9.4 Statistical Reasoning

In our day-to-day experience all of us encounter arguments that rest on statistical evidence. An especially prolific source of such arguments is the advertising industry. We are constantly told that we ought to smoke a certain brand of cigarettes because it has 20 percent less tar, buy a certain kind of car because it gets 5 percent better gas mileage, and use a certain cold remedy because it is recommended by four out of five physicians. But the advertising industry is not the only source. We often read in the newspapers that some union is asking an increase in pay because its members earn less than the average or that a certain region is threatened with floods because rainfall has been more than the average.

To evaluate such arguments, we must be able to interpret the statistics upon which they rest, but doing so is not always easy. Statements expressing averages and percentages are often ambiguous and can mean any number of things, depending on how the average or percentage is computed. These difficulties are compounded by the fact that statistics provide a highly convenient way for people to deceive one another. Such deceptions can be effective even though they fall short of being outright lies. Thus, to evaluate arguments based on statistics one must be familiar not only with the ambiguities that occur in the language but with the devices that unscrupulous individuals use to deceive others.

This section touches on five areas that are frequent sources of such ambiguity and deception: problems in sampling, the meaning of "average," the importance of dispersion in a sample, the use of graphs and pictograms, and the use of percentages for the purpose of comparison. By becoming acquainted with these topics and with some of the misuses that occur, we are better able to determine whether a conclusion follows probably from a set of statistical premises.

Samples

Much of the statistical evidence presented in support of inductively drawn conclusions is gathered from analyzing samples. When a sample is found to possess a certain characteristic, it is argued that the group as a whole (the population) possesses that characteristic. For example, if we wanted to know the opinion of the student body at a certain university about whether to adopt an academic honor code, we could take a poll of 10 percent of the students. If the results of the poll showed that 80 percent of those sampled favored the code, we might draw the conclusion that 80 percent of the entire student body favored it. Such an argument would be classified as an inductive generalization.

The problem that arises with the use of samples has to do with whether the sample is representative of the population. Samples that are not representative are said to be **biased.** Depending on what the population consists of, whether machine parts or human beings, different considerations enter into determining whether a sample is biased. These considerations include (1) whether the sample is randomly selected, (2) the size of the sample, and (3) psychological factors.

A sample is **random** if and only if every member of the population has an equal chance of being selected. The requirement that a sample be randomly selected applies to practically all samples, but sometimes it can be taken for granted. For example, when a physician draws a blood sample to test for blood sugar, there is no need to take a little bit from the finger, a little from the arm, and a little from the leg. Because blood is a circulating fluid, it can be assumed that it is homogenous in regard to blood sugar.

The randomness requirement must be given more attention when the population consists of discrete units. Suppose, for example, that a quality control engineer for a manufacturing firm needed to determine whether the components on a certain conveyor belt were within specifications. To do so, let us suppose the engineer removed every tenth component for measurement. The sample obtained by such a procedure would not be random if the components were not randomly arranged on the conveyor belt. As a result of some malfunction in the manufacturing process it is quite possible that every tenth component turned out perfect and the rest imperfect. If the engineer happened to select only the perfect ones, the sample would be biased. A selection procedure that would be more likely to ensure a random sample would be to roll a pair of dice and remove every component corresponding to a roll of ten. Since the outcome of a roll of dice is a random event, the selection would also be random. Such a procedure would be more likely to include defective components that turn up at regular intervals.

The randomness requirement presents even greater problems when the population consists of human beings. Suppose, for example, that a public opinion poll is to be conducted on the question of excessive corporate profits. It would hardly do to ask such a question randomly of the people encountered on Wall Street in New York City. Such a sample would almost certainly be biased in favor of the corporations. A less biased sample could be obtained by randomly selecting phone numbers from the telephone directory, but even this procedure would not yield a completely random sample. Among other things, the time of day in which a call is placed influences the kind of responses obtained. Most people who are employed full time are not available during the day, and even if calls are made at night, approximately 30 percent of the population have unlisted numbers.

A poll conducted by mail based on the addresses listed in the city directory would also yield a fairly random sample, but this method, too, has shortcomings. Many apartment dwellers are not listed, and others move before the directory is printed. Furthermore, none of those who live in rural areas are listed. In short, it is both difficult and expensive to conduct a large-scale public opinion poll that succeeds in obtaining responses from anything approximating a random sample of individuals.

A classic case of a poll that turned out to be biased in spite of a good deal of effort and expense was conducted by *Literary Digest* magazine to predict the outcome of the 1936 presidential election. The sample consisted of a large number of the magazine's subscribers together with a number of others selected from the telephone directory. Because four similar polls had picked the winner in previous years, the results of this poll were highly respected. As it turned out, however, the Republican candidate, Alf Landon, got a significant majority in the poll, but Franklin D. Roosevelt won the election by a landslide. The incorrect prediction is explained by the fact that the 1936 election occurred in the middle of the Depression, at a time when many people could afford neither a telephone nor a subscription to the *Digest*. These were the people who were overlooked in the poll, and they were also the ones who voted for Roosevelt.

Size is also an important factor in determining whether a sample is representative. Given that a sample is randomly selected, the larger the sample, the more closely it replicates the population. In statistics, this degree of closeness is expressed in terms of **sampling error.** The sampling error is the difference between the relative frequency with which some characteristic occurs in the sample and the relative frequency with which the same characteristic occurs in the population. If, for example, a poll were taken of a labor union and 60 percent of the members sampled expressed their intention to vote for Smith for president but in fact only 55 percent of the whole union intended to vote for Smith, the sampling error would be 5 percent. If a larger sample were taken, the error would be less.

Just how large a sample should be is a function of the size of the population and of the degree of sampling error that can be tolerated. For a sampling error of, say, 5 percent, a population of 10,000 would require a larger sample than would a population of 100. However, the ratio is not linear. The sample for the larger population need not be 100 times as large as the one for the smaller population to obtain the same precision. When the population is very large, the size of the sample needed to ensure a certain precision levels off to a constant figure. Studies based on the Gallup poll show that a random sample of 400 will yield results of plus or minus 6 percent whether the population is 100,000 or 100 million. Additional figures for large populations are given in Table 9.6:

Table 9.6 Sample Size and Sampling Error*

Numbers of interviews	Margin of error (in percentage points)
4,000	±2
1,500	±3
1,000	±4
750	±4
600	±5
400	±6
200	±8
100	±11

As the table indicates, reducing the sampling error below 5 percent requires rather substantial increases in the size of the sample. The cost of obtaining large samples may not justify an increase in precision. The table also points up the importance of randomness. The sample in the 1936 *Literary Digest* poll was based on 2 million responses, yet the sampling error was huge because the sample was not randomly selected.

Statements of sampling error are often conspicuously absent from surveys used to support advertising claims. Marketers of products such as patent medicines have been known to take a number of rather small samples until they obtain one that gives the "right" result. For example, twenty polls of twenty-five people might be taken inquiring about the preferred brand of aspirin. Even though the samples might be randomly selected, one will eventually be found in which twenty of the twenty-five respondents indicate their preference for Alpha brand aspirin. Having found such a sample, the mar-

*From Charles W. Roll Jr. and Albert H. Cantril, *Polls: Their Use and Misuse in Politics* (New York: Basic Books, 1972), p. 72.

keting firm proceeds to promote this brand as the one preferred by four out of five of those sampled. The results of the other samples are, of course, discarded, and no mention is made of sampling error.

Psychological factors can also have a bearing on whether the sample is representative. When the population consists of inanimate objects, such as cans of soup or machine parts, psychological factors are usually irrelevant, but they can play a significant role when the population consists of human beings. If the people composing the sample think that they will gain or lose something by the kind of answer they give, it is to be expected that their involvement will affect the outcome. For example, if the residents of a neighborhood were to be surveyed for annual income with the purpose of determining whether the neighborhood should be ranked among the fashionable areas in the city, it would be expected that the residents would exaggerate their answers. But if the purpose of the study were to determine whether the neighborhood could afford a special levy that would increase property taxes, one might expect the incomes to be underestimated.

The kind of question asked can also have a psychological bearing. Questions such as "How often do you brush your teeth?" and "How many books do you read in a year?" can be expected to generate responses that overestimate the truth, while "How many times have you been intoxicated?" and "How many extramarital affairs have you had?" would probably receive answers that underestimate the truth. Similar exaggerations can result from the way a question is phrased. For example, "Do you favor a reduction in welfare benefits as a response to rampant cheating?" would be expected to receive more affirmative answers than simply "Do you favor a reduction in welfare benefits?"

Another source of psychological influence is the personal interaction between the surveyor and the respondent. Suppose, for example, that a door-to-door survey were taken to determine how many people believe in God or attend church on Sunday. If the survey were conducted by priests and ministers dressed in clerical garb, one might expect a larger number of affirmative answers than if the survey were taken by nonclerics. The simple fact is that many people like to give answers that please the questioner.

To prevent this kind of interaction from affecting the outcome, scientific studies are often conducted under "double blind" conditions in which neither the surveyor nor the respondent knows what the "right" answer is. For example, in a double blind study to determine the effectiveness of a drug, bottles containing the drug would be mixed with other bottles containing a placebo (sugar tablet). The contents of each bottle would be matched with a code number on the label, and neither the person distributing the bottles nor the person recording the responses would know what the code is. Under these conditions the persons conducting the study would not be able to influence, by some smile or gesture, the response of the persons to whom the drugs are given.

Most of the statistical evidence encountered in ordinary experience contains no reference to such factors as randomness, sampling error, or the conditions under which the sample was taken. In the absence of such information, the person faced with evaluating the evidence must use his or her best judgment. If either the organization conducting the study or the persons composing the sample have something to gain by the kind of answer that is given, the results of the survey should be regarded as suspect. And if the questions that are asked concern topics that would naturally elicit distorted answers, the results should probably be rejected. In either event, the mere fact that a study *appears* scientific or is expressed in mathematical language should never intimidate a person into accepting the results. Numbers and scientific terminology are no substitute for an unbiased sample.

The Meaning of "Average"

In statistics the word "average" is used in three different senses: mean, median, and mode. In evaluating arguments and inferences that rest upon averages, it is often important to know in precisely what sense the word is being used.

The **mean** value of a set of data is the arithmetical average. It is computed by dividing the sum of the individual values by the number of data in the set. Suppose, for example, that we are given Table 9.7 listing the ages of a group of people:

Table 9.7

Number of People	Age
1	16
4	17
1	18
2	19
3	23

To compute the mean age, we divide the sum of the individual ages by the number of people:

$$\text{mean age} = \frac{(1 \times 16) + (4 \times 17) + (1 \times 18) + (2 \times 19) + (3 \times 23)}{11}$$
$$= 19$$

The **median** of a set of data is the middle point when the data are arranged in ascending order. In other words, the median is the point at which there are an equal number of data above and below. In Table 9.7 the median age is 18 because there are five people above this age and five below.

The **mode** is the value that occurs with the greatest frequency. Here the mode is 17, because there are four people with that age and fewer people with any other age.

In this example, the mean, median, and mode, while different from one another, are all fairly close together. The problem for induction occurs when there is a great disparity between these values. This sometimes occurs in the case of salaries. Consider, for example, Table 9.8, which reports the salaries of a hypothetical architectural firm:

Table 9.8

Capacity	Number of Personnel	Salary
president	1	$275,000
senior architect	2	150,000
junior architect	2	80,000
senior engineer	1	65,000 ← mean
junior engineer	4	55,000
senior draftsman	1	45,000 ← median
junior draftsman	10	30,000 ← mode

Since there are twenty-one employees and a total of \$1,365,000 is paid in salaries, the mean salary is \$1,365,000/21, or \$65,000. The median salary is \$45,000 because ten employees earn less than this and ten earn more, and the mode, which is the salary that occurs most frequently, is \$30,000. Each of these figures represents the "average" salary of the firm, but in different senses. Depending on the purpose for which the average is used, different figures might be cited as the basis for an argument.

For example, if the senior engineer were to request a raise in salary, the president could respond that his or her salary is already well above the average (in the sense of median and mode) and that therefore that person does not deserve a raise. If the junior draftsmen were to make the same request, the president could respond that they are presently earning the firm's average salary (in the sense of mode), and that for draftsmen to be earning the average salary is excellent. Finally, if someone from outside the firm were to make the allegation that the firm pays subsistence-level wages, the president could respond that the average salary of the firm is a hefty \$65,000. All of the president's responses would be true, but if the reader or listener is not sophisticated enough to distinguish the various senses of "average," he or she might be persuaded by the arguments.

In some situations, the mode is the most useful average. Suppose, for example, that you are in the market for a three-bedroom house. Suppose further that a real estate agent assures you that the houses in a certain complex have an average of three bedrooms and that therefore you will certainly want to see them. If the salesman has used "average" in the sense of mean, it is possible that half the houses in the complex are four-bedroom, the other half are two-bedroom, and there are no three-bedroom houses at all. A similar result is possible if the salesman has used average in the sense of median. The only sense of average that would be useful for your purposes is mode: If the modal average is three bedrooms, there are more three-bedroom houses than any other kind.

On other occasions a mean average is the most useful. Suppose, for example, that you have taken a job as a pilot on a plane that has nine passenger seats and a maximum carrying capacity of 1,350 pounds (in addition to yourself). Suppose further that you have arranged to fly a group of nine passengers over the Grand Canyon and that you must determine whether their combined weight is within the required limit. If a representative of the group tells you that the average weight of the passengers is 150 pounds, this by itself tells you nothing. If he means average in the sense of median, it could be the case that the four heavier passengers weigh 200 pounds and the four lighter ones weigh 145, for a combined weight of 1,530 pounds. Similarly, if the passenger representative means average in the sense of mode, it could be that two passengers weigh 150 pounds and that the others have varying weights in excess of 200 pounds, for a combined weight of over 1,700 pounds. Only if the representative means average in the sense of mean do you know that the combined weight of the passengers is 9×150 or 1,350 pounds.

Finally, sometimes a median average is the most meaningful. Suppose, for example, that you are a manufacturer of a product that appeals to an age group under thirty-five. To increase sales you decide to run an ad in a national magazine, but you want some assurance that the ad will be read by the right age group. If the advertising director of a magazine tells you that the average age of the magazine's readers is 35, you know virtually nothing. If the director means average in the sense of mean, it could be that 90 percent of the readership is over 35 and that the remaining 10 percent bring the average down to 35. Similarly, if the director means average in the sense of mode, it could be that 3 percent of the readership are exactly 35 and that the remaining 97 percent have ages ranging from 35 to 85. Only if the director means average in the sense of median do you know that half the readership is 35 or less.

Dispersion

Although averages often yield important information about a set of data, there are many cases in which merely knowing the average, in any sense of the term, tells us very little. The reason for this is that the average says nothing about how the data are distributed. For this, we need to know something about **dispersion,** which refers to how spread out the data are in regard to numerical value. Three important measures of dispersion are *range, variance,* and *standard deviation.*

Let us first consider the **range** of a set of data, which is the difference between the largest and the smallest values. For an example of the importance of this parameter, suppose that after living for many years in an intemperate climate, you decide to relocate in an area that has a more ideal mean temperature. Upon discovering that the annual mean temperature of Oklahoma City is 60°F you decide to move there, only to find that you roast in the summer and freeze in the winter. Unfortunately, you had ignored the fact that Oklahoma City has a temperature *range* of 130°, extending from a record low of −17° to a record high of 113°. In contrast, San Nicholas Island, off the coast of California, has a mean temperature of 61° but a range of only 40 degrees, extending from 47° in the winter to 87° in the summer. The temperature ranges for these two locations are approximated in Figure 1:

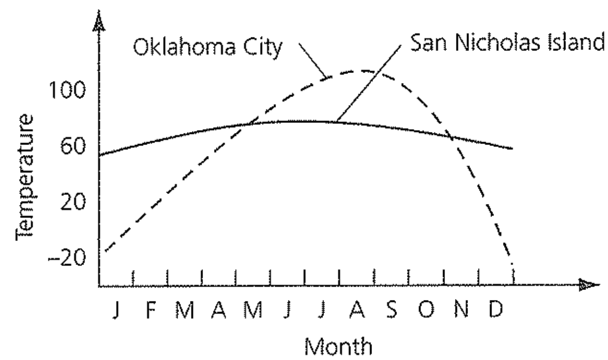


Figure 1*

Even granting the importance of the range of the data in this example, however, range really tells us relatively little because it comprehends only two data points, the maximum and minimum. It says nothing about how the other data points are distributed. For this

*This example is taken from Darrell Huff, *How to Lie with Statistics* (New York: W. W. Norton, 1954), p. 52.

we need to know the **variance,** or the **standard deviation,** which measure how every data point varies or deviates from the mean.

For an example of the importance of these two parameters in describing a set of data, suppose you have a four-year-old child and you are looking for a day-care center that will provide plenty of possible playmates about the same age as your child. After calling several centers on the phone, you narrow the search down to two: the Rumpus Center and the Bumpus Center. Both report that they regularly care for nine children, that the mean and median age of the children is four, and that the range in ages of the children is six. Unable to decide between these two centers, you decide to pay them a visit. Once having done so, it is obvious that the Rumpus Center will meet your needs better than the Bumpus Center. The reason is that the ages of the children in the two centers are distributed differently.

The ages of the children in the two centers are as follows:

Rumpus Center: 1, 3, 3, 4, 4, 4, 5, 5, 7
Bumpus Center: 1, 1, 2, 2, 4, 6, 6, 7, 7

To illustrate the differences in distribution, these ages can be plotted on a certain kind of bar graph, called a *histogram,* as shown in Figures 2a and 2b.

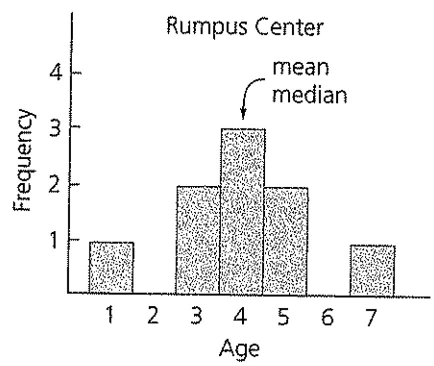


Figure 2a

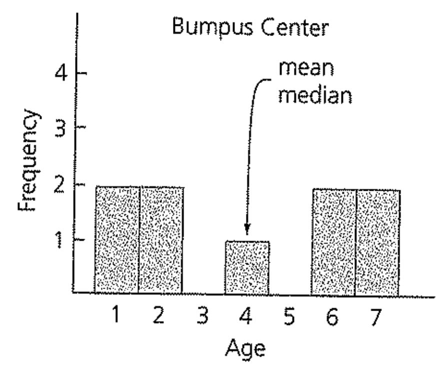


Figure 2b

Obviously the reason why the Rumpus Center comes closer to meeting your needs is that it has seven children within one year of your own child, whereas the Bumpus Center has only one such child. This difference in distribution is measured by the variance and the standard deviation. Computing the value of these parameters for the two centers is quite easy, but first we must introduce some symbols. The standard deviation is represented in statistics by the Greek letter σ (sigma), and the variance, which is the square of the standard deviation, is represented by σ^2. We compute the variance first, which is defined as follows:

$$\sigma^2 = \frac{\Sigma (x - \mu)^2}{n}$$

In this expression (which looks far more complicated than it is), Σ (upper case sigma) means the sum of, x is a variable that ranges over the ages of the children, the Greek letter μ (mu) is the mean age, (4), and n is the number of children (9). Thus, to compute the variance, we take each of the ages of the children, subtract the mean age (4) from each, square the result of each, add up the squares, and then divide the sum by the number of children (9). The first three steps of this procedure for the Rumpus Center are reported in Table 9.9:

Table 9.9

x	$(x - \mu)$	$(x - \mu)^2$
1	−3	9
3	−1	1
3	−1	1
4	0	0
4	0	0
4	0	0
5	+1	1
5	+1	1
7	+3	9
		Total = 22

First, the column for x (the children's ages) is entered, next the column for $(x - \mu)$, and last the column for $(x - \mu)^2$. After adding up the figures in the final column, we obtain the variance by dividing the sum (22) by n (9):

$$\text{Variance} = \sigma^2 = \frac{22}{9} = 2.44$$

Finally, to obtain the standard deviation, we take the square root of the variance:

$$\text{Standard deviation} = \sigma = \sqrt{2.44} = 1.56$$

Next, we can perform the same operation on the ages of the children in the Bumpus Center. The figures are expressed in Table 9.10.

Table 9.10

x	$(x - \mu)$	$(x - \mu)^2$
1	−3	9
1	−3	9
2	−2	4
2	−2	4
4	0	0
6	+2	4
6	+2	4
7	+3	9
7	+3	9
		Total = 52

Now, for the variance, we have

$$\sigma^2 = \frac{52}{9} = 5.78$$

And for the standard deviation, we have

$$\sigma = \sqrt{5.78} = 2.40$$

These figures for the variance and standard deviation reflect the difference in distribution shown in the two histograms. In the histogram for the Rumpus Center, the ages of most of the children are clumped around the mean age (4). In other words, they vary or deviate relatively slightly from the mean, and this fact is reflected in relatively small figures for the variance (2.44) and the standard deviation (1.56). On the other hand, in the histogram for the Bumpus center, the ages of most of the children vary or deviate relatively greatly from the mean, so the variance (5.78) and the standard deviation (2.40) are larger.

One of the more important kinds of distribution used in statistics is called the **normal distribution,** which expresses the distribution of random phenomena in a population. Such phenomena include (approximately) the heights of adult men or women in a city, the useful life of a certain kind of tire or light bulb, and the daily sales figures of a certain grocery store. To illustrate this concept, suppose that a certain college has 2000 female students. The heights of these students range from 57 inches to 73 inches. If we divide these heights into one-inch intervals and express them in terms of a histogram, the resulting graph would probably look like the one in Figure 3a.

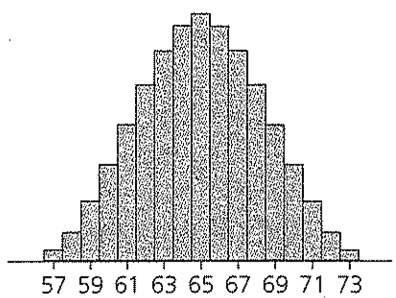


Figure 3a

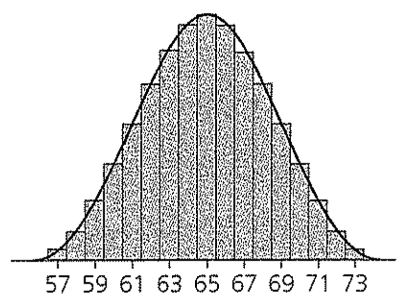


Figure 3b

This histogram has the shape of a bell. When a continuous curve is superimposed on top of this histogram, the result appears in Figure 3b. This curve is called a normal curve, and it represents a normal distribution. The heights of all of the students fit under the curve, and each vertical slice under the curve represents a certain subset of these heights. The number of heights trails off toward a zero at the extreme left and right ends of the curve, and it reaches a maximum in the center. The peak of the curve reflects the average height in the sense of mean, median, and mode.

The parameters of variance and standard deviation apply to normal distributions in basically the same way as they do for the histograms relating to the day-care centers.

Normal curves with a relatively small standard deviation tend to be relatively narrow and pointy, with most of the population clustered close to the mean, while curves with a relatively large standard deviation tend to be relatively flattened and stretched out, with most of the population distributed some distance from the mean. This idea is expressed in Figure 4. As usual, σ represents the standard deviation, and μ represents the mean.

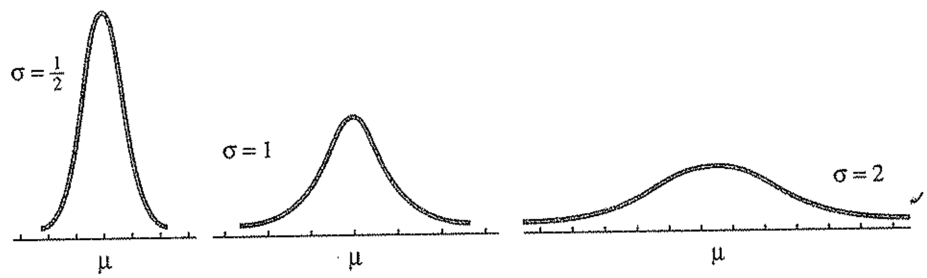


Figure 4

For a final example that illustrates the importance of dispersion, suppose that you decide to put your life savings into a business that designs and manufactures women's dresses. As corporation president you decide to save money by restricting production to dresses that fit the average woman. Because the average size in the sense of mean, median, and mode is 12, you decide to make only size 12 dresses. Unfortunately, you later discover that while size 12 is indeed the average, 95 percent of women fall outside this interval, as Figure 5 shows:

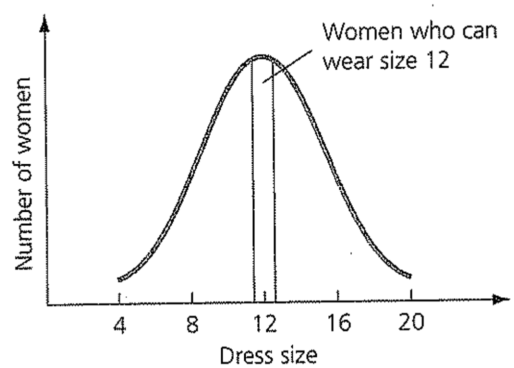


Figure 5

The problem is that you failed to take into account the standard deviation. If the standard deviation were relatively small, then most of the dress sizes would be clustered about the mean (size 12). But in fact the standard deviation is relatively large, so most of the dress sizes fall outside this interval.

Graphs and Pictograms

Graphs provide a highly convenient and informative way to represent statistical data, but they are also susceptible to misuse and misinterpretation. Here we will confine our attention to some of the typical ways in which graphs are misused.

First of all, if a graph is to represent an actual situation, it is essential that both the vertical and horizontal axes be scaled. Suppose, for example, that the profit level of a corporation is represented by a graph such as Figure 6. Such a graph is practically meaningless because it fails to show how much the profits increased over what period of time. If the curve represents a 10 percent increase over twenty years, then, of course, the picture is not very bright. Although they convey practically no information, graphs of this kind are used quite often in advertising. A manufacturer of vitamins, for example, might print such a graph on the label of the bottle to suggest that a person's energy level is supposed to increase dramatically after taking the tablets. Such ads frequently make an impression because they look scientific, and the viewer rarely bothers to check whether the axes are scaled or precisely what the curve is supposed to signify.

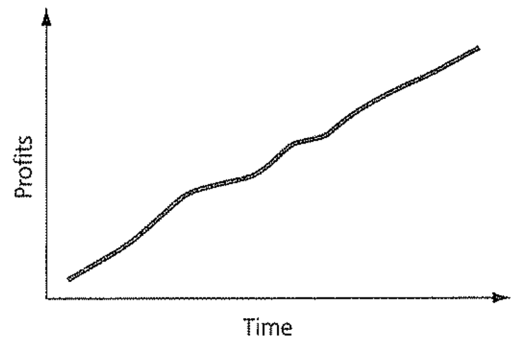


Figure 6

A graph that more appropriately represents corporate profits is given in Figure 7 (the corporation is fictitious):

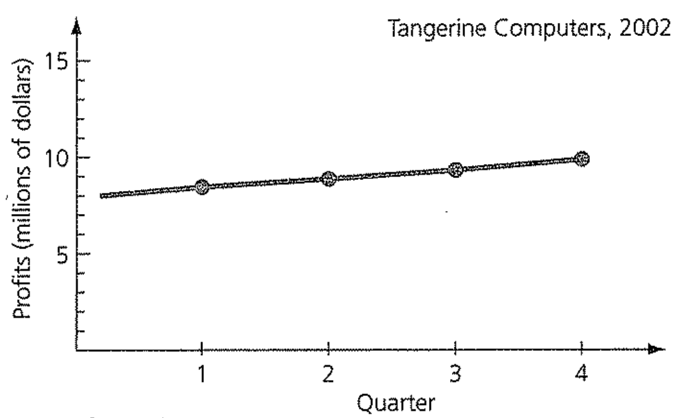


Figure 7

Inspection of the graph reveals that between January and December profits rose from $8 to $10 million, which represents a respectable 25 percent increase. This increase can be made to *appear* even more impressive by chopping off the bottom of the graph and altering the scale on the vertical axis while leaving the horizontal scale as is:

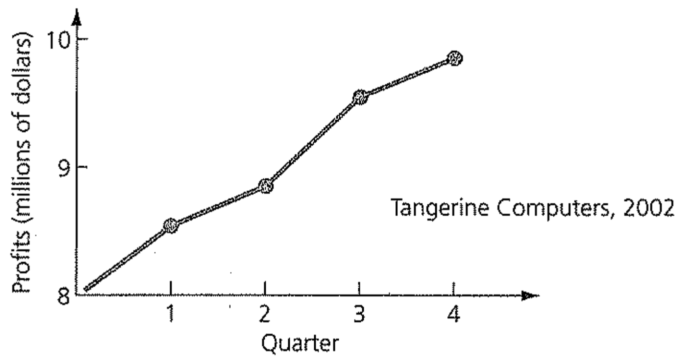


Figure 8

Again, strictly speaking, the graph accurately represents the facts, but if the viewer fails to notice what has been done to the vertical scale, he or she is liable to derive the impression that the profits have increased by something like a hundred percent or more.

The same strategy can be used with bar graphs. The graphs in Figure 9 compare sales volume for two consecutive years, but the one on the right conveys the message more dramatically:

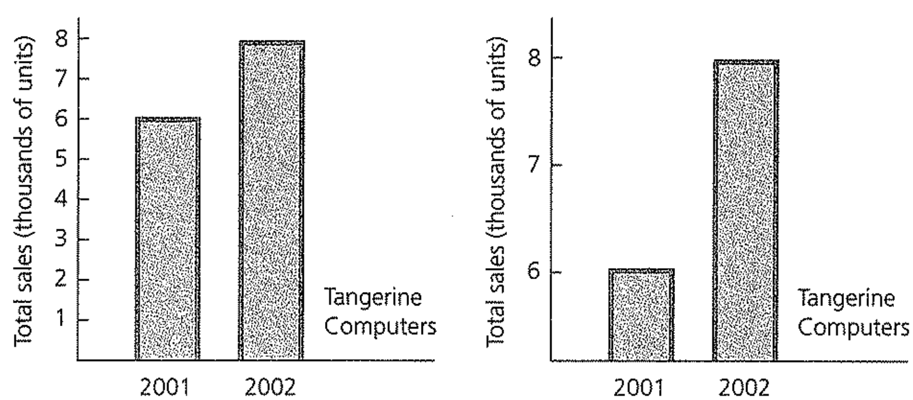


Figure 9

Of course, if the sales volume has decreased, the corporate directors would probably want to minimize the difference, in which case the design on the left is preferable.

An even greater illusion can be created with the use of pictograms. A **pictogram** is a diagram that compares two situations through drawings that differ either in size or in the number of entities depicted. Consider Figure 10, which illustrates the increase in production of an oil company between 1990 and 2000.

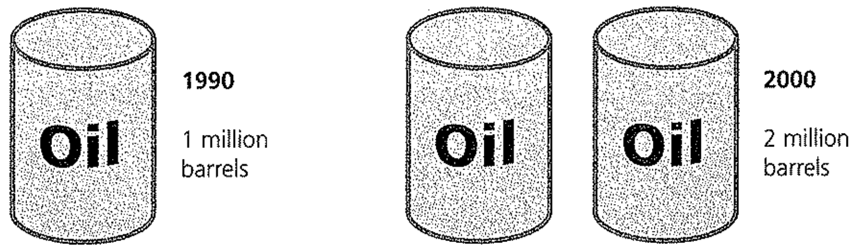


Figure 10

This pictogram accurately represents the increase because it unequivocally shows that the amount doubled between the years represented. But the effect is not especially dramatic. The increase in production can be exaggerated by representing the 1999 level with an oil barrel twice as tall:

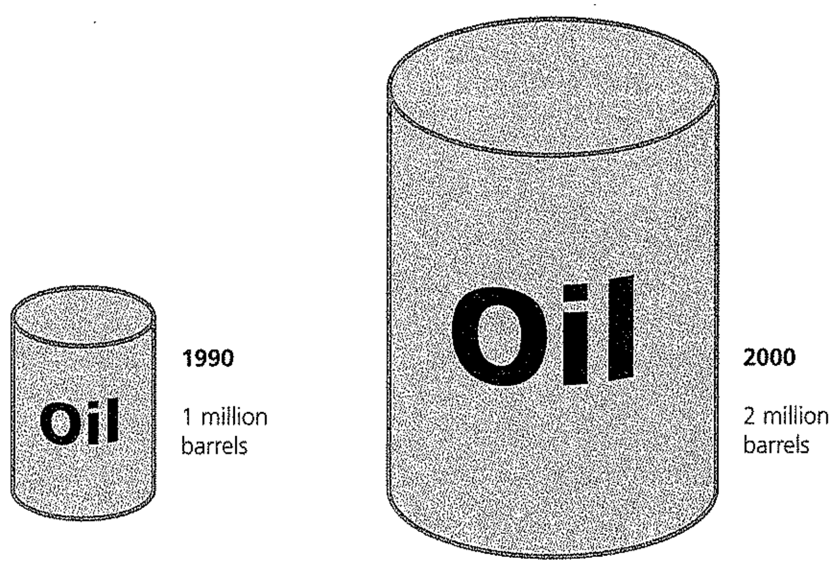


Figure 11

Even though the actual production is stated adjacent to each drawing, this pictogram creates the illusion that production has much more than doubled. While the drawing on the right is exactly twice as high as the one on the left, it is also twice as wide. Thus, it occupies four times as much room on the page. Furthermore, when the viewer's three-dimensional judgment is called into play, the barrel on the right is perceived as having eight times the volume of the one on the left. Thus, when the third dimension is taken into account, the increase in consumption is exaggerated by 600 percent.

Percentages

The use of percentages to compare two or more situations or quantities is another source of illusion in statistics. A favorite of advertisers is to make claims such as "Zesty Cola has 20 percent fewer calories" or "The price of the new Computrick computer has been reduced by 15 percent." These claims are virtually meaningless. The question is, 20 percent less than *what*, and 15 percent reduced from *what*? If the basis of the comparison or reduction is not mentioned, the claim tells us nothing. Yet such claims are often effective because they leave us with the impression that the product is in some way superior or less expensive.

Another strategy sometimes used by governments and businesses involves playing sleight-of-hand tricks with the base of the percentages. Suppose, for example, that you are a university president involved with a funding drive to increase the university's endowment. Suppose further that the endowment currently stands at $15 million and that the objective is to increase it to $20 million. To guarantee the success of the drive, you engage the services of a professional fund-raising organization. At the end of the allotted time the organization has increased the endowment to $16 million. They justify their effort by stating that, since $16 million of the $20 million has been raised, the drive was 80 percent successful (16/20 × 100%).

In fact, of course, the drive was nowhere near that successful. The objective was not to raise $20 million but only $5 million, and of that amount only $1 million has actually been raised. Thus, at best the drive was only 20 percent successful. Even this figure is probably exaggerated, though, because $1 million might have been raised without any special drive. The trick played by the fund-raising organization consisted in switching the numbers by which the percentage was to be computed.

This same trick, incidentally, was allegedly used by Joseph Stalin to justify the success of his first five-year plan.* Among other things, the original plan called for an increase in steel output from 4.2 million tons to 10.3 million. After five years the actual output rose to 5.9 million, whereupon Stalin announced that the plan was 57 percent successful (5.9/10.3 × 100%). The correct percentage, of course, is much less. The plan called for an increase of 6.1 million tons and the actual increase was only 1.7 million. Thus, at best, the plan was only 28 percent successful.

Similar devices have been used by employers on their unsuspecting employees. When business is bad, an employer may argue that salaries must be reduced by 20 percent. Later, when business improves, salaries will be raised by 20 percent, thus restoring them to their original level. Such an argument, of course, is fallacious. If a person earns $10 per hour and that person's salary is reduced by 20 percent, the adjusted salary is $8. If that figure is later increased by 20 percent, the final salary is $9.60. The problem, of course, stems from the fact that a different base is used for the two percentages. The

*Stephen K. Campbell, *Flaws and Fallacies in Statistical Thinking* (Englewood Cliffs, N.J.: Prentice-Hall, 1974) p. 8. The original reference is to Eugene Lyons's *Workers' Paradise Lost.*

fallacy committed by such arguments is a variety of equivocation. Percentages are relative terms, and they mean different things in different contexts.

A different kind of fallacy occurs when a person attempts to add percentages as if they were cardinal numbers. Suppose, for example, that a baker increases the price of a loaf of bread by 50 percent. To justify the increase the baker argues that it was necessitated by rising costs: the price of flour increased by 10 percent, the cost of labor by 20 percent, utility rates went up 10 percent, and the cost of the lease on the building increased 10 percent. This adds up to a 50 percent increase. Again, the argument is fallacious. If *everything* had increased by 20 percent, this would justify only a 20 percent increase in the price of bread. As it is, the justified increase is less than that. The fallacy committed by such arguments would probably be best classified as a case of missing the point (*ignoratio elenchi*). The arguer has failed to grasp the significance of his own premises.

Statistical variations of the suppressed evidence fallacy are also quite common. One variety consists in drawing a conclusion from a comparison of two different things or situations. For example, persons running for political office sometimes cite figures indicating that crime in the community has increased by, let us say, 20 percent during the past three or four years. What is needed, they conclude, is an all-out war on crime. But they fail to mention the fact that the population in the community has also increased by 20 percent during the same period. The number of crimes per capita, therefore, has not changed. Another example of the same fallacy is provided by the ridiculous argument that 90 percent more traffic accidents occur in clear weather than in foggy weather and that therefore it is 90 percent more dangerous to drive in clear than in foggy weather. The arguer ignores the fact that the vast percentage of vehicle miles are driven in clear weather, which accounts for the greater number of accidents.

A similar misuse of percentages is committed by businesses and corporations that, for whatever reason, want to make it appear that they have earned less profit than they actually have. The technique consists of expressing profit as a percentage of sales volume instead of as a percentage of investment. For example, during a certain year a corporation might have a total sales volume of $100 million, a total investment of $10 million, and a profit of $5 million. If profits are expressed as a percentage of investment, they amount to a hefty 50 percent; but as a percentage of sales they are only 5 percent. To appreciate the fallacy in this procedure, consider the case of the jewelry merchant who buys one piece of jewelry each morning for $9 and sells it in the evening for $10. At the end of the year the total sales volume is $3650, the total investment $9, and the total profit $365. Profits as a percentage of sales amount to only 10 percent, but as a percentage of investment they exceed 4000 percent.

EXERCISE 9.4

I. Criticize the following arguments in light of the material presented in this section:

★1. To test the algae content in a lake, a biologist took a sample of the water at one end. The algae in the sample registered 5 micrograms per liter. Therefore, the algae in the lake at that time registered 5 micrograms per liter.

2. To estimate public support for a new municipality-funded convention center, researchers surveyed 100 homeowners in one of the city's fashionable neighborhoods. They found that 89 percent of those sampled were enthusiastic about the project. Therefore, we may conclude that 89 percent of the city's residents favor the convention center.

3. A quality-control inspector for a food-processing firm needed assurance that the cans of fruit in a production run were filled to capacity. He opened every tenth box in the warehouse and removed the can in the left front corner of each box. He found that all of these cans were filled to capacity. Therefore, it is probable that all of the cans in the production run were filled to capacity.

★4. When a random sample of 600 voters was taken on the eve of the presidential election, it was found that 51 percent of those sampled intended to vote for the Democrat and 49 percent for the Republican. Therefore, the Democrat will probably win.

5. To determine the public's attitude toward TV soap operas, 1,000 people were contacted by telephone between 8 A.M. and 5 P.M. on week days. The numbers were selected randomly from the phone directories of cities across the nation. The researchers reported that 43 percent of the respondents said that they were avid viewers. From this we may conclude that 43 percent of the public watches TV soap operas.

6. To predict the results of a U.S. Senate race in New York State, two polls were taken. One was based on a random sample of 750 voters, the other on a random sample of 1,500 voters. Since the second sample was twice as large as the first, the results of the second poll were twice as accurate as the first.

★7. In a survey conducted by the manufacturers of Ultrasheen toothpaste, 65 percent of the dentists randomly sampled preferred that brand over all others. Clearly Ultrasheen is the brand preferred by most dentists.

8. To determine the percentage of adult Americans who have never read the U.S. Constitution, surveyors put this question to a random sample of 1,500 adults. Only 13 percent gave negative answers. Therefore, since the sampling error for such a sample is 3 percent, we may conclude that no more than 16 percent of American adults have not read the Constitution.

9. To determine the percentage of patients who follow the advice of their personal physician, researchers asked 200 randomly chosen physicians to put the question to their patients. Of the 4,000 patients surveyed, 98 percent replied that they did indeed follow their doctor's advice. We may therefore conclude that at least 95 percent of the patients across the nation follow the advice of their personal physician.

★10. Janet Ryan can afford to pay no more than $15 for a birthday gift for her eight-year-old daughter. Since the average price of a toy at General Toy Company is $15, Janet can expect to find an excellent selection of toys within her price range at that store.

11. Anthony Valardi, who owns a fish market, pays $2 per pound to fishermen for silver salmon. A certain fisherman certifies that the average size of the salmon

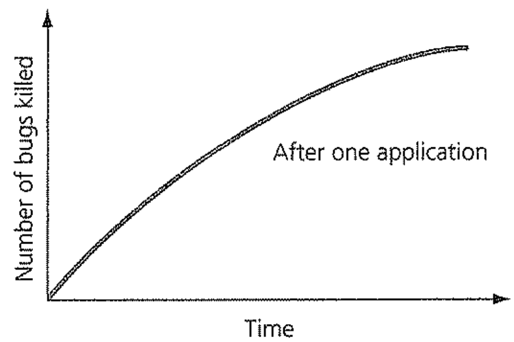
in his catch of the day is 10 pounds, and that the catch numbers 100 salmon. Mr. Valardi is therefore justified in paying the fisherman $2,000 for the whole catch.

12. Pamela intends to go shopping for a new pair of shoes. She wears size 8. Since the average size of the shoes carried by the Bon Marche is size 8, Pamela can expect to find an excellent selection of shoes in her size at that store.

★13. Tim Cassidy, who works for a construction company, is told to load a pile of rocks onto a truck. The rocks are randomly sized, and the average piece weighs 50 pounds. Thus, Tim should have no trouble loading the rocks by hand.

14. The average IQ (in the sense of mean, median, and mode) of the students in Dr. Jacob's symbolic logic class is 120. Thus, none of the students should have any trouble mastering the subject matter.

15. An insecticide manufacturer prints the following graph on the side of its spray cans:



Obviously, the insecticide is highly effective at killing bugs, and it keeps working for a long time.

★16. A corporation's sales for two consecutive years are represented in a bar graph. Since the bar for the later year is twice as high as the one for the previous year, it follows that sales for the later year were double those for the previous year.

17. Forced to make cutbacks, the president of a manufacturing firm reduced certain costs as follows: advertising by 4 percent, transportation by 5 percent, materials by 2 percent, and employee benefits by 3 percent. The president thus succeeded in reducing total costs by 14 percent.

18. During a certain year, a grocery store chain had total sales of $100 million and profits of $10 million. The profits thus amounted to a modest 10 percent for that year.

★19. There were 40 percent more traffic accidents in 1990 than there were in 1960. Therefore, it was 40 percent more dangerous to drive a car in 1990 than it was in 1960.

20. An efficiency expert was hired to increase the productivity of a manufacturing firm and was given three months to accomplish the task. At the end of the

period the productivity had increased from 1,500 units per week to 1,700. Since the goal was 2,000 units per week, the effort of the efficiency expert was 85 percent successful (1,700/2,000).

II. Compute the answers to the following questions.

★1. Given the following group of people together with their weights, what is the average weight in the sense of mean, median, and mode?

Number of People	Weight
2	150
4	160
3	170
1	180
1	190
1	200
1	220
2	230

2. Given the following group of people together with their salaries, what is the average salary in the sense of mean, median, and mode?

Number of People	Salary
1	$95,000
2	85,000
1	70,000
3	40,000
1	30,000
2	20,000
5	15,000

3. A small company has five employees who missed work during a certain month. The number of days missed were: 1, 1, 2, 4, 7. What is the mean number of days missed? What is the variance and standard deviation of this set of data?

★4. A day-care center cares for 10 children. Their ages are 1, 1, 2, 2, 2, 3, 3, 4, 6, 6. Construct a histogram that represents the distribution of ages. What is the mean age? What is the variance and standard deviation of these ages?

5. An instructor gave a ten-question multiple choice quiz to twelve students. The scores were 10, 10, 9, 9, 8, 8, 8, 7, 7, 7, 7, 6. What is the mean score? What is the variance and standard deviation of these scores?

III. Answer "true" or "false" to the following statements:

★1. If a sample is very large, it need not be randomly selected.

2. If a population is randomly arranged, a sample obtained by selecting every tenth member would be a random sample.

3. If a sample is randomly selected, the larger the sample is, the more closely it replicates the population.

★4. To ensure the same precision, a population of 1 million would require a much larger random sample than would a population of 100,000.

5. In general, if sample A is twice as large as sample B, then the sampling error for A is one-half that for B.

6. When a sample consists of human beings, the purpose for which the sample is taken can affect the outcome.

★7. The personal interaction between a surveyor and a respondent can affect the outcome of a survey.

8. The mean value of a set of data is the value that occurs with the greatest frequency.

9. The median value of a set of data is the middle point when the data are arranged in ascending order.

★10. The modal value of a set of data is the arithmetical average.

11. If one needed to know whether a sizable portion of a group were above or below a certain level, the most useful sense of average would be mode.

12. Data reflecting the results of a random sample conform fairly closely to the normal probability distribution.

★13. If a set of data conform to the normal probability distribution, then the mean, median, and mode have the same value.

14. The range, variance, and standard deviation are measurements of dispersion.

15. Statements about averages often present an incomplete picture without information about the dispersion.

★16. Data reflecting the size of full-grown horses would exhibit greater dispersion than data reflecting the size of full-grown dogs.

17. The visual impression made by graphs can be exaggerated by changing one of the scales while leaving the other unchanged.

18. Data reflecting a 100 percent increase in housing construction could be accurately represented by a pictogram of two houses, one twice as high as the other.

★19. If a certain quantity is increased by 10 percent and later decreased by 10 percent, the quantity is restored to what it was originally.

20. Expressing profits as a percentage of sales volume presents an honest picture of the earnings of a corporation.

9.5 Hypothetical/Scientific Reasoning

Hypothetical reasoning is most immediately applied to the production of explanations. You will recall from Chapter 1 of this book that an explanation is a kind of expression