Data Science for Causal Inference

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The Lab @ DC

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About Me

- Associate Prof of Government
  (American University)
- Associate Director, Center for Data Science
  (American University)
- Senior Social Scientist
  (The Lab @ DC)
- Fellow in Methodology
  (US Office of Evaluation Sciences: “OES”)

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- Fellow in Methodology
  (US Office of Evaluation Sciences: "OES")
- Research agenda: political methodology, causal inference, experimental design, experiments in public policy

About You!

- Name?

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- Role?

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- Interests?
- Olympic sport you look forward to?

Plan

- Data Science in Causal Inference

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- Data Science in Causal Inference
  - Models

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- Data Science in Causal Inference
  - Models
  - Heterogeneous treatment effects

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- Modern difference-in-difference designs

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  - Canonical DiD

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  - Staggered adoption

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  - Calloway-Sant'Anna approach

Data Science in Causal Inference

Causal Inference Approaches

The "potential outcomes" framework:

Causal Inference Approaches

The "potential outcomes" framework:

Citizen	Canvass?	Would Enroll if Canvass?	Would Enroll if No Canvass?	Enroll
1	Yes	Yes		Yes
2	Yes			Yes
3	No			No
4	No			No

Causal Inference Approaches

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3	No	(Yes)	No	No
4	No	(No)	No	No

Causal Inference Approaches

The "potential outcomes" framework, more abstractly:

Unit i	Treatment T	$Y(1)$	$Y(0)$	Y^{obs}	True τ $Y(1) - Y(0)$
1	1	10		10	
2	1	20		20	
3	0		15	15	
4	0		5	5	

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				$\widehat{ATE} = \hat{\bar{\tau}} =$	$15 - 10 = 5$

Causal Inference Approaches

The "potential outcomes" framework, notation:

- Units indexed by i
- Treatment T_i or D_i or Z_i
- Outcome if treated $Y_i(1)$
- Outcome if control $Y_i(0)$
- True treatment effect $\tau_i = Y_i(1) - Y_i(0)$
- True average treatment effect $\bar{\tau} = \frac{1}{n}\sum_{i=1}^{n}(Y_i(1) - Y_i(0))$
- Pre-treatment covariates $\mathbf{X}$

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(and we'll draw some DAG's, too)

Data Science Approaches

Three tasks of data science:

- Description

Data Science Approaches

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Models/algorithms central to all three.

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Models/algorithms central to all three.

Hernán, Hsu, and Healy (2019)

Data Science Approaches

Description

- Identifying patterns, etc.

Data Science Approaches

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- Identifying patterns, etc.
- E.g., clustering to discover groups

Data Science Approaches

Prediction

- Components

Data Science Approaches

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  - Inputs/outputs (predictors/outcomes, features/responses, ...)

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- With these, model machine learning does the work

Data Science Approaches

Prediction

- Components
  - Inputs/outputs (predictors/outcomes, features/responses, ...)
  - Mapping from inputs to outputs (linear model, decision tree, ...)
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- With these, model machine learning does the work
- E.g., regression, random forests, neural networks, ...

Data Science Approaches

Causal Inference

- Potential outcomes/counterfactual/interventionist perspective

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- Requires *expertise* different to description/prediction

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  - Not all inputs treated same

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  - T v. $\mathbf{X}$ – very different!

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  - (alternative: solve fundamental problem of causal inference!)

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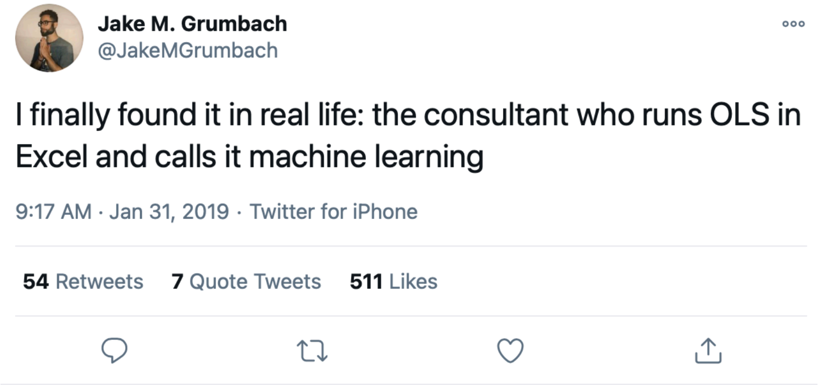
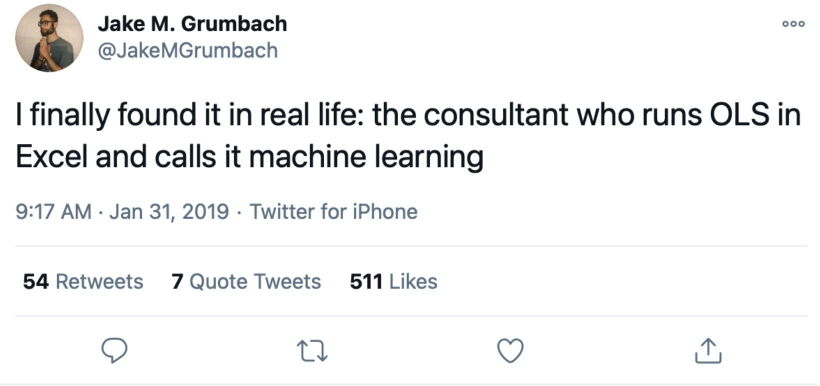
  - Not all inputs treated same
  - T v. $\mathbf{X}$ – very different!
  - (the more knowledge, the better!)
  - (alternative: solve fundamental problem of causal inference!)
- E.g., experiments, observational causal designs, ...

Causal Inference with Machine Learning

Causal Inference with Machine Learning



Causal Inference with Machine Learning



(OK, not "machine learning", perhaps, but *models* at least ...)

Causal Inference with Models

Loaded two datasets:

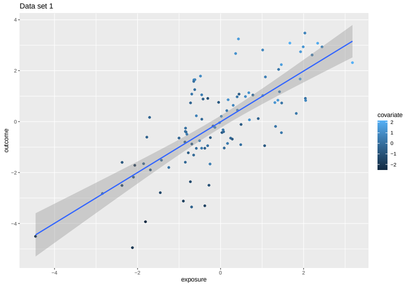
```
str(df1)
```

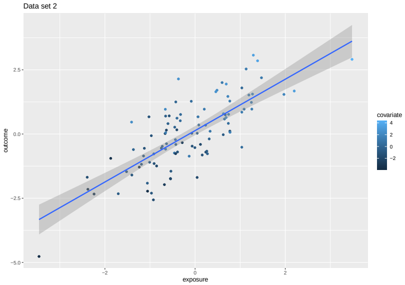
```
tibble [100 x 3] (S3: tbl_df/tbl/data.frame)
 $ covariate: num [1:100] -0.622 1.137 -0.238 1.529 -0.154
 $ exposure : num [1:100] 0.0332 0.3627 0.2422 1.4633 0.779
 $ outcome  : num [1:100] -0.429 2.675 -0.647 2.238 1.044
```

```
str(df2)
```

```
tibble [100 x 3] (S3: tbl_df/tbl/data.frame)
 $ exposure : num [1:100] 0.4862 0.0653 -1.4021 -0.546 -0.
 $ outcome  : num [1:100] 1.706 0.669 -1.597 -1.733 0.617
 $ covariate: num [1:100] 2.24 0.924 -0.999 -2.343 0.207 ..
```

Causal Inference with Models





Causal Inference with Models

Model each

```r
lm_df1 <- lm(outcome ~ exposure, data = df1)
lm_df2 <- lm(outcome ~ exposure, data = df2)
```

```
# A tibble: 4 x 4
  data  term        estimate std.error
  <chr> <chr>          <dbl>     <dbl>
1 df1   (Intercept) -0.00671    0.120
2 df1   exposure     0.996      0.0927
3 df2   (Intercept)  0.133      0.0890
4 df2   exposure     1.00       0.0841
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- Both cases: effect of exposure ≈ 1.

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```

- Both cases: effect of exposure ≈ 1.
- Is this good?
- What if we adjust for covariate?

Causal Inference with Models

```r
lm_df1_adj <- lm(outcome ~ exposure + covariate, data = df1
lm_df2_adj <- lm(outcome ~ exposure + covariate, data = df2
```

```
# A tibble: 4 x 4
  data  term      estimate std.error
  <chr> <chr>        <dbl>     <dbl>
1 df1   exposure     0.501    0.108
2 df1   covariate    0.970    0.147
3 df2   exposure     0.554    0.0990
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```

- Both cases: effect of exposure ≈ 0.5.

Causal Inference with Models

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- Which is correct? $\beta = 1$? $\beta = 0.5$?

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- Both cases: effect of exposure ≈ 0.5.
- Is this good?
- Which is correct? $\beta = 1$? $\beta = 0.5$?
- *Should* we adjust for covariate?

Causal Inference with Models

There is nothing in the data that tells us.

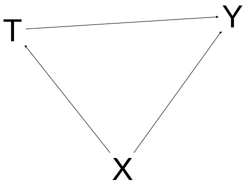
Causal Inference with Models

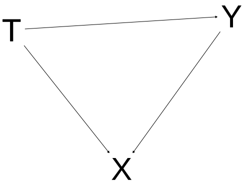
There is nothing in the data that tells us. ☹

Causal Inference with Models

There is nothing in the data that tells us. ☹

Here are the true structures:





Causal Inference with Models

When know structures, adjustment sets for unbiasedness differ:

- `df1`: confounding $\Rightarrow$ **adjust for** X
- `df2`: collider $\Rightarrow$ **do not adjust for** X

```
g_conf <- dagitty("dag{ x -> y ; x <- c -> y }")
g_coll <- dagitty("dag{ x -> y ; x -> c <- y }")

adjustmentSets(g_conf, "x", "y")
```

```
{ c }
```

```
adjustmentSets(g_coll, "x", "y")
```

```
{}
```

Causal Inference with Models

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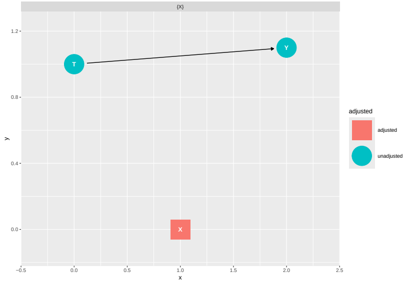

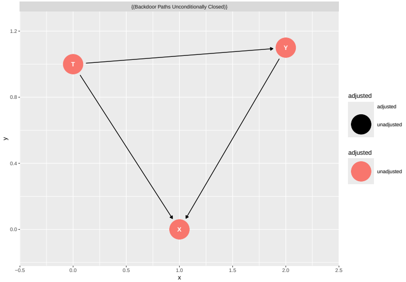

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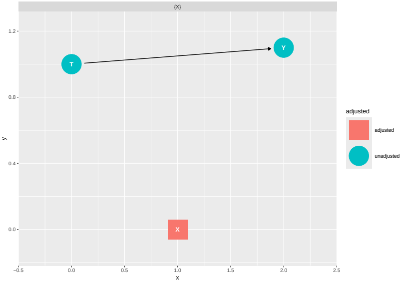

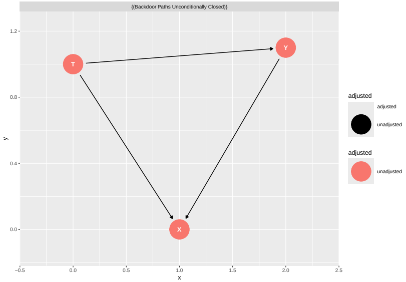

(Data from D'Agostino McGowan (2023))

Causal Inference with Models

- Importance of identifying "pre-treatment covariates", "proper covariates"; doing "design before analysis"

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- Though, methods from prediction can aid causal inference

Causal Inference with Models

- Importance of identifying “pre-treatment covariates”, “proper covariates”; doing “design before analysis”
- Importance of experiments: strong knowledge about (part of) causal structure
- Causal inference is critical to scientific questions, and separate from prediction
- Though, methods from prediction can aid causal inference
- (A perspective on “causal euphimisms”: Hernán (2018))

Approaches of Prediction and Causal Inference

Two Cultures, (Breiman 2001)

- *Data Models*: our "social science modeling"
- *Algorithmic Models*: our "data science algorithms"

Methods for Prediction and Causal Inference

- Cross-validation
- Regression/Decision trees
- Random forests

James et al. (2021)

Cross-validation

k-fold cross-validation

- Randomly partition data into k groups
- Apply method to $k-1$ groups
- Use result to predict for left-out group
- Calculate $\text{MSE}_i = \frac{1}{n}\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2$
- Calculate test error as average of the k MSE's:

$$CV_{(k)} = \frac{1}{k}\sum_{i=1}^{k}\text{MSE}_i$$

- Select model that minimises $CV_{(k)}$

CV for Linear Model

```r
library(tidyverse)

## Make data

mk_data <- function(n = 90, n_folds = 10){

  df <- tibble(
    x1 = rnorm(n),
    x2 = rnorm(n),
    x3 = rnorm(n),
    y = 0.1 * x1 + 0.2 * x2 + 0.5 * x3 + rnorm(n),
    cv_fold = sample(rep(1:n_folds, (n / n_folds)))
  )

}

df <- mk_data()
```

CV for Linear Model

```r
head(df)
```

```
# A tibble: 6 x 5
      x1     x2      x3      y cv_fold
   <dbl>  <dbl>   <dbl>  <dbl>   <int>
1  1.81   0.827 -0.0417 -0.841       2
2  0.336 -1.42   0.314   0.197       7
3  0.372  0.602 -0.601   0.424       1
4  0.648  1.27  -0.122   1.22        2
5 -1.62  -0.670  1.25    0.158       4
6  0.841 -0.318 -0.104   0.561       6
```

CV for Linear Model

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5 -1.62  -0.670  1.25    0.158       4
6  0.841 -0.318 -0.104   0.561       6
```

```r
table(df$cv_fold)
```

```

 1  2  3  4  5  6  7  8  9 10 
 9  9  9  9  9  9  9  9  9  9 
```

CV for Linear Model

```
cv_lm <- function(data, fmla){

  n_folds <- max(data$cv_fold)
  store_mses <- vector("numeric", length = n_folds)

  for(idx in 1:n_folds){

    df_train <- data |> filter(cv_fold != idx)
    df_test <- data |> filter(cv_fold == idx)

    lm_out <- lm(fmla, data = df_train)

    predictions <- predict(lm_out, newdata = df_test)

    store_mses[idx] <- mean((df_test$y - predictions)^2)}

  test_error_cv_k <- mean(store_mses)
  return(test_error_cv_k)
```

CV for Linear Model

```r
cv_lm(data = df, fmla = y ~ x1 + x2)
```

```
[1] 1.323677
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```
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```

```
df <- mk_data()
cv_lm(df, y ~ x1 + x2 + x3)
```

```
[1] 0.9505413
```

CV for Linear Model

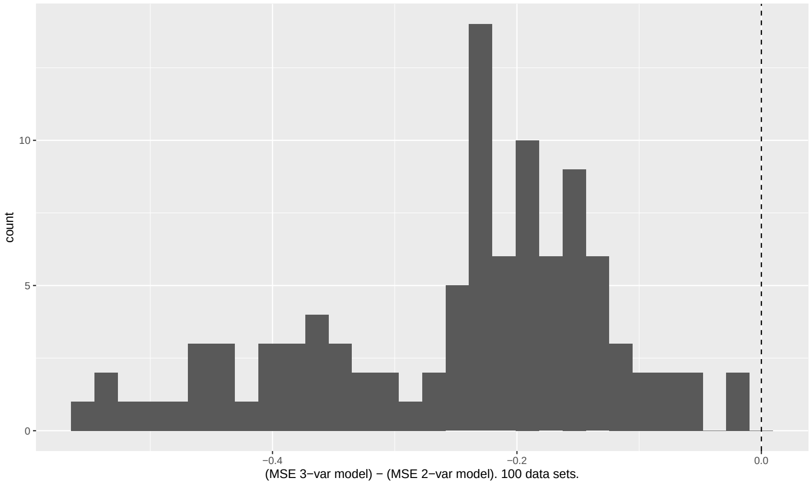

Figure 1: MSE always less (better) for 3-variable model.

Regression Trees

- Partition predictor space into regions $R_1, R_2, \ldots, R_J$.
- If unit falls in region R_j, use average outcome in R_j as predicted value: $\hat{y}_{R_j}$
- (For *decision* on discrete outcome, count votes in R_j)
- Goal: minimise residual sum of squares (RSS), just like LS regression:

$$\sum_{j=1}^{J} \sum_{i \in R_j} \left(y_i - \hat{y}_{R_j} \right)$$

Regression Trees

How to define regions R_j?

Regression Trees

How to define regions R_j?

- Top-down, greedy recursive binary split
- At each step, find predictor and cut-point that minimise

$$\sum_{i: x \in R_1(j,s)} \left(y_i - \hat{y}_{R_1(j,s)}\right)^2 + \sum_{i: x \in R_2(j,s)} \left(y_i - \hat{y}_{R_2(j,s)}\right)^2$$

Regression Trees

- Overfitting is a potential problem

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- Can we increase predictive quality by only using *part* of a tree?

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- Can we increase predictive quality by only using *part* of a tree?
- "Pruning"

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- Select the subtree that gives least prediction error (via cross-validation)

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$$\sum_{m=1}^{|T|} \sum_{i:x_i \in R_m} \left(y_i - \hat{y}_{R_m}\right)^2 + \alpha|T|$$

Sum squared pred. error (plus penalty that grows with tree size) across units in region, then regions.

Regression Trees

But, how to choose α? (Use cross-validation.)

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4. Using that α, select best subtree from Step 2

Example: Regression Tree

```
library(qss)
library(rsample)
library(tree)

data("MPs")
mps <- MPs |> mutate(age = yod - yob,
                     is_labour = if_else(party == "labour",
                     is_london = if_else(region == "Greater
                     is_winner = if_else(margin > 0, 1, 0))
  select(ln.net, age, is_labour, is_london, is_winner) |>
  na.omit()

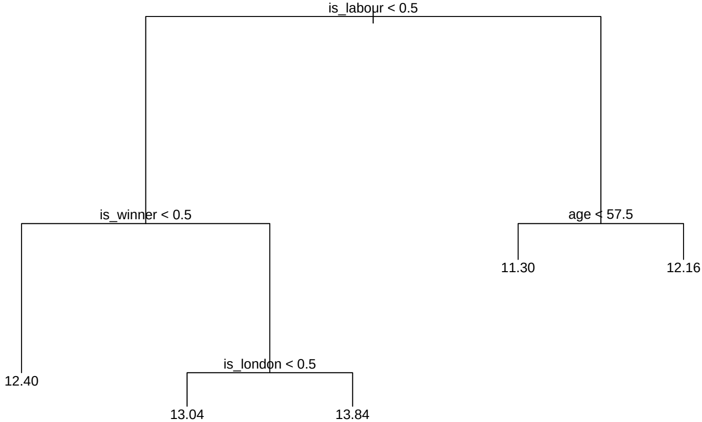
set.seed(765076184)

mp_split <- initial_split(mps, prop = 0.7)

mp_train <- training(mp_split)
mp_test <- testing(mp_split)
```

Example: Regression Tree

```
plot(tree_mp)
text(tree_mp)
```



Example: Regression Tree

Would pruning help?

```
cv_mps <- cv.tree(tree_mp, K = 10)

plot(cv_mps$size, cv_mps$dev, type = "b")
```

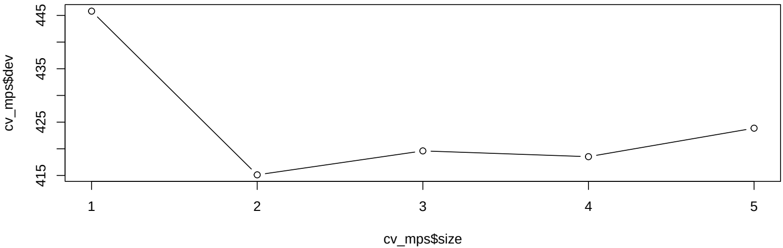


Figure 3: Subtree size 2 minimises SSR

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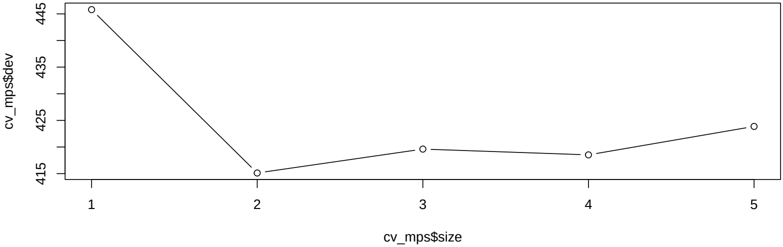

Figure 3: Subtree size 2 minimises SSR

Example: Regression Tree

```
prune_mps <- prune.tree(tree_mp, best = 2)

plot(prune_mps)
text(prune_mps)
```

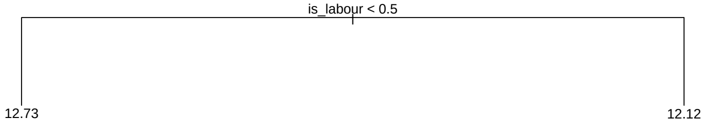

Figure 4: The pruned tree

Example: Regression Tree

Predict for test set:

- MSE for pruned: 1.922
- MSE for full: 1.945

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(Pretty good for 1 split!?)

Random Forests

Next: random forest algorithm

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Ensemble learning algorithms:

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- Boosting: models build on prior models $\rightsquigarrow$ pick feature, predict, upweight mispredicted data, ... Do several times and combine.

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Bagging: *b*ootstrap *agg*regation

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- Bagging averages over data subsets, reducing variance
- (Linear regression: lower variance)

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- (Often choose $m \approx \sqrt{p}$)
- So, different splits consider different predictors
- So, trees will look very different to each other

Example: Random Forests

```r
library(randomForest)

# Full bag:
bag_mps <- randomForest(ln.net ~ ., data = mp_train,
                        ntree = 500, mtry = 4,
                        importance = TRUE)

# Decorrelate:
rf_mps <- randomForest(ln.net ~ ., data = mp_train,
                        ntree = 500, mtry = 2,
                       importance = TRUE)
```

Example: Random Forests

Predict:

```r
preds_bag <- predict(bag_mps, newdata = mp_test)

preds_rf <- predict(rf_mps, newdata = mp_test)
```

- MSE for RF: 1.995
- MSE for full bag: 2.536

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So, decorrelating helped us avoid some overfitting to each bootstrap subsample (and thus, reduced variance).

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(Typical pred error of $\sqrt{1.995} \approx 1.412$)

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Heterogeneous Treatment Effects

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Homogeneous and Heterogeneous Effects: Estimation

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$$\text{Outcome} = \beta_0 + \beta_1 \text{Treatment} + \epsilon$$

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```r
lm_out <- lm(ln.net ~ is_winner, data = mps)
lm_out
```

```

Call:
lm(formula = ln.net ~ is_winner, data = mps)

Coefficients:
(Intercept)    is_winner
    12.2464       0.5176
```

Homogeneous and Heterogeneous Effects: Estimation

Homogeneous effects:

```r
t.test(ln.net ~ is_winner, data = mps)
```

```

    Welch Two Sample t-test

data:  ln.net by is_winner
t = -3.9552, df = 287.65, p-value = 9.636e-05
alternative hypothesis: true difference in means betwee
95 percent confidence interval:
 -0.7751044 -0.2599998
sample estimates:
mean in group 0 mean in group 1
       12.24641        12.76396
```

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$$\text{Outcome} = \beta_0 + \beta_1 \text{Treatment} + \sum \beta_j X_j + \epsilon$$

```
lm_out <- lm(ln.net ~ is_winner + is_labour +
               is_london + age, data = mps)
lm_out
```

```
Call:
lm(formula = ln.net ~ is_winner + is_labour + is_london + a
    data = mps)

Coefficients:
(Intercept)    is_winner    is_labour    is_london
  12.078838     0.398818    -0.477549     0.161134     0.00
```

Homogeneous and Heterogeneous Effects: Estimation

Homogeneous effects:

```
lm_lin(ln.net ~ is_winner, covariates = ~ is_labour + is_lo
```

```
                        Estimate  Std. Error        t valu
(Intercept)          1.226687e+01 0.078894901 155.4836617
is_winner            3.459885e-01 0.131207672   2.6369536
is_labour_c         -1.613663e-01 0.152608515  -1.0573871
is_london_c          2.427360e-01 0.250214401   0.9701118
age_c                4.740367e-03 0.007031323   0.6741786
is_winner:is_labour_c -9.104022e-01 0.264395760  -3.4433313
is_winner:is_london_c -8.847770e-02 0.426241818  -0.2075763
is_winner:age_c     -4.778657e-05 0.012753800  -0.0037468
                       CI Lower    CI Upper  DF
(Intercept)          12.111785723 12.42195044 416
is_winner             0.088075873  0.60390123 416
is_labour_c          -0.461346226  0.13861367 416
is_london_c          -0.249106208  0.73457813 416
```

Homogeneous and Heterogeneous Effects: Detection

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CATEs: Conditional ATEs

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- *Conditional average treatment effect* (CATE): avg treatment effect for subset of population
- Sometimes "CACE"
- Inference: not "evidence against TE = 0?", but "evidence against $\text{CATE}_1 = \text{CATE}_2$?"

Homogeneous and Heterogeneous Effects: Estimation

Heterogeneous effects:

$$\text{Outcome} = \beta_0 + \beta_1 \text{Treatment} + \beta_2 \text{Group} + \beta_3 \text{Treatment} \cdot \text{Group} + \epsilon$$

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$$\text{Outcome} = \beta_0 + \beta_1 \text{Treatment} + \beta_2 \text{Group} + \beta_3 \text{Treatment} \cdot \text{Group} + \epsilon$$

- β_1 gives TE for `Group == 0`

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Heterogeneous effects:

$$\text{Outcome} = \beta_0 + \beta_1 \text{Treatment} + \beta_2 \text{Group} + \beta_3 \text{Treatment} \cdot \text{Group} + \epsilon$$

- β_1 gives TE for `Group == 0`
- $\beta_1 + \beta_3$ gives TE for `Group == 1`

Homogeneous and Heterogeneous Effects: Estimation

Heterogeneous effects:

```
lm_out <- lm(ln.net ~ is_winner * is_labour +
               is_london + age, data = mps)
coef(lm_out) |> round(3)
```

```
         (Intercept)            is_winner           is_labou
              11.959                0.780             -0.16
                 age is_winner:is_labour
               0.005               -0.914
```

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- Our regression trees had terminal nodes ("leaves") that were sufficiently homogeneous for prediction.
- Use $\hat{y}_{R_j}$ as pred value for obs in R_j
- (Tory, winner, London $\rightarrow$ 13.84)
- $$\hat{y}_{R_j} = \frac{1}{|R_j|} \sum_{i \in R_j} Y_i$$

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- Treateds in R_j provide good estimates of what controls in R_j would have done under treatment
- Controls in R_j provide good estimates of what treateds in R_j would have done under control
- Assignment w/in leaf R_j is as-good-as-random

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- Treateds in R_j provide good estimates of what controls in R_j would have done under treatment
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  - Splitting cannot use y_i from $\mathcal{I}$
  - Prediction, estimation of $\hat{\hat{\tau}}$ uses only $\mathcal{I}$
- Build a random forest (decorrelated deep trees picking from m predictors) of causal trees

Example: Causal Forests

```r
library(grf)

X <- mp_train |> select(age, is_labour, is_london)

W <- mp_train |> select(is_winner) |>
  unlist() |> as.numeric()

Y <- mp_train |> select(ln.net) |> unlist()

cf_out <- causal_forest(X, Y, W)
```

Example: Causal Forests

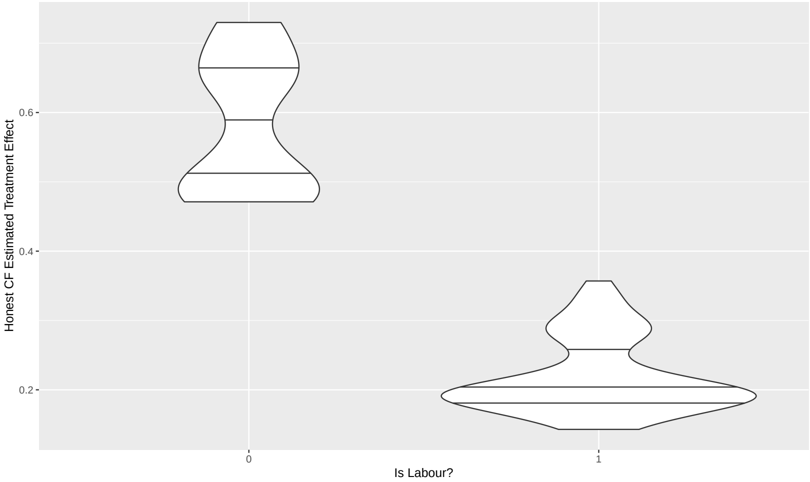
```
X_test <- mp_test |> select(age, is_labour, is_london)

cf_pred_est_var <- predict(cf_out, X_test,
                           estimate.variance = TRUE)

cf_preds <- cf_pred_est_var$predictions

df_cf <- tibble(X_test,
                cf_te = cf_preds,
                cf_se = sqrt(cf_pred_est_var$variance.e
                te_1se_lower = cf_te - cf_se,
                te_1se_upper = cf_te + cf_se)
```

Example: Causal Forests Results, Party



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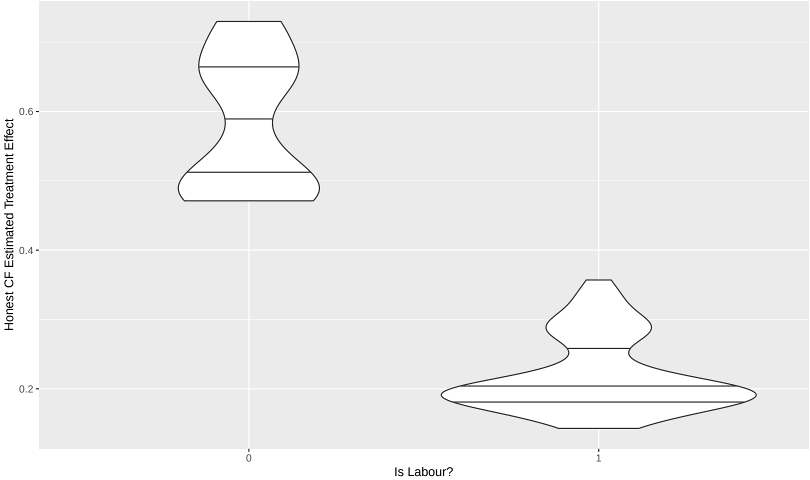

- Mean CF TE, Tory: 0.58

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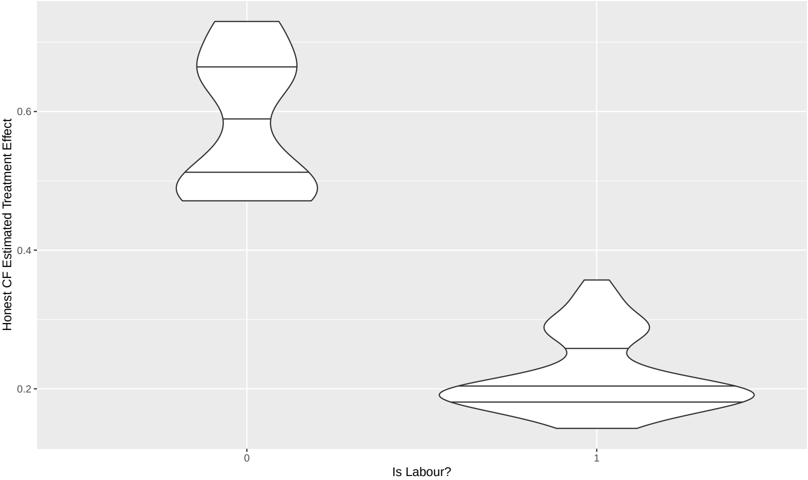

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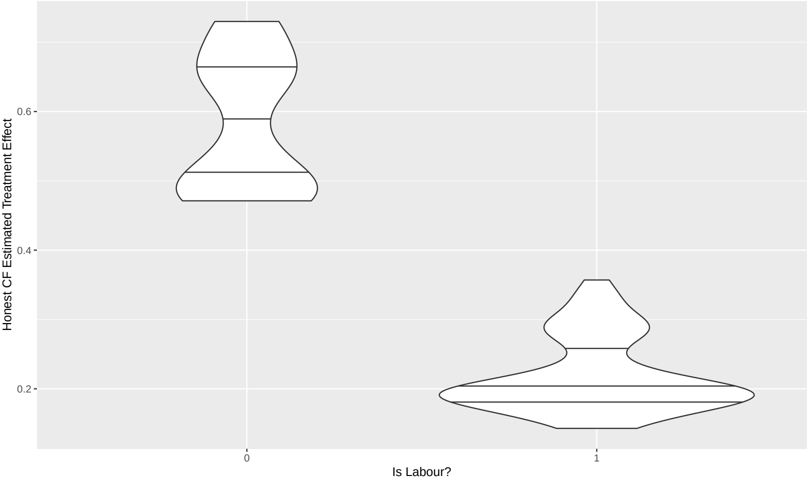

- Mean CF TE, Tory: $0.58 \rightsquigarrow £192,000$
- Mean CF TE, Labour: 0.219

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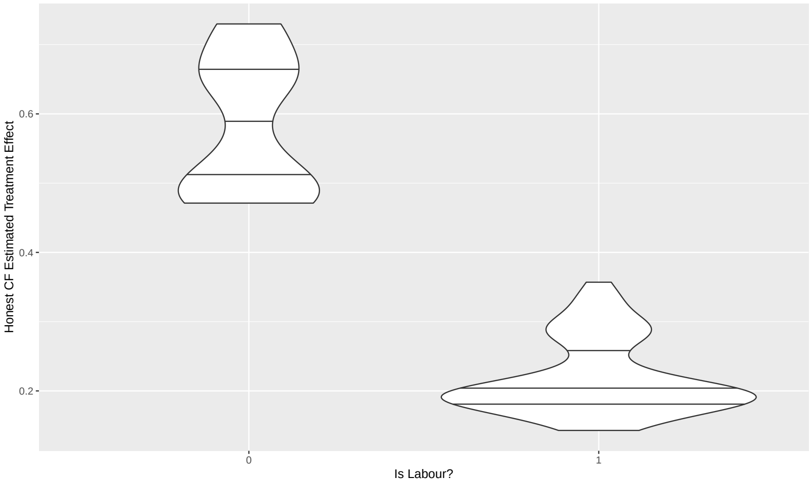
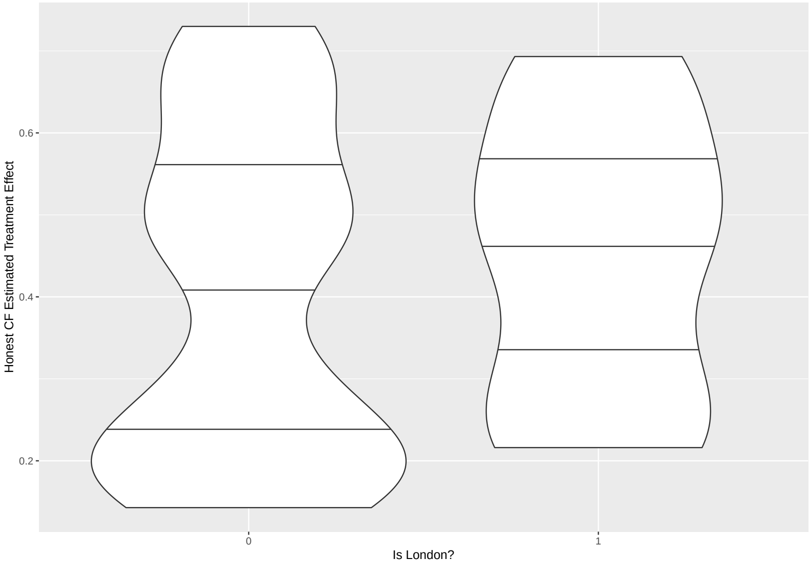
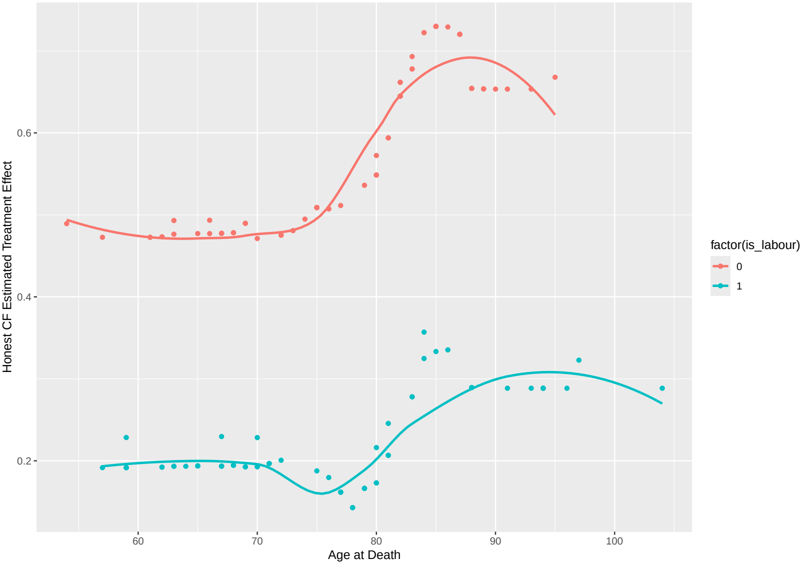


- Mean CF TE, Tory: $0.58 \rightsquigarrow £192,000$
- Mean CF TE, Labour: $0.219 \rightsquigarrow £60,000$

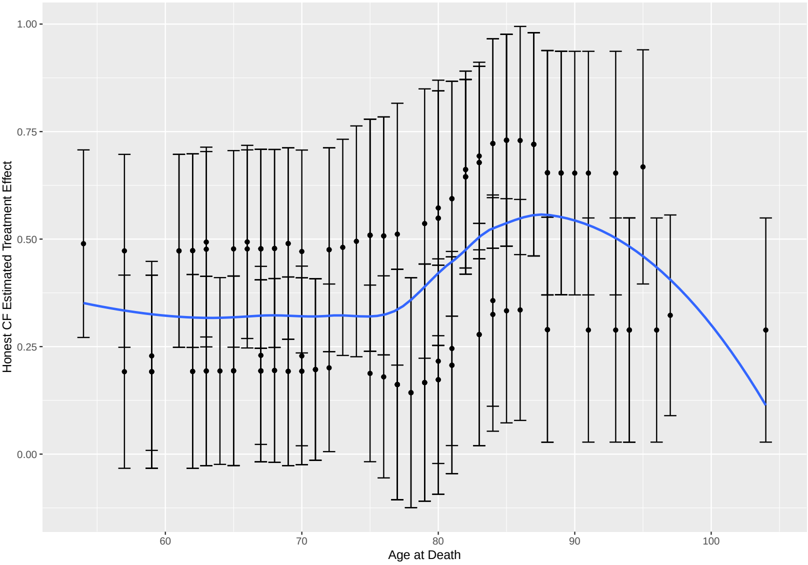
Example: Causal Forests Results, London



Example: Causal Forests Results, Age



Example: Causal Forests Results, Age



Variable Selection

Feature Selection

- Wrappers: pick subset of covars, train on data (estimate model), test on hold-out, score predictions. Keep best-scoring subset.

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- Filters: correlate covars with outcome. Keep strongest.
- Embeds: select features and estimate model at same time. Penalize using more predictors.

Regularization Methods

OLS reminder

Minimize SSR:

$$\text{argmin}_\beta \sum_{i=1}^{n} \left(y_i - \hat{y}_i\right)^2$$

$$\text{argmin}_\beta \sum_{i=1}^{n} \left(\mathbf{y} - \mathbf{X}\hat{\beta}\right)^2$$

Embedded Regularization Methods

L1 regularization: the LASSO (Least Absolute Shrinkage and Selection Operator)

$$\text{argmin}_{\beta} \left[\sum_{i=1}^{n} \left(y_i - \mathbf{X}\hat{\beta} \right)^2 + \lambda \sum_{j=1}^{k} |\hat{\beta}_j| \right]$$

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L2 regularization: Ridge regression

$$\text{argmin}_{\beta}\left[\sum_{i=1}^{n}\left(y_i - \mathbf{X}\hat{\beta}\right)^2 + \lambda\sum_{j=1}^{k}\hat{\beta}_j^2\right]$$

Embedded Regularization Methods

Mix L1 and L2: Elastic net

$$\mathrm{argmin}_{\beta}\left(\frac{\sum_{i=1}^{n}\left(y_i - \mathbf{X}\widehat{\beta}\right)^2}{2n} + \lambda\left[\alpha\sum_{j=1}^{k}|\widehat{\beta}_j| + \frac{1-\alpha}{2}\sum_{j=1}^{k}\widehat{\beta}_j^2\right]\right)$$

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$$\operatorname{argmin}_{\beta} \left(\frac{\sum_{i=1}^{n} \left(y_i - \mathbf{X}\hat{\beta} \right)^2}{2n} + \lambda \left[\alpha \sum_{j=1}^{k} |\hat{\beta}_j| + \frac{1-\alpha}{2} \sum_{j=1}^{k} \hat{\beta}_j^2 \right] \right)$$

Regularized trees, ...

Embedded Regularization Methods

How to choose λ, α?

Embedded Regularization Methods

How to choose λ, α?

The LASSO

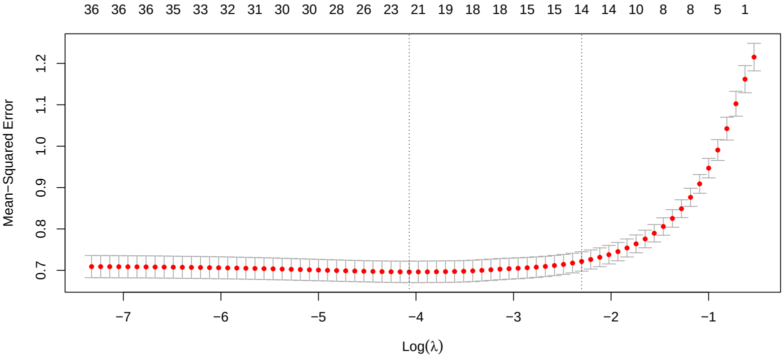
The LASSO

Cross-validation for λ:

```r
df_lasso <- read_csv("~/Desktop/lasso.csv")

X <- as.matrix(df_lasso[, 2:ncol(df_lasso)])

Y <- as.matrix(df_lasso[, "y"])

library(glmnet)

cv_lasso <- cv.glmnet(X, Y, alpha = 1)
```

The LASSO

```r
plot(cv_lasso)
```




```r
cv_lasso$lambda.min
```

```
[1] 0.0170891
```

The LASSO

Implement:

```
lasso_out <- glmnet(X, Y, alpha = 1,
                    lambda = cv_lasso$lambda.min)

lasso_out
```

```
Call:  glmnet(x = X, y = Y, alpha = 1, lambda = cv_lasso$la

  Df  %Dev  Lambda
1 21 45.32 0.01709
```

The LASSO

Coefficients:

```r
coef_lasso <- coef(lasso_out)
round(coef_lasso, 3)
```

```
37 x 1 sparse Matrix of class "dgCMatrix"
                  s0
(Intercept)    0.000
x1             0.112
x2             0.095
x3             0.086
x4             0.147
x5             0.002
x6             0.063
x7             0.051
x8             0.074
x9             0.042
x10            .
```

The LASSO

Coefficients:

```
round(coef_lasso[, ], 3)
```

```
(Intercept)          x1          x2          x3          x4
      0.000       0.112       0.095       0.086       0.147
         x6          x7          x8          x9         x10
      0.063       0.051       0.074       0.042       0.000
        x12         x13         x14         x15         x16
      0.039       0.000       0.026       0.000       0.000
        x18         x19         x20         x21         x22
      0.010       0.127      -0.015       0.030       0.000
        x24         x25         x26         x27         x28
      0.000       0.000       0.000       0.000      -0.010
        x30         x31         x32         x33         x34
      0.000       0.028       0.032       0.000      -0.041
        x36
      0.048
```

The LASSO

Implement, alternative λ:

```
lasso_1se <- glmnet(X, Y, alpha = 1,
                    lambda = cv_lasso$lambda.1se)

coef(lasso_1se)
```

```
37 x 1 sparse Matrix of class "dgCMatrix"
                       s0
(Intercept) -0.0003034087
x1           0.1051188782
x2           0.0898842045
x3           0.0742522801
x4           0.1513883536
x5           .
x6           0.0603811184
x7           0.0389489143
x8           0.0575738993
x9           0.0374420416
```

The LASSO

Coefficients:

```r
round(coef(lasso_1se)[, ], 3)
```

```
(Intercept)          x1          x2          x3          x4
      0.000       0.105       0.090       0.074       0.15
         x6          x7          x8          x9         x1
      0.060       0.039       0.058       0.037       0.00
        x12         x13         x14         x15         x1
      0.029       0.000       0.008       0.000       0.00
        x18         x19         x20         x21         x2
      0.004       0.039       0.000       0.000       0.00
        x24         x25         x26         x27         x2
      0.000       0.000       0.000       0.000       0.00
        x30         x31         x32         x33         x3
      0.000       0.000       0.013       0.000       0.00
        x36
      0.030
```

The Double LASSO for Treatment Effects

The idea:

- covariates may $\rightsquigarrow Y$ or $\rightsquigarrow T$

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- $\approx$ "double robust", "AIPW" estimators

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- covariates may $\rightsquigarrow Y$ or $\rightsquigarrow T$
- $\approx$ “double robust”, “AIPW” estimators
- (different to just “doing LASSO twice” for regularization + shrinkage)

The Double LASSO for Treatment Effects

1. Model $Y = f(X)$ using LASSO

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The Double LASSO for Treatment Effects

```r
library(hdm)
```

```r
data(social)

df_social <- social |> mutate(is_male = if_else(sex == "mal
                                  age = 2006 - yearofbirth,
                                  is_neighbors = if_else(messag
  filter(messages %in% c("Neighbors", "Control"))

rlasso_out <- rlassoATE(primary2006 ~ age + is_male + prima
```

The Double LASSO for Treatment Effects

```r
summary(rlasso_out)
```

```
Estimation and significance testing of the treatment effect
Type: ATE
Bootstrap: not applicable
     coeff.      se. t-value p-value
TE 0.080091 0.002625   30.51  <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 '
```

The Double LASSO for Treatment Effects

```
X <- as.matrix(df_social[, c("age", "is_male", "primary2004

Y <- as.matrix(df_social[, "primary2006"])

D <- as.matrix(df_social[, "is_neighbors"])

summary(rlassoEffects(X, Y, method = "double selection"))
```

```
[1] "Estimates and significance testing of the effect of ta
              Estimate. Std. Error t value Pr(>|t|)
age           0.0038449  0.0000681  56.456  < 2e-16 ***
is_male       0.0086763  0.0018889   4.593 4.36e-06 ***
primary2004   0.1474364  0.0019924  74.000  < 2e-16 ***
hhsize        0.0004260  0.0012618   0.338    0.736
is_neighbors  0.0802361  0.0026278  30.534  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 '
```

R packages for Regularization, etc.

- `glmnet`
- `caret`

See also `tidymodels`, `parsnip`, ...

Embedded Regularization Methods

Thanks!

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