# ECE 549 Homework 2

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#### 1 A featrue tracker

## **Keypoint Selection** 1.1

```
Algorithm 1 Get keypoints using harris detector
```

**Input:** Input image, threshold for selecting keypoints  $\tau$ .

Output: x and y coordinates of the keypoints

Steps:

- 1.Smooth the image with gaussian filter before taking the derivative.
- 2. Computet the x and y gradient value of the smoothed image  $I_x, I_y$ .
- 3.Calculate the second moment matrix

$$\begin{bmatrix} I_x I_x & I_x I_y \\ I_x I_y & I_y I_y \end{bmatrix}$$

4. Further gaussian filtering of the image.

$$\begin{bmatrix} g(I_x I_x) & g(I_x I_y) \\ g(I_x I_y) & g(I_y I_y) \end{bmatrix}$$

5. Construct the cornerness function 
$$har = \det \begin{vmatrix} g(I_xI_x) & g(I_xI_y) \\ g(I_xI_y) & g(I_yI_y) \end{vmatrix} - \alpha[g(I_xI_x) + g(I_yI_y)]^2$$
6. Thresholding using harris criteria for selecting the key points

 $har > \tau$ 

7. Non-maxima suppression of 5 by 5 window centered at each key point.

Here  $\alpha$  is chosen to be 0.05 and threshold  $\tau$  is  $4 \times 10^{-8}$ . The Gaussian filter that had been used to smooth the image is chosen to be the size of 7 by 7 with  $\sigma = 2$ .



Figure 1: Keypoints detected by the harris detector

# 1.2 Tracking

```
Algorithm 2 Kanade-Lucas-Tomasi tracking procedure
Input: x and y coordinates of the keypoints.
Output: path of the key points over the 50 frames
   for frame t and t+1 from 1 to 50
      for each key point x_i, y_i from 1 to N
         Consider a interpolated 15 by 15 window centered at each keypoint
         if the window has points outside the image
            stop tracking window
         else
            calculate the second moment matrix of the window
            While the refinement is not converged
               1. Calculate the temporal gradient I_t by taking the difference between the windows of the adjacent
   frames.
               2. Solve displacement u, v by
               \begin{bmatrix} \sum_{W} I_x I_x & \sum_{W} I_x I_y \\ \sum_{W} I_x I_y & \sum_{W} I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} \sum_{W} I_x I_t \\ \sum_{W} I_y I_t \end{bmatrix}
3. Updated predicted (x'_{i+1}, y'_{i+1}) = (x'_i, y'_i) + (u, v)
            end
         end
      \quad \text{end} \quad
   end
```

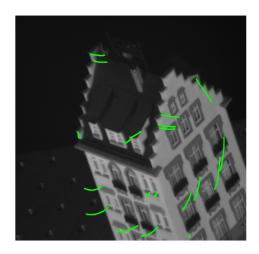


Figure 2: 20 Key points path over 50 frames



Figure 3: Points moved out of the frame during the tracking

# 2 Shape alignment

Summary of the algorithm with pseudocode

# Algorithm 3 ICP Algorithm with affine transformation

Input: The original image and the image that needs to be aligned.

Output: Alignment result.

## Steps:

1. Initialization: Calculate the mean, standard deviation of the boundary pixels in image 1 (Set1) and image 2 (Set2)

2. Align two sets of pixels by doing  $x_i' = \frac{x_i - \mu_{x_1}}{\sigma_{x_1}} \sigma_{x_2} + \mu_{x_2}$ 

$$x_i' = \frac{x_i - \mu_{x_1}}{\sigma_{x_1}} \sigma_{x_2} + \mu_{x_2}$$

Same for y coordinates.

While the refinement is not converged > threshold (0.0000001)

- 3. Assign each point in the Set1 to its nearest neighbor in Set2
- 4. Set up the affine transformation model and calculate the parameters
- 5. Solve the affine transformation matrix by least squares method.
- 6. Transform the points in Set1 using the affine transformation matrix
- 7. Calculate the difference between the transformed points and the points that were transformed.

## end

This algorithm uses Iterative Closest Points Algorithm to iteratively estimate the transform between two sets of points in order to achieve the shape alignment task. The transformation model used is the affine transform which combines the translation, scaling, rotation and shearing that give 6 degree of freedom so it is a suitable transformation model in this question. The affine matrix is solved by least squares method.

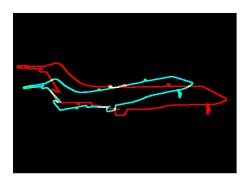


Figure 4: align object2 to object2t

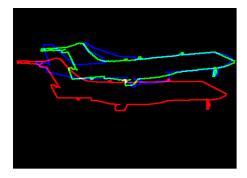


Figure 5: align object 2 to object 1

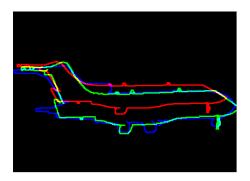


Figure 6: align object2 to object3

Image	Error	Runtime
object2t	0.26	0.58s
object1	5.39	1.60s
object3	4.52	1.61s

## 3 Object instance recogition

### 3.1 Keypoint matching

## Algorithm 4 key point matching

**Input:** A key point descriptor g from one image, a set of keypoint descriptors  $f_1 ldots f_n$  from a second image. Output: The keypoint matches g.

Steps:

for  $f_i = f_1 : f_n$ 

Calculate the euclidean distance between g and each individual descriptor  $f_i ||g - f_i||$ 

end

Find two nearest neighbors  $f_m$  and  $f_n$  of g with distances  $d_m$  and  $d_n$  $f_m$  matches g if  $\frac{d_m}{d_n} < 0.7$  where 0.7 is the optimized threshold.

#### 3.2 Object alignment

Known:  $(x_1, y_1, w_1, h_1), (u_1, v_1, s_1, \theta_1), (u_2, v_2, s_2, \theta_2).$ 

We wish to transform  $(x_1, y_1)$  to  $(x_2, y_2)$  and find  $w_2, h_2, o_2$ .

In order to determine  $(x_2, y_2)$  and we already know keypoints pairs  $(u_1, v_1)$  and  $(u_2, v_2)$  they are matched. We start with  $(x_1, y_1)$  and setup the transformation between them by first subtracting bounding box center coordinates  $\begin{bmatrix} u_1 - x_1 \\ v_1 - y_1 \end{bmatrix}$ , then we multiply rotation matrix R  $\begin{bmatrix} cos(o_2) & -sin(o_2) \\ sin(o_2) & cos(o_2) \end{bmatrix}$  and scaling matrix  $\begin{bmatrix} \frac{s_2}{s_1} \\ 0 \end{bmatrix}$ 

Now we need to do subtraction from (u, v) to center and we put all together we get the center of the bounding box in image 2

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} u_2 \\ v_2 \end{bmatrix} + \begin{bmatrix} \frac{s_2}{s_1} & 0 \\ 0 & \frac{s_2}{s_1} \end{bmatrix} \begin{bmatrix} cos(o_2) & -sin(o_2) \\ sin(o_2) & cos(o_2) \end{bmatrix} \begin{bmatrix} x_1 - u_1 \\ y_1 - v_1 \end{bmatrix}$$
Notice that the rotation angle  $o_2 = \theta_2 - \theta_1$ .

 $w_2 = s_2 \frac{w_1}{s_1}, h_2 = s_2 \frac{h_1}{s_1}$  by scaling.