1) $m(a+bX) = a + b\cdot m(X)$ $\qquad m(a+bx) = \frac{1}{N}\sum_{i=1}^{N}(a+bx_i)$

$$m(a+bX) = \cancel{\frac{1}{N}\sum_{i=1}^{n}} a + \frac{1}{N}\sum_{i=1}^{N} b x_i$$

$$a + b\,\frac{1}{N}\sum_{i=1}^{N} x_i$$

$$\boxed{m(a+bx) = a + b\cdot m(X)}$$

2) $cov(X, a+bY) = b\cdot cov(X,Y)$

$$cov(X, a+bY) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))((a+by_i) - m(a+bY))$$

a's cancel

$$\cancel{a} + b\cdot m(Y)$$

$$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(b(y_i - m(Y)))$$

$$= b\times\frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(y_i - m(Y))$$

$$cov(X, a+bY) = b\times cov(X,Y) \checkmark$$

3) $cov(a+bX, a+bX) = b^2\, cov(X,X)$

$$\frac{1}{N}\sum_{i=1}^{N}((a+bx_i) - m(a+bX))((a+bx_i) - m(a+bX))$$

simplify → $\cancel{a}+bx_i - \cancel{a} + bm(X)$ $\qquad \cancel{a}+bx_i - \cancel{a} - bm(X)$

$$\frac{1}{N}\sum_{i=1}^{N} b(x_i - m(X))\quad b(x_i - m(Y))$$

$cov(X,X) = s^2$ because $\frac{1}{N}\sum_{i=1}^{N}[x_i - m(x)]^2$

$$\frac{1}{N}\sum_{i=1}^{N} b^2(x_i - m(X))(x_i - m(X))$$

$$= b^2\, cov(X,X) \checkmark \qquad\qquad = s^2 \checkmark$$

sample variance = cov(X,X)

4) median → half the data is below, half above (50%)

$F(m(X)) \geq 0.5$

$g(x) = 2 + 5X$ → because $g(x)$ preserves the order of values, then apply g to all x values does not change the 50% split

$m(X)$ = median of X, then $g(m(X))$ is the median of $g(X)$

Yes, a non-decreasing transformation of the median is always equal to the median of the transformed variable.

This applies to any quintile, but not the IQR or range because these do not preserve the order of the values (subtraction happens)

5) $m(g(X)) \overset{?}{=} g(m(X))$ is this always true? (non-decreasing)

$g(x) = 2 + 5X$ - for linear transformations and smooth monotonic functions it holds that $m(g(X)) = g(m(X))$

$m(2+5X) = 2 + 5(m(X))$ ✓