Modern Difference-in-Difference Designs

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  - pre-tr outcome affected by (obs/unobs) predictors of post-tr outcome

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Before-After Designs:

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This is **not**

$$ATT = \overline{[Y(1)|T=1]} - \overline{[Y(0)|T=1]}$$

Why not?

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- Change in Control: removes part of change in emp that would have occurred anyway, in absence of law change.
- (Economy growing, or fast food restaurants hit by health scares and lay off workers, ...)

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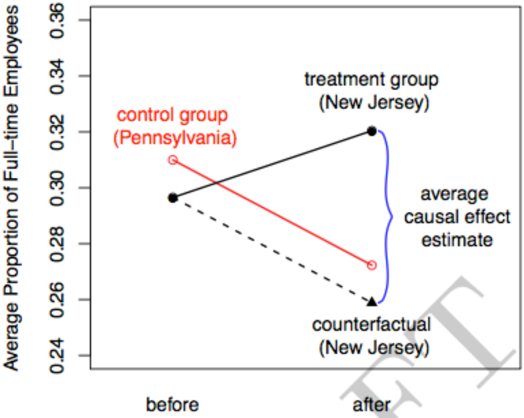
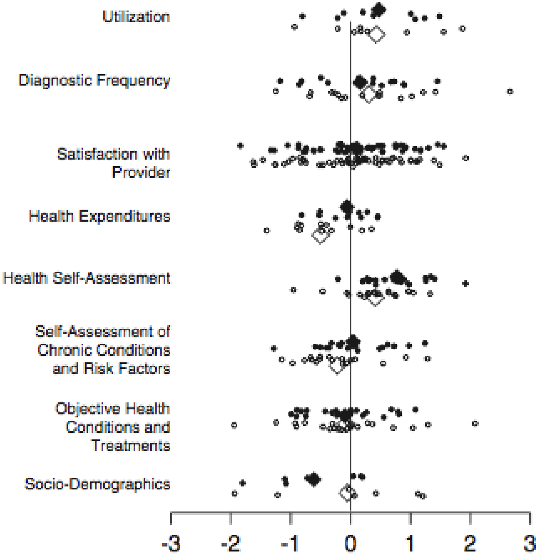
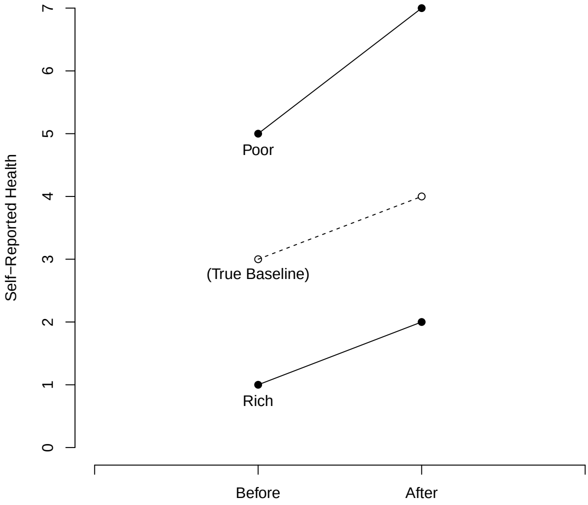


Self-Reported Health in *Seguro Popular* Evaluation

King, et al. (2007)



Simplified Example



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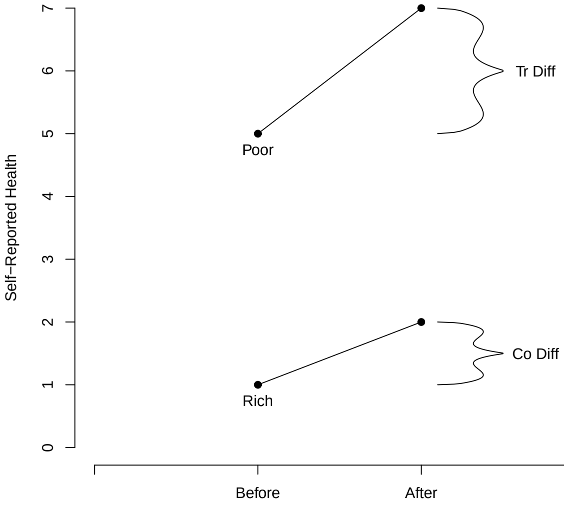
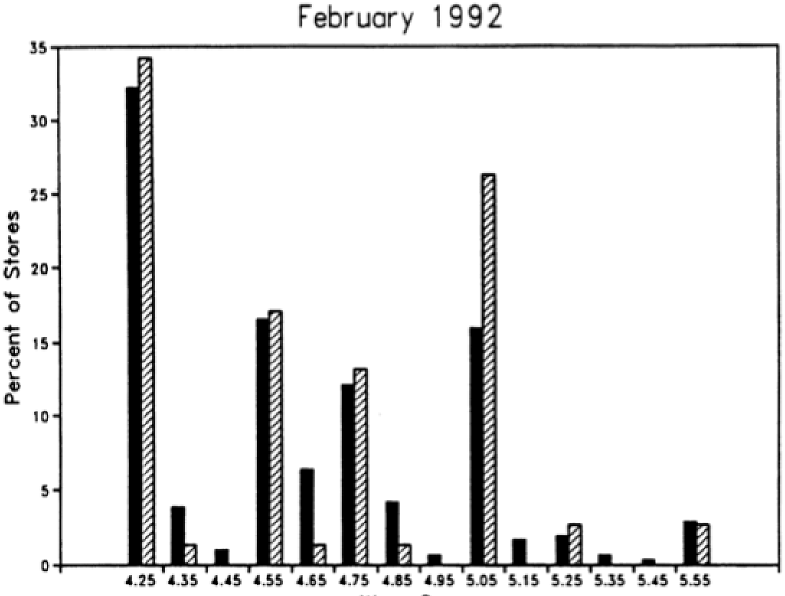


The Difference-in-Differences Estimator

Units	Outcome Before	Outcome After	Difference
Treated	$\bar{Y}^{T_B}$	$\bar{Y}^{T_A}$	$\bar{Y}^{T_A} - \bar{Y}^{T_B}$
Control	$\bar{Y}^{C_B}$	$\bar{Y}^{C_A}$	$\bar{Y}^{C_A} - \bar{Y}^{C_B}$
	$\bar{Y}^{T_B} - \bar{Y}^{C_B}$	$\bar{Y}^{T_A} - \bar{Y}^{C_A}$	$(\bar{Y}^{T_A} - \bar{Y}^{T_B}) - (\bar{Y}^{C_A} - \bar{Y}^{C_B})$

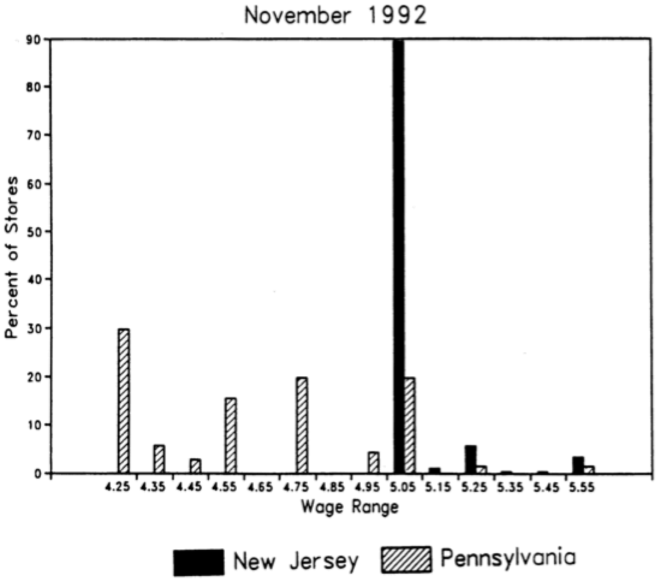
Example: Effect of Minimum Wage on Employment

The Baseline



Example: Effect of Minimum Wage on Employment

The Policy Change: Significant Shock



Example: Effect of Minimum Wage on Employment

Variable	PA (i)	NJ (ii)	Difference, NJ − PA (iii)
1. FTE employment before, all available observations	23.33 (1.35)	20.44 (0.51)	−2.89 (1.44)
2. FTE employment after, all available observations	21.17 (0.94)	21.03 (0.52)	−0.14 (1.07)
3. Change in mean FTE employment	−2.16 (1.25)	0.59 (0.54)	2.76 (1.36)

Extension 1: Regression Difference-in-Differences

Let

- $T_i \in \{0, 1\}$ be treatment status ("ever-treated")
- $A_i \in \{0, 1\}$ be "after" period status

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Model

$$y_i = \beta_0 + \beta_1 T_i + \beta_2 A_i + \beta_3 T_i A_i + \epsilon_i$$

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$$y_i = \beta_0 + \beta_1 T_i + \beta_2 A_i + \beta_3 T_i A_i + \epsilon_i$$

Then

$$\begin{aligned}
\bar{Y}^{T_A} &= \beta_0 + \beta_1 + \beta_2 + \beta_3 \\
\bar{Y}^{T_B} &= \beta_0 + \beta_1 \\
\bar{Y}^{C_A} &= \beta_0 + \beta_2 \\
\bar{Y}^{C_B} &= \beta_0
\end{aligned}$$

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\bar{Y}^{C_A} &= \beta_0 + \beta_2 \\
\bar{Y}^{C_B} &= \beta_0
\end{aligned}$$

And

$$\begin{aligned}
(\bar{Y}^{T_A} - \bar{Y}^{T_B}) - (\bar{Y}^{C_A} - \bar{Y}^{C_B}) &= ([\beta_0 + \beta_1 + \beta_2 + \beta_3] - [\beta_0 + \beta_1]) \\
&\quad -([\beta_0 + \beta_2] - \beta_0) \\
&= (\beta_2 + \beta_3) - (\beta_2) \\
&= \beta_3
\end{aligned}$$

Extension 1: Regression Difference-in-Differences

Note: $T_i \in \{0, 1\}$ needs to be "ever-treated" (not "currently treated")

Simulation:

- 40 units (20 Tr, 20 Co)
- 2 time periods (2018, 2022)
- Baseline outcome difference Tr $-$ Co $= 1$
- True TE $= 2$
- (TE occurs in 2022, only for Tr)

Extension 1: Regression Difference-in-Differences

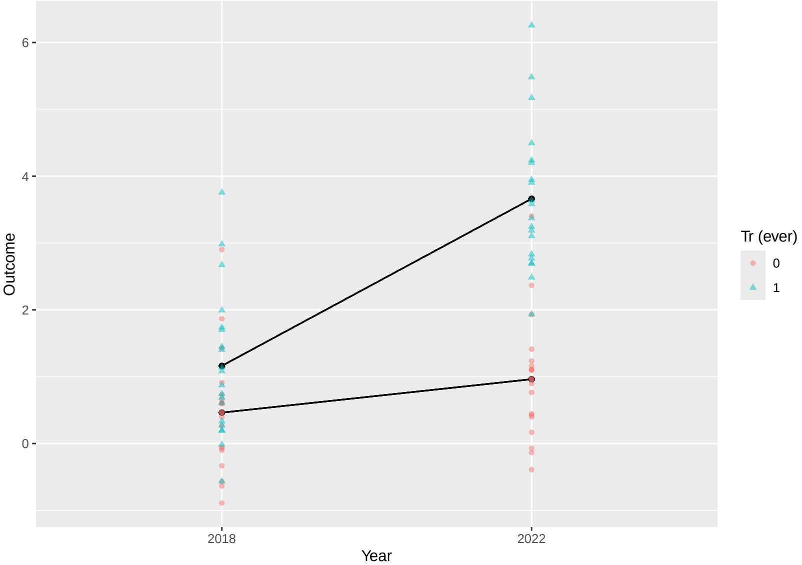

Figure 2: Simulated Data DiD

Extension 1: Regression Difference-in-Differences

Two possible regressions:

$$Y = \beta_0 + \beta_1(2022) + \beta_2(\text{TrEver}) + \beta_3(2022 \times \text{TrEver})$$

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Two possible regressions:

$$Y = \beta_0 + \beta_1(2022) + \beta_2(\text{TrEver}) + \beta_3(2022 \times \text{TrEver})$$

versus

$$Y = \beta_0 + \beta_1(2022) + \beta_2(\text{TrNow}) + \beta_3(2022 \times \text{TrNow})$$

Extension 1: Regression Difference-in-Differences

Table 1: Ever-treated vs. Currently-treated DiD Regressions

	Outcome	
	y	
	(1)	(2)
2022	0.500	0.150
	(0.313)	(0.278)
Tr (Ever)	0.700^{**}	
	(0.313)	
2022 x Tr (Ever)	2.000^{***}	
	(0.442)	
Tr (Now)		2.700^{***}
		(0.321)
2022 x Tr (Now)		
(Intercept)	0.462^{**}	0.812^{***}
	(0.221)	(0.160)
Observations	80	80
R^2	0.623	0.598
Adjusted R^2	0.608	0.588
Residual Std. Error	0.989 (df = 76)	1.015 (df = 77)
F Statistic	41.827^{***} (df = 3; 76)	57.261^{***} (df = 2; 77)
Note:		$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

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$$Y = \underbrace{0.46}_{\text{Co Mean 2018}} + \underbrace{0.5}_{\text{Time Trend}}(2022) + \underbrace{0.7}_{\text{Baseline Diff}}(\text{TrEver}) + \underbrace{2}_{\widehat{ATT}}(2022 \times \text{TrEver})$$

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versus

$$\begin{aligned} Y \quad = & \underbrace{0.81}_{\text{Part of Co Mean 2018 + Time Trend??}} + \underbrace{0.15}_{\text{Rest of Time Trend??}}(2022) + \\ & \underbrace{2.7}_{\text{Baseline Diff } + \widehat{ATT}\text{??}}(\text{TrNow}) + \underbrace{NA}_{\text{Unest Interaction for } \widehat{ATT}\text{??}}(2022 \times \text{TrNow}) \end{aligned}$$

Extension 2: Regression Difference-in-Differences

Related: Use change scores or regressor variable method? (Allison 90)

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$$y_{i2} = \alpha + \beta_1 y_{i1} + \beta_2 T_i + \epsilon_i \qquad (1)$$

Extension 2: Regression Difference-in-Differences

Related: Use change scores or regressor variable method? (Allison 90)

$$y_{i2} = \alpha + \beta_1 y_{i1} + \beta_2 T_i + \epsilon_i \tag{1}$$

$$y_{i1} = \alpha + \gamma G_i + \epsilon_{i1} \tag{2}$$

$$y_{i2} = \alpha + \underbrace{\tau}_{\text{secular trend}} + \gamma G_i + \delta T_i + \epsilon_{i2} \tag{3}$$

$$y_{i2} - y_{i1} = \tau + \delta T_i + (\epsilon_{i2} - \epsilon_{i1}) \tag{4}$$

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Related: Use change scores or regressor variable method? (Allison 90)

$$y_{i2} = \alpha + \beta_1 y_{i1} + \beta_2 T_i + \epsilon_i \tag{1}$$

$$\begin{aligned} y_{i1} &= \alpha + \gamma G_i + \epsilon_{i1} && (2) \\ y_{i2} &= \alpha + \underbrace{\tau}_{\text{secular trend}} + \gamma G_i + \delta T_i + \epsilon_{i2} && (3) \\ y_{i2} - y_{i1} &= \tau + \delta T_i + (\epsilon_{i2} - \epsilon_{i1}) && (4) \end{aligned}$$

- Change score is a DiD estimate
- Worse if pretest not very strong, and restriction of $\beta_1 = 1$ creates bias.
- Change score is more restrictive on parameter, but makes different assumption about $Cor(y_1, \epsilon)$.

Extension 3: Regression Difference-in-Differences

Add other covariates:

$$y_i = \beta_0 + \beta_1 T_i + \beta_2 A_i + \beta_3 T_i A_i + \gamma' \mathbf{x_i} + \ {}_{\mathbf{i}}$$

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But, careful about inference from SE's with many time periods (Bertrand, Duflo, and Mullainathan (2004)).

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Actually, be careful about bias with many time periods, too!

(See here.)

Extension 4: Difference in (Differences-in-Differences)

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- Useful for estimating effect across two subgroups
- (E.g., Empl vs. unempl'd; or placebo robustness check if one subgroup should *not* be affected by treatment)

Extension 4: Difference in (Differences-in-Differences)

Group 1:

Units	Outcome Before	Outcome After	Difference
Treated	T_{1B}	T_{1A}	$T_{1A} - T_{1B}$
Control	C_{1B}	C_{1A}	$C_{1A} - C_{1B}$
	$T_{1B} - C_{1B}$	$T_{1A} - C_{1A}$	$(T_{1A} - T_{1B}) - (C_{1A} - C_{1B})$

Group 2:

Units	Outcome Before	Outcome After	Difference
Treated	T_{2B}	T_{2A}	$T_{2A} - T_{2B}$
Control	C_{2B}	C_{2A}	$C_{2A} - C_{2B}$
	$T_{2B} - C_{2B}$	$T_{2A} - C_{2A}$	$(T_{2A} - T_{2B}) - (C_{2A} - C_{2B})$

DiDiD Estimate:

$$[(T_{2A} - T_{2B}) - (C_{2A} - C_{2B})] - [(T_{1A} - T_{1B}) - (C_{1A} - C_{1B})]$$

Extension 5: Multiple Time Periods, Staggered Designs

- Callaway and Sant'Anna (2021b) on multiple time periods

Extension 5: Multiple Time Periods, Staggered Designs

- Callaway and Sant'Anna (2021b) on multiple time periods
- Goodman-Bacon (2021) on staggered treatment assignment

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- NMAAHC experiment

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- Start with all untreated
- Over $\approx$ 1.5-2 years, treat all officers
- Measure admin data (use of force, etc.) and three waves of survey

Calloway-Sant'anna Approach: Multiple Time Periods

With *many* time periods, challenging to defining **the** estimand (like ATT).

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- Decide how to aggregate them
- Estimate them

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Central disaggregated component:

the *group-time average treatment effect*.

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- Exposure/Dosage ("effect for those w/ 2 years treatment")
- Group
  ("effect for those first treated in 2020")
- Cumulative
  ("total effect up to 2020")
- Overall
  ("total effect up to 2024")

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  ("total effect up to 2020")
- Overall
  ("total effect up to 2024")
- Overall, group as unit
  ("total effect up to 2024: get state avgs, avg those")

Calloway-Sant'Anna Approach: Multiple Time Periods

Key assumptions:

1. No treated at $t = 1$. If treated at t^*, then treated at $t^* + 1$. Once treated, stay "treated".

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3/4/5 define valid comparison group, thus analysis sample.

C-S Example

Effect of US state min wages on *teen* employment.

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- Annual data, 2003-2007

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C-S Example

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- Annual data, 2003-2007
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- Panel data structure: (5 yrs) $\times$ (500 cntys) = (2500 cnty-yrs)

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Callaway and Sant'Anna (2021a): `did`

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Calculate all the $ATT(g, t)$ effects, for each (state $\times$ year)

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cs_out <- att_gt(yname = "lemp",
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                 tname = "year",
                 xformla = ~ 1,
                 data = mpdta,
                 est_method = "reg"
)
```

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- `est_method`: regression, IPW, doubly-robust, etc.

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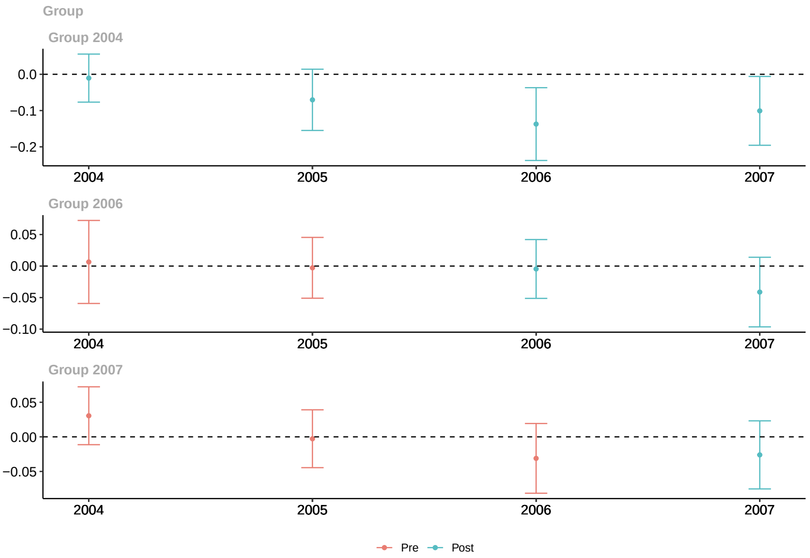
- `est_method`: regression, IPW, doubly-robust, etc.
- `xformla`: formula for covariate adjustment
- `control_group`: `"nevertreated"`, `"notyettreated"`

C-S Example

```
ggdid(cs_out)
```



C-S Example

- "Pre" C-S $ATT(g,t)$ values: *pre*-test of parallel trends assumption

C-S Example

- "Pre" C-S $ATT(g,t)$ values: *pre*-test of parallel trends assumption
- "Post" C-S $ATT(g,t)$ values: effects of min wage policy

C-S Example: Aggregate by g, t

Instead of fully-disaggregated $ATT(g, t)$'s, summarise into various effects.

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Instead of fully-disaggregated $ATT(g,t)$'s, summarise into various effects.

First, avg $ATT(g,t)$'s (weight by group size):

```r
aggte(cs_out, type = "simple")
```

```


Call:
aggte(MP = cs_out, type = "simple")

Reference: Callaway, Brantly and Pedro H.C. Sant'Anna.


   ATT    Std. Error     [ 95%  Conf. Int.]
 -0.04        0.0124    -0.0643     -0.0156 *


---
```

C-S Example: Aggregate by group g

Second, get ATE for each group, avg across groups.

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```
group_effects <- aggte(cs_out, type = "group")
```

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Second, get ATE for each group, avg across groups.

```
group_effects <- aggte(cs_out, type = "group")
```

```
Overall summary of ATT's based on group/cohort aggregation:
    ATT    Std. Error     [ 95%  Conf. Int.]
 -0.031        0.0127     -0.056      -0.006 *


Group Effects:
 Group Estimate Std. Error [95% Simult.  Conf. Band]
  2004  -0.0797     0.0301       -0.1455     -0.0140 *
  2006  -0.0229     0.0172       -0.0606      0.0148
  2007  -0.0261     0.0169       -0.0631      0.0109
```

C-S Example: Aggregate by Dosage

Third, by amt of treatment/exposure length/dosage ("event study"):

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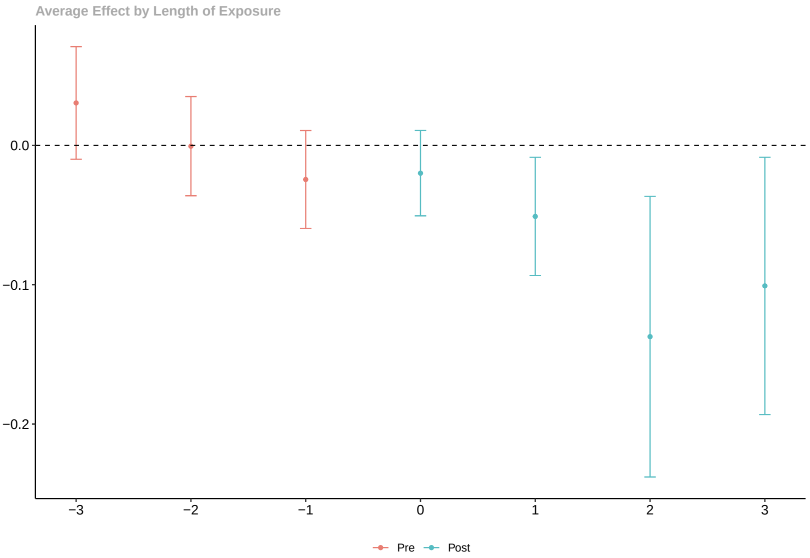
```
cs_es_out <- aggte(cs_out, type = "dynamic")
```

C-S Example: Aggregate by Dosage

```
ggdid(cs_es_out)
```



C-S Example: Aggregate by Dosage

```
summary(cs_es_out)
```

C-S Example: Aggregate by Dosage

```r
summary(cs_es_out)
```

```
Overall summary of ATT's based on event-study/dynamic aggregation:
     ATT    Std. Error     [ 95%  Conf. Int.]
 -0.0772        0.0222    -0.1207     -0.0338 *
```

C-S Example: Aggregate by Dosage

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summary(cs_es_out)
```

```
 Overall summary of ATT's based on event-study/dynamic aggregation:
      ATT    Std. Error     [ 95%  Conf. Int.]
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```

"Overall" effect of participating: get avg effect at each exposure length, avg those across different exposure lengths.

C-S Example: Aggregate by Time t

Fourth, get avg effect for each time period, avg across periods.

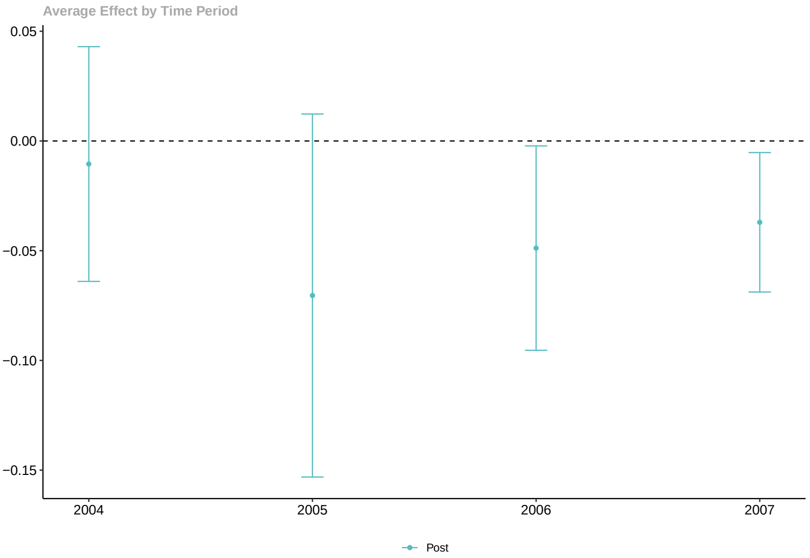
```
cs_cal_out <- aggte(cs_out, type = "calendar")
```

```
Overall summary of ATT's based on calendar time aggregation:
     ATT    Std. Error     [ 95%  Conf. Int.]
 -0.0417         0.018    -0.0769     -0.0065 *


Time Effects:
 Time Estimate Std. Error [95% Simult.  Conf. Band]
 2004  -0.0105     0.0240       -0.0656      0.0446
 2005  -0.0704     0.0300       -0.1392     -0.0016 *
 2006  -0.0488     0.0215       -0.0980      0.0003
 2007  -0.0371     0.0144       -0.0700     -0.0041 *
```

C-S Example: Aggregate by Time t

```
ggdid(cs_cal_out)
```



Linear Models and Two-Way Fixed-Effects

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Consider

$$Y_{i,t} = \alpha_i + \gamma_t + \beta \cdot T_{i,t} + \epsilon_{i,t}$$

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- More generally, β is weighted avg of *all possible* 2×2 DiD estimators comparing C-S "groups"
- (Among these, some use "never treated", others use "not yet treated")

Linear Models and Two-Way Fixed-Effects

- Weights: prop to

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- highest at $p = 0.5$

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Goodman-Bacon (2021)

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So, use C-S aggregation!

What if effect constant over time?

Linear Models and Two-Way Fixed-Effects

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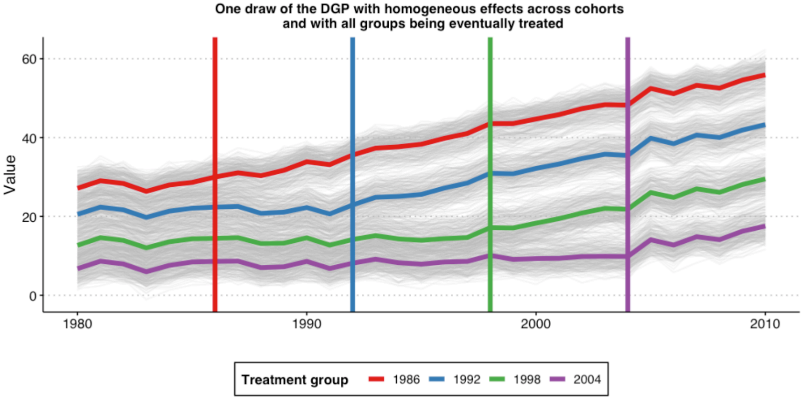
Callaway and Sant'Anna (2023)

Linear Models and Two-Way Fixed-Effects



The above plot shows the 1,000 individual values of $Y_{i,t}$, as well as the average by treatment-group (the thicker colorful lines), and the vertical lines show the period when treatment begins for each group. Because treatment effects here grows linearly with time elapsed since treatment started, we can see that the earliest treated units end up with the highest value of Y, and that the difference grows by the gap between treatment years.

Linear Models and Two-Way Fixed-Effects Event Study

Model

$$Y_{i,t} = \alpha_i + \beta_t + \gamma(Pre) + \delta_j(Lag_j) + \zeta_k(Lead_k) + \epsilon_{i,t}$$

- Indicators for units
- Indicators for time periods
- Indicators for lags ("tr how long ago?")
- Indicators for leads ("tr how far into future?")
- (Cluster SE's on "states" of assignment)

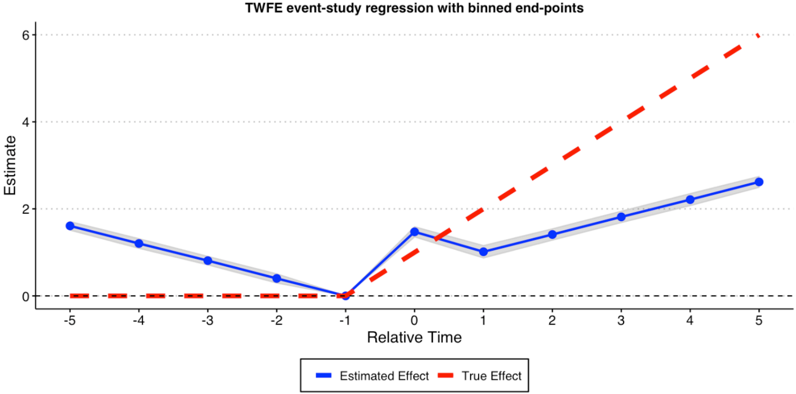
Linear Models and Two-Way Fixed-Effects Event Study

Model

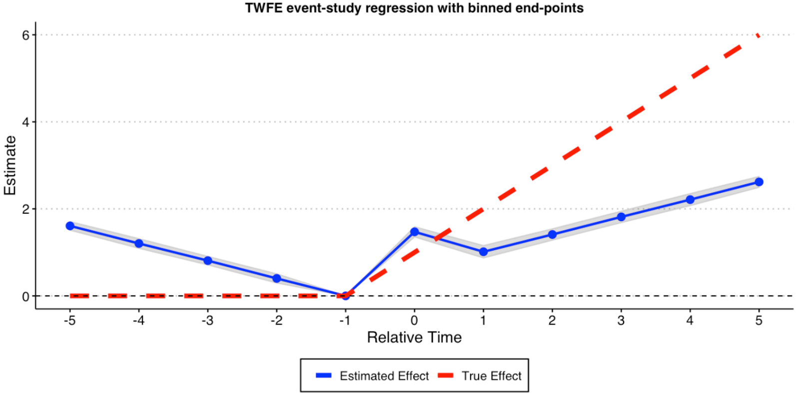
$$Y_{i,t} = \alpha_i + \beta_t + \gamma(Pre) + \delta_j(Lag_j) + \zeta_k(Lead_k) + \epsilon_{i,t}$$

- δ_j often interpreted as "ATE of j periods of treatment"
- ζ_j often interpreted as "pre-tr trends"

Linear Models and Two-Way Fixed-Effects Event Study



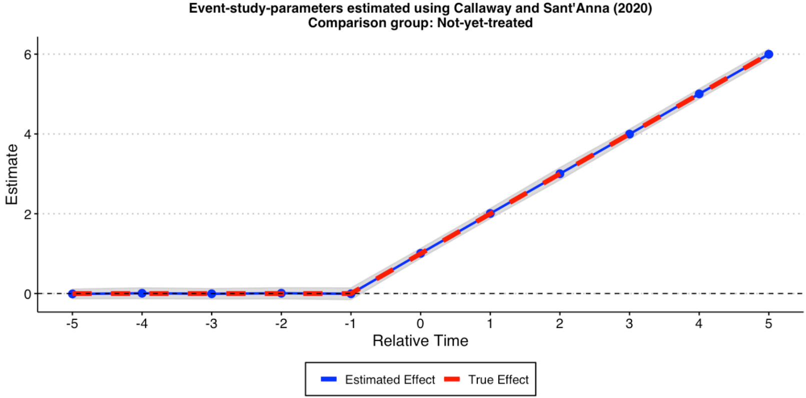
Linear Models and Two-Way Fixed-Effects Event Study



Severe bias for "effect of 3 years of Tr", e.g.!

Linear Models and Two-Way Fixed-Effects Event Study

Instead, using C-S approach correctly ID's identical lead/pre-tr trends and cumulative effects of Tr!



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- But, if there are heterogeneous treatment effects, TWFE-ES won't recover truth (even with never-treated group)

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- But, if there are heterogeneous treatment effects, TWFE-ES won't recover truth (even with never-treated group)
- Calloway-Sant'Anna does
- (So, use C-S aggregation!)

Thanks!

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References I

Bertrand, Marianne, Esther Duflo, and Sendhil Mullainathan. 2004. "How Much Should We Trust Differences-in-Differences Estimates?" *The Quarterly Journal of Economics* 119 (1): 249–75.

Callaway, Brantly, and Pedro H. C. Sant'Anna. 2021a. "Did: Difference in Differences." https://bcallaway11.github.io/did/.

———. 2021b. "Difference-in-Differences with Multiple Time Periods." *Journal of Econometrics* 225 (2): 200–230.

———. 2023. "Problems with Two-Way Fixed-Effects Event-Study Regressions." https://bcallaway11.github.io/did/articles/TWFE.html.

Card, David, and Alan B. Krueger. 1994. "Minimum Wages and Employment: A Case Study of the Fast-Food Industry in New Jersey and Pennsylvania." *The American Economic Review* 90 (5): 1397–1420.

Goodman-Bacon, Andrew. 2021. "Difference-in-Differences with Variation in Treatment Timing." *Journal of Econometrics* 225: 254–77. https://doi.org/10.1016/j.jeconom.2021.03.014.