Exercises

I. For each of the following arguments, state the rule of inference by which its conclusion follows from its premiss.

*1. $(A \supset B) \bullet (C \supset D)$
$\therefore (A \supset B) \bullet (\sim D \supset \sim C)$

2. $(E \supset F) \bullet (G \supset \sim H)$
$\therefore (\sim E \vee F) \bullet (G \supset \sim H)$

3. $[I \supset (J \supset K)] \bullet (J \supset \sim I)$
$\therefore [(I \bullet J) \supset K] \bullet (J \supset \sim I)$

4. $[L \supset (M \vee N)] \vee [L \supset (M \vee N)]$
$\therefore L \supset (M \vee N)$

*5. $O \supset [(P \supset Q) \bullet (Q \supset P)]$
$\therefore O \supset (P \equiv Q)$

6. $\sim(R \vee S) \supset (\sim R \vee \sim S)$
$\therefore (\sim R \bullet \sim S) \supset (\sim R \vee \sim S)$

7. $(T \vee \sim U) \bullet [(W \bullet \sim V) \supset \sim T]$
$\therefore (T \vee \sim U) \bullet [W \supset (\sim V \supset \sim T)]$

8. $(X \vee Y) \bullet (\sim X \vee \sim Y)$
$\therefore [(X \vee Y) \bullet \sim X] \vee [(X \vee Y) \bullet \sim Y]$

9. $Z \supset (A \supset B)$
$\therefore Z \supset (\sim\sim A \supset B)$

*10. $[C \bullet (D \bullet \sim E)] \bullet [(C \bullet D) \bullet \sim E]$
$\therefore [(C \bullet D) \bullet \sim E] \bullet [(C \bullet D) \bullet \sim E]$

11. $(\sim F \vee G) \bullet (F \supset G)$
$\therefore (F \supset G) \bullet (F \supset G)$

12. $(H \supset \sim I) \supset (\sim I \supset \sim J)$
$\therefore (H \supset \sim I) \supset (J \supset I)$

13. $(\sim K \supset L) \supset (\sim M \vee \sim N)$
$\therefore (\sim K \supset L) \supset \sim(M \bullet N)$

14. $[(\sim O \vee P) \vee \sim Q] \bullet [\sim O \vee (P \vee \sim Q)]$
$\therefore [\sim O \vee (P \vee \sim Q)] \bullet [\sim O \vee (P \vee \sim Q)]$

*15. $[(R \vee \sim S) \bullet \sim T] \vee [(R \vee \sim S) \bullet U]$
$\therefore (R \vee \sim S) \bullet (\sim T \vee U)$

16. $[V \supset \sim(W \vee X)] \supset (Y \vee Z)$
$\therefore \{[V \supset \sim(W \vee X)] \bullet [V \supset \sim(W \vee X)]\} \supset (Y \vee Z)$

17. $[(\sim A \bullet B) \bullet (C \vee D)] \vee [\sim(\sim A \bullet B) \bullet \sim(C \vee D)]$
$\therefore (\sim A \bullet B) \equiv (C \vee D)$

18. $[\sim E \vee (\sim\sim F \supset G)] \bullet [\sim E \vee (F \supset G)]$
$\therefore [\sim E \vee (F \supset G)] \bullet [\sim E \vee (F \supset G)]$

19. $[H \bullet (I \vee J)] \vee [H \bullet (K \supset \sim L)]$
$\therefore H \bullet [(I \vee J) \vee (K \supset \sim L)]$

*20. $(\sim M \vee \sim N) \supset (O \supset \sim\sim P)$
$\therefore \sim(M \bullet N) \supset (O \supset \sim\sim P)$

II. Each of the following is a formal proof of validity for the indicated argument. State the "justification" for each numbered line that is not a premiss.

*1.
1. $A \supset B$
2. $C \supset \sim B$
$\therefore A \supset \sim C$
3. $\sim\sim B \supset \sim C$
4. $B \supset \sim C$
5. $A \supset \sim C$

2.
1. $(D \bullet E) \supset F$
2. $(D \supset F) \supset G$
$\therefore E \supset G$
3. $(E \bullet D) \supset F$
4. $E \supset (D \supset F)$
5. $E \supset G$

3. 1. $(H \vee I) \supset [J \bullet (K \bullet L)]$
2. I
$\therefore J \bullet K$
3. $I \vee H$
4. $H \vee I$
5. $J \bullet (K \bullet L)$
6. $(J \bullet K) \bullet L$
7. $J \bullet K$

***5.** 1. $(Q \vee \sim R) \vee S$
2. $\sim Q \vee (R \bullet \sim Q)$
$\therefore R \supset S$
3. $(\sim Q \vee R) \bullet (\sim Q \vee \sim Q)$
4. $(\sim Q \vee \sim Q) \bullet (\sim Q \vee R)$
5. $\sim Q \vee \sim Q$
6. $\sim Q$
7. $Q \vee (\sim R \vee S)$
8. $\sim R \vee S$
9. $R \supset S$

7. 1. $Y \supset Z$
2. $Z \supset [Y \supset (R \vee S)]$
3. $R \equiv S$
4. $\sim(R \bullet S)$
$\therefore \sim Y$
5. $(R \bullet S) \vee (\sim R \bullet \sim S)$
6. $\sim R \bullet \sim S$
7. $\sim(R \vee S)$
8. $Y \supset [Y \supset (R \vee S)]$
9. $(Y \bullet Y) \supset (R \vee S)$
10. $Y \supset (R \vee S)$
11. $\sim Y$

9. 1. $(D \bullet E) \supset \sim F$
2. $F \vee (G \bullet H)$
3. $D \equiv E$
$\therefore D \supset G$
4. $(D \supset E) \bullet (E \supset D)$
5. $D \supset E$
6. $D \supset (D \bullet E)$
7. $D \supset \sim F$
8. $(F \vee G) \bullet (F \vee H)$
9. $F \vee G$
10. $\sim\sim F \vee G$
11. $\sim F \supset G$
12. $D \supset G$

4. 1. $(M \vee N) \supset (O \bullet P)$
2. $\sim O$
$\therefore \sim M$
3. $\sim O \vee \sim P$
4. $\sim(O \bullet P)$
5. $\sim(M \vee N)$
6. $\sim M \bullet \sim N$
7. $\sim M$

6. 1. $T \bullet (U \vee V)$
2. $T \supset [U \supset (W \bullet X)]$
3. $(T \bullet V) \supset \sim(W \vee X)$
$\therefore W \equiv X$
4. $(T \bullet U) \supset (W \bullet X)$
5. $(T \bullet V) \supset (\sim W \bullet \sim X)$
6. $[(T \bullet U) \supset (W \bullet X)] \bullet$
$[(T \bullet V) \supset (\sim W \bullet \sim X)]$
7. $(T \bullet U) \vee (T \bullet V)$
8. $(W \bullet X) \vee (\sim W \bullet \sim X)$
9. $W \equiv X$

8. 1. $A \supset B$
2. $B \supset C$
3. $C \supset A$
4. $A \supset \sim C$
$\therefore \sim A \bullet \sim C$
5. $A \supset C$
6. $(A \supset C) \bullet (C \supset A)$
7. $A \equiv C$
8. $(A \bullet C) \vee (\sim A \bullet \sim C)$
9. $\sim A \vee \sim C$
10. $\sim(A \bullet C)$
11. $\sim A \bullet \sim C$

***10.** 1. $(I \vee \sim\sim J) \bullet K$
2. $[\sim L \supset \sim(K \bullet J)] \bullet$
$[K \supset (I \supset \sim M)]$
$\therefore \sim(M \bullet \sim L)$
3. $[(K \bullet J) \supset L] \bullet$
$[K \supset (I \supset \sim M)]$
4. $[(K \bullet J) \supset L] \bullet$
$[(K \bullet I) \supset \sim M]$
5. $(I \vee J) \bullet K$
6. $K \bullet (I \vee J)$
7. $(K \bullet I) \vee (K \bullet J)$
8. $(K \bullet J) \vee (K \bullet I)$
9. $L \vee \sim M$
10. $\sim M \vee L$
11. $\sim M \vee \sim\sim L$
12. $\sim(M \bullet \sim L)$

III. For each of the following, adding just two statements to the premisses will produce a formal proof of validity. Construct a formal proof of validity for each of the following arguments.

*1. $A \supset {\sim}A$
$\therefore {\sim}A$

2. $B \bullet (C \bullet D)$
$\therefore C \bullet (D \bullet B)$

3. E
$\therefore (E \vee F) \bullet (E \vee G)$

4. $H \vee (I \bullet J)$
$\therefore H \vee I$

*5. ${\sim}K \vee (L \supset M)$
$\therefore (K \bullet L) \supset M$

6. $(N \bullet O) \supset P$
$\therefore (N \bullet O) \supset [N \bullet (O \bullet P)]$

7. $Q \supset [R \supset (S \supset T)]$
$Q \supset (Q \bullet R)$
$\therefore Q \supset (S \supset T)$

8. $U \supset {\sim}V$
V
$\therefore {\sim}U$

9. $W \supset X$
${\sim}Y \supset {\sim}X$
$\therefore W \supset Y$

*10. $Z \supset A$
${\sim}A \vee B$
$\therefore Z \supset B$

11. $C \supset {\sim}D$
${\sim}E \supset D$
$\therefore C \supset {\sim}{\sim}E$

12. $F \equiv G$
${\sim}(F \bullet G)$
$\therefore {\sim}F \bullet {\sim}G$

13. $H \supset (I \bullet J)$
$I \supset (J \supset K)$
$\therefore H \supset K$

14. $(L \supset M) \bullet (N \supset M)$
$L \vee N$
$\therefore M$

*15. $(O \vee P) \supset (Q \vee R)$
$P \vee O$
$\therefore Q \vee R$

16. $(S \bullet T) \vee (U \bullet V)$
${\sim}S \vee {\sim}T$
$\therefore U \bullet V$

17. $(W \bullet X) \supset Y$
$(X \supset Y) \supset Z$
$\therefore W \supset Z$

18. $(A \vee B) \supset (C \vee D)$
${\sim}C \bullet {\sim}D$
$\therefore {\sim}(A \vee B)$

19. $(E \bullet F) \supset (G \bullet H)$
$F \bullet E$
$\therefore G \bullet H$

*20. $I \supset [J \vee (K \vee L)]$
${\sim}[(J \vee K) \vee L]$
$\therefore {\sim}I$

21. $(M \supset N) \bullet ({\sim}O \vee P)$
$M \vee O$
$\therefore N \vee P$

22. $({\sim}Q \supset {\sim}R) \bullet ({\sim}S \supset {\sim}T)$
${\sim}{\sim}({\sim}Q \vee {\sim}S)$
$\therefore {\sim}R \vee {\sim}T$

23. ${\sim}[(U \supset V) \bullet (V \supset U)]$
$(W \equiv X) \supset (U \equiv V)$
$\therefore {\sim}(W \equiv X)$

24. $(Y \supset Z) \bullet (Z \supset Y)$
$\therefore (Y \bullet Z) \vee ({\sim}Y \bullet {\sim}Z)$

*25. $A \vee B$
$C \vee D$
$\therefore [(A \vee B) \bullet C] \vee [(A \vee B) \bullet D]$

26. $[(E \vee F) \bullet (G \vee H)] \supset (F \bullet I)$
$(G \vee H) \bullet (E \vee F)$
$\therefore F \bullet I$

27. $(J \bullet K) \supset [(L \bullet M) \vee (N \bullet O)]$
${\sim}(L \bullet M) \bullet {\sim}(N \bullet O)$
$\therefore {\sim}(J \bullet K)$

28. $(P \supset Q) \supset [(R \vee S) \bullet (T \equiv U)]$
$(R \vee S) \supset [(T \equiv U) \supset Q]$
$\therefore (P \supset Q) \supset Q$

29. $[V \bullet (W \vee X)] \supset (Y \supset Z)$
$\sim(Y \supset Z) \vee (\sim W \equiv A)$
$\therefore [V \bullet (W \vee X)] \supset (\sim W \equiv A)$

*30. $\sim[(B \supset \sim C) \bullet (\sim C \supset B)]$
$(D \bullet E) \supset (B \equiv \sim C)$
$\therefore \sim(D \bullet E)$

IV. For each of the following, adding just three statements to the premisses will produce a formal proof of validity. Construct a formal proof of validity for each of the following arguments.

*1. $\sim A \supset A$
$\therefore A$

2. $\sim B \vee (C \bullet D)$
$\therefore B \supset C$

3. $E \vee (F \bullet G)$
$\therefore E \vee G$

4. $H \bullet (I \bullet J)$
$\therefore J \bullet (I \bullet H)$

*5. $[(K \vee L) \vee M] \vee N$
$\therefore (N \vee K) \vee (L \vee M)$

6. $O \supset P$
$P \supset \sim P$
$\therefore \sim O$

7. $Q \supset (R \supset S)$
$Q \supset R$
$\therefore Q \supset S$

8. $T \supset U$
$\sim(U \vee V)$
$\therefore \sim T$

9. $W \bullet (X \vee Y)$
$\sim W \vee \sim X$
$\therefore W \bullet Y$

*10. $(Z \vee A) \vee B$
$\sim A$
$\therefore Z \vee B$

11. $(C \vee D) \supset (E \bullet F)$
$D \vee C$
$\therefore E$

12. $G \supset H$
$H \supset G$
$\therefore (G \bullet H) \vee (\sim G \bullet \sim H)$

13. $(I \supset J) \bullet (K \supset L)$
$I \vee (K \bullet M)$
$\therefore J \vee L$

14. $(N \bullet O) \supset P$
$(\sim P \supset \sim O) \supset Q$
$\therefore N \supset Q$

*15. $[R \supset (S \supset T)] \bullet [(R \bullet T) \supset U]$
$R \bullet (S \vee T)$
$\therefore T \vee U$

V. The exercises in this set represent frequently recurring patterns of inference found in longer formal proofs of validity. Familiarity with them will be useful in subsequent work. Construct a formal proof of validity for each of the following arguments.

*1. $\sim A$
$\therefore A \supset B$

2. C
$\therefore D \supset C$

3. $E \supset (F \supset G)$
$\therefore F \supset (E \supset G)$

4. $H \supset (I \bullet J)$
$\therefore H \supset I$

*5. $K \supset L$
$\therefore K \supset (L \vee M)$

6. $N \supset O$
$\therefore (N \bullet P) \supset O$

7. $(Q \vee R) \supset S$
$\therefore Q \supset S$

8. $T \supset U$
$T \supset V$
$\therefore T \supset (U \bullet V)$

9. $W \supset X$
$Y \supset X$
$\therefore (W \vee Y) \supset X$

*10. $Z \supset A$
$Z \vee A$
$\therefore A$

VI. Construct a formal proof of validity for each of the following arguments.

*1. $A \supset {\sim}B$
${\sim}(C \bullet {\sim}A)$
$\therefore C \supset {\sim}B$

2. $(D \bullet {\sim}E) \supset F$
${\sim}(E \vee F)$
$\therefore {\sim}D$

3. $(G \supset {\sim}H) \supset I$
${\sim}(G \bullet H)$
$\therefore I \vee {\sim}H$

4. $(J \vee K) \supset {\sim}L$
L
$\therefore {\sim}J$

*5 $[(M \bullet N) \bullet O] \supset P$
$Q \supset [(O \bullet M) \bullet N]$
$\therefore {\sim}Q \vee P$

6. $R \vee (S \bullet {\sim}T)$
$(R \vee S) \supset (U \vee {\sim}T)$
$\therefore T \supset U$

7. $({\sim}V \supset W) \bullet (X \supset W)$
${\sim}({\sim}X \bullet V)$
$\therefore W$

8. $[(Y \bullet Z) \supset A] \bullet [(Y \bullet B) \supset C]$
$(B \vee Z) \bullet Y$
$\therefore A \vee C$

9. ${\sim}D \supset ({\sim}E \supset {\sim}F)$
${\sim}(F \bullet {\sim}D) \supset {\sim}G$
$\therefore G \supset E$

*10. $[H \vee (I \vee J)] \supset (K \supset J)$
$L \supset [I \vee (J \vee H)]$
$\therefore (L \bullet K) \supset J$

11. $M \supset N$
$M \supset (N \supset O)$
$\therefore M \supset O$

12. $(P \supset Q) \bullet (P \vee R)$
$(R \supset S) \bullet (R \vee P)$
$\therefore Q \vee S$

13. $T \supset (U \bullet V)$
$(U \vee V) \supset W$
$\therefore T \supset W$

14. $(X \vee Y) \supset (X \bullet Y)$
${\sim}(X \vee Y)$
$\therefore {\sim}(X \bullet Y)$

*15. $(Z \supset Z) \supset (A \supset A)$
$(A \supset A) \supset (Z \supset Z)$
$\therefore A \supset A$

16. ${\sim}B \vee [(C \supset D) \bullet (E \supset D)]$
$B \bullet (C \vee E)$
$\therefore D$

17. ${\sim}F \vee {\sim}[{\sim}(G \bullet H) \bullet (G \vee H)]$
$(G \supset H) \supset [(H \supset G) \supset I]$
$\therefore F \supset (F \bullet I)$

18. $J \vee ({\sim}J \bullet K)$
$J \supset L$
$\therefore (L \bullet J) \equiv J$

19. $(M \supset N) \bullet (O \supset P)$
${\sim}N \vee {\sim}P$
${\sim}(M \bullet O) \supset Q$
$\therefore Q$

*20. $(R \vee S) \supset (T \bullet U)$
${\sim}R \supset (V \supset {\sim}V)$
${\sim}T$
$\therefore {\sim}V$

VII. Construct a formal proof of validity for each of the following arguments, in each case using the suggested notation.

*1. Either the manager didn't notice the change or else he approves of it. He noticed it all right. So he must approve of it. *(N, A)*

2. The oxygen in the tube either combined with the filament to form an oxide or else it vanished completely. The oxygen in the tube could not have vanished completely. Therefore the oxygen in the tube combined with the filament to form an oxide. *(C, V)*