (intercept)					t(a)				
Size quintile	Low	2	3	4	Higher	Low	2	3	4	Higher
Small	-0.012	-0.002	0.000	-0.002	-0.001	**-4.586**	-1.090	0.191	-1.275	-0.507
2	0.001	0.001	0.000	0.001	0.001	0.513	0.562	0.090	0.438	0.904
3	0.000	0.001	0.003	0.001	-0.001	0.047	0.963	1.520	0.268	-0.440
4	0.000	0.000	0.002	-0.002	0.001	0.298	-0.244	1.004	-1.215	0.623
Big	0.001	0.000	0.000	-0.001	-0.002	0.964	0.137	-0.007	-1.048	-1.079
	b (slope for Rm-Rf)					t(b)				
Small	0.924	0.841	0.878	0.906	0.858	11.729	12.502	15.269	16.929	16.009
2	0.892	0.931	1.018	0.982	1.109	15.635	18.619	23.090	19.409	26.002
3	1.007	1.056	1.036	1.062	1.082	23.131	22.986	20.588	18.617	18.898
4	1.028	1.047	0.975	1.028	1.002	25.102	25.113	21.216	18.034	16.250
Big	1.034	0.930	0.990	0.903	0.909	41.850	27.809	24.789	23.178	17.639
	s (slope for SMB)					t(s)				
Small	1.301	1.268	1.142	0.941	1.254	13.361	15.254	16.071	14.234	18.914
2	1.179	0.953	0.889	0.884	1.002	16.716	15.413	16.317	14.132	19.010
3	0.782	0.615	0.740	0.617	0.579	14.543	10.827	11.893	8.748	8.181
4	0.454	0.313	0.331	0.256	0.410	8.980	6.082	5.823	3.628	5.376
Big	**-0.180**	**-0.090**	**-0.051**	**-0.033**	**-0.133**	**-5.878**	**-2.170**	**-1.033**	**-0.682**	**-2.080**
	h (slope for HML)					t(h)				
Small	-0.170	**-0.054**	0.174	0.310	0.793	-1.845	**-0.686**	2.578	4.939	12.628
2	-0.485	**-0.089**	0.259	0.292	0.979	-7.258	**-1.525**	5.011	4.928	19.603
3	-0.349	**0.085**	0.238	0.404	1.063	-6.839	**1.583**	4.040	6.039	15.858
4	-0.244	**-0.026**	0.197	0.299	1.074	-5.093	**-0.539**	3.665	4.479	14.877
Big	-0.253	**0.053**	0.172	0.521	0.632	-8.747	**1.361**	3.669	11.421	10.470
	R2					AIC				
Small	0.873	0.893	0.914	0.915	0.937	-422.97	-451.36	-479.70	-492.63	-492.17
2	0.923	0.926	0.945	0.926	0.963	-480.99	-504.71	-527.50	-502.64	-533.43
3	0.942	0.928	0.922	0.898	0.917	-529.76	-519.96	-503.56	-480.96	-480.43
4	0.933	0.923	0.899	0.858	0.888	-540.78	-537.55	-520.00	-481.23	-467.08
Big	0.961	0.914	0.898	0.900	0.840	-631.57	-577.17	-545.23	-549.70	-499.32
	Adjusted R2									
Small	0.868	0.889	0.911	0.912	0.935					
2	0.920	0.924	0.943	0.923	0.962					
3	0.940	0.926	0.919	0.895	0.914					
4	0.931	0.920	0.896	0.853	0.884					
Big	0.959	0.911	0.895	0.896	0.834					

In order to prove stronger explanatory power of FF 3-factor model, regression analysis has also been conducted on single factor CAPM. Results are tabulated below. Generally, t-statistics for intercept are much higher than they are in FF 3 in absolute term. This implies abnormal returns vanish with introduction of SMB and HML. On the other hand, b values for CAPM tend to be higher than when they are in FF 3. Last but not least, discrepancies of R2 in Table 5 and Table 6 indicates FF 3-factor model improves the explanatory power from 50%-90% range (CAPM) to 84%-96% range. Adjusted R2 results double confirm the additional explanatory power that SMB and HML bring to the model. FF 3-factor models have lower AIC in all 25 portfolios than CAPM, suggesting its explanatory power comes without overfitting.

Table 6: Regression Summary for CAPM 2011-2018

	Book-to-market equity (BE/ME) quintiles									
	a (intercept)					t(a)				
Size quintile	Low	2	3	4	Higher	Low	2	3	4	Higher
Small	-0.023	-0.014	-0.010	-0.012	-0.015	-5.418	-3.417	-2.895	-3.676	-3.290
2	-0.008	-0.007	-0.009	-0.008	-0.011	-2.102	-2.447	-3.012	-2.713	-2.596
3	-0.006	-0.004	-0.005	-0.006	-0.009	-2.201	-1.962	-1.841	-2.331	-2.429
4	-0.003	-0.003	-0.002	-0.006	-0.006	-1.544	-1.993	-1.163	-2.668	-1.539
Big	0.003	0.001	0.000	-0.003	-0.003	2.753	0.736	-0.078	-1.439	-1.119
	b (slope for Rm-Rf)					t(b)				
Small	1.412	1.314	1.296	1.246	1.300	11.636	11.577	12.653	13.759	10.227
2	1.345	1.288	1.340	1.301	1.451	12.371	15.136	16.365	15.270	12.194
3	1.308	1.282	1.304	1.279	1.264	17.380	20.481	17.367	16.579	11.421
4	1.204	1.164	1.091	1.113	1.121	22.683	26.638	21.584	18.796	10.355
Big	0.976	0.895	0.966	0.874	0.840	29.345	29.477	25.562	16.144	12.256
	R2					AIC				
Small	0.606	0.604	0.645	0.683	0.543	-325.38	-337.48	-355.89	-378.18	-317.14
2	0.635	0.722	0.753	0.726	0.628	-345.16	-389.30	-396.28	-389.06	-329.03
3	0.774	0.827	0.774	0.757	0.597	-411.42	-444.57	-411.88	-406.94	-342.06
4	0.854	0.890	0.841	0.801	0.549	-474.29	-509.31	-483.01	-454.54	-346.06
Big	0.907	0.908	0.881	0.748	0.631	-558.48	-574.83	-535.49	-470.63	-428.25
	Adjusted R2									
Small	0.602	0.599	0.641	0.679	0.538					
2	0.631	0.719	0.750	0.723	0.624					
3	0.772	0.825	0.772	0.755	0.593					
4	0.852	0.888	0.839	0.798	0.544					
Big	0.906	0.907	0.880	0.745	0.626					

3 Interpretation

Fama and French explained size and BE/ME are not ad hoc variables for explaining average stock return(1992b). They believed both variables are related to economic fundamentals. Firms that have high BE/ME ratio tend to have low earnings on assets. The intuition is that investors would not be attracted by firms that have poor earning performance recently. Reversely, investors tend to invest in firms with strong earning/profitability figure recently (1992b). Size is also related to profitability. Fama and French attributed size effect to smalls firms not able to participate in economic boom of the middle and late 1980s, which pushed small firms to a long earnings depression. However, their paper in 1992b and 1993 did not explain how size and BE/MEs relationship with earning leads to their relationship with average excess returns (of 25 portfolios).

Practitioners have been debating on whether the outperformance tendency is due to market efficiency or inefficiency. The inefficiency proponents believe the outperformance is explained by incorrectly value pricing of companies by market participants. However, the inefficiency rhetoric does not explain why small firms were mispriced higher instead of lower.

Market efficiency seems to explain this more logically. Intuition comes as: in old days such as 1963-1991, it was not easy for investors to access to or liquidate small-cap stocks. In addition, small firms normally have higher cost of capital and greater business risk. All of these made small-cap stocks riskier to invest, hence higher return in old days. On the other hand, it is much easier for investors to access to small-cap stocks and liquidate nowadays. More transparent market also reduces risk of investing in small firms. These could explain why small cap stocks does not outperform large cap stocks as they used to be.

We are still able to see News nowadays saying samll cap's outperformance tendency exists. This is not true for US equities tested between 2011-2018 in this paper. Flaw of these News was their neglect of "Survivorship Bias". According to Elton, Gruber, and Blake (1996), survivorship bias is larger in the small-fund sector than in large mutual funds. Small caps are easier to go bankrupcy and being excluded from analysis.