Patagonia parrots density analysis

Francisco Denes and Peter Solymos

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Motivation

We want to model the density of the austral parakeet *Enicognathus ferrugineus* and the slender-billed parakeet *Enicognathus leptorhynchus* from count data obtained with road transect surveys in Patagonia. Because these parrots are gregarious, we want to assess whether group size and size of groups vary across habitats (classified as 'urban', 'agropastoral' and 'other' (i.e. various natural forest formations), and between breeding (Nov-Dec) and non-breeding seasons (Jan-Oct). We use distance sampling methods to account for detectability of parrots.

These animals tend to form smaller or larger groups, but groups of size 2 are also often observed as a result of mating pairs, as shown in Figure 1.

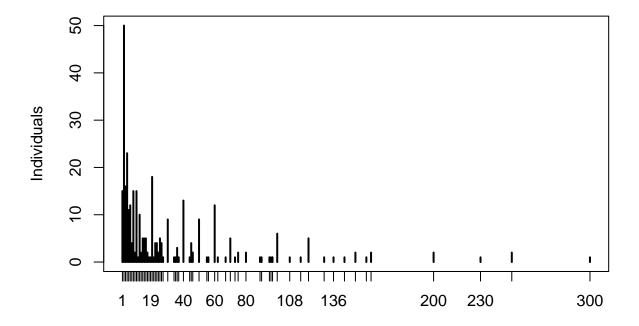


Figure 1: Enicognathus ferrugineus count frequencies

Austral parakeet Enicognathus ferrugineus

Estimating effective detection radius (EDR)

We fit distance sampling models with detection predictor variables, including the average group size, the number of groups that were observed (to account for the possibility that grouping behaviour influences detectability), and also habitat type. We used forward-stepwise model selection, starting with single covariate models and eliminating covariates that do not improve model parsimony (i.e. result in dAIC < 2) in relation to the null model.

Table 1: E. ferrugineous EDR models AIC

	df	AIC	dAIC
EDR.habitatype	3	1434.532	0.00
EDR.null	1	1438.166	3.63
EDR.avggroupsize	2	1439.144	4.61
EDR.numbergroups	2	1440.166	5.63

The model (EDR.habitatype) has the lowest AIC (Table 1), indicating that habitat type affects the effective detection radius (EDR) of *Enicognathus ferrugineus*. Specifically, detection radius is higher in agropastoral than urban and other habitats (Table 2). This means that detectability is higher in agropastoral habitats, and also that the area surveyed in a given sample unit is larger if the habitat therein is agropastoral vs. urban or other, presumably because it is possible to see further in pastures and planted fields than in forest or urban environments.

Table 2: E. ferrugineous top-ranked EDR model estimates

	Estimate	Std. Error	z value	Pr(> z)
log.tau_(Intercept)	4.589	0.038	119.365	0.000
log.tau_Urban	0.057	0.058	0.988	0.323
$log.tau_Agropastoral$	0.263	0.103	2.556	0.011

Table 3: E. ferrugineous habitat-specific EDR (m)

Habitat	EDR
Other	98.36847
Urban	90.30047
Agropastoral	127.92056
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Models for number of groups

We then model the number of groups as a function of covariates. The model for number of groups is $G_i \sim \text{Poisson}(D_i A_i)$, where $D_i = \text{covariates}$ and $A_i = \text{area sampled in site } i$. A_i is calculated using the habitat-specific estimated EDR as $A_i = L_i * (2EDR_i)$, where L is the length of each site (i.e. the transect). A_i is added to the model as an offset.

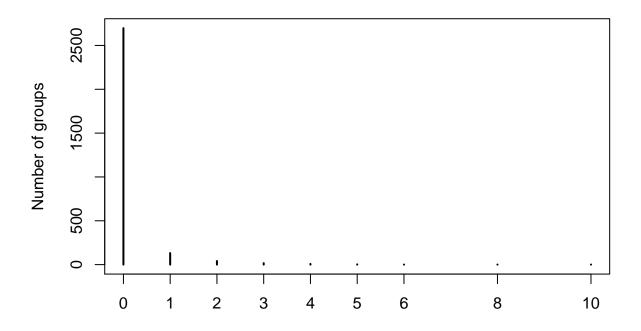


Figure 2: Enicognathus ferrugineus group numbers

Model selection

For number of groups, we use a stage-wise selection procedure. First, we build a first set of models to evaluate the effect of covariates related to habitat (habitat type and elevation).

Table 4: E. ferrugineous number of group models

	df	AIC	dAIC
ngroup.hab.ele2	5	2082.963	0.00
ngroup.hab.ele	4	2114.437	31.47
ngroup.hab	3	2133.717	50.75
ngroup.ele2	3	2275.565	192.60
ngroup.ele	2	2323.806	240.84

The model with both habitat type and elevation (quadratic effect) has the lowest AIC (Table 4), indicating that both covariates affect the number of groups:

Table 5: E. ferrugineous 'ngroup.hab.ele2' model estimates

	Estimate	Std. Error	z value	$\Pr(> z)$
(Intercept)	-3.108	0.233	-13.356	0

	Estimate	Std. Error	z value	$\Pr(> z)$
habitatOther	1.507	0.202	7.455	0
habitatUrban	2.390	0.202	11.823	0
elevation	-0.004	0.001	-7.032	0
I(elevation^2)	0.000	0.000	5.947	0

Table 6: Deviance partitioning of 'ngroup.hab.ele2' model for $\it E. ferrugineous$

	Df	Deviance	Resid. Df	Resid. Dev
NULL	NA	NA	2900	1865.338
habitat	2	207.965	2898	1657.373
elevation	1	21.279	2897	1636.094
I(elevation^2)	1	33.475	2896	1602.619

We then proceed by adding within-year temporal covariates (breeding/non-breeding season and Julian date) and their interactions with habitat.

Table 7: $E.\ ferrugineous$ number of group models (within-year temporal predictors) AIC

	df	AIC	dAIC
ngroup.habXseason.ele2	8	2065.594	0.00
ngroup.hab.season.ele2	6	2067.555	1.96
ngroup.hab X j date.ele 2	8	2081.546	15.95
ngroup.hab.ele2	5	2082.963	17.37
ngroup.hab.jdate.ele2	6	2084.491	18.90

The model with the season*habitat interaction (Table 8) is equally parsimonious with the model with only the additive effects (Table 9). Given these are nested models, this indicates the interaction only marginally improves the model fit, so will continue with model with season and no interaction:

Table 8: $E.\ ferrugineous$ 'ngroup.hab Xseason.ele2' interaction model estimates

	Estimate	Std. Error	z value	$\Pr(> z)$
(Intercept)	-18.187	558.012	-0.033	0.974
elevation	-0.004	0.001	-6.547	0.000
I(elevation ²)	0.000	0.000	5.500	0.000
habitatOther	15.797	558.012	0.028	0.977
habitatUrban	2.008	908.375	0.002	0.998
seasonnon-breeding	15.186	558.012	0.027	0.978
habitatOther:seasonnon-breeding	-14.357	558.012	-0.026	0.979
habitatUrban:seasonnon-breeding	0.266	908.375	0.000	1.000

Table 9: $E.\ ferrugineous$ 'ngroup.hab.season.ele2' additive model estimates

	Estimate	Std. Error	z value	$\Pr(> z)$
(Intercept)	-4.250	0.412	-10.318	0.000
habitatOther	1.510	0.204	7.411	0.000
habitatUrban	2.314	0.203	11.424	0.000
elevation	-0.004	0.001	-6.408	0.000
I(elevation^2)	0.000	0.000	5.365	0.000
seasonnon-breeding	1.175	0.340	3.451	0.001

Finally, we assess year effects by adding a year covariate (2013-2016).

Table 10: $E.\ ferrugineous$ number of group models (year predictor) AIC table

	df	AIC	dAIC
ngroup.hab.season.ele2.year	9	2022.265	0.00
ngroup.hab.season.ele2	6	2067.555	45.29

The model with lowest AIC indicates that the number of groups is affected by habitat type, elevation, season (breeding/non breeding) and year.

Table 11: $E.\ ferrugineous$ top-ranked (ngroup.hab.season.ele2.year) model estimates

	Estimate	Std. Error	z value	$\Pr(> z)$
(Intercept)	-4.716	0.417	-11.311	0.000
habitatOther	1.739	0.206	8.434	0.000
habitatUrban	2.460	0.204	12.075	0.000
elevation	-0.003	0.001	-5.712	0.000
I(elevation ²)	0.000	0.000	4.588	0.000
seasonnon-breeding	1.022	0.348	2.939	0.003
as.factor(year)2014	0.092	0.168	0.549	0.583
as.factor(year)2015	0.918	0.161	5.688	0.000
as.factor(year)2016	0.612	0.205	2.979	0.003

Table 12: Deviance partitioning of top-ranked 'ngroup.hab.season.ele2.year' model for $E.\ ferrugineous$

	Df	Deviance	Resid. Df	Resid. Dev
NULL	NA	NA	2900	1865.338
habitat	2	207.965	2898	1657.373
elevation	1	21.279	2897	1636.094
I(elevation^2)	1	33.475	2896	1602.619
season	1	17.408	2895	1585.211
as.factor(year)	3	51.290	2892	1533.921

Models for group size

The count distribution is characterized with a spike at 2 (Figure 1), and by the absence of 0s due to group size being conditional on having >0 birds to consider it a group.

We start developing a general V-Inflated Poisson (VIP) model ("V" stands for "variable", in reference to the "Z" representing 0 in a zero-inflated Poisson model, or ZIP), then we add the >0 condition.

R functions for the VIP model are presented at the end of this document, including simulations to check the estimating procedure.

Maximum likelihood

Let Y be a random variable, and y are observations, V is the count value that has some extra probability mass $(V=0 \text{ is the ZIP model}), f(y;\lambda)$ is the Poisson density $(f(y;\lambda)=e^{-\lambda}\frac{\lambda^y}{y!})$.

The V-Inflated density can be written as $P(Y = y) = \phi I(Y = V) + (1 - \phi)f(y; \lambda)$ which is $\phi + (1 - \phi)f(V; \lambda)$ when Y = V and $(1 - \phi)f(y; \lambda)$ otherwise.

We define the extra probability mass at V=2 to account for the group size peak in pairs. The model is also truncated at 0, because there are no groups with 0 individuals. We include habitat type as a covariate for the count (Poisson) component, and compare models with and without season (breeding/non-breeding) as a covariate for the V-inflation probability.

Goodness-of-fit

Model goodness-of-fit can be evaluated visually, inspecting the proportions of fitted values against the count distribution. Deviation from the 1:1 line can be used as a goodness-of-fit metric, with the better model showing smaller deviation.

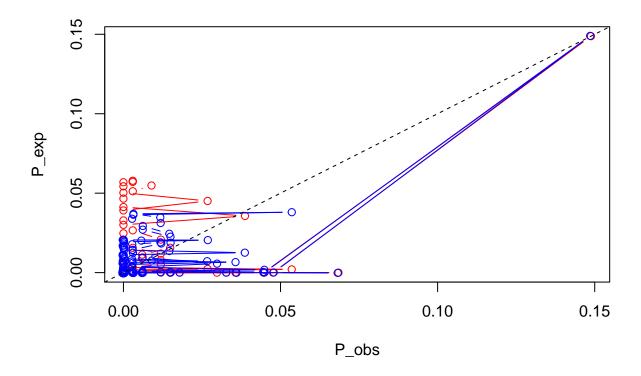


Figure 3: *Enicognathus ferrugineus* group size model goodness-of-fit (red: null model; blue: habitat.season model). Better model is closer to 1:1 line.

The deviation of null model is 1.38, and of the VIP.habitat.season model is 1.17.

Model estimates and confidence intervals

Estimated model coefficients (Table 13) indicate that group sizes are larger in urban ($\beta = 0.877$) and agropastoral ($\beta = 0.622$) habitats when compared to other natural habitats. Moreover, the probability of there being extra pairs (i.e. the V-inflation probability, ϕ) is smaller in the non-breeding season ($\beta = -2.058$).

Table 13: E. ferrugineous group size model estimates

	Estimate	Std. Error	z value	$\Pr(> z)$
P_(Intercept)	3.027	0.019	161.500	0.000
P_Urban	0.877	0.023	38.534	0.000
P_Agropastoral	0.622	0.036	17.167	0.000
$V_{\underline{\hspace{0.5cm}}}(Intercept)$	0.223	0.671	0.333	0.739
V _seasonnon-breeding	-2.058	0.690	-2.984	0.003

Confidence intervals (CI) based on estimated standard errors can be obtained (Table 14):

Table 14: E. ferrugineous group size model CI

	2.5%	97.5%
P_(Intercept)	2.990	3.064
P_Urban	0.832	0.922
P_Agropastoral	0.551	0.694
V_(Intercept)	-1.092	1.538
V _seasonnon-breeding	-3.410	-0.706

Alternatively, we can estimate confidence intervals based on quantiles using bootstrap samples (with n=250) for the estimated coefficients (Table 15).

Table 15: Estimated coefficients and 95% CI based on bootstrap sample (n=250) quantiles for *E. ferrugineous* group size models

	Estimate	2.5%	97.5%
P_(Intercept)	3.027	2.924	2.924
P_Urban	0.877	0.956	0.956
P_Agropastoral	0.622	0.512	0.512
V_(Intercept)	0.223	0.693	0.693
V _seasonnon-breeding	-2.058	-2.554	-2.554

Density Predictions

The mean expected abundance (E) can be predicted for each surveyed site i as $E_i = (\phi_i 2 + (1 - \phi_i)\mu_i)N_i$, where N is the mean number of groups, μ is the mean group size and ϕ is the V(=2)-inflation probability (i.e. the probability that a given group will be a pair).

We can then obtain the expected density stratified by habitat type, year and season, as $D = \frac{\sum E}{\sum A}$, where A is the area surveyed.

Table 16: $E.\ ferrugineous$ mean density predictions for each habitat type, year and season

Habitat	Year	Season	Density (individuals/ha)	Density (2.5% CI)	Density (97.5% CI)
Agropastoral	2013	breeding	0.001	0.000	0.002
Other	2013	breeding	0.002	0.001	0.004
Urban	2013	breeding	0.007	0.003	0.017
Agropastoral	2015	breeding	0.002	0.001	0.004
Other	2015	breeding	0.005	0.002	0.011
Urban	2015	breeding	0.022	0.008	0.054
Agropastoral	2013	non-breeding	0.002	0.001	0.004
Other	2013	non-breeding	0.011	0.008	0.015
Urban	2013	non-breeding	0.041	0.028	0.057
Agropastoral	2014	non-breeding	0.003	0.002	0.004
Other	2014	non-breeding	0.012	0.009	0.016
Urban	2014	non-breeding	0.048	0.038	0.062
Agropastoral	2015	non-breeding	0.008	0.005	0.011
Other	2015	non-breeding	0.027	0.021	0.036
Urban	2015	non-breeding	0.111	0.085	0.143

Habitat	Year	Season	Density (individuals/ha)	Density $(2.5\% \text{ CI})$	Density (97.5% CI)
Agropastoral	2016	non-breeding	0.006	0.003	0.009
Other	2016	non-breeding	0.019	0.013	0.027
Urban	2016	non-breeding	0.082	0.057	0.115

Slender-billed parakeet $Enicognathus\ leptorhynchus$

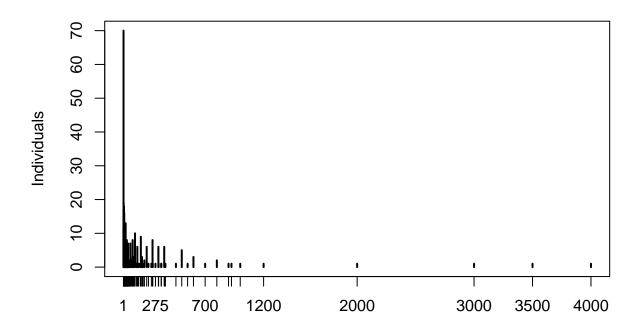


Figure 4: $Enicognathus\ leptorhynchus\ count\ frequencies$

Estimating effective detection radius (EDR) $\,$

We follow the same procedure used for the austral parakeet above.

Table 17: E. leptorhynchus EDR models AIC

	df	AIC	dAIC
EDR.avggroupsize.habitat	4	1054.464	0.00
EDR.habitat.avggroupsize.numbergroups	5	1059.062	4.60

	df	AIC	dAIC
EDR.avggroupsize	2	1063.875	9.41
EDR.avggroupsize.numbergroups	3	1064.553	10.09
EDR.habitat.numbergroups	4	1069.376	14.91
EDR.habitatype	3	1070.755	16.29
EDR.null	1	1086.330	31.87
EDR.numbergroups	2	1087.172	32.71

The model (EDR.avggroupsize.habitat) has the lowest AIC (Table 16), indicating that habitat type and average group size affect the effective detection radius (EDR) of *Enicognathus leptorhynchus*. The mean EDR for each habitat, predicted using model coefficients and the habitat-specific means of average group size, is shown in Table 17.

Table 18: E. leptorhynchus top-ranked EDR model estimates

	Estimate	Std. Error	z value	$\Pr(> z)$
log.tau_(Intercept)	5.361	0	1.273654e + 51	0
$log.tau_gavg$	0.001	0	9.610881e + 50	0
$log.tau_Urban$	-0.266	0	-6.308165e+49	0
$log.tau_Agropastoral$	-0.029	0	-4.843080e+48	0

Table 19: E. leptorhynchus habitat-specific mean EDR (m)

Habitat	EDR
Other	235.4183
Urban	224.2124
Agropastoral	180.7002

Models for number of groups

The model for number of groups is $G_i \sim Poisson(D_iA_i)$, where $D_i = covariates$ and $A_i = area$ sampled in site i. A_i is calculated using the habitat-specific estimated EDR, and is added to the model as an offset.

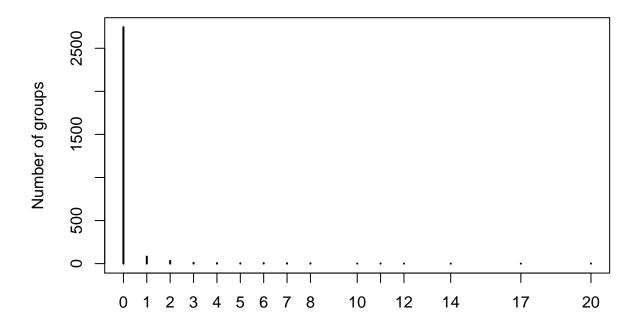


Figure 5: Enicognathus leptorhynchus group numbers

Model selection

First set of models to evaluate the effect of habitat type and elevation covariates (Table 18):

Table 20: E. leptorhynchus number of group models

	df	AIC	dAIC
ngroup.hab.ele2	5	2486.990	0.00
ngroup.hab.ele	4	2487.834	0.84
ngroup.hab	3	2493.346	6.36
ngroup.ele2	3	2746.031	259.04
ngroup.ele	2	2761.122	274.13

The models 'ngroup.hab.ele' (with linear elevation effect) and 'ngroup.hab.ele2' (with quadratic effect) are equally parsimonious (Table 18), but the quadratic term in the latter is mostly uninformative (Tables 19-20). Given these are nested models, we drop the quadratic term and continue with the model with habitat and linear elevation effects.

Table 21: E. leptorhynchus 'ngroup.hab.ele' model estimates

	Estimate	Std. Error	z value	$\Pr(> z)$
(Intercept)	-2.781	0.123	-22.568	0.000
habitatOther	-2.035	0.147	-13.874	0.000
habitatUrban	-0.734	0.143	-5.116	0.000
elevation	0.000	0.000	2.736	0.006

Table 22: Deviance partitioning of 'ngroup.hab.ele' model for $\it E.\ leptorhynchus$

	Df	Deviance	Resid. Df	Resid. Dev
NULL	NA	NA	2900	2358.856
habitat	2	270.326	2898	2088.530
elevation	1	7.512	2897	2081.018

Adding within-year temporal covariates (breeding/non-breeding season and Julian date) and their interactions with habitat:

Table 23: $E.\ leptorhynchus$ number of group models (within-year temporal predictors) AIC

	df	AIC	dAIC
ngroup.habXjdate.ele	7	2110.653	0.00
ngroup.hab.ele.jdate	5	2230.345	119.69
ngroup.habXseason.ele	7	2472.731	362.08
ngroup.hab.ele	4	2487.834	377.18
ngroup.hab.ele.season	5	2489.059	378.41

The model 'ngroup.habXjdate.ele' has the lowest AIC (Table 21).

Table 24: E. leptorhynchus 'ngroup.habXjdate.ele' model estimates

	Estimate	Std. Error	z value	$\Pr(> z)$
(Intercept)	-1.463	0.167	-8.741	0.000
elevation	0.000	0.000	1.939	0.053
habitatOther	2.234	0.590	3.788	0.000
habitatUrban	0.412	0.310	1.329	0.184
jdate	-0.006	0.001	-10.029	0.000
habitatOther:jdate	-0.040	0.007	-5.589	0.000
habitatUrban:jdate	-0.009	0.002	-4.095	0.000

Finally, we assess year effects by adding a year covariate (2013-2016).

Table 25: $E.\ leptorhynchus$ number of group models (year predictor) AIC table

	df	AIC	dAIC
ngroup.habXjdate.ele.year	10	1907.516	0.00
ngroup.hab X j date.ele	7	2110.653	203.14

The best-ranked model indicates that the number of groups is affected by habitat type, elevation, Julian date and year (Tables 23-24).

Table 26: $E.\ leptorhynchus$ 'ngroup.hab Xjdate.ele.year' model estimates

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	-3.424	0.553	-6.189	0.000
elevation	0.001	0.000	3.208	0.001
habitatOther	0.677	0.350	1.932	0.053
habitatUrban	0.566	0.257	2.206	0.027
jdate	-0.002	0.002	-1.291	0.197
as.factor(year)2014	-0.201	0.352	-0.570	0.569
as.factor(year)2015	1.388	0.233	5.953	0.000
as.factor(year)2016	1.909	0.442	4.322	0.000
habitatOther:jdate	-0.020	0.004	-5.091	0.000
habitatUrban:jdate	-0.008	0.002	-4.327	0.000

Table 27: Deviance partitioning of 'ngroup.habXjdate.ele.year' model for $E.\ leptorhynchus$

	Df	Deviance	Resid. Df	Resid. Dev
NULL	NA	NA	2900	2358.856
elevation	1	0.551	2899	2358.306
habitat	2	277.287	2897	2081.018
jdate	1	259.490	2896	1821.528
as.factor(year)	3	239.982	2893	1581.547
habitat:jdate	2	92.847	2891	1488.700

Models for group size

Goodness-of-fit

Model goodness-of-fit can be evaluated visually, inspecting the proportions of fitted values against the count distribution. Deviation from the 1:1 line can be used as a goodness-of-fit metric, with the better model showing smaller deviation.

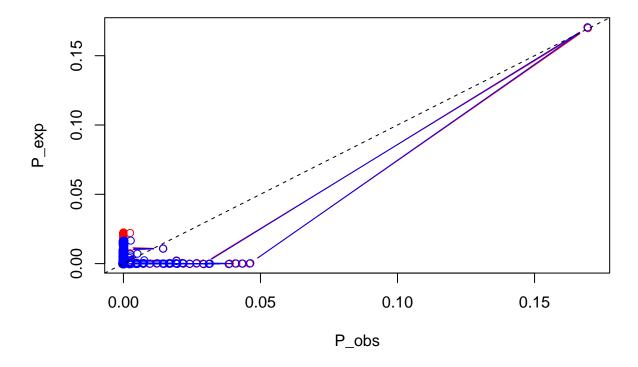


Figure 6: *Enicognathus leptorhynchus* group size model goodness-of-fit (red: null model; blue: habitat.season model). Better model is closer to 1:1 line.

The deviation of null model is 1.62, and of the VIP.habitat.season model is 1.59.

Model estimates and confidence intervals

Table 28: E. leptorhynchus group size model estimates

	Estimate	Std. Error	z value	$\Pr(> z)$
P_(Intercept)	5.231	0	6.213682e + 54	0
P_Urban	-0.055	0	-1.311872e+49	0
P_Agropastoral	0.154	0	3.659220e+49	0
V_(Intercept)	-0.432	0	-7.254354e+49	0
V _seasonnon-breeding	-1.293	0	-2.172534e+50	0

Confidence intervals (CI) based on estimated standard errors can be obtained (Table 26):

Estimated model coefficients (Table 25) indicate that group sizes are slightly smaller in urban ($\beta = -0.055$) and larger in agropastoral ($\beta = 0.154$) habitats when compared to other natural habitats. Moreover, the probability of there being extra pairs (i.e. the V-inflation probability, ϕ) is smaller in the non-breeding season ($\beta = -1.293$).

The very low estimated standard errors indicate that there may be some issues with this model. This is further corroborated by the different coefficient and CI values obtained using the bootstrap mean and quantile approach.

Table 29: Coefficients and 95% CI based on bootstrap mean and quantiles for *E. leptorhynchus* group size models

	Estimate	2.5%	97.5%
P_(Intercept)	4.478	4.478	4.478
P_Urban	0.698	0.698	0.698
P_Agropastoral	0.454	0.454	0.454
V_(Intercept)	-0.061	-0.062	-0.059
V _seasonnon-breeding	-1.685	-1.687	-1.684

Because of the lack of concordance between direct estimates and the bootstrap approach, indicative of issues with the model, and since there is only a very small difference in goodness-of-fit compared to the null model (Figure 6), we report density prediction using the null model.

Density Predictions

The mean expected abundance (E) can be predicted for each surveyed site i as $E_i = (\phi_i 2 + (1 - \phi_i)\mu_i)N_i$, where N is the mean number of groups, μ is the mean group size and ϕ is the V(=2)-inflation probability (i.e. the probability that a given group will be a pair).

We can then obtain the expected density stratified by habitat type, year and season, as $D = \frac{\sum E}{\sum A}$, where A is the area surveyed.

Table 30: E. leptorhynchus mean density predictions for each habitat type, year and season

Habitat	Year	Season	Density (individuals/ha)	Density (2.5% CI)	Density (97.5% CI)
Agropastoral	2013	breeding	0.059	0.038	0.092
Other	2013	breeding	0.000	0.000	0.002
Urban	2013	breeding	0.010	0.004	0.022
Agropastoral	2015	breeding	0.258	0.201	0.326
Other	2015	breeding	0.001	0.000	0.008
Urban	2015	breeding	0.036	0.017	0.079
Agropastoral	2013	non-breeding	0.073	0.048	0.112
Other	2013	non-breeding	0.000	0.000	0.003
Urban	2013	non-breeding	0.013	0.006	0.028
Agropastoral	2014	non-breeding	0.075	0.052	0.110
Other	2014	non-breeding	0.014	0.008	0.025
Urban	2014	non-breeding	0.044	0.028	0.068
Agropastoral	2015	non-breeding	0.295	0.234	0.369
Other	2015	non-breeding	0.003	0.001	0.015
Urban	2015	non-breeding	0.058	0.032	0.110
Agropastoral	2016	non-breeding	0.795	0.652	0.973
Other	2016	non-breeding	0.486	0.364	0.661
Urban	2016	non-breeding	0.730	0.531	0.990

VIP model - R functions and simulations

Peter Solymos

Functions

The vip function does the optimization. method argument can take values listed for optim and also "DE" for differential evolution algorithm. If there are convergence issues with "Nelder-Mead", try "SANN" and "DE".

```
library(DEoptim)
library(Matrix)
vip <-
function(Y, X, Z, V=0,
offsetx, offsetz, weights, linkz="logit",
truncate=FALSE, hessian=TRUE, method="Nelder-Mead", init=NULL, ...) {
    if (missing(Y))
        stop("C'mon, you must have some data?!")
    if (truncate && any(Y < 1))</pre>
        stop("Y must be >0 when truncate=TRUE")
    n <- length(Y)
    id0 <- Y == V
    id1 <- !id0
    if (missing(X)) {
        X <- matrix(1, n, 1)</pre>
        colnames(X) <- "(Intercept)"</pre>
    }
    if (missing(Z)) {
        Z <- matrix(1, n, 1)</pre>
        colnames(Z) <- "(Intercept)"</pre>
    }
    kx \leftarrow ncol(X)
    kz \leftarrow ncol(Z)
    if (missing(offsetx))
        offsetx <- 0
    if (missing(offsetz))
        offsetz <- 0
    if (missing(weights))
        weights <- rep(1, n)
    linkinvx <- poisson("log")$linkinv</pre>
    linkinvz <- binomial(linkz)$linkinv</pre>
    good.num.limit <- c(.Machine$double.xmin, .Machine$double.xmax)^(1/3)</pre>
    ## VIP model full likelihood
    nll_VIP_ML <- function(parms) {</pre>
        mu <- as.vector(linkinvx(X %*% parms[1:kx] + offsetx))</pre>
        phi <- as.vector(linkinvz(Z %*% parms[(kx + 1):(kx + kz)] + offsetz))</pre>
        loglik0 <- log(phi + (1 - phi) * dpois(V, lambda = mu, log = FALSE))</pre>
        loglik1 <- log(1 - phi) + dpois(Y, lambda = mu, log = TRUE)</pre>
        loglik <- sum(weights[id0] * loglik0[id0]) + sum(weights[id1] * loglik1[id1])</pre>
        if (!is.finite(loglik) || is.na(loglik))
             loglik <- -good.num.limit[2]</pre>
        -loglik
    }
    ## 0-truncated VIP model full likelihood
```

```
nll_VIP_TR <- function(parms) {</pre>
    mu <- as.vector(linkinvx(X %*% parms[1:kx] + offsetx))</pre>
    phi <- as.vector(linkinvz(Z %*% parms[(kx + 1):(kx + kz)] + offsetz))</pre>
    loglik0 <- log(phi + (1 - phi) * dpois(V, lambda = mu, log = FALSE) / (1-exp(-mu)))
    loglik1 <- log((1 - phi) * dpois(Y, lambda = mu, log = FALSE) / (1-exp(-mu)))</pre>
    loglik <- sum(weights[id0] * loglik0[id0]) + sum(weights[id1] * loglik1[id1])</pre>
    if (!is.finite(loglik) | is.na(loglik))
        loglik <- -good.num.limit[2]</pre>
    -loglik
}
.solvenear <-
function(x)
    xinv <- try(solve(x), silent = TRUE)</pre>
    if (inherits(xinv, "try-error"))
        xinv <- as.matrix(solve(Matrix::nearPD(x)$mat))</pre>
    xinv
if (is.null(init))
    init \leftarrow rep(0, kx+kz)
nll <- if (truncate) nll_VIP_TR else nll_VIP_ML</pre>
if (method == "DE") {
    DElimit <- 10
    up <- rep(DElimit, length(init))</pre>
    lo <- -up
    opt <- DEoptim(fn=nll, lower=lo, upper=up,</pre>
        control=list(trace=FALSE, itermax=length(init)*200))
    par <- opt$optim$bestmem</pre>
    names(par) <- c(paste0("P_", colnames(X)), paste0("V_", colnames(Z)))</pre>
    11 <- -opt$optim$bestval</pre>
    if (hessian) {
        hess <- optimHess(opt$optim$bestmem, nll)
        vc <- .solvenear(hess)</pre>
        matrix(NA, length(par), length(par))
} else {
    opt <- optim(init, nll,</pre>
        hessian=hessian, method=method, ...)
    par <- opt$par
    names(par) <- c(paste0("P_", colnames(X)), paste0("V_", colnames(Z)))</pre>
    vc <- if (hessian)
        .solvenear(opt$hessian) else matrix(NA, length(par), length(par))
    11 <- -opt$value</pre>
}
dimnames(vc) <- list(names(par), names(par))</pre>
out <- list(call=match.call(),</pre>
    coefficients=par, loglik=ll, vcov=vc, nobs=n,
    truncate=truncate, Y=Y, X=X, Z=Z, V=V,
    offsetx=offsetx, offsetz=offsetz, weights=weights,
    linkz=linkz, method=method, init=init)
```

```
class(out) <- "vip"</pre>
}
vcov.vip <- function(object, ...) object$vcov</pre>
logLik.vip <- function (object, ...)</pre>
    structure(object$loglik, df = object$nobs - length(object$coef),
        nobs = object$nobs, class = "logLik")
summary.vip <- function (object, ...) {</pre>
    k <- length(object$coefficients)</pre>
    coefs <- coef(object)</pre>
    se <- sqrt(diag(vcov(object)))</pre>
    tstat <- coefs/se
    pval <- 2 * pnorm(-abs(tstat))</pre>
    coefs <- cbind(coefs, se, tstat, pval)</pre>
    colnames(coefs) <- c("Estimate", "Std. Error", "z value", "Pr(>|z|)")
    coefs <- coefs[1:k, , drop = FALSE]</pre>
    rownames(coefs) <- names(coef(object))</pre>
    out <- list(call = object$call, coefficients=coefs, loglik = object$loglik,</pre>
        bic=BIC(object), truncate=object$truncate)
    class(out) <- "summary.vip"</pre>
    return(out)
print.summary.vip <- function (x, digits, ...)</pre>
    if (missing(digits))
        digits <- max(3, getOption("digits") - 3)</pre>
    cat("\nCall:", deparse(x$call,
        width.cutoff = floor(getOption("width") * 0.85)), "", sep = "\n")
    cat("V-Inflated", if (x$truncate) "(Zero-Truncated)" else "", "Poisson Model\n\n")
    cat(paste("Coefficients:\n", sep = ""))
    printCoefmat(x$coefficients, digits = digits, signif.legend = FALSE)
    if (!any(is.na(array(x$coefficients)))) {
        if (getOption("show.signif.stars") & any(x$coefficients[,4] < 0.1))</pre>
             cat("---\nSignif. codes: ", "0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1", "\n")
    }
    cat("\nLog-likelihood:", formatC(x$loglik, digits = digits),
         "\nBIC =", formatC(x$bic, digits = digits), "\n")
    cat("\n")
    invisible(x)
confint.vip <-</pre>
function (object, parm, level = 0.95, ...)
    cf <- coef(object)</pre>
    pnames <- names(cf)</pre>
    if (missing(parm)) {
        parm <- pnames
    } else {
        if (is.numeric(parm))
            parm <- pnames[parm]</pre>
    }
    a <- (1 - level)/2
    a <- c(a, 1 - a)
```

```
pct <- paste(format(100 * a, trim = TRUE, scientific = FALSE, digits = 3), "%", sep="")
ci <- array(NA, dim = c(length(parm), 2), dimnames = list(parm, pct))
fac <- qnorm(a)
ses <- sqrt(diag(vcov(object, model, type)))
ci[] <- cf[parm] + ses[parm] %o% fac
ci
}</pre>
```

Simple case

Est=coef(mod))

```
set.seed(123)
n <- 1000
lam <- 2 \# poisson mean, can be a vector of length n
phi <- 0.4 # V-inflation probability, can be a vector of length n
V \leftarrow 2 \# V  is the count value, can be 0, 2, etc
y <- y0 <- rpois(n, lam)
a <- rbinom(n, 1, phi)
y[a > 0] \leftarrow V
table(Poisson=y0, Vinflated=y)
##
          Vinflated
## Poisson
                     2
                                 5
            0
                 1
##
         0
           81
                 0 51
                         0
                             0
                                 0
                                         0
             0 151 126
                         0
##
         1
                             0
         2
                 0 274
                         0
                             0
                                 0
##
             0
         3
                 0 65 112
                             0
##
            0
                                 0
                                    0 0
           0
                                     0 0
##
         4
                 0 39
                         0
                            43
                                 0
         5
                         0
                                29
##
            0
                 0 12
                             0
                                     0
         6
           0
                         0
                             0
                                     9
##
                 0
                    6
                                 0
##
         7
             0
                 0
                         0
                             0
                                 0
                                     0
                                        0
                     1
##
         8
             0
                 0
                     0
                         0
                             0
                                 0
                                     0
mod \leftarrow vip(Y=y, V=2)
summary(mod)
##
## Call:
## vip(Y = y, V = 2)
## V-Inflated Poisson Model
##
## Coefficients:
                 Estimate Std. Error z value Pr(>|z|)
## P_(Intercept) 0.70472
                             0.02909 24.224 < 2e-16 ***
## V_(Intercept) -0.33900
                             0.08824 -3.842 0.000122 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Log-likelihood: -1345
## BIC = 9585
cbind(True=c(log_lam=log(lam), logit_phi=qlogis(phi)),
```

```
##
                   True
## log_lam
             0.6931472 0.7047243
## logit_phi -0.4054651 -0.3389963
```

Covariates for the non-V part

##

True

Est

```
set.seed(123)
n <- 1000
x \leftarrow rnorm(n)
df <- data.frame(x=x)</pre>
X <- model.matrix(~x, df)</pre>
beta \leftarrow c(-0.5,-0.5) # Intercept and beta values for covariate
lam <- exp(X %*% beta) # poisson mean, can be a vector of length n
phi <- 0.4 # V-inflation probability, can be a vector of length n
V \leftarrow 2 \# V is the count value, can be 0, 2, etc
y <- y0 <- rpois(n, lam)
a <- rbinom(n, 1, phi)
y[a > 0] \leftarrow V
table(Poisson=y0, Vinflated=y)
         Vinflated
## Poisson 0 1
                    2
                       3
                               5
                           4
##
        0 299
                0 228
                       0
                           0
                              0
##
        1
            0 201 131
                      0
                           0
        2
##
               0 95
                      0
           0
                           0
                              0
                                  0
        3
##
           0
                0 14 18 0
        4 0
                           6
                             0
##
               0
                     0
##
        5
            0
               0 0
                       0
                           0
                              4
##
        6
            0
               0
                    0
                       0
                           0
                               0
mod \leftarrow vip(Y=y, X=X, V=2)
summary(mod)
##
## Call:
## vip(Y = y, X = X, V = 2)
## V-Inflated Poisson Model
##
## Coefficients:
               Estimate Std. Error z value Pr(>|z|)
0.05331 -9.198 < 2e-16 ***
## P x
                -0.49031
## V_(Intercept) -0.42040
                         0.07813 -5.381 7.42e-08 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Log-likelihood: -1141
## BIC = 9169
cbind(True=c(beta=beta, logit_phi=qlogis(phi)),
     Est=coef(mod))
```

Methods

```
coef(mod)
## P_(Intercept)
                        P_x V_(Intercept)
## -0.3967122 -0.4903124 -0.4203965
vcov(mod)
##
               P_(Intercept)
                                     P_x V_(Intercept)
## P_(Intercept) 0.004073534 0.0018566813 -0.0014632502
## P_x
                 0.001856681 0.0028417922 -0.0006105607
## V_(Intercept) -0.001463250 -0.0006105607 0.0061042545
summary(mod)
##
## Call:
## vip(Y = y, X = X, V = 2)
## V-Inflated Poisson Model
##
## Coefficients:
               Estimate Std. Error z value Pr(>|z|)
##
## P_x
               -0.49031 0.05331 -9.198 < 2e-16 ***
## V_(Intercept) -0.42040 0.07813 -5.381 7.42e-08 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Log-likelihood: -1141
## BIC = 9169
confint(mod)
                     2.5%
                              97.5%
## P_(Intercept) -0.5218054 -0.2716189
               -0.5947950 -0.3858297
## V_(Intercept) -0.5735279 -0.2672650
nobs (mod)
## [1] 1000
logLik(mod)
## 'log Lik.' -1140.751 (df=997)
AIC(mod)
## [1] 4275.502
BIC(mod)
## [1] 9168.534
```

Zero-truncated VIP

We can truncate counts to be larger than 0. We also need V > 0 (for V = 0 case, look into ZIP or conditional Poisson model). Conceptually, the V-Inflation follows the 0-truncation (because we cannot observe 0, real truncated distribution).

The 0-truncated PDF is $P(Y=y\mid Y>0)=\frac{P(Y=y)}{1-P(Y=0)}$. The 0-truncated V-Inflated density is $P(Y=y\mid Y>0)=\frac{P(Y=y)}{1-P(Y=0)}$. $y \mid Y > 0, V > 0$ = $\phi I(Y = V) + (1 - \phi) \frac{f(y;\lambda)}{1 - f(0;\lambda)}$. This can be achieved in the vip call by the argument truncate=TRUE.

```
Here we use covariates for both the V and non-V part.
set.seed(1)
n <- 1000
x \leftarrow rnorm(n)
z \leftarrow runif(n, -1, 1)
df <- data.frame(x=x, z=z)</pre>
X <- model.matrix(~x, df)</pre>
Z <- model.matrix(~z, df)</pre>
beta <- c(-0.5, -0.5)
alpha <- c(0, 0.5)
lam <- exp(X %*% beta)
phi <- plogis(Z %*% alpha)
V \leftarrow 2 \# V  is the count value, cannot be 0
y <- y0 <- rpois(n, lam)
a <- rbinom(n, 1, phi)
keep <- y0>0
y <- y[keep] # conditioning (i.e. exclude Os)
y0 \leftarrow y0[\text{keep}]
X <- X[keep,]</pre>
Z \leftarrow Z[keep,]
y[a[keep] > 0] \leftarrow V
table(Poisson=y0, Vinflated=y)
##
            Vinflated
## Poisson
               1
                         3
                              4
                                  6
##
          1 155 141
                         0
                                  0
##
                                  0
```

```
2
              0 127
                        0
##
          3
              0
                  21
                       16
                            0
                                 0
          4
              0
                   4
                        0
                            7
                                 0
##
          5
                   2
                                 0
##
              0
                        0
##
          6
                   0
                        0
                            0
                                 1
mod <- vip(Y=y, X=X, Z=Z, V=2, truncate=TRUE)</pre>
summary(mod)
```

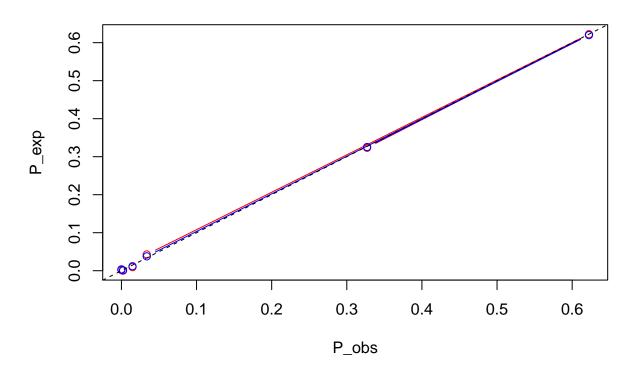
```
##
## Call:
## vip(Y = y, X = X, Z = Z, V = 2, truncate = TRUE)
## V-Inflated (Zero-Truncated) Poisson Model
##
## Coefficients:
                 Estimate Std. Error z value Pr(>|z|)
## P_(Intercept) -0.50814
                             0.14803 -3.433 0.000598 ***
                             0.10170 -5.639 1.71e-08 ***
## P_x
                 -0.57344
```

```
## V_(Intercept) 0.02131
                           0.12572 0.170 0.865387
## V_z
                 0.47041
                           0.20691 2.273 0.022999 *
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Log-likelihood: -384.1
## BIC = 3664
cbind(True=c(beta=beta, alpha=alpha),
     Est=coef(mod))
##
         True
                      Est.
## beta1 -0.5 -0.50813933
## beta2 -0.5 -0.57343540
## alpha1 0.0 0.02131236
## alpha2 0.5 0.47040670
```

Goodness of fit

```
goodness <- function(object, maxcount=NULL) {</pre>
    if (is.null(maxcount))
        maxcount <- max(object$Y)</pre>
    COUNTS <- if (object$truncate)
         1L:max(object$Y) else OL:maxcount
    P_obs <- table(object$Y) / nobs(object)</pre>
    P_obs <- as.numeric(P_obs[match(COUNTS, names(P_obs))])</pre>
    P_obs[is.na(P_obs)] <- 0
    names(P_obs) <- COUNTS</pre>
    P_exp <- P_obs
    P_{exp}[] \leftarrow 0
    linkinvx <- poisson("log")$linkinv</pre>
    linkinvz <- binomial(object$linkz)$linkinv</pre>
    parms <- coef(object)</pre>
    kx <- ncol(object$X)</pre>
    kz <- ncol(object$Z)</pre>
    mu <- as.vector(linkinvx(object$X %*% parms[1:kx] + object$offsetx))</pre>
    phi <- as.vector(linkinvz(object$Z %*% parms[(kx + 1):(kx + kz)] + object$offsetz))</pre>
    \#id0 \leftarrow object\$Y == object\$V
    Pmat <- matrix(0, nobs(object), length(COUNTS))</pre>
    colnames(Pmat) <- COUNTS</pre>
    PV <- if (object$truncate) {
         log(phi + (1 - phi) * dpois(object$V,
             lambda = mu, log = FALSE) / (1-exp(-mu)))
    } else {
        log(phi + (1 - phi) * dpois(object$V, lambda = mu, log = FALSE))
    }
    for (i in COUNTS) {
        PC <- if (object$truncate) {</pre>
             log((1 - phi) * dpois(i, lambda = mu, log = FALSE) / (1-exp(-mu)))
```

```
} else {
            log(1 - phi) + dpois(i, lambda = mu, log = TRUE)
        Pmat[,as.character(i)] <- if (i == object$V)</pre>
            exp(PV) else exp(PC)
    P_exp <- colMeans(Pmat)</pre>
    cbind(P_obs=P_obs, P_exp=P_exp)
}
## fit null model
mod0 <- vip(Y=y, X=X[,1,drop=FALSE], Z=Z[,1,drop=FALSE], V=2, truncate=TRUE)</pre>
## calculate GoF for null and other model
(gof0 <- goodness(mod0))</pre>
##
           P_{obs}
                         P_exp
## 1 0.327004219 0.3234032597
## 2 0.622362869 0.6223385125
## 3 0.033755274 0.0427550867
## 4 0.014767932 0.0095197408
## 5 0.00000000 0.0016957134
## 6 0.002109705 0.0002517089
(gof <- goodness(mod))</pre>
           P_{obs}
                        P_exp
## 1 0.327004219 0.325425662
## 2 0.622362869 0.619861184
## 3 0.033755274 0.037772839
## 4 0.014767932 0.011711772
## 5 0.00000000 0.003622341
## 6 0.002109705 0.001119974
## better model is closer to the 1:1 line
plot(gof0, type="b", col=2)
points(gof, type="b", col=4)
abline(0, 1, lty=2)
```



```
## better model will give smaller absolute deviation
sum(abs(apply(gof0, 1, diff)))
## [1] 0.02142703
sum(abs(apply(gof, 1, diff)))
```

[1] 0.01576604