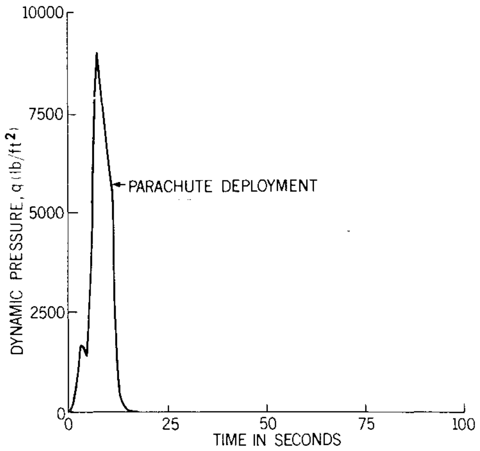

Fig. 4 Variation of dynamic pressure with time for test 158-13.

range of 1.92 to 0.44 and values obtained from the upper deceleration curve by using $C_DS = F/q$ agree fairly well with the contraves tracking camera trajectory data.

The decay in dynamic pressure with time from launch for test 158–13 is shown in Fig. 4. The sharp trajectory alteration at parachute deployment is graphically illustrated in Fig. 5 as obtained from the Contraves Cinetheodolites.[3]

Two coats of RTV silicone rubber cement were applied to the guide surface pilot chute and the suspension lines of the 20-ft chute on all three tests to act as an ablator for the aerodynamic heating resulting from the approximate 500°F stagnation temperature.

Conclusions

A series of three Nike/Nike rocket-boosted tests of a specially designed 20-ft-diam ribbon parachute has been conducted. It is concluded that 1) the chute is suitable for use up to $M = 2.43$ and a dynamic pressure of 5700 psf using a 1100-lb total weight test vehicle system and 2) both the 18-in.-diam pilot chute and reefed main 20-ft chute operated in a stable configuration and without damage during deployment.

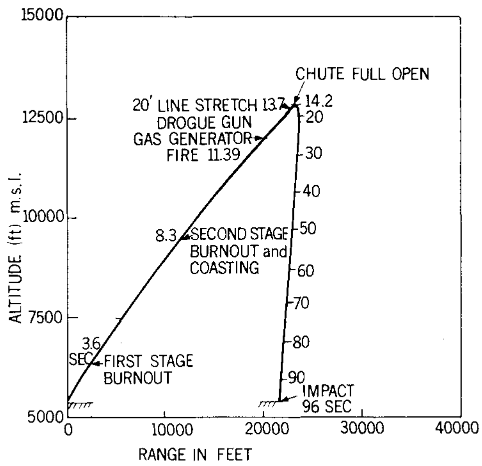

Fig. 5 Variation of altitude with range for test 158-13.

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A New Solution for the Skin-Friction Drag on a Cylinder

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Introduction

THE generally accepted formulation for the skin-friction drag on a right circular cylinder is due to Thom.[1,2] He calculated the skin friction up to 60° from the forward stagnation point by using a closed-form approximate solution of the boundary-layer equations, and by taking values between 60° and 90° from experiment, thus deducing $3.84(Re)^{-1/2}$ as the skin-friction drag coefficient for the front half of the cylinder. With a small addition for the contribution of the rear half, Thom gave $4.0(Re)^{-1/2}$ as a close estimate of this coefficient. The analysis given in this paper is formulated on the basis of Blasius' solution for flow past a right circular cylinder. The integration is carried out from the stagnation point to the point of separation. The results of the integration can be arranged as a sum of easily computed terms, which heretofore have gone unnoticed.

Analysis

The Blasius velocity distribution[3] for the flow around a right circular cylinder of radius R is

$$u = 2U_\infty \left\{ \left(\frac{x}{R}\right) f_1'(\eta) - \frac{4}{3!}\left(\frac{x}{R}\right)^3 f_3'(\eta) + \frac{6}{5!}\left(\frac{x}{R}\right)^5 f_5'(\eta) - \ldots \right\} \quad (1)$$

where x denotes the distance from the stagnation point measured along the contour of the cylinder, U_∞ is the free-stream velocity and

$$\eta = y(Re)^{1/2}/R \quad (2)$$

where the Reynolds number Re is equal to $2U_\infty R/\nu$, and y is measured perpendicular to the cylinder surface. The function $f'(\eta)$ is tabulated elsewhere.[4]

The viscous drag per unit length on the cylinder may be calculated by evaluating the following integral which is the defining equation for the skin-friction drag:

$$D_F = 2\int_0^\beta \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} \cos\Phi dx \quad (3)$$

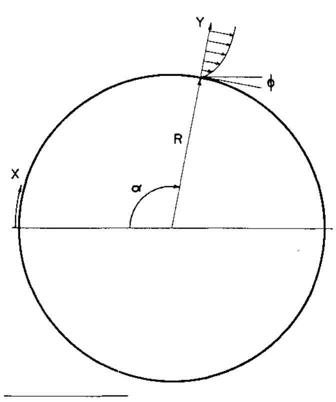

Fig. 1 Cylinder cross section showing coordinate references and angles.

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Table 1 Skin-friction drag coefficient for various degrees of approximation

$C_{DF}\ (Re)^{1/2}$	Approximation
15.3880	through x
3.4016	through x^3
6.5448	through x^5
6.0096	through x^7
6.0136	through x^9
5.9856	through x^{11}

where μ is the dynamic viscosity, β is the point of separation and Φ is the angle between the tangent to the surface and the freestream velocity, Fig. 1. The position of the point of separation β can be found from the condition that the shearing stress must vanish at that point. For a right circular cylinder in cross flow, the point of separation has been found experimentally to be around 109° from the stagnation point. Use of the Blasius' series terminated at x^{11} leads to a separation angle of 108.8°, as shown in Ref. 3.

Substitution of Eqs. (1) and (2) into (3) gives

$$D_F = \frac{4\mu U_\infty}{R}(Re)^{1/2}\int_0^\beta\left[\left(\frac{x}{R}\right)f_1'' - \frac{4}{3!}\left(\frac{x}{R}\right)^3 f_3'' + \ldots\right]\cos\Phi dx \qquad (4)$$

This integral may be evaluated by noting that $\cos\Phi = \sin\alpha$ (Fig. 1), and $\alpha = x/R$; therefore,

$$D_F = 4\mu U_\infty (Re)^{1/2}\int_0^\beta\left[f_1''\alpha - \frac{4}{3!}\alpha^3 f_3'' + \ldots\right]\sin\alpha d\alpha \qquad (5)$$

or

$$D_F = 4\mu U_\infty (Re)^{1/2}\left[f_1''\int_0^\beta \alpha \sin\alpha d\alpha - \frac{4}{3!}f_3''\int_0^\beta \alpha^3 \sin\alpha d\alpha + \ldots\right] \qquad (6)$$

The integrals in Eq. (6) may be evaluated by successive integration by parts, which leads to

$$D_F = 4\mu U_\infty(Re)^{1/2}[f_1''(-\beta\cos\beta + \sin\beta) - (4/3!) \times f_3''(-\beta^3\cos\beta + 3\beta^2\sin\beta + 6\beta\cos\beta - 6\sin\beta) + (6/5!)f_5''(-\beta^5\cos\beta + 5\beta^4\sin\beta + 20\beta^3\cos\beta - 60\beta^2\sin\beta - 120\beta\cos\beta + 120\sin\beta) + \ldots] \qquad (7)$$

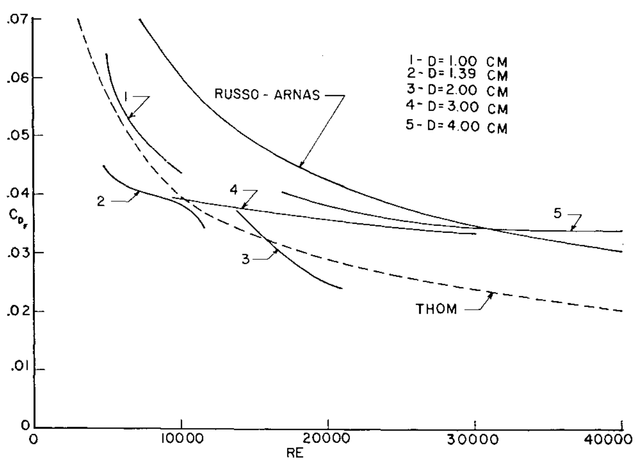

Fig. 2 Comparison of Russo-Arnas' theory and Thom's theory with data of Linke.

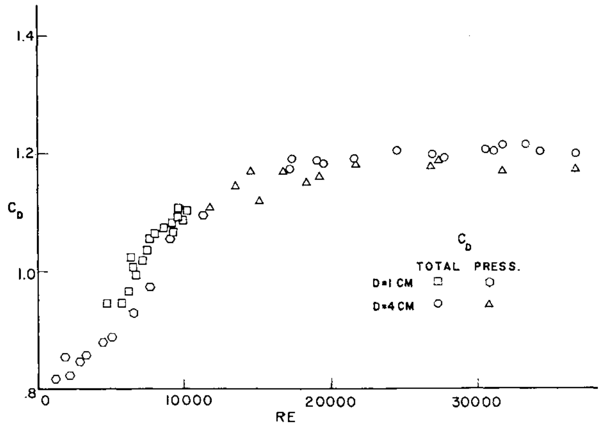

Fig. 3 Linke's data for total and pressure drag coefficients

Equation (7) may be rearranged to give

$$D_F = 4\mu U_\infty(Re)^{1/2}\Bigg[f_1''(\sin\beta - \beta\cos\beta) + 2\left(\frac{\sin\beta}{0!} - \frac{\beta\cos\beta}{1!}\right)(2f_3'' + 3f_5'' + 4f_7'' + 5f_9'' + 6f_{11}'' + \ldots) - 2\beta^2\left(\frac{\sin\beta}{2!} - \frac{\beta\cos\beta}{3!}\right) \times (2f_3'' + 3f_5'' + 4f_7'' + 5f_9'' + 6f_{11}'' + \ldots) + 2\beta^4 \times \left(\frac{\sin\beta}{4!} - \frac{\beta\cos\beta}{5!}\right)(3f_5'' + 4f_7'' + 5f_9'' + 6f_{11}'' + \ldots) - \ldots\Bigg] \qquad (8)$$

Equation (7) may also be written using summation notation as

$$D_F = 4\mu U_\infty(Re)^{1/2}\left\{f_1''(\sin\beta - \beta\cos\beta) + 2\sum_{m=1}^{m=\infty}\left[(m+1)f_{2m+1}''\left(\sum_{n=0}^{n=m}(-1)^n\beta^{2n} \times \frac{\sin\beta}{(2n)!} - \frac{\beta\cos\beta}{(2n+1)!}\right)\right]\right\} \qquad (9)$$

The viscous drag coefficient is defined as

$$C_{DF} = \frac{D_F}{\frac{1}{2}\rho U_\infty^2 D} \qquad (10)$$

where ρ is the density. Hence, substituting Eq. (9) into (10) gives

$$C_{DF} = 8(Re)^{-1/2}\left\{f_1''(\sin\beta - \beta\cos\beta) + 2\sum_{m=1}^{m=\infty}\left[(m+1)f_{2m+1}''\left(\sum_{n=0}^{n=m}(-1)^n\beta^{2n} \times \frac{\sin\beta}{(2n)!} - \frac{\beta\cos\beta}{(2n+1)!}\right)\right]\right\} \qquad (11)$$

The foregoing coefficient is tabulated in Table 1 for various degrees of approximation. As can be seen from the table, the coefficient appears to be converging to 6 $(Re)^{-1/2}$, which is depicted in Fig. 2.

Conclusion

This analysis gives a value of 6 $(Re)^{-1/2}$ for the skin-friction drag coefficient, whereas, Thom's approximate theory gives a value of 4 $(Re)^{-1/2}$. Both values are plotted in Fig. 2 along with the experimental data of Linke.[5] Caution should be taken in drawing conclusions from the data of Linke, as drawn in Fig. 2, since it is calculated from experimentally obtained results for the total and pressure drag coefficients, and not by direct measurements of the skin-friction. Figure 3 is a typical plot of the data of Linke for total and pressure drag coefficients. As can be seen from this figure, calculation of the skin-friction drag is very arbitrary because of the scatter in the data.

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Turbulent Skin-Friction Coefficient and Momentum Thickness in Adverse Pressure Gradient

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Introduction

NEARLY all the available methods for evaluating the skin-friction coefficient are based on the measured mean velocity parameters. Theoretically, it is a function of both local and upstream conditions. If local (surface roughness) conditions are specified, skin friction becomes dependent mainly upon the pressure distribution of the flowfield, and the relation between skin friction and the pressure distribution can be considered unique. The complex nature of the turbulent flow has deterred development of either a theoretical or an empirical relationship between the skin friction and the pressure distribution. This Note is intended to present an empirical relation. The development of momentum thickness is also presented.

Analysis

Sandborn and Liu[1] have given an equation to predict the turbulent boundary-layer separation in a two dimensional flow. It's form is

$$C_p^{(2n-1)/4}[x(dC_p/dx)]^{1/2} = C \tag{1}$$

where C_p is the pressure coefficient, x is the distance measured in the stream direction, C is a constant which depends on n, and n is the $1/n$th power law representation of the velocity distribution at the beginning of the adverse pressure. At such a point, the left-hand term of Eq. (1) is zero and the skin-friction coefficient C_{f0} is assumed known (it can be assumed to be the flat plate condition). As the adverse pressure increases downstream, the position of separation is found at which both terms of Eq. (1) balance, i.e., where the skin-friction coefficient is zero. By this reasoning, the following approximate linear relation may be assumed:

$$C_f/C_{f0} = 1 - \{C_p^{(2n-1)/4}[x(dC_p/dx)]^{1/2}\}/C \tag{2}$$

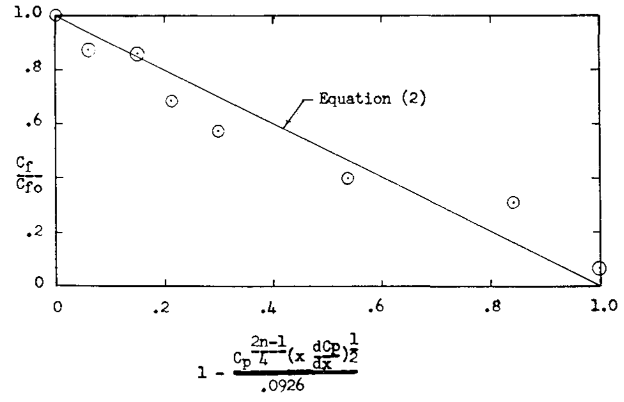

Fig. 1 Variation of skin-friction coefficient.

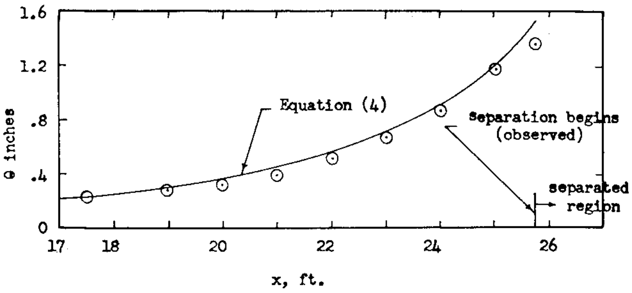

Fig. 2 Development of momentum thickness along the flow.

Figure 1 indicates Schubauer and Klebanoff's[2] experimental data are approximately a straight line.

The momentum integral equation is used to find the momentum thickness development with distance. In a two dimensional flow, its form is

$$d\theta/dx = (H + 2)(\theta/U)(dU/dx) + C_f/2 \tag{3}$$

where θ is the momentum thickness, H is the form factor, and U is the freestream velocity.

Substituting Eq. (2) into Eq. (3) and assuming that H takes a mean value, Eq. (3) can be integrated easily. The final form is

$$\theta = \exp\left(-\int_{x_0}^{x} \frac{H+2}{U}\frac{dU}{dx}\,dx\right)\left\{\theta_0 + \frac{1}{2}C_{f0} \times \int_{x_0}^{x}\left[1 - \frac{C_p^{(2n-1)/4}(x[dC_p/dx])^{1/2}}{C}\right] \times \exp\left(\int_{x_0}^{x}\frac{H+2}{U}\frac{dU}{dx}\,dx\right)\right\} \tag{4}$$

where x_0 and θ_0 are, respectively, the length and momentum thickness at the starting position of adverse pressure.

Using Schubauer and Klebanoff's experimental values, Eq. (4) is plotted in Fig. 2. The measured data are also shown. The result is reasonable.

The mean value of H is found as follows. At the beginning of adverse pressure, H is about 1.30. As separation is approached, H usually assumes a value around 2.6 for an unsteady separation. Hence, $H = 2.0$ is assumed. The constant mean value of H can induce a local maximum error about 15% in H; however, the over-all error in the region of calculation is small.

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