Problem Statement

$P_X^A(\cdot), P_X^B(\cdot)$

$$w(x) := \frac{P_X^B(x)}{P_X^A(x)}.$$

N^A

$$\{x_i^A, y_i^A\}$$

$$X_i^A, Y_i^A \sim P_{X,Y}^A(\cdot) = P_X^A(\cdot) P_{Y|X}(\cdot)$$

$h(\cdot)$

$$P_{X,Y}^B(\cdot) = P_X^B(\cdot)P_{Y|X}(\cdot)$$

$$L(h, P_{X,Y}^{B}) = \sum_{i=1}^{N^{A}} w(x_i^A) L(y_i^A, h(x_i^A)), \tag{1}$$

$$\hat{w}(x_i^A)$$

$w(x_i^A)$

$$P_X^A(\cdot) \neq P_X^B(\cdot)$$

$$P_{Y|X}^{A}(\cdot) = P_{Y|X}^{B}(\cdot)$$

Past work

Ratio Matching Methods

General Formulation

$$\hat{w}(\cdot)$$

$$E_{X \sim P_X^A(\cdot)}[d_f(w(X), \hat{w}(X))]$$

$$d_f(t, \hat{t}) := f(t) - (f(\hat{t}) + \nabla f(t - \hat{t}))$$

f

$$
\begin{aligned}
E_{X \sim P_X^A(\cdot)}[d_f(w(X), \hat{w}(X))] &= E_{X \sim P_X^A(\cdot)}[f(w(X)) - f(\hat{w}(X)) - \nabla f(\hat{w}(X))(w(X) - \hat{w}(X))] && (2)\\
&= -E_{X \sim P_X^A(\cdot)}[f(\hat{w}(X))] - E_{X \sim P_X^A(\cdot)}[\nabla f(\hat{w}(X)) w(X)] + E_{X \sim P_X^A(\cdot)}[\nabla f(\hat{w}(X))\hat{w}(X)] && (3)\\
&= -E_{X \sim P_X^A(\cdot)}[f(\hat{w}(X))] - E_{X \sim P_X^B(\cdot)}[\nabla f(\hat{w}(X))] + E_{X \sim P_X^A(\cdot)}[\nabla f(\hat{w}(X))\hat{w}(X)] && (4)
\end{aligned}
$$

$$\hat{w}(\cdot)$$

$$\hat{E}_{X\sim P_X^A(\cdot)}[d_f(w(X),\hat{w}(X))] = -\frac{1}{N^A}\sum_i f(\hat{w}(x_i^A))] - \frac{1}{N^B}\sum_i \nabla f(\hat{w}(x_i^B))] + \frac{1}{N^B}\sum_i \nabla f(\hat{w}(x_i^A))\hat{w}(x_i^A)] \tag{5}$$

$P_X^A(\cdot)$

$$P_X^B(\cdot)$$

$w(\cdot)$

Formulation with KL loss

d_f

$$f(t) = t \log t - t$$

$$\nabla f(t) = \log t$$

$$\hat{E}_{X\sim P_X^A(\cdot)}[d_f(w(X),\hat{w}(X))] = \tfrac{1}{N^A}\sum_i \hat{w}(x_i^A) - \tfrac{1}{N^B}\sum_i \log \hat{w}(x_i^B) \tag{6}$$

$$E_{X\sim P_X^A(\cdot)}[w(X)] = \int_x P_X^A(x)\frac{P_X^B(x)}{P_X^A(x)}dx = \int_x P_X^B(x)dx = 1$$

$$\hat{w}(x)$$

$$\frac{1}{N^B}\sum_i \hat{w}(x_i^B) = 1$$

$$\min_{\{\hat{w}(x_i^B)\}_{i=1}^{N^B}} -\tfrac{1}{N^B}\sum_i \log \hat{w}(x_i^B) \quad \text{subject to} \tag{7}$$

$$\tfrac{1}{N^B}\sum_i \hat{w}(x_i^B) = 1 \tag{8}$$

$$\hat{w}(x_i^B) > 0 \tag{9}$$

N^B

$$\{x_i^B\}$$

$$\{x_i^A\}$$

$$\hat{w}(\cdot)$$

$w(\cdot)$

K

$$\{\phi_k(\cdot)\}$$

$$\hat{w}(\cdot) = \sum_k \alpha_k \phi_k(\cdot). \tag{10}$$

$$\min_{\{\alpha_k\}} -\tfrac{1}{N^B}\sum_i \log \sum_k \alpha_k \phi_k(x_i^B) \quad \text{subject to} \tag{11}$$

$$\tfrac{1}{N^B}\sum_i \sum_k \alpha_k \phi_k(x_i^B) = 1 \tag{12}$$

$$\alpha_k \geq 0 \tag{13}$$

Formulation with squared loss

f

d_f

$$f(t) = \frac{1}{2}t^2$$

$$\nabla f(t) = t$$

$$\hat{E}_{X\sim P_X^A(\cdot)}[d_f(w(X),\hat{w}(X))] = \frac{1}{2N^A}\sum_i \hat{w}(x_i^A)^2 - \frac{1}{N^B}\sum_i \hat{w}(x_i^B) \tag{14}$$

$$\phi(x) := (\phi_1(x), \ldots, \phi_K(x))'$$

$$\alpha := (\alpha_1, \ldots, \alpha_K)$$

$$\min_{\alpha} \frac{1}{2}\alpha'\Big(\frac{1}{N^A}\sum_i \phi(x_i^A)\phi(x_i^A)'\Big)\alpha - \Big(\frac{1}{N^B}\sum_i \phi(x_i^B)\Big)'\alpha \quad \text{subject to} \tag{15}$$

$$\alpha \geq 0 \tag{16}$$

α

α

λ

α

λ

α

λ

Cross-validation

$$\{\phi_k(\cdot)\}$$

λ

$$P_X^A(\cdot)$$

$$P_X^B(\cdot)$$

k

$$P_X^A(\cdot)$$

$$P_X^B(\cdot)$$

Formulation with dimension reduction

$$w(x)$$

$x \in \mathbb{R}^D$

d

$U \in \mathbb{R}^{D \times d}$

x

$$x = \mathrm{Proj}_U(x) + \mathrm{Proj}_V(x)$$

$$V = U^{\perp}$$

$P_X^A(\cdot)$

$$P_X^A(x) = P_U^A(\mathrm{Proj}_U(x))P_{V|X}^A(\mathrm{Proj}_V(x))$$

$P_X^B(\cdot)$.

$$P^A_{V|X}(\cdot) = P^B_{V|X}(\cdot), \quad \text{then} \tag{17}$$

$$w(x) = \frac{P^A_U(\mathrm{Proj}_U(x))P^A_{V|X}(\mathrm{Proj}_V(x))}{P^B_U(\mathrm{Proj}_U(x))P^B_{V|X}(\mathrm{Proj}_V(x))} = \frac{P^A_U(\mathrm{Proj}_U(x))}{P^B_U(\mathrm{Proj}_U(x))} \tag{18}$$

U

$$P_U^A(\cdot) \neq P_U^B(\cdot)$$

d

d

U

$$P_U^A(\cdot)$$

$$P_U^B(\cdot)$$

$$PD(P_U^A(\cdot), P_U^B(\cdot)) = E_{U \sim P_U^A(\cdot)}\left[\left(\frac{P_U^B(U)}{P_U^A(U)} - 1\right)^2\right] \tag{19}$$

Kernel Mean Matching

Formulation

$$\phi : \mathcal{X} \to \mathcal{F}$$

$x \in \mathcal{X}$

$$k : \mathcal{X} \times \mathcal{X} \to \mathbb{R}$$

$$P_A^X(\cdot), P_B^X(\cdot)$$

$$w(\cdot) : x \rightarrow \frac{P_X^B(x)}{P_X^A(x)} :$$

$$\min_{w(\cdot)} \left| E_{X \sim P_X^B(\cdot)}[\phi(X)] - E_{X \sim P_X^A(\cdot)}[w(X)\phi(X)] \right| \quad \text{subject to} \tag{20}$$

$$w(x) \geq 0 \tag{21}$$

$$E_{X \sim P_X^A(\cdot)}[w(x)] = 1 \tag{22}$$

$$x_i^A \sim P_X^A(\cdot)$$

$$x_i^B \sim P_X^B(\cdot)$$

$$\hat{w}(\cdot)$$

$$\min_{B(\cdot)} \Big| \tfrac{1}{N^B} \sum_i \phi(x_i^B) - \tfrac{1}{N^A} \sum_i w(x_i^A)\phi(x_i^A) \Big| \quad \text{subject to} \tag{23}$$

$$w(x_i^A) \in [0, W_{\max}] \tag{24}$$

$$\big| \tfrac{1}{N^A} \sum_i w(x_i^A)] \big| \le 1 - \epsilon \tag{25}$$

$$\hat{w}(\cdot)$$

$$W_{\max}$$

$$w(x) \leq W_{\max}$$

$$w(\cdot)$$

$$|\frac{1}{N^A}\sum_i w(x_i^A)]|$$

1

$$w(\cdot)$$

Drawbacks

$$\hat{w}(\cdot)$$

$$P_X^A(\cdot)$$

$$\{\hat{B}(x_i^A)\}$$

$\phi(\cdot), W_{\max}, \epsilon$

Classifier-based methods

Standalone Approaches

Z, X

Z

$$Z = 1$$

X

$P_X^B(\cdot)$

X

$$P_X^A(\cdot)$$

$P_{Z,X}(\cdot)$

$$w(x) = \frac{P_X^B(x)}{P_X^A(x)} = \frac{P_{X|Z}(x|z=1)}{P_{X|Z}(x|z=0)} = \frac{\frac{P_{Z|X}(z=1|x)P_X(x)}{P_Z(z=1)}}{\frac{P_{Z|X}(z=0|x)P_X(x)}{P_Z(z=0)}} = \frac{P_{Z|X}(z=1|x)}{P_{Z|X}(z=0|x)} \frac{P_Z(z=0)}{P_Z(z=1)} \tag{26}$$

$P_Z(\cdot)$

$$P_Z(z=1) = \frac{N^B}{N^A+N^B}$$

$P_{Z|X}(\cdot)$

$$Z = 1$$

$$P_X^B(\cdot)$$

$$Z = 0$$

$$P_{Z|X}(\cdot)$$

Joint Approaches

$P_X^B(\cdot)$

$$\{x_i^A\}, \{x_i^B\}$$

$$w^*(\cdot) = \operatorname{argmin}_{w(\cdot)} L_w(w(\cdot); \{x_i^A\}, \{x_i^B\})$$

L_w

$w(\cdot)$

$f^*(\cdot)$

$$f^*(\cdot) = \operatorname{argmin}_{f(\cdot)} \sum_i w^*(x_i^A) L_f(f(x_i^A), y_i^a)$$

$$x_i^A$$

$$y_i^A$$

L_f

$$\operatorname{argmin}_{w(\cdot),f(\cdot)} L_w(w(\cdot);\{x_i^A\},\{x_i^B\}) + \sum_i w(x_i^A) L_f(f(x_i^A), y_i^a) \tag{27}$$

$$f(\cdot)$$

$$f(\cdot)$$

$w(\cdot)$

$f(\cdot)$

$w(\cdot)$

Supervised Dimension Reduction for Ratio Estimation

U

$$f(\cdot)$$

U

$$f(\cdot)$$

Standalone or Joint Approach?

U

U, Y

$P_{U,Y}^{B}(\cdot)$

$$\mathrm{MI}(U, Y)$$

$$P_{U,Y}^{B}(\cdot)$$

$w(\cdot)$

$P^{B}_{U,Y}(\cdot)$

$$P_{U,Y}^{A}(\cdot)$$

U

$$\operatorname{argmin}_{f(\cdot)} \sum_i w(x_i^A) L_f(f(\mathrm{proj}_U(x_i^A)), y_i^a)$$

$w(\cdot)$

$$w(x) = \frac{P_U^B(\mathrm{proj}_U(x))}{P_U^A(\mathrm{proj}_U(x))}$$

$$w(\cdot)$$

U

U

$w(\cdot)$