The robust face recognition problem is given by

$$\min_{w \in \mathbb{R}^{m+n}} \frac{1}{2}\|Bw - b\|_2^2 + \lambda\|w\|_1$$

where

- $\lambda \in \mathbb{R}_{>0}$ is a regularization parameter,
- $B = [A \quad I] \in \mathbb{R}^{m\times(m+n)}$ with $A \in \mathbb{R}^{m\times n}$, $I \in \mathbb{R}^{m\times m}$ a highly correlated dictionary of m face images $v \in \mathbb{R}^n$, stacked column-wise;
- I the identity matrix $\in \mathbb{R}^{n\times n}$, $b = Ax + e$ a face image to be recognized not in the dictionary A, with $e \in \mathbb{R}^n$ an unknown error;
- w the minimization variable.

The term $F(w) := \frac{1}{2}\|Bw - b\|_2^2$ is differentiable,

$$\nabla F(w) = B^T(Bw - b) \tag{1}$$

with Lipschitz constant $L := \|B^T B\|_F$,

$$\begin{aligned}
\|B^T(Bw - b) - B^T(Bz - b)\|_2 &= \|B^T Bw - B^T b - B^T Bz + B^T b\|_2 \\
&= \|B^T Bw - B^T Bz\|_2 \\
&\le \|B^T B\|_F \|w - z\|_2.
\end{aligned}$$

The norm $\|w\|_1$ is proper, convex and lsc and has as proximal operator **soft shrinkage**, given component-wise by

$$\left[\mathrm{Prox}_{\gamma\|\cdot\|}(w)\right]_i = \begin{cases} w_i + \gamma & \text{if } w_i < \gamma, \\ 0 & \text{if } -\gamma \le w_i \le \gamma, \\ w_i - \gamma & \text{if } w_i > \gamma. \end{cases}$$

Multiplying by $\lambda > 0$ we have:

$$\begin{aligned}
\left[\mathrm{Prox}_{\gamma R}(w)\right]_i &= \lambda^{-1}\left[\mathrm{Prox}_{\lambda^2\gamma\|\cdot\|}(\lambda w)\right]_i \\
&= \begin{cases} w_i + \lambda\gamma & \text{if } w_i < \lambda\gamma, \\ 0 & \text{if } -\lambda\gamma \le w_i \le \lambda\gamma, \\ w_i - \lambda\gamma & \text{if } w_i > \lambda\gamma. \end{cases}
\end{aligned}$$

The non-zero entries of x_* in the solution $w_* = [x_*^T, e_*^T]^T$ of problem (1) represent the person the image belongs to.