20. 1. $I \supset J$
2. $I \vee ({\sim}{\sim}K \bullet {\sim}{\sim}J)$
3. $L \supset {\sim}K$
4. ${\sim}(I \bullet J)$
$\therefore {\sim}L \vee {\sim}J$
5. $I \supset (I \bullet J)$
6. ${\sim}I$
7. ${\sim}{\sim}K \bullet {\sim}{\sim}J$
8. ${\sim}{\sim}K$
9. ${\sim}L$
10. ${\sim}L \vee {\sim}J$

III. For each of the following, adding just two statements to the premises will produce a formal proof of validity. Construct a formal proof of validity for each of the following arguments.

21. $D \supset E$
$D \bullet F$
$\therefore E$

22. $J \supset K$
J
$\therefore K \vee L$

23. $P \bullet Q$
R
$\therefore P \bullet R$

24. $V \vee W$
${\sim}V$
$\therefore W \vee X$

25. $Y \supset Z$
Y
$\therefore Y \bullet Z$

26. $D \supset E$
$(E \supset F) \bullet (F \supset D)$
$\therefore D \supset F$

27. ${\sim}(K \bullet L)$
$K \supset L$
$\therefore {\sim}K$

28. $(T \supset U) \bullet (T \supset V)$
T
$\therefore U \vee V$

29. $(Z \bullet A) \supset (B \bullet C)$
$Z \supset A$
$\therefore Z \supset (B \bullet C)$

30. $D \supset E$
$[D \supset (D \bullet E)] \supset (F \supset {\sim}G)$
$\therefore F \supset {\sim}G$

31. $(K \supset L) \supset M$
${\sim}M \bullet {\sim}(L \supset K)$
$\therefore {\sim}(K \supset L)$

32. $[T \supset (U \vee V)] \bullet [U \supset (T \vee V)]$
$(T \vee U) \bullet (U \vee V)$
$\therefore (U \vee V) \vee (T \vee V)$

33. $A \supset B$
$A \vee C$
$C \supset D$
$\therefore B \vee D$

34. $J \vee {\sim}K$
$K \vee (L \supset J)$
${\sim}J$
$\therefore L \supset J$

35. $(M \supset N) \bullet (O \supset P)$
$N \supset P$
$(N \supset P) \supset (M \vee O)$
$\therefore N \vee P$

IV. For each of the following, adding just three statements to the premises will produce a formal proof of validity. Construct a formal proof of validity for each of the following arguments.

36. $(D \vee E) \supset (F \bullet G)$
D
$\therefore F$

37. $(H \supset I) \bullet (H \supset J)$
$H \bullet (I \vee J)$
$\therefore I \vee J$

38. $(K \bullet L) \supset M$
$K \supset L$
$\therefore K \supset [(K \bullet L) \bullet M]$

39. $Q \supset R$
$R \supset S$
$\sim S$
$\therefore \sim Q \bullet \sim R$

40. $T \supset U$
$V \vee \sim U$
$\sim V \bullet \sim W$
$\therefore \sim T$

41. $\sim X \supset Y$
$Z \supset X$
$\sim X$
$\therefore Y \bullet \sim Z$

42. $(A \vee B) \supset \sim C$
$C \vee D$
A
$\therefore D$

43. $(H \supset I) \bullet (J \supset K)$
$K \vee H$
$\sim K$
$\therefore I$

44. $(P \supset Q) \bullet (Q \supset P)$
$R \supset S$
$P \vee R$
$\therefore Q \vee S$

45. $(T \supset U) \bullet (V \supset W)$
$(U \supset X) \bullet (W \supset Y)$
T
$\therefore X \vee Y$

V. Construct a formal proof of validity for each of the following arguments.

46. $(F \supset G) \bullet (H \supset I)$
$J \supset K$
$(F \vee J) \bullet (H \vee L)$
$\therefore G \vee K$

47. $(K \vee L) \supset (M \vee N)$
$(M \vee N) \supset (O \bullet P)$
K
$\therefore O$

48. $W \supset X$
$(W \bullet X) \supset Y$
$(W \bullet Y) \supset Z$
$\therefore W \supset Z$

49. $A \supset B$
$C \supset D$
$A \vee C$
$\therefore (A \bullet B) \vee (C \bullet D)$

50. $J \supset K$
$K \vee L$
$(L \bullet \sim J) \supset (M \bullet \sim J)$
$\sim K$
$\therefore M$

51. $[(A \vee B) \supset C] \bullet [(X \bullet Y) \supset Z]$
$\sim C$
$(A \vee B) \vee (Y \supset X)$
$\sim X$
$\therefore \sim Y \vee (X \equiv Y)$

VI. Construct a formal proof of validity for each of the following arguments. Translate using the abbreviations suggested.

52. If Adams joins, then the club's social prestige will rise; and if Baker joins, then the club's financial position will be more secure.

V. (46) 1. $(F \supset G) \cdot (H \supset I)$
2. $J \supset K$
3. $(F \vee J) \cdot (H \vee L)$ / $G \vee K$
4. $F \supset G$ — 1 simp
5. $F \vee J$ — 3 Simp
6 $(F \supset G) \cdot (J \supset K)$ — 2,4 Conj
7. $G \vee K$ — 5,6 CD

(47) 1. $(K \vee L) \supset (M \vee N)$
2. $(M \vee N) \supset (O \cdot P)$
3. K / O
4 $K \vee L$ — 3 add
5. $M \vee N$ — 1,4 MP
6 $O \cdot P$ — 2,5 MP
7 O — 6 Simp

(48) 1. $W \supset X$
2. $(W \cdot X) \supset Y$
3. $(W \cdot Y) \supset Z$ / $W \supset Z$
4. $W \supset (W \cdot X)$ — 1 absorption (a rule we don't have)
5. $W \supset Y$ — 2,5 HS
6. $W \supset (W \cdot Y)$ — 5 abs
7 $W \supset Z$ — 3,6 HS

(50)
1. $J \supset K$
2. $K \vee L$
3. $(L \cdot \sim J) \supset (M \cdot \sim J)$
4. $\sim K$ / M
5. L 2,4 DS
6. $\sim J$ 1,4 MT
7. $L \cdot \sim J$ 5,6 Conj
8 $M \cdot \sim J$ 3,7 MP
9 M 8 Simp

(51)
1. $[(A \vee B) \supset C] \cdot [(X \cdot Y) \supset Z]$
2. $\sim C$
3. $(A \vee B) \vee (Y \supset X)$
4. $\sim X$ / $\sim Y \vee (X \equiv Y)$
5. $(A \vee B) \supset C$ 1 Simp
6 $\sim (A \vee B)$ 2,5 MT
7. $Y \supset X$ 3,6 DS
8 $\sim Y$ 4,7 MT
9 $\sim Y \vee (X \equiv Y)$ 8 add

III. For each of the following, adding just two statements to the premisses will produce a formal proof of validity. Construct a formal proof of validity for each of the following arguments.

*1. A
B
$\therefore (A \vee C) \bullet B$

2. $D \supset E$
$D \bullet F$
$\therefore E$

3. G
H
$\therefore (G \bullet H) \vee I$

4. $J \supset K$
J
$\therefore K \vee L$

*5. $M \vee N$
$\sim M \bullet \sim O$
$\therefore N$

6. $P \bullet Q$
R
$\therefore P \bullet R$

7. $S \supset T$
$\sim T \bullet \sim U$
$\therefore \sim S$

8. $V \vee W$
$\sim V$
$\therefore W \vee X$

9. $Y \supset Z$
Y
$\therefore Y \bullet Z$

*10. $A \supset B$
$(A \bullet B) \supset C$
$\therefore A \supset C$

11. $D \supset E$
$(E \supset F) \bullet (F \supset D)$
$\therefore D \supset F$

12. $(G \supset H) \bullet (I \supset J)$
G
$\therefore H \vee J$

13. $\sim(K \bullet L)$
$K \supset L$
$\therefore \sim K$

14. $(M \supset N) \bullet (M \supset O)$
$N \supset O$
$\therefore M \supset O$

*15. $(P \supset Q) \bullet (R \supset S)$
$(P \vee R) \bullet (Q \vee R)$
$\therefore Q \vee S$

16. $(T \supset U) \bullet (T \supset V)$
T
$\therefore U \vee V$

17. $(W \vee X) \supset Y$
W
$\therefore Y$

18. $(Z \bullet A) \supset (B \bullet C)$
$Z \supset A$
$\therefore Z \supset (B \bullet C)$

19. $D \supset E$
$[D \supset (D \bullet E)] \supset (F \supset \sim G)$
$\therefore F \supset \sim G$

*20. $(\sim H \vee I) \vee J$
$\sim(\sim H \vee I)$
$\therefore J \vee \sim H$

21. $(K \supset L) \supset M$
$\sim M \bullet \sim(L \supset K)$
$\therefore \sim(K \supset L)$

22. $(N \supset O) \supset (P \supset Q)$
$[P \supset (N \supset O)] \bullet [N \supset (P \supset Q)]$
$\therefore P \supset (P \supset Q)$

premisses will
of validity for

23. $R \supset S$
$S \supset (S \bullet R)$
$\therefore [R \supset (R \bullet S)] \bullet [S \supset (S \bullet R)]$

24. $[T \supset (U \vee V)] \bullet [U \supset (T \vee V)]$
$(T \vee U) \bullet (U \vee V)$
$\therefore (U \vee V) \vee (T \vee V)$

*25. $(W \bullet X) \supset (Y \bullet Z)$
$\sim[(W \bullet X) \bullet (Y \bullet Z)]$
$\therefore \sim(W \bullet X)$

26. $A \supset B$
$A \vee C$
$C \supset D$
$\therefore B \vee D$

27. $(E \bullet F) \vee (G \supset H)$
$I \supset G$
$\sim(E \bullet F)$
$\therefore I \supset H$

28. $J \vee \sim K$
$K \vee (L \supset J)$
$\sim J$
$\therefore L \supset J$

29. $(M \supset N) \bullet (O \supset P)$
$N \supset P$
$(N \supset P) \supset (M \vee O)$
$\therefore N \vee P$

*30. $Q \supset (R \vee S)$
$(T \bullet U) \supset R$
$(R \vee S) \supset (T \bullet U)$
$\therefore Q \supset R$

IV. For each of the following, adding just three statements to the premisses will produce a formal proof of validity. Construct a formal proof of validity for each of the following arguments.

*1. $A \vee (B \supset A)$
$\sim A \bullet C$
$\therefore \sim B$

2. $(D \vee E) \supset (F \bullet G)$
D
$\therefore F$

3. $(H \supset I) \bullet (H \supset J)$
$H \bullet (I \vee J)$
$\therefore I \vee J$

4. $(K \bullet L) \supset M$
$K \supset L$
$\therefore K \supset [(K \bullet L) \bullet M]$

*5. $N \supset [(N \bullet O) \supset P]$
$N \bullet O$
$\therefore P$

6. $Q \supset R$
$R \supset S$
$\sim S$
$\therefore \sim Q \bullet \sim R$

7. $T \supset U$
$V \vee \sim U$
$\sim V \bullet \sim W$
$\therefore \sim T$

8. $\sim X \supset Y$
$Z \supset X$
$\sim X$
$\therefore Y \bullet \sim Z$

9. $(A \vee B) \supset \sim C$
$C \vee D$
A
$\therefore D$

*10. $E \vee \sim F$
$F \vee (E \vee G)$
$\sim E$
$\therefore G$

11. $(H \supset I) \bullet (J \supset K)$
$K \vee H$
$\sim K$
$\therefore I$

12. $L \vee (M \supset N)$
$\sim L \supset (N \supset O)$
$\sim L$
$\therefore M \supset O$

13. $(P \supset Q) \bullet (Q \supset P)$
$R \supset S$
$P \vee R$
$\therefore Q \vee S$

14. $(T \supset U) \bullet (V \supset W)$
$(U \supset X) \bullet (W \supset Y)$
T
$\therefore X \vee Y$

***15.** $(Z \bullet A) \supset B$
$B \supset A$
$(B \bullet A) \supset (A \bullet B)$
$\therefore (Z \bullet A) \supset (A \bullet B)$

V. Construct a formal proof of validity for each of the following arguments.

***1.** $A \supset B$
$A \vee (C \bullet D)$
$\sim B \bullet \sim E$
$\therefore C$

2. $(F \supset G) \bullet (H \supset I)$
$J \supset K$
$(F \vee J) \bullet (H \vee L)$
$\therefore G \vee K$

3. $(\sim M \bullet \sim N) \supset (O \supset N)$
$N \supset M$
$\sim M$
$\therefore \sim O$

4. $(K \vee L) \supset (M \vee N)$
$(M \vee N) \supset (O \bullet P)$
K
$\therefore O$

***5.** $(Q \supset R) \bullet (S \supset T)$
$(U \supset V) \bullet (W \supset X)$
$Q \vee U$
$\therefore R \vee V$

6. $W \supset X$
$(W \bullet X) \supset Y$
$(W \bullet Y) \supset Z$
$\therefore W \supset Z$

7. $A \supset B$
$C \supset D$
$A \vee C$
$\therefore (A \bullet B) \vee (C \bullet D)$

8. $(E \vee F) \supset (G \bullet H)$
$(G \vee H) \supset I$
E
$\therefore I$

9. $J \supset K$
$K \vee L$
$(L \bullet \sim J) \supset (M \bullet \sim J)$
$\sim K$
$\therefore M$

***10.** $(N \vee O) \supset P$
$(P \vee Q) \supset R$
$Q \vee N$
$\sim Q$
$\therefore R$

VI. Construct a formal proof of validity for each of the following arguments, using the abbreviations suggested.

***1.** If either Gertrude or Herbert wins, then both Jane and Kenneth lose. Gertrude wins. Therefore Jane loses. (*G*—Gertrude wins; *H*—Herbert wins; *J*—Jane loses; *K*—Kenneth loses.)

2. If Adams joins, then the club's social prestige will rise; and if Baker joins, then the club's financial position will be more secure. Either Adams or Baker will join. If the club's social prestige rises, then Baker will join; and if the club's financial position becomes more secure, then Wilson will join. Therefore either Baker or Wilson will join. (*A*—Adams joins; *S*—The club's social prestige rises; *B*—Baker joins; *F*—The club's financial position is more secure; *W*—Wilson joins.)

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(1)
1. $A \supset B$
2. $A \vee (C \cdot D)$
3. $\sim B \cdot \sim E$ / C
4. $\sim B$ — 3 Simp
5. $\sim A$ — 1,4 MT
6. $C \cdot D$ — 2,5 DS
7. C — 6 Simp

(3)
1. $(\sim M \cdot \sim N) \supset (O \supset N)$
2. $N \supset M$
3. $\sim M$ / $\sim O$
4. $\sim N$ — 2,3 MT
5. $\sim M \cdot \sim N$ — 3,4 Conj
6. $O \supset N$ — 1,5 MP
7. $\sim O$ — 4,6 MT

(5)
1. $(Q \supset R) \cdot (S \supset T)$
2. $(U \supset V) \cdot (W \supset X)$
3. $Q \vee U$ / $R \vee V$
4. $Q \supset R$ — 1 Simp
5. $U \supset V$ — 2 Simp
6. $(Q \supset R) \cdot (U \supset V)$ — 4,5 Conj
7. $R \vee V$ — 3,6 CD

(8) 1. $(E \vee F) \supset (G \cdot H)$
2. $(G \vee H) \supset I$
3. E / I
4. $E \vee F$ — 3 add
5. $G \cdot H$ — 1,4 mp
6 G — 5 simp
7. $G \vee H$ — 6 add
8. I — 2,7 mp

(10) 1. $(N \vee O) \supset P$
2. $(P \vee Q) \supset R$
3. $Q \vee N$
4. $\sim Q$ / R
5. N — 3,4 DS
6 $N \vee O$ — 5 add
7. P — 1,6 mp
8. $P \vee Q$ — 7 add
9 R — 2,8 mp