Data Science for Causal Inference

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The Lab @ DC

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Introductions

About Me

- Associate Prof of Government
  (American University)
- Associate Director, Center for Data Science
  (American University)
- Senior Social Scientist
  (The Lab @ DC)
- Fellow in Methodology
  (US Office of Evaluation Sciences: “OES”)

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- Associate Director, Center for Data Science
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- Senior Social Scientist
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- Fellow in Methodology
  (US Office of Evaluation Sciences: "OES")
- Research agenda: political methodology, causal inference, experimental design, experiments in public policy

About You!

- Name?

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About You!

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- Olympic sport you look forward to?

Plan

- Data Science in Causal Inference

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  - Models, tasks, methods

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  - Models, tasks, methods
  - Heterogeneous treatment effects

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  - Variable selection

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- Modern difference-in-difference designs

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  - Multiple time periods
  - Staggered adoption
  - Calloway-Sant'Anna approach

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- Introduction to several approaches

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- A survey

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- Examples:

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- A survey
- Examples:
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- Materials here: https://github.com/ryantmoore/new-directions-berlin

Data Science in Causal Inference

Causal Inference Approaches

The “potential outcomes” framework:

Causal Inference Approaches

The "potential outcomes" framework:

Citizen	Canvass?	Would Enroll if Canvass?	Would Enroll if No Canvass?	Enroll
1	Yes			Yes
2	Yes			Yes
3	No			No
4	No			No

Causal Inference Approaches

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Causal Inference Approaches

The "potential outcomes" framework, more abstractly:

Unit i	Treatment T	$Y(1)$	$Y(0)$	Y^{obs}	True τ $Y(1) - Y(0)$
1	1	10		10	
2	1	20		20	
3	0		15	15	
4	0		5	5	

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Unit i	Treatment T	$Y(1)$	$Y(0)$	Y^{obs}	True τ $Y(1) - Y(0)$
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2	1	20	(10)	20	10
3	0	(40)	15	15	25
4	0	(20)	5	5	15

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				$\text{ATE} = \bar{\tau} =$	$\frac{50}{4} = 12.5$
				$\widehat{ATE} = \hat{\bar{\tau}} =$	$15 - 10 = 5$

Causal Inference Approaches

The "potential outcomes" framework, notation:

- Units indexed by i

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(and we'll draw some DAG's, too)

Data Science Approaches

Three tasks of data science:

Data Science Approaches

Three tasks of data science:

- Description

Data Science Approaches

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Models/algorithms central to all three.

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Models/algorithms central to all three.

Hernán, Hsu, and Healy (2019)

Data Science Approaches

Description

- Identifying patterns, etc.

Data Science Approaches

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- Identifying patterns, etc.
- E.g., clustering to discover groups

Data Science Approaches

Prediction

- Components

Data Science Approaches

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  - Inputs/outputs
    (predictors/outcomes, features/responses, ...)

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Data Science Approaches

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Data Science Approaches

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    (linear model, decision tree, ...)
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- With these, model/machine learning algorithm does the work
- E.g., regression, random forests, neural networks, ...

Data Science Approaches

Causal Inference

- Potential outcomes/counterfactual/interventionist perspective

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- Requires *expertise* different to description/prediction

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  - (the more knowledge, the better!)

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  - (alternative: solve fundamental problem of causal inference! ☺)

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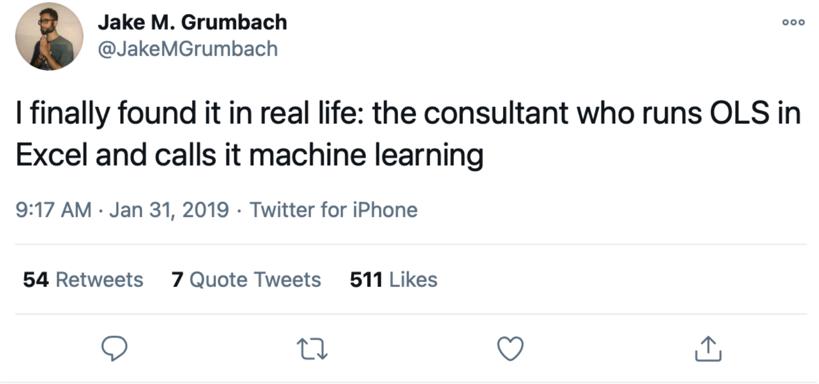
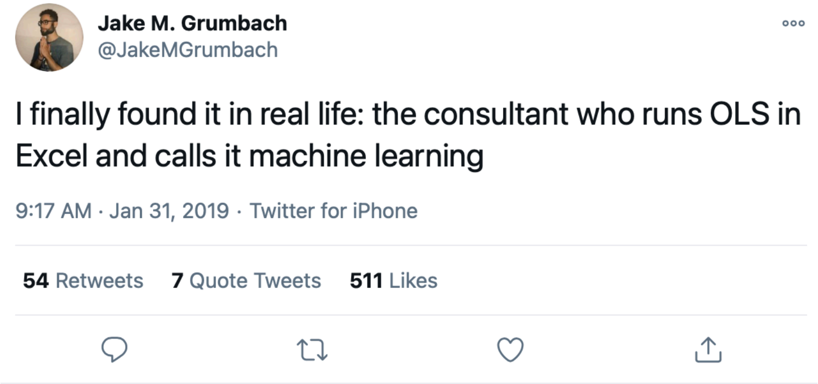
  - (alternative: solve fundamental problem of causal inference! ☺)
- E.g., experiments, observational causal designs, ...

Causal Inference with Machine Learning

Causal Inference with Machine Learning

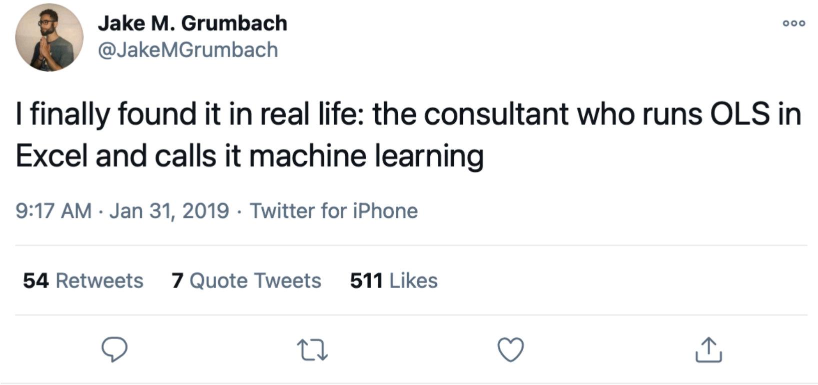


Causal Inference with Machine Learning



Don't do this.

Causal Inference with Machine Learning



Don't do this.

(Not "machine learning", probably, but *models* at least ...)

Causal Inference with Models

Consider two loaded datasets:

Causal Inference with Models

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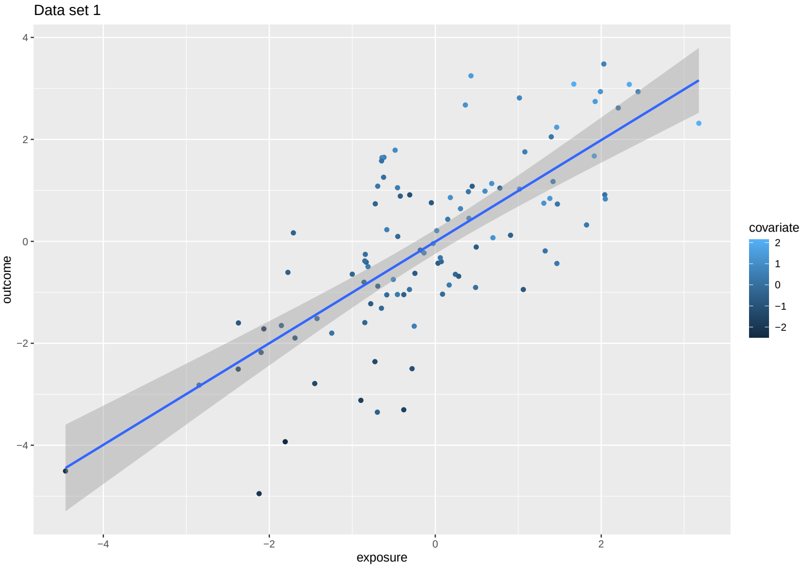
```
str(df1)
```

```
tibble [100 x 3] (S3: tbl_df/tbl/data.frame)
 $ covariate: num [1:100] -0.622 1.137 -0.238 1.529 -0.154
 $ exposure : num [1:100] 0.0332 0.3627 0.2422 1.4633 0.779
 $ outcome  : num [1:100] -0.429 2.675 -0.647 2.238 1.044
```

```
str(df2)
```

```
tibble [100 x 3] (S3: tbl_df/tbl/data.frame)
 $ exposure : num [1:100] 0.4862 0.0653 -1.4021 -0.546 -0.
 $ outcome  : num [1:100] 1.706 0.669 -1.597 -1.733 0.617
 $ covariate: num [1:100] 2.24 0.924 -0.999 -2.343 0.207 ..
```

Causal Inference with Models



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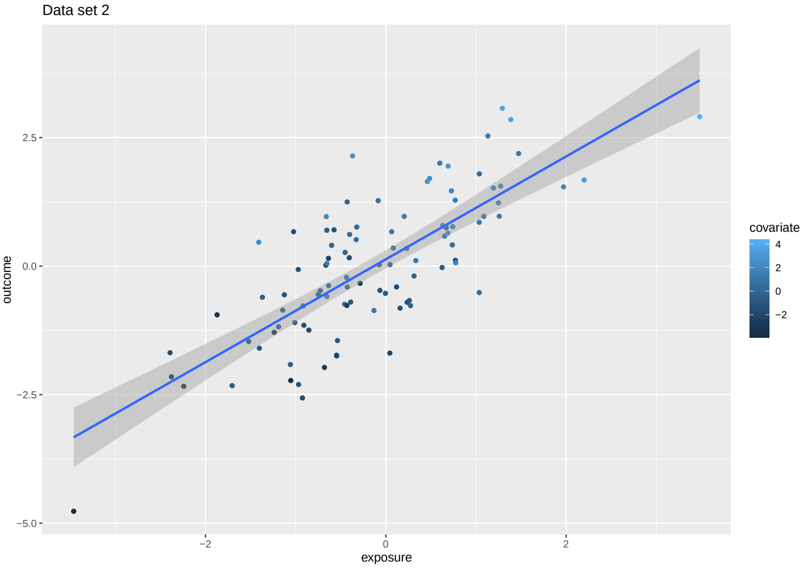

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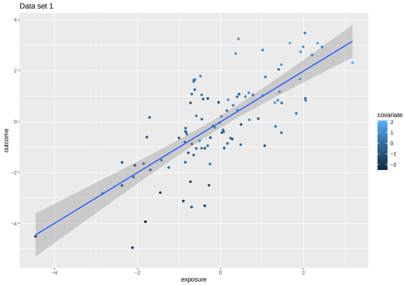

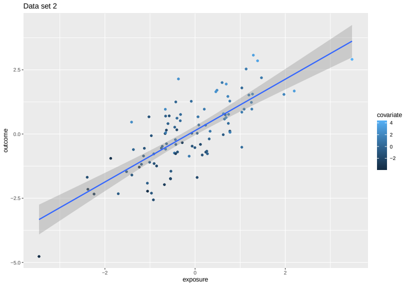

Causal Inference with Models

Model each

```r
lm_df1 <- lm(outcome ~ exposure, data = df1)
lm_df2 <- lm(outcome ~ exposure, data = df2)
```

```
# A tibble: 4 x 4
  data  term        estimate std.error
  <chr> <chr>          <dbl>     <dbl>
1 df1   (Intercept) -0.00671    0.120
2 df1   exposure     0.996      0.0927
3 df2   (Intercept)  0.133      0.0890
4 df2   exposure     1.00       0.0841
```

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- Both cases: effect of exposure ≈ 1.

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```

- Both cases: effect of exposure ≈ 1.
- Is this good? Is it correct?
- What if we adjust for covariate?

Causal Inference with Models

```
lm_df1_adj <- lm(outcome ~ exposure + covariate, data = df1
lm_df2_adj <- lm(outcome ~ exposure + covariate, data = df2
```

```
# A tibble: 4 x 4
  data  term       estimate std.error
  <chr> <chr>         <dbl>     <dbl>
1 df1   exposure      0.501    0.108
2 df1   covariate     0.970    0.147
3 df2   exposure      0.554    0.0990
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- Both cases: effect of exposure ≈ 0.5.

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- Is this good? Is it correct?
- Which is correct? $\beta = 1$? $\beta = 0.5$?

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Causal Inference with Models

There is nothing in the data that tells us.

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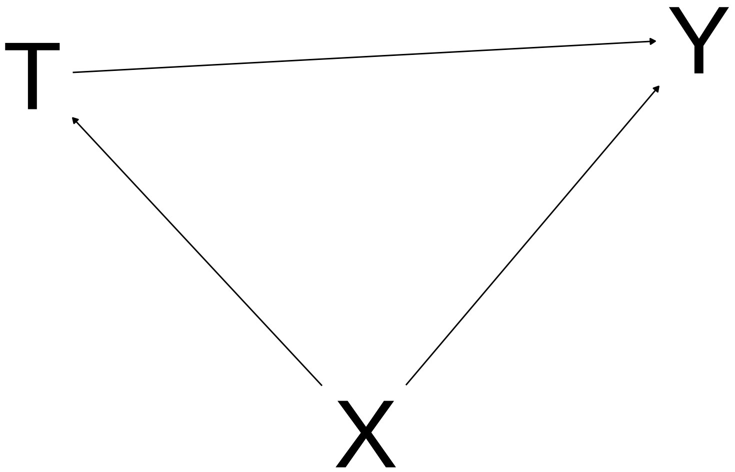
Here are the true structures: First



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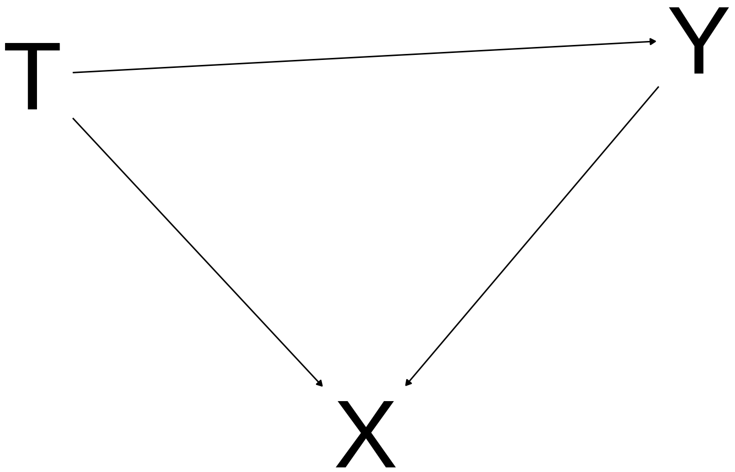

Causal Inference with Models

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- `df1`: confounding ⇒ **adjust for** X

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When know structures, adjustment sets for unbiasedness differ:

- `df1`: confounding $\Rightarrow$ **adjust for** X
- `df2`: collider $\Rightarrow$ **do not adjust for** X

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```
g_conf <- dagitty("dag{ x -> y ; x <- c -> y }")
g_coll <- dagitty("dag{ x -> y ; x -> c <- y }")
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g_coll <- dagitty("dag{ x -> y ; x -> c <- y }")
```

```
adjustmentSets(g_conf, "x", "y")
```

```
{ c }
```

Causal Inference with Models

When know structures, adjustment sets for unbiasedness differ:

- `df1`: confounding ⇒ **adjust for** X
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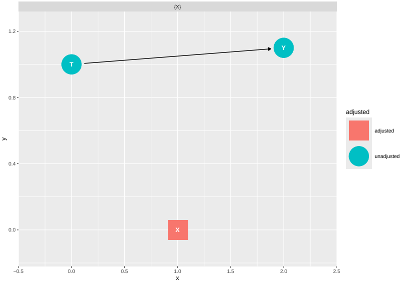

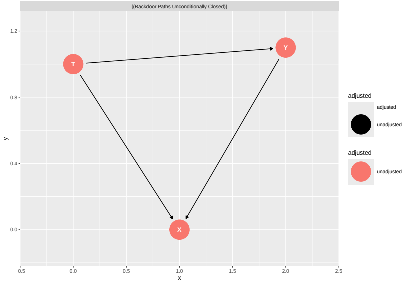

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  <chr> <chr>       <dbl>     <dbl>
1 df1   exposure    0.501     0.108
```

`df2`, do not adjust for X, $\beta = 1$:

```
# A tibble: 1 x 4
  data  term     estimate std.error
  <chr> <chr>       <dbl>     <dbl>
1 df2   exposure     1.00    0.0841
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(Data from D'Agostino McGowan (2023))

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- Though, methods from prediction can aid causal inference
- “Causal euphimisms” don’t help (Hernán 2018)

Approaches of Prediction and Causal Inference

Two Cultures, (Breiman 2001)

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Methods for Prediction and Causal Inference

- Cross-validation
- Regression/Decision trees
- Random forests

James et al. (2021)

Cross-validation

k-fold cross-validation to select method

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$$CV_{(k)} = \frac{1}{k}\sum_{i=1}^{k}\text{MSE}_i$$

- Select model that minimises $CV_{(k)}$

CV for Linear Model

```r
library(tidyverse)

## Make data

mk_data <- function(n = 90, n_folds = 10){

  df <- tibble(
    x1 = rnorm(n),
    x2 = rnorm(n),
    x3 = rnorm(n),
    y = 0.1 * x1 + 0.2 * x2 + 0.5 * x3 + rnorm(n),
    cv_fold = sample(rep(1:n_folds, (n / n_folds)))
  )

}

df <- mk_data()
```

CV for Linear Model

```r
head(df)
```

```
# A tibble: 6 x 5
      x1     x2      x3      y cv_fold
   <dbl>  <dbl>   <dbl>  <dbl>   <int>
1  0.175 -0.720 -0.399  -1.18        5
2 -0.275  0.620  1.32    2.01        3
3 -0.204 -2.47  -0.147   0.355       4
4 -1.95  -1.99   0.543   0.565       7
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```

```r
table(df$cv_fold)
```

```
 1  2  3  4  5  6  7  8  9 10
 9  9  9  9  9  9  9  9  9  9
```

CV for Linear Model

```r
cv_lm <- function(data, fmla){

  n_folds <- max(data$cv_fold)
  store_mses <- vector("numeric", length = n_folds)

  for(idx in 1:n_folds){

    df_train <- data |> filter(cv_fold != idx)
    df_test <- data |> filter(cv_fold == idx)

    lm_out <- lm(fmla, data = df_train)

    predictions <- predict(lm_out, newdata = df_test)

    store_mses[idx] <- mean((df_test$y - predictions)^2)}

  test_error_cv_k <- mean(store_mses)
  return(test_error_cv_k)
```

CV for Linear Model

```r
cv_lm(data = df, fmla = y ~ x1 + x2)
```

```
[1] 1.06925
```

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```
df <- mk_data()
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```

```
[1] 0.7021406
```

CV for Linear Model

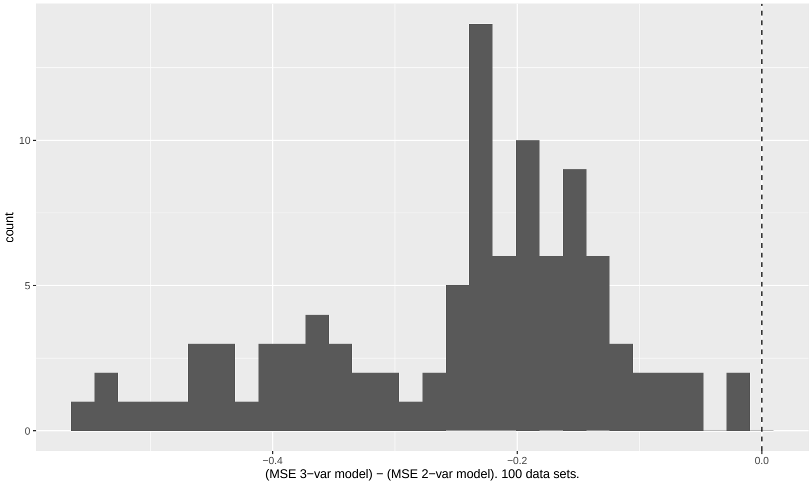

Figure 1: MSE always less (better) for 3-variable model.

Regression Trees

- Partition predictor space into regions $R_1, R_2, \ldots, R_J$.

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$$\sum_{j=1}^{J} \sum_{i \in R_j} \left(y_i - \widehat{y}_{R_j} \right)$$

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How to define regions R_j?

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$$\sum_{i:x\in R_1(j,s)} \left(y_i - \hat{y}_{R_1(j,s)}\right)^2 + \sum_{i:x\in R_2(j,s)} \left(y_i - \hat{y}_{R_2(j,s)}\right)^2$$

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- Can we increase predictive quality by only using *part* of a tree?

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Sum squared pred. error (plus penalty that grows with tree size) across units in region, then regions.

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4. Using that α, select best subtree from Step 2

Example: Regression Tree

Effect of office-holding on wealth
(Eggers and Hainmueller 2009):

```
library(qss)
library(rsample)
library(tree)

data("MPs")
mps <- MPs |> mutate(age = yod - yob,
                     is_labour = if_else(party == "labour",
                     is_london = if_else(region == "Greate
                     is_winner = if_else(margin > 0, 1, 0))
  select(ln.net, age, is_labour, is_london, is_winner) |>
  na.omit()
```

Example: Regression Tree

```
set.seed(765076184)

mp_split <- initial_split(mps, prop = 0.7)

mp_train <- training(mp_split)
mp_test <- testing(mp_split)
```

Example: Regression Tree

```
tree_mp <- tree(ln.net ~ ., data = mp_train)
plot(tree_mp)
text(tree_mp)
```

Figure 2: The regression tree (for training data)

Example: Regression Tree

Would pruning help?

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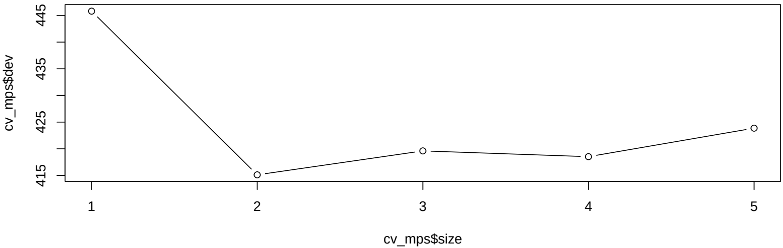
```
cv_mps <- cv.tree(tree_mp, K = 10)

plot(cv_mps$size, cv_mps$dev, type = "b")
```



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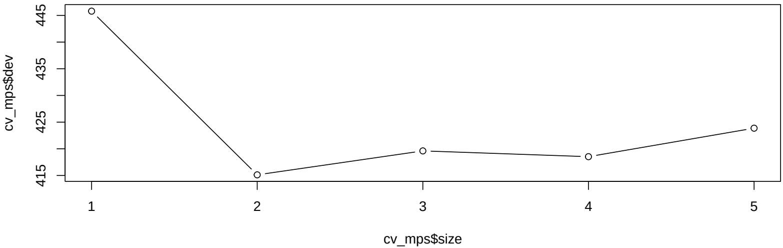

Figure 3: Subtree size 2 minimises SSR

Example: Regression Tree

```r
prune_mps <- prune.tree(tree_mp, best = 2)

plot(prune_mps)
text(prune_mps)
```

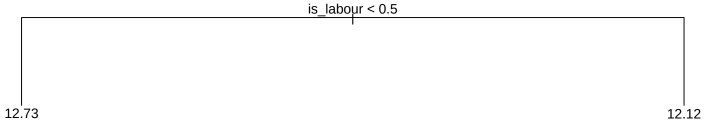

Figure 4: The pruned tree

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Predict for test set:

- MSE for pruned: 1.922
- MSE for full: 1.945

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(Pretty good for 1 split!?)

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Next: random forest algorithm

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Ensemble learning algorithms:

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Bagging: *b*ootstrap *agg*regation

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- (Linear regression: lower variance)

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- (Often choose $m \approx \sqrt{p}$)
- So, different splits consider different predictors
- So, trees will look very different to each other

Example: Random Forests

```
library(randomForest)

# Full bag:
bag_mps <- randomForest(ln.net ~ ., data = mp_train,
                        ntree = 500, mtry = 4,
                        importance = TRUE)

# Decorrelate:
rf_mps <- randomForest(ln.net ~ ., data = mp_train,
                       ntree = 500, mtry = 2,
                       importance = TRUE)
```

Example: Random Forests

Predict:

```
preds_bag <- predict(bag_mps, newdata = mp_test)

preds_rf <- predict(rf_mps, newdata = mp_test)
```

- MSE for RF: 1.995
- MSE for full bag: 2.536

Example: Random Forests

Predict:

```
preds_bag <- predict(bag_mps, newdata = mp_test)

preds_rf <- predict(rf_mps, newdata = mp_test)
```

- MSE for RF: 1.995
- MSE for full bag: 2.536

So, decorrelating helped us avoid some overfitting to each bootstrap subsample (and thus, reduced variance).

Example: Random Forests

Predict:

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preds_rf <- predict(rf_mps, newdata = mp_test)
```

- MSE for RF: 1.995
- MSE for full bag: 2.536

So, decorrelating helped us avoid some overfitting to each bootstrap subsample (and thus, reduced variance).

(Typical pred error of $\sqrt{1.995} \approx 1.412$)

Example: Random Forests

Predict:

```
preds_bag <- predict(bag_mps, newdata = mp_test)

preds_rf <- predict(rf_mps, newdata = mp_test)
```

- MSE for RF: 1.995
- MSE for full bag: 2.536

So, decorrelating helped us avoid some overfitting to each bootstrap subsample (and thus, reduced variance).

(Typical pred error of $\sqrt{1.995} \approx 1.412$)

(Bigger than IQR of 1.183, but range covers $[6.98, 16.3]$.)

Heterogeneous Treatment Effects

Homogeneous and Heterogeneous Effects

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  - Constituency effects (public policy)
- Notationally, $\exists i : \tau_i \neq \tau$

Homogeneous and Heterogeneous Effects: Estimation

Homogeneous effects:

$$\text{Outcome} = \beta_0 + \beta_1 \text{Treatment} + \epsilon$$

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```r
lm_out <- lm(ln.net ~ is_winner, data = mps)
lm_out
```

```

Call:
lm(formula = ln.net ~ is_winner, data = mps)

Coefficients:
(Intercept)    is_winner
    12.2464       0.5176
```

Homogeneous and Heterogeneous Effects: Estimation

Homogeneous effects:

```r
t.test(ln.net ~ is_winner, data = mps)
```

```
	Welch Two Sample t-test

data:  ln.net by is_winner
t = -3.9552, df = 287.65, p-value = 9.636e-05
alternative hypothesis: true difference in means betwe
95 percent confidence interval:
 -0.7751044 -0.2599998
sample estimates:
mean in group 0 mean in group 1
       12.24641        12.76396
```

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```
lm_out <- lm(ln.net ~ is_winner + is_labour +
               is_london + age, data = mps)
lm_out
```

```
Call:
lm(formula = ln.net ~ is_winner + is_labour + is_london + a
    data = mps)

Coefficients:
(Intercept)    is_winner    is_labour    is_london
  12.078838     0.398818    -0.477549     0.161134     0.00
```

Homogeneous and Heterogeneous Effects: Estimation

Homogeneous effects:

```
lm_lin(ln.net ~ is_winner, covariates = ~ is_labour + is_lo
```

```
                          Estimate  Std. Error       t valu
(Intercept)           1.226687e+01 0.078894901 155.4836617
is_winner             3.459885e-01 0.131207672   2.6369536
is_labour_c          -1.613663e-01 0.152608515  -1.0573871
is_london_c           2.427360e-01 0.250214401   0.9701118
age_c                 4.740367e-03 0.007031323   0.6741786
is_winner:is_labour_c -9.104022e-01 0.264395760  -3.4433313
is_winner:is_london_c -8.847770e-02 0.426241818  -0.2075763
is_winner:age_c      -4.778657e-05 0.012753800  -0.0037468
                         CI Lower    CI Upper  DF
(Intercept)           12.111785723 12.42195044 416
is_winner              0.088075873  0.60390123 416
is_labour_c           -0.461346226  0.13861367 416
is_london_c           -0.249106208  0.73457813 416
```

CATEs: Conditional ATEs

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- Sometimes "CACE"
- Inference: not "evidence against TE = 0?", but "evidence against $\text{CATE}_1 = \text{CATE}_2$?"

Homogeneous and Heterogeneous Effects: Estimation

Heterogeneous effects:

$$\text{Outcome} = \beta_0 + \beta_1 \text{Treatment} + \beta_2 \text{Group} + \beta_3 \text{Treatment} \cdot \text{Group} + \epsilon$$

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Heterogeneous effects:

$$\text{Outcome} = \beta_0 + \beta_1 \text{Treatment} + \beta_2 \text{Group} + \beta_3 \text{Treatment} \cdot \text{Group} + \epsilon$$

- β_1 gives TE for `Group == 0`

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Heterogeneous effects:

$$\text{Outcome} = \beta_0 + \beta_1 \text{Treatment} + \beta_2 \text{Group} + \beta_3 \text{Treatment} \cdot \text{Group} + \epsilon$$

- β_1 gives TE for `Group == 0`
- $\beta_1 + \beta_3$ gives TE for `Group == 1`

Homogeneous and Heterogeneous Effects: Estimation

Heterogeneous effects:

```
lm_out <- lm(ln.net ~ is_winner * is_labour +
                 is_london + age, data = mps)
coef(lm_out) |> round(3)
```

```
        (Intercept)           is_winner           is_labou
             11.959               0.780              -0.16
                age is_winner:is_labour
              0.005              -0.914
```

Causal Forests

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- Use $\hat{y}_{R_j}$ as pred value for obs in R_j
- (Tory, winner, London $\rightarrow$ 13.84)
- $$\hat{y}_{R_j} = \frac{1}{|R_j|} \sum_{i \in R_j} Y_i$$

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- Similarly, consider each leaf R_j small, homogeneous enough that potential outcomes independent

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$$Y(0), Y(1) \perp\!\!\!\perp T | \mathbf{X}$$

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  - Prediction, estimation of $\hat{\hat{\tau}}$ uses only $\mathcal{I}$
- Build a random forest (decorrelated deep trees picking from m predictors) of causal trees

Example: Causal Forests

```r
library(grf)

X <- mp_train |> select(age, is_labour, is_london)

W <- mp_train |> select(is_winner) |>
  unlist() |> as.numeric()

Y <- mp_train |> select(ln.net) |> unlist()

cf_out <- causal_forest(X, Y, W)
```

Example: Causal Forests

```r
cf_out
```

```
GRF forest object of type causal_forest
Number of trees: 2000
Number of training samples: 296
Variable importance:
    1     2     3
0.654 0.307 0.039
```

Example: Causal Forests

```
cf_out
```

```
GRF forest object of type causal_forest
Number of trees: 2000
Number of training samples: 296
Variable importance:
    1     2     3
0.654 0.307 0.039
```

(“How frequently was i the split feature?”)

Example: Causal Forests

```r
X_test <- mp_test |> select(age, is_labour, is_london)

cf_pred_est_var <- predict(cf_out, X_test,
                           estimate.variance = TRUE)
```

Example: Causal Forests

```r
X_test <- mp_test |> select(age, is_labour, is_london)

cf_pred_est_var <- predict(cf_out, X_test,
                           estimate.variance = TRUE)
```

```r
cf_preds <- cf_pred_est_var$predictions

df_cf <- tibble(X_test,
                cf_te = cf_preds,
                cf_se = sqrt(cf_pred_est_var$variance.e
                te_1se_lower = cf_te - cf_se,
                te_1se_upper = cf_te + cf_se)
```

Example: Causal Forests

Avg pred treatment effect in test sample:

```
mean(cf_preds)
```

```
[1] 0.405236
```

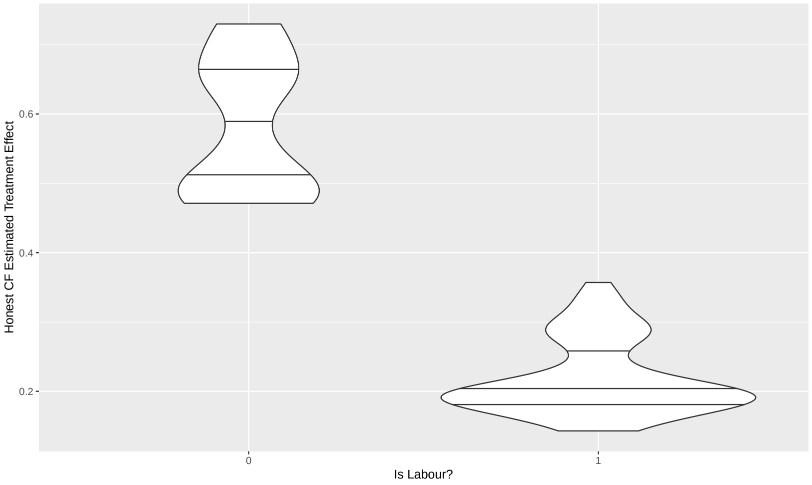
Example: Causal Forests

A doubly-robust ATE:

```
average_treatment_effect(cf_out)
```

```
 estimate   std.err
0.3465627 0.1715298
```

Example: Causal Forests Results, Party



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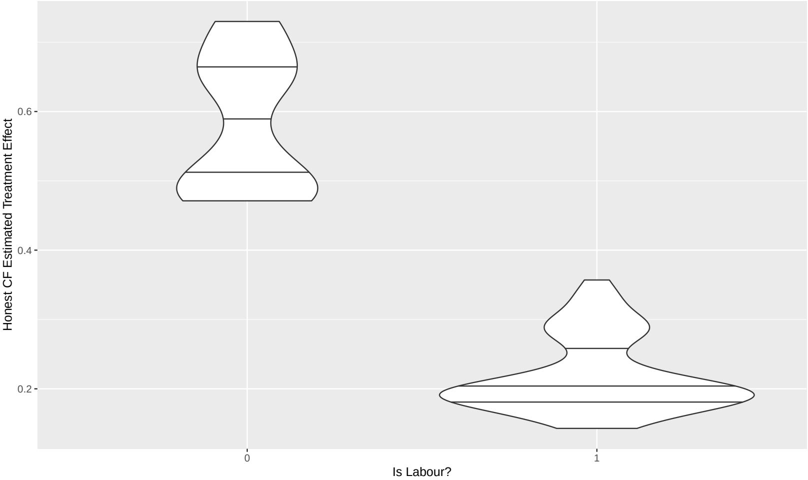

- Mean CF TE, Tory: 0.58

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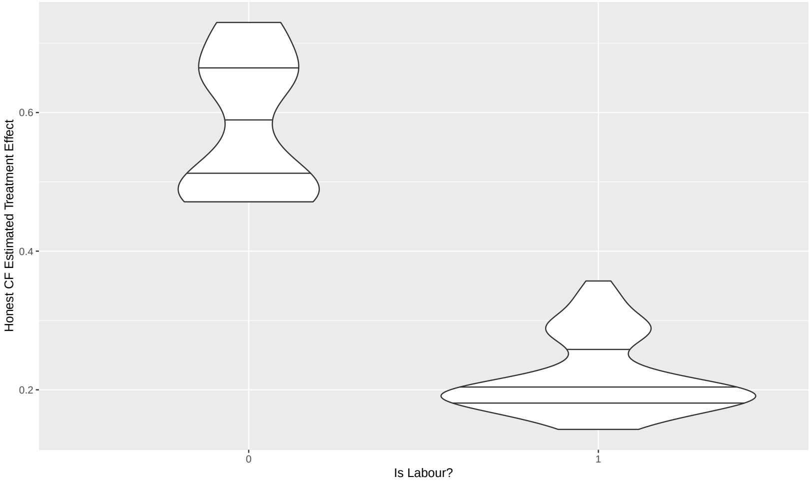

- Mean CF TE, Tory: $0.58 \rightsquigarrow £192,000$

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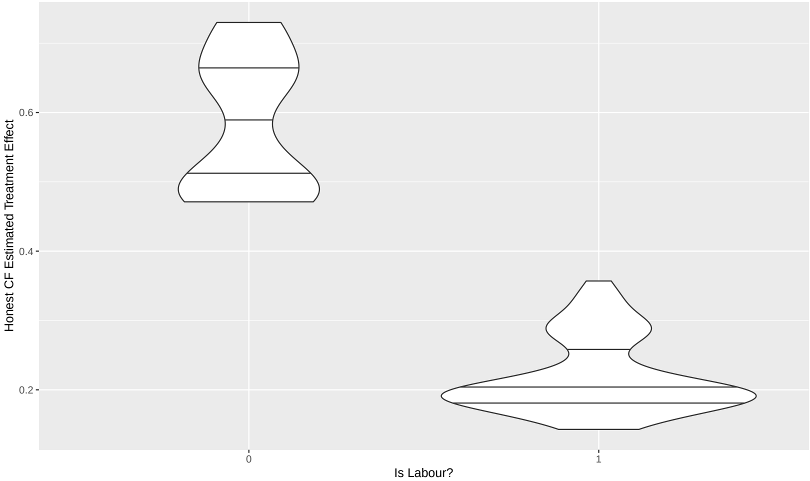
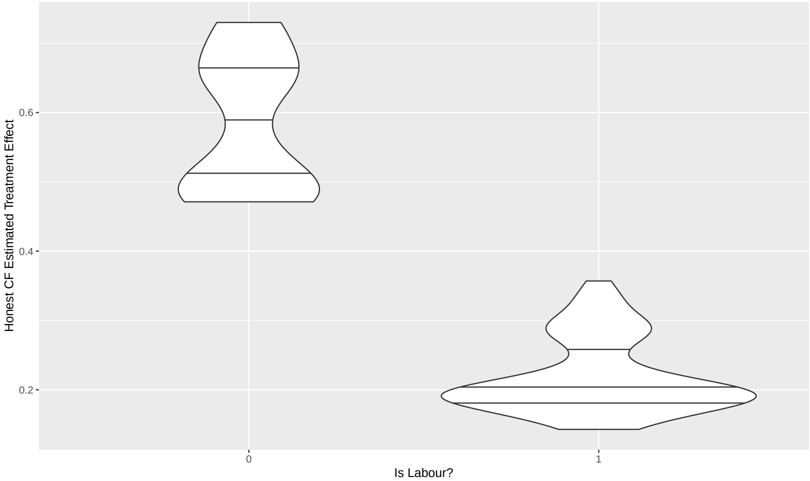


- Mean CF TE, Tory: 0.58 $\rightsquigarrow$ £192, 000
- Mean CF TE, Labour: 0.219

Example: Causal Forests Results, Party



- Mean CF TE, Tory: $0.58 \rightsquigarrow £192,000$
- Mean CF TE, Labour: $0.219 \rightsquigarrow £60,000$

Example: Causal Forests Results, Party

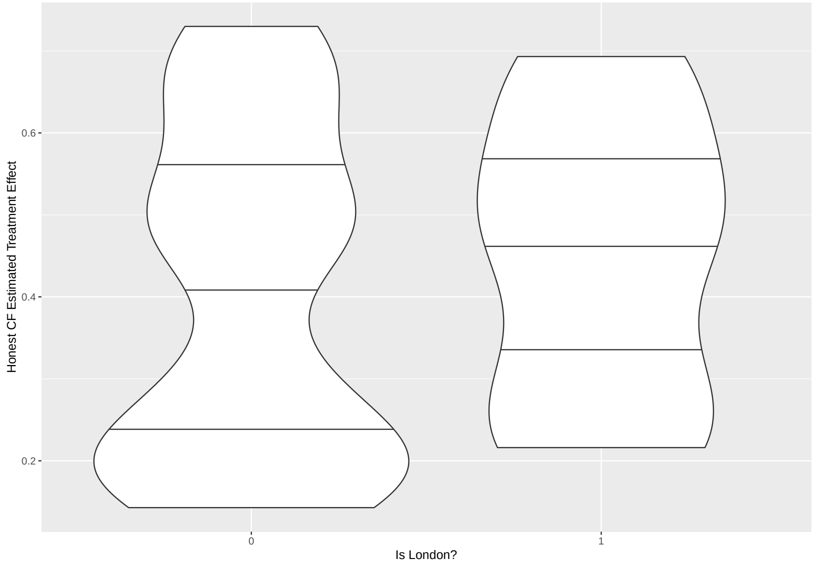
```r
average_treatment_effect(
  cf_out,
  subset = X$is_labour == 0)
```

```
 estimate   std.err
0.7503034 0.2237473
```

```r
average_treatment_effect(
  cf_out,
  subset = X$is_labour == 1)
```

```
  estimate    std.err
-0.1033197  0.2589494
```

Example: Causal Forests Results, London



Example: Causal Forests Results, London

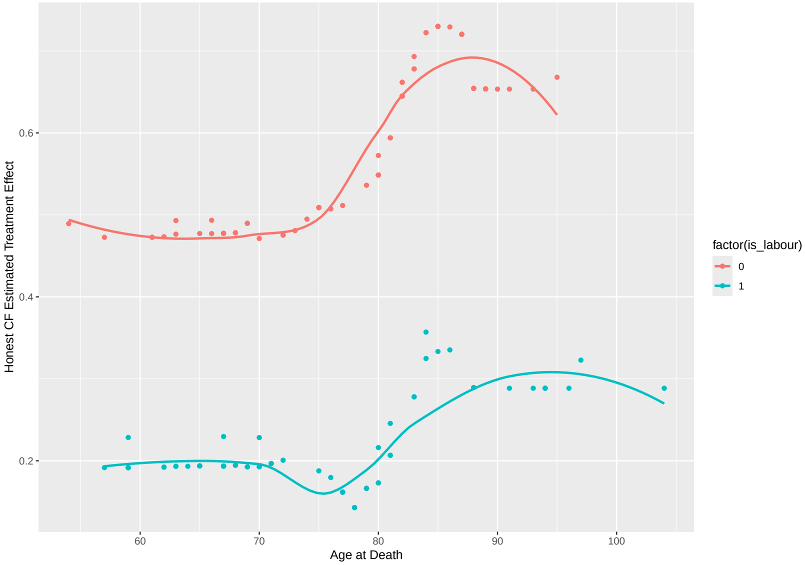
```
average_treatment_effect(
  cf_out,
  subset = X[, "is_london"] == 1)
```

```
 estimate   std.err
1.1156002 0.3778383
```

```
average_treatment_effect(
  cf_out,
  subset = X[, "is_london"] == 0)
```

```
 estimate   std.err
0.2400806 0.1874061
```

Example: Causal Forests Results, Age



Variable Selection

Feature Selection

- Wrappers: pick subset of covars, train on data (estimate model), test on hold-out, score predictions. Keep best-scoring subset.

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Feature Selection

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- Filters: correlate covars with outcome. Keep strongest.
- Embeds: select features and estimate model at same time. Penalize using more predictors.

Regularization Methods

OLS reminder

Minimize SSR:

$$\text{argmin}_\beta \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$\text{argmin}_\beta \sum_{i=1}^{n} \left(\mathbf{y} - \mathbf{X}\hat{\beta}\right)^2$$

Embedded Regularization Methods

L1 regularization: the LASSO (Least Absolute Shrinkage and Selection Operator)

$$\text{argmin}_{\beta}\left[\sum_{i=1}^{n}\left(y_i - \mathbf{X}\hat{\beta}\right)^2 + \lambda\sum_{j=1}^{k}|\hat{\beta}_j|\right]$$

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L2 regularization: Ridge regression

$$\mathrm{argmin}_{\beta}\left[\sum_{i=1}^{n}\left(y_i - \mathbf{X}\hat{\beta}\right)^2 + \lambda\sum_{j=1}^{k}\hat{\beta}_j^2\right]$$

Embedded Regularization Methods

Mix L1 and L2: Elastic net

$$\text{argmin}_\beta \left(\frac{\sum_{i=1}^{n} \left(y_i - \mathbf{X}\hat{\beta} \right)^2}{2n} + \lambda \left[\alpha \sum_{j=1}^{k} |\hat{\beta}_j| + \frac{1-\alpha}{2} \sum_{j=1}^{k} \hat{\beta}_j^2 \right] \right)$$

Embedded Regularization Methods

Mix L1 and L2: Elastic net

$$\mathrm{argmin}_{\beta}\left(\frac{\sum_{i=1}^{n}\left(y_i-\mathbf{X}\hat{\beta}\right)^2}{2n}+\lambda\left[\alpha\sum_{j=1}^{k}|\hat{\beta}_j|+\frac{1-\alpha}{2}\sum_{j=1}^{k}\hat{\beta}_j^2\right]\right)$$

Regularized trees, ...

Embedded Regularization Methods

How to choose λ, α?

Embedded Regularization Methods

How to choose λ, α?

The LASSO

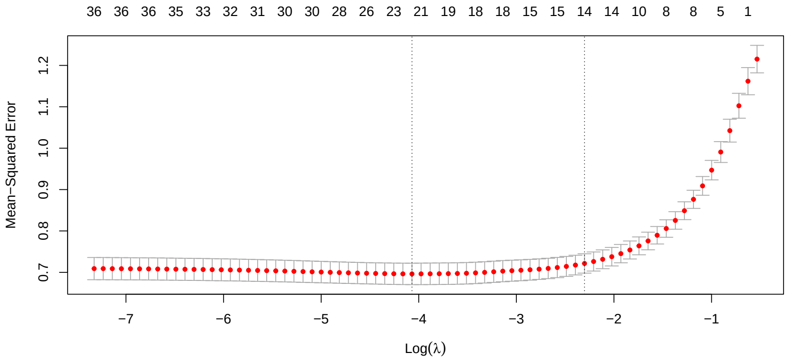
The LASSO

Cross-validation for λ:

```r
df_lasso <- read_csv("../data/01-lasso.csv")

X <- as.matrix(df_lasso[, 2:ncol(df_lasso)])

Y <- as.matrix(df_lasso[, "y"])

library(glmnet)

cv_lasso <- cv.glmnet(X, Y, alpha = 1)
```

The LASSO

```r
plot(cv_lasso)
```




```r
cv_lasso$lambda.min
```

```
[1] 0.0170891
```

The LASSO

Implement:

```
lasso_out <- glmnet(X, Y, alpha = 1,
                    lambda = cv_lasso$lambda.min)

lasso_out
```

```
Call:  glmnet(x = X, y = Y, alpha = 1, lambda = cv_lasso$la

  Df  %Dev  Lambda
1 21 45.32 0.01709
```

The LASSO

Coefficients:

```
coef_lasso <- coef(lasso_out)
round(coef_lasso, 3)
```

```
37 x 1 sparse Matrix of class "dgCMatrix"
                   s0
(Intercept)  0.000
x1           0.112
x2           0.095
x3           0.086
x4           0.147
x5           0.002
x6           0.063
x7           0.051
x8           0.074
x9           0.042
x10          .
```

The LASSO

Coefficients:

```
round(coef_lasso[, ], 3)
```

```
(Intercept)          x1          x2          x3          x4
      0.000       0.112       0.095       0.086       0.147
         x6          x7          x8          x9         x10
      0.063       0.051       0.074       0.042       0.000
        x12         x13         x14         x15         x16
      0.039       0.000       0.026       0.000       0.000
        x18         x19         x20         x21         x22
      0.010       0.127      -0.015       0.030       0.000
        x24         x25         x26         x27         x28
      0.000       0.000       0.000       0.000      -0.010
        x30         x31         x32         x33         x34
      0.000       0.028       0.032       0.000      -0.04[illegible]
        x36
      0.048
```

The LASSO

Implement, alternative λ:

```r
lasso_1se <- glmnet(X, Y, alpha = 1,
                    lambda = cv_lasso$lambda.1se)

coef(lasso_1se)
```

```
37 x 1 sparse Matrix of class "dgCMatrix"
                       s0
(Intercept) -0.0003034087
x1           0.1051188782
x2           0.0898842045
x3           0.0742522801
x4           0.1513883536
x5           .
x6           0.0603811184
x7           0.0389489143
x8           0.0575738993
x9           0.0374420416
```

The LASSO

Coefficients:

```
round(coef(lasso_1se)[, ], 3)
```

```
(Intercept)          x1          x2          x3          x4
      0.000       0.105       0.090       0.074       0.15
         x6          x7          x8          x9          x1
      0.060       0.039       0.058       0.037       0.00
        x12         x13         x14         x15         x1
      0.029       0.000       0.008       0.000       0.00
        x18         x19         x20         x21         x2
      0.004       0.039       0.000       0.000       0.00
        x24         x25         x26         x27         x2
      0.000       0.000       0.000       0.000       0.00
        x30         x31         x32         x33         x3
      0.000       0.000       0.013       0.000       0.00
        x36
      0.030
```

The Double LASSO for Treatment Effects

The idea:

- covariates may $\rightsquigarrow Y$ or $\rightsquigarrow T$

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- $\approx$ "double robust", "AIPW" estimators

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- covariates may $\rightsquigarrow Y$ or $\rightsquigarrow T$
- $\approx$ "double robust", "AIPW" estimators
- (different to just "doing LASSO twice" for regularization + shrinkage)

The Double LASSO for Treatment Effects

1. Model $Y = f(X)$ using LASSO

The Double LASSO for Treatment Effects

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2. Model $T = f(X)$, using LASSO

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3. Let X_{LASSO} be set of imp covariates identified s.t. each $\beta_{X_{\text{LASSO}}} > 0$

The Double LASSO for Treatment Effects

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The Double LASSO for Treatment Effects

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The Double LASSO for Treatment Effects: Example

The Double LASSO for Treatment Effects: Example

Dear Registered Voter:

WHAT IF YOUR NEIGHBORS KNEW WHETHER YOU VOTED?

Why do so many people fail to vote? We've been talking about the problem for years, but it only seems to get worse. This year, we're taking a new approach. We're sending this mailing to you and your neighbors to publicize who does and does not vote.

The chart shows the names of some of your neighbors, showing which have voted in the past. After the August 8 election, we intend to mail an updated chart. You and your neighbors will all know who voted and who did not.

DO YOUR CIVIC DUTY — VOTE!

MAPLE DR	Aug 04	Nov 04	Aug 06
9995 JOSEPH JAMES SMITH	Voted	Voted	______
9995 JENNIFER KAY SMITH		Voted	______
9997 RICHARD B JACKSON		Voted	______
9999 KATHY MARIE JACKSON		Voted	______
9999 BRIAN JOSEPH JACKSON		Voted	______
9991 JENNIFER KAY THOMPSON		Voted	______
9991 BOB R THOMPSON		Voted	______
9993 BILL S SMITH			______
9989 WILLIAM LUKE CASPER		Voted	______
9989 JENNIFER SUE CASPER		Voted	______
9987 MARIA S JOHNSON	Voted	Voted	

The Double LASSO for Treatment Effects: Example

```r
library(hdm)
library(qss)
data(social)
```

```r
df_social <- social |>
  mutate(is_male = if_else(sex == "male", 1, 0),
         age = 2006 - yearofbirth,
         is_neighbors = if_else(messages == "Neighbors", 1
  filter(messages %in% c("Neighbors", "Control"))

df_social |> select(-yearofbirth) |> head()
```

```
     sex primary2004 messages primary2006 hhsize is_male ag
1   male           0  Control           0      3       1 2
2 female           0  Control           1      3       0 4
3   male           0  Control           1      3       1 5
4 female           0  Control           0      2       0 3
5   male           0  Control           0      2       1 3
```

The Double LASSO for Treatment Effects: Example

```
rlasso_out <- rlassoATE(
  primary2006 ~ age + is_male + primary2004 + hhsize +
    is_neighbors | age + is_male + primary2004 + hhsize,
  data = df_social)
```

The Double LASSO for Treatment Effects: Example

```r
summary(rlasso_out)
```

```
Estimation and significance testing of the treatment effect
Type: ATE
Bootstrap: not applicable
     coeff.      se. t-value p-value
TE 0.080091 0.002625   30.51  <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 '
```

The Double LASSO for Treatment Effects

```
X <- as.matrix(df_social[, c("age", "is_male", "primary2004
                             "hhsize", "is_neighbors")])

Y <- as.matrix(df_social[, "primary2006"])

D <- as.matrix(df_social[, "is_neighbors"])

summary(rlassoEffects(X, Y, method = "double selection"))
```

```
[1] "Estimates and significance testing of the effect of ta
             Estimate. Std. Error t value Pr(>|t|)
age          0.0038449  0.0000681  56.456  < 2e-16 ***
is_male      0.0086763  0.0018889   4.593 4.36e-06 ***
primary2004  0.1474364  0.0019924  74.000  < 2e-16 ***
hhsize       0.0004260  0.0012618   0.338    0.736
is_neighbors 0.0802361  0.0026278  30.534  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 '
```

R packages for Regularization, etc.

- `glmnet`
- `caret`

See also `tidymodels`, `parsnip`, ...

Thanks!

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