Sensitivity Analyses

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Sensitivity

What is “sensitivity”?

Sensitivity to Model Specification

Should we trust our model?

Estimating all possible regressions

Idea

Example 1

Moore, Powell, and Reeves (2013)

Implementation

Hebbali (2024)

```r
library(olsrr)
```

Example 2

```r
library(qss)
data(social)

social <- social |> mutate(
  age = 2006 - yearofbirth,
  age_c = age - mean(age),
  messages = fct_relevel(messages, "Control")
)

head(social)
```

```
     sex yearofbirth primary2004   messages primary2006 hhs
1   male        1941           0 Civic Duty           0
2 female        1947           0 Civic Duty           0
3   male        1951           0  Hawthorne           1
4 female        1950           0  Hawthorne           1
5 female        1982           0  Hawthorne           1
6   male        1981           0    Control           0
```

Example 2

```
lm_out <- lm(primary2006 ~ messages + sex + age_c +
               primary2004 + hhsize, data = social)

all_lm_social <- ols_step_all_possible(lm_out)$result
```

Example 2

```
all_lm_social_coefs <- ols_step_all_possible_betas(lm_out)

all_lm_social_coefs
```

```
   model          predictor         beta
1      1        (Intercept)  0.2966383083
2      1 messagesCivic Duty  0.0178993441
3      1  messagesHawthorne  0.0257363121
4      1  messagesNeighbors  0.0813099129
5      2        (Intercept)  0.3059095493
6      2            sexmale  0.0126509479
7      3        (Intercept)  0.3122445777
8      3              age_c  0.0041515670
9      4        (Intercept)  0.2508820413
10     4        primary2004  0.1528795252
11     5        (Intercept)  0.3763534949
12     5             hhsize -0.0293482475
13     6        (Intercept)  0.2902800648
```

Example 2

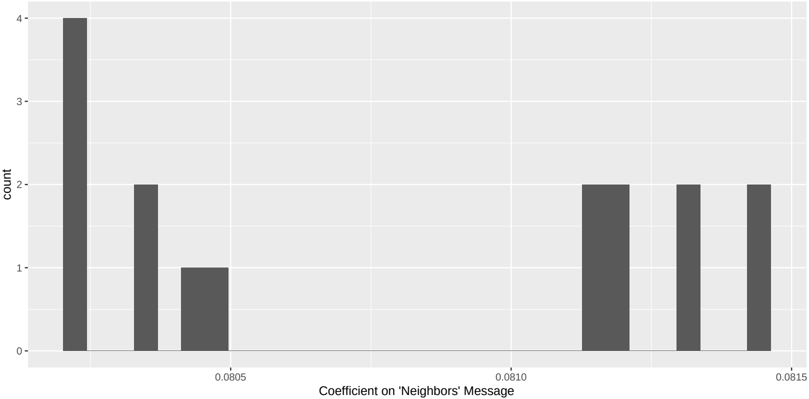

Figure 1: Coefficients from All Possible Regressions

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
0.08023	0.08032	0.08081	0.08080	0.08122	0.08145

All Coefficients

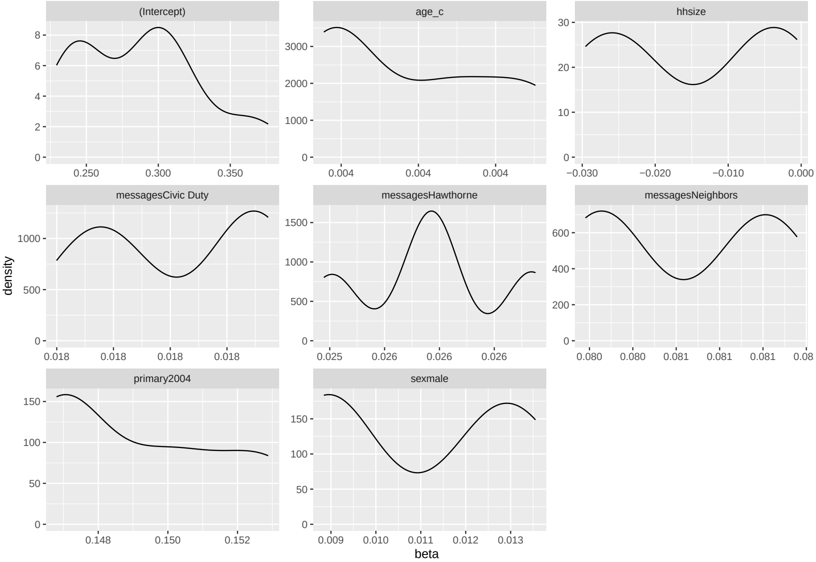

Figure 2

Matching as Preprocessing

- minimize effects of model-based adjustment (subclassify, match)

"model-based adjustments ...will give basically the same point estimates"

Matching as Preprocessing

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"model-based adjustments ...will give basically the same point estimates"

What does this mean?

Ho et al. (2007)

"Matching as Nonparametric Preprocessing for Reducing Model Dependence in Parametric Causal Inference"

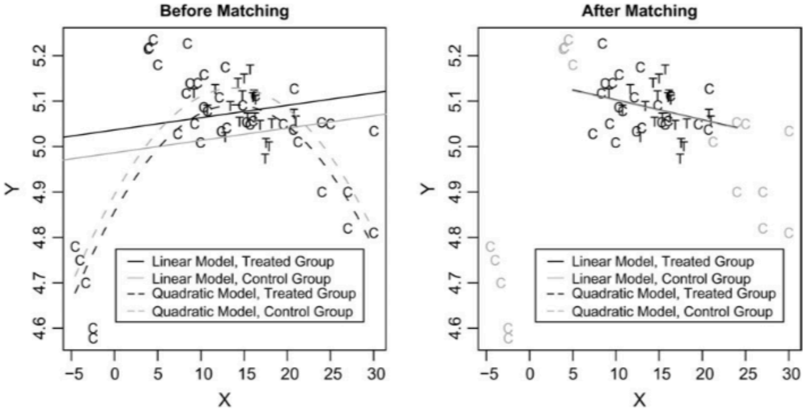


Figure 3: Here

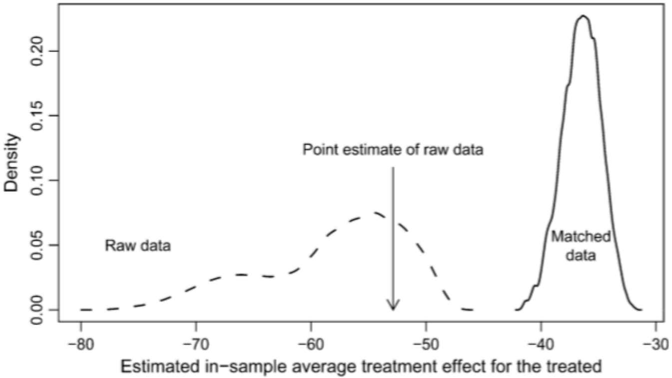


Fig. 2 Kernel density plot (a smoothed histogram) of point estimates of the in-sample ATT of the Democratic Senate majority on FDA drug approval time across 262,143 specifications. The solid line

Figure 4: Here

How to Identify Problem?

Different distributions; non-overlap

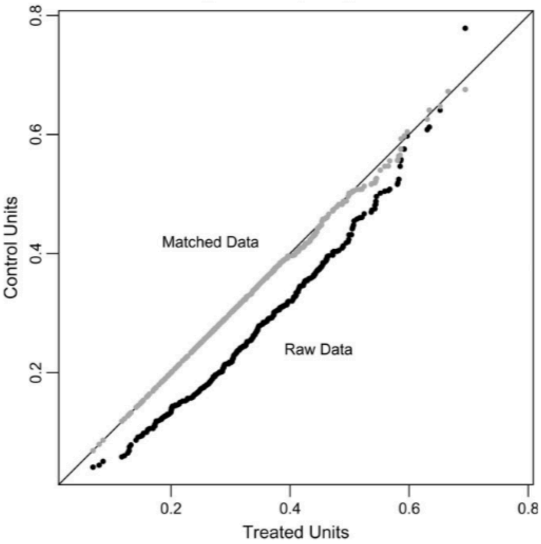


Fig. 3 QQ plot of propensity score for candidate visibility. The black dots represent empirical QQ

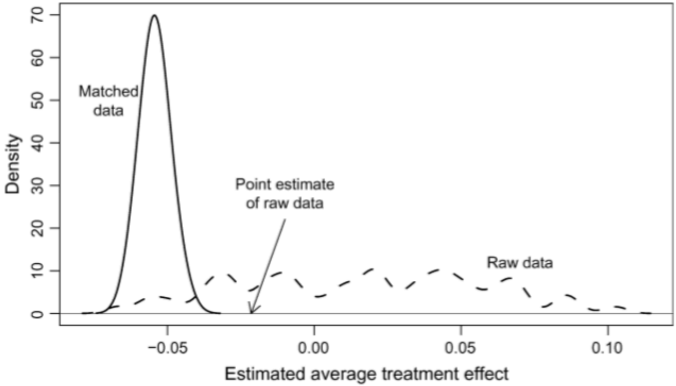


Fig. 4 Kernel density plot of point estimates of the effect of being a less visible male Republican candidate across 63 possible specifications with the Koch data. The dashed line presents estimates for

Figure 6: Here

Paradox of Regression for causal inference?

- If diffs large, regression not enough, very sensitive
- If diffs small, regression won't matter much
- Ho et al. (2007)

Matching as Preprocessing for Dynamic Treatment Regimes

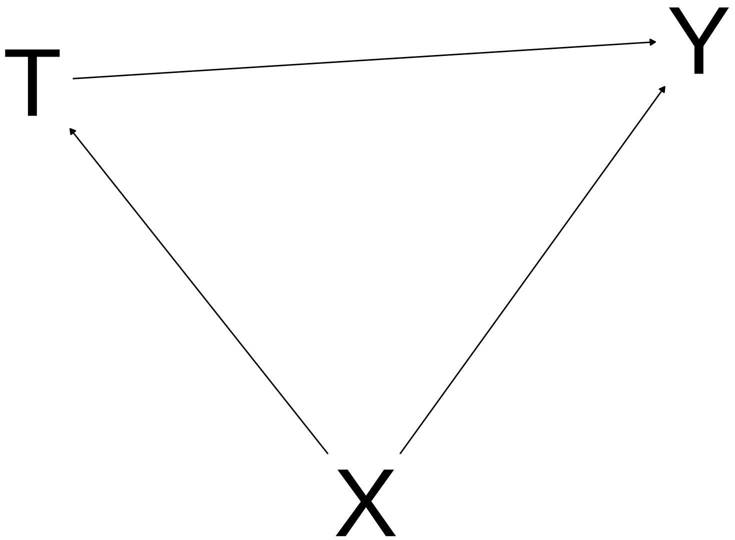
Blackwell and Strezhnev (2022)

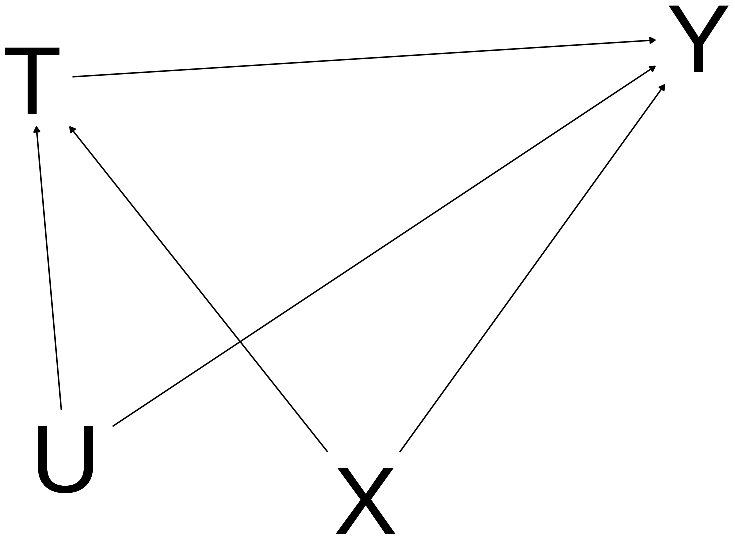
Sensitivity to an Unidentifiable Parameter

Mediation Analysis

Sensitivity to an Unobserved Covariates

Confounding in Observational Studies




T
Y
U
X

Addressing Confounding

To break confounding,

- can't break $X \to Y$
- break $X \to T$
- I.e., make $X \perp\!\!\!\perp T$
- But this doesn't address $U \to T$ (or $U \to Y$).

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(Of course, if no causal effect of $U \to Y$, no problem.)

Hidden Bias

Where there is $U \to T$ and $U \to Y$, there is *hidden bias*.

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but are different in prop score:

$$\pi_i \neq \pi_j$$

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Sociability affects whether called (know more people) and turnout.

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Sociability affects whether called (know more people) and turnout.

Sensitivity: how strong must sociability be to invalidate inference about phone calls?

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$$A_1 : A_2 = \frac{p(A_1)}{p(A_2)}$$

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Application: Measuring Group Differences (JP Scanlon)

	% Below Pov Line			% Above Pov Line		
	B	W	$\frac{B}{W}$	B	W	$\frac{B}{W}$
t_1	90	80	1.1	10	20	0.5

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Absolute Differences: 10, 10, 10, 10

Clearly, huge absolute improvements.

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- Key: it's not clear whether relative disparities getting better/worse/neither by below/above measures.
- (Easy to produce examples of OR's same and AbsDiffs slightly diff.)
- (Diffs betwn groups real, importnt, but how we meas. changes is tricky)

King’s Conjecture

Gary King @kinggary

the "odds ratio" is a lame way to communicate statistical results; I conjecture that there's *always* a better way

Expand Reply Retweet Favorite

17 October 2012

Rosenbaum's Model

Modeling Hidden Bias

Odds of treatment for i and j:

$$\frac{\pi_i}{1-\pi_i}, \frac{\pi_j}{1-\pi_j}$$

Modeling Hidden Bias

Odds of treatment for i and j:

$$\frac{\pi_i}{1-\pi_i}, \frac{\pi_j}{1-\pi_j}$$

OR of i versus j:

$$\begin{aligned} OR &= \frac{\pi_i}{1-\pi_i} \div \frac{\pi_j}{1-\pi_j} \\ &= \frac{\pi_i(1-\pi_j)}{\pi_j(1-\pi_i)} \end{aligned}$$

Modeling Hidden Bias

Let Γ be upper bound on OR of treatment.

$$\frac{1}{\Gamma} \leq \frac{\pi_i(1-\pi_j)}{\pi_j(1-\pi_i)} \leq \Gamma \qquad \forall i, j \text{ s.t. } \mathbf{x}_i = \mathbf{x}_j$$

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$$\frac{1}{\Gamma} \leq \frac{\pi_i(1-\pi_j)}{\pi_j(1-\pi_i)} \leq \Gamma \qquad \forall i, j \text{ s.t. } \mathbf{x}_i = \mathbf{x}_j$$

By what factor does the odds of treatment differ? (No more than Γ)

Modeling Hidden Bias

Rosenbaum (2020) shows that this is same as

$$\begin{aligned}
\log\left(\frac{\pi_i}{1-\pi_i}\right) &= \kappa(\mathbf{x}_i) + \gamma u_i \\
\log\left(\frac{\pi_j}{1-\pi_j}\right) &= \kappa(\mathbf{x}_j) + \gamma u_j
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s.t. $0 \leq u_i \leq 1$.

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s.t. $0 \leq u_i \leq 1$.

Interpretation: first rewrite

$$\log\left(\frac{\pi_j}{1-\pi_j}\right) = \kappa(\mathbf{x}_i) + \gamma u_j$$

Exponentiate:

$$\begin{aligned}\left(\frac{\pi_i}{1-\pi_i}\right) &= e^{\kappa(\mathbf{x}_i)+\gamma u_i} \\ \left(\frac{\pi_j}{1-\pi_j}\right) &= e^{\kappa(\mathbf{x}_i)+\gamma u_j}\end{aligned}$$

Exponentiate:

$$
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\left(\frac{\pi_j}{1-\pi_j}\right) &= e^{\kappa(\mathbf{x}_i)+\gamma u_j}
\end{aligned}
$$

Calculate OR:

$$
\begin{aligned}
OR &= \frac{\pi_i(1-\pi_j)}{\pi_j(1-\pi_i)} \\
&= \frac{e^{\kappa(\mathbf{x}_i)+\gamma u_i}}{e^{\kappa(\mathbf{x}_i)+\gamma u_j}} \\
&= e^{(\kappa(\mathbf{x}_i)+\gamma u_i)-(\kappa(\mathbf{x}_i)+\gamma u_j)} \\
&= e^{(\gamma u_i-\gamma u_j)} \\
&= e^{\gamma(u_i-u_j)}
\end{aligned}
$$

Interpreting Γ

$$OR = e^{\gamma(u_i - u_j)}$$

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Log odds differ by factor of γ times diff in unobs confounder.

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Shows $\Gamma = e^{\gamma}$.

TABLE 4.1. Sensitivity Analysis for Hammond's Study of Smoking and Lung Cancer: Range of Significance Levels for Hidden Biases of Various Magnitudes.

Γ	Minimum	Maximum
1	< 0.0001	< 0.0001
2	< 0.0001	< 0.0001
3	< 0.0001	< 0.0001
4	< 0.0001	0.0036
5	< 0.0001	0.03
6	< 0.0001	0.1

Figure 7: Here

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Figure 7: Here

- Groups: smokers/nonsmokers
- Outcome: lung cancer
- Something must increase smoking by $6\times$ to change inference.
- If exists, maybe it's that factor, not smoking directly.

(Bias from $U \to T$; effectively $U \to Y$ nearly perfect.)

Table 1: Example Table

Γ	Minimum	Maximum
1	≤ 0.0001	≤ 0.0001
2	≤ 0.0001	0.0018
3	≤ 0.0001	0.0136
4	≤ 0.0001	0.0388
4.25	≤ 0.0001	0.0468
5	≤ 0.0001	0.0740

Table 4.2: Signed-Rank Statistic p-value Sensitivity for Lead in Children's Blood

- Groups: parents occupationally exposed/unexposed
- Outcome: children's levels
- Something must increase parents' exposure by $5\times$ to change inference.
- If exists, maybe it's that, not parental exposure directly.

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(one-sided)

Table 2: Example Table

Γ	Minimum	Maximum
1	15	15
2	10.25	19.5
3	8	23
4	6.5	25
5	5	26.5

Table 4.3: Point Estimate Sensitivity for Lead in Children's Blood

Table 2: Example Table

Γ	Minimum	Maximum
1	15	15
2	10.25	19.5
3	8	23
4	6.5	25
5	5	26.5

Table 4.3: Point Estimate Sensitivity for Lead in Children's Blood

- HL point estimate: 15 (median of all $m \times n$ possible matched pairs)
- With confounding, wider range of possible effects.

Table 3: Example Table with 95% Confidence Intervals

Γ	95% CI
1	(9.5, 20.5)
2	(4.5, 27.5)
3	(1.0, 32.0)
4	(-1.0, 36.5)
5	(-3.0, 41.5)

Table 4.4: Confidence Interval Sensitivity for Lead in Children's Blood

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1	(9.5, 20.5)
2	(4.5, 27.5)
3	(1.0, 32.0)
4	(-1.0, 36.5)
5	(-3.0, 41.5)

Table 4.4: Confidence Interval Sensitivity for Lead in Children's Blood

- Inverted NHST CI's
- If something increases parental exposure by $4\times$, negative estimates of parents on children are reasonable.

(two-sided)

Implementation

Packages

- `sensitivitymw`
- `sensitivitymv`

Example

```r
anes <- read_csv("../data/anes_pilot_2016.csv")
dim(anes)
```

```
[1] 1200  594
```

```r
anes <- anes |> mutate(age = 2016 - birthyr,
                       pid_rep = as.numeric(pid3 == 3),
                       pid_dem = as.numeric(pid3 == 1))
```

```r
lm_out <- lm(turnout12 ~ pid_rep, data = anes)
summary(lm_out)
```

```

Call:
lm(formula = turnout12 ~ pid_rep, data = anes)

Residuals:
    Min      1Q  Median      3Q     Max 
-0.3395 -0.2451 -0.2451 -0.2451  1.7549 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  1.24512    0.01868  66.641  < 2e-16 ***
pid_rep      0.09435    0.03320   2.842  0.00456 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 '

Residual standard error: 0.535 on 1198 degrees of freedom
Multiple R-squared:  0.006695,	Adjusted R-squared:  0.005
```

```r
library(konfound)
konfound(lm_out, pid_rep)
```

```r
library(konfound)
konfound(lm_out, pid_rep)
```

```
Robustness of Inference to Replacement (RIR):
To invalidate an inference, 30.959 % of the estimate would
have to be due to bias.
This is based on a threshold of 0.065 for statistical
significance (alpha = 0.05).

To invalidate an inference, 372 observations would have to
be replaced with cases
for which the effect is 0 (RIR = 372).

See Frank et al. (2013) for a description of the method.

Citation: Frank, K.A., Maroulis, S., Duong, M., and Kelcey,
B. (2013).
What would it take to change an inference?
Using Rubin's causal model to interpret the
robustness of causal inferences
```

```r
lm_out <- lm(turnout12 ~ pid_rep + age, data = anes)
summary(lm_out)
```

```

Call:
lm(formula = turnout12 ~ pid_rep + age, data = anes)

Residuals:
    Min      1Q  Median      3Q     Max 
-0.5825 -0.3388 -0.1711  0.0301  1.9831 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)  1.678649   0.045960  36.524  < 2e-16 ***
pid_rep      0.082685   0.031870   2.594  0.00959 ** 
age         -0.008943   0.000873 -10.244  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 '

Residual standard error: 0.5132 on 1197 degrees of freedom
```

```r
konfound(lm_out, pid_rep)
```

```
Robustness of Inference to Replacement (RIR):
To invalidate an inference, 24.379 % of the estimate would
have to be due to bias.
This is based on a threshold of 0.063 for statistical
significance (alpha = 0.05).

To invalidate an inference, 293 observations would have to
be replaced with cases
for which the effect is 0 (RIR = 293).

See Frank et al. (2013) for a description of the method.

Citation: Frank, K.A., Maroulis, S., Duong, M., and Kelcey,
B. (2013).
What would it take to change an inference?
Using Rubin's causal model to interpret the
robustness of causal inferences.
Education, Evaluation and
```

```r
cor(anes[,c("pid_rep", "turnout12", "econnow")])
```

```
             pid_rep   turnout12     econnow
pid_rep   1.00000000 0.081825966 0.141257803
turnout12 0.08182597 1.000000000 0.008599061
econnow   0.14125780 0.008599061 1.000000000
```

```r
lm_out <- lm(turnout12 ~ pid_rep + age + econnow, data = an
summary(lm_out)
```

```
Call:
lm(formula = turnout12 ~ pid_rep + age + econnow, data = an

Residuals:
     Min       1Q   Median       3Q      Max
-0.60257 -0.33748 -0.17138  0.04458  1.96702

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.6290966  0.0565381  28.814   <2e-16 ***
pid_rep      0.0755031  0.0322095   2.344   0.0192 *
age         -0.0091496  0.0008833 -10.358   <2e-16 ***
econnow      0.0202398  0.0134633   1.503   0.1330
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 '
```

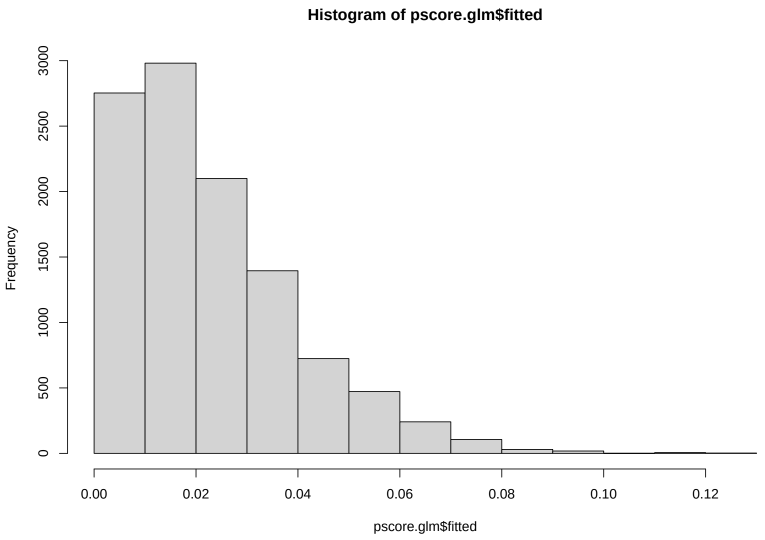
```r
konfound(lm_out, pid_rep)
```

```
Robustness of Inference to Replacement (RIR):
To invalidate an inference, 16.303 % of the estimate would
have to be due to bias.
This is based on a threshold of 0.063 for statistical
significance (alpha = 0.05).

To invalidate an inference, 196 observations would have to
be replaced with cases
for which the effect is 0 (RIR = 196).

See Frank et al. (2013) for a description of the method.

Citation: Frank, K.A., Maroulis, S., Duong, M., and Kelcey,
B. (2013).
What would it take to change an inference?
Using Rubin's causal model to interpret the
robustness of causal inferences.
Education, Evaluation and
```

Implementation in rbounds

```
library(Matching)
data(GerberGreenImai)

# Estimate Propensity Score
pscore.glm <- glm(PHN.C1 ~ PERSONS + VOTE96.1 +
                    NEW + MAJORPTY + AGE + WARD +
                    PERSONS:VOTE96.1 + PERSONS:NEW +
                    AGE2, family = binomial(logit),
                  data = GerberGreenImai)
```

```r
hist(pscore.glm$fitted)
```



Implementation in rbounds

```
# Match - without replacement
m.obj <- Match(Y = GerberGreenImai$VOTED98,
               Tr = GerberGreenImai$PHN.C1,
               X = fitted(pscore.glm), M = 1, replace = FAI

summary(m.obj)
```

```
Estimate...  0.036437
SE.........  0.040216
T-stat.....  0.90604
p.val......  0.36492

Original number of observations..............  10829
Original number of treated obs...............  247
Matched number of observations...............  247
Matched number of observations  (unweighted).  247
```

Implementation in `rbounds`

```r
library(rbounds)

# Sensitivity Test
# binarysens(m.obj, Gamma = 2, GammaInc = .1)
```

Implementation in `rbounds`

```
#hlsens(m.obj, Gamma = 5, GammaInc = 1)
```

Thanks!

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References I

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