Practice with Spinors

The first thing to do is to make the writing a little more compact. Follow the first few lines to get a better handle on how to format the rest of your document. Once you have implemented some of these changes, delete the direction lines.

I Algebra with Dirac γ matrices

In order to make your equations more compact use the equation or eqnarray environments. Look above the \begin{document}

1. $(\overline{u}_1\gamma^\mu u_2)^* \;=\; (u_1^\dagger\gamma^0\gamma^\mu u_2)^\dagger$

   Because $(\overline{u}_1\gamma^\mu u_2)$ is a 1x1 matrix, the complex conjugate is the same as the Hermitian conjugate

   $(\overline{u}_1\gamma^\mu u_2)^* \;=\; (u_2^\dagger(\gamma^\mu)^\dagger(\gamma^0)^\dagger(u_1)$

   Note: $(\gamma^0)^\dagger \;=\; \gamma^0$ and $(\gamma^\mu)^\dagger \;=\; \gamma^0\gamma^\mu\gamma^0$

   $(\overline{u}_1\gamma^\mu u_2)^* \;=\; (u_2)^\dagger\gamma^0\gamma^\mu\gamma^0\gamma^0 u_1$

   $(\overline{u}_1\gamma^\mu u_2)^* \;=\; (u_2)^\dagger\gamma^0\gamma^\mu(1)u_1$

   $(\overline{u}_1\gamma^\mu u_2)^* \;=\; \overline{u}_2\gamma^\mu u_1$

   Let's change this item to look like the following:

   $(\overline{u}_1\gamma^\mu u_2)$ is a 1×1 matrix. Therefore, its complex conjugate is the same as its Hermitian conjugate, i.e. if we call $L^\mu = (\overline{u}_1\gamma^\mu u_2)$, then $(L^\mu)^* = (L^\mu)^\dagger$. We can then express this quantity as follows:

$$\begin{aligned} L^\mu &= \overline{u}_1\gamma^\mu u_2 \;, && (1)\\ \Rightarrow (L^\mu)^* &= (L^\mu)^\dagger \;, \\ &= (\overline{u}_1\gamma^\mu u_2)^\dagger \;, \\ &= ((u_1)^\dagger\gamma^0\gamma^\mu u_2)^\dagger \quad (\text{using } (A\ldots Z)^\dagger = Z^\dagger\ldots A^\dagger \;, \\ &= (u_2^\dagger)(\gamma^\mu)^\dagger(\gamma^0)^\dagger(u_1) \quad (\text{using } \ldots) && (2) \end{aligned}$$

2. $(\overline{u}_1\gamma^\mu\gamma^5 u_2)^* \;=\; (u_1^\dagger\gamma^0\gamma^\mu\gamma^5 u_2)^\dagger$

   $(\overline{u}_1\gamma^\mu\gamma^5 u_2)^* \;=\; u_2^\dagger(\gamma^5)^\dagger(\gamma^\mu)^\dagger(\gamma^0)^\dagger u_1$

   Note: $(\gamma^5)^\dagger = \gamma^5$

   $(\overline{u}_1\gamma^\mu\gamma^5 u_2)^* \;=\; u_2^\dagger\gamma^5\gamma^0\gamma^\mu\gamma^0\gamma^0 u_1$

   $(\overline{u}_1\gamma^\mu\gamma^5 u_2)^* \;=\; u_2^\dagger\gamma^5\gamma^0\gamma^\mu(1)u_1$

   $(\overline{u}_1\gamma^\mu\gamma^5 u_2)^* \;=\; -\,u_2^\dagger\gamma^0\gamma^5\gamma^\mu u_1$

   $(\overline{u}_1\gamma^\mu\gamma^5 u_2)^* \;=\; -\,\overline{u}_2\gamma^5\gamma^\mu u_1$

   $(\overline{u}_1\gamma^\mu\gamma^5 u_2)^* \;=\; \overline{u}_2\gamma^\mu\gamma^5 u_1$

3. $(\overline{u}_1 u_2)^* \;=\; (u_1^\dagger \gamma^0 u_2)^\dagger$

$$(\overline{u}_1 u_2)^* \;=\; u_2^\dagger (\gamma^0)^\dagger u_1$$
$$(\overline{u}_1 u_2)^* \;=\; u_2^\dagger \gamma^0 u_1$$
$$(\overline{u}_1 u_2)^* \;=\; \overline{u}_2 u_1$$

4. $(\overline{u}_1 \gamma^5 u_2)^* \;=\; (u_1^\dagger \gamma^0 \gamma^5 u_2)^\dagger$

$$(\overline{u}_1 \gamma^5 u_2)^* \;=\; u_2^\dagger (\gamma^5)^\dagger (\gamma^0)^\dagger u_1$$
$$(\overline{u}_1 \gamma^5 u_2)^* \;=\; u_2^\dagger (-\gamma^5) \gamma^0 u_1$$
$$(\overline{u}_1 \gamma^5 u_2)^* \;=\; u_2^\dagger \gamma^0 \gamma^5 u_1$$
$$(\overline{u}_1 \gamma^5 u_2)^* \;=\; \overline{u}_2 \gamma^5 u_1$$

5. $(\overline{u}_1 \sigma^{\mu\nu} u_2)^* \;=\; ((u_1)^\dagger \gamma^0 \sigma^{\mu\nu} u_2)^\dagger$

$$(\overline{u}_1 \sigma^{\mu\nu} u_2)^* \;=\; (u_2)^\dagger (\sigma^{\mu\nu})^\dagger (\gamma^0)^\dagger u_1$$

Note: $(\sigma^{\mu\nu})^\dagger = (\frac{i}{2}[\gamma^\mu, \gamma^\nu])^\dagger$

$$(\sigma^{\mu\nu})^\dagger = \tfrac{i}{2}([\gamma^\mu, \gamma^\nu])^\dagger$$
$$(\sigma^{\mu\nu})^\dagger = \tfrac{i}{2}(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu)^\dagger$$
$$(\sigma^{\mu\nu})^\dagger = \tfrac{i}{2}((\gamma^\mu)^\dagger (\gamma^\nu)^\dagger - (\gamma^\nu)^\dagger (\gamma^\mu)^\dagger)$$
$$(\sigma^{\mu\nu})^\dagger = \tfrac{i}{2}(\gamma^0 \gamma^\mu \gamma^0 \gamma^0 \gamma^\nu \gamma^0 - \gamma^0 \gamma^\nu \gamma^0 \gamma^0 \gamma^\mu \gamma^0)$$
$$(\sigma^{\mu\nu})^\dagger = \tfrac{i}{2}(\gamma^0 \gamma^\mu \gamma^\nu \gamma^0 - \gamma^0 \gamma^\nu \gamma^\mu \gamma^0)$$
$$(\sigma^{\mu\nu})^\dagger = \tfrac{i}{2}((-1)^2 \gamma^\mu \gamma^\nu - (-1)^2 \gamma^\nu \gamma^\mu)$$
$$(\sigma^{\mu\nu})^\dagger = \tfrac{i}{2}(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu) = \sigma^{\mu\nu}$$

Thus: $(\overline{u}_1 \sigma^{\mu\nu} u_2)^* \;=\; (u_2)^\dagger \sigma^{\mu\nu} \gamma^0 u_1$

$$(\overline{u}_1 \sigma^{\mu\nu} u_2)^* \;=\; (u_2)^\dagger \gamma^0 \sigma^{\mu\nu} u_1$$
$$(\overline{u}_1 \sigma^{\mu\nu} u_2)^* \;=\; \overline{u}_2) \sigma^{\mu\nu} u_1$$

Absolute Values Squared

6. $\mid \overline{u}_1 \gamma^\mu u_2 \mid^2 \;=\; (\overline{u}_1 \gamma^\mu u_2)(\overline{u}_1 \gamma^\mu u_2)^*$

$$\mid \overline{u}_1 \gamma^\mu u_2 \mid^2 \;=\; (\overline{u}_1 \gamma^\mu u_2)(\overline{u}_2 \gamma^\mu u_1)$$

I know that both of the expressions in the parentheses are 1x1 matrices but I fail to see how it can be simplified anymore.

Or is this the way to proceed: $\mid \overline{u}_1 \gamma^\mu u_2 \mid^2 \;=\; \mid \overline{u}_1 \mid^2 \mid \gamma^\mu \mid^2 \mid u_2 \mid^2$

$$\mid \overline{u}_1 \gamma^\mu u_2 \mid^2 \;=\; (\overline{u}_1)(\overline{u}_1)^* \gamma^\mu (\gamma^\mu)^* (u_2)(u_2)^*$$
$$\mid \overline{u}_1 \gamma^\mu u_2 \mid^2 \;=\; (\overline{u}_1)(\overline{u}_1)^* \gamma^\mu \gamma^\mu (u_2)(u_2)^*$$
$$\mid \overline{u}_1 \gamma^\mu u_2 \mid^2 \;=\; (\overline{u}_1)(\overline{u}_1)^* (u_2)(u_2)^*$$