

Easy-to-implement hp -adaptivity for non-elliptic goal-oriented problems

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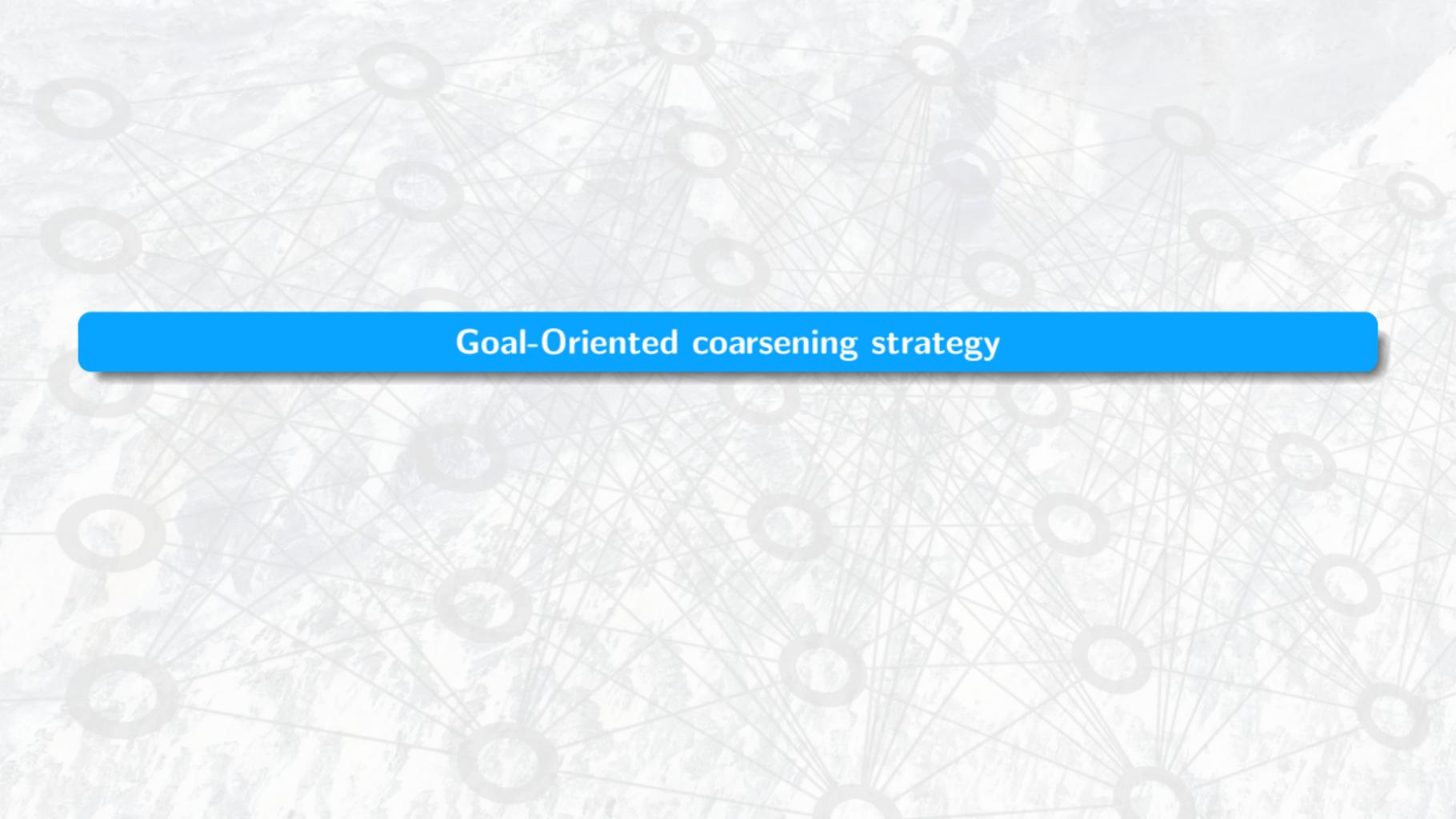
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Outline

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Goal-Oriented hp -adaptivity for non-parametric PDEs



Goal-Oriented coarsening strategy

1D Numerical results for Goal-Oriented h - and p -adaptivity

2D Numerical results for *hp*-adaptivity

2D Numerical results for hp -adaptivity

Singular Poisson example

Find u satisfying:

$$-\Delta u = \mathbf{1}_{\Omega_f} \quad \text{in } \Omega, \tag{1}$$

$$u = 0 \quad \text{on } \partial\Omega. \tag{2}$$

Domain Definitions:

- $\Omega_f = \left(\frac{1}{4}, \frac{1}{2}\right)^2 \subset \Omega,$
- $\Omega_I = \left(\frac{1}{2}, \frac{3}{4}\right)^2 \subset \Omega,$
- $a(\cdot, \cdot) := \langle \nabla \cdot, \nabla \cdot \rangle_{L^2(\Omega)}.$

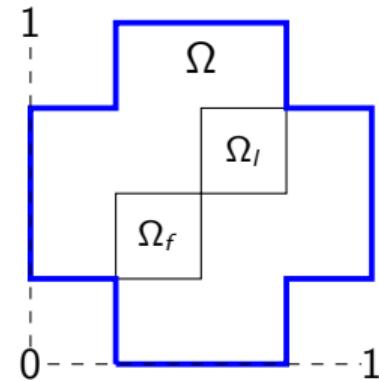
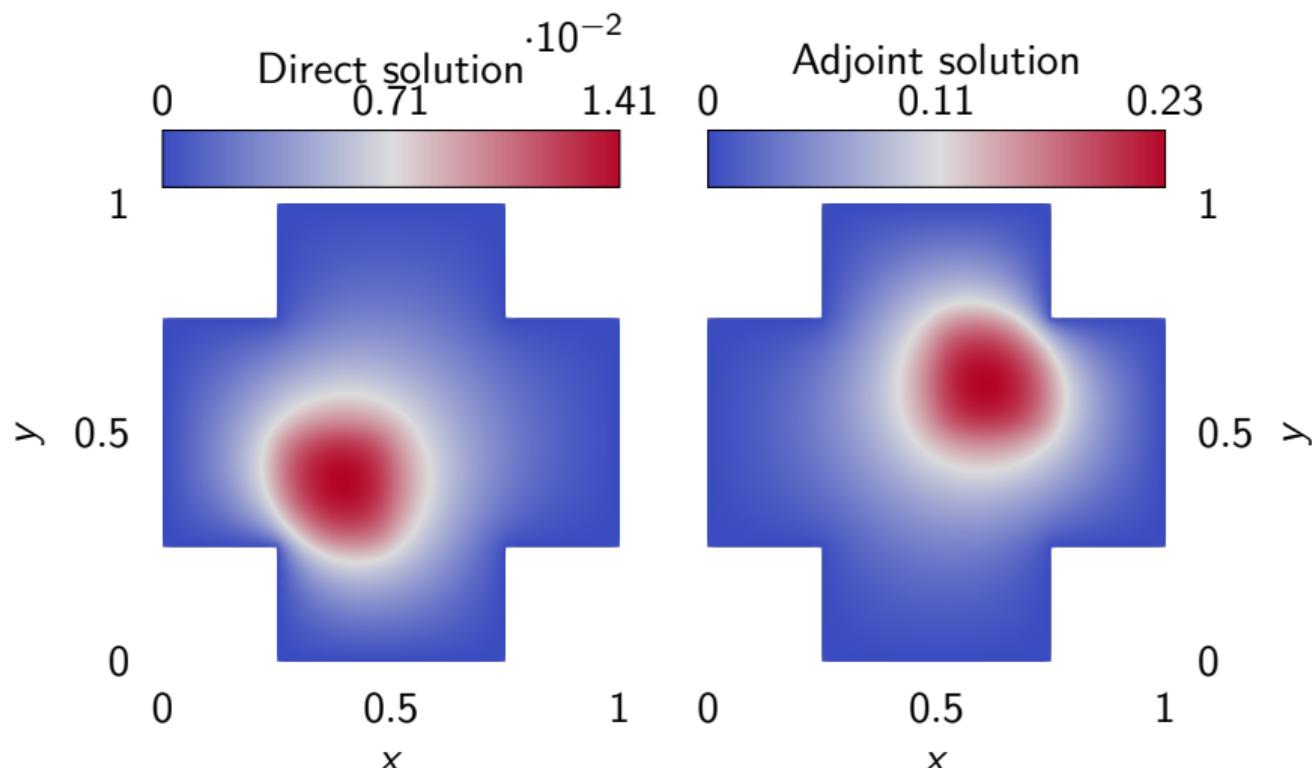


Figure: Domain
 $\Omega = ((0, 1) \times (\frac{1}{4}, \frac{3}{4})) \cup ((\frac{1}{4}, \frac{3}{4}) \times (0, 1)).$

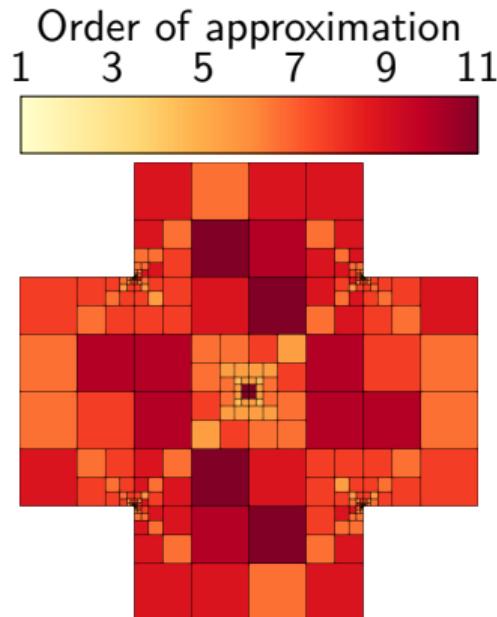
Singular Poisson example



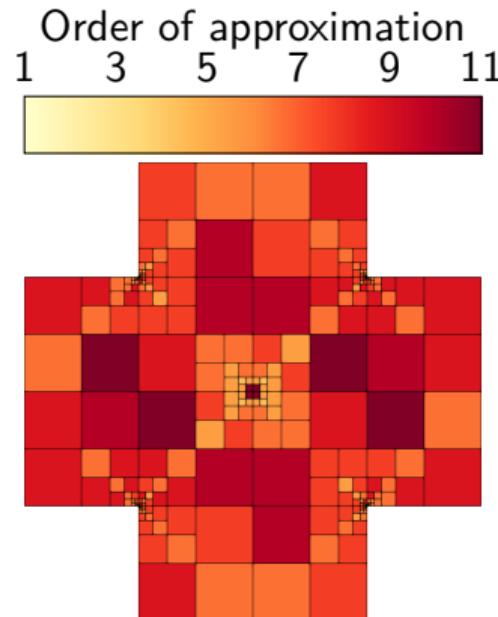
(a) Solution to the direct problem.

(b) Solution to the adjoint problem.

Singular Poisson example



(a) Final *hp*-adapted mesh with polynomial orders in the x-direction.



(b) Final *hp*-adapted mesh with polynomial orders in the y-direction.

Singular Poisson Example

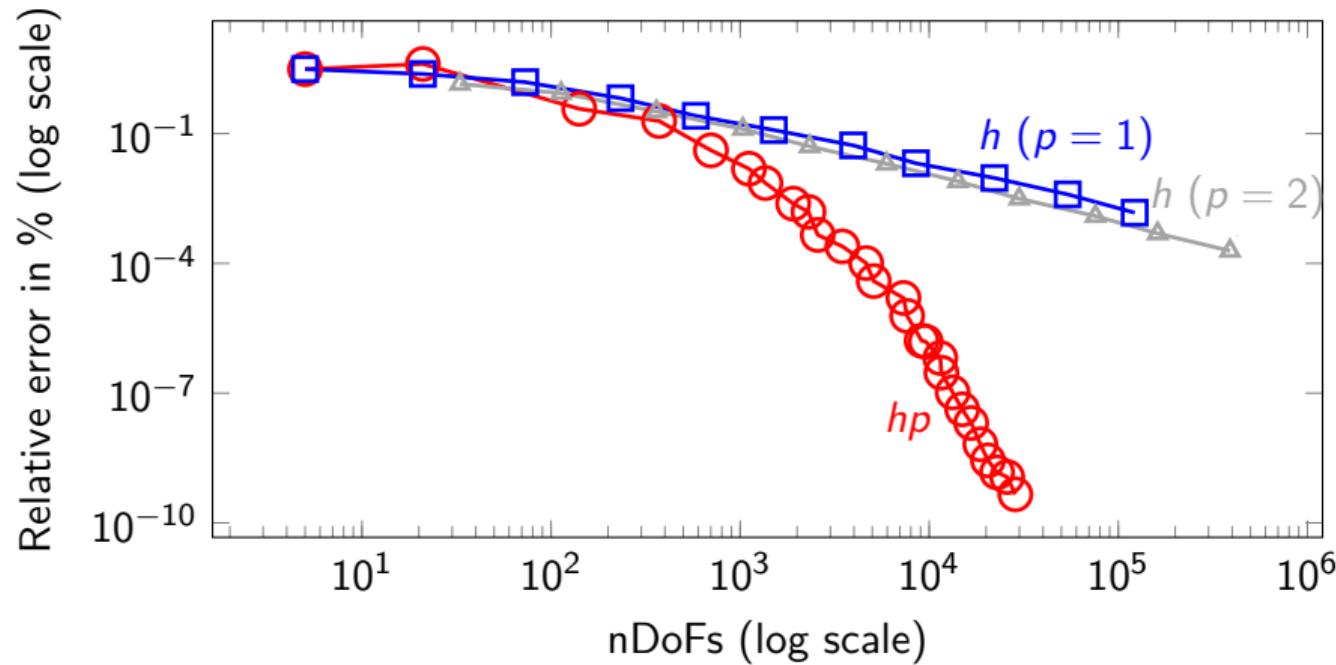


Figure: Evolution of $e_{\text{rel}}^{\text{QoI}}$ in the adaptive process.

2D Numerical results for hp -adaptivity

Wave propagation problem

Find u satisfying:

$$-\Delta u - k^2 u = \mathbb{1}_{\Omega_f} \quad \text{in } \Omega, \tag{3}$$

$$u = 0 \quad \text{on } \Gamma_D, \tag{4}$$

$$\nabla u \cdot \vec{n} = 0 \quad \text{on } \Gamma_N. \tag{5}$$

Domain Definitions:

- $\Omega_f = \left(0, \frac{1}{4}\right)^2 \subset \Omega,$
- $k = (8 \cdot 2\pi, 2\pi),$
- $\Omega_I = \left(\frac{3}{4}, 1\right)^2 \subset \Omega,$
- $a(\cdot, \cdot) :=$

$$\left| \langle \nabla \cdot, \nabla \cdot \rangle_{L^2(\Omega)} \right| + |k^2| \left| \langle \cdot, \cdot \rangle_{L^2(\Omega)} \right|.$$

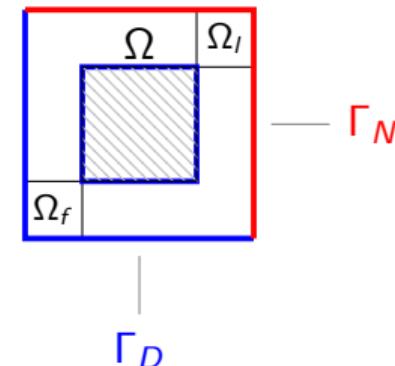
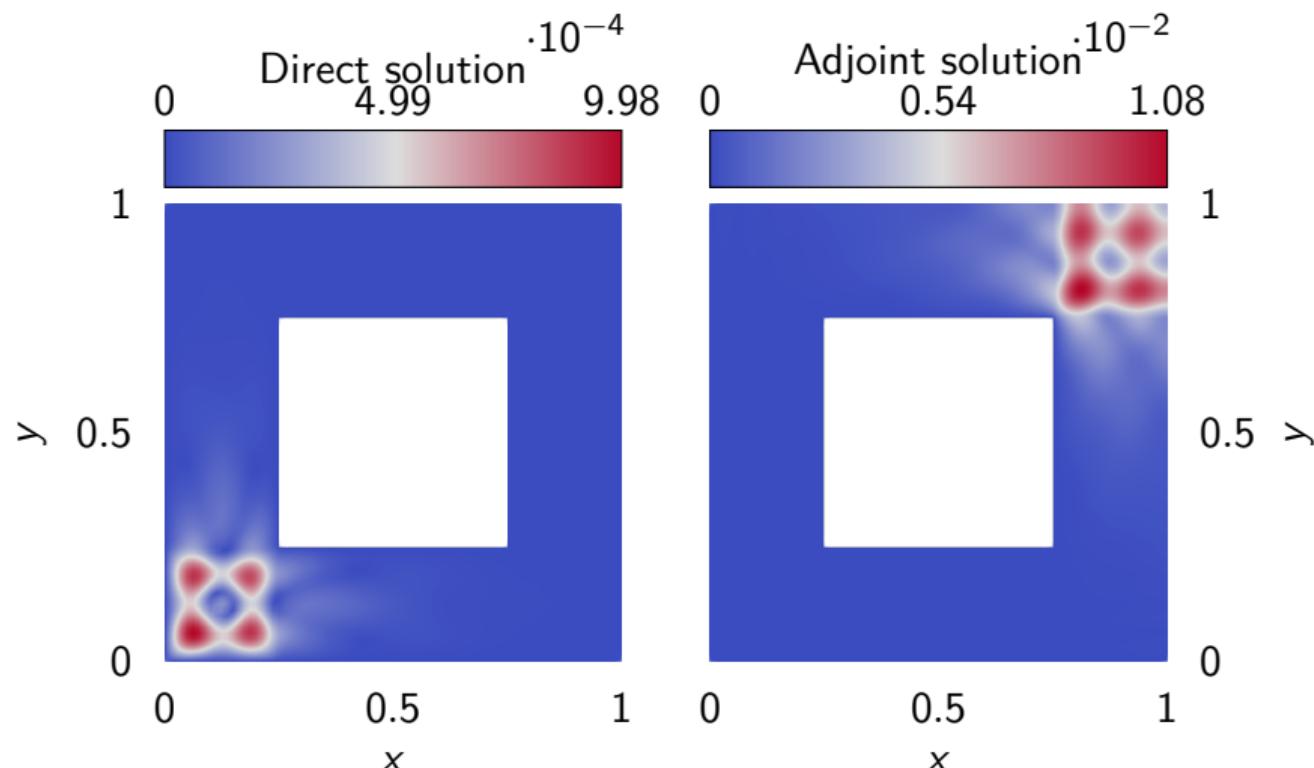
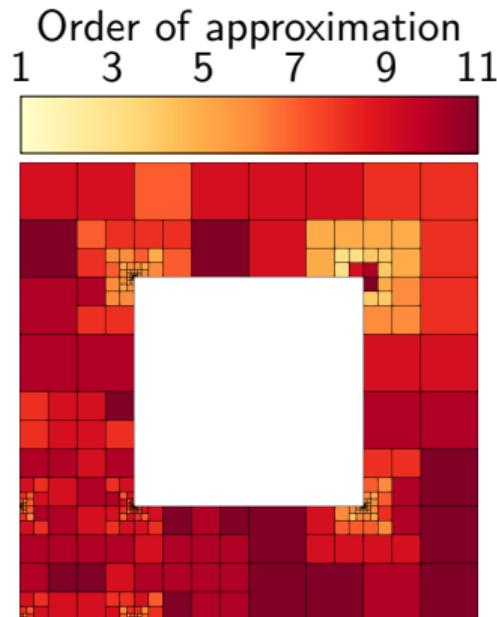


Figure: Domain $\Omega = (0, 1)^2 \setminus \left(\frac{1}{4}, \frac{3}{4}\right)^2 \subset \mathbb{R}^2$.

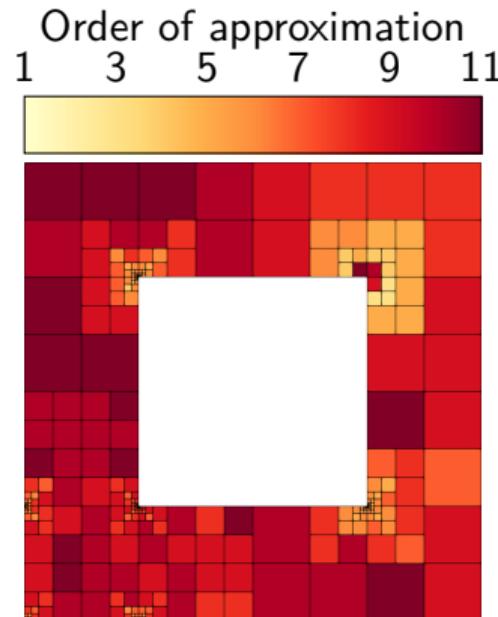
Wave propagation problem



Energy-norm adaptivity



(a) Final hp -adapted mesh with polynomial orders in the x-direction.



(b) Final hp -adapted mesh with polynomial orders in the y-direction.

Energy-norm adaptivity

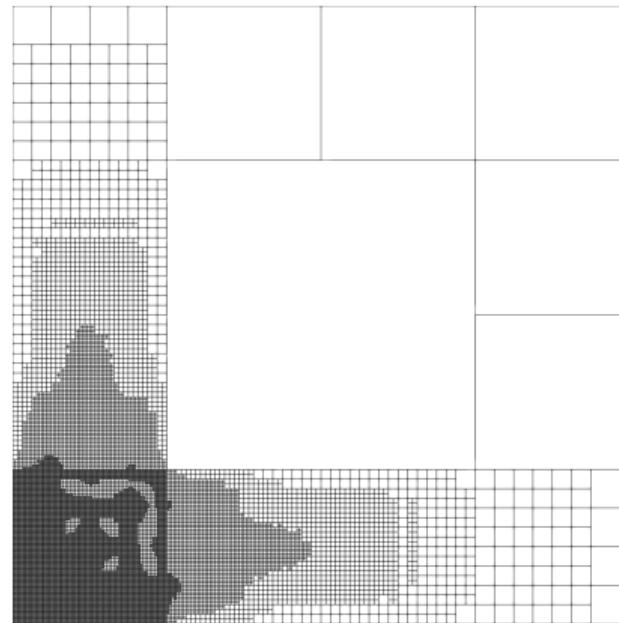
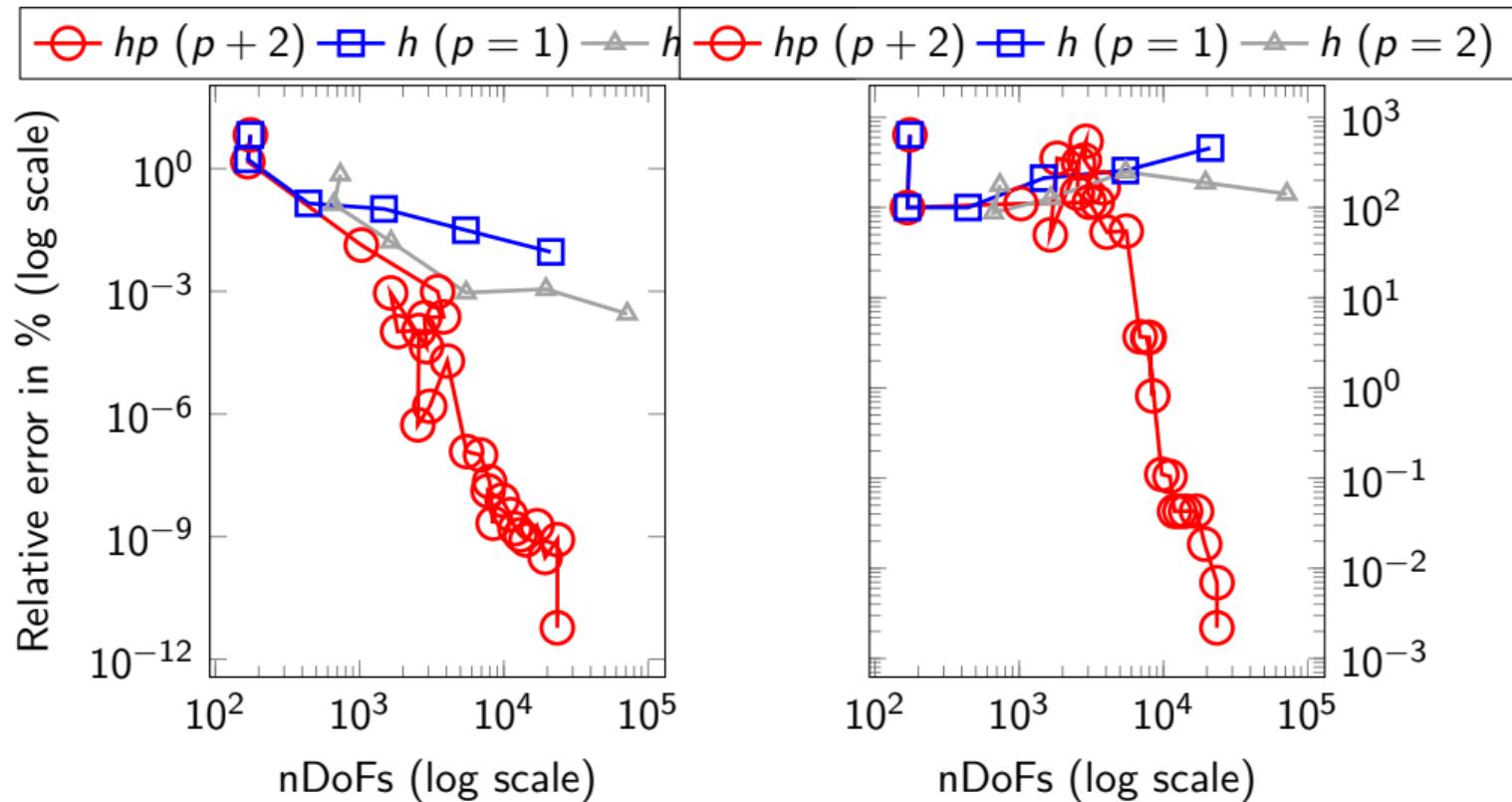


Figure: Final h -adapted mesh for $p = 1$.

Testing ERRORS



(a) Evolution of $\tilde{e}_{\text{rel}}^{\text{energy}}$ in the process.

(b) Evolution of $e_{\text{rel}}^{\text{QoI}}$ in the process.

Main Achievements

Main Achievements

Peer-Reviewed Publications

-  F. V. Caro, V. Darrigrand, J. Alvarez-Aramberri, and D. Pardo. "A Multi-Adaptive-Goal-Oriented Strategy to Generate Massive Databases of Parametric PDEs," To be submitted to *Computer Methods in Applied Mechanics and Engineering*, December 2023.
-  F. V. Caro, V. Darrigrand, J. Alvarez-Aramberri, E. Alberdi, and D. Pardo. "A Painless Multi-Level Automatic Goal-Oriented hp -Adaptive Coarsening Strategy for Elliptic and Non-Elliptic Problems," *Computer Methods in Applied Mechanics and Engineering*, vol. 401, 115641, 2022. <https://doi.org/10.1016/j.cma.2022.115641>
-  F. V. Caro, V. Darrigrand, J. Alvarez-Aramberri, E. A. Celaya, and D. Pardo. "1D Painless Multi-Level Automatic Goal-Oriented h and p Adaptive Strategies Using a Pseudo-Dual Operator," In *Computational Science – ICCS 2022*, pp. 347–357, 2022.
https://doi.org/10.1007/978-3-031-08754-7_43

Main Achievements

Conference Talks

- [1] F. V. Caro, V. Darrigrand, J. Alvarez-Aramberri, and D. Pardo.
Databases for Deep Learning Inversion Using A Goal-Oriented hp-Adaptive Strategy.
XI International Conference on Adaptive Modeling and Simulation,
Gothenburg, Sweden, June 19-21, 2023.
- [2] F. V. Caro, V. Darrigrand, J. Alvarez-Aramberri, E. Alberdi, and D. Pardo.
*A Painless Automatic hp-Adaptive Coarsening Strategy For Non-SPD problems:
A Goal-Oriented Approach.* 15th World Congress on Computational Mechanics
& 8th Asian Pacific Congress on Computational Mechanics,
Yokohama, Japan, July 31 - August 5, 2022.
- [3] F. V. Caro, V. Darrigrand, J. Alvarez-Aramberri, E. Alberdi, and D. Pardo.
*1D Painless Multi-Level Automatic Goal-Oriented h and p Adaptive Strategies using
a Pseudo-Dual Operator.* 22nd International Conference on Computational Science,
London, United Kingdom, June 21-23, 2022.

Main Achievements

Conference Talks

- [4] F. V. Caro, V. Darrigrand, J. Alvarez-Aramberri, E. Alberdi, and D. Pardo.
Goal-Oriented hp-Adaptive Finite Element Methods: A Painless Multilevel Automatic Coarsening Strategy For Non-SPD Problems. 8th European Congress on Computational Methods in Applied Sciences and Engineering, Oslo, Norway, June 5-9, 2022.
- [5] F. V. Caro, V. Darrigrand, J. Alvarez-Aramberri, E. Alberdi, and D. Pardo.
A Painless Goal-Oriented hp-Adaptive Strategy for Indefinite Problems.
16th U.S. National Congress on Computational Mechanics,
Chicago, U.S.A, July 25-29, 2021.
- [6] F. V. Caro, V. Darrigrand, J. Alvarez-Aramberri, E. Alberdi, and D. Pardo.
Goal-Oriented hp-Adaptive Finite Element Methods: A Painless Multi-level Automatic Coarsening Strategy. 10th International Conference on Adaptive Modeling and Simulation, Gothenburg, Sweden, June 21-23, 2021.

Main Achievements

Research Stays

FEB. 2023 – MAR. 2023
(2 months) University of Science and Technology (AGH),
Krakow (Poland).
Supervisor: Maciej Paszynski.

SEP. 2021 – Nov. 2021
(2 months) CNRS-IRIT-ENSEEIHT (N7),
Toulouse (France).

Supervisor: Vincent Darrigrand.

Nov. 2020 – DEC. 2020
(1 month) CNRS-IRIT-ENSEEIHT (N7),
Toulouse (France).

Supervisor: Vincent Darrigrand.

Main Achievements

Bilbao



Toulouse



Kraków



Conclusions and Future Work

Conclusions

- We employ hierarchical basis functions that effectively address the challenge of *hanging nodes*.
- We have developed **simple-to-implement** h - and p -GOA strategies that use an unconventional symmetric and positive definite bilinear form for possibly non-elliptic goal-oriented problems.
- We have expanded upon a painless automatic hp strategy, initially developed for energy-norm adaptivity, to both non-elliptic and goal-oriented problems.
- We have extended the applicability of a coarsening strategy to encompass parametric PDEs.

Future Work

- Extend algorithms to address multi-physics problems, notably $H(\text{curl})$ and $H(\text{div})$.
- Validate the efficacy of our algorithms in real-world scenarios such as Magnetotellurics, Controlled Sources, and Logging While Drilling.
- Analyze the impact of the nature and distribution of various random samples on DL inversion for optimization.
- Enhance parallelization and factorization techniques to reduce computational resource requirements in future applications.