DOTE 6635: Artificial Intelligence for Business Research

Inference

Renyu (Philip) Zhang

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OpenAI API Prices

OpenAI Prices: https://openai.com/api/pricing/

Similar pricing scheme for Claude, Grok, DeepSeek, Qwen, etc.

OpenAI o1
Frontier reasoning model that supports tools, Structured Outputs, and vision | 200k context length

Price
Input:
$15.00 / 1M tokens
Cached input:
$7.50 / 1M tokens
Output:
$60.00 / 1M tokens

OpenAI o3-mini
Small cost-efficient reasoning model that's optimized for coding, math, and science, and supports tools and Structured Outputs | 200k context length

Price
Input:
$1.10 / 1M tokens
Cached input:
$0.55 / 1M tokens
Output:
$4.40 / 1M tokens

Save 50% on inputs and outputs with the Batch API and run tasks asynchronously over 24 hours.

GPT-4.5
Largest GPT model designed for creative tasks and agentic planning, currently available in a research preview. | 128k context length

Price
Input:
$75.00 / 1M tokens
Cached input:
$37.50 / 1M tokens
Output:
$150.00 / 1M tokens

GPT-4o
High-intelligence model for complex tasks | 128k context length

Price
Input:
$2.50 / 1M tokens
Cached input:
$1.25 / 1M tokens
Output:
$10.00 / 1M tokens

GPT-4o mini
Affordable small model for fast, everyday tasks | 128k context length

Price
Input:
$0.150 / 1M tokens
Cached input:
$0.075 / 1M tokens
Output:
$0.600 / 1M tokens

ChatGPT's User Experience: What is Behind the Decline in Intelligence?
ChatGPT Bugs chatgpt

daixin0906 Jan 6

Since the beginning of this year, I have noticed some significant changes in the functionality and performance of ChatGPT, especially in terms of its intelligence and depth of reasoning. Once, whether as a work assistant or a daily conversational partner, ChatGPT left a deep impression on me. But now, with the use of the GPT-4o model and GPT-01, I can't help but feel that their performance is far below the previous versions. This article will discuss this change from several perspectives.

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https://community.openai.com/t/chatgpts-user-experience-what-is-behind-the-decline-in-intelligence/1081511

- **Key question**: How to make LLM inference more efficient and cost-effective?

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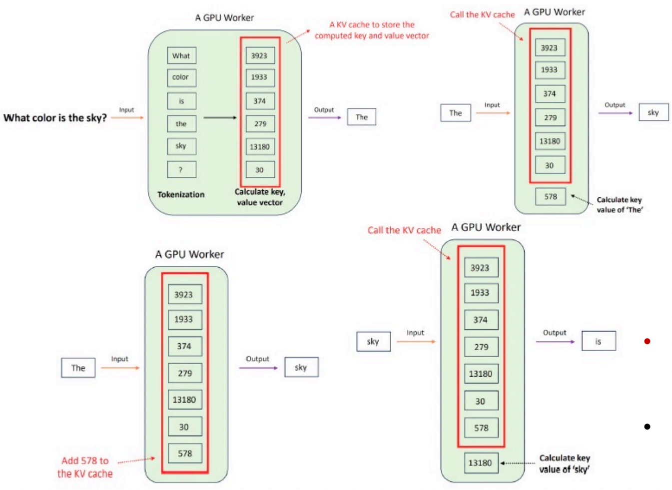
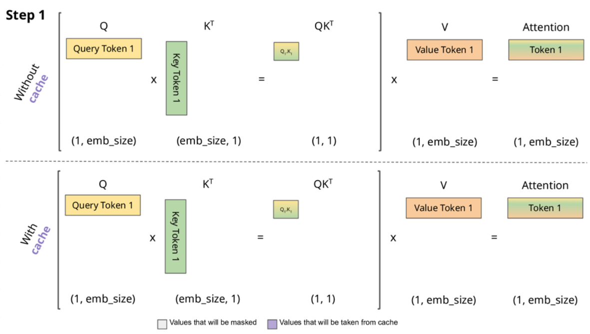
Agenda

- KV-Cache
- Quantization
- DeepSeek Inference System
- OR for LLM Inference

KV Cache

Hugging Face KV Cache Intro: https://huggingface.co/blog/not-lain/kv-caching
https://medium.com/@plienhar/llm-inference-series-2-the-two-phase-process-behind-llms-responses-1ff1ff021cd5

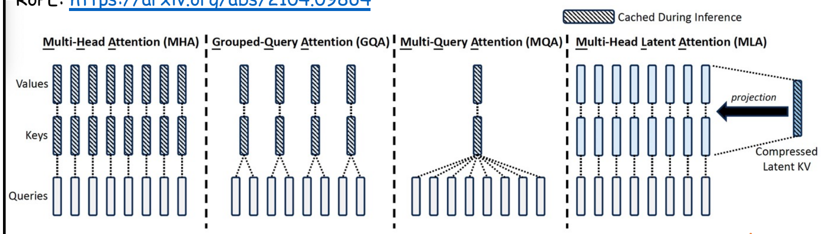




- **Essence of KV Cache**: Use memory to substitute compute.
- KV Cache is linear in context window length.

Multi-Head Latent Attention

DeepSeek-V3: https://arxiv.org/abs/2412.19437v1; https://www.bilibili.com/video/BV1HqFQezEMt
RoPE: https://arxiv.org/abs/2104.09864



Down-Projection Matrix

$$\mathbf{c}_t^{KV} = W^{DKV}\mathbf{h}_t,$$
$$\mathbf{k}_t^{C} = W^{UK}\mathbf{c}_t^{KV},$$
$$\mathbf{v}_t^{C} = W^{UV}\mathbf{c}_t^{KV},$$

Up-Projection Matrix

A low-rank joint compression of keys and values.

Compress the query to reduce activation memory.

$$\mathbf{c}_t^{Q} = W^{DQ}\mathbf{h}_t,$$
$$[\mathbf{q}_{t,1}^{C};\mathbf{q}_{t,2}^{C};\ldots;\mathbf{q}_{t,n_h}^{C}] = \mathbf{q}_t^{C} = W^{UQ}\mathbf{c}_t^{Q},$$
$$[\mathbf{q}_{t,1}^{R};\mathbf{q}_{t,2}^{R};\ldots;\mathbf{q}_{t,n_h}^{R}] = \mathbf{q}_t^{R} = \mathrm{RoPE}(W^{QR}\mathbf{c}_t^{Q}),$$
$$\mathbf{q}_{t,i} = [\mathbf{q}_{t,i}^{C};\mathbf{q}_{t,i}^{R}],$$

Attention Mechanism	KV Cache per Token (# Element)	Capability
Multi-Head Attention (MHA)	$2n_h d_h l$	Strong
Grouped-Query Attention (GQA)	$2n_g d_h l$	Moderate
Multi-Query Attention (MQA)	$2d_h l$	Weak
MLA (Ours)	$(d_c + d_h^R)l \approx \frac{9}{2}d_h l$	Stronger

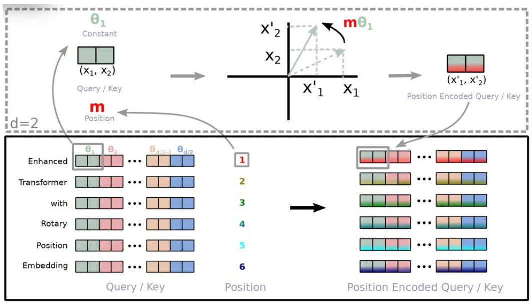


Figure 1: Implementation of Rotary Position Embedding(RoPE).

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- OR for LLM Inference

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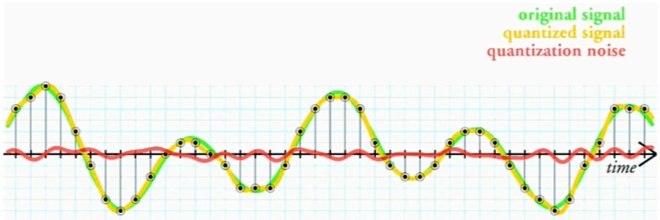
Quantization

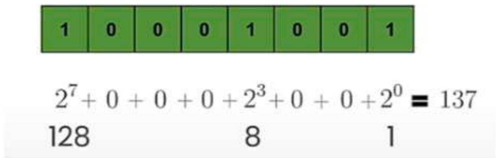
MIT Efficient DL Computing: https://hanlab.mit.edu/courses/2024-fall-65940
Quantization Fundamentals with Hugging Face: https://learn.deeplearning.ai/courses/quantization-fundamentals/

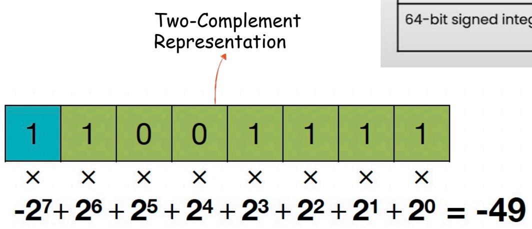
- Quantization: Mapping an input from a large (and continuous) set of values to a smaller (and discrete) set of values.
  - We do quantization to save memory and energy and accelerate compute, especially for LLM inference.



Data Type	torch.dtype
8-bit signed integer	torch.int8
8-bit unsigned integer	torch.uint8
16-bit signed integer	torch.int16
32-bit signed integer	torch.int32
64-bit signed integer	torch.int64

- For unsigned integer data types, $[0, 2^n-1]$.
- For signed integer data types, $[-2^n, 2^{n-1}-1]$.





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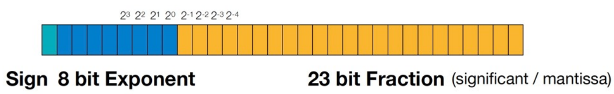
Floating Number Representations

MIT Efficient DL Computing: https://hanlab.mit.edu/courses/2024-fall-65940
Quantization Fundamentals with Hugging Face: https://learn.deeplearning.ai/courses/quantization-fundamentals/

- Floating-point numbers:
  - Sign: +/-
  - Exponent: Range
  - Fraction/mantissa: Precision

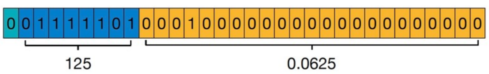
Data Type	torch.dtype	torch.dtype alias
16-bit floating point	torch.float16	torch.half
16-bit brain floating point	torch.bfloat16	
32-bit floating point	torch.float32	torch.float
64-bit floating point	torch.float64	torch.double

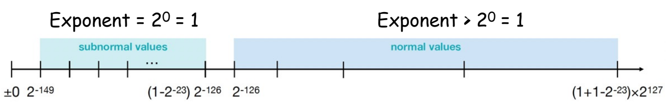


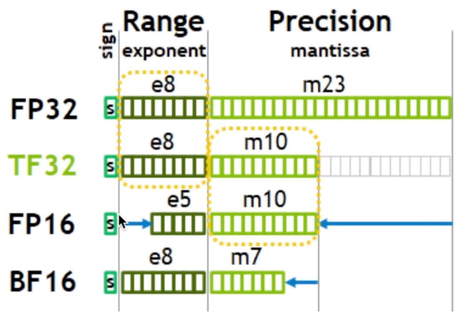
$(-1)^{\text{sign}} \times (1 + \textbf{Fraction}) \times 2^{\textbf{Exponent}-127}$ ⟵ Exponent Bias = 127 = $2^{8-1}-1$

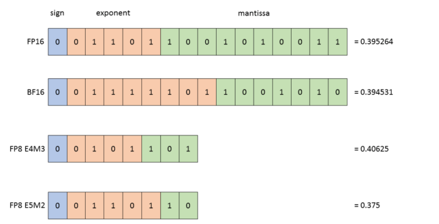
How to represent **0.265625**?

0.265625 = 1.0625 × 2^{-2} = (1 + 0.0625) × $2^{125-127}$









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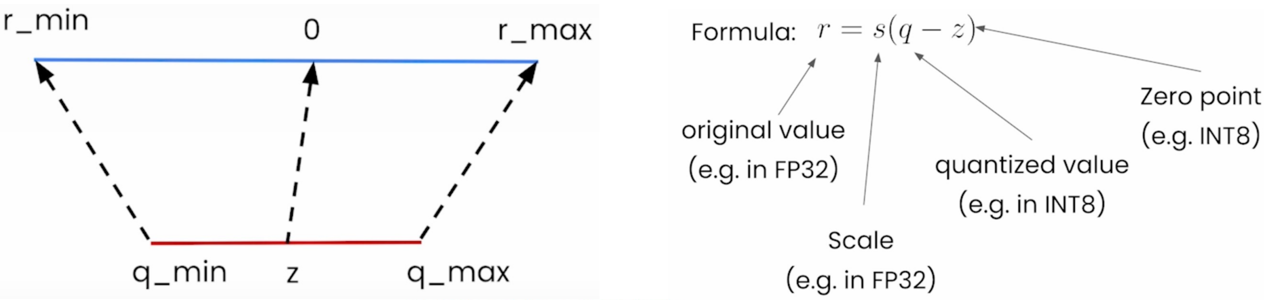
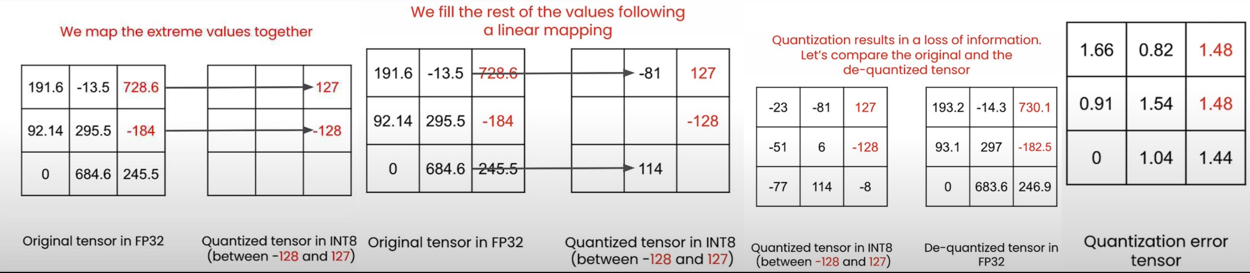
Linear Quantization

MIT Efficient DL Computing: https://hanlab.mit.edu/courses/2024-fall-65940
Quantization Fundamentals with Hugging Face: https://learn.deeplearning.ai/courses/quantization-fundamentals/

- Use a linear mapping to represent a number in high-precision type (FP32) in low-precision type (INT8).





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Linear Quantization

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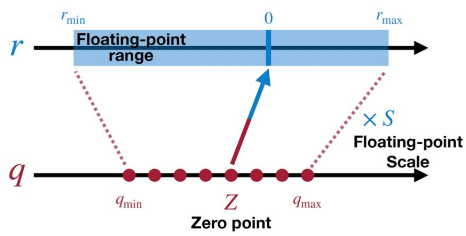
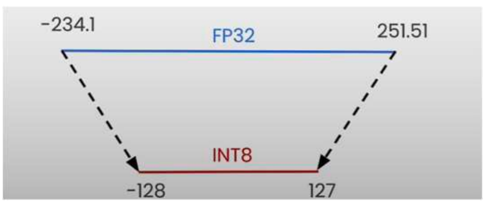
- How do we determine the scale s and zero point z?

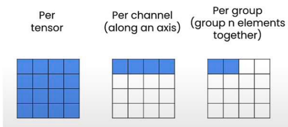


$$r_{\max} = S\left(q_{\max} - Z\right)$$
$$r_{\min} = S\left(q_{\min} - Z\right)$$
$$\downarrow$$
$$r_{\max} - r_{\min} = S\left(q_{\max} - q_{\min}\right)$$
$$S = \frac{r_{\max} - r_{\min}}{q_{\max} - q_{\min}}$$

$$r_{\min} = S\left(q_{\min} - Z\right)$$
$$\downarrow$$
$$Z = q_{\min} - \frac{r_{\min}}{S}$$
$$\downarrow$$
$$Z = \text{round}\left(q_{\min} - \frac{r_{\min}}{S}\right)$$



- Per-channel and per-group quantization.
- Quantization of weights and activations.
- Quantization-aware training.



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Quantization + LoRA: QLoRA

Stanford CS224N: https://web.stanford.edu/class/cs224n/
QLoRA Paper: https://arxiv.org/abs/2305.14314

Qlora: Efficient finetuning of quantized llms
T Dettmers, A Pagnoni, A Holtzman… - Advances in neural …, 2023 - proceedings.neurips.cc
… We present **QLORA**, an efficient finetuning approach that reduces … **QLORA** backpropagates gradients through a frozen, 4-bit … **QLORA** introduces a number of innovations to save memory …
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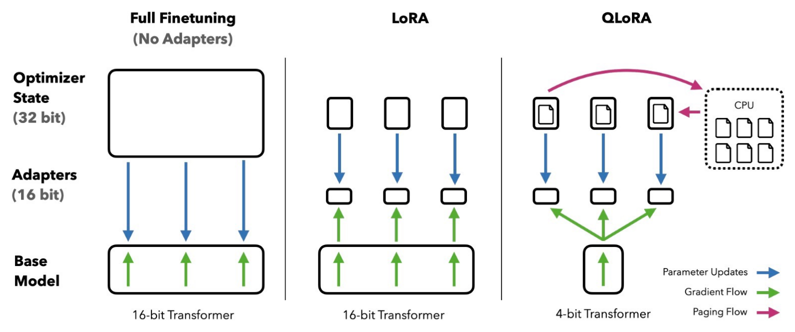
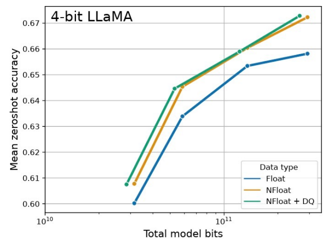


Figure 1: Different finetuning methods and their memory requirements. QLoRA improves over LoRA by quantizing the transformer model to 4-bit precision and using paged optimizers to handle memory spikes.

To further save memory, adopt double-quantization (DQ).

- QLoRA improves over LoRA by quantizing the transformer to 4-bit precision and using paged optimizer to handle memory.
- 4-bit NormalFloat (NF4)
  - Data type suitable for normally distributed weights.



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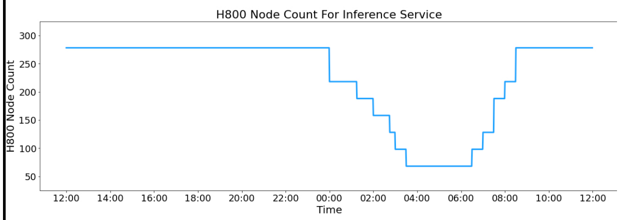
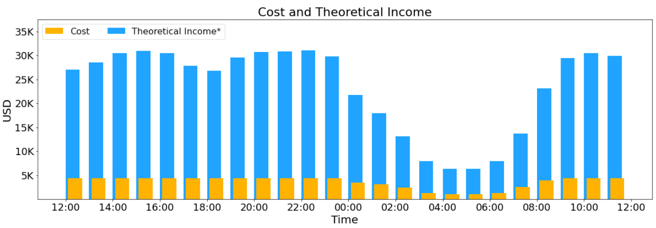
DeepSeek-V3/R1 Inference System

GitHub: https://github.com/deepseek-ai/open-infra-index/blob/main/202502OpenSourceWeek/day_6_one_more_thing_deepseekV3R1_inference_system_overview.md

- Extremely optimized for high throughput and low latency: cross-node Expert Parallelism (EP).
  - Leverage EP to scale batch size.
  - Hide communication latency behind computation.
  - Perform load balancing.
- Served with 278 8-H800 GPU nodes; average occupancy 226.75 nodes; each with throughput ~73.7k tokens/s for input during prefilling and ~14.8k tokens/s for output during decoding.
- Daily input tokens: 608B (342B hit the KV cache)
- Daily output tokens: 168B; 20-22 tokens/s; average KV-cache length per output: 4,989 tokens.





* The theoretical income is calculated based on R1's standard API pricing, taking into account all tokens across web, APP, and API. It is not our actual income.

Daily cost = $87,072; Daily Revenue under the R1-API pricing = $562,027, i.e., 545% profit margin

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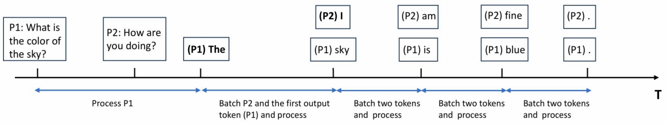
OR for LLM Inference

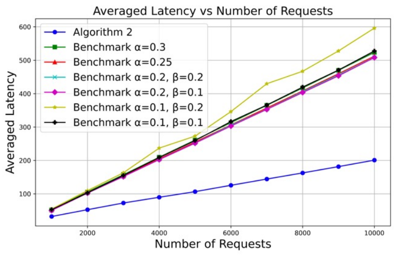
Fundamental Modeling for LLM Inference with Exploding KV Cache Demands

Patrick Jaillet
Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology, jaillet@mit.edu
Jiashuo Jiang
HKUST, jsjiang@ust.hk
Chara Podimata
Sloan School of Management, Massachusetts Institute of Technology podimata@mit.edu
Zijie Zhou*
Operations Research Center, Massachusetts Institute of Technology, zhou98@mit.edu



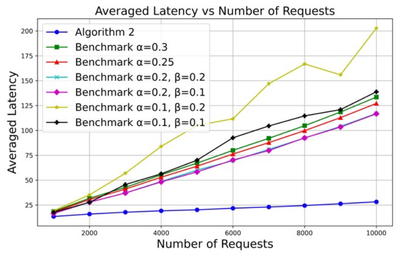
- Given the KV-cache memory limit, how to batch different prompts and output tokens to minimize total end-to-end latency.

- Proposed scheduling algorithm (with provable constant regret): Prioritize the prompt with the smallest predicted number of output tokens, subject to the KV-cache limit constraint.
  - Benchmark: alpha-protection first-come-first-serve, beta-clearing when reaching KV-cache limit.



High-Demand

Low-Demand



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