Logic Notes

Keshav Ramamurthy

February 11, 2026

Contents

1 Introduction

1.1 Proposition

- $\sqrt{3}$ is irrational (true)
- $6 - 2 = 3$ (false)

1.2 Combining Propositions

Connectives

- $P \wedge Q$ (and)
- $P \vee Q$ (or)
- $\neg P$ (not)
- $P \implies Q$ (implies)

1.3 Logical Equivalences

Notice that $P \implies Q$ is equivalent to $\neg P \vee Q$
Proof: Merely Notice that

$$\neg P \vee Q \equiv (\neg P) \vee (Q \wedge P)$$

Along with this, we have the contrapositive: $P \implies Q$ is

$$\neg Q \implies \neg P$$

If $P \implies Q$, the contrapositive is also true.
We also have the converse of $P \implies Q$, which is

$$Q \implies P$$

1.4 Predicates and Quantifiers

For All:

$$(\forall x \in \mathbb{Z}) P(x)$$

There Exists:

$$(\exists x \in \mathbb{Z}) P(x)$$

$\forall x \in \mathbb{R}(\exists x \in \mathbb{R}(x^3 = k) \wedge (\neg(\exists y \in \mathbb{R}((y \neq x) \wedge (y^3 = k)))))$