Modern Difference-in-Difference Designs

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  - pre-tr outcome affected by (obs/unobs) predictors of post-tr outcome

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Before-After Designs:

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This is **not**

$$ATT = \overline{[Y(1)|T=1]} - \overline{[Y(0)|T=1]}$$

Why not?

Canonical Difference-in-Difference

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&= (\underbrace{\overline{Y}_{NJ}^{\text{After}} - \overline{Y}_{NJ}^{\text{Before}}}_{\text{Change in Treated}}) - (\underbrace{\overline{Y}_{PA}^{\text{After}} - \overline{Y}_{PA}^{\text{Before}}}_{\text{Change in Control}})
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- Change in Control: removes part of change in emp that would have occurred anyway, in absence of law change.
- (Economy growing, or fast food restaurants hit by health scares and lay off workers, ...)

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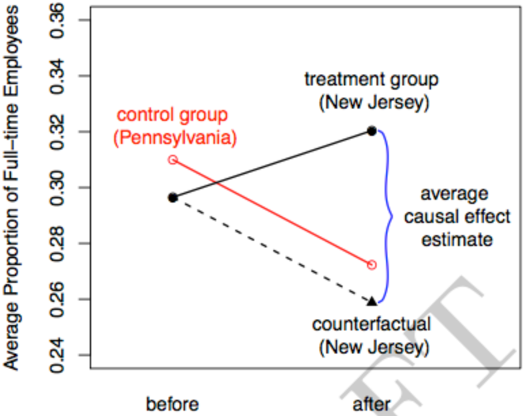
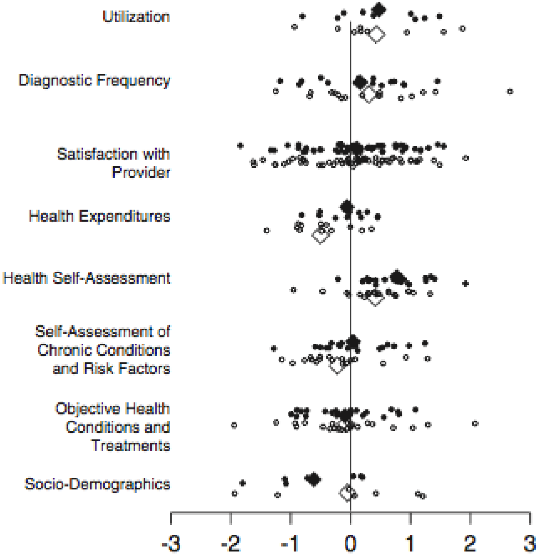
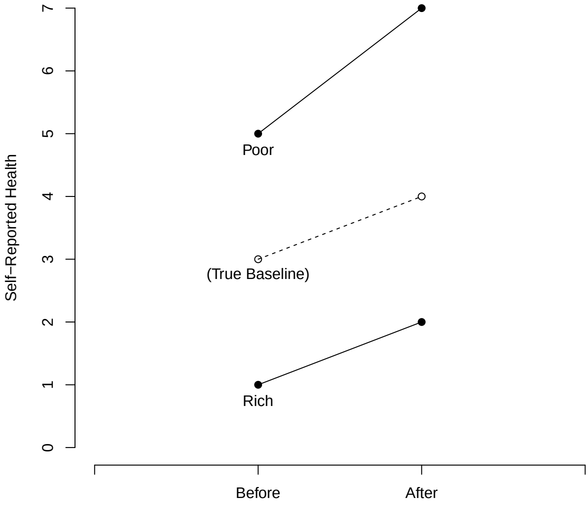


Self-Reported Health in *Seguro Popular* Evaluation

King, et al. (2007)



Simplified Example



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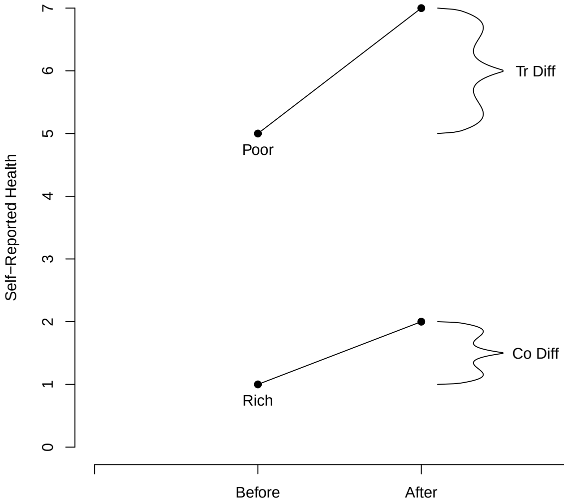
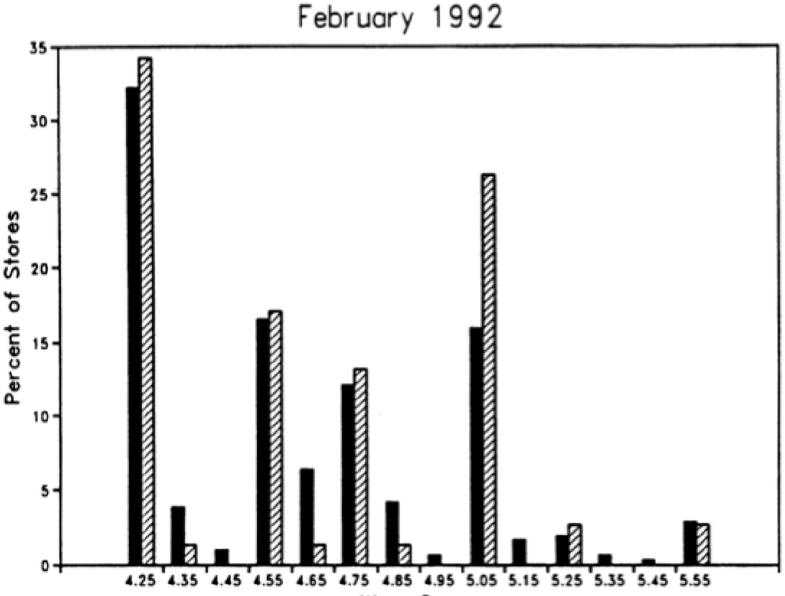


The Difference-in-Differences Estimator

Units	Outcome Before	Outcome After	Difference
Treated	$\bar{Y}^{T_B}$	$\bar{Y}^{T_A}$	$\bar{Y}^{T_A} - \bar{Y}^{T_B}$
Control	$\bar{Y}^{C_B}$	$\bar{Y}^{C_A}$	$\bar{Y}^{C_A} - \bar{Y}^{C_B}$
	$\bar{Y}^{T_B} - \bar{Y}^{C_B}$	$\bar{Y}^{T_A} - \bar{Y}^{C_A}$	$(\bar{Y}^{T_A} - \bar{Y}^{T_B}) - (\bar{Y}^{C_A} - \bar{Y}^{C_B})$

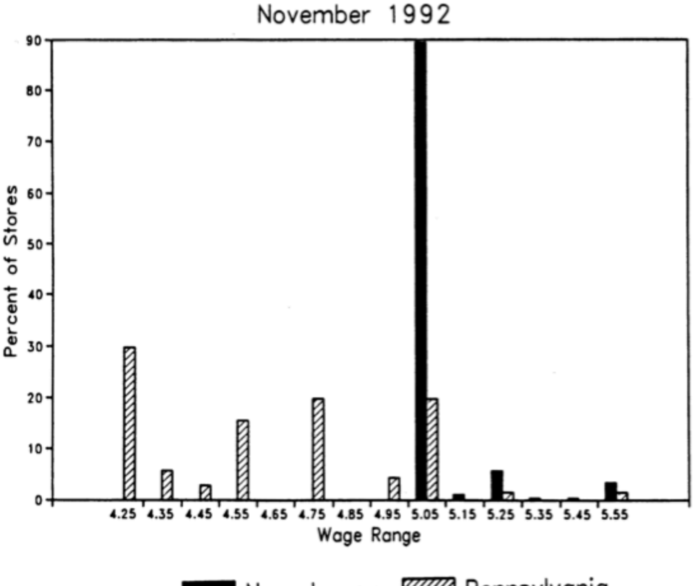
Example: Effect of Minimum Wage on Employment

The Baseline



Example: Effect of Minimum Wage on Employment

The Policy Change: Significant Shock



Example: Effect of Minimum Wage on Employment

Variable	PA (i)	NJ (ii)	Difference, NJ − PA (iii)
1. FTE employment before, all available observations	23.33 (1.35)	20.44 (0.51)	−2.89 (1.44)
2. FTE employment after, all available observations	21.17 (0.94)	21.03 (0.52)	−0.14 (1.07)
3. Change in mean FTE employment	−2.16 (1.25)	0.59 (0.54)	2.76 (1.36)

Extension 1: Regression Difference-in-Differences

Let

- $T_i \in \{0, 1\}$ be treatment status ("ever-treated")
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Model

$$y_i = \beta_0 + \beta_1 T_i + \beta_2 A_i + \beta_3 T_i A_i + \epsilon_i$$

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$$y_i = \beta_0 + \beta_1 T_i + \beta_2 A_i + \beta_3 T_i A_i + \epsilon_i$$

Then

$$\begin{aligned}
\bar{Y}^{T_A} &= \beta_0 + \beta_1 + \beta_2 + \beta_3 \\
\bar{Y}^{T_B} &= \beta_0 + \beta_1 \\
\bar{Y}^{C_A} &= \beta_0 + \beta_2 \\
\bar{Y}^{C_B} &= \beta_0
\end{aligned}$$

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\bar{Y}^{C_A} &= \beta_0 + \beta_2 \\
\bar{Y}^{C_B} &= \beta_0
\end{aligned}$$

And

$$\begin{aligned}
(\bar{Y}^{T_A} - \bar{Y}^{T_B}) - (\bar{Y}^{C_A} - \bar{Y}^{C_B}) &= ([\beta_0 + \beta_1 + \beta_2 + \beta_3] - [\beta_0 + \beta_1]) \\
&\quad -([\beta_0 + \beta_2] - \beta_0) \\
&= (\beta_2 + \beta_3) - (\beta_2) \\
&= \beta_3
\end{aligned}$$

Extension 1: Regression Difference-in-Differences

Note: $T_i \in \{0, 1\}$ needs to be "ever-treated" (not "currently treated")

Simulation:

- 40 units (20 Tr, 20 Co)
- 2 time periods (2018, 2022)
- Baseline outcome difference $\text{Tr} - \text{Co} = 1$
- True $\text{TE} = 2$
- (TE occurs in 2022, only for Tr)

Extension 1: Regression Difference-in-Differences

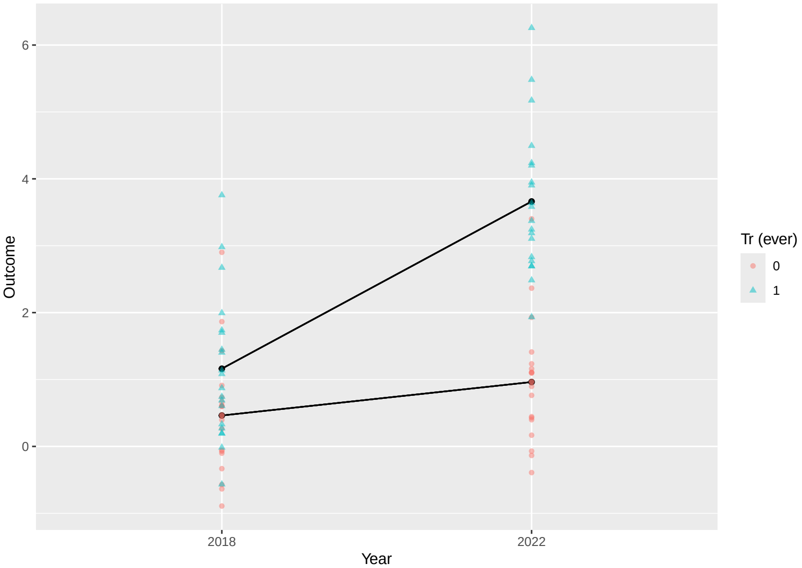

Figure 2: Simulated Data DiD

Extension 1: Regression Difference-in-Differences

Two possible regressions:

$$Y = \beta_0 + \beta_1(2022) + \beta_2(\text{TrEver}) + \beta_3(2022 \times \text{TrEver})$$

Extension 1: Regression Difference-in-Differences

Two possible regressions:

$$Y = \beta_0 + \beta_1(2022) + \beta_2(\text{TrEver}) + \beta_3(2022 \times \text{TrEver})$$

versus

$$Y = \beta_0 + \beta_1(2022) + \beta_2(\text{TrNow}) + \beta_3(2022 \times \text{TrNow})$$

Extension 1: Regression Difference-in-Differences

Table 1: Ever-treated vs. Currently-treated DiD Regressions

	Outcome	
	y	
	(1)	(2)
2022	0.500	0.150
	(0.313)	(0.278)
Tr (Ever)	0.700**	
	(0.313)	
2022 x Tr (Ever)	2.000***	
	(0.442)	
Tr (Now)		2.700***
		(0.321)
2022 x Tr (Now)		
(Intercept)	0.462**	0.812***
	(0.221)	(0.160)
Observations	80	80
R^2	0.623	0.598
Adjusted R^2	0.608	0.588
Residual Std. Error	0.989 (df = 76)	1.015 (df = 77)
F Statistic	41.827*** (df = 3; 76)	57.261*** (df = 2; 77)
Note:		*p<0.1; **p<0.05; ***p<0.01

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$$Y = \underbrace{0.46}_{\text{Co Mean 2018}} + \underbrace{0.5}_{\text{Time Trend}}(2022) + \underbrace{0.7}_{\text{Baseline Diff}}(\text{TrEver}) + \underbrace{2}_{\widehat{ATT}}(2022 \times \text{TrEver})$$

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versus

$$Y = \underbrace{0.81}_{\text{Part of Co Mean 2018 + Time Trend??}} + \underbrace{0.15}_{\text{Rest of Time Trend??}}(2022) + \underbrace{2.7}_{\text{Baseline Diff }+\widehat{ATT}\text{??}}(\text{TrNow}) + \underbrace{NA}_{\text{Unest Interaction for }\widehat{ATT}\text{??}}(2022 \times \text{TrNow})$$

Extension 2: Regression Difference-in-Differences

Related: Use change scores or regressor variable method?
(Allison 90)

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$$y_{i2} = \alpha + \beta_1 y_{i1} + \beta_2 T_i + \epsilon_i \quad (1)$$

Extension 2: Regression Difference-in-Differences

Related: Use change scores or regressor variable method? (Allison 90)

$$y_{i2} = \alpha + \beta_1 y_{i1} + \beta_2 T_i + \epsilon_i \tag{1}$$

$$\begin{aligned} y_{i1} &= \alpha + \gamma G_i + \epsilon_{i1} && (2) \\ y_{i2} &= \alpha + \underbrace{\tau}_{\text{secular trend}} + \gamma G_i + \delta T_i + \epsilon_{i2} && (3) \\ y_{i2} - y_{i1} &= \tau + \delta T_i + (\epsilon_{i2} - \epsilon_{i1}) && (4) \end{aligned}$$

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\end{aligned}
$$

- Change score is a DiD estimate
- Worse if pretest not very strong, and restriction of $\beta_1 = 1$ creates bias.
- Change score is more restrictive on parameter, but makes different assumption about $Cor(y_1, \epsilon)$.

Extension 3: Regression Difference-in-Differences

Add other covariates:

$$y_i = \beta_0 + \beta_1 T_i + \beta_2 A_i + \beta_3 T_i A_i + \gamma' \mathbf{x_i} + \ _{\mathbf{i}}$$

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But, careful about inference from SE's with many time periods (Bertrand, Duflo, and Mullainathan (2004)).

(At $\alpha = 0.05$, false findings actually 0.45!)

Actually, be careful about bias with many time periods, too!

(See here.)

Extension 4: Difference in (Differences-in-Differences)

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- Useful for estimating effect across two subgroups
- (E.g., Empl vs. unempl'd; or placebo robustness check if one subgroup should *not* be affected by treatment)

Extension 4: Difference in (Differences-in-Differences)

Group 1:

Units	Outcome Before	Outcome After	Difference
Treated	T_{1B}	T_{1A}	$T_{1A} - T_{1B}$
Control	C_{1B}	C_{1A}	$C_{1A} - C_{1B}$
	$T_{1B} - C_{1B}$	$T_{1A} - C_{1A}$	$(T_{1A} - T_{1B}) - (C_{1A} - C_{1B})$

Group 2:

Units	Outcome Before	Outcome After	Difference
Treated	T_{2B}	T_{2A}	$T_{2A} - T_{2B}$
Control	C_{2B}	C_{2A}	$C_{2A} - C_{2B}$
	$T_{2B} - C_{2B}$	$T_{2A} - C_{2A}$	$(T_{2A} - T_{2B}) - (C_{2A} - C_{2B})$

DiDiD Estimate:

$$[(T_{2A} - T_{2B}) - (C_{2A} - C_{2B})] - [(T_{1A} - T_{1B}) - (C_{1A} - C_{1B})]$$

Extension 5: Multiple Time Periods, Staggered Designs

- Callaway and Sant'Anna (2021) on multiple time periods

Extension 5: Multiple Time Periods, Staggered Designs

- Callaway and Sant'Anna (2021) on multiple time periods
- Goodman-Bacon (2021) on staggered treatment assignment

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- Callaway and Sant'Anna (2021) on multiple time periods
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- Package `did`

The Stepped Wedge Design

Calloway-Sant'anna Approach: Multiple Time Periods

With *many* time periods, challenging to defining **the** estimand (like ATT).

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Central disaggregated component:

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- $ATT(g, t)$

Calloway-Sant'anna Approach: Multiple Time Periods

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Many ways!

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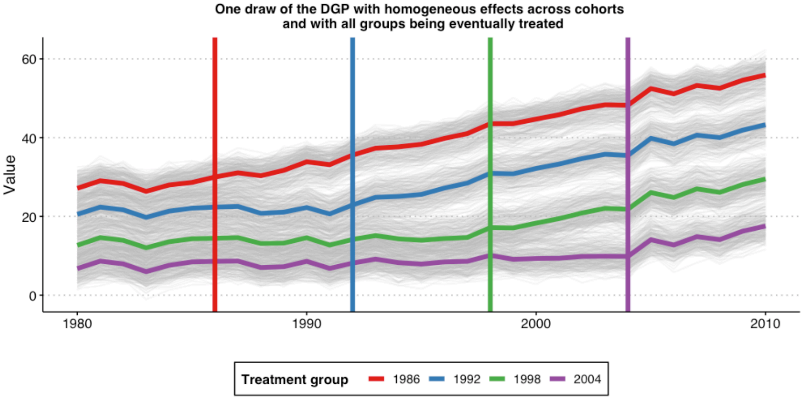
6. Overlap: $\epsilon < \Pr(\mathrm{Tr}) < 1 - \epsilon$ for each $t > 2$, g

3/4/5 define valid comparison group, thus analysis sample.

Linear Models and Two-Way Fixed-Effects

Callaway and Sant'Anna (2023)



The above plot shows the 1,000 individual values of $Y_{i,t}$, as well as the average by treatment-group (the thicker colorful lines), and the vertical lines show the period when treatment begins for each group. Because treatment effects here grows linearly with time elapsed since treatment started, we can see that the earliest treated units end up with the highest value of Y, and that the difference grows by the gap between treatment years.

Linear Models and Two-Way Fixed-Effects Event Study

Model

$$Y_{i,t} = \alpha_i + \beta_t + \gamma(Pre) + \delta_j(Lag_j) + \zeta_k(Lead_k) + \epsilon_{i,t}$$

- Indicators for units
- Indicators for time periods
- Indicators for lags (“tr how long ago?”)
- Indicators for leads (“tr how far into future?”)
- (Cluster SE’s on “states” of assignment)

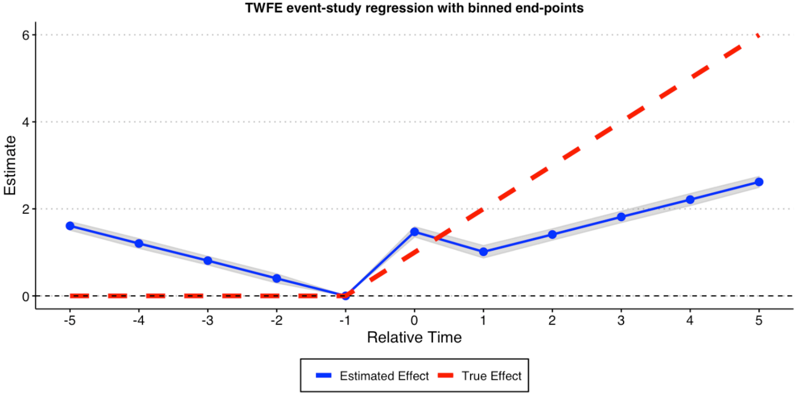
Linear Models and Two-Way Fixed-Effects Event Study

Model

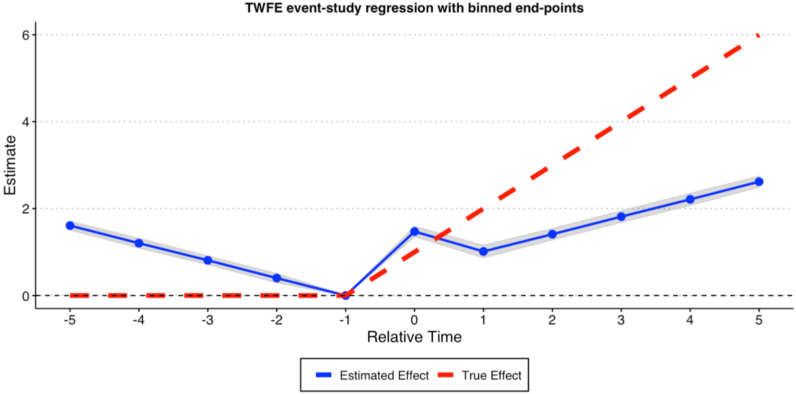
$$Y_{i,t} = \alpha_i + \beta_t + \gamma(Pre) + \delta_j(Lag_j) + \zeta_k(Lead_k) + \epsilon_{i,t}$$

- δ_j often interpreted as "ATE of j periods of treatment"
- ζ_j often interpreted as "pre-tr trends"

Linear Models and Two-Way Fixed-Effects Event Study



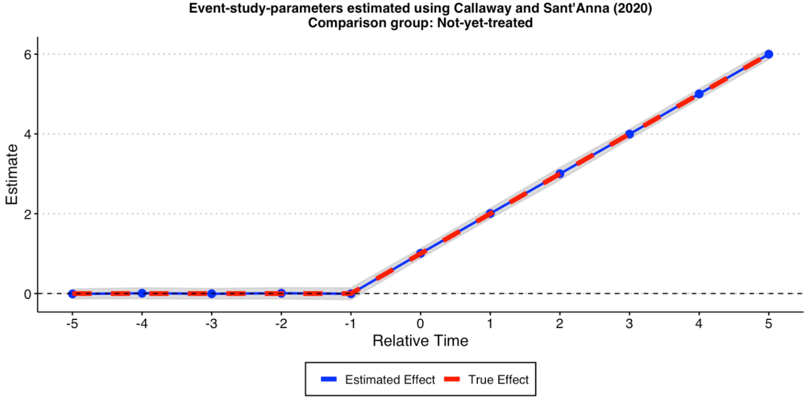
Linear Models and Two-Way Fixed-Effects Event Study



Severe bias for "effect of 3 years of Tr", e.g.!

Linear Models and Two-Way Fixed-Effects Event Study

Instead, using C-S approach correctly ID's identical lead/pre-tr trends and cumulative effects of Tr!



Thanks!

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