<u>Practice with Spinors</u>

I Algebra with Dirac γ matrices

Notation:

1. $S = \overline{u}u$
2. $P = \overline{u}\gamma^5 u$
3. $V^\mu = \overline{u}\gamma^\mu u$
4. $A^\mu = \overline{u}\gamma^\mu\gamma^5 u$
5. $T^{\mu\nu} = \overline{u}\sigma^{\mu\nu}u$

1. $(\overline{u}_1\gamma^\mu u_2)^* \ = ?$

   Note: $(\gamma^0)^\dagger \ = \ \gamma^0$ and $(\gamma^\mu)^\dagger \ = \ \gamma^0\gamma^\mu\gamma^0$

   $(\overline{u}_1\gamma^\mu u_2)$ is a 1×1 matrix. Therefore, its complex conjugate is the same as its Hermitian conjugate, i.e. if we call $V^\mu = (\overline{u}_1\gamma^\mu u_2)$, then $(V^\mu)^* = (V^\mu)^\dagger$. We can then express this quantity as follows:

$$
\begin{aligned}
V^\mu &= \overline{u}_1\gamma^\mu u_2 \ , && (1)\\
\Rightarrow (V^\mu)^* &= (V^\mu)^\dagger \ , && (2)\\
&= (\overline{u}_1\gamma^\mu u_2)^\dagger \ , && (3)\\
&= ((u_1)^\dagger\gamma^0\gamma^\mu u_2)^\dagger \quad \text{using } (A\ldots Z)^\dagger = Z^\dagger\ldots A^\dagger \ , && (4)\\
&= (u_2^\dagger)(\gamma^\mu)^\dagger(\gamma^0)^\dagger(u_1) && (5)\\
&= (u_2^\dagger)\gamma^0\gamma^\mu\gamma^0\gamma^0(u_1) && (6)\\
&= (u_2^\dagger)\gamma^0\gamma^\mu(u_1) && (7)\\
&= \overline{u}_2\gamma^\mu(u_1) && (8)
\end{aligned}
$$

   Therefore $(\overline{u}_1\gamma^\mu u_2)^* \ = \ \overline{u}_2\gamma^\mu u_1$. To solve for $|V^\mu|^2$ we simply use $|V^\mu|^2 \ = \ \mathrm{Tr}[\overline{u}_1\gamma^\mu u_2\overline{u}_2\gamma^\nu u_1]$.

   <u>Question:</u>
   Why is $|V^\mu|^2 \ \neq \ \mathrm{Tr}[\overline{u}_1\gamma^\mu u_2\overline{u}_2\gamma_\mu u_1]$?
   Why is $|V^\mu|^2 \ = \ \mathrm{Tr}[\overline{u}_1\gamma^\mu u_2\overline{u}_2\gamma^\nu u_1]$?

   You start with one index μ. When you square why do you get two indices and not a sum over two of the same index?
   When you square you have two indices because you must increase the number of components. The number of components when you square should go as n^2 not simply n. When you have one index, you restrict the number of components, because you have 4 components and not 16.

Note: $\text{Tr}[\gamma^\mu\gamma^\nu] = 4g^{\mu\nu}$, $\text{Tr}[\gamma^\mu\gamma^\nu\gamma^\lambda\gamma^\sigma] = 4(g^{\mu\nu}g^{\lambda\sigma} - g^{\mu\lambda}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\lambda})$, The trace over the product of an odd number of gamma matrices is zero.

$$
\begin{aligned}
|V^\mu|^2 &= \text{Tr}[\overline{u}_1\gamma^\mu u_2\overline{u}_2\gamma^\nu u_1] && (9)\\
&= \text{Tr}[\overline{u}_1\gamma^\mu(\not{p}_2 + m)\gamma^\nu u_1] && (10)\\
&= \text{Tr}[u_1\overline{u}_1\gamma^\mu(\not{p}_2 + m)\gamma^\nu] && (11)\\
&= \text{Tr}[(\not{p}_1 + m)\gamma^\mu(\not{p}_2 + m)\gamma^\nu] && (12)\\
&= \text{Tr}[\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu] + m[\text{Tr}(\gamma^\mu\not{p}_1\gamma^\nu) + \text{Tr}(\gamma^\mu\gamma^\nu\not{p}_2)] + m^2\text{Tr}[\gamma^\mu\gamma^\nu] && (13)\\
&= \text{Tr}[\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu] + m^2\text{Tr}[\gamma^\mu\gamma^\nu] && (14)\\
&= \text{Tr}[(p_1)_\lambda\gamma^\lambda\gamma^\mu(p_2)_\sigma\gamma^\sigma\gamma^\nu] + 4m^2g^{\mu\nu} && (15)\\
&= (p_1)_\lambda(p_2)_\sigma\text{Tr}[\gamma^\lambda\gamma^\mu\gamma^\sigma\gamma^\nu] + 4m^2g^{\mu\nu} && (16)\\
&= (p_1)_\lambda(p_2)_\sigma 4(g^{\mu\nu}g^{\lambda\sigma} - g^{\mu\lambda}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\lambda}) + 4m^2g^{\mu\nu} && (17)\\
&= 4[p_1^\mu p_2^\nu - g^{\mu\nu}(p_1\cdot p_2) + p_2^\mu p_1^\nu] + 4m^2g^{\mu\nu} && (18)
\end{aligned}
$$

2. $(\overline{u}_1\gamma^\mu\gamma^5u_2)^*$ is also a 1×1 Matrix so the same reasoning applies as above in 1. Note: $(\gamma^5)^\dagger = \gamma^5$ We define: A^μ as $\overline{u}_1\gamma^\mu\gamma^5u_2$ thus:

$$
\begin{aligned}
(A^\mu)^* &= (A^\mu)^\dagger && (19)\\
&= (\overline{u}_1\gamma^\mu\gamma^5u_2)^\dagger && (20)\\
&= ((u_1)^\dagger\gamma^0\gamma^\mu\gamma^5u_2)^\dagger && (21)\\
&= (u_2^\dagger)(\gamma^5)^\dagger(\gamma^\mu)^\dagger(\gamma^0)^\dagger(u_1) && (22)\\
&= (u_2^\dagger)\gamma^5\gamma^0\gamma^\mu\gamma^0\gamma^0u_1 && (23)\\
&= (u_2^\dagger)\gamma^5\gamma^0\gamma^\mu(1)u_1 && (24)\\
&= -(u_2^\dagger)\gamma^0\gamma^5\gamma^\mu u_1 && (25)\\
&= -\overline{u}_2\gamma^5\gamma^\mu u_1 && (26)\\
&= \overline{u}_2\gamma^\mu\gamma^5u_1 && (27)
\end{aligned}
$$

Therefore $(\overline{u}_1\gamma^\mu\gamma^5u_2)^* = \overline{u}_2\gamma^\mu\gamma^5u_1$

We also are able to calculate $|A^\mu|^2$

$$
\begin{aligned}
|A^\mu|^2 &= \text{Tr}[(\overline{u}_1\gamma^\mu\gamma^5u_2)(\overline{u}_2\gamma^\nu\gamma^5u_1)] && (28)\\
&= \text{Tr}[\overline{u}_1\gamma^\mu\gamma^5(\not{p}_2 + m)\gamma^\nu\gamma^5u_1] && (29)\\
&= \text{Tr}[u_1\overline{u}_1\gamma^\mu\gamma^5(\not{p}_2 + m)\gamma^\nu\gamma^5] && (30)\\
&= \text{Tr}[(\not{p}_1 + m)\gamma^\mu\gamma^5(\not{p}_2 + m)\gamma^\nu\gamma^5] && (31)\\
&= \text{Tr}[\not{p}_1\gamma^\mu\gamma^5\not{p}_2\gamma^\nu\gamma^5 + m(\not{p}_1\gamma^\mu\gamma^5\gamma^\nu\gamma^5 + \gamma^\mu\gamma^5\not{p}_2\gamma^\nu\gamma^5) + m^2(\gamma^\mu\gamma^5\gamma^\nu\gamma^5)] && (32)\\
&= \text{Tr}[\not{p}_1\gamma^\mu\gamma^5\not{p}_2\gamma^\nu\gamma^5 + m(\not{p}_1\gamma^\mu(-\gamma^5\gamma^5)\gamma^\nu + \not{p}_2\gamma^\mu(-\gamma^5\gamma^5)\gamma^\nu) + m^2(\gamma^\mu\gamma^5\gamma^\nu\gamma^5)]\\
&= \text{Tr}[\not{p}_1\gamma^\mu\gamma^5\not{p}_2\gamma^\nu\gamma^5 - m(\not{p}_1\gamma^\mu\gamma^\nu + \not{p}_2\gamma^\mu\gamma^\nu) + m^2(\gamma^\mu\gamma^5\gamma^\nu\gamma^5)] && (33)\\
&= \text{Tr}[\not{p}_1\gamma^\mu\gamma^5\not{p}_2\gamma^\nu\gamma^5] - mTr[\not{p}_1\gamma^\mu\gamma^\nu] - mTr[\not{p}_2\gamma^\mu\gamma^\nu] + m^2(\gamma^\mu\gamma^5\gamma^\nu\gamma^5)] && (34)
\end{aligned}
$$

$$
\begin{aligned}
&= \mathrm{Tr}[\not{p}_1\gamma^\mu\gamma^5\not{p}_2\gamma^\nu\gamma^5 + m^2(\gamma^\mu\gamma^5\gamma^\nu\gamma^5)] && (35)\\
&= \mathrm{Tr}[(p_1)_\lambda\gamma^\lambda\gamma^\mu\gamma^5(p_2)_\sigma\gamma^\sigma\gamma^\nu\gamma^5] + m^2\mathrm{Tr}[\gamma^\mu\gamma^5\gamma^\nu\gamma^5] && (36)\\
&= (p_1)_\lambda(p_2)_\sigma\mathrm{Tr}[\gamma^\lambda\gamma^\mu\gamma^5\gamma^\sigma\gamma^\nu\gamma^5] - m^2\mathrm{Tr}[\gamma^\mu\gamma^5\gamma^5\gamma^\nu] && (37)\\
&= (p_1)_\lambda(p_2)_\sigma\mathrm{Tr}[\gamma^\lambda\gamma^\mu\gamma^5\gamma^5\gamma^\sigma\gamma^\nu] - m^2\mathrm{Tr}[\gamma^\mu\gamma^\nu] && (38)\\
&= (p_1)_\lambda(p_2)_\sigma\mathrm{Tr}[\gamma^\lambda\gamma^\mu\gamma^\sigma\gamma^\nu] - m^2(g^{\mu\nu}) && (39)\\
&= (p_1)_\lambda(p_2)_\sigma 4(g^{\mu\nu}g^{\lambda\sigma} - g^{\mu\lambda}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\lambda}) - 4m^2 g^{\mu\nu} && (40)\\
&= 4[p_1^\mu p_2^\nu - g^{\mu\nu}(p_1\cdot p_2) + p_2^\mu p_1^\nu] - 4m^2 g^{\mu\nu} && (41)
\end{aligned}
$$

3. $(\overline{u}_1 u_2)^* = ?$ We let $S = \overline{u}_1 u_2$

$$
\begin{aligned}
(S)^* &= (S)^\dagger && (42)\\
&= (\overline{u}_1 u_2)^\dagger\ , && (43)\\
&= ((u_1)^\dagger\gamma^0 u_2)^\dagger && (44)\\
&= (u_2)^\dagger(\gamma^0)^\dagger(u_1) && (45)\\
&= (u_2)^\dagger\gamma^0(u_1) && (46)\\
&= \overline{u}_2(u_1) && (47)
\end{aligned}
$$

Therefore$(\overline{u}_1 u_2)^* = \overline{u}_2 u_1$. In order to find $|S|^2$ we simply do the following:

$$
\begin{aligned}
|S|^2 &= \mathrm{Tr}[\overline{u}_1 u_2 \overline{u}_2 u_1] && (48)\\
&= \mathrm{Tr}[(\not{p}_1 + m)(\not{p}_2 + m)] && (49)\\
&= \mathrm{Tr}[\not{p}_1\not{p}_2 + m(\not{p}_1 + \not{p}_2) + m^2] && (50)\\
&= \mathrm{Tr}[\not{p}_1\not{p}_2] + \mathrm{Tr}[m(\not{p}_1 + \not{p}_2)] + \mathrm{Tr}[m^2] && (51)\\
&= \mathrm{Tr}[\not{p}_1\not{p}_2] + m(\mathrm{Tr}[\not{p}_1] + \mathrm{Tr}[\not{p}_2]) + m^2\mathrm{Tr}[1] && (52)\\
&= \mathrm{Tr}[\not{p}_1\not{p}_2] + 4m^2 && (53)\\
&= 4(p_1\cdot p_2) + 4m^2 && (54)
\end{aligned}
$$

4. By the same reasoning as shown above it can be shown that $(\overline{u}_1\gamma^5 u_2)^* = \overline{u}_2\gamma^5 u_1$
If we let $P = \overline{u}_1\gamma^5 u_2$ then:

$$
\begin{aligned}
(P)^* &= (P)^\dagger && (55)\\
&= (\overline{u}_1\gamma^5 u_2)^\dagger\ , && (56)\\
&= ((u_1)^\dagger\gamma^0\gamma^5 u_2)^\dagger && (57)\\
&= (u_2)^\dagger(\gamma^5)^\dagger(\gamma^0)^\dagger(u_1) && (58)\\
&= (u_2)^\dagger(\gamma^5)\gamma^0(u_1) && (59)\\
&= -(u_2)^\dagger\gamma^0\gamma^5(u_1) && (60)\\
&= -\overline{u}_2\gamma^5(u_1) && (61)
\end{aligned}
$$

Therefore $(\overline{u}_1\gamma^5 u_2)^* = -\overline{u}_2\gamma^5 u_1$

In order to square P we do the following:

$$|P|^2 = \mathrm{Tr}[\overline{u}_1\gamma^5 u_2(-\overline{u}_2\gamma^5 u_1)] \qquad (62)$$

$$
\begin{aligned}
&= \mathrm{Tr}[u_1\overline{u}_1\gamma^5(-\not{p}_2 - m)\gamma^5] && (63)\\
&= \mathrm{Tr}[(\not{p}_1 + m)\gamma^5(-\not{p}_2 - m)\gamma^5] && (64)\\
&= \mathrm{Tr}[((p_1)_\mu\gamma^\mu + m)\gamma^5((-p_2)_\nu\gamma^\nu - m)\gamma^5] && (65)\\
&= \mathrm{Tr}[((p_1)_\mu\gamma^\mu\gamma^5 + m\gamma^5)((-p_2)_\nu\gamma^\nu\gamma^5 - m\gamma^5)] && (66)\\
&= \mathrm{Tr}[(p_1)_\mu\gamma^\mu\gamma^5(-p_2)_\nu\gamma^\nu\gamma^5 + m\gamma^5(-p_2)_\nu\gamma^\nu\gamma^5 - m\gamma^5(p_1)_\mu\gamma^\mu\gamma^5 - \gamma^5\gamma^5 m^2)] &&\\
&= \mathrm{Tr}[(p_1)_\mu\gamma^\mu\gamma^5(-p_2)_\nu\gamma^\nu\gamma^5] + Tr[m\gamma^5(-\gamma^5)(-p_2)_\nu\gamma^\nu] - Tr[m\gamma^5(-\gamma^5)(p_1)_\mu\gamma^\mu] - Tr[m^2] &&\\
&= \mathrm{Tr}[(p_1)_\mu\gamma^\mu\gamma^5(-p_2)_\nu\gamma^\nu\gamma^5] - Tr[m(-p_2)_\nu\gamma^\nu] + Tr[m(p_1)_\mu\gamma^\mu] - 4m^2 && (67)\\
&= \mathrm{Tr}[(p_1)_\mu\gamma^\mu\gamma^5(-p_2)_\nu\gamma^\nu\gamma^5] - 4m^2 && (68)\\
&= (p_1)_\mu(-p_2)_\nu\mathrm{Tr}[\gamma^\mu\gamma^5\gamma^\nu\gamma^5] - 4m^2 && (69)\\
&= (p_1)_\mu(-p_2)_\nu\mathrm{Tr}[\gamma^\mu\gamma^5(-\gamma^5\gamma^\nu)] - 4m^2 && (70)\\
&= (p_1)_\mu(-p_2)_\nu(-\mathrm{Tr}[\gamma^\mu\gamma^\nu]) - 4m^2 && (71)\\
&= (p_1)_\mu(-p_2)_\nu(-4g^{\mu\nu}) - 4m^2 && (72)\\
&= 4(p_1)(p_2) - 4m^2 && (73)
\end{aligned}
$$

5. While the above identities could be shown to be trivial, the identity: $(\overline{u}_1\sigma^{\mu\nu}u_2)^* = \overline{u}_2\sigma^{\nu\mu}u_1$ is more difficult to solve. The identity:$(\sigma^{\mu\nu})^\dagger = \sigma^{\mu\nu}$ is needed

$$
\begin{aligned}
(\sigma^{\mu\nu})^\dagger &= (\frac{i}{2}[\gamma^\mu,\gamma^\nu])^\dagger && (74)\\
&= -\frac{i}{2}([\gamma^\mu,\gamma^\nu])^\dagger && (75)\\
&= -\frac{i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)^\dagger && (76)\\
&= -\frac{i}{2}((\gamma^\nu)^\dagger(\gamma^\mu)^\dagger - (\gamma^\mu)^\dagger(\gamma^\nu)^\dagger) && (77)\\
&= -\frac{i}{2}(\gamma^0\gamma^\nu\gamma^0\gamma^0\gamma^\mu\gamma^0 - \gamma^0\gamma^\mu\gamma^0\gamma^0\gamma^\nu\gamma^0) && (78)\\
&= -\frac{i}{2}(\gamma^0\gamma^\nu\gamma^\mu\gamma^0 - \gamma^0\gamma^\mu\gamma^\nu\gamma^0) && (79)\\
&= -\frac{i}{2}((-1)^2\gamma^\nu\gamma^\mu - (-1)^2\gamma^\mu\gamma^\nu) && (80)\\
&= -\frac{i}{2}(\gamma^\nu\gamma^\mu - \gamma^\mu\gamma^\nu) && (81)\\
&= -\sigma^{\nu\mu} && (82)
\end{aligned}
$$

After showing $(\sigma^{\mu\nu})^\dagger = \sigma^{\nu\mu}$ is true it is trivial to show $(\overline{u}_1\sigma^{\mu\nu}u_2)^* = -\overline{u}_2\sigma^{\nu\mu}u_1$. We let $T^{\mu\nu} = \overline{u}_1\sigma^{\mu\nu}u_2$

$$
\begin{aligned}
(T^{\mu\nu})^* &= (T^{\mu\nu})^\dagger\ , && (83)\\
&= (\overline{u}_1\sigma^{\mu\nu}u_2)^\dagger && (84)\\
&= ((u_1)^\dagger\gamma^0\sigma^{\mu\nu}u_2)^\dagger && (85)\\
&= (u_2^\dagger)(\sigma^{\mu\nu})^\dagger(\gamma^0)^\dagger(u_1) && (86)\\
&= (u_2^\dagger)(-\sigma^{\nu\mu})\gamma^0(u_1) && (87)
\end{aligned}
$$

$$
\begin{aligned}
&= (u_2^\dagger)(-\gamma^0)(-\sigma^{\nu\mu})(u_1) && (88)\\
&= \overline{u}_2\sigma^{\mu\nu}(u_1) && (89)
\end{aligned}
$$

In order to find the value of $|T^{\mu\nu}|^2$ one needs to find the value of $Tr[\sigma^{\sigma\lambda}\sigma^{\mu\nu}]$

$$
\begin{aligned}
Tr[\sigma^{\sigma\lambda}\sigma^{\mu\nu}] &= Tr[\frac{i}{2}(\gamma^\sigma\gamma^\lambda-\gamma^\lambda\gamma^\sigma)\frac{i}{2}(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu)] && (90)\\
&= Tr[\frac{i}{2}(\gamma^\sigma\gamma^\lambda-\gamma^\lambda\gamma^\sigma)\frac{i}{2}(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu)] && (91)\\
&= -\frac{1}{4}Tr[(\gamma^\sigma\gamma^\lambda-\gamma^\lambda\gamma^\sigma)(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu)] && (92)\\
&= -\frac{1}{4}Tr[\gamma^\sigma\gamma^\lambda\gamma^\mu\gamma^\nu]+\frac{1}{4}Tr[\gamma^\sigma\gamma^\lambda\gamma^\nu\gamma^\mu]+\frac{1}{4}Tr[\gamma^\lambda\gamma^\sigma\gamma^\mu\gamma^\nu]-\frac{1}{4}Tr[\gamma^\lambda\gamma^\sigma\gamma^\nu\gamma^\mu]
\end{aligned}
$$

Here we must label each of the traces individually:

$$
\begin{aligned}
A &= -\frac{1}{4}Tr[\gamma^\sigma\gamma^\lambda\gamma^\mu\gamma^\nu] && (93)\\
&= -(g^{\sigma\lambda}g^{\mu\nu}-g^{\sigma\mu}g^{\lambda\nu}+g^{\sigma\nu}g^{\lambda\mu}) && (94)
\end{aligned}
$$

$$
\begin{aligned}
B &= +\frac{1}{4}Tr[\gamma^\sigma\gamma^\lambda\gamma^\nu\gamma^\mu] && (95)\\
&= +(g^{\sigma\lambda}g^{\nu\mu}-g^{\sigma\nu}g^{\lambda\mu}+g^{\sigma\mu}g^{\lambda\nu}) && (96)
\end{aligned}
$$

$$
\begin{aligned}
C &= \frac{1}{4}Tr[\gamma^\lambda\gamma^\sigma\gamma^\mu\gamma^\nu] && (97)\\
&= (g^{\lambda\sigma}g^{\mu\nu}-g^{\lambda\mu}g^{\sigma\nu}+g^{\lambda\nu}g^{\sigma\mu}) && (98)
\end{aligned}
$$

$$
\begin{aligned}
D &= -\frac{1}{4}Tr[\gamma^\lambda\gamma^\sigma\gamma^\nu\gamma^\mu] && (99)\\
&= -(g^{\lambda\sigma}g^{\nu\mu}-g^{\lambda\nu}g^{\sigma\mu}+g^{\lambda\mu}g^{\sigma\nu}) && (100)
\end{aligned}
$$

$$
\begin{aligned}
Tr[\sigma^{\sigma\lambda}\sigma^{\mu\nu}] &= A+B++C+D && (101)\\
&= 2g^{\sigma\mu}g^{\lambda\nu}-2g^{\sigma\nu}g^{\lambda\mu}-2g^{\lambda\mu}g^{\sigma\nu}+2g^{\lambda\nu}g^{\sigma\mu}) && (102)
\end{aligned}
$$

In order to find the value of $|T^{\mu\nu}|^2$ one needs to do the following:

$$
\begin{aligned}
|T^{\mu\nu}|^2 &= \mathrm{Tr}[\overline{u}_1\sigma^{\mu\nu}u_2\overline{u}_2\sigma^{\sigma\lambda}u_1] && (103)\\
&= \mathrm{Tr}[\overline{u}_1\sigma^{\mu\nu}(\not{p}_2+m)\sigma^{\sigma\lambda}u_1] && (104)\\
&= \mathrm{Tr}[(\not{p}_1+m)\sigma^{\mu\nu}(\not{p}_2+m)\sigma^{\sigma\lambda}] && (105)\\
&= \mathrm{Tr}[((p_1)_\kappa\gamma^\kappa+m)\sigma^{\mu\nu}((p_2)_\gamma\gamma^\gamma+m)\sigma^{\sigma\lambda}] && (106)
\end{aligned}
$$

$$
\begin{aligned}
&= \mathrm{Tr}[((p_1)_\kappa\gamma^\kappa\sigma^{\mu\nu} + m\sigma^{\mu\nu})((p_2)_\gamma\gamma^\gamma\sigma^{\sigma\lambda} + m\sigma^{\sigma\lambda})] && (107)\\
&= \mathrm{Tr}[((p_1)_\kappa\gamma^\kappa\sigma^{\mu\nu}(p_2)_\gamma\gamma^\gamma\sigma^{\sigma\lambda} + (p_1)_\kappa\gamma^\kappa\sigma^{\mu\nu}m\sigma^{\sigma\lambda} + (p_2)_\gamma\gamma^\gamma\sigma^{\sigma\lambda}m\sigma^{\mu\nu} + m\sigma^{\sigma\lambda}m\sigma^{\mu\nu})] && (108)\\
&= \mathrm{Tr}[((p_1)_\kappa\gamma^\kappa\sigma^{\mu\nu}(p_2)_\gamma\gamma^\gamma\sigma^{\sigma\lambda}] + Tr[(p_1)_\kappa\gamma^\kappa\sigma^{\mu\nu}m\sigma^{\sigma\lambda} + (p_2)_\gamma\gamma^\gamma\sigma^{\sigma\lambda}m\sigma^{\mu\nu}] + Tr[m\sigma^{\sigma\lambda}m\sigma^{\mu\nu}]\\
&= \mathrm{Tr}[((p_1)_\kappa\gamma^\kappa\sigma^{\mu\nu}(p_2)_\gamma\gamma^\gamma\sigma^{\sigma\lambda}] + Tr[m\sigma^{\sigma\lambda}m\sigma^{\mu\nu}] && (108)\\
&= \mathrm{Tr}[((p_1)_\kappa\gamma^\kappa\sigma^{\mu\nu}(p_2)_\gamma\gamma^\gamma\sigma^{\sigma\lambda}] + m^2Tr[\sigma^{\sigma\lambda}\sigma^{\mu\nu}] \text{ Let B } = \mathrm{m}^2\mathrm{Tr}[\sigma^{\sigma\lambda}\sigma^{\mu\nu}]\\
&= \mathrm{Tr}[((p_1)_\kappa\gamma^\kappa\sigma^{\mu\nu}(p_2)_\gamma\gamma^\gamma\sigma^{\sigma\lambda}] + B\\
&= (p_1)_\kappa(p_2)_\gamma\mathrm{Tr}[\gamma^\kappa\sigma^{\mu\nu}\gamma^\gamma\sigma^{\sigma\lambda}] + B\\
&= (p_1)_\kappa(p_2)_\gamma\mathrm{Tr}[\gamma^\kappa(\frac{i}{2}((\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)\gamma^\gamma(\frac{i}{2}(\gamma^\sigma\gamma^\lambda - \gamma^\lambda\gamma^\sigma))] + B && (106)\\
&= -\frac{1}{4}(p_1)_\kappa(p_2)_\gamma\mathrm{Tr}[\gamma^\kappa(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)\gamma^\gamma(\gamma^\sigma\gamma^\lambda - \gamma^\lambda\gamma^\sigma)] + B \text{ (Let A } = -\frac{1}{4}(\mathrm{p}_1)_\kappa(\mathrm{p}_2)_\gamma)\\
&= (A)\mathrm{Tr}[(\gamma^\kappa\gamma^\mu\gamma^\nu - \gamma^\kappa\gamma^\nu\gamma^\mu)(\gamma^\gamma\gamma^\sigma\gamma^\lambda - \gamma^\gamma\gamma^\lambda\gamma^\sigma)] + B\\
&= (A)\mathrm{Tr}[(\gamma^\kappa\gamma^\mu\gamma^\nu\gamma^\gamma\gamma^\sigma\gamma^\lambda - \gamma^\kappa\gamma^\mu\gamma^\nu\gamma^\gamma\gamma^\lambda\gamma^\sigma - \gamma^\kappa\gamma^\nu\gamma^\mu\gamma^\gamma\gamma^\sigma\gamma^\lambda + \gamma^\kappa\gamma^\nu\gamma^\mu\gamma^\gamma\gamma^\lambda\gamma^\sigma)] + B\\
&= (A)\mathrm{Tr}[(\gamma^\kappa\gamma^\mu\gamma^\nu\gamma^\gamma\gamma^\sigma\gamma^\lambda - \gamma^\kappa\gamma^\mu\gamma^\nu\gamma^\gamma(-\gamma^\sigma\gamma^\lambda) - \gamma^\kappa\gamma^\nu\gamma^\mu\gamma^\gamma\gamma^\sigma\gamma^\lambda + \gamma^\kappa\gamma^\nu\gamma^\mu\gamma^\gamma(-\gamma^\sigma\gamma^\lambda))] + B\\
&= (A)\mathrm{Tr}[(\gamma^\kappa\gamma^\mu\gamma^\nu\gamma^\gamma\gamma^\sigma\gamma^\lambda + \gamma^\kappa\gamma^\mu\gamma^\nu\gamma^\gamma\gamma^\sigma\gamma^\lambda - \gamma^\kappa\gamma^\nu\gamma^\mu\gamma^\gamma\gamma^\sigma\gamma^\lambda - \gamma^\kappa\gamma^\nu\gamma^\mu\gamma^\gamma\gamma^\sigma\gamma^\lambda))] + B\\
&= (A)\mathrm{Tr}[2\gamma^\kappa\gamma^\mu\gamma^\nu\gamma^\gamma\gamma^\sigma\gamma^\lambda - 2\gamma^\kappa\gamma^\nu\gamma^\mu\gamma^\gamma\gamma^\sigma\gamma^\lambda] + B && (102)\\
&= (A)\mathrm{Tr}[2\gamma^\kappa(-\gamma^\nu\gamma^\mu)\gamma^\gamma\gamma^\sigma\gamma^\lambda - 2\gamma^\kappa\gamma^\nu\gamma^\mu\gamma^\gamma\gamma^\sigma\gamma^\lambda] + B && (103)\\
&= (A)\mathrm{Tr}[-2\gamma^\kappa\gamma^\nu\gamma^\mu\gamma^\gamma\gamma^\sigma\gamma^\lambda - 2\gamma^\kappa\gamma^\nu\gamma^\mu\gamma^\gamma\gamma^\sigma\gamma^\lambda] + B && (104)\\
&= (A)\mathrm{Tr}[-4\gamma^\kappa\gamma^\nu\gamma^\mu\gamma^\gamma\gamma^\sigma\gamma^\lambda] + B && (105)\\
&= -\frac{1}{4}(p_1)_\kappa(p_2)_\gamma\mathrm{Tr}[-4\gamma^\kappa\gamma^\mu\gamma^\nu\gamma^\gamma\gamma^\sigma\gamma^\lambda] + B && (106)\\
&= (p_1)_\kappa(p_2)_\gamma\mathrm{Tr}[\gamma^\kappa\gamma^\gamma] + B && (107)\\
&= 4(p_1)_\kappa(p_2)_\gamma g^{\kappa\gamma} + B && (108)\\
&= 4p_1p_2 + m^2Tr[\sigma^{\sigma\lambda}\sigma^{\mu\nu}] && (109)\\
&= 4p_1p_2 + m^24(g^{\mu\nu}g^{\lambda\sigma} - g^{\mu\lambda}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\lambda}) && (110)
\end{aligned}
$$

Branching Ratios

II The branching Ratio of a Pion

In order to calculate the Branching ratio of a charged pion, one must be familiar with the Feynman Rules of calculating amplitudes and the trace identities.

We must start with recognizing that the pion decay is a charged, weak, interaction which arises from the fact that a pion is made of quarks, and the decay is mediated by a massive W boson.

A diagram of the decay may been seen below:

Picture courtesy of Qora.com

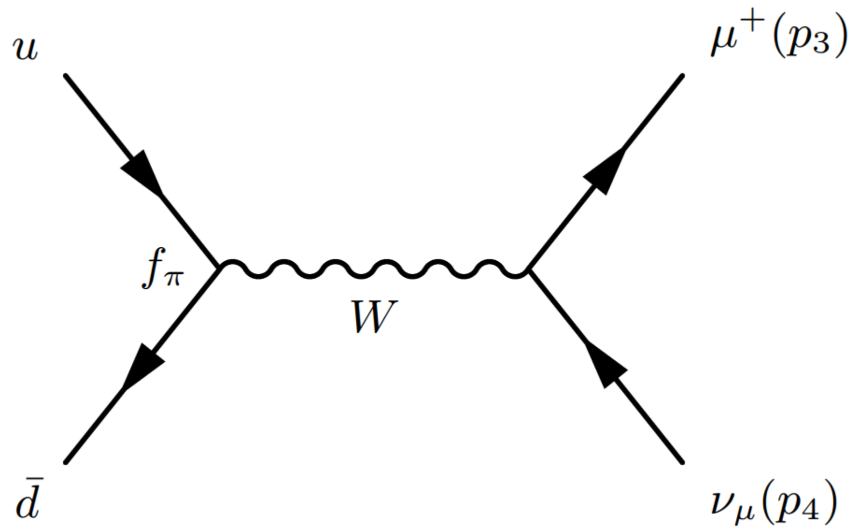

Figure 1: Pion Decay

Where the up and ani-down quark are the pion and the muon and muon neutrino are on the right. However, the muon and the muon neutrino could be the electron and electron neutrino.

The formula given to describe the branching ratio of the decay is given by [1]:

$$\Gamma \quad = \quad \frac{S|\mathbf{p}|}{8\pi\hbar m_1^2 c}|\mathcal{M}|^2 \tag{111}$$

Where $|\mathbf{p}|$ is the outgoing momentum, S is the product of statistical factors (in our case it will be equal to 1), and $\mathcal{M}$ is the Feynman amplitude. In our notation, we will use the Natural Units so our expression becomes:

$$\Gamma \quad = \quad \frac{S|\mathbf{p}|}{8\pi m_1^2}|\mathcal{M}|^2 \tag{112}$$

The next step is to determine the Feynman Amplitude which can be done in a few steps In the charged weak lepton decays, we have different notation for the vertices and propagators.

(a) For each vertex add a factor of $\frac{-ig_w}{2\sqrt{2}}(\gamma^\nu(1-\gamma^5))$ where $g_w = \sqrt{4\pi\alpha_w}$

(b) For each propagator we add a factor of $\frac{-ig_{\mu\nu}-\frac{q_\mu q_\nu}{m^2}}{q^2-m^2}$ where m is the mass of the boson. In our case, $m_w \gg q$ so the expression simplifies to $\frac{ig_{\mu\nu}}{m_w^2}$

From these rules and Figure 1 we are able to calculate the value of $\mathcal{M}$

$$-i\mathcal{M} \quad = \quad \left[\overline{u}(3)\left(\frac{-ig_w}{2\sqrt{2}}(\gamma^\nu(1-\gamma^5)\right)v(2)\right]\left[\frac{ig_{\mu\nu}}{m_w^2}\right]\left[\frac{-ig_w}{2\sqrt{2}}F^\mu\right] \tag{113}$$

We have a factor of -i with the $\mathcal{M}$ so that we obtain the real part of the expression and F^μ is the form factor of the coupling of the pion to the W boson. F^μ has the form of $f_\pi p^\mu$

$$-i\mathcal{M} = \frac{-ig_w^2}{8m_w^2}[\overline{u}(3)(\gamma^\mu(1-\gamma^5))v(2)]F^\mu \tag{114}$$
$$\mathcal{M} = \frac{g_w^2}{8m_w^2}[\overline{u}(3)(\gamma^\mu(1-\gamma^5))v(2)]F^\mu \tag{115}$$

In order to square the amplitude we do the following:

$$\langle|\mathcal{M}^2|\rangle = \left(\frac{g_w^2}{8m_w^2}f_\pi\right)^2 Tr[(\overline{u}(3)(\gamma^\mu(1-\gamma^5))v(2)p_\mu(\overline{v}(2)(\gamma^\nu(1-\gamma^5)u(3))]p_\nu \tag{116}$$
$$\langle|\mathcal{M}^2|\rangle = \left(\frac{g_w^2}{8m_w^2}f_\pi\right)^2 p_\mu p_\nu Tr[(u(3)\overline{u}(3)(\gamma^\mu(1-\gamma^5))v(2)\overline{v}(2)(\gamma^\nu(1-\gamma^5))] \tag{117}$$
$$\langle|\mathcal{M}^2|\rangle = \left(\frac{g_w^2}{8m_w^2}f_\pi\right)^2 p_\mu p_\nu Tr[((\not{p}_3+m_l)(\gamma^\mu(1-\gamma^5))\not{p}_2(\gamma^\nu(1-\gamma^5))] \tag{118}$$
$$\langle|\mathcal{M}^2|\rangle = \left(\frac{g_w^2}{8m_w^2}f_\pi\right)^2 p_\mu p_\nu(2Tr[\not{p}_3\gamma^\mu\not{p}_2\gamma^\nu] - 2Tr[\not{p}_3\gamma^\mu\not{p}_2\gamma^\nu\gamma^5]) \tag{119}$$
$$\langle|\mathcal{M}^2|\rangle = \left(\frac{g_w^2}{8m_w^2}f_\pi\right)^2 p_\mu p_\nu(8[p_3^\mu p_2^\nu + p_2^\mu p_3^\nu - (p_3\cdot p_2)g^{\mu\nu}] + 8i\epsilon^{\mu\lambda\nu\sigma}p_{3\lambda}p_{2\sigma} \tag{120}$$

Summing over the spins gives us:

$$\langle|\mathcal{M}^2|\rangle = 8\left(\frac{g_w^2}{8m_w^2}f_\pi\right)^2[2(p_1\cdot p_2)(p_1\cdot p_3) - p^2(p_2\cdot p_3)] \tag{121}$$

Since $p = p_2 + p_3$, we can simplify the equation further
For simplicity and consistency we will use the following notation: $p_1 = p_\pi, p_2 = p_l, p_3 = p_{\nu_l}$
We begin with showing the value of the 4-momentum squared:

$$p_1 = (E, \vec{p}_1) \tag{122}$$
$$(p_1)^2 = p_\mu p_\nu g_{\mu\nu} \tag{123}$$
$$(p_1)^2 = p_1p_1(1) + p_2p_2(-1) + ... \tag{124}$$
$$(p_1)^2 = E^2 - (\vec{p}_1)^2 \tag{125}$$
$$(p_1)^2 = m_1^2 \tag{126}$$
$$\tag{127}$$

This can be also shown to be true for the other 4-Momenta thus:
$(p_1)^2 = (p_\pi)^2 = m_1^2, (p_2)^2 = (p_l)^2 = m_l^2, (p_3)^2 = (p_{\nu_l})^2 = (m_{\nu_l})^2 = 0$ Using this we can further simplify Equation 121

$$p_1 = p_2 + p_3 \tag{128}$$

$$
\begin{aligned}
(p_1)^2 &= (p_2+p_3)(p_2+p_3) && (129)\\
(p_1)^2 &= (p_2)^2+(p_3)^2+2(p_2\cdot p_3) && (130)\\
(p_1)^2-(p_2)^2 &= 2(p_2\cdot p_3) && (131)\\
\frac{1}{2}[(m_\pi)^2-(m_l)^2] &= (p_2\cdot p_3) && (132)
\end{aligned}
$$

$$
\begin{aligned}
p_1 &= p_2+p_3 && (133)\\
(p_2)^2 &= (p_1-p_3)(p_1-p_3) && (134)\\
(p_2)^2 &= (p_1)^2+(p_3)^2-2(p_1\cdot p_3) && (135)\\
(p_2)^2-(p_1)^2 &= -2(p_1\cdot p_3) && (136)\\
\frac{1}{2}[(m_\pi)^2-(m_l)^2] &= (p_1\cdot p_3) && (137)
\end{aligned}
$$

$$
\begin{aligned}
p_1 &= p_2+p_3 && (138)\\
(p_3)^2 &= (p_1-p_2)(p_1-p_2) && (139)\\
(p_3)^2 &= (p_1)^2+(p_2)^2-2(p_1\cdot p_2) && (140)\\
-(p_1)^2-(p_2)^2 &= -2(p_1\cdot p_2) && (141)\\
\frac{1}{2}[(m_\pi)^2+(m_l)^2] &= (p_1\cdot p_2) && (142)
\end{aligned}
$$

Returning to Equation 121 we now have: (With $8(\frac{g_w^2}{8m_w^2}f_\pi)^2 = C$)

$$
\begin{aligned}
\langle|\mathcal{M}^2|\rangle &= C[2(p_1\cdot p_2)(p_1\cdot p_3)-p^2(p_2\cdot p_3)] && (143)\\
\langle|\mathcal{M}^2|\rangle &= C[\frac{1}{2}[(m_\pi)^2+(m_l)^2][(m_\pi)^2-(m_l)^2]-(m_\pi)^2(\frac{1}{2}((m_\pi)^2-(m_l)^2)] && (144)\\
\langle|\mathcal{M}^2|\rangle &= C[[\frac{1}{2}[(m_\pi)^4-(m_l)^4)]-[\frac{1}{2}((m_\pi)^4+(m_\pi)^2(m_l)^2)] && (145)\\
\langle|\mathcal{M}^2|\rangle &= C[\frac{1}{2}(m_\pi)^4-\frac{1}{2}(m_l)^4-\frac{1}{2}(m_\pi)^4+\frac{1}{2}(m_\pi)^2(m_l)^2] && (146)\\
\langle|\mathcal{M}^2|\rangle &= C[-\frac{1}{2}(m_l)^4+\frac{1}{2}(m_\pi)^2(m_l)^2] && (147)\\
\langle|\mathcal{M}^2|\rangle &= C[-\frac{1}{2}(m_l)^4+\frac{1}{2}(m_\pi)^2(m_l)^2)] && (148)\\
\langle|\mathcal{M}^2|\rangle &= C[\frac{1}{2}(m_\pi)^2(m_l)^2-\frac{1}{2}(m_l)^4] && (149)\\
\langle|\mathcal{M}^2|\rangle &= C[\frac{1}{2}(m_l)^2((m_\pi)^2-(m_l)^2)] && (150)\\
& && (151)
\end{aligned}
$$

In this way we are able to calculate the branching ratio of a pion, since we know have the Feyneman Amplitude we simply return to equation (112)

$$
\Gamma = \frac{S|\mathbf{p}|}{8\pi m_1^2}|\mathcal{M}|^2 \qquad (152)
$$

$$\Gamma = \frac{S|\mathbf{p}|}{8\pi m_\pi^2} 8\left(\frac{g_w^2}{8m_w^2} f_\pi\right)^2 \left[\frac{1}{2}(m_l)^2((m_\pi)^2 - (m_l)^2)\right] \tag{153}$$

We can remove S because in this case S=1
We then need to find the value of$|\mathbf{p}|$

$$|\mathbf{p}| = \frac{\sqrt{(m_1+m_2+m_3)(m_1-m_2-m_3)(m_1+m_2-m_3)(m_1-m_2+m_3)}}{2m_1} \tag{154}$$

$$|\mathbf{p}| = \frac{\sqrt{m_1^4+m_2^4+m_3^4-2m_1^2m_2^2-2m_1^2m_3^2-2m_2^2m_3^2}}{2m_1} \tag{155}$$

$$|\mathbf{p}| = \frac{\sqrt{m_1^4+m_2^4-m_1^2m_2^2}}{2m_1} \tag{156}$$

(Because the neutrino is massless)

$$|\mathbf{p}| = \frac{\sqrt{(m_1^2-m_2^2)^2}}{2m_1} \tag{157}$$

$$\tag{158}$$

For our case $m_1 = m_\pi$ and $m_2 = m_l$

$$|\mathbf{p}| = \frac{\sqrt{(m_\pi^2-m_l^2)^2}}{2m_\pi} \tag{159}$$

$$|\mathbf{p}| = \frac{(m_\pi^2-m_l^2)}{2m_\pi} \tag{160}$$

$$\tag{161}$$

Combining with Equation 153:

$$\Gamma = \frac{|\mathbf{p}|}{8\pi m_1^2}\left(\frac{g_w^2}{8m_w^2} f_\pi\right)^2 \left[\frac{1}{2}(m_l)^2((m_\pi)^2 - (m_l)^2)\right] \tag{162}$$

$$\Gamma = \frac{\frac{(m_\pi^2-m_l^2)}{2m_\pi}}{\pi m_\pi^2}\left(\frac{g_w^2}{m_w^2} f_\pi\right)^2 \left[\frac{1}{2}(m_l)^2((m_\pi)^2 - (m_l)^2)\right] \tag{163}$$

$$\Gamma = \frac{1}{4\pi}\left(\frac{g_w^2 f_\pi}{8m_w^2}\right)^2 \frac{1}{m_\pi^3} m_l^2 (m_\pi^2 - m_l^2)^2 \tag{164}$$

Expanding on this idea, we are able to graph $\frac{\Gamma_l}{\Gamma_\pi}$ by:

$$\Gamma = \frac{1}{4\pi}\left(\frac{g_w^2 f_\pi}{8m_w^2}\right)^2 \frac{1}{m_\pi^3} m_l^2 (m_\pi^2 - m_l^2)^2 \tag{165}$$

$$\Gamma = \frac{1}{4\pi}\left(\frac{g_w^2 f_\pi}{8m_w^2}\right)^2 \frac{1}{m_\pi^3} m_l^2 m_\pi^4 \left(1 - \frac{m_l^2}{m_\pi^2}\right)^2 \tag{166}$$

$$\Gamma = \frac{1}{4\pi}\left(\frac{g_w^2 f_\pi}{8m_w^2}\right)^2 m_l^2 m_\pi \left(1 - \frac{m_l^2}{m_\pi^2}\right)^2 \tag{167}$$

$$\Gamma = \frac{1}{4\pi}\left(\frac{g_w^2 f_\pi}{8m_w^2}\right)^2 m_\pi^3 \frac{m_l^2}{m_\pi^2}\left(1-\frac{m_l^2}{m_\pi^2}\right)^2 \tag{168}$$

$$\Gamma = \frac{1}{4\pi}\left(\frac{g_w^2 f_\pi}{8m_w^2}\right)^2 m_\pi^3 \left(\frac{m_l}{m_\pi}\right)^2\left(1-\left(\frac{m_l}{m_\pi}\right)^2\right)^2 \tag{169}$$

$$\Gamma = \frac{1}{4\pi}\left(\frac{G_f f_\pi}{\sqrt{2}}\right)^2 m_\pi^3 \left(\frac{m_l}{m_\pi}\right)^2\left(1-\left(\frac{m_l}{m_\pi}\right)^2\right)^2 \tag{170}$$

$$\Gamma = \frac{1}{8\pi}(G_f f_\pi)^2 m_\pi^3 \left(\frac{m_l}{m_\pi}\right)^2\left(1-\left(\frac{m_l}{m_\pi}\right)^2\right)^2 \tag{171}$$

In order to find $\frac{\Gamma_l}{\Gamma_\pi}$ we need to divide the above expression by $\frac{1}{\tau_\pi}$ but normalize it with $\hbar$ so our final expression is:

$$\frac{\Gamma_l}{\Gamma_\pi} = \frac{1}{8\pi}(G_f f_\pi)^2 m_\pi^3 \left(\frac{\tau_\pi}{\hbar}\right)\left(\frac{m_l}{m_\pi}\right)^2\left(1-\left(\frac{m_l}{m_\pi}\right)^2\right)^2 \tag{172}$$

The graph of the equation (with $m_l = m_e$) is:

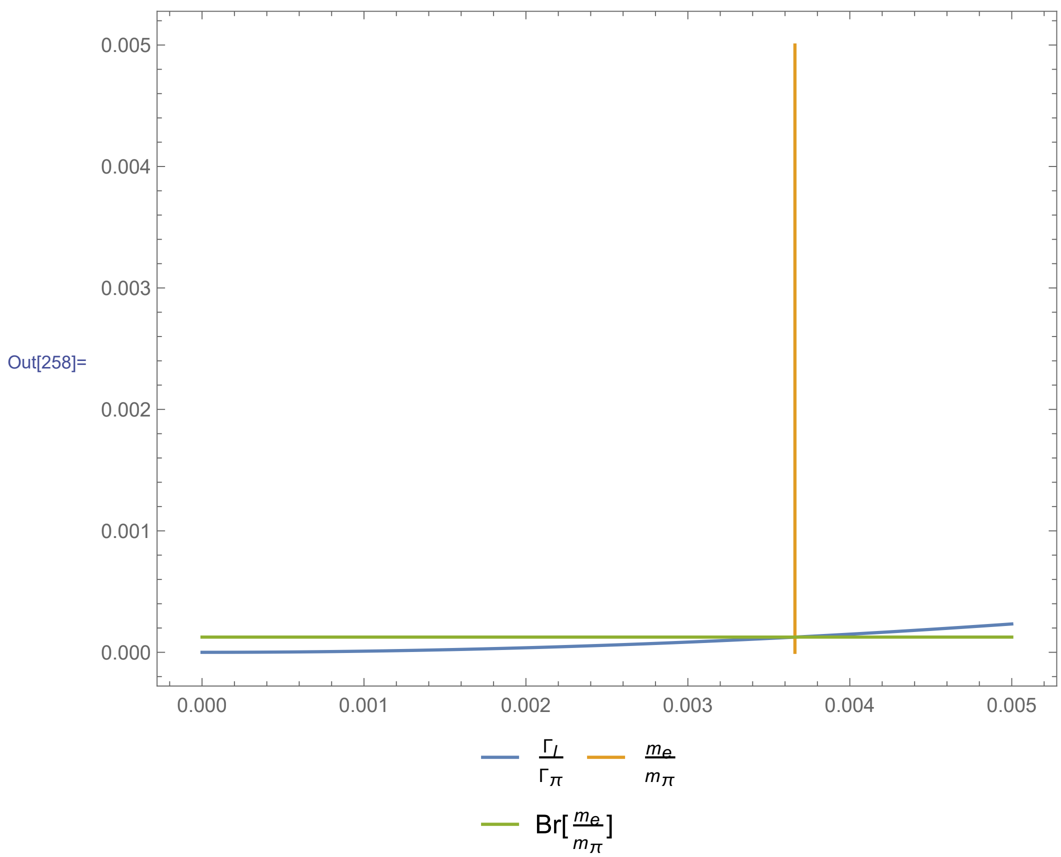

Figure 2: Graph of $\frac{\Gamma_{m_e}}{\Gamma_\pi}$ and the plots of $\frac{m_e}{m_\pi}$ and the value of $Br(\pi^- \to e^- + \nu_e)$

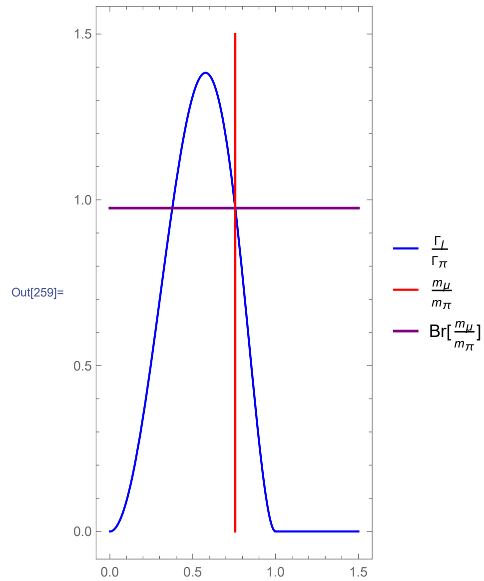

Figure 3: Graph of $\frac{\Gamma_{m_\mu}}{\Gamma_\pi}$ and the plots of $\frac{m_\mu}{m_\pi}$ and the value of $Br(\pi^- \to \mu^- + \nu_\mu)$

Using the values from PDG [2] we can do calculations with the branching ratios.

Observables	e	μ	π
τ	6.6×10^{28} yr	$2.1969811(22) \times 10^-6$ s	$2.6033(5) \times 10^-8$ s
Mass(MeV)	0.5109989461(31)	105.6583745(24)	139.57061(24)

If we would like to calculate the ratio of the $\pi^- \to e^- + \overline{\nu}_e$ and $\pi^- \to \mu^- + \overline{\nu}_\mu$ we simply do the following:

$$\frac{\Gamma_e}{\Gamma_\mu} = \frac{m_e^2(m_\pi^2 - m_e^2)^2}{m_\mu^2(m_\pi^2 - m_\mu^2)^2} \tag{173}$$

$$\frac{\Gamma_e}{\Gamma_\mu} = 1.28334(73) \times 10^{-4} \tag{174}$$

III The Branching Ratio of the $B_s \to \mu^+\mu^-$ decay

Following the procedure outlined above and comparing to [3] we are able to find an expression for the decay rate of the$B_s \to \mu^+\mu^-$ with some coefficients of new physics included.
The decay rate is as follows:

$$\mathcal{B}(B_s \to \mu^+\mu^-) = \frac{G_F^2 \alpha_{em}^2 m_{B_s}^5 f_{B_s}^2 \tau_{B_s}}{64\pi^3} \sqrt{1 - \frac{4m_\mu^2}{m_{B_s}^2}} \times$$
$$\left\{ \left(1 - \frac{4m_\mu^2}{m_{B_s}^2}\right) \left| \zeta \frac{C_S - C_S'}{m_b + m_s} \right|^2 + \left| \zeta \frac{C_P - C_P'}{m_b + m_s} + \frac{2m_\mu}{m_{B_s}^2} [|V_{tb}V_{ts}^*| C_{10} + \zeta(C_A - C_A')] \right|^2 \right\} \quad (174)$$

Where $\zeta \equiv (\frac{g_{NP}^2}{\Lambda^2})(\frac{\sqrt{2}}{4G_F})(\frac{4\pi}{\alpha_{em}})$

We seek to find the constraints on the parameters $C_S, C_S', C_P, C_P', C_A$, and C_A'

Setting the equation equal to the branching ratio given in [4] and using the values for $C_{10}, |V_{tb}V_{ts}^*|, g_{NP}$, and Λ given in [3] We are able to find the constraints on the parameters.

The values for the Branching ratio of $B_s \to \mu^+\mu^-$ decay is equal to $2.4^{+0.9}_{-0.7} \times 10^{-9}$ [2]

In order to solve for one parameter, we allow the other two to be equal to 0, this will simply our calculations.

We wish to make three measurements of each parameter and solve for the average and the standard deviation. We solve for each coefficient by setting the expression with the unknown coefficient equal to the branching ratio, the upper limit of the branching ratio, and its lower limit.

$$C_S = \pm 1.6215i \times 10^{-4} \text{ For the lower BR value} \quad (175)$$
$$= \pm 4.63981 \times 10^{-5} \text{ For the upper BR value} \quad (176)$$

$$C_P = 6.3832 \times 10^{-5} \text{ or } 4.1125 \times 10^{-4} \text{ For the lower BR value} \quad (177)$$
$$= -4.482 \times 10^{-6} \text{ or } 4.7957 \times 10^{-4} \text{ For the upper BR value} \quad (178)$$

$$C_A = 2.0348 \times 10^{-3} \text{ or } 1.31095 \times 10^{-2} \text{ For the lower BR value} \quad (179)$$
$$= 1.48237 \times 10^{-4} \text{ or } 1.5287 \times 10^{-2} \text{ For the upper BR value} \quad (180)$$

Notes about the Wilson Coefficients:
The C_S coefficient is equal to $\pm(0.811i + 0.232) \times 10^{-4}$ (We are able to calculate the mean by a simply average)

The C_P has two values the first value is equal to 2.967×10^{-5} and the second value is equal to 4.454×10^{-4}

The last coefficient C_A also has two values, the first one is equal to 1.089×10^{-3} and the second value is equal to 1.420×10^{-2}

After finding the Wilson Coefficients, I was able to use the Python package Flavio [4] to compute the NP values of $B(B_s \to \mu^+\mu^-)$

We again only apply one Wilson Coefficient at a time to find the branching ratio:
For the C_S:

$$C_S = \pm(0.811i + 0.232) \times 10^{-4}$$
$$\text{Flavio Prediction: BR} = 3.610 \times 10^{-9}$$

For C_P:

$$C_{P1} = 2.967 \times 10^{-5}$$
$$\text{Flavio Prediction: } Br = 3.603 \times 10^{-9}$$
$$C_{P2} = 4.454 \times 10^{-4}$$
$$\text{Flavio Prediction: } Br = 3.508 \times 10^{-9}$$

For C_A:

$$C_{A1} = 1.089 \times 10^{-3}$$
$$\text{Flavio Prediction: } Br = 3.608 \times 10^{-9}$$
$$C_{A2} = 1.420 \times 10^{-2}$$
$$\text{Flavio Prediction: } Br = 3.584 \times 10^{-9}$$

IV Plots of $Br(WC)$

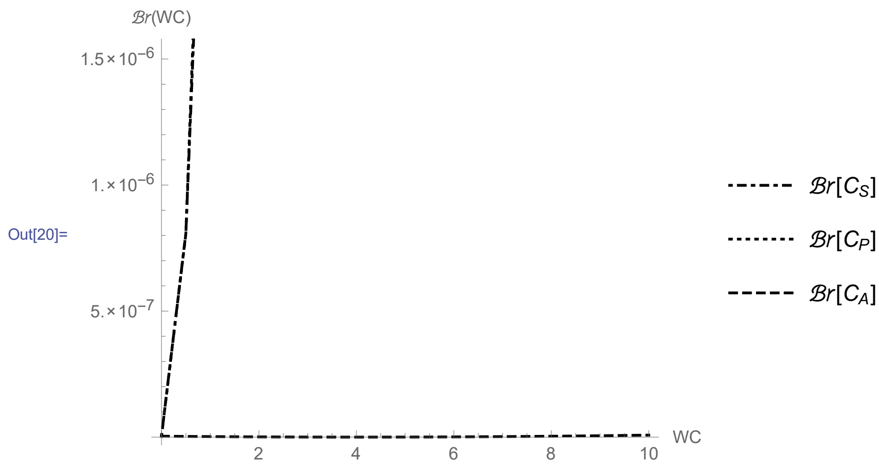

Figure 4: Graph of $Br(B_s \to \mu^+\mu^-)$ as functions of the Wilson Coefficients

The above graph gives an indication of how drastically the variation of a Wilson Coefficient could change the value of the Branching ratio.
For each of the plots I have conducted a fit to the lines in order to find the coefficient values to the lines.
$Br(C_S) = 3.60932 \times 10^{-9} - (9.45 \times 10^{-14})x + (3.22041 \times 10^{-6})x^2$
$Br(C_P) = 3.60975 \times 10^{-9} - (2.30203 \times 10^{-7})x + (3.67017 \times 10^{-6})x^2$
$Br(C_A) = 3.60975 \times 10^{-9} - (1.79093 \times 10^{-9})x + (2.22135 \times 10^{-10})$

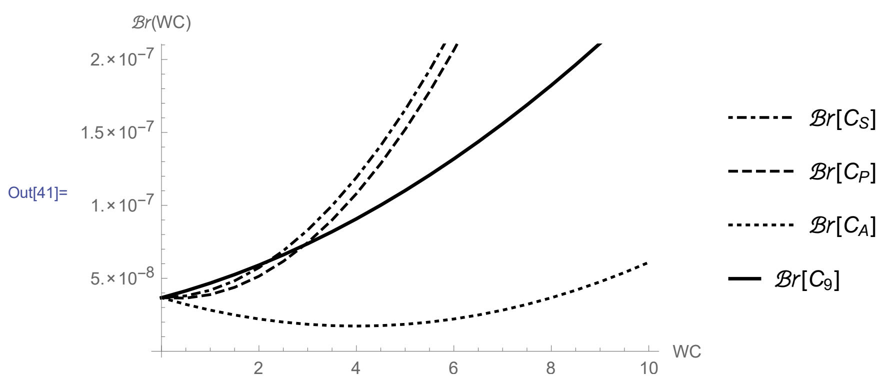

Figure 5: Graph of $Br(B \to K\mu^+\mu^-)$ as functions of the Wilson Coefficients

Here, in this decay I was also able to compute fits to the plots.

$Br(C_S) = 3.66906 \times 10^{-8} + (2.58706 \times 10^{-20})x + (5.15254 \times 10^{-9})x^2$
$Br(C_P) = 3.66906 \times 10^{-8} - (3.08941 \times 10^{-9})x + (5.23039 \times 10^{-9})x^2$
$Br(C_A) = 3.66906 \times 10^{-8} - (9.70883 \times 10^{-9})x + (1.21345 \times 10^{-9})x^2$
$Br(C_9) = 3.66906 \times 10^{-8} + (8.83595 \times 10^{-9})x + (1.17121 \times 10^{-9})x^2$

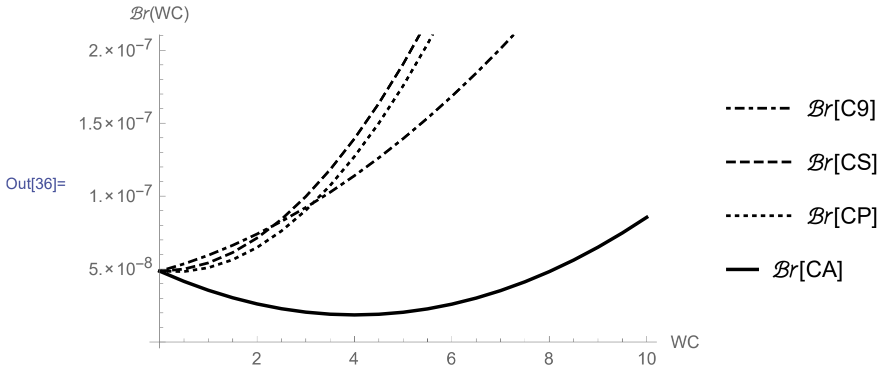

Figure 6: Graph of $Br(B \to K^* \mu^+ \mu^-)$ as functions of the Wilson Coefficients

The fits for the plots are as follows:
$Br(C_S) = 4.857 \times 10^{-8} + (9.43947 \times 10^{-21})x + (5.68001 \times 10^{-9})x^2$
$Br(C_P) = 4.857 \times 10^{-8} - (3.40568 \times 10^{-9})x + (5.76584 \times 10^{-9})x^2$
$Br(C_A) = 4.857 \times 10^{-8} - (1.4957 \times 10^{-8})x + (1.86477 \times 10^{-9})x^2$
$Br(C_9) = 4.857 \times 10^{-8} + 9.1123 \times 10^{-9})x + 1.81191 \times 10^{-9})x^2$

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