

# Some notes on tenebris

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## Abstract

A small and simple Python library that supports automatic differentiation using dual numbers and solving some simple equations.

## Getting started

The code of **tenebris** was originally published on



`https://github.com/fwidmaier/tenebris`

To install the library, simply use

```
pip install dist/tenebris-1.0-py2.py3-none-any.whl
```

To build the wheel file of the library, use

```
python setup.py bdist_wheel -universal
```

**Dependencies.** The required packages to work with **tenebris** are listed in `requirements.txt`. **Currently, there are no dependencies. Only standard libraries are used.**

# 1 Dual numbers

Define  $\mathcal{D} = \mathbb{R}[\varepsilon]/\langle \varepsilon^2 \rangle$  to be the polynomial ring with one variable  $\varepsilon$  modulo the ideal generated by  $\varepsilon^2$ . So any element in  $\mathcal{D}$  is of the form

$$d = a + b\varepsilon$$

for some  $a, b \in \mathbb{R}$ . We call  $a$  the *real part* of  $d$  and  $b$  the *dual part* of  $d$ . Let us consider some analytic function  $f: \mathbb{R} \rightarrow \mathbb{R}$ . Then by the Taylor expansion of  $f$  we have for some  $x + b\varepsilon \in \mathcal{D}$

$$\begin{aligned} f(x + b\varepsilon) &= f(x) + f'(x)b\varepsilon + \frac{f''(x)}{2}b^2\varepsilon^2 + \frac{f'''(x)}{6}b^3\varepsilon^3 + \dots \\ &= f(x) + f'(x)b\varepsilon. \end{aligned}$$

Hence  $f(x + \varepsilon) = f(x) + f'(x)\varepsilon$ . And et voilà! This is basically all we need for automatic differentiation using dual numbers! We only need to know the dual part of  $f(x + \varepsilon)$  and we obtain  $f'(x)$ .

**Example.** Let us go through a little toy example in order to get some better grasp on dual numbers. For instance, let  $f: x \mapsto x^n$  for some  $n \in \mathbb{N}$ . Then

$$\begin{aligned} f(x + \varepsilon) &= (x + \varepsilon)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} \varepsilon^k \\ &= x^n + nx^{n-1}\varepsilon + \binom{n}{2}x^{n-2}\varepsilon^2 + \dots = x^n + nx^{n-1}\varepsilon. \end{aligned}$$

Just as expected.

Basic computations show

$$\begin{aligned} (a + b\varepsilon) + (c + d\varepsilon) &= a + c + (b + d)\varepsilon, & (a + b\varepsilon)(c + d\varepsilon) &= ac + (ad + bc)\varepsilon \\ \text{and} \quad (a + b\varepsilon)(c + d\varepsilon)^{-1} &= \frac{a}{c} + \frac{bc - ad}{c^2} \quad \text{where } c \neq 0. \end{aligned}$$

This arithmetic of dual numbers is basically encoded in the class `Dual`. The differentiating operator is implemented as `d`. It evaluates the given function on  $x + \varepsilon$  (or rather `Dual(x, 1)`) and returns the dual part of the result.