

# LocalFuzzyHistogramHyperbolization

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Contrast enhancement based on a fuzzy logic histogram hyperbolization algorithm. [\[more\]](#)

**Keywords:** contrast enhancement, fuzzy logic, fuzzy membership function, histogram hyperbolization, histogram transformation

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## 1 Introduction

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Fuzzy logic is a mathematical theory which was developed to deal with fuzzy data. In Boolean logic a term is either true or false, whereas in fuzzy logic a term can be partial true. The degree of truth is described with the aid of membership functions. Typically, fuzzy logic will be applied in the field of signal processing and image processing is an essential part of this.

### 1.1 Fuzzy Histogram Hyperbolization

Histogram hyperbolization is a classical algorithm for contrast enhancement and the way to generalize it to a fuzzy logic algorithm was proposed by H. R. Tizhoosh and M. Fochem [\[1\]](#). For an image of size  $M \times N$  any kind of histogram manipulation can be described by a function  $f$  which transforms the gray level  $g_{m,n}$  of the  $(m, n)^{th}$  pixel like

$$g'_{m,n} = \lambda \cdot f(g_{m,n}) \quad [1]$$

or for a fuzzy processing

$$g'_{m,n} = \lambda \cdot f(\mu(g_{m,n})), \quad [2]$$

where  $\mu(g_{m,n})$  is the **membership** of  $g_{m,n}$ .

For fuzzy histogram hyperbolization (FHH) the parameters can be determined as follows

$$\lambda = \frac{1}{e^{-1} - 1} \quad [3]$$

and

$$f(\mu(g_{m,n})) = e^{-\mu(g_{m,n})^\beta} - 1. \quad [4]$$

So finally, the equation for FHH will be:

$$g'_{m,n} = \left( \frac{1}{e^{-1} - 1} \right) \cdot \left( e^{-\mu(g_{m,n})^\beta} - 1 \right). \quad [5]$$

The fuzzifier  $\beta$  modifies the membership values additionally, which allows to control the gray level dynamics of the resulting image.

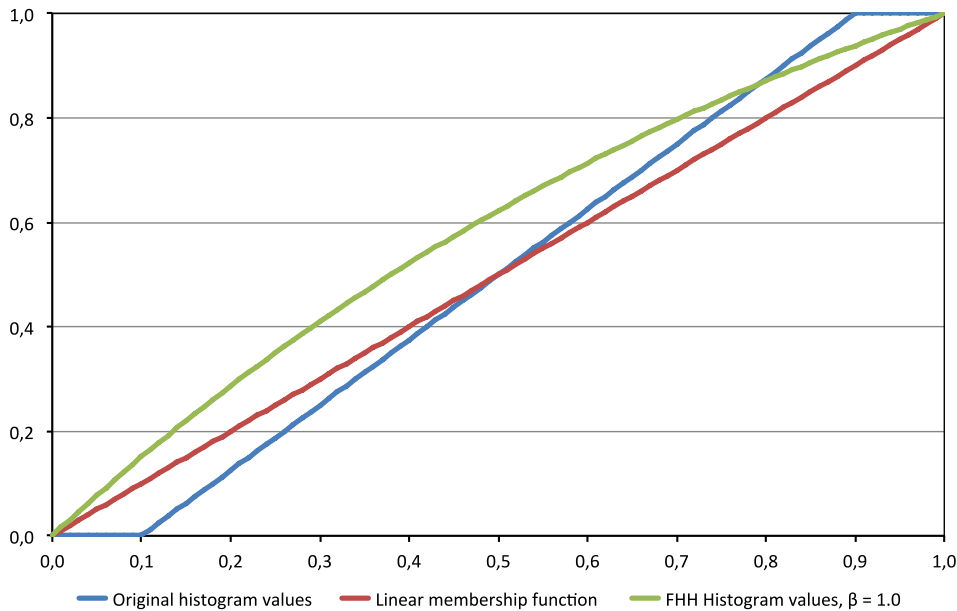
## 1.2 Membership Functions

As mentioned, the fuzzy membership function of a fuzzy set represents the degree of truth as an extension of valuation. For a given fuzzy set, in the case under consideration the gray levels of an astronomical image, the most suitable membership function can not be determined automatically but must be discovered by trial and error. For that reason the script offers four different alternative membership functions.

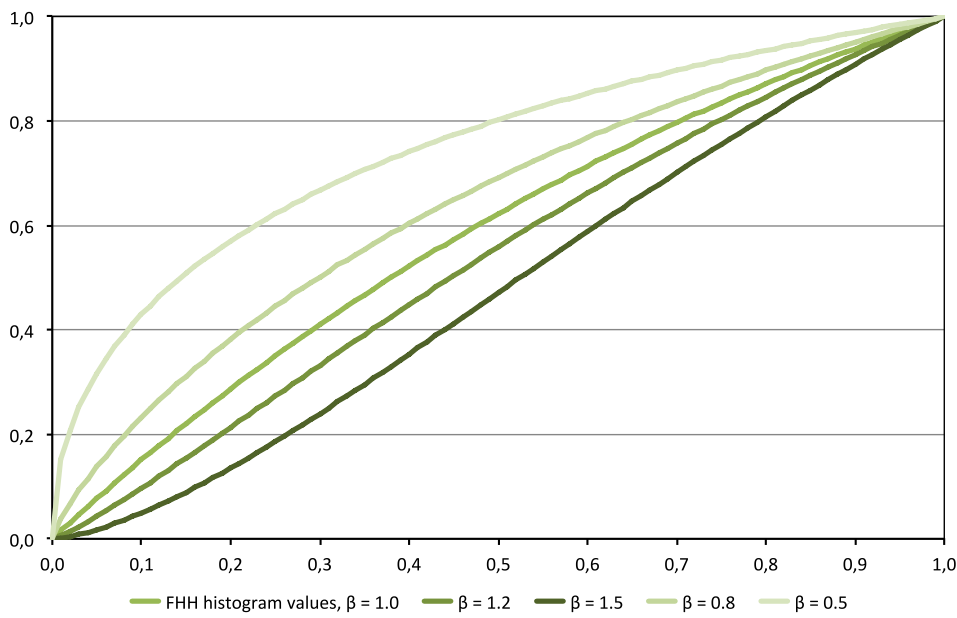
- **Linear function**, as used in the original paper [1]:

$$\mu(g_{m,n}) = \frac{g_{m,n} - g_{min}}{g_{max} - g_{min}}, \quad [6]$$

where  $g_{min}$  and  $g_{max}$  are the minimal and maximal values of the image histogram. With the linear membership function the FHH algorithm transfers the original histogram values (example with  $g_{min} = 0.1$  and  $g_{max} = 0.9$ ) as follows:



As can easily be seen in the figure below, the variation of the fuzzifier  $\beta$  has a wide influence on the FHH algorithm.

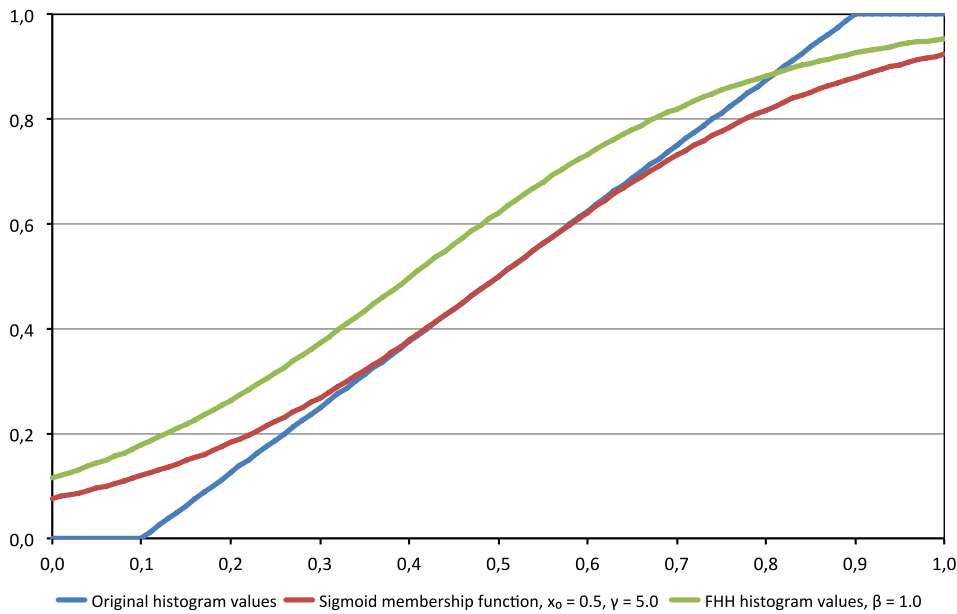


- **Sigmoid function**, as a typical S-shape function:

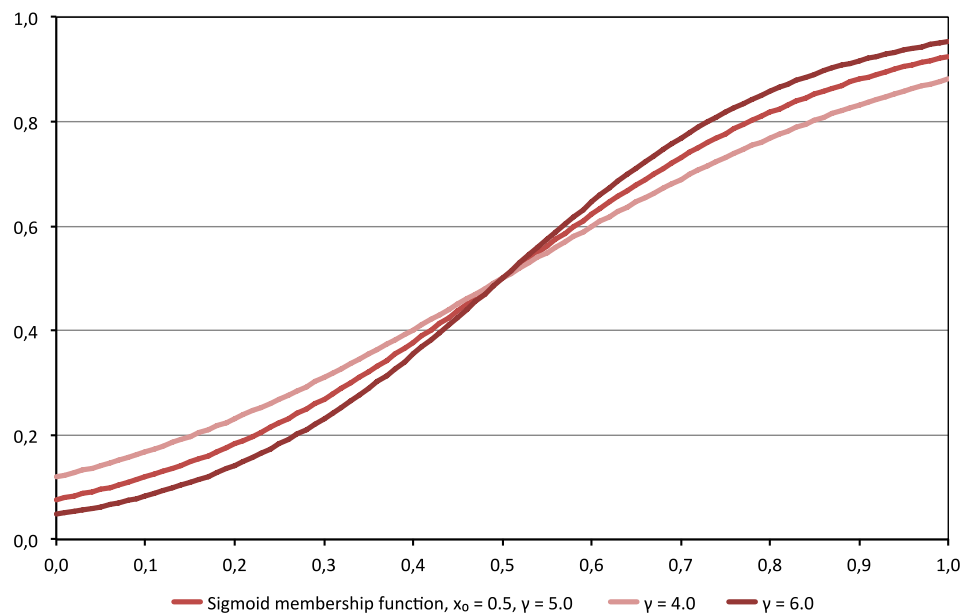
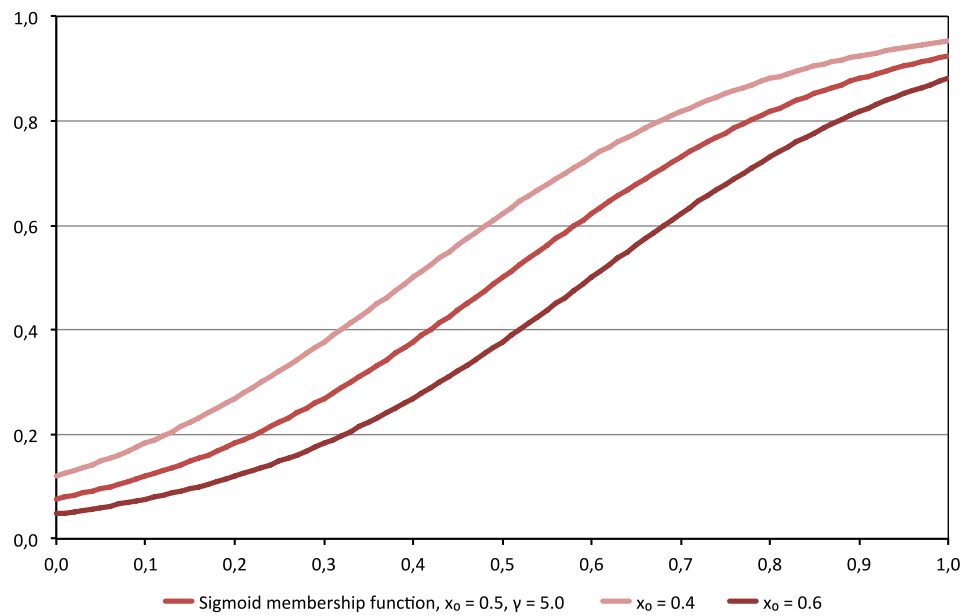
$$\mu(g_{m,n}) = \frac{1}{1 + e^{\gamma \cdot \left(x_0 - \frac{g_{m,n} - g_{min}}{g_{max} - g_{min}}\right)}},$$

[7]

where  $g_{min}$  and  $g_{max}$  are again the minimal and maximal values of the image histogram. With the Sigmoid membership function the original histogram values (same example as above) will be transferred as follows:



The specific parameters  $x_0$  and  $\gamma$  modify the shape of the Sigmoid function, whereas  $\beta$  works in the same way as for the linear membership function. The impact of  $x_0$  and  $\gamma$  is shown in the following diagrams:

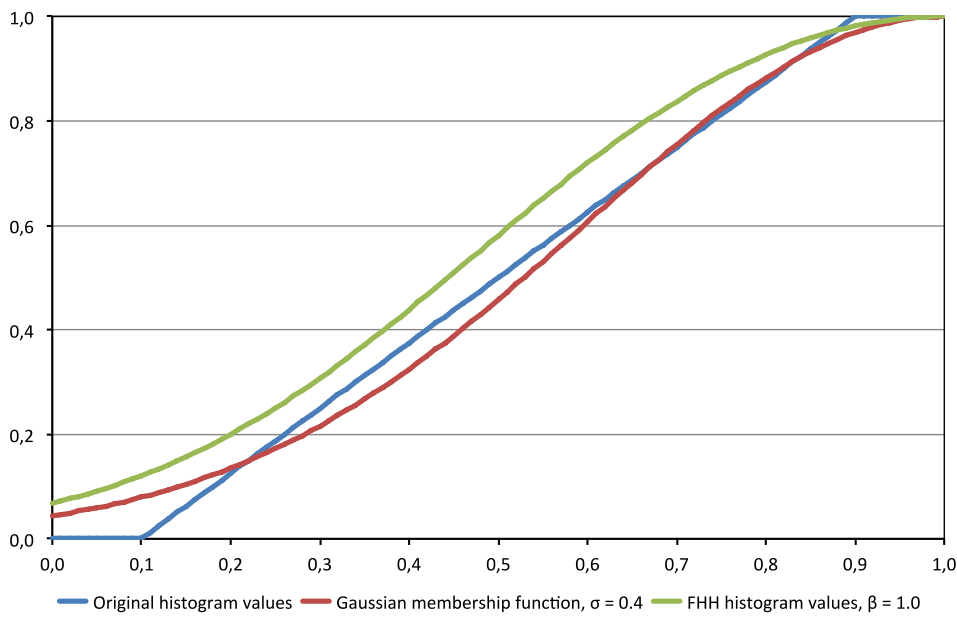


Typically, a fuzzy membership function is normalized, which means  $\mu(0) = 0$  and  $\mu(1) = 1$ . The Sigmoid membership function is not normalized, so that the pixel values of the resulting image not fully utilize the range from zero to one. On the other hand the gaps on both tails of the curve are not really serious, therefore the Sigmoid membership function works very efficient anyway.

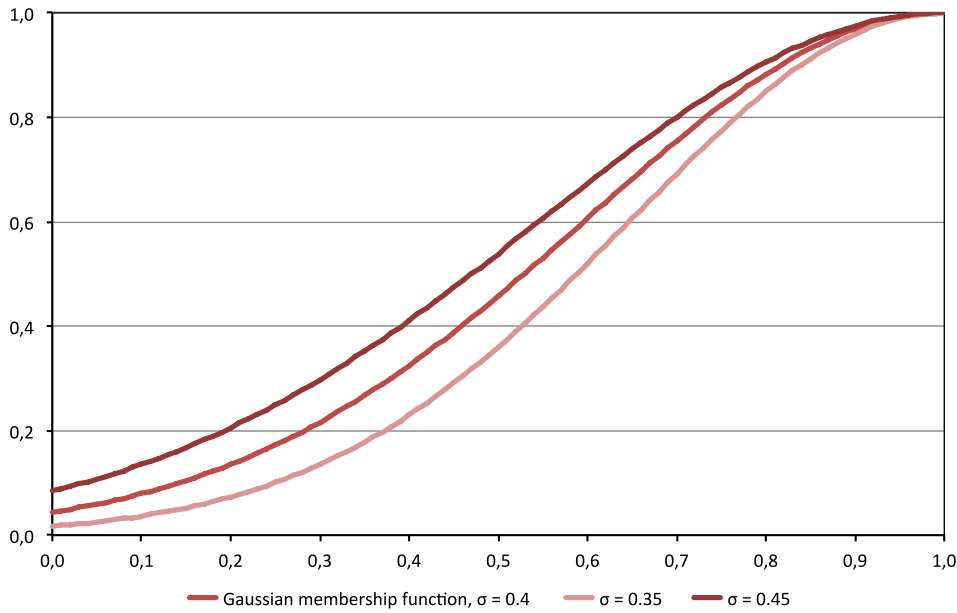
- **Gaussian function**, or to be more precise the left part of a Gaussian bell curve as another S-shape function:

$$\mu(g_{m,n}) = e^{-\frac{1}{2\sigma^2} \cdot \left(\frac{g_{max} - g_{m,n}}{g_{max} - g_{min}}\right)^2}, \quad [8]$$

$g_{min}$  and  $g_{max}$  as above. The FHH transformation with a Gaussian membership function is illustrated in the figure below:



The parameter  $\sigma$  represents the standard deviation of the Gaussian distribution. The impact of  $\sigma$  is shown in the following diagram:



The Gaussian membership function is also not normalized, because  $\mu(0) \neq 0$ . But similar to the Sigmoid membership function this has only a small effect on the quality of the FHH algorithm using a Gaussian membership function.

- **Normalized Gaussian function**, similar to the Gaussian membership function, but with an additional term  $y_0$  in the equations [3] and [4] to meet the stipulation  $\mu(0) = 0$ :

$$\lambda = \frac{1}{e^{-1} - 1 - y_0} \quad [9]$$

and

$$f(\mu(g_{m,n})) = e^{-\mu(g_{m,n})^\beta} - 1 - y_0. \quad [10]$$

with

$$y_0 = e^{-\left(e^{-\frac{1}{2\sigma^2}}\right)^\beta} - 1, \quad [11]$$

In comparison with the Gaussian membership function, the normalized Gaussian membership function utilizes the full range from zero to one, so that the contrast enhancement is more powerful than for the ordinary Gaussian membership function.

## 1.3 Local Algorithm

Generally, the FHH algorithm can be applied on an image as a whole. However, to increase the impact of the contrast enhancement, the algorithm should be applied locally to discrete segments of the image. In this case the individual histogram information of each segment is used for the FHH algorithm [2]. For the local fuzzy histogram hyperbolization (LFHH) algorithm the target image will be evenly divided into a grid pattern of rectangles. The LFHH algorithm also includes a final interpolation procedure which avoids a bordering between the segments.

## 2 Description

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LocalFuzzyHistogramHyperbolization Script

LocalFuzzyHistogramHyperbolization Script 1.0 — A script for contrast enhancement.

The script applies a Fuzzy Histogram Hyperbolization (FHH) algorithm locally to the target image. Different membership functions can be chosen for the FHH algorithm. The intensity of the FHH algorithm as well as the shape of the membership functions can be modified individually by the user. An automatic Histogram Transformation is available to adjust the brightness of the final image.

The local algorithm divides the image into discrete segments and applies the FHH algorithm separately to each segment. A user-defined specification of the segmentation offers the possibility to consider the target image characteristics. Additionally, a special clustering algorithm can be chosen to qualify the impact of the segmentation. An optional star protection is provided by selecting a star mask.

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Image Selection

Target Image: <No View Selected>

Star Mask Image: <No View Selected>

Segmentation Parameters

Image segmentation: 15 x 15

Clustering: ☒

Clustering threshold: 0.05

Fuzzy Parameters

FHH greatest lower bound: 0.35

FHH least upper bound: 0.85

Local FHH beta: ☒

Local FHH beta strength: 1.20

Constant FHH beta: 1.20

Membership Function Parameters

Fuzzy membership function: Gauss

Sigmoid gamma: 5.00

Sigmoid x0: 0.50

Gauss sigma: 0.55

Output Options

Automatic HT: ☒

HT target median: 0.20

Replace Target Image: ☐

Execute

Exit

This script applies a FHH algorithm locally to a grayscale image. The specification of the parameters is divided into five sections:

Image Selection

Segmentation Parameters

Fuzzy Parameters

Membership Function Parameters

Output Options

The tool bar at the bottom includes a button to reset the parameters to the default settings as well as a button to show this documentation.

### 2.1 Image Selection

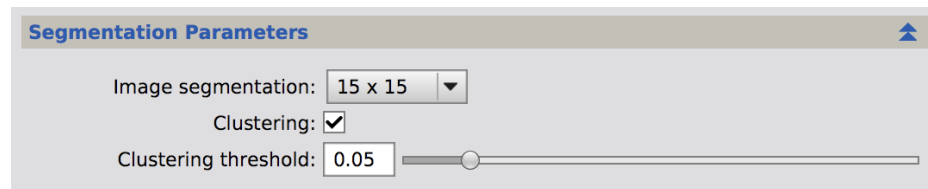
Image Selection

Target Image: <No View Selected>

Star Mask Image: <No View Selected>

In the image selection section a nonlinear grayscale image is needed to be selected as target image. Additionally, an optional star mask can be chosen. The application of a star mask excludes stars from the contrast enhancement.

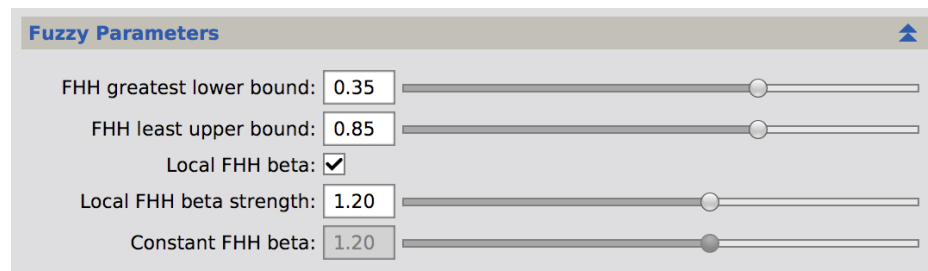
## 2.2 Segmentation Parameters



The 'Segmentation Parameters' window contains three controls: 'Image segmentation' is a dropdown menu set to '15 x 15'; 'Clustering' is a checked checkbox; and 'Clustering threshold' is a slider set to 0.05.

To apply the FHH algorithm locally to the target image the size of the local image segments is needed to be specified. The segmentation can be chosen from 5 x 5 up to 25 x 25. If clustering is selected, the fuzzy parameters of bordering segments with similar characteristics will be harmonized. Otherwise individual fuzzy parameters will be applied to each segment. The threshold parameter is used to define the sensitivity of the clustering method.

## 2.3 Fuzzy Parameters

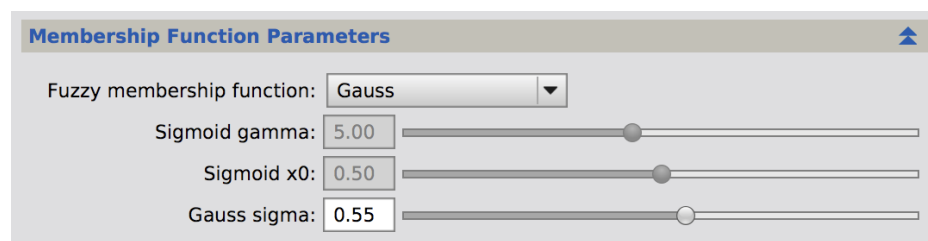


The 'Fuzzy Parameters' window contains five controls: 'FHH greatest lower bound' is a slider set to 0.35; 'FHH least upper bound' is a slider set to 0.85; 'Local FHH beta' is a checked checkbox; 'Local FHH beta strength' is a slider set to 1.20; and 'Constant FHH beta' is a slider set to 1.20.

$g_{min}$  and  $g_{max}$  are the essential parameters of every of the four [membership functions](#) which are offered by the script. The [local algorithm](#) identifies  $g_{min}$  and  $g_{max}$  individually for each single segment of the target image. In particular, for a granular segmentation the individual extrem values can vary in a wide range. For that reason it might be useful to define a greatest lower bound (glb) for  $g_{min}$  respectively a least upper bound (lub) for  $g_{max}$  to limit the variation of the extrem values.

The fuzzifier  $\beta$  modifies the dynamics of the [FHH](#) algorithm. This parameter can be used either globally or locally. In the case of a global use, a constant value for  $\beta$  will be applied to every image segment. Whereas for a local use  $\beta$  is calculated individually for each segment depending on the mean segment brightness. In both cases the strength of the method can be controlled by a user-defined specification of  $\beta$ .

## 2.4 Membership Function Parameters



The 'Membership Function Parameters' window contains four controls: 'Fuzzy membership function' is a dropdown menu set to 'Gauss'; 'Sigmoid gamma' is a slider set to 5.00; 'Sigmoid x0' is a slider set to 0.50; and 'Gauss sigma' is a slider set to 0.55.

In this section the [membership function](#) can be chosen. Four membership functions are available:

- [Linear function](#)
- [Sigmoid function](#)
- [Gaussian function](#)
- [Normalized Gaussian function](#)

For the Sigmoid function the parameters  $x_0$  and  $\gamma$  can be specified as well as  $\sigma$  for the Gaussian and normalized Gaussian function.

## 2.5 Output Options



The output options allow to select an automatic Histogram Transformation (HT) process to optimize the brightness of the resulting image. Therefore the HT target median can be defined.

Additionally, it can be selected, whether the script will replace the target image or that the modified image will be provided as a newly created image window and the target image won't be affected.

## 3 Usage

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### 3.1 Typical Workflow

A careful preparation of the target image and its star mask is essential for an effective application of the script. As every contrast enhancement algorithm FHH responds very sensitively to the existing noise of the target image. For that reason it is an important factor to reduce the noise before starting the script. The additional use of a star mask is strongly recommended, otherwise the contrast enhancement would strengthen the stars too much. However, the size of the stars in the mask must be determined very carefully to avoid ring artefacts in the final image.

The next step is the selection of a reasonable segmentation. The local application causes a very strong contrast enhancement, therefore it is recommended not to choose a too granular segmentation. Another possibility to regulate the effect of the segmentation is the clustering option. Usually, the clustering of neighboring segments with similar characteristics will lead to better results. The clustering threshold determines the sensitivity of that method. Values between 0.05 and 0.15 are typical.

Before starting the determination of the particular FHH parameters it is useful to define the target median of the resulting image in the output options. Without that the manipulation of the histogram can lead to unsatisfying results which are too dark or too bright. The parameter works comparable to those in other intensity transformation processes. Indeed the script applies a final HistogramTransformation using the specified target median.

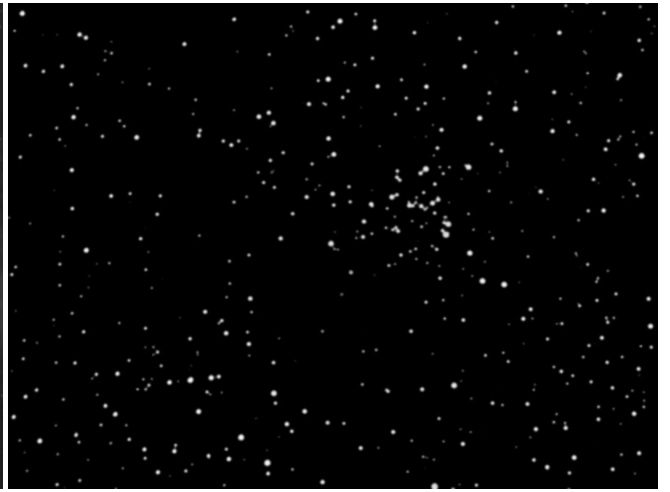
Trying the different membership functions and assess their effect on the target image is a necessary labor. For this purpose the variation of the FHH parameters has a wider influence than the membership function parameters. Usually the determination of the greatest lower bound for  $g_{min}$  respectively the least upper bound for  $g_{max}$  has the highest impact. Typical values for the greatest lower bound are between 0.2 and 0.4, for the least upper bound between 0.7 and 0.9. In most cases the local  $\beta$ -method leads to better results than the global one. Typical values for  $\beta$  are between 1.1 and 1.5. The specific parameters for the single membership functions can be used for a final fine tuning.

All used parameter values are documented in the console output. Additionally, the console output includes a measurement of the local contrast and the signal-to-noise ratio of the resulting image. These key figures may be helpful to assess the image quality.

### 3.2 Examples

In the following example LFHH is applied to a H $\alpha$ -image of M16. On the left you can see the original target image and on the right the star mask used for the algorithm.





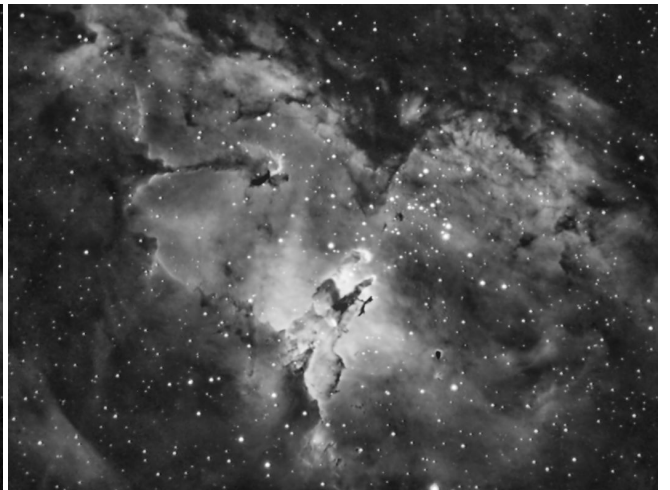
Typical results for the different membership function can be seen below.

Top left: Linear/15x15/Clustering 0.05/FHH glb 0.45/FHH lub 0.55/Local FHH beta 1.10/HT median 0.23

Top right: Sigmoid/15x15/Clustering 0.05/FHH glb 0.4/FHH lub 0.6/Local FHH beta 1.05/gamma 5.0/x0 0.50/HT median 0.23

Bottom left: Gauss/15x15/Clustering 0.05/FHH glb 0.45/FHH lub 0.55/Local FHH beta 1.20/sigma 0.60/HT median 0.23

Bottom right: NormalizedGauss/15x15/Clustering 0.05/FHH glb 0.45/FHH lub 0.55/Local FHH beta 1.10/sigma 0.75/HT median 0.23



And finally a simple PixelMath-combination of the four images.



## References

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**[1]** Hamid R. Tizhoosh, Manfred Fochem, *Fuzzy Histogram Hyperbolization for Image Enhancement*, Proc. of EUFIT'95, vol. 3, pp. 1695 - 1698, 1995

**[2]** H. R. Tizhoosh, G. Krell, B. Michaelis, *Locally adaptive Fuzzy Image Enhancement*, Computational Intelligence, Theory and Applications, Proc. of 5<sup>th</sup> Fuzzy Days, pp. 272 - 276, 1997

## Related Tools

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[LocalHistogramTransformation](#), [HistogramTransformation](#), [StarMask](#)

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