

UNIVERSIDAD AUTÓNOMA DE NUEVO LEÓN

FACULTAD DE INGENIERÍA MECÁNICA Y ELÉCTRICA

SUBDIRECCIÓN DE ESTUDIOS DE POSGRADO



STOCHASTIC SIRS EPIDEMICS ON NETWORKS

POR

GERARDO PALAFOX CASTILLO

COMO REQUISITO PARCIAL PARA OBTENER EL GRADO DE

DOCTORADO EN INGENIERÍA DE SISTEMAS

OCTUBRE 2013

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Los miembros del Comité de Tesis recomendamos que la Tesis *Stochastic SIRS epidemics on networks*, realizada por el alumno Gerardo Palafox Castillo, con número de matrícula 1649275, sea aceptada para su defensa como requisito parcial para obtener el grado de Doctorado en Ingeniería de Sistemas .

El Comité de Tesis

---

Dr. César Emilio Villarreal Rodríguez  
Asesor

---

Nombre del revisor A  
Revisor

---

Nombre del revisor B  
Revisor

---

Nombre del revisor C  
Revisor

---

Nombre del revisor D  
Revisor

Vo. Bo.

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Dr. Simón Martínez Martínez  
Subdirector de Estudios de Posgrado

San Nicolás de los Garza, Nuevo León, octubre 2013

*A mis padres, y a Fabiola.*

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# ACKNOWLEDGMENTS

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Aquí puedes poner tus agradecimientos. (No olvides agradecer a tu comité de tesis, a tus profesores, a la facultad y a CONACyT en caso de que hallas sido beneficiado con una beca).

# RESUMEN

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Gerardo Palafox Castillo.

Candidato para obtener el grado de Doctorado en Ingeniería de Sistemas .

Universidad Autónoma de Nuevo León.

Facultad de Ingeniería Mecánica y Eléctrica.

Título del estudio: STOCHASTIC SIRS EPIDEMICS ON NETWORKS.

Número de páginas: 12.

OBJETIVOS Y MÉTODO DE ESTUDIO: Aquí debes poner tus objetivos y métodos de estudio. (Este es el formato).

CONTRIBUCIONES Y CONCLUSIONES: Y aquí tus contribuciones y conclusiones. (También es parte del formato).

Firma del asesor: \_\_\_\_\_  
Dr. César Emilio Villarreal Rodríguez

## CHAPTER 1

# INTRODUCTION

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## 1.1 BACKGROUND

Mathematical models for the spread of infectious disease have been long studied Bailey [1975]. Early work was usually done for specific diseases, often with deterministic models. However, as assumptions are dropped for more realistic scenarios, stochastic models are better suited to study these phenomena.

A commonality among the different models is the compartmentalization of the population. In the more common models, the population is divided in three classes: susceptible (S), infected and infectious (I) and recovered and immune (R). A SIR epidemic model is one in which susceptible people can be infected by an infectious individual, and infected individuals eventually heal and become permanently immune to the disease. Other models studied are those in which recovered individuals immediately become infectious again (SIS). Models with a latent state when people are infected but not immediately infectious are called SEIR (E for exposed). Models where recovered individuals' immunity ends after some time are called SIRS. Besides partitioning the population, we distinguish our models by how the epidemic spreads and how the population is organized. In *scalar models*, it is typically assumed that individuals in the population have the same chances of interacting with each other

Mei et al. [2017]. In contrast, a *network epidemic model* specifies, as a graph, the structure of the community upon which an epidemic spreads Britton [2010]. Regarding the spread of the epidemic, we distinguish whether the propagation dynamics is assumed to be *stochastic* or *deterministic* Mei et al. [2017]. To have a more accurate depiction of our population, we will focus on network models. Particularly our goal is to address stochastic, SIRS epidemic models on networks.

## 1.2 LITERATURE REVIEW

While our work will focus on stochastic epidemics, there is an interest on deterministic epidemics too. Results from each area have the potential to inspire results in the other, and both have the common goal of understanding the spread of epidemics. Typical models such as SI, SIS and SIR over networks are studied in its deterministic form at Mei et. al. Mei et al. [2017]. A great survey on general stochastic epidemic models is one by T. Britton Britton [2010]. A survey focusing on epidemic models on networks is one by Pastor-Satorras et. al. Pastor-Satorras et al. [2015]. Here, the authors try to unify some of the important work in the area that has spread across multiple disciplines (mainly, statistical physics, computer science and mathematical biology). A modern treatment of the mathematics of epidemics on networks is a book by Kiss et. al. Kiss et al. [2017]. A survey on how spectral and optimal control have been, and can be further, used in controlling the spread of epidemics in networks was done by Nowzari et. al. Nowzari et al. [2016]. Liu and Buss study the optimal control problems of information epidemics on social networks Liu and Buss [2020].

A recent work by Britton Britton [2019] surveys epidemics on common random network models (e.g., Erdős-Rényi, or the Configuration Model Newman [2018]), and uses statistical inference to address and pose questions of interest, such as prevention strategies or using virus sequences to obtain epidemic data.

Common ways to slow down the spread of epidemics are imposing some restriction on the population, such as curfews, travel bans, banning of gatherings, etc. Vaccination, when available, is also an effective measure, but comes with resource-allocation and cost challenges. The effects that vaccination have on epidemics and its duration have been studied recently Ball and Britton [2020], Lashari et al. [2020], Fransson and Trapman [2019].

Sometimes, we need to consider our network dynamic, in the sense that edges can be added or removed. This is useful to study the impact of measures that can be taken to stop or slow an epidemic, or to model changes in behaviour of the population. Preventive rewiring Ball and Britton [2020] and other graph theoretic methods Wijayanto and Murata [2019] have been recently addressed.

Finally, it is worth noting, that even if we did not state this studies in particular, there is a lot of work on the general modeling of contagion processes over networks. Our interests and this literature review have focused on biological aspects (actual epidemics). However, the spread of rumors in social networks, of viruses in computer networks, of crisis in financial networks, etc., all share similarities between them and are active areas of research.

## CHAPTER 2

# BACKGROUND

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## CHAPTER 3

# LITERATURE REVIEW

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## CHAPTER 4

# METHODS

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## CHAPTER 5

# RESULTS

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## CHAPTER 6

# CONCLUSIONS

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## APPENDIX A

# ESTE ES UN APÉNDICE

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## A.1 CITAS BIBLIOGRÁFICAS

En principio tienes total libertad de incluir tu bibliografía con el entorno `thebibliography` nativo de `LATEX` o mediante la herramienta `BIBTEX`. En caso de que optes por esto último (recomendado), puedes usar alguno de los archivos `mighelbib.bst` o `mighelnat.bst` incluidos en el paquete `Tesis-FIME`, pues sus diseños están basados en el estilo bibliográfico estándar del español, además de que armoniza con el estilo de tesis provisto por `fime.cls`.

El estilo bibliográfico `mighelbib` es numérico, es decir cita con un número entre corchetes, por ejemplo una cita `\cite{Dan82}` genera una etiqueta del tipo [13], mientras que el estilo `mighelnat` es tipo autor-año y requiere que el paquete `natbib` sea cargado (sin opciones) para su correcto funcionamiento, cita con el apellido del autor y el año, por ejemplo una cita `\citet{Dan82}` genera una etiqueta del tipo Dantzig (1982), mientras que una cita `\citep{Dan82}` genera una etiqueta del tipo (Dantzig, 1982).

Como muestra del estilo, unas citas: un libro clásico de epidemiología matemática en redes [Kiss et al., 2017], o un survey como el de Britton [2010].

## A.2 COMILLAS

El objetivo de esta sección era provocar otra página para que se vea el encabezado. Pero aprovechamos para decir que la clase `fime.cls` carga el paquete `babel` con la opción `spanish`, por lo que cambiará automáticamente los dobles signos `<<` y `>>` por `¡¡` y `!!`. Estas comillas angulares son las correctas en el idioma español, y son las que se usan en la clase `fime.cls`, por lo que se sugiere sean las usadas en el texto cada que quieras `¡¡entrecomillar!!` algo.

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# RESUMEN AUTOBIOGRÁFICO

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Gerardo Palafox Castillo

Candidato para obtener el grado de  
Doctorado en Ingeniería de Sistemas

Universidad Autónoma de Nuevo León  
Facultad de Ingeniería Mecánica y Eléctrica

Tesis:

STOCHASTIC SIRS EPIDEMICS ON NETWORKS

Aquí va tu historia. Recuerda que debe incluir: lugar y fecha de nacimiento, nombre de los padres, escuelas y universidades en las que se graduó después de la preparatoria, títulos o grados obtenidos (no incluir los estudios que se están concluyendo), experiencia profesional y organizaciones profesionales a las que pertenece (no incluir lista de publicaciones).