IFT6135 – Apprentissage de Représentations (Deep Learning)

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Introduction

These are my personal notes from the course IFT6135 — Representation Learning — taught by Aaron Courville at MILA in Montreal, Quebec, during the Winter 2025 semester. The course ran from January to April, and it was a deep (pun intended) dive into the theory and practice of learning useful representations from data.

That said — fair warning! These notes are incomplete, sometimes messy (I often switch between French and English), and occasionally wrong. They reflect my own understanding of the material, and I've tried my best to make sense of complex ideas. Some parts were generated or cleaned up with the help of ChatGPT, others go slightly beyond what was taught in class. There are definitely missing citations, and I still plan to add more figures and references.

I'm sharing these in case they help others going through this intense (but rewarding!) course. If you spot errors, have suggestions, or want to expand a section, feel free to fork the repo and open a pull request. I'll do my best to review changes promptly. If you do contribute, please don't forget to add your name to the author list!

A sincere thank you to Professor Aaron Courville for his inspiring lectures, to the TAs for crafting such challenging and thought-provoking assignments, and to all the students — especially Thomas, Maël and Olivier — whose discussions, questions, and collaboration made the learning process far more engaging. It was a real privilege to be part of such a passionate and driven group.

And for those reading this outside the context of MILA — welcome! You'll probably get the most out of these notes if you already have some background in math, computer science, and deep learning. A solid starting point is the Deep Learning book: `https://www.deeplearningbook.org/`.

Enjoy the ride — and good luck with your learning journey!

Contents

1 History

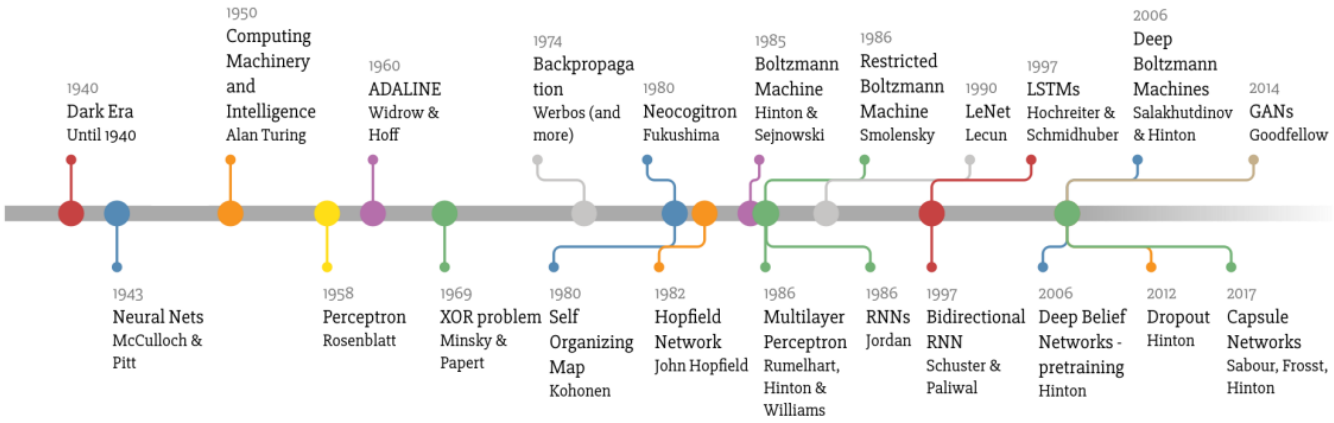


Figure 1: Contexte historique

Naissance de l'IA (années 1950) Le début du domaine de l'IA (intelligence artificielle) a eu lieu vers 1950.

- Perceptron de McCulloch et Pitts (modèle de neurone artificiel).
- Apprentissage de Rosenblatt (introduction du calcul de l'erreur).
- Théorie du calcul de Turing.
- Théorie de l'information de Shannon.

Le neurone artificiel est une inspiration de son homologue biologique. Il est composé d'entrées, de poids et d'une fonction d'activation. Initialement, il n'existait pas de mécanisme d'apprentissage. Rosenblatt a introduit un apprentissage basé sur le calcul de l'erreur, mais sans descente de gradient, limitant les réseaux à une seule couche.

Premier hiver de l'IA (années 1960-1970) Le problème du XOR soulevé par Minsky et Papert a mis en évidence une limitation majeure du perceptron, menant à un désintérêt général pour l'IA et à une réduction des investissements dans le domaine. C'est à cette époque que les Multi-Layer Perceptrons (MLP) ont été introduits, permettant de dépasser la contrainte de séparabilité linéaire des données. L'introduction de la **rétropropagation** a marqué une avancée majeure :

- **Efficace** : Complexité de $O(N)$, où N est le nombre d'exemples.
- **Générique** : Basé sur la descente de gradient, encore utilisé aujourd'hui.
- **Local** : Aucune garantie de trouver une solution optimale globale.

Retour de l'IA (années 1990) Dans les années 1990, l'intelligence artificielle a connu un renouveau avec l'apparition de nouveaux modèles :

- **LeNet** de Yann LeCun, utilisé pour la reconnaissance automatique de caractères.
- **Réseaux récurrents** développés par Hochreiter et Schmidhuber, permettant la représentation de séquences.

Deuxième hiver de l'IA (années 2000) Un nouvel hiver de l'IA est survenu, causé principalement par :

- Le problème du *vanishing gradient*, empêchant l'entraînement des réseaux de plus de 3-4 couches.
- Le manque de justification théorique des performances des réseaux de neurones.
- L'apparition des SVMs (machines à vecteurs de support) qui offraient des performances comparables avec moins de complexité et d'hyper-paramètres.

Début de l'apprentissage profond (2006) À partir de 2006, des méthodes ont été développées pour entraîner efficacement des réseaux de neurones profonds :

- Deep Boltzmann Machines.
- Deep Belief Networks.
- Pré-entraînement non supervisé et apprentissage par couche.

L'entraînement des autoencodeurs par couche était alors fastidieux, nécessitant des sous-entraînements successifs.

Explosion de l'apprentissage profond (2012-2013) L'année 2012 marque un tournant majeur avec l'introduction d'AlexNet (SuperVision) lors de la compétition ImageNet. Plusieurs innovations ont contribué à ce succès :

- Fonction d'activation **ReLU**.
- **Augmentation des données** pour améliorer la robustesse des modèles.
- **Réseaux convolutifs** pour extraire des caractéristiques hiérarchiques des images.
- **Utilisation des GPUs**, rendant l'entraînement des modèles plus rapide et efficace.
- Accès à de grandes bases de données comme ImageNet.

Aujourd'hui

L'apprentissage profond est omniprésent et trouve des applications variées dans de nombreux domaines.

2 Fundamental Concepts

2.1 Définition de l'apprentissage machine

La programmation 'manuelle' est un processus/algorithme qui a comme entrée des données et des instructions (règles) et qui produit une sortie.

À l'inverse, l'apprentissage machine est un système où l'on fournit en entrée des données et la réponse attendue pour une certaine donnée. La sortie de ce système est un ensemble de règles (modèle) qui cherche à généraliser une relation entre les données et leur sortie respective.

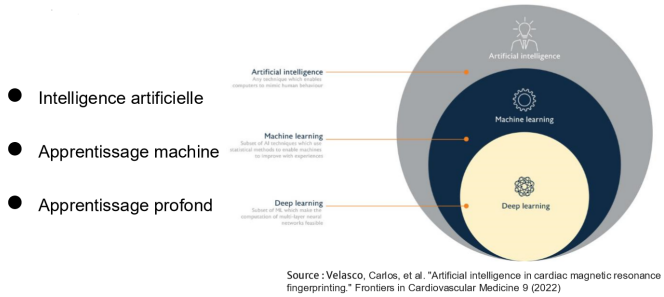


Figure 2: Terminologie de l'IA

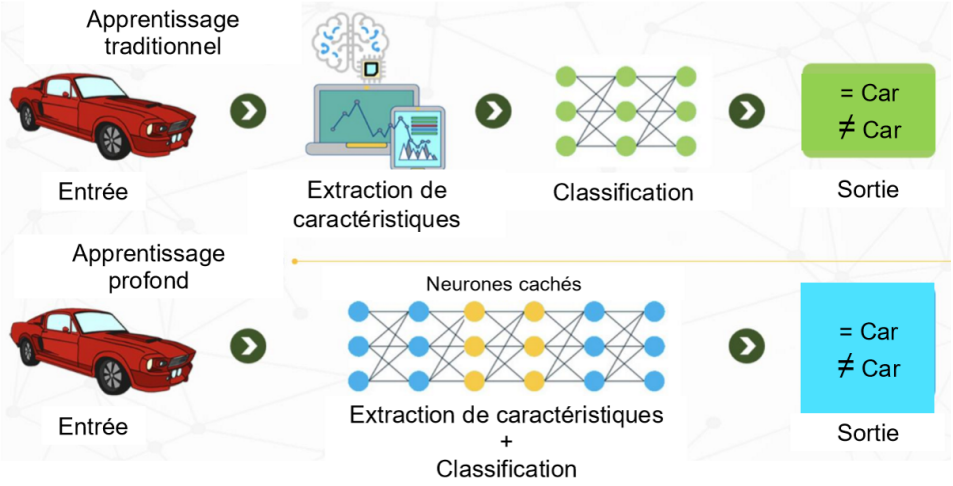


Figure 3: Différence entre l'apprentissage machine et l'apprentissage profond

2.2 Le Perceptron

Le perceptron est le bloc de construction de base dans les réseaux de neurones. Deux composants essentiels :

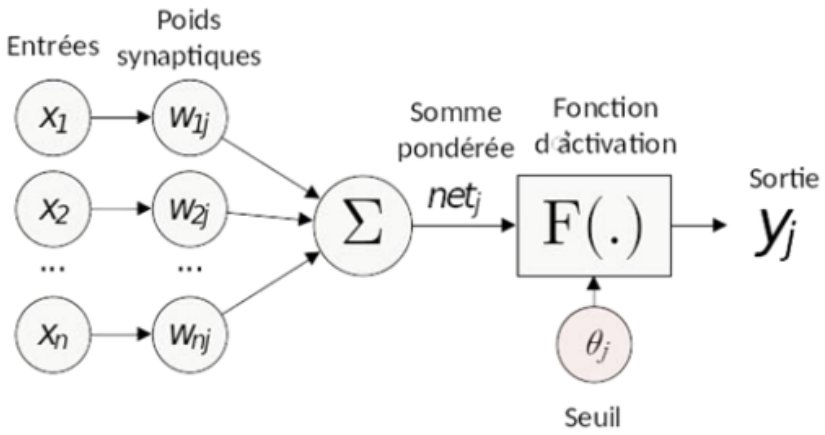
1. **La somme pondérée (net_j) :**

$$net_j = \sum_{i=0}^{n} x_i w_{ij} = \vec{w}_j \cdot \vec{x} = \mathbf{w}_j \mathbf{x}$$

ou, en termes du produit scalaire et de l'angle α,

$$net_j = |\mathbf{w}_j||\mathbf{x}| \cos(\alpha).$$

2. **La fonction d'activation (F) :** introduit la non-linéarité en activant ou désactivant le neurone selon l'agrégation pondérée des entrées.



$$net_j = \sum_{i=1}^{n} x_i w_{ij}$$

$$y_j = F(net_j, \theta_j)$$

Figure 4: Schéma d'un perceptron

Note importante : Une caractéristique importante du perceptron est qu'il est sensible à l'ordre des données d'entrée. Deux ensembles de données identiques mais présentés dans un ordre différent peuvent produire des ajustements de poids différents.

L'ajustement des poids par la correction de l'erreur a permis au perceptron d'apprendre, mais il n'était pas capable de résoudre certains problèmes comme le problème du XOR. Cette limitation a été surmontée avec l'introduction de méthodes d'optimisation plus avancées, telles que la descente de gradient. Cependant, le choix de la fonction d'activation dans ces méthodes est crucial pour assurer un apprentissage efficace.

2.3 Neural Networks Layer (NEW)

In a deep neural network, we can find multiple layers of different functions. The most basic one is a Fully Connected Layer.

For an input of N_{in} neurons and an output of N_{out} neurons:

$$\text{Total Parameters} = N_{in} \times N_{out} + N_{out}.$$

2.4 Conception d'un Modèle

La conception d'un modele d'apprentissage profond peut être décomposée en 4 étapes :

- Definition de la taches et du dataset
- **Sélection du modèle**
- **Choix de la fonction de coût**
- Optimisation du modele

2.4.1 Le Modèle

Un modèle efficace doit généraliser et représenter les données. Le choix dépend aussi des contraintes de déploiement (puissance de calcul, rapidité, etc.) et il peut être pertinent de se demander si l'utilisation de l'IA est nécessaire ou si un algorithme plus simple pourrait suffire.

2.4.2 L'Optimisation

Cette phase cherche la configuration optimale des paramètres dans l'espace de la fonction de coût. La validation permet d'éviter les problèmes de surapprentissage et de sous-apprentissage.

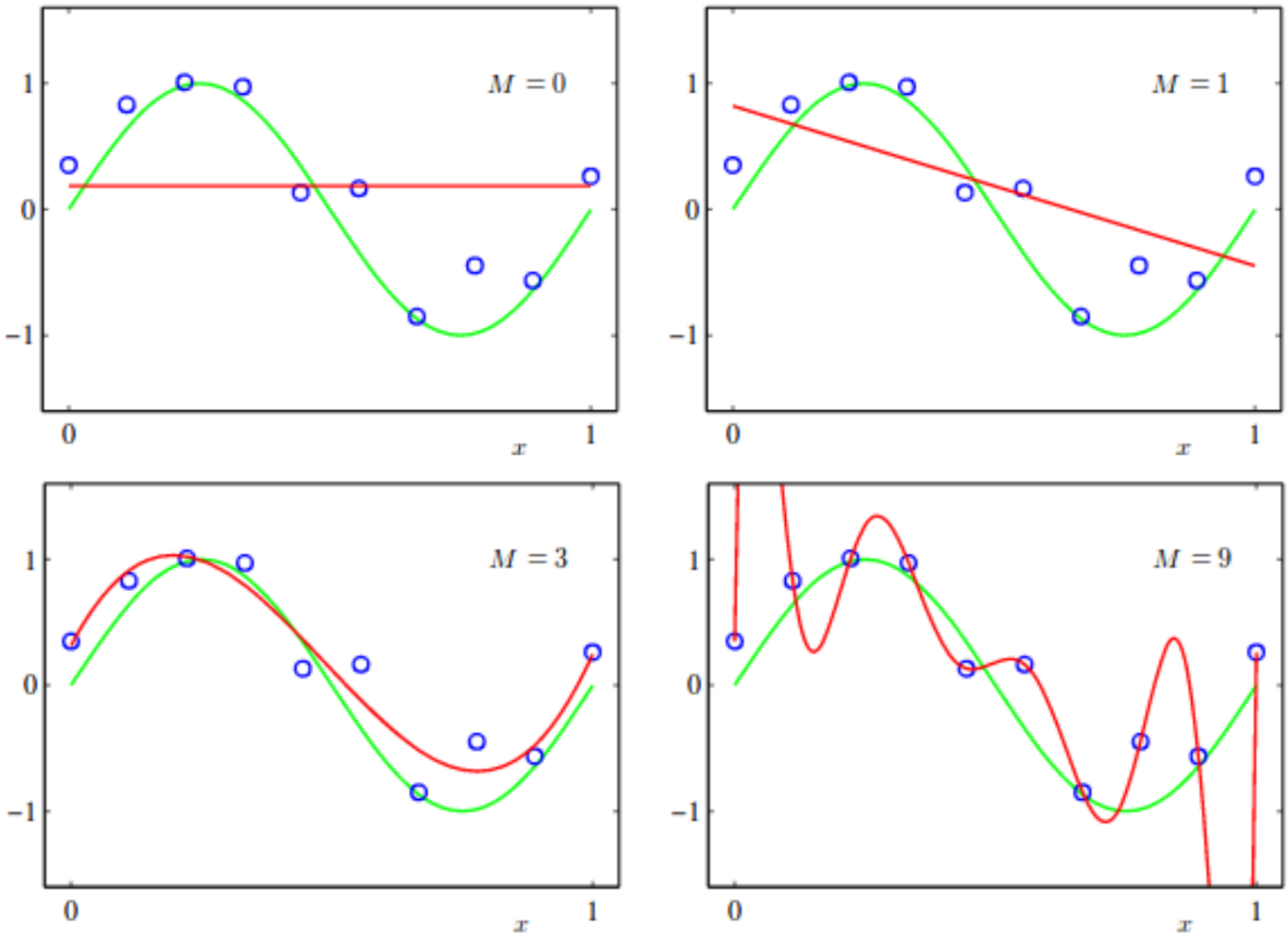


Figure 5: Exemple de surapprentissage (overfit) et de sous-apprentissage (underfit)

2.4.3 Bias-Variance Tradeoff

The overall error can be decomposed as:

$$\text{Total Error} = \text{Bias}^2 + \text{Variance} + \sigma^2,$$

with

$$\text{Bias}[\tilde{f}] = E[\tilde{f}] - f \quad \text{and} \quad \text{Variance}[\tilde{f}] = E\big[(\tilde{f} - E[\tilde{f}])^2\big].$$

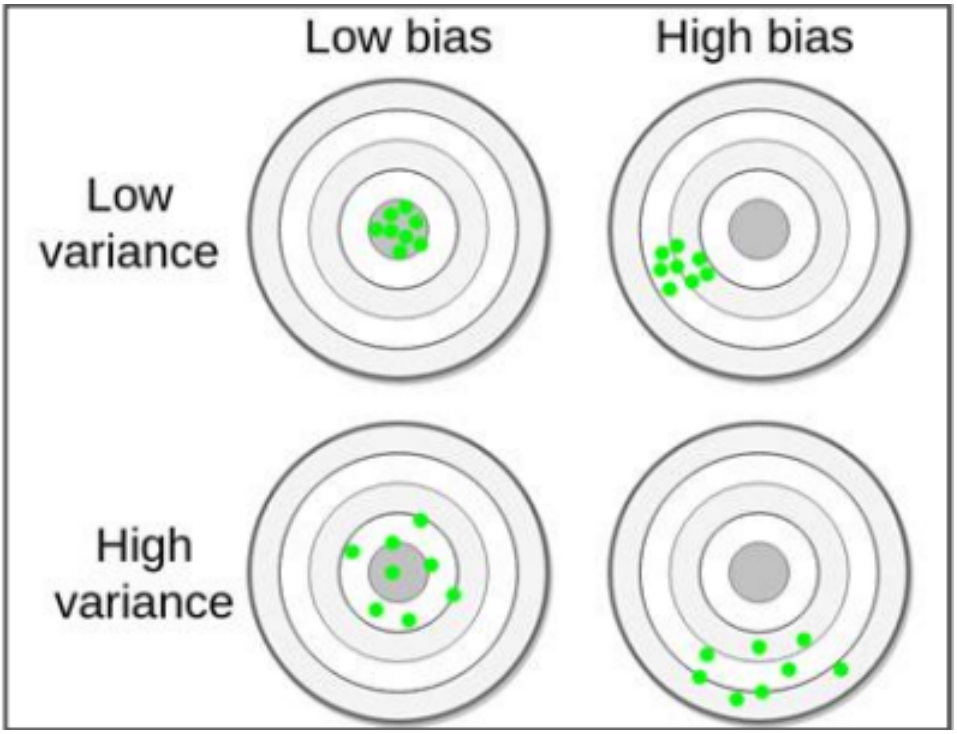


Figure 6: Bias-Variance Tradeoff

2.5 Types de Problèmes

- **Régression :** $x \to \mathbb{R}$
- **Classification :** $x \to \mathbb{N}$

2.6 Types de Modèles

Les approches de l'apprentissage machine se classent en différentes familles :

- **Supervisé :** Toutes les données sont étiquetées.
- **Non-supervisé :** Aucune étiquette n'est disponible.
- **Semi-supervisé :** Seule une partie des données est étiquetée.
- **Auto-supervisé :** Des étiquettes peu coûteuses sont générées pour des tâches auxiliaires.
- **Apprentissage par renforcement :**

2.7 Inductive Bias in Machine Learning

Inductive bias refers to the assumptions a model makes to generalize from training data to unseen data. Different architectures (e.g., CNNs, RNNs, Transformers) incorporate unique biases that affect how they learn and represent information.

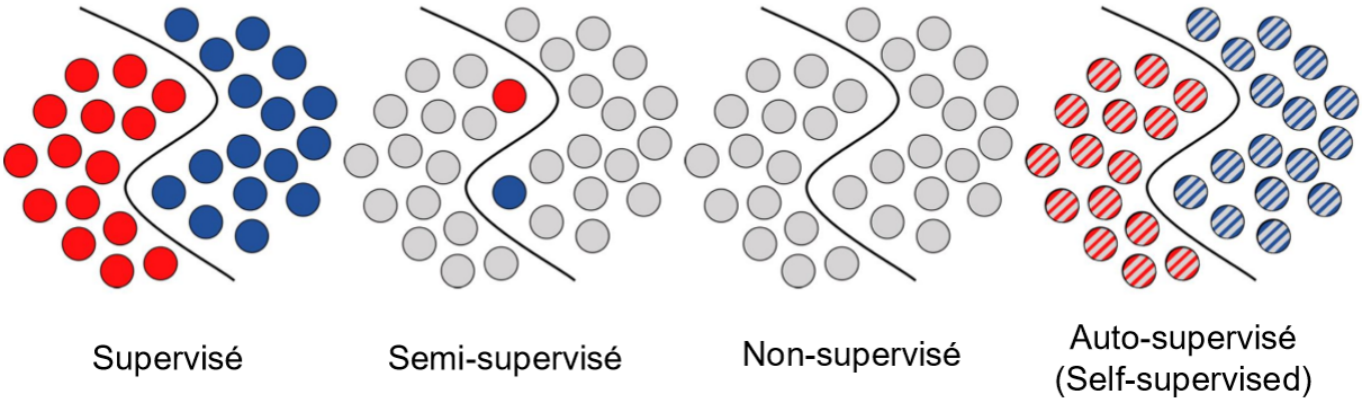


Figure 7: Différents types d'apprentissage

Given that real-world data is limited, an inductive bias enables a model to make reasonable predictions by imposing constraints on the possible functions it can learn.

Deep learning architectures have specific biases that give them particular strengths and limitations. For instance, CNNs exploit spatial locality, while Transformers capture long-range dependencies using attention mechanisms.

- **Smoothness Bias**: Similar inputs yield similar outputs.
- **Linear Separability**: Data can be separated by a linear boundary (used in logistic regression and SVMs).
- **Locality Bias**: Local structures are emphasized (as in CNNs).
- **Sequential Dependence**: Past information influences future predictions (as in RNNs).
- **Attention-Based Bias**: Certain input features are weighted more heavily (as in Transformers).
- **Manifold Hypothesis**:

Depending on the level of bias imposed on the modele, we get different results.

- **Strong Inductive Bias:** Imposes strong assumptions for faster learning but reduced flexibility (e.g., linear regression).
- **Weak Inductive Bias:** Fewer assumptions allow greater flexibility but require more data (e.g., deep neural networks).

3 Optimization Methods

Optimization methods search for the best parameter configuration by minimizing the loss function. Generally, optimization techniques will focus on improving the training accuracy of the model and tit's convergeance speed. Asopposed to regularisation techniques (that we will cover lated that are more focused on the ability of the model to generlise . Therefor soetimes slwolying the training. En d'autre mot, elle va determiné si on réussis a trouvé le minimum de notre fonction de cout (loss function).

- Evolutionary algorithms.
- Closed-form solutions.
- Gradient-based methods (first and second order).

In this section, we will mostly focus on first-order gradient-based methods.

3.1 Optimizers

3.1.1 Stochastic Gradient Descent (SGD)

SGD updates parameters using mini-batches:

Algorithm 1 Stochastic Gradient Descent

1: **Input:** Learning rate η, initial parameter θ
2: **while** Stopping criterion not met **do**
3: Sample a mini-batch of m examples $(x^{(i)}, y^{(i)})$
4: Compute gradient estimate:

$$\hat{g} = \frac{1}{m} \sum_{i=1}^{m} \nabla_\theta L(f(x^{(i)}; \theta), y^{(i)})$$

5: Update parameters:

$$\theta \leftarrow \theta - \eta \hat{g}$$

6: **end while**

3.1.2 Momentum

Le **gradient avec moment** est une méthode d'optimisation qui améliore la convergence des algorithmes de descente de gradient en ajoutant une notion d'accumulation des gradients passés. Cela aide à surmonter les oscillations et à accélérer la convergence dans des directions où le gradient varie peu. Cette méthode peut aussi être combinée avec d'autres techniques d'optimisation, comme le redémarrage à chaud et l'ajustement adaptatif du taux d'apprentissage.

La mise à jour des poids en utilisant le gradient avec moment se fait selon la règle suivante :

$$v_{t+1} = \rho v_t - \eta \nabla L(f(x; \theta_t), y)$$

$$\theta^{t+1} = \theta^t + v_{t+1}$$

where ρ is the momentum coefficient.

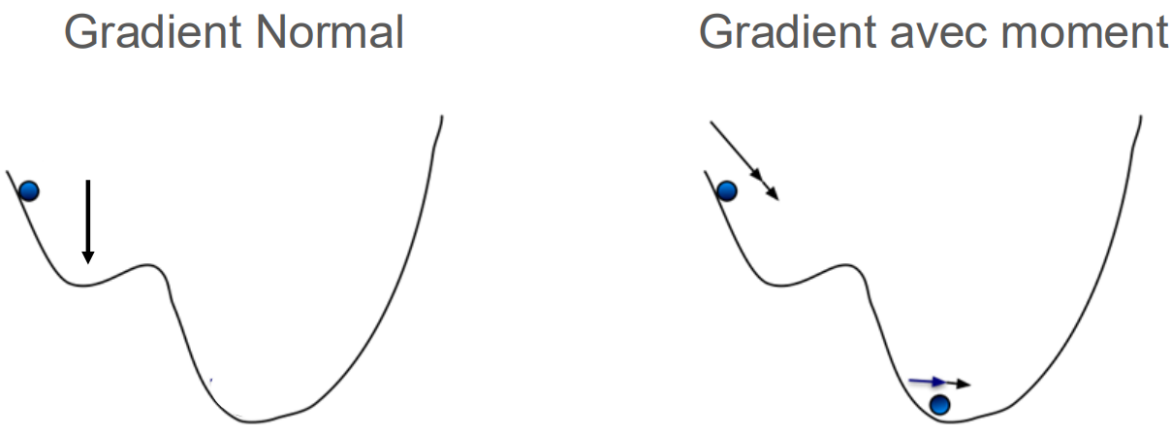


Figure 8: Gradient avec moment

3.1.3 Nesterov Accelerated Gradient (NAG)

Une variante du gradient avec moment est le **moment de Nesterov**, qui prévoit le déplacement avant de calculer le gradient. Cette approche permet d'anticiper le comportement de la fonction de perte et d'améliorer encore davantage la convergence. Elle aide à réduire les oscillations et à améliorer la vitesse d'apprentissage dans des zones planes de l'espace des solutions.

Nesterov Momentum refines the momentum method by first applying a partial update before computing the gradient, allowing for more accurate gradient adjustments.

$$\tilde{\theta} = \theta + \alpha v,$$

$$g = \nabla_\theta L(\tilde{\theta}),$$

$$v \leftarrow \alpha v - \eta g, \quad \theta \leftarrow \theta + v.$$

3.1.4 ADAGRAD (Adaptive Gradient)

Adagrad adapts the learning rate for each parameter for each parameter using accumulated squared gradients, making it suitable for convex problems but potentially slowing down over time due to accumulated gradients.

$$r \leftarrow r + g \odot g,$$

Here r accumulates past squared gradients element-wise.

$$\Delta\theta = \frac{\eta}{\sqrt{r} + \epsilon} \odot g,$$

Here ϵ is a small constant for numerical stability.

$$\theta \leftarrow \theta - \Delta\theta.$$

3.1.5 RMSPROP

TODO

3.1.6 ADAM (Adaptive Moment Estimation)

This optimiser combines RMSPROP and momentum, computing adaptive learning rates for each parameter by using both first-order moment estimates (momentum) and second-order moment estimates (variance).

We compute the first moment estimate m (moving average of gradients) and the second moment estimate v (moving average of squared gradients) as

$$m \leftarrow \beta_1 m + (1 - \beta_1)g, \quad v \leftarrow \beta_2 v + (1 - \beta_2)g \odot g,$$

Where β_* is is the decay rate.

$$\hat{m} = \frac{m}{1 - \beta_1^t}, \quad \hat{v} = \frac{v}{1 - \beta_2^t},$$

The bias correction terms $\hat{m}$ and $\hat{v}$ are then used to adjust for the initialization bias.

$$\Delta\theta = \frac{\eta}{\sqrt{\hat{v}} + \epsilon} \odot \hat{m}, \quad \theta \leftarrow \theta - \Delta\theta.$$

Adam is widely used due to its robustness and ability to handle sparse gradients.

3.1.7 ADAMW

The succesor of ADAM, who is also very popular. This method decouples weight decay from gradient updates, improving generalization.

3.1.8 AMSGRAD

This optimiser ensures non-increasing second moment estimates to address convergence issues in ADAM.

3.1.9 ADABOUND

This optimiser dynamically bounds the learning rate to transition smoothly between adaptive methods and SGD.

3.2 Normalization

(NEW) Normalization accelerates training and stabilizes gradient flow by standardizing activations.

Normalizing inputs (or activations) helps reduce the impact of differing value ranges, smoothing the loss landscape and improving gradient flow.

As we can see in figure 11, the landscape of normalized data will be much easier to traverse during training. Without it, there are some locations that could be complicated to get out of and some points are much further away than others. In the normalized case, everything is "relatively" close.

This will also make initialization less critical and leads to faster convergence from various starting points.

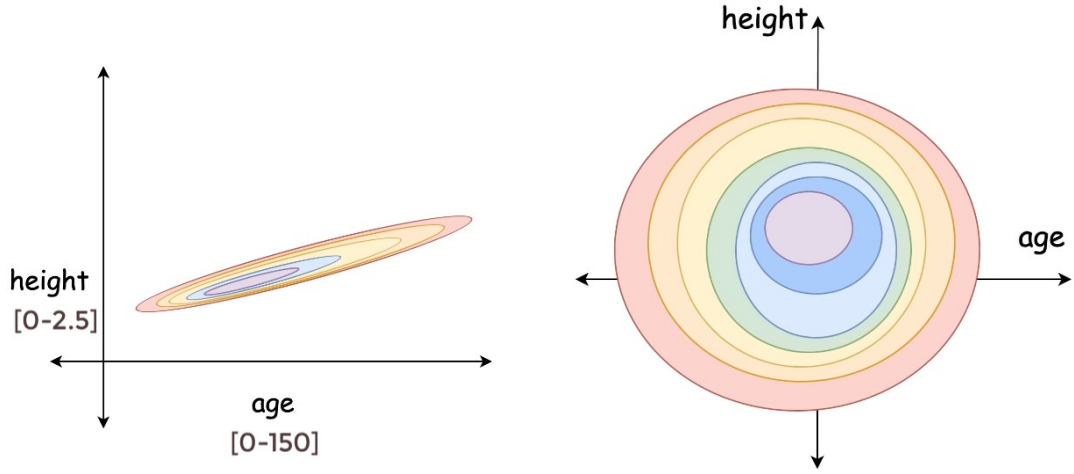


Figure 9: Gradient landscape without and with normalization

Once the parameters are normalised, the learned parameters γ and β are applied:

$$\hat{x}_i = \frac{x_i - \mu}{\sqrt{\sigma^2 + \epsilon}}, \quad y_i = \gamma \hat{x}_i + \beta.$$

In neural networks, we can apply layers of normalisation to the data. In such a layer, for D features, there are $2 \times D$ trainable parameters (γ and β). **Parameters:**

$$\text{Total Parameters} = 2 \times D,$$

accounting for γ and β per feature (aside from non-trainable running statistics).

3.2.1 Batch Normalization

Reference: Ioffe, Szegedy (ICML 2015)

Batch Normalization (BN) normalizes activations across each channel for all samples in a batch. Given batch of examples of size B, with channels C and features F, we compute:

$$\mu_c = \frac{1}{B * F} \sum_{B,F} x_{bcf}, \quad \sigma_c^2 = \frac{1}{B * F} \sum_{B,F} (x_{bcf} - \mu_c)^2.$$

- Works well in image processing. Typically inserted between linear/convolution layers and the nonlinearity.
- Works well for large mini-batch sizes.

3.2.2 Layer Normalization

Reference: Ba, Kiros, Hinton

Layer Normalization (LN) normalizes across the features (hidden units) within a single layer for each sample. Given batch of examples of size B, with channels C and features F, we compute:

$$\mu_b = \frac{1}{C * F} \sum_{C,F} x_{bcf}, \quad \sigma_b^2 = \frac{1}{C * F} \sum_{C,F} (x_{bcf} - \mu_b)^2.$$

- Often used in RNNs and Transformers (especially when mini-batches are small or variable in size).
- Normalization is independent for each training example.

3.2.3 Instance Normalization

Instance Normalization (IN) normalizes across spatial dimensions for each channel and for each sample independently. It is frequently used in style transfer tasks. Given batch of examples of size B, with channels C and features F, we compute:

$$\mu_{bc} = \frac{1}{F}\sum_{F} x_{bcf}, \quad \sigma_{bc}^2 = \frac{1}{F}\sum_{F}(x_{bcf} - \mu_{bc})^2.$$

- Each sample-channel is normalized independently.
- Helps in style transfer by normalizing per-image and per-channel statistics.

3.2.4 RMS Normalization

RMS Normalization (RMSNorm) is similar to LayerNorm but normalizes based on the root mean square of the features rather than the variance:

$$\mu_{\mathrm{b}}^2 = \frac{1}{C * F}\sum_{C,F} x_{bcf}^2 \quad \text{and} \quad \hat{x}_{bcf} = \frac{x_{bcf}}{\sqrt{\mu_b^2 + \epsilon}}$$

- Does not subtract a mean; only divides by the root mean square.
- Useful in large-scale models due to reduced computational cost compared to LayerNorm.

3.2.5 Group Normalization

Reference: Wu and He (2018)

Group Normalization (GN) normalizes by splitting channels into G groups, then normalizing each group's mean and variance.

- Bridges the gap between LayerNorm and InstanceNorm.
- Works well when mini-batch sizes are small.

3.2.6 Weight Normalization

Weight Normalization (WN) normalizes the weights (rather than activations). It reparameterizes a weight vector $\mathbf{w}$ as

$$\mathbf{w} = \frac{g}{\|\mathbf{v}\|}\mathbf{v},$$

where $\mathbf{v}$ is a learned direction and g is a learned scalar. Then the output is:

$$y = \phi(\mathbf{w} \cdot \mathbf{x} + b).$$

- Decouples the magnitude of the weights from their direction.
- Simplifies optimization by controlling the length of the weight vector explicitly.

4 Regularization

A central problem in machine learning is how to make an algorithm that will perform well not just on the training data, but also on new inputs. Many strategies used in machine learning are explicitly designed to reduce the test error, possibly at the expense of increased training error. These strategies are known collectivelyas regularization. A great many forms of regularization are available to the deep learning practitioner.

4.1 L2 Regularization

TODO..

4.2 L1 Regularization

TODO..

4.3 Early Stopping

This algorithm allows us to stop the training when we see the performance on the validation set reducing.

4.3.1 Variants

There exist techniques that are not very used recently but are still worth mentioning such as

- **Early Stopping with Retraining:** This technique is made to not waste certain example in the validation set and uses validated examples for a minor performance boost. It's not very popular because it's not the goal to grab this little performance gain and the data base are very big now.
- **Early Stopping with Surrogate Loss:** Employs an alternate loss function.

4.4 Dropout Training

Randomly disables certain connections during training to reduce overfitting.

4.5 Stochastic Depth

Randomly drop entire layers during training, particularly effective in ResNets.

4.6 Transfer Learning

Train a general model and fine-tune it for specific tasks.

4.7 Multi-Task Learning

Build a general model and plug in smaller task-specific modules.

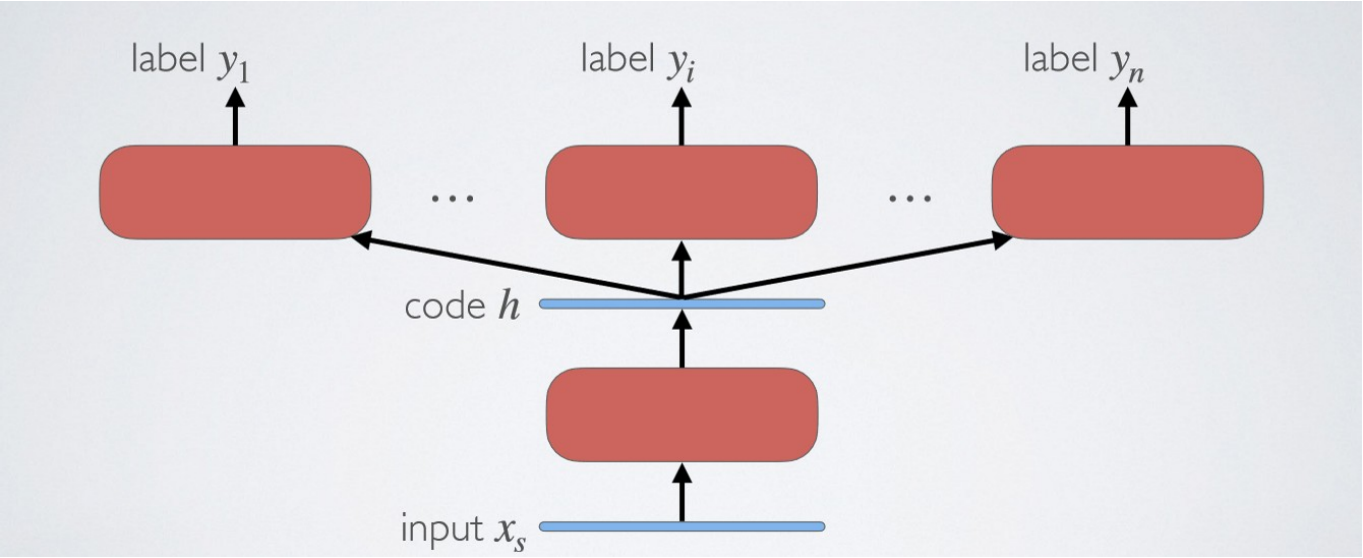


Figure 10: Multi-Task Learning architecture

4.8 Label Smoothing

Replace hard labels (0 and 1) with smoothed values to improve generalization. This results in a cluster of labels together.

4.9 Data Augmentation

Increase dataset size by applying transformations (translation, rotation, etc.). This technique had a HUGE impact on the image, audio, video and other tasks with those type of data but NONE to others like text because there is no way found so far to augment text.

4.9.1 Data Augmentation for Images

RandAugment This technique is among the most popular. In general, it applies transformations such as: translation, rotation, scaling, equalization, posterization and solarization, etc...

However, more aggressive augmentation techniques exist..

MixUp Blends two images and their labels. This technique generally yields low performance.

Random Erasing / Cutout Removes a random portion of the image.

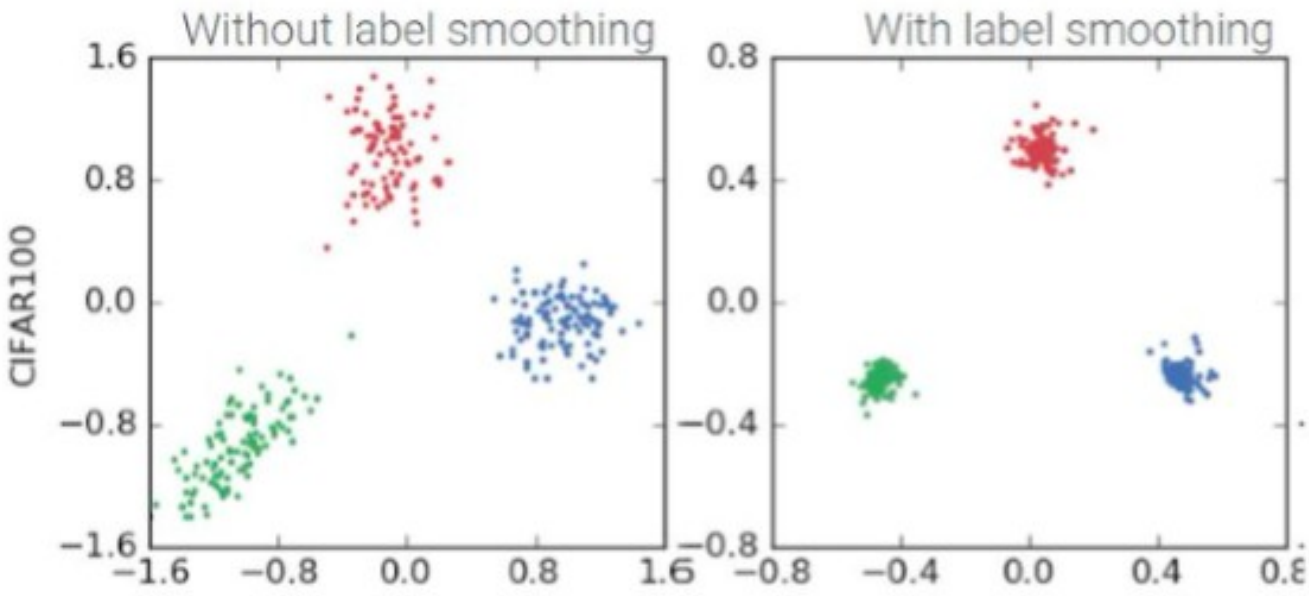


Figure 11: Source: When does label smoothing help?

CutMix Combines regions from different images with proportional labels.

5 Convolutional Neural Networks (CNNs)

A Convolutional Neural Network (CNN) is a type of neural network designed specifically for pattern recognition in image or audio data (mimicking the way the visual cortex processes visual information). CNNs leverage convolution operations to extract hierarchical features from images, efficiently handling high-dimensional data while exploiting spatial structures, such as the relationship between nearby pixels. This architecture builds invariance to transformations like translation, making it highly effective for image-related tasks. A typical CNN consists of alternating convolutional and pooling layers, followed by fully connected layers.

5.1 Key Features

Local Connectivity Each neuron connects only to a local region of the input, called the *receptive field*. This allows CNNs to focus on localized patterns (e.g., edges, textures), reducing the number of parameters compared to fully connected networks (FFNs).

Parameter Sharing The same set of weights (kernel) is used across different spatial locations of the input, significantly reducing computational complexity.

Pooling (Dimensionality Reduction) Reduces the number of hidden units in the model and helps achieve translational invariance, improving generalization.

For an image x, a kernel k, and output y:

$$(y * k)_{i,j} = \sum_{p,q} x_{i+p,j+q} \cdot k_{r-p,r-q}.$$

5.2 Convolution Layer

For an input of size $N \times N \times D_{in}$, kernel of size $K \times K$, stride S, padding P, and D_{out} filters:

$$N_{out} = \left\lfloor \frac{N - K + 2P}{S} \right\rfloor + 1,$$

with total parameters:

$$\text{Total Parameters} = D_{out} \times (K \times K \times D_{in} + (\text{bias term})) .$$

5.3 Pooling Layer

For input of size $N \times N \times D$, pooling window $F \times F$, and stride S:

$$N_{out} = \left\lfloor \frac{N - F}{S} \right\rfloor + 1,$$

with zero trainable parameters. There are two main types of pooling

- **Max Pooling:**

$$y_{i,j} = \max_{(p,q) \in R} x_{i+p,j+q}.$$

- **Average Pooling:**

$$y_{i,j} = \frac{1}{|R|} \sum_{(p,q)\in R} x_{i+p,j+q}.$$

5.4 Stride and Padding

Stride Controls the step size of the convolutional filter when moving across the input.

Padding Adds extra pixels around the input to control output size. Special cases include:

- **Valid Convolution** ($P = 0$): no padding, spatial dimensions shrink.
- **Same Convolution** ($P = \lfloor d\,(K-1)/2 \rfloor$): half (or "same") padding, preserves spatial dimensions.
- **Full Convolution** ($P = K - 1$): full padding, maximizes overlap so the output is larger than the input.

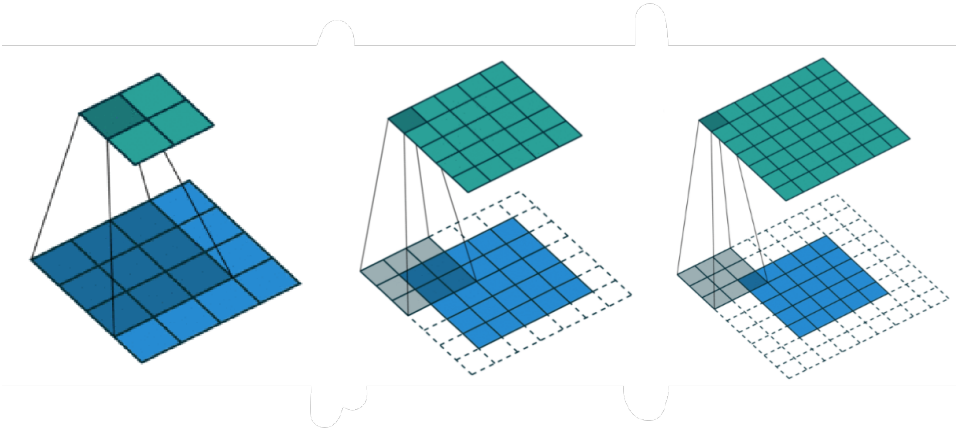


Figure 12: Special padding cases for a 2D convolution: to the left valid convolution (no padding), in the middle same convolution (input and output dimensions are the equal), and to the right full convolution (maximum padding).

5.5 Receptive Field

The *receptive field* of a neuron in a convolutional neural network (CNN) is the region of the input that influences that neuron's output. As you progress deeper into the network, the receptive field generally increases, allowing the network to capture larger context. Understanding the receptive field is essential because it helps you design architectures that capture enough context from the input.

In simpler terms, it is the size of the patch in the input image that contributes to a particular feature in a deeper layer. For layer i with kernel k_i and stride s_i, the receptive field can be computed recursively

$$r_i = r_{i-1} + (k_i - 1) \times S_{i-1},$$

where $S_{i-1} = \prod_{j=1}^{i-1} s_j$ is the effective stride. Also, for the first layer applied directly on the input, the receptive field is simply

$$r_1 = k_1.$$

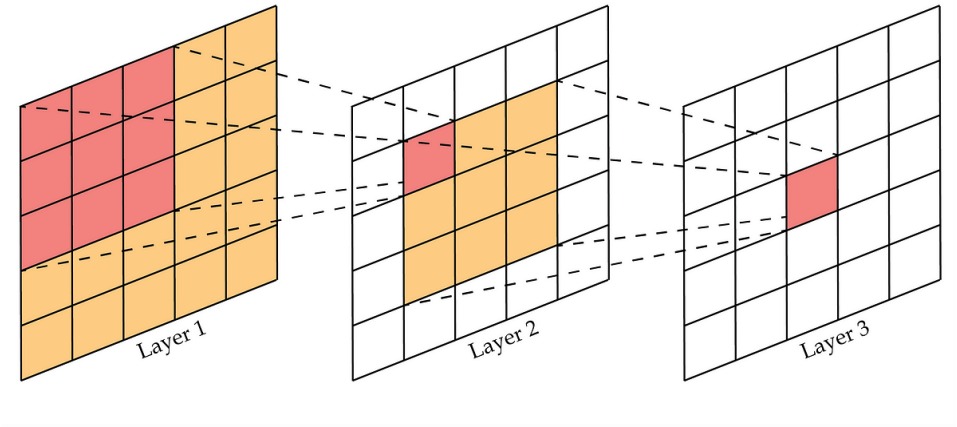


Figure 13: Receptive field expansion in a deep CNN using consecutive 3×3 kernels. Source: Reza Kalantar, "Receptive Fields in Deep Convolutional Networks," Medium (2019).

5.6 Backpropagation (TODO)

5.7 Dilated Convolutions

Dilated convolutions "inflate" the kernel by inserting spaces between its elements, thereby expanding the network's receptive field without extra parameters or computation.

They are especially useful when nearby inputs are highly redundant, and have been employed in sequence–modeling architectures such as WaveNet (see 5.10.3) and ByteNet (see 5.10.4).

Dilation factor A hyperparameter $d \geq 1$ that inserts $d - 1$ "holes" between successive kernel elements.

Effective kernel size Denote the original kernel size by k. A dilated kernel of size k and rate d has an effective size

$$\hat{k} = k + (k - 1)(d - 1) = d(k - 1) + 1.$$

Larger $\hat{k}$ directly increases the receptive field.

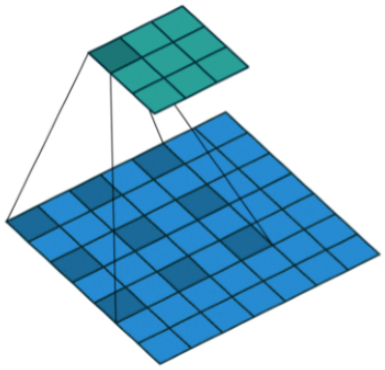


Figure 14: Dilated convolution with $i = 7$, kernel $k = 3$, dilation rate $d = 2$, stride $s = 1$, and padding $p = 0$, yielding an effective kernel size $\hat{k} = 5$.

5.8 Transpose Convolutions

Transpose convolutions (also called "deconvolutions") is the natural "inverse" of the convolution operation. It perform learnable upsampling by reversing the forward pass of a convolution. They are widely used in decoder architectures for image generation or semantic segmentation.

5.9 Separable Convolutions

Separable convolutions factorize a standard convolution into two smaller operations—depthwise and pointwise—to reduce parameters and computation.

Depthwise convolution Applies a $k \times k$ filter to each input channel separately:

$$\text{FLOPs}_{\text{dw}} = H_{\text{in}} W_{\text{in}} \times k^2 C_{\text{in}}.$$

Pointwise convolution Follows with a 1×1 convolution to linearly combine the C_{in} channels into C_{out}:

$$\text{FLOPs}_{\text{pw}} = H_{\text{in}} W_{\text{in}} \times C_{\text{in}} C_{\text{out}}.$$

Parameter reduction Compared to a standard $k \times k$ convolution ($k^2 C_{\text{in}} C_{\text{out}}$ parameters), depthwise separable uses

$$k^2 C_{\text{in}} + C_{\text{in}} C_{\text{out}},$$

often yielding a 5–10× reduction in computation for typical network widths.

5.10 Architectural Taxonomy

5.10.1 ResNet (Residual Networks)

Deep networks suffer from the *vanishing gradient problem*, where gradients become too small to update weights effectively. ResNet introduces skip connections, allowing layers to learn residual mappings:

$$F(x) = H(x) - x,$$

where $H(x)$ is the desired underlying mapping. This enables stable training of very deep models (e.g. ResNet-50, ResNet-152).

There are two types of residual blocks:

Basic Block Used in shallower ResNets (e.g. ResNet-18/34). It consists of two 3×3 convolutions, each followed by BatchNorm and ReLU, plus an identity shortcut.

Bottleneck Block Used in deeper variants (e.g. ResNet-50/101/152). It stacks a 1×1 projection (reduce channels), a 3×3 convolution, and a 1×1 expansion, all with BatchNorm and ReLU, plus a shortcut.

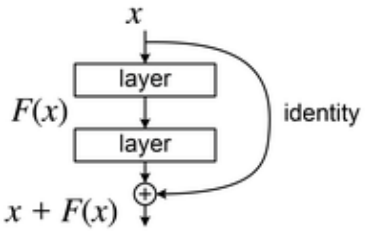


Figure 15: Structure of a ResNet residual block. The input x is added to the output of the residual function $F(x)$ via a skip connection.

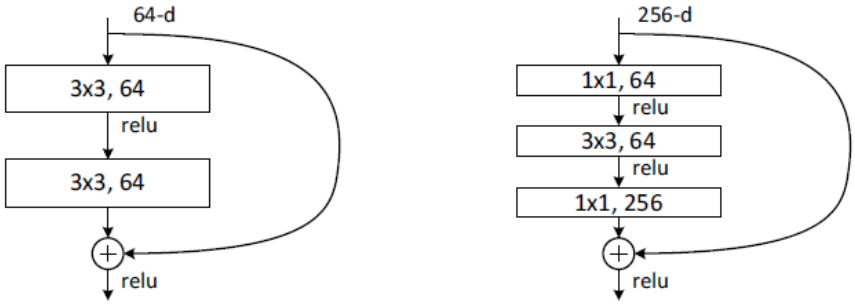


Figure 16: Comparison of ResNet block types. **Left**: basic block (two 3×3 convolutions). **Right**: bottleneck block (1×1, 3×3, 1×1 convolutions);

5.10.2 SENet (Squeeze-and-Excitation Networks)

SENet introduces a lightweight channel-wise attention mechanism:

1. **Squeeze:** Global average pooling produces a per-channel descriptor.
2. **Excitation:** Two small FC layers (with bottleneck) learn channel weights via a sigmoid.
3. **Recalibration:** These weights rescale the original feature maps.

This adaptively emphasises informative channels with minimal cost.

5.10.3 WaveNet

A generative model for raw audio signals. WaveNet stacks causal, dilated 1D convolutions with exponentially increasing dilation rates, interleaved with gated activations and residual/skip connections. It models long-range dependencies without recurrence and outputs sample-by-sample distributions via a softmax over quantised amplitudes.

5.10.4 ByteNet

A fully convolutional sequence-to-sequence model for text. ByteNet uses two stacks of dilated convolutions: an encoder to build representations of the source sequence, and an autoregressive decoder that conditions on previous outputs. Residual connections and causal masking ensure efficient, parallelisable training and linear inference time.

5.10.5 EfficientNet

EfficientNet achieves state-of-the-art accuracy through *compound scaling*: jointly scaling network depth, width, and input resolution by a set of coefficients found via small-grid search. It builds on MobileNetV2's MBConv blocks (inverted residual + squeeze-and-excitation) to maximise performance per FLOP.

5.10.6 ConvNeXt

ConvNeXt modernises the classic CNN by incorporating design elements inspired by Vision Transformers:

- Replace 3×3 convs with large-kernel depth-wise convs (e.g. 7×7).
- Use LayerNorm in "channels-last" format.
- Swap ReLU+BN for GELU+LayerNorm.
- Maintain a simple macro-architecture (ResNet-style stages and downsampling).

These changes yield competitive accuracy on ImageNet with similar compute to large transformer models.

6 Recurrent Neural Networks (RNN)

Recurrent Neural Networks (RNNs) are a class of neural networks specifically designed to handle sequential data, making them ideal for tasks such as language modeling, speech recognition, and machine translation. Their strength lies in their ability to maintain a persistent hidden state across time steps, allowing them to capture temporal dependencies within the data. Despite their early success, RNNs have been largely overshadowed by transformers (see **??**), which have proven to be more efficient and effective in handling long-range dependencies and parallel computation.

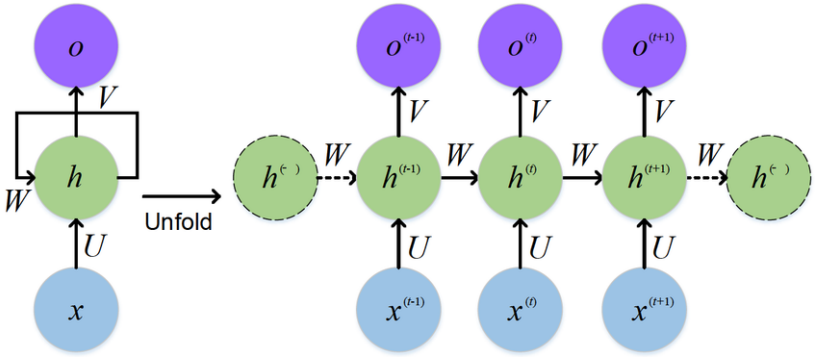


Figure 17: Unrolled recurrent neural network over multiple time steps, depicting hidden state propagation and input–output dependencies in an audio-visual speech recognition model. Source: Feng et al. (2017).

RNNs process sequential data by maintaining a hidden state that evolves over time. At each time step t, the hidden state h_t is updated based on the previous hidden state h_{t-1} and the current input x_t:

$$h_t = f(h_{t-1}, x_t)$$

The output y_t is then computed from the hidden state:

$$y_t = \text{softmax}(Wh_t + b)$$

where W is the weight matrix and b is the bias term.

This recurrence allows the model to retain information from earlier time steps, making it suitable for tasks where context plays a crucial role in understanding the sequence.

6.1 Backpropagation Through Time (BPTT)

Backpropagation Through Time (BPTT) is the method used to train RNNs by applying backpropagation to the unrolled network over time. The network is "unrolled" for the number of time steps corresponding to the input sequence. The gradients are computed and propagated backward through the unrolled graph.

6.1.1 Truncated BPTT

Truncated BPTT is a practical modification where, instead of unrolling the network for the entire sequence, the network is unrolled for a fixed number of time steps, k. This approximation helps reduce computational complexity and memory usage, especially when dealing with long sequences. Truncated BPTT also helps mitigate issues like vanishing gradients by limiting the range over which gradients are propagated.

6.2 Exploding Gradient Problem

The exploding gradient problem occurs when the gradients grow exponentially during backpropagation. This is typically caused by repeated multiplication of gradients by values greater than 1. In such cases, the gradients can become very large, causing the model's weights to update in large, unstable steps. This can lead to numerical overflow or an inability to converge during training.

Mathematically, the exploding gradient problem is the opposite of the vanishing gradient problem. If the term $\frac{\partial h_{t+1}}{\partial h_t}$ is greater than 1, the gradients will increase exponentially as they are propagated back through the network:

$$\frac{\partial L}{\partial h_t} = \frac{\partial L}{\partial h_{t+1}} \cdot \frac{\partial h_{t+1}}{\partial h_t}$$

This problem leads to instability in the training process, where the weights may fluctuate wildly, preventing the model from finding an optimal solution.

6.2.1 Gradient Clipping

Gradient clipping is a solution to the exploding gradient problem. It involves rescaling the gradients if their norm exceeds a specified threshold, preventing them from growing too large and causing instability in the learning process:

$$\mathbf{g} = \frac{\mathbf{g}}{\|\mathbf{g}\|} \cdot \text{threshold}, \quad \text{if} \quad \|\mathbf{g}\| > \text{threshold}$$

This technique ensures that the gradients do not cause the model's weights to make large, unstable updates.

6.3 Vanishing Gradient Problem

The vanishing gradient problem occurs when the gradients shrink exponentially as they are propagated back through time. This happens because the gradients are repeatedly multiplied by values smaller than 1 (e.g., activation functions like tanh or sigmoid squash their inputs to a small range). Over many time steps, this multiplication causes the gradients to decay towards zero, making it difficult for the model to learn long-range dependencies.

Mathematically, for an RNN, the gradients are propagated back through the network as:

$$\frac{\partial L}{\partial h_t} = \frac{\partial L}{\partial h_{t+1}} \cdot \frac{\partial h_{t+1}}{\partial h_t}$$

If the term $\frac{\partial h_{t+1}}{\partial h_t}$ is less than 1 for each time step, the gradient will diminish as it is propagated back through many layers, leading to a vanishing gradient.

This problem makes it very difficult for RNNs to capture long-term dependencies. As a result, models struggle to remember important information from earlier time steps in a sequence.

6.3.1 Orthogonal Initialization

Orthogonal initialization helps mitigate the vanishing gradient problem. By initializing the weight matrices U as orthogonal matrices, we ensure that the gradients do not shrink exponentially as they propagate through the network. This helps maintain stable learning over many time steps:

$$U^\top U = UU^\top = I$$

where I is the identity matrix. This ensures that the weight matrix does not distort the signal as it is passed through the network, helping to preserve the magnitude of gradients during training.

6.4 Architectural Taxonomy

RNNs have several important variants that enhance their ability to model complex temporal dependencies. Below, we discuss some of the most popular architectures used in practice.

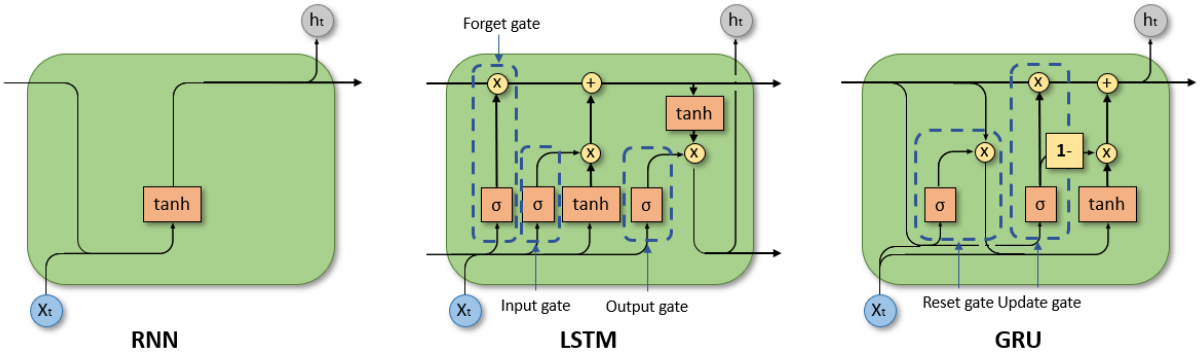


Figure 18: Comparison of standard RNN, LSTM, and GRU architectures, illustrating their recurrent connections and gating mechanisms. Source: Suvankar Maity, LinkedIn.

6.4.1 LSTM (Long Short-Term Memory)

LSTM is a type of RNN designed to avoid the vanishing gradient problem by introducing memory cells. These cells maintain information over time and use gates to control the flow of data. The key components of an LSTM are the input gate, forget gate, and output gate:

$$i_t = \sigma(W_i x_t + U_i h_{t-1}), \quad f_t = \sigma(W_f x_t + U_f h_{t-1}), \quad o_t = \sigma(W_o x_t + U_o h_{t-1})$$

$$c_t = f_t \cdot c_{t-1} + i_t \cdot \tilde{c}_t, \quad h_t = o_t \cdot \tanh(c_t)$$

where c_t is the cell state, i_t is the input gate, f_t is the forget gate, and o_t is the output gate.

For more in-depth learning about LSTMs, the following resources are highly recommended:

- Colah's Blog - Understanding LSTMs
- Josh Starmer's YouTube Video Explaining LSTMs

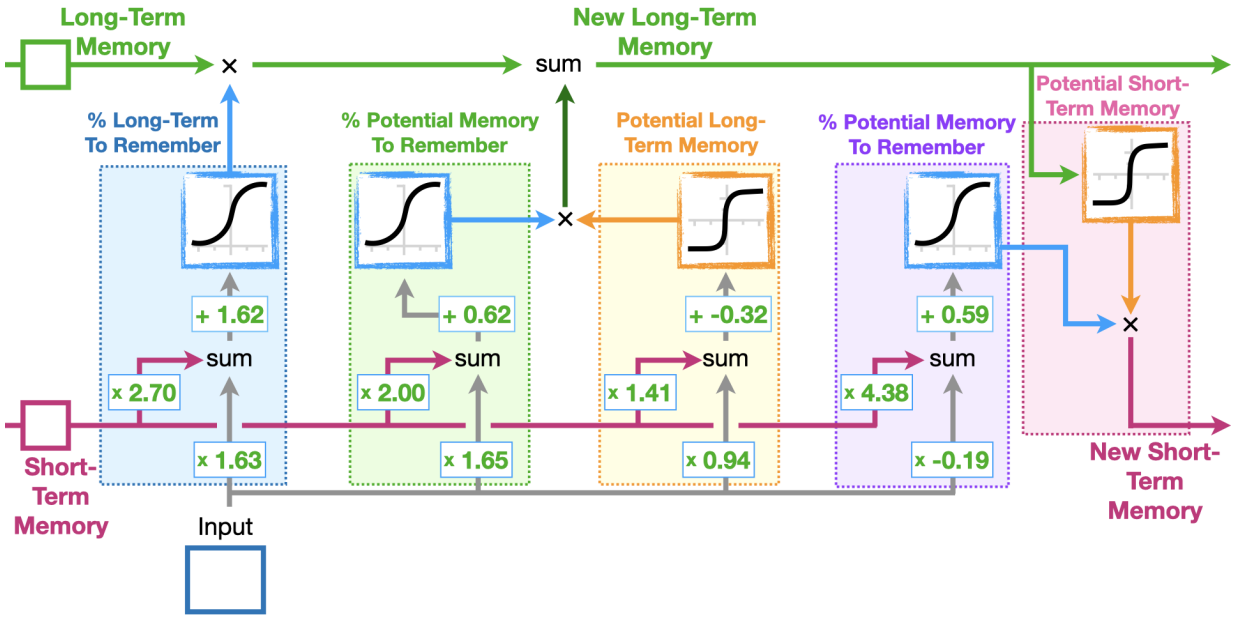


Figure 19: Structure of a Long Short-Term Memory (LSTM) unit showing the input, forget, and output gates along with the cell state update. Source: Josh Starmer, Lightning AI.

6.4.2 GRU (Gated Recurrent Unit)

GRU is a variant of LSTM with fewer gates. It uses two gates: the update gate z_t and the reset gate r_t. These gates control the amount of information to be retained or discarded at each time step:

$$z_t = \sigma(W_z x_t + U_z h_{t-1}), \quad r_t = \sigma(W_r x_t + U_r h_{t-1})$$

$$h_t = (1 - z_t) \cdot h_{t-1} + z_t \cdot \tilde{h}_t$$

GRUs are simpler than LSTMs but perform similarly in many tasks.

6.4.3 Seq2Seq (Sequence to Sequence)

Seq2Seq models use RNNs to map one sequence to another, making them ideal for tasks like machine translation. The architecture consists of an encoder RNN, which processes the input sequence, and a decoder RNN, which generates the output sequence:

$$h_t = f(h_{t-1}, x_t), \quad y_t = \text{Decoder}(h_t)$$

Seq2Seq models can use LSTMs or GRUs for both the encoder and decoder.

6.4.4 Bidirectional RNN

Bidirectional RNNs (BiRNNs) process each sequence in two directions—forward and backward—allowing the model to incorporate both past and future context when making predictions. At each time step t, the forward and backward hidden states are computed as

$$h_t^{(f)} = \tanh\big(W^{(f)} x_t + U^{(f)} h_{t-1}^{(f)}\big), \qquad h_t^{(b)} = \tanh\big(W^{(b)} x_t + U^{(b)} h_{t+1}^{(b)}\big),$$

and the output is formed by combining these two states:

$$y_t = V^{(f)} h_t^{(f)} \; + \; V^{(b)} h_t^{(b)}.$$

Here, $W^{(f)}, W^{(b)}, U^{(f)}, U^{(b)}, V^{(f)}, V^{(b)} \in \mathbb{R}^{d \times d}$, and $x_t, h_t^{(f)}, h_t^{(b)} \in \mathbb{R}^d$ for all $t \in [1, T]$.

The forward RNN (f) captures information from the past, while the backward RNN (b) captures information from the future; their concatenation (or sum) enriches the representation used for each prediction.

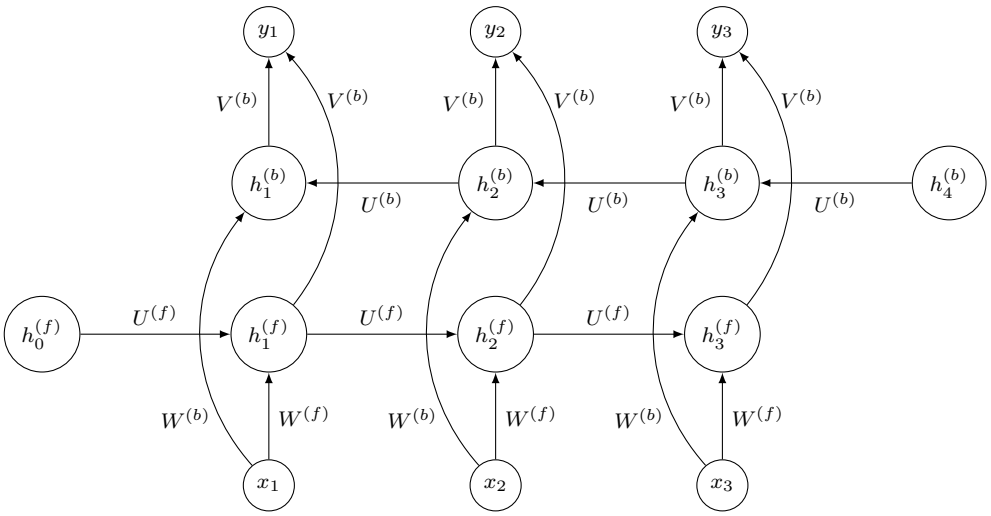


Figure 20: Unrolled bidirectional RNN over three time steps, showing the forward and backward hidden states ($h^{(f)}$ and $h^{(b)}$), their recurrent connections ($U^{(\cdot)}$), input connections ($W^{(\cdot)}$), and output projections ($V^{(\cdot)}$).

6.4.5 State Space Models (SSM)

State Space Models (SSMs) generalize RNNs by representing the system dynamics as continuous state transitions. SSMs efficiently model long-term dependencies by leveraging structured latent states.

HIPPO (Harmonic Infinite Order Polynomial ODE) The HIPPO framework formulates SSMs using polynomial Ordinary Differential Equations (ODEs). It introduces a harmonic structure that improves the modeling of long-range dependencies by providing a smooth transition between states across time.

6.4.6 S4 (Structured State Space Model)

The S4 model builds on SSMs by leveraging structured state spaces for efficient sequence modeling. By employing matrix decompositions, S4 can handle long sequences more effectively than traditional RNNs, providing better scalability and computational efficiency.

6.4.7 MAMBA (Masked Multi-Head Attention RNN)

MAMBA is a hybrid architecture that combines the benefits of attention mechanisms with RNNs. It uses masked attention heads to focus on relevant parts of the input sequence while processing it, improving performance on tasks like sequence labeling and machine translation.

7 Transformers

Transformers are effective for language modeling due to their ability to process data in parallel and capture long-range dependencies through attention.

Here are some key points when fine-tuning this type of model:

- The dimension of attention keys is critical.
- Larger models generally achieve better performance.
- Dropout is beneficial.
- Positional encodings (sinusoidal or learned) produce similar results.

7.1 Attention Mechanism

TODO: This concept needs a dedicated sub section to clearly explain it

7.2 Temperature in Transformers

Temperature is a scaling factor used to control the probability distribution in the self-attention mechanism and during text generation. It modulates the model's "confidence" in its predictions by adjusting the sharpness of the distributions. Temperature scales the softmax in both self-attention and text generation:

Self-Attention

$$e_{ij} = \frac{Q_i \cdot K_j^T}{T\sqrt{d_k}},$$

where T adjusts the sharpness.

Text Generation

$$P(y) = \text{softmax}\left(\frac{z}{T}\right).$$

In other words, the parameter T influences the attention distribution:

- **Low Temperature ($T < 1$)**: Results in a sharper distribution, concentrating more strongly on a few tokens with high scores.
- **High Temperature ($T > 1$)**: Produces a more diffuse distribution, allowing broader but less pronounced attention.

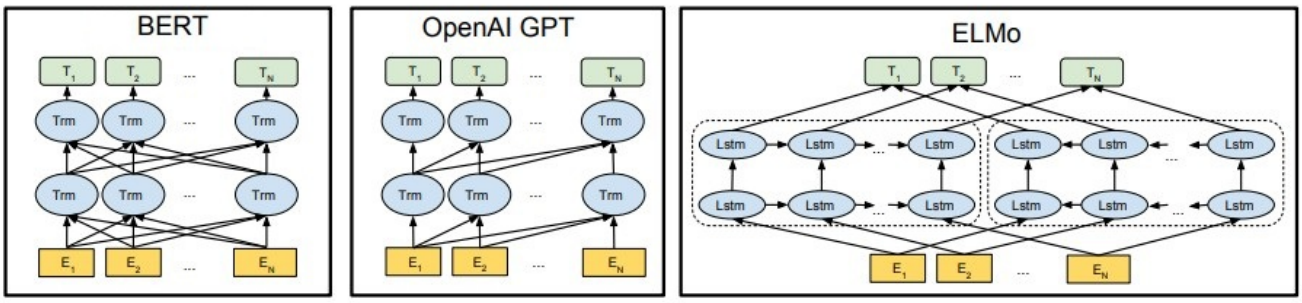


Figure 21: Architecture des Transformers. Source : *BERT: Pre-training of Deep Bidirectional Transformers for Language Understanding* (Devlin et al.)

7.3 Architectural Taxonomy

7.3.1 GPT (Generative Pre-trained Transformer)

GPT est un modèle basé sur un décodeur Transformer unidirectionnel. Il est entraîné de manière autoregressive pour prédire le token suivant dans une séquence. Son principal atout est la capacité à générer du texte de façon cohérente et fluide.

Les versions plus récentes (GPT-2, GPT-3) utilisent des contextes de taille fixe mais augmentent en taille de modèle pour améliorer la qualité de génération.

GPT3 Emerging abilities - Zero-shot & Few-shot Prompting - Scale-up at some point make the performance explode

7.3.2 BERT (Bidirectional Encoder Representations from Transformers)

BERT (Bidirectional Encoder Representations from Transformers) is a Transformer-based model introduced by Google AI [**?**]. It revolutionized NLP by introducing a powerful bidirectional language representation model. It is designed to pre-train deep bidirectional representations by using self-supervised learning, allowing it to achieve state-of-the-art performance on a variety of natural language processing (NLP) tasks.

Ce modele est pré-entraîné à l'aide de la tâche de *Masked Language Modeling* (MLM) ainsi que de la prédiction de la prochaine phrase (NSP).

Masked Language Modeling (MLM) BERT is pre-trained using a self-supervised learning approach called *Masked Language Modeling* (MLM). In this method:

- 15% of the input tokens are randomly selected for masking.
- Of these, 80% are replaced with a special `[MASK]` token, 10% are replaced with a random token, and 10% remain unchanged.
- The model is trained to predict the original tokens based on their surrounding context.

This forces BERT to develop a deep bidirectional understanding of text.

Next Sentence Prediction (NSP) To enhance sentence-level understanding, BERT also uses the *Next Sentence Prediction* (NSP) task:

- The model is given two sentences: A and B.
- It predicts whether B is the actual next sentence following A or a randomly selected sentence.

This task helps BERT learn sentence relationships, which is useful for tasks like question answering and natural language inference.

Fine-tuning After pre-training, BERT can be fine-tuned for specific NLP applications by adding task-specific layers on top of the Transformer encoders. Common applications include:

- **Text Classification**: Sentiment analysis, spam detection, topic classification.
- **Named Entity Recognition (NER)**: Identifying people, locations, and organizations in text.
- **Question Answering (QA)**: Extracting answers from passages, such as in the SQuAD dataset.
- **Text Summarization**: Generating concise versions of longer documents.

Several improved versions of BERT have been introduced over the years.

RoBERTa Removes NSP and trains with larger datasets and longer sequences.

ALBERT Reduces parameter size using factorized embeddings and cross-layer parameter sharing.

DistilBERT A smaller, faster, and more efficient variant of BERT with fewer layers.

That said, at the moment there isn't much followup on that model as all research are going toward decorder only style architectures like ChatGPT and company. That said, Prof. Aaron mentioned that if he had to bet what is the futur for LLMs in long term, he thinks BERT is because of it's much better performance for same model size.

7.3.3 ELMo (Embeddings from Language Models)

ELMo est basé sur des réseaux LSTM bidirectionnels plutôt que sur des Transformers. Il génère des embeddings contextuels pour chaque token en prenant en compte l'ensemble de la phrase. Bien qu'il ait été révolutionnaire pour fournir des représentations riches en contexte, il est moins efficace pour traiter de très longues séquences par rapport aux modèles Transformer modernes.

7.3.4 Transformer-XL

Transformer-XL étend le modèle autoregressif en introduisant une récursion au niveau des segments, ce qui permet de réutiliser les états cachés d'un segment à l'autre. Ce mécanisme, associé aux embeddings positionnels relatifs, permet de traiter des contextes beaucoup plus longs que les Transformers standards.

7.3.5 T5 (Text-to-Text Transfer Transformer)

T5 adopte une approche encodeur-décodeur en formulant toutes les tâches NLP comme un problème de conversion de texte en texte. Il est particulièrement efficace pour des tâches telles que la traduction, le résumé et la question-réponse. Sa flexibilité en fait un modèle très pertinent dans de nombreux contextes d'applications actuels.

7.3.6 VIT

TODO

7.3.7 SWIN

TODO

8 Variational Auto-Encoders (VAEs)

This section introduces the central concepts of Variational Auto-Encoders (VAEs). We discuss how high-dimensional data is modeled using a low-dimensional latent space, derive a tractable variational objective, and present the methods that enable end-to-end learning in these models.

8.1 Manifold Hypothesis

In a machine learning context, the manifod hypothesis is the idea that although real world data (like image, audio or text) may exist in a high-dimensional space, the meaningful variations in the data are atually confined to a much lower-dimensional plane or manifold embedded within that high dimensional space. In other words, while each data point is represented by a large number of features, the true degree of freedo (the intrinsic dimensions) are far fewer... There is a limite number of possible values the data points can actually take.

8.2 Latent Variable Representation

We represent data x using latent variables z via a deterministic mapping:

$$x = g(z), \quad z \sim \mathcal{N}(0, I)$$

Here, z encapsulates the underlying factors of variation, and the decoder g reconstructs the high-dimensional data from z.

8.2.1 Maximum Likelihood

The goal would then be to maximise a mapping such that

$$\log p(x) = \log \int p(x|z)p(z)\, dz$$

However, the integral over z is generally intractable.

8.3 The Variational Objective

To bypass the intractability of directly computing $\log p(x)$, VAEs optimize a approximated lower bound on the log-likelihood.

8.3.1 ECLL (Expected Complete-Data Log-Likelihood)

An intermediate formulation approximates the complete-data log-likelihood:

$$\mathbb{E}_{q(z|x)}[\log p(x, z)]$$

This expectation serves as a proxy by averaging over the latent variable distribution.

8.3.2 KL Divergence

The KL divergence measures the difference between the approximate posterior and the prior:

$$D_{\mathrm{KL}}(q(z|x)\|p(z|x)) = -\int q(z|x)\,\log\frac{q(z|x)}{p(z|x)}\,dz$$

This term regularizes the latent space, ensuring that $q(z|x)$ remains close to the simple prior $p(z|x)$.

8.3.3 ELBO

The Evidence Lower Bound (ELBO) is defined as:

$$\mathrm{ELBO} = \mathbb{E}_{q(z|x)}[\log p(x|z)] - D_{\mathrm{KL}}(q(z|x)\|p(z))$$

Maximizing the ELBO leads to a practical training objective that both reconstructs x and regularizes $q(z|x)$.

8.4 Backpropagation

The entire VAE, including both encoder and decoder, is trained end-to-end via backpropagation using gradients derived from the variational objective.

8.4.1 The Reparameterization Trick

To enable gradient flow through sampling, we reparameterize z as:

$$z = \mu(x) + \sigma(x)\epsilon, \quad \epsilon \sim \mathcal{N}(0, I)$$

This neat transformation allows us to treat the random sampling as a differentiable operation.

8.5 Architectural Taxonomy

8.5.1 VAE-IAF

To achieve a richer posterior, we apply a series of invertible transformations:

$$z_T = f_T \circ \cdots \circ f_1(z_0)$$

Inverse Autoregressive Flow (IAF) enables the approximate posterior to capture complex dependencies.

8.5.2 IWAE (Weighted Importances)

An improved variational bound is obtained by importance weighting:

$$\log p(x) \approx \log \frac{1}{K}\sum_{k=1}^{K}\frac{p(x, z_k)}{q(z_k|x)}$$

Multiple samples tighten the bound and enhance the model's performance.

8.5.3 SVG / VRNN

For sequential data, stochastic variational recurrent networks incorporate latent variables into recurrent architectures. This approach, known as SVG or VRNN, models temporal dependencies alongside latent representations.

9 Generative Adversarial Networks (GANs)

TODO

10 Normalizing Flows

Normalizing flows are a powerful class of generative models that transform complex distributions into simpler, known distributions, and vice versa, through a series of invertible transformations. These models are particularly advantageous due to their ability to compute the exact likelihood of the data, a challenging task for many other generative models like GANs and VAEs. By applying a sequence of invertible mappings to a simple base distribution (often Gaussian), normalizing flows allow for efficient density estimation and generative modeling.

The golden age of normalizing flows occurred between the eras of **Generative Adversarial Networks (GANs)** (see 9) and **Diffusion Models** (see 11). During this period, normalizing flows were at the forefront of generative modeling, as they were capable of learning complex distributions while maintaining tractable likelihood estimation. This made them a valuable tool for tasks like density estimation, generative modeling, and variational inference. However, their popularity has been somewhat overshadowed by the rise of diffusion models in recent years. Nevertheless, normalizing flows remain a valuable tool for modeling complex data distributions efficiently.

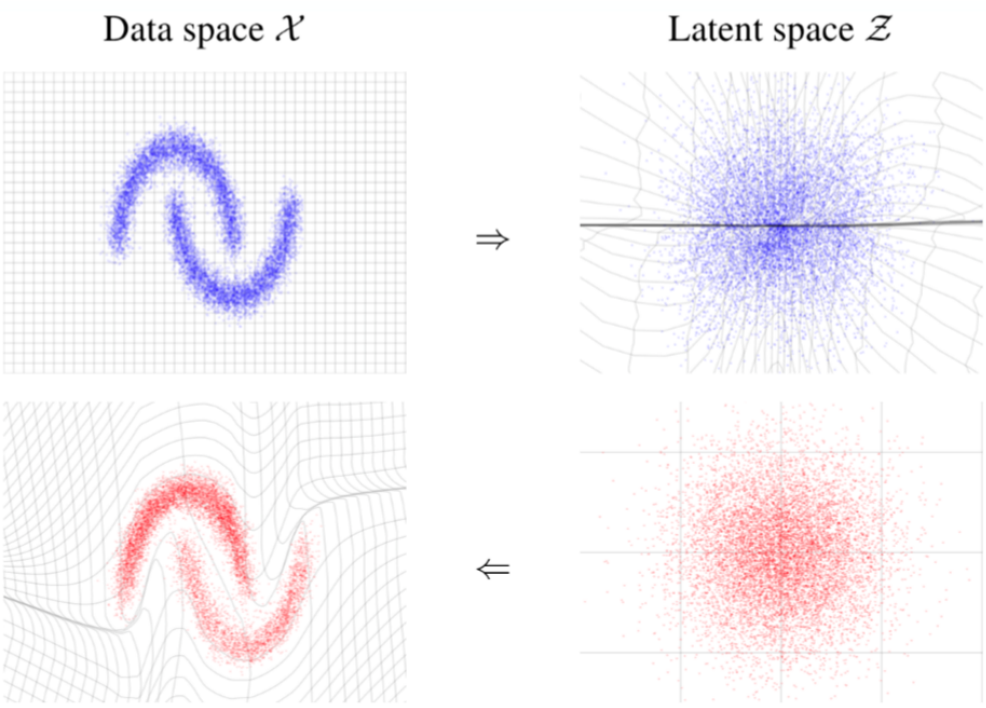


Figure 22: Visualization of normalizing flows: The top part shows the inference process where data points in the data space X are transformed into the latent space Z through the invertible transformation f. The bottom part illustrates the generation process, where samples are drawn from the latent space Z and mapped back to the data space X using the inverse transformation f^{-1}.

10.1 Change of Variables and the Jacobian Determinant

Given an invertible transformation f that maps an input x from a base distribution $p_X(x)$ to a transformed space $y = f(x)$, the probability density function $p_Y(y)$ of the transformed variable can be computed using the **change of variables formula**:

$$p_Y(y) = p_X(x) \left| \det \frac{df(x)}{dx} \right|$$

Where:

- $p_X(x)$ is the probability density of the base distribution,
- $\frac{df(x)}{dx}$ is the Jacobian matrix of the transformation f,
- $\left|\det \frac{df(x)}{dx}\right|$ is the absolute value of the determinant of the Jacobian, which accounts for how the transformation scales volumes in the data space.

This formula allows us to compute the likelihood of data under the transformed distribution p_Y, which is a central task in generative modeling.

10.2 Criteria for Designing Normalizing Flows

When designing normalizing flows, there are key criteria to consider:

Invertible Architecture: The architecture must be invertible to ensure that the transformation between the data space and the latent space is bijective. This can be achieved using structures like **coupling layers** or **autoregressive transformations**.

Efficient Computation of the Change of Variables Equation: The Jacobian determinant, which is critical for calculating the likelihood, must be efficiently computed. This ensures that normalizing flows can scale to high-dimensional data and complex distributions. The likelihood is computed using the formula:

$$\log p_{\text{model}}(x) = \log p_Y(f(x)) + \log \left|\det \frac{df(x)}{dx}\right|$$

Where $p_{\text{model}}(x)$ represents the model distribution, and $p_Y(f(x))$ is the base distribution.

The key property that allows normalizing flows to work effectively is that neural networks are composed of functions that can be chained together. This compositional structure ensures that the transformation from the data space to the latent space (and vice versa) remains invertible, as long as each individual function in the chain is invertible. By using this property, normalizing flows can learn complex transformations while preserving the ability to compute exact likelihoods, making them powerful tools for density estimation and generative modeling.

10.3 Training Normalizing Flows

Training normalizing flows involves maximizing the likelihood of the data under the transformed distribution. This is done by minimizing the negative log-likelihood, which can be written as:

$$\mathcal{L}(\theta) = -\sum_{i=1}^{N} \log p_Y(y_i)$$

Where $y_i = f(x_i)$ is the transformed data and θ represents the parameters of the flow.

The likelihood is computed using the change of variables formula, and the parameters are optimized using gradient-based methods.

10.4 Architectural Taxonomy

There are various architectures for normalizing flows, each designed to ensure invertibility and enable efficient computation of the Jacobian determinant. Some notable models include:

10.4.1 Planar Flows

Planar flows are a simple class of flows where the transformation f is parameterized as:

$$f(x) = x + u \tanh(Wx + b)$$

Where u, W, and b are learned parameters. The Jacobian determinant for planar flows can be computed exactly, making them an attractive choice for simple tasks.

10.4.2 RealNVP (Real-valued Non-Volume Preserving)

RealNVP employs a coupling layer where the input is split into two parts: one part is transformed using a simple neural network, while the other is left unchanged. The transformation ensures that the Jacobian determinant is easy to compute. The general form of the transformation is:

$$y_1 = x_1, \quad y_2 = x_2 \odot \exp(s(x_1)) + t(x_1)$$

Where $s(x_1)$ and $t(x_1)$ are scale and translation functions, respectively, and $\odot$ denotes element-wise multiplication.

10.4.3 Glow (Generative Flow with Invertible 1x1 Convolutions)

Glow extends RealNVP by using invertible 1x1 convolutions, allowing for a more flexible transformation of the data. This allows the model to perform better on high-dimensional data while maintaining efficient computation of the likelihood.

10.4.4 Invertible Residual Networks (i-ResNets)

Invertible Residual Networks (i-ResNets) extend residual networks to ensure invertibility. They are particularly useful for modeling high-dimensional data, as they offer efficient training and likelihood computation. The transformation in i-ResNets is parameterized by:

$$y = x + f(x; \theta)$$

Where $f(x; \theta)$ is a residual function that is designed to be invertible.

10.5 Applications of Normalizing Flows

- **Density Estimation:** Learning complex distributions directly from data.
- **Generative Modeling:** Sampling from complex distributions, such as image and audio generation.
- **Anomaly Detection:** Using the model's likelihood to detect outliers in data.
- **Variational Inference:** Approximating complex posterior distributions in Bayesian inference.

11 Diffusion models

One interesting note in the course was diffusion models as spectral noise. You saw in that plot that when you do the process of adding the gaussian noise to the image, you loose the high frequency information of the image and you just keep the low frequency information.

The difference of the model diffusion and VAE is that VAE takes from the latent space in one shot but the diffusion models go from noisy to no noise iteratively. This gives time to the model to correct himself, one of the reason why the diffusion models are so performing at the moment.

11.1 Controlling Diffusion models (Part 2)

Il y at un tradeoff entre la quality et la quantite des images generer

Tu plug un encoder text avant par exemple stable diffusion alors les deux captions "An image of a loaf of bread in the shape of a cat." "An image of a cat in the shape of a loaf of bread"

You get almost the same thing which is because the word encoder mix them up and cause the caption to loose the nuances.

Difficulte avec la spacialite dans la generation des images. Genre quelque chose a gauche et un autre a droite.

Aussi difficulte dans les interactions entre les objets et choses.

Dalle essaye toujours de toujours generer 10:10 dans les montres. C'est un biais pcq les photos de montre sur le net c'est tres tres souvent 10:10.

Diffusion discret sont le future?? Mais pas encore de resultats empirique

Les modele de diffusion discret opperere sur les nombres reel??

Toujours chain de markov

12 Reinforcement Learning

12.1 Reward Hacking

12.2 Markov Decision Process

12.3 Policies

12.4 Bellman Equations

12.5 Value-Based RL

12.5.1 Goal

Learn value functions

12.5.2 Policy

You derive the policy from the value function, typically by choosing the action with the highest value (greedy policy).

12.5.3 Q-Learning

12.5.4 Deep Q-Learning

12.6 Policy-Based RL

12.6.1 Goal

Learn policy parameters

12.6.2 Policy

You directly learn a parameterized policy (e.g., neural network) that outputs actions or action probabilities.

12.7 Actor-Critic

12.7.1 Policy

You learn both a policy (the actor) and a value function (the critic). The actor selects actions, while the critic evaluates them.

13 Reinforcement Learning from Human Feedback (RLHF)

Language models are not aligned with user intent [Ouyang et al., 2022]. Finetuning can help!

There are two steps to this situation.

First step is pretraining on language modeling (lots of text; learn general things!)

Second step is fietuning with human feedback (many tasks, many labels, adapt to the tasks!)

We can generally do two type of finetuning. One is supervised training and the other by reinforcement (RLHF).

13.1 Flan-T5

13.2 Limitations of Finetuning

One problem is that we have open-ended questions that are difficult to evaluate what is "good" and what is "bad". Especially in creative tasks. If we finetune too much on those tasks there will be overfitting quickly.

Another problem is that language modeling penalizes all token-level mistakes equally, but some errors are worse than others.

For exemple when you evaluate the output of the model, we will use cross-entropy based on the probabilities output of the model. Why it doesn't punish the model too much even if he output a synonym is that in the probabilities, the synonimes will generally both at the same %. Therefor the model is not too much punished.

13.3 Human-in-the-loop

This method is by far the best way to finetune models but there are some big problems with this technique too.

One of the main problem is that this process is expensive. The solution that companies have been using is to model their preferences as a separate (NLP) problem. [Knox and Stone, 2009] - One exemple is chatgpt options to thumbs up/down the awnsers, regenerate awnsers and make the user choose between two awnsers.

Another problem is that human judgment is noisy and misscalibrated. A solution that was found was that instead of asking for direct ratings, the system should ask for pairwise comparisons, which can be more reliable. [Phelps et al., 2015; Clark et al., 2018]

13.4 Reward

Almost always this for RLHF

13.5 InstructGPT

Step 1 (Supervised training): Collect demonstration data and train a supervised policy. Step 2... Step 3...

13.6 LLM-as-a-judge

- PAS UTILISER POUR REINFORCEMENT!! JUST BENCHMARKING

Instead of using a human in the feed-back-loop, we can use a LLM to judge the awnsers. This method has been increasing in popularity in recent years. We generally choose the biggest model to make the judgments on the awnsers.

Here are some notes on that: - Models are heavily positionally biased - Models often rate on syntax & response length

13.7 Benchmarks and comparisons

TODO

13.7.1 MT Bench

TODO

14 Self-Supervised Learning

The main idea behind self-supervision is to design auxiliary ("pretext") tasks that generate supervisory signals directly from the structure of the data, thus eliminating the need for manual labels. These pretext tasks force the model to learn meaningful representations that are useful when transferred to downstream tasks.

14.1 Pretext Task Paradigm

At its core, the self-supervised framework leverages pretext tasks to generate supervisory signals from the data itself.

- **Data Transformation:** Raw inputs (e.g., images) are modified using operations such as cropping, permutation, or geometric transformation.
- **Task Definition:** The network is trained to predict the transformation, the relative context, or to compare different views of the same instance.
- **Representation Learning:** In solving the pretext task, the network learns high-level semantic or structural features that generalize well.

14.2 Spatial Context and Structural Tasks

14.2.1 Context Prediction

- **Task:** Given two patches from the same image (with a spatial gap and random jitter), the network predicts their relative spatial configuration.
- **Architecture:** Utilizes a *Siamese network* structure with shared weights to process each patch, and a late-fusion module that combines features (often from a fully connected layer) for classification into one of several spatial configurations.
- **Design Consideration:** Incorporates mechanisms such as patch gaps, jitter, and color preprocessing to prevent the network from learning trivial (low-level) solutions.

14.2.2 Jigsaw Puzzles

- **Task:** An image is split into tiles (e.g., a 3×3 grid) and then randomly permuted. The network is tasked with reassembling the puzzle.
- **Architecture:** Implements a multi-stream network where each tile is processed by a branch with tied weights, followed by a permutation classification layer that predicts the correct arrangement from a carefully chosen set of possibilities.
- **Outcome:** This method forces the model to understand both object parts and the overall structure.

14.2.3 Rotation Prediction

- **Task:** The network must predict the rotation applied to an image (typically one of 0°, 90°, 180°, or 270°).
- **Architecture:** Consists of a single-stream CNN that extracts features from the rotated image, feeding into a classification head that outputs a probability distribution over rotations.

- **Intuition:** The task compels the network to understand the configuration of objects, as accurate rotation prediction requires awareness of object semantics.

14.3 Contrastive Learning and Beyond

Contrastive methods and their extensions drive learning by comparing different views of the same instance against other samples, enforcing invariance across augmentations.

14.3.1 MoCo (Momentum Contrast)

- **Idea:** Build a dynamic dictionary using a large queue of negative examples and a momentum-updated encoder.
- **Architecture:**
  - Dual encoders: a query encoder for the current mini-batch and a key encoder updated via a momentum mechanism.
  - A contrastive loss (e.g., InfoNCE) that encourages the query representation to align with its positive key while differentiating from negatives stored in the queue.
- **Advantage:** Decouples the dictionary size from the mini-batch size while ensuring stable key representations.

14.3.2 SimCLR (Simple Framework for Contrastive Learning)

- **Idea:** Leverage strong data augmentations to create varied views of the same image and pull their representations together.
- **Architecture:** A standard CNN backbone is augmented with a projection head and trained with a contrastive loss very similar to InfoNCE using large batch sizes and temperature scaling.
- **Outcome:** Establishes a simplified yet highly competitive baseline in self-supervised learning.

14.3.3 CLIP (Contrastive Language–Image Pre-training)

- **Idea:** Align image representations with natural language descriptions using a bi-encoder architecture.
- **Architecture:** Includes an image encoder and a separate text encoder. A contrastive loss is used to maximize the similarity of matching image–text pairs and minimize it for non-matching pairs.
- **Impact:** Enables learning of rich, multi-modal representations that transfer effectively across vision and language tasks.

14.3.4 BYOL (Bootstrap Your Own Latent)

- **Idea:** Achieves self-supervised learning without explicit negative examples by maintaining an online and a target network.
- **Architecture:** Both networks (often with an added projection head) process augmented views of the same image. The target network is updated as a moving average of the online network.
- **Outcome:** Yields robust representations through careful design and momentum updates despite the absence of negative pairs.

14.3.5 Barlow Twins

- **Idea:** Reduce redundancy in feature dimensions by pushing the cross-correlation matrix between different augmentations toward the identity matrix.
- **Architecture:** Uses a shared backbone with a projection head for two augmented views. The loss function combines terms for invariance and decorrelation.
- **Advantage:** Avoids the need for explicit negative sampling while still enforcing meaningful representation learning.

14.3.6 DINO (Self-Distillation with No Labels)

- **Idea:** Employ a teacher–student framework, where the teacher (an exponential moving average of the student) provides soft targets that stabilize training.
- **Architecture:** Both the student and teacher networks process multiple augmented views, and a distillation loss aligns the student's outputs with the teacher's.
- **Outcome:** Generates strong clustering and attention maps, leading to highly transferable features.

14.3.7 JEPA (Joint Embedding Predictive Architecture) and I-JEPA (Improved JEPA)

- **Idea:** Go beyond simple instance discrimination by predicting the representation of one part of an image from its surrounding context.
- **Architecture:** Consists of a context encoder and a predictor network that work on partially observed images (or masked regions) to predict the representations for missing parts.
- **Enhancements in I-JEPA:** Refinements in both prediction and embedding strategies yield more robust and transferable features.
- **Significance:** Aligns with modern trends emphasizing joint embedding frameworks without the explicit need for negative sampling.

14.4 Knowledge Distillation

Knowledge Distillation has traditionally been used to transfer information from a "teacher" model to a "student" model in supervised settings. It allows further refinements by predicting content based on context and transferring rich teacher knowledge to student models.

Methods like DINO utilize a teacher–student formulation in a label-free setting, where the student is trained to mimic the softened outputs of the teacher.

This process generally enhances feature robustness and improves transferability by internalizing richer representations.