Data Science for Causal Inference

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The Lab @ DC

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- Associate Prof of Government
  (American University)
- Associate Director, Center for Data Science
  (American University)
- Senior Social Scientist
  (The Lab @ DC)
- Fellow in Methodology
  (US Office of Evaluation Sciences: “OES”)

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- Senior Social Scientist
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- Fellow in Methodology
  (US Office of Evaluation Sciences: "OES")
- Research agenda: political methodology, causal inference, experimental design, experiments in public policy

About You!

- Name?

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- Name?
- Role?

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- Name?
- Role?
- Interests?

About You!

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- Interests?
- Olympic sport you look forward to?

Plan

- Data Science in Causal Inference

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- Data Science in Causal Inference
  - Models

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- Data Science in Causal Inference
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  - Heterogeneous treatment effects

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- Modern difference-in-difference designs

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  - Staggered adoption

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  - Staggered adoption
  - Calloway-Sant'Anna approach

Data Science in Causal Inference

Causal Inference Approaches

The "potential outcomes" framework:

Causal Inference Approaches

The "potential outcomes" framework:

Citizen	Canvass?	Would Enroll if Canvass?	Would Enroll if No Canvass?	Enroll
1	Yes	Yes		Yes
2	Yes			Yes
3	No			No
4	No			No

Causal Inference Approaches

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Causal Inference Approaches

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2	Yes	Yes	(No)	Yes
3	No	(Yes)	No	No
4	No	(No)	No	No

Causal Inference Approaches

The "potential outcomes" framework, more abstractly:

Unit i	Treatment T	$Y(1)$	$Y(0)$	Y^{obs}	True τ $Y(1) - Y(0)$
1	1	10		10	
2	1	20		20	
3	0		15	15	
4	0		5	5	

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4	0	(20)	5	5	15

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				$\text{ATE} = \bar{\tau} =$	$\frac{50}{4} = 12.5$
				$\widehat{ATE} = \hat{\bar{\tau}} =$	$15 - 10 = 5$

Causal Inference Approaches

The "potential outcomes" framework, notation:

- Units indexed by i
- Treatment T_i or D_i or Z_i
- Outcome if treated $Y_i(1)$
- Outcome if control $Y_i(0)$
- True treatment effect $\tau_i = Y_i(1) - Y_i(0)$
- True average treatment effect $\bar{\tau} = \frac{1}{n}\sum_{i=1}^{n}(Y_i(1) - Y_i(0))$
- Pre-treatment covariates $\mathbf{X}$

Causal Inference Approaches

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(and we'll draw some DAG's, too)

Data Science Approaches

Three tasks of data science:

- Description

Data Science Approaches

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- Prediction

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Models/algorithms central to all three.

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Models/algorithms central to all three.

Hernán, Hsu, and Healy (2019)

Data Science Approaches

Description

- Identifying patterns, etc.

Data Science Approaches

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- Identifying patterns, etc.
- E.g., clustering to discover groups

Data Science Approaches

Prediction

- Components

Data Science Approaches

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  - Inputs/outputs (predictors/outcomes, features/responses, ...)

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- With these, model machine learning does the work

Data Science Approaches

Prediction

- Components
  - Inputs/outputs (predictors/outcomes, features/responses, ...)
  - Mapping from inputs to outputs (linear model, decision tree, ...)
  - Metric for evaluating mapping
- With these, model machine learning does the work
- E.g., regression, random forests, neural networks, ...

Data Science Approaches

Causal Inference

- Potential outcomes/counterfactual/interventionist perspective

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    - Not all inputs treated same

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  - T v. $\mathbf{X}$ – very different!

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  - (the more knowledge, the better!)

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  - (alternative: solve fundamental problem of causal inference!)

Data Science Approaches

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  - Not all inputs treated same
  - T v. $\mathbf{X}$ – very different!
  - (the more knowledge, the better!)
  - (alternative: solve fundamental problem of causal inference!)
- E.g., experiments, observational causal designs, ...

Causal Inference with Machine Learning

Causal Inference with Machine Learning

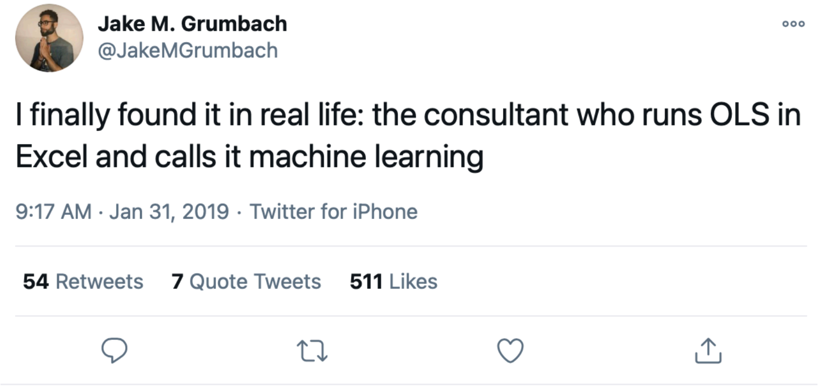
Jake M. Grumbach
@JakeMGrumbach

I finally found it in real life: the consultant who runs OLS in Excel and calls it machine learning

9:17 AM · Jan 31, 2019 · Twitter for iPhone

54 Retweets **7** Quote Tweets **511** Likes

Causal Inference with Machine Learning



(OK, not “machine learning”, perhaps, but *models* at least ...)

Causal Inference with Models

Loaded two datasets:

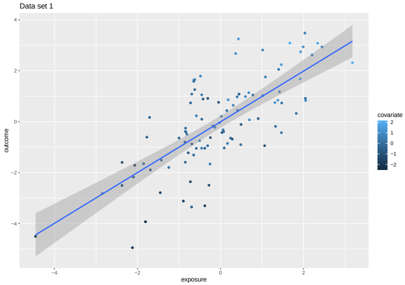
```r
str(df1)
```

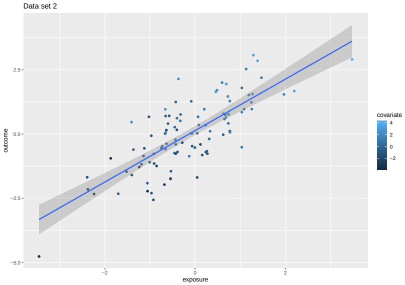
```
tibble [100 x 3] (S3: tbl_df/tbl/data.frame)
 $ covariate: num [1:100] -0.622 1.137 -0.238 1.529 -0.154
 $ exposure : num [1:100] 0.0332 0.3627 0.2422 1.4633 0.779
 $ outcome  : num [1:100] -0.429 2.675 -0.647 2.238 1.044 .
```

```r
str(df2)
```

```
tibble [100 x 3] (S3: tbl_df/tbl/data.frame)
 $ exposure : num [1:100] 0.4862 0.0653 -1.4021 -0.546 -0.4
 $ outcome  : num [1:100] 1.706 0.669 -1.597 -1.733 0.617 .
 $ covariate: num [1:100] 2.24 0.924 -0.999 -2.343 0.207 ..
```

Causal Inference with Models





Causal Inference with Models

Model each

```r
lm_df1 <- lm(outcome ~ exposure, data = df1)
lm_df2 <- lm(outcome ~ exposure, data = df2)
```

```
# A tibble: 4 x 4
  data  term        estimate std.error
  <chr> <chr>          <dbl>     <dbl>
1 df1   (Intercept) -0.00671    0.120
2 df1   exposure     0.996      0.0927
3 df2   (Intercept)  0.133      0.0890
4 df2   exposure     1.00       0.0841
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- Both cases: effect of exposure ≈ 1.

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```

- Both cases: effect of exposure ≈ 1.
- Is this good?
- What if we adjust for covariate?

Causal Inference with Models

```
lm_df1_adj <- lm(outcome ~ exposure + covariate, data = df1
lm_df2_adj <- lm(outcome ~ exposure + covariate, data = df2
```

```
# A tibble: 4 x 4
  data  term       estimate std.error
  <chr> <chr>         <dbl>     <dbl>
1 df1   exposure      0.501    0.108
2 df1   covariate     0.970    0.147
3 df2   exposure      0.554    0.0990
4 df2   covariate     0.385    0.0598
```

- Both cases: effect of exposure ≈ 0.5.

Causal Inference with Models

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- Which is correct? $\beta = 1$? $\beta = 0.5$?

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- Both cases: effect of exposure ≈ 0.5.
- Is this good?
- Which is correct? $\beta = 1$? $\beta = 0.5$?
- *Should* we adjust for covariate?

Causal Inference with Models

There is nothing in the data that tells us.

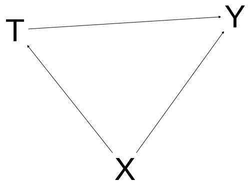
Causal Inference with Models

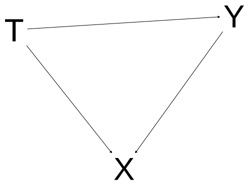
There is nothing in the data that tells us. ☹

Causal Inference with Models

There is nothing in the data that tells us. ☹

Here are the true structures:





Causal Inference with Models

When know structures, adjustment sets for unbiasedness differ:

- `df1`: confounding ⇒ **adjust for** X
- `df2`: collider ⇒ **do not adjust for** X

```
g_conf <- dagitty("dag{ x -> y ; x <- c -> y }")
g_coll <- dagitty("dag{ x -> y ; x -> c <- y }")

adjustmentSets(g_conf, "x", "y")
```

```
{ c }
```

```
adjustmentSets(g_coll, "x", "y")
```

```
 {}
```

Causal Inference with Models

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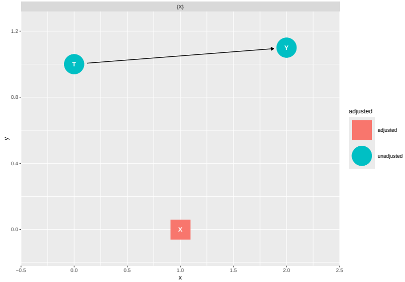

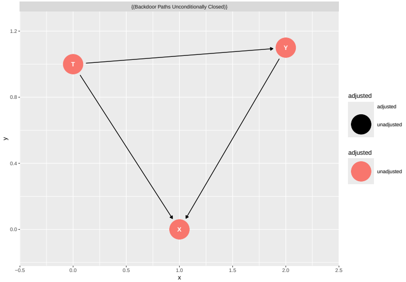

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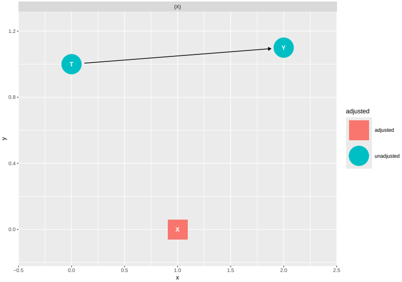

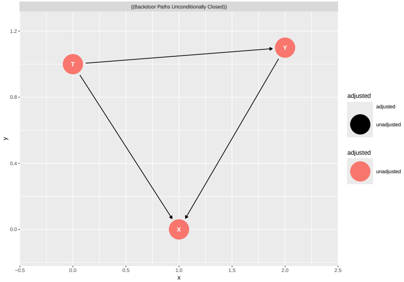

(Data from D'Agostino McGowan (2023))

Causal Inference with Models

- Importance of identifying "pre-treatment covariates", "proper covariates"; doing "design before analysis"

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- Though, methods from prediction can aid causal inference

Causal Inference with Models

- Importance of identifying "pre-treatment covariates", "proper covariates"; doing "design before analysis"
- Importance of experiments: strong knowledge about (part of) causal structure
- Causal inference is critical to scientific questions, and separate from prediction
- Though, methods from prediction can aid causal inference
- (A perspective on "causal euphimisms": Hernán (2018))

Approaches of Prediction and Causal Inference

Two Cultures, (Breiman 2001)

- *Data Models*: our "social science modeling"
- *Algorithmic Models*: our "data science algorithms"

Methods for Prediction and Causal Inference

- Cross-validation
- Regression/Decision trees
- Random forests

James et al. (2021)

Cross-validation

k-fold cross-validation

- Randomly partition data into k groups
- Apply method to $k-1$ groups
- Use result to predict for left-out group
- Calculate $\mathrm{MSE}_i = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$
- Calculate test error as average of the k MSE's:

$$CV_{(k)} = \frac{1}{k}\sum_{i=1}^{k}\mathrm{MSE}_i$$

- Select model that minimises $CV_{(k)}$

CV for Linear Model

```r
library(tidyverse)

## Make data

mk_data <- function(n = 90, n_folds = 10){

  df <- tibble(
    x1 = rnorm(n),
    x2 = rnorm(n),
    x3 = rnorm(n),
    y = 0.1 * x1 + 0.2 * x2 + 0.5 * x3 + rnorm(n),
    cv_fold = sample(rep(1:n_folds, (n / n_folds)))
  )

}

df <- mk_data()
```

CV for Linear Model

```
head(df)
```

```
# A tibble: 6 x 5
      x1      x2     x3      y cv_fold
   <dbl>   <dbl>  <dbl>  <dbl>   <int>
1 -0.403  1.54    1.48   1.66        7
2 -0.164 -2.71   -0.897 -2.15        1
3 -0.594 -0.398  -0.591  0.147       4
4  1.33   0.129  -1.75  -0.793       8
5 -0.249 -0.544   1.40  -0.507       8
6  2.02   0.0630  1.05  -0.130       2
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CV for Linear Model

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6  2.02   0.0630  1.05  -0.130       2
```

```r
table(df$cv_fold)
```

```
 1  2  3  4  5  6  7  8  9 10
 9  9  9  9  9  9  9  9  9  9
```

CV for Linear Model

```
cv_lm <- function(data, fmla){

  n_folds <- max(data$cv_fold)
  store_mses <- vector("numeric", length = n_folds)

  for(idx in 1:n_folds){

    df_train <- data |> filter(cv_fold != idx)
    df_test <- data |> filter(cv_fold == idx)

    lm_out <- lm(fmla, data = df_train)

    predictions <- predict(lm_out, newdata = df_test)

    store_mses[idx] <- mean((df_test$y - predictions)^2)}

  test_error_cv_k <- mean(store_mses)
  return(test_error_cv_k)
```

CV for Linear Model

```r
cv_lm(data = df, fmla = y ~ x1 + x2)
```

```
[1] 1.468833
```

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```

```
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```

```r
df <- mk_data()
cv_lm(df, y ~ x1 + x2 + x3)
```

```
[1] 0.9552452
```

CV for Linear Model

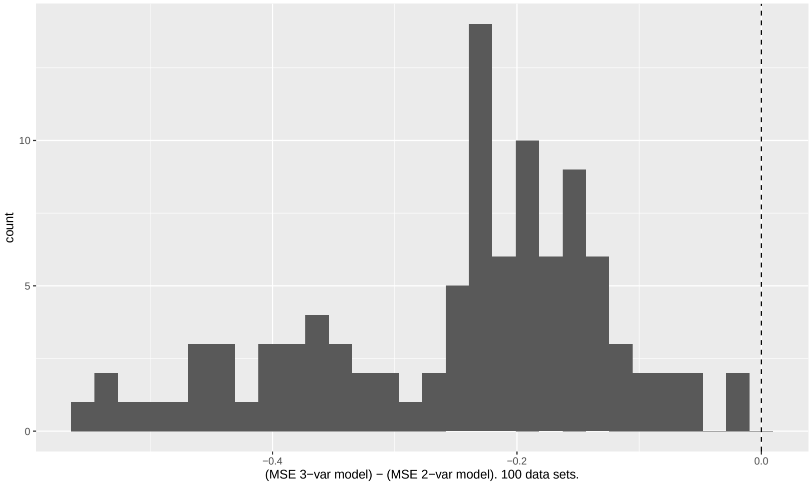

Figure 1: MSE always less (better) for 3-variable model.

Regression Trees

- Partition predictor space into regions $R_1, R_2, \dots, R_J$.
- If unit falls in region R_j, use average outcome in R_j as predicted value: $\hat{y}_{R_j}$
- (For *decision* on discrete outcome, count votes in R_j)
- Goal: minimise residual sum of squares (RSS), just like LS regression:

$$\sum_{j=1}^{J} \sum_{i \in R_j} \left(y_i - \hat{y}_{R_j} \right)$$

Regression Trees

How to define regions R_j?

Regression Trees

How to define regions R_j?

- Top-down, greedy recursive binary split
- At each step, find predictor and cut-point that minimise

$$\sum_{i:x \in R_1(j,s)} \left(y_i - \hat{y}_{R_1(j,s)}\right)^2 + \sum_{i:x \in R_2(j,s)} \left(y_i - \hat{y}_{R_2(j,s)}\right)^2$$

Regression Trees

- Overfitting is a potential problem

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- Can we increase predictive quality by only using *part* of a tree?

Regression Trees

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- Can we increase predictive quality by only using *part* of a tree?
- “Pruning”

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- Select the subtree that gives least prediction error (via cross-validation)

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$$\sum_{m=1}^{|T|} \sum_{i: x_i \in R_m} \left(y_i - \hat{y}_{R_m}\right)^2 + \alpha|T|$$

Sum squared pred. error (plus penalty that grows with tree size) across units in region, then regions.

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But, how to choose α? (Use cross-validation.)

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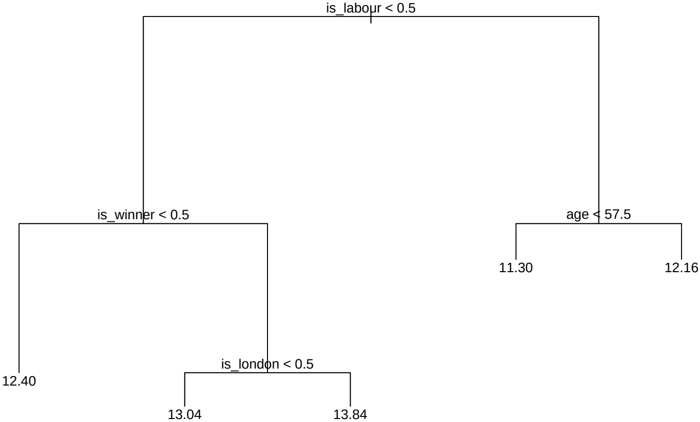
4. Using that α, select best subtree from Step 2

Example: Regression Tree

```
library(qss)
library(rsample)
library(tree)

data("MPs")
mps <- MPs |> mutate(age = yod - yob,
                     is_labour = if_else(party == "labour"
                     is_london = if_else(region == "Greate
                     is_winner = if_else(margin > 0, 1, 0))
  select(ln.net, age, is_labour, is_london, is_winner) |>
  na.omit()

set.seed(765076184)

mp_split <- initial_split(mps, prop = 0.7)

mp_train <- training(mp_split)
mp_test <- testing(mp_split)
```

Example: Regression Tree

```
plot(tree_mp)
text(tree_mp)
```



Example: Regression Tree

Would pruning help?

```
cv_mps <- cv.tree(tree_mp, K = 10)

plot(cv_mps$size, cv_mps$dev, type = "b")
```

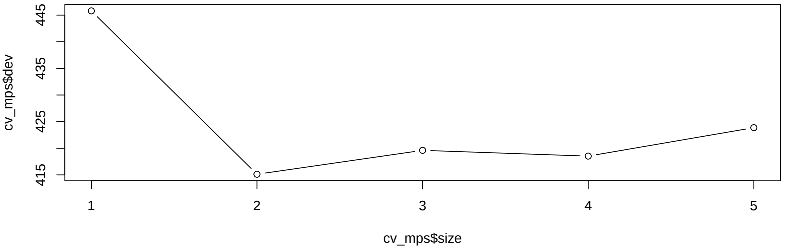


Figure 3: Subtree size 2 minimises SSR

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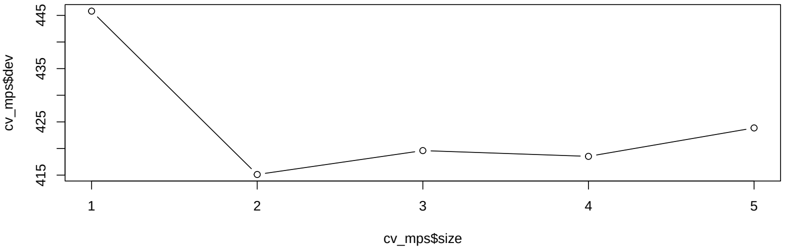

Figure 3: Subtree size 2 minimises SSR

Example: Regression Tree

```
prune_mps <- prune.tree(tree_mp, best = 2)

plot(prune_mps)
text(prune_mps)
```

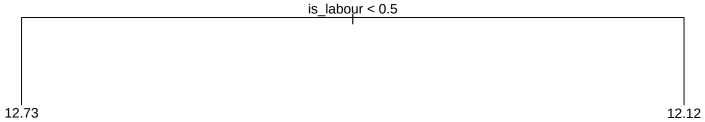

Figure 4: The pruned tree

Example: Regression Tree

Predict for test set:

- MSE for pruned: 1.922
- MSE for full: 1.945

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(Pretty good for 1 split!?)

Random Forests

Next: random forest algorithm

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Ensemble learning algorithms:

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Bagging: *b*ootstrap *agg*regation

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- (Linear regression: lower variance)

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- (Often choose $m \approx \sqrt{p}$)
- So, different splits consider different predictors
- So, trees will look very different to each other

Example: Random Forests

```r
library(randomForest)

# Full bag:
bag_mps <- randomForest(ln.net ~ ., data = mp_train,
                        ntree = 500, mtry = 4,
                        importance = TRUE)

# Decorrelate:
rf_mps <- randomForest(ln.net ~ ., data = mp_train,
                        ntree = 500, mtry = 2,
                       importance = TRUE)
```

Example: Random Forests

Predict:

```
preds_bag <- predict(bag_mps, newdata = mp_test)

preds_rf <- predict(rf_mps, newdata = mp_test)
```

- MSE for RF: 1.995
- MSE for full bag: 2.536

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Heterogeneous Treatment Effects

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Homogeneous and Heterogeneous Effects: Estimation

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```r
lm_out <- lm(ln.net ~ is_winner, data = mps)
lm_out
```

```
Call:
lm(formula = ln.net ~ is_winner, data = mps)

Coefficients:
(Intercept)    is_winner
    12.2464       0.5176
```

Homogeneous and Heterogeneous Effects: Estimation

Homogeneous effects:

```r
t.test(ln.net ~ is_winner, data = mps)
```

```

    Welch Two Sample t-test

data:  ln.net by is_winner
t = -3.9552, df = 287.65, p-value = 9.636e-05
alternative hypothesis: true difference in means betwe
95 percent confidence interval:
 -0.7751044 -0.2599998
sample estimates:
mean in group 0 mean in group 1
       12.24641        12.76396
```

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```
lm_out <- lm(ln.net ~ is_winner + is_labour +
               is_london + age, data = mps)
lm_out
```

```
Call:
lm(formula = ln.net ~ is_winner + is_labour + is_london + a
    data = mps)

Coefficients:
(Intercept)    is_winner    is_labour    is_london
  12.078838     0.398818    -0.477549     0.161134     0.00
```

Homogeneous and Heterogeneous Effects: Estimation

Homogeneous effects:

```
lm_lin(ln.net ~ is_winner, covariates = ~ is_labour + is_lo
```

```
                      Estimate  Std. Error      t valu
(Intercept)          1.226687e+01 0.078894901 155.483661
is_winner            3.459885e-01 0.131207672   2.636953
is_labour_c         -1.613663e-01 0.152608515  -1.057387
is_london_c          2.427360e-01 0.250214401   0.970111
age_c                4.740367e-03 0.007031323   0.674178
is_winner:is_labour_c -9.104022e-01 0.264395760  -3.443331
is_winner:is_london_c -8.847770e-02 0.426241818  -0.207576
is_winner:age_c     -4.778657e-05 0.012753800  -0.003746
                      CI Lower    CI Upper  DF
(Intercept)         12.111785723 12.42195044 416
is_winner            0.088075873  0.60390123 416
is_labour_c         -0.461346226  0.13861367 416
is_london_c         -0.249106208  0.73457813 416
```

Homogeneous and Heterogeneous Effects: Detection

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CATEs: Conditional ATEs

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- *Conditional average treatment effect* (CATE): avg treatment effect for subset of population
- Sometimes “CACE”
- Inference: not “evidence against TE $= 0$?”, but “evidence against $\text{CATE}_1 = \text{CATE}_2$?”

Homogeneous and Heterogeneous Effects: Estimation

Heterogeneous effects:

$$\text{Outcome} = \beta_0 + \beta_1 \text{Treatment} + \beta_2 \text{Group} + \beta_3 \text{Treatment} \cdot \text{Group} + \epsilon$$

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$$\text{Outcome} = \beta_0 + \beta_1 \text{Treatment} + \beta_2 \text{Group} + \beta_3 \text{Treatment} \cdot \text{Group} + \epsilon$$

- β_1 gives TE for `Group == 0`

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Heterogeneous effects:

$$\text{Outcome} = \beta_0 + \beta_1 \text{Treatment} + \beta_2 \text{Group} + \beta_3 \text{Treatment} \cdot \text{Group} + \epsilon$$

- β_1 gives TE for `Group == 0`
- $\beta_1 + \beta_3$ gives TE for `Group == 1`

Homogeneous and Heterogeneous Effects: Estimation

Heterogeneous effects:

```r
lm_out <- lm(ln.net ~ is_winner * is_labour +
               is_london + age, data = mps)
coef(lm_out) |> round(3)
```

```
        (Intercept)           is_winner           is_labou
             11.959               0.780             -0.16
                age is_winner:is_labour
              0.005              -0.914
```

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- Use $\hat{y}_{R_j}$ as pred value for obs in R_j
- (Tory, winner, London $\rightarrow$ 13.84)
- $$\hat{y}_{R_j} = \frac{1}{|R_j|} \sum_{i \in R_j} Y_i$$

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- Treateds in R_j provide good estimates of what controls in R_j would have done under treatment
- Controls in R_j provide good estimates of what treateds in R_j would have done under control
- Assignment w/in leaf R_j is as-good-as-random

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- Build a random forest (decorrelated deep trees picking from m predictors) of causal trees

Example: Causal Forests

```r
library(grf)

X <- mp_train |> select(age, is_labour, is_london)

W <- mp_train |> select(is_winner) |>
  unlist() |> as.numeric()

Y <- mp_train |> select(ln.net) |> unlist()

cf_out <- causal_forest(X, Y, W)
```

Example: Causal Forests

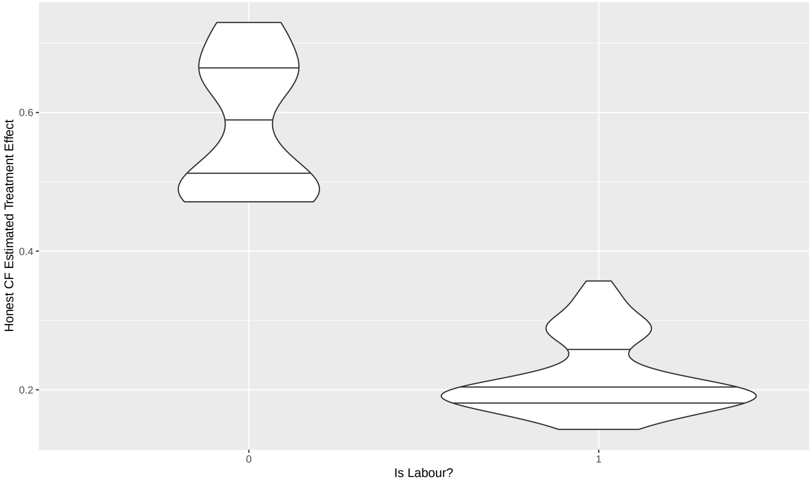
```
X_test <- mp_test |> select(age, is_labour, is_london)

cf_pred_est_var <- predict(cf_out, X_test,
                           estimate.variance = TRUE)

cf_preds <- cf_pred_est_var$predictions

df_cf <- tibble(X_test,
                cf_te = cf_preds,
                cf_se = sqrt(cf_pred_est_var$variance.e
                te_1se_lower = cf_te - cf_se,
                te_1se_upper = cf_te + cf_se)
```

Example: Causal Forests Results, Party



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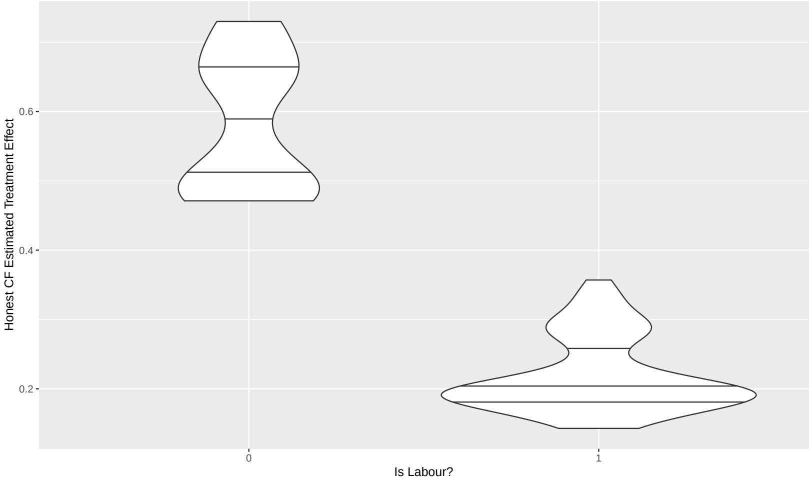

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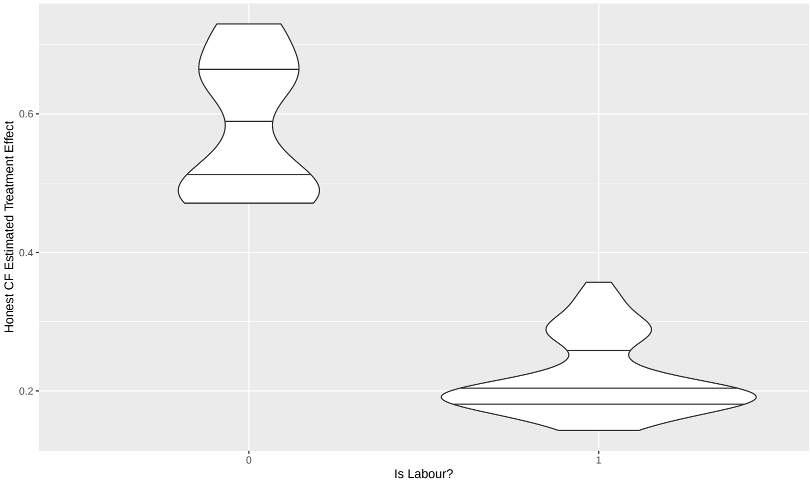

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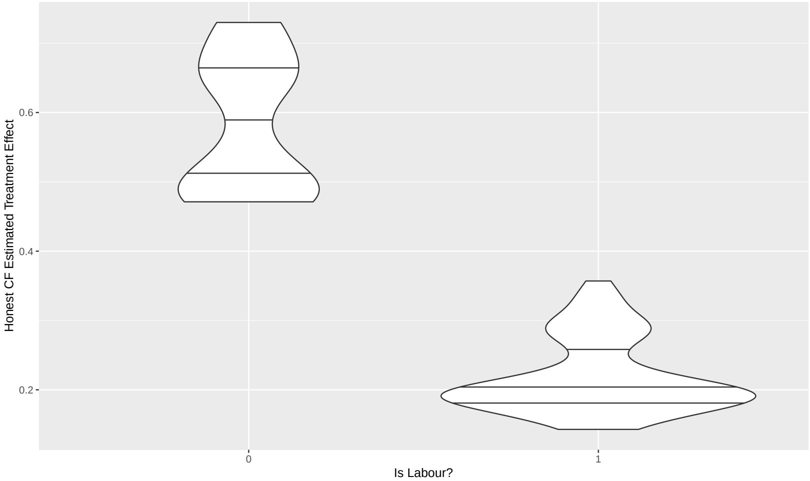

- Mean CF TE, Tory: $0.58 \rightsquigarrow £192,000$
- Mean CF TE, Labour: 0.219

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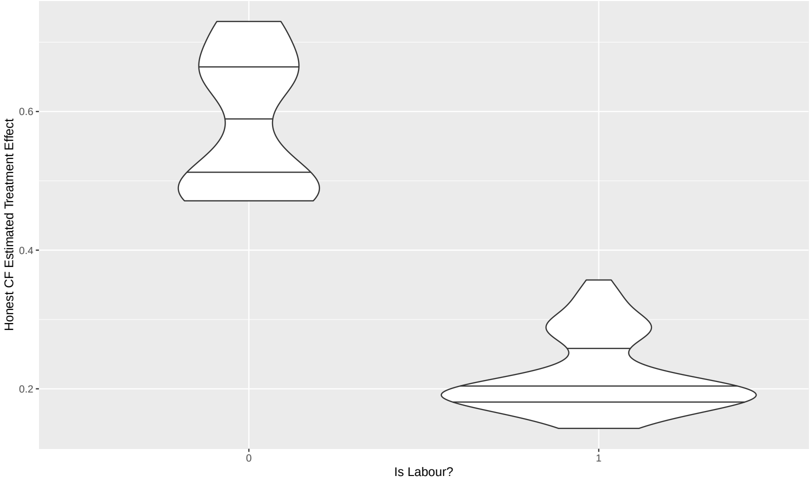
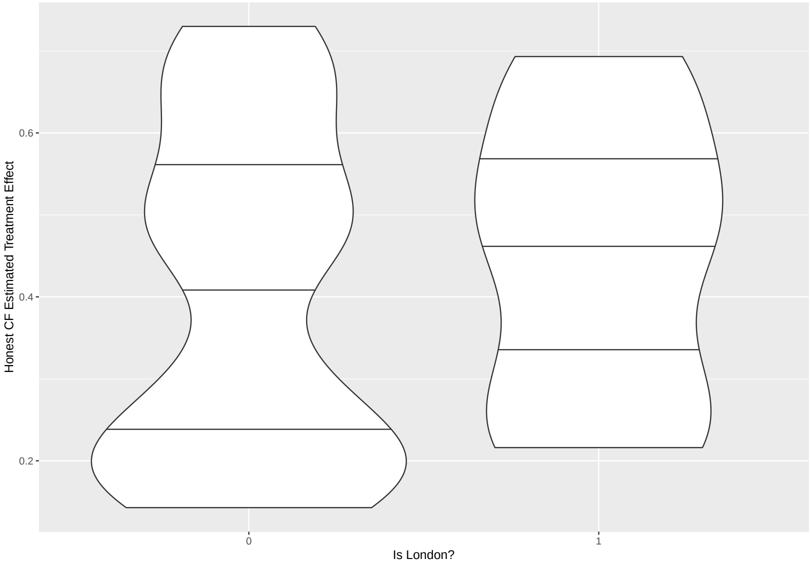
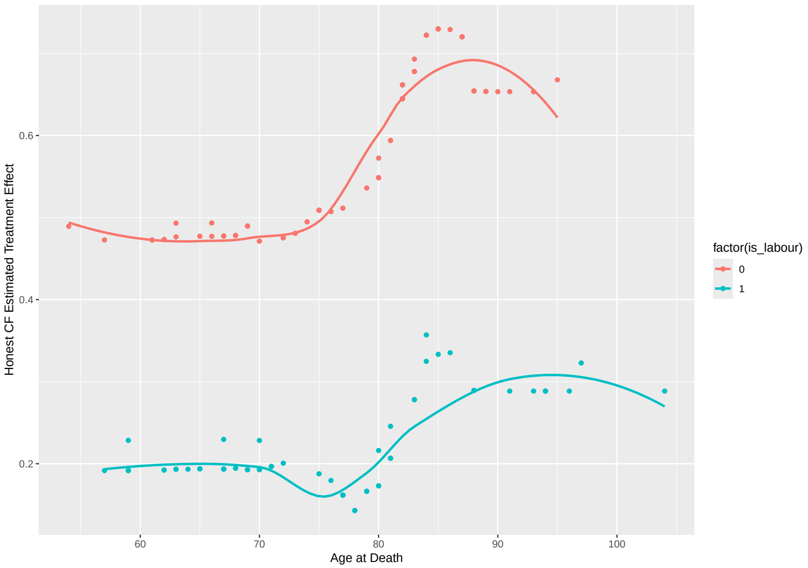


- Mean CF TE, Tory: $0.58 \rightsquigarrow £192,000$
- Mean CF TE, Labour: $0.219 \rightsquigarrow £60,000$

Example: Causal Forests Results, London



Example: Causal Forests Results, Age



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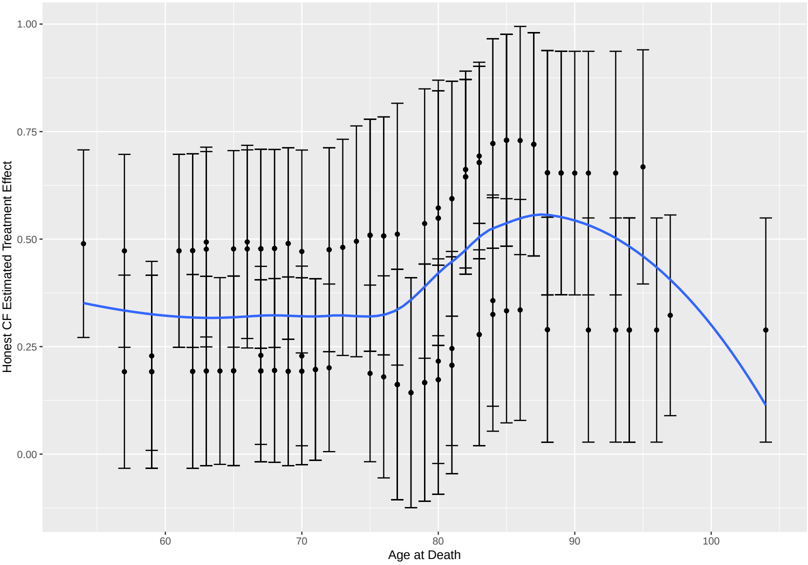

Variable Selection

Slide Title

Material.

Thanks!

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