Data Science for Causal Inference

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The Lab @ DC

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About Me

- Associate Prof of Government
  (American University)
- Associate Director, Center for Data Science
  (American University)
- Senior Social Scientist
  (The Lab @ DC)
- Fellow in Methodology
  (US Office of Evaluation Sciences: “OES”)

About Me

- Associate Prof of Government (American University)
- Associate Director, Center for Data Science (American University)
- Senior Social Scientist (The Lab @ DC)
- Fellow in Methodology (US Office of Evaluation Sciences: "OES")
- Research agenda: political methodology, causal inference, experimental design, experiments in public policy

About You!

- Name?

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- Name?
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About You!

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- Olympic sport you look forward to?

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- Data Science in Causal Inference

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- Data Science in Causal Inference
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  - Heterogeneous treatment effects

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- Modern difference-in-difference designs

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  - Multiple time periods

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  - Unobserved confounders
- Modern difference-in-difference designs
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  - Staggered adoption

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- Sensitivity
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- Modern difference-in-difference designs
  - Canonical DiD
  - Multiple time periods
  - Staggered adoption
  - Calloway-Sant'Anna approach

Data Science in Causal Inference

Causal Inference Approaches

The "potential outcomes" framework:

Causal Inference Approaches

The "potential outcomes" framework:

Citizen	Canvass?	Would Enroll if Canvass?	Would Enroll if No Canvass?	Enroll
1	Yes			Yes
2	Yes			Yes
3	No			No
4	No			No

Causal Inference Approaches

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Causal Inference Approaches

The "potential outcomes" framework, more abstractly:

Unit i	Treatment T	$Y(1)$	$Y(0)$	Y^{obs}	True τ $Y(1)-Y(0)$
1	1	10		10	
2	1	20		20	
3	0		15	15	
4	0		5	5	

Causal Inference Approaches

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2	1	20	(10)	20	10
3	0	(40)	15	15	25
4	0	(20)	5	5	15

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				$\widehat{ATE} = \hat{\bar{\tau}} =$	$15 - 10 = 5$

Causal Inference Approaches

The “potential outcomes” framework, notation:

- Units indexed by i

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- True average treatment effect
  $\bar{\tau} = \frac{1}{n}\sum_{i=1}^{n}\left(Y_i(1) - Y_i(0)\right)$

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(and we'll draw some DAG's, too)

Data Science Approaches

Three tasks of data science:

- Description

Data Science Approaches

Three tasks of data science:

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- Prediction

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Models/algorithms central to all three.

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Models/algorithms central to all three.

Hernán, Hsu, and Healy (2019)

Data Science Approaches

Description

- Identifying patterns, etc.

Data Science Approaches

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- Identifying patterns, etc.
- E.g., clustering to discover groups

Data Science Approaches

Prediction

- Components

Data Science Approaches

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  - Inputs/outputs (predictors/outcomes, features/responses, ...)

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Data Science Approaches

Prediction

- Components
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  - Mapping from inputs to outputs (linear model, decision tree, ...)
  - Metric for evaluating mapping
- With these, model machine learning does the work
- E.g., regression, random forests, neural networks, ...

Data Science Approaches

Causal Inference

- Potential outcomes/counterfactual/interventionist perspective

Data Science Approaches

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- Requires *expertise* different to description/prediction

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Data Science Approaches

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Data Science Approaches

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  - T v. $\mathbf{X}$ – very different!

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  - (the more knowledge, the better!)

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  - (alternative: solve fundamental problem of causal inference!)

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  - Not all inputs treated same
  - T v. $\mathbf{X}$ – very different!
  - (the more knowledge, the better!)
  - (alternative: solve fundamental problem of causal inference!)
- E.g., experiments, observational causal designs, ...

Causal Inference with Machine Learning

Causal Inference with Machine Learning

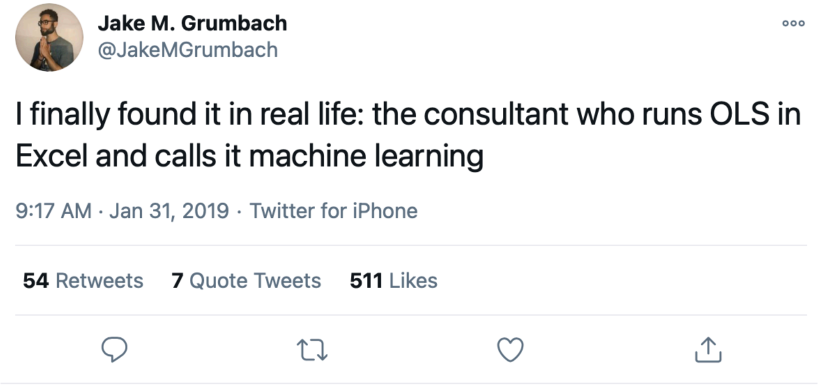
Jake M. Grumbach
@JakeMGrumbach

I finally found it in real life: the consultant who runs OLS in Excel and calls it machine learning

9:17 AM · Jan 31, 2019 · Twitter for iPhone

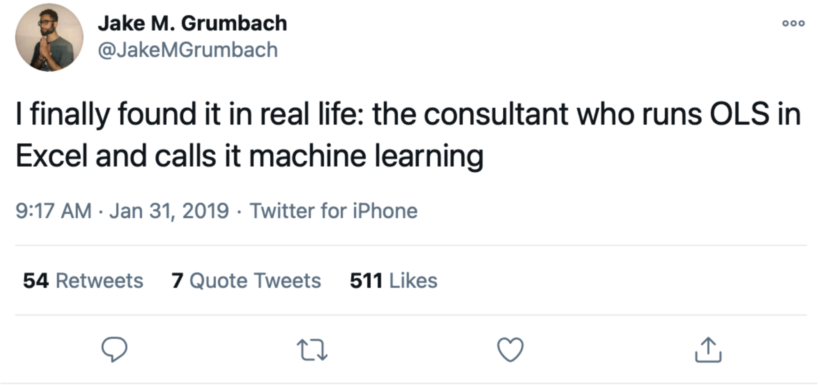
54 Retweets **7** Quote Tweets **511** Likes

Causal Inference with Machine Learning



Don't do this.

Causal Inference with Machine Learning



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Don’t do this.

(Not “machine learning”, probably, but *models* at least ...)

Causal Inference with Models

Loaded two datasets:

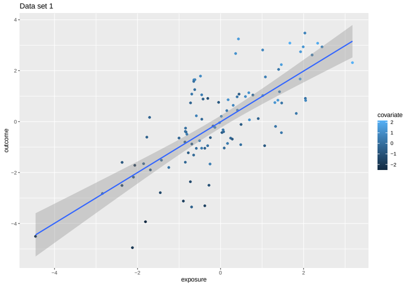
```r
str(df1)
```

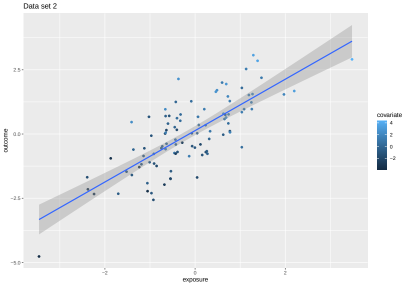
```
tibble [100 x 3] (S3: tbl_df/tbl/data.frame)
 $ covariate: num [1:100] -0.622 1.137 -0.238 1.529 -0.154
 $ exposure : num [1:100] 0.0332 0.3627 0.2422 1.4633 0.779
 $ outcome  : num [1:100] -0.429 2.675 -0.647 2.238 1.044
```

```r
str(df2)
```

```
tibble [100 x 3] (S3: tbl_df/tbl/data.frame)
 $ exposure : num [1:100] 0.4862 0.0653 -1.4021 -0.546 -0.
 $ outcome  : num [1:100] 1.706 0.669 -1.597 -1.733 0.617
 $ covariate: num [1:100] 2.24 0.924 -0.999 -2.343 0.207 ..
```

Causal Inference with Models





Causal Inference with Models

Model each

```
lm_df1 <- lm(outcome ~ exposure, data = df1)
lm_df2 <- lm(outcome ~ exposure, data = df2)
```

```
# A tibble: 4 x 4
  data  term        estimate std.error
  <chr> <chr>          <dbl>     <dbl>
1 df1   (Intercept) -0.00671    0.120
2 df1   exposure     0.996      0.0927
3 df2   (Intercept)  0.133      0.0890
4 df2   exposure     1.00       0.0841
```

Causal Inference with Models

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- Both cases: effect of exposure ≈ 1.

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- Is this good? Is it correct?

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```

- Both cases: effect of exposure ≈ 1.
- Is this good? Is it correct?
- What if we adjust for covariate?

Causal Inference with Models

```r
lm_df1_adj <- lm(outcome ~ exposure + covariate, data = df1
lm_df2_adj <- lm(outcome ~ exposure + covariate, data = df2
```

```
# A tibble: 4 x 4
  data  term      estimate std.error
  <chr> <chr>        <dbl>     <dbl>
1 df1   exposure     0.501    0.108
2 df1   covariate    0.970    0.147
3 df2   exposure     0.554    0.0990
4 df2   covariate    0.385    0.0598
```

- Both cases: effect of exposure ≈ 0.5.

Causal Inference with Models

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- Is this good? Is it correct?
- Which is correct? $\beta = 1$? $\beta = 0.5$?

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```

- Both cases: effect of exposure ≈ 0.5.
- Is this good? Is it correct?
- Which is correct? $\beta = 1$? $\beta = 0.5$?
- *Should* we adjust for covariate?

Causal Inference with Models

There is nothing in the data that tells us.

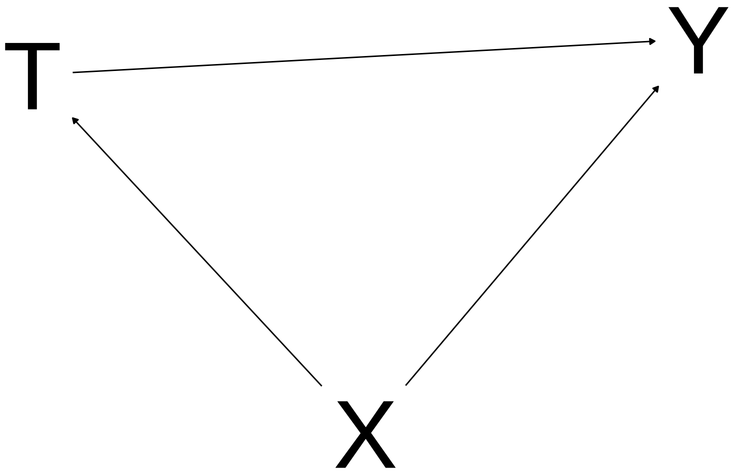
Causal Inference with Models

There is nothing in the data that tells us. ☹

Causal Inference with Models

There is nothing in the data that tells us. ☹

Here are the true structures: First



Causal Inference with Models

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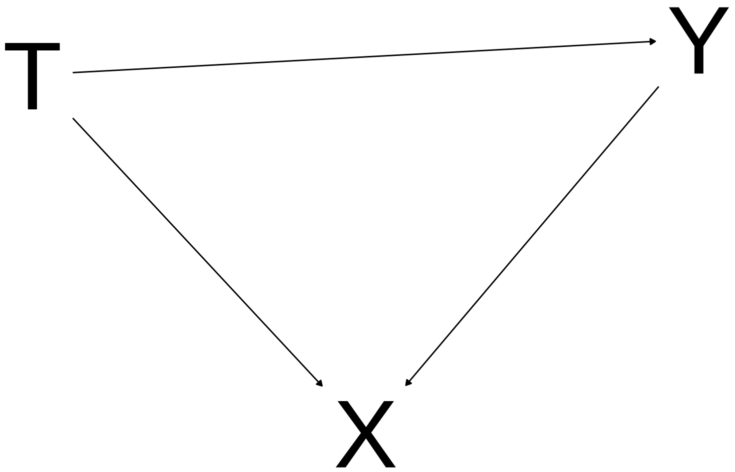
Here are the true structures: Second



Causal Inference with Models

When know structures, adjustment sets for unbiasedness differ:

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- `df1`: confounding ⇒ **adjust for** X

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When know structures, adjustment sets for unbiasedness differ:

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- `df2`: collider $\Rightarrow$ **do not adjust for** X

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g_conf <- dagitty("dag{ x -> y ; x <- c -> y }")
g_coll <- dagitty("dag{ x -> y ; x -> c <- y }")
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```
adjustmentSets(g_conf, "x", "y")
```

```
{ c }
```

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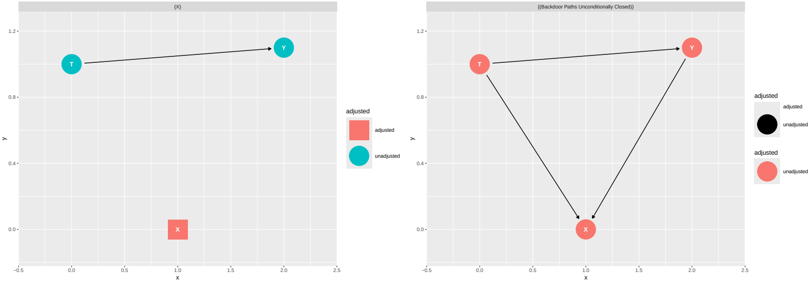

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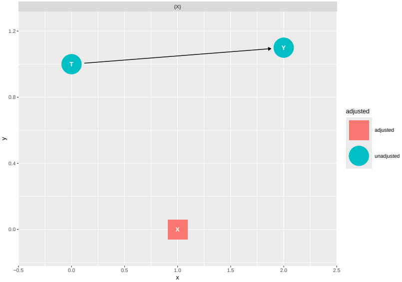

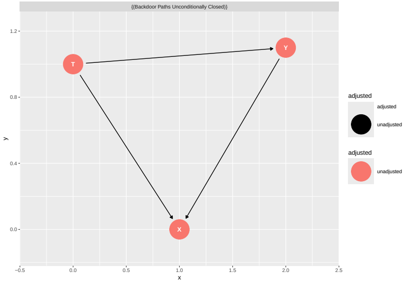

(Data from D'Agostino McGowan (2023))

Causal Inference with Models

- Importance of identifying “pre-treatment covariates”, “proper covariates”; doing “design before analysis”

Causal Inference with Models

- Importance of identifying “pre-treatment covariates”, “proper covariates”; doing “design before analysis”
- Importance of experiments: strong knowledge about (part of) causal structure – assgn mechanism

Causal Inference with Models

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- "Causal euphimisms" don't help (Hernán 2018)

Approaches of Prediction and Causal Inference

Two Cultures, (Breiman 2001)

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- *Algorithmic Models*: our "data science algorithms"

Methods for Prediction and Causal Inference

- Cross-validation
- Regression/Decision trees
- Random forests

James et al. (2021)

Cross-validation

k-fold cross-validation to select method

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$$CV_{(k)} = \frac{1}{k}\sum_{i=1}^{k}\text{MSE}_i$$

- Select model that minimises $CV_{(k)}$

CV for Linear Model

```r
library(tidyverse)

## Make data

mk_data <- function(n = 90, n_folds = 10){

  df <- tibble(
    x1 = rnorm(n),
    x2 = rnorm(n),
    x3 = rnorm(n),
    y = 0.1 * x1 + 0.2 * x2 + 0.5 * x3 + rnorm(n),
    cv_fold = sample(rep(1:n_folds, (n / n_folds)))
  )

}

df <- mk_data()
```

CV for Linear Model

```r
head(df)
```

```
# A tibble: 6 x 5
      x1     x2      x3      y cv_fold
   <dbl>  <dbl>   <dbl>  <dbl>   <int>
1 -1.36  -0.778 -0.0372 -2.34        7
2  0.536  0.504  1.21    0.270      10
3 -0.791 -0.142  0.0307 -0.214       7
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```

```r
table(df$cv_fold)
```

```

 1  2  3  4  5  6  7  8  9 10 
 9  9  9  9  9  9  9  9  9  9 
```

CV for Linear Model

```
cv_lm <- function(data, fmla){

  n_folds <- max(data$cv_fold)
  store_mses <- vector("numeric", length = n_folds)

  for(idx in 1:n_folds){

    df_train <- data |> filter(cv_fold != idx)
    df_test <- data |> filter(cv_fold == idx)

    lm_out <- lm(fmla, data = df_train)

    predictions <- predict(lm_out, newdata = df_test)

    store_mses[idx] <- mean((df_test$y - predictions)^2)}

  test_error_cv_k <- mean(store_mses)
  return(test_error_cv_k)
```

CV for Linear Model

```r
cv_lm(data = df, fmla = y ~ x1 + x2)
```

```
[1] 1.710041
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```

```
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```

CV for Linear Model

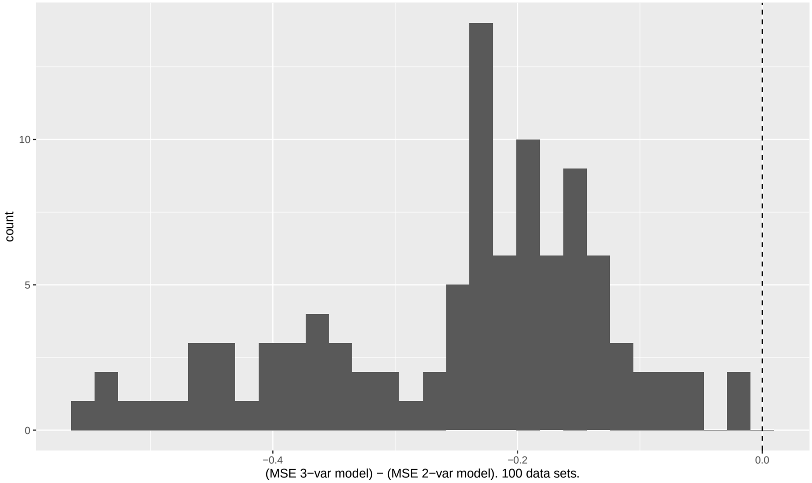

Figure 1: MSE always less (better) for 3-variable model.

Regression Trees

- Partition predictor space into regions $R_1, R_2, \ldots, R_J$.

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$$\sum_{j=1}^{J} \sum_{i \in R_j} \left(y_i - \hat{y}_{R_j} \right)$$

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How to define regions R_j?

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$$\sum_{i:x \in R_1(j,s)} \left(y_i - \hat{y}_{R_1(j,s)}\right)^2 + \sum_{i:x \in R_2(j,s)} \left(y_i - \hat{y}_{R_2(j,s)}\right)^2$$

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$$\sum_{m=1}^{|T|} \sum_{i: x_i \in R_m} \left(y_i - \hat{y}_{R_m}\right)^2 + \alpha|T|$$

Sum squared pred. error (plus penalty that grows with tree size) across units in region, then regions.

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4. Using that α, select best subtree from Step 2

Example: Regression Tree

```
library(qss)
library(rsample)
library(tree)

data("MPs")
mps <- MPs |> mutate(age = yod - yob,
                     is_labour = if_else(party == "labour",
                     is_london = if_else(region == "Greater
                     is_winner = if_else(margin > 0, 1, 0))
  select(ln.net, age, is_labour, is_london, is_winner) |>
  na.omit()

set.seed(765076184)

mp_split <- initial_split(mps, prop = 0.7)

mp_train <- training(mp_split)
mp_test <- testing(mp_split)
```

Example: Regression Tree

```
tree_mp <- tree(ln.net ~ ., data = mp_train)
plot(tree_mp)
text(tree_mp)
```

Figure 2: The regression tree (for training data)

Example: Regression Tree

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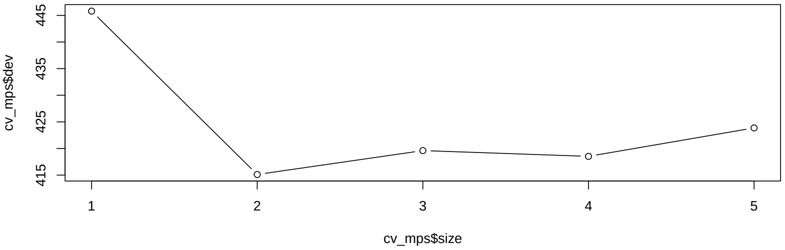
```
cv_mps <- cv.tree(tree_mp, K = 10)

plot(cv_mps$size, cv_mps$dev, type = "b")
```



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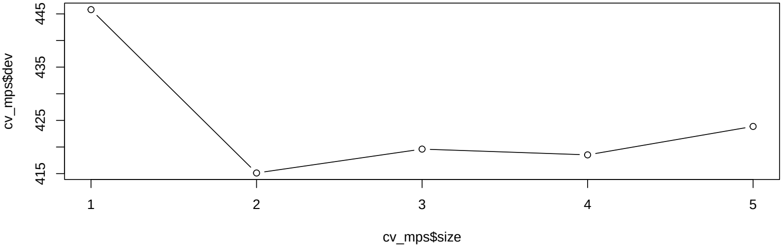

Figure 3: Subtree size 2 minimises SSR

Example: Regression Tree

```r
prune_mps <- prune.tree(tree_mp, best = 2)

plot(prune_mps)
text(prune_mps)
```

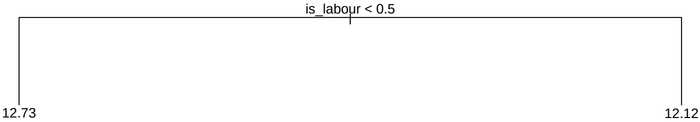

Figure 4: The pruned tree

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Predict for test set:

- MSE for pruned: 1.922
- MSE for full: 1.945

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(Pretty good for 1 split!?)

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Next: random forest algorithm

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Bagging: *b*ootstrap *agg*regation

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- (Often choose $m \approx \sqrt{p}$)
- So, different splits consider different predictors
- So, trees will look very different to each other

Example: Random Forests

```r
library(randomForest)

# Full bag:
bag_mps <- randomForest(ln.net ~ ., data = mp_train,
                        ntree = 500, mtry = 4,
                        importance = TRUE)

# Decorrelate:
rf_mps <- randomForest(ln.net ~ ., data = mp_train,
                       ntree = 500, mtry = 2,
                      importance = TRUE)
```

Example: Random Forests

Predict:

```r
preds_bag <- predict(bag_mps, newdata = mp_test)

preds_rf <- predict(rf_mps, newdata = mp_test)
```

- MSE for RF: 1.995
- MSE for full bag: 2.536

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  - Targeting resources (campaigns, marketing)
  - Constituency effects (public policy)
- Notationally, $\exists i : \tau_i \neq \tau$

Homogeneous and Heterogeneous Effects: Estimation

Homogeneous effects:

$$\text{Outcome} = \beta_0 + \beta_1 \text{Treatment} + \epsilon$$

Homogeneous and Heterogeneous Effects: Estimation

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$$\text{Outcome} = \beta_0 + \beta_1 \text{Treatment} + \epsilon$$

```r
lm_out <- lm(ln.net ~ is_winner, data = mps)
lm_out
```

```
Call:
lm(formula = ln.net ~ is_winner, data = mps)

Coefficients:
(Intercept)    is_winner
    12.2464       0.5176
```

Homogeneous and Heterogeneous Effects: Estimation

Homogeneous effects:

```r
t.test(ln.net ~ is_winner, data = mps)
```

```

    Welch Two Sample t-test

data:  ln.net by is_winner
t = -3.9552, df = 287.65, p-value = 9.636e-05
alternative hypothesis: true difference in means betwee
95 percent confidence interval:
 -0.7751044 -0.2599998
sample estimates:
mean in group 0 mean in group 1
       12.24641        12.76396
```

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Homogeneous effects:

$$\text{Outcome} = \beta_0 + \beta_1 \text{Treatment} + \sum \beta_j X_j + \epsilon$$

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$$\text{Outcome} = \beta_0 + \beta_1 \text{Treatment} + \sum \beta_j X_j + \epsilon$$

```
lm_out <- lm(ln.net ~ is_winner + is_labour +
               is_london + age, data = mps)
lm_out
```

```
Call:
lm(formula = ln.net ~ is_winner + is_labour + is_london + a
    data = mps)

Coefficients:
(Intercept)    is_winner    is_labour    is_london
  12.078838     0.398818    -0.477549     0.161134     0.00
```

Homogeneous and Heterogeneous Effects: Estimation

Homogeneous effects:

```
lm_lin(ln.net ~ is_winner, covariates = ~ is_labour + is_lo
```

```
                          Estimate  Std. Error       t valu
(Intercept)           1.226687e+01 0.078894901 155.4836617
is_winner             3.459885e-01 0.131207672   2.6369536
is_labour_c          -1.613663e-01 0.152608515  -1.0573871
is_london_c           2.427360e-01 0.250214401   0.9701118
age_c                 4.740367e-03 0.007031323   0.6741786
is_winner:is_labour_c -9.104022e-01 0.264395760  -3.4433313
is_winner:is_london_c -8.847770e-02 0.426241818  -0.2075763
is_winner:age_c      -4.778657e-05 0.012753800  -0.0037468
                         CI Lower    CI Upper  DF
(Intercept)           12.111785723 12.42195044 416
is_winner              0.088075873  0.60390123 416
is_labour_c           -0.461346226  0.13861367 416
is_london_c           -0.249106208  0.73457813 416
```

CATEs: Conditional ATEs

- *Conditional average treatment effect* (CATE): avg treatment effect for subset of population

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- *Conditional average treatment effect* (CATE): avg treatment effect for subset of population
- Sometimes "CACE"
- Inference: not "evidence against $\text{TE} = 0$?", but "evidence against $\text{CATE}_1 = \text{CATE}_2$?"

Homogeneous and Heterogeneous Effects: Estimation

Heterogeneous effects:

$$\text{Outcome} = \beta_0 + \beta_1 \text{Treatment} + \beta_2 \text{Group} + \beta_3 \text{Treatment} \cdot \text{Group} + \epsilon$$

Homogeneous and Heterogeneous Effects: Estimation

Heterogeneous effects:

$$\text{Outcome} = \beta_0 + \beta_1 \text{Treatment} + \beta_2 \text{Group} + \beta_3 \text{Treatment} \cdot \text{Group} + \epsilon$$

- β_1 gives TE for `Group == 0`

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Heterogeneous effects:

$$\text{Outcome} = \beta_0 + \beta_1 \text{Treatment} + \beta_2 \text{Group} + \beta_3 \text{Treatment} \cdot \text{Group} + \epsilon$$

- β_1 gives TE for `Group == 0`
- $\beta_1 + \beta_3$ gives TE for `Group == 1`

Homogeneous and Heterogeneous Effects: Estimation

Heterogeneous effects:

```
lm_out <- lm(ln.net ~ is_winner * is_labour +
               is_london + age, data = mps)
coef(lm_out) |> round(3)
```

```
        (Intercept)           is_winner           is_labou
             11.959               0.780              -0.16
                age is_winner:is_labour
              0.005              -0.914
```

Causal Forests

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- Use $\hat{y}_{R_j}$ as pred value for obs in R_j
- (Tory, winner, London $\rightarrow$ 13.84)
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$$Y(0), Y(1) \perp\!\!\!\perp T | \mathbf{X}$$

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$$\hat{\hat{\tau}}_{R_j} = \frac{1}{|\{T, R_j\}|} \sum_{\{T, R_j\}} Y_i - \frac{1}{|\{C, R_j\}|} \sum_{\{C, R_j\}} Y_i$$

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☺

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  - Splitting cannot use y_i from $\mathcal{I}$
  - Prediction, estimation of $\hat{\hat{\tau}}$ uses only $\mathcal{I}$
- Build a random forest (decorrelated deep trees picking from m predictors) of causal trees

Example: Causal Forests

```r
library(grf)

X <- mp_train |> select(age, is_labour, is_london)

W <- mp_train |> select(is_winner) |>
  unlist() |> as.numeric()

Y <- mp_train |> select(ln.net) |> unlist()

cf_out <- causal_forest(X, Y, W)
```

Example: Causal Forests

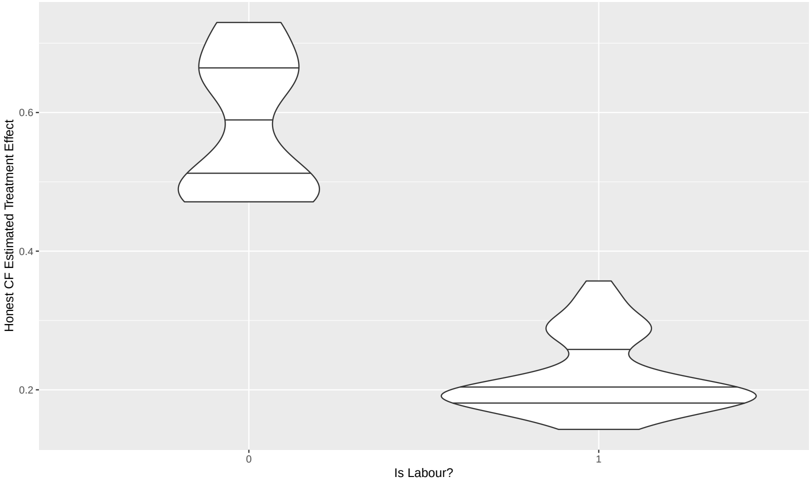
```
X_test <- mp_test |> select(age, is_labour, is_london)

cf_pred_est_var <- predict(cf_out, X_test,
                           estimate.variance = TRUE)

cf_preds <- cf_pred_est_var$predictions

df_cf <- tibble(X_test,
                cf_te = cf_preds,
                cf_se = sqrt(cf_pred_est_var$variance.e
                te_1se_lower = cf_te - cf_se,
                te_1se_upper = cf_te + cf_se)
```

Example: Causal Forests Results, Party



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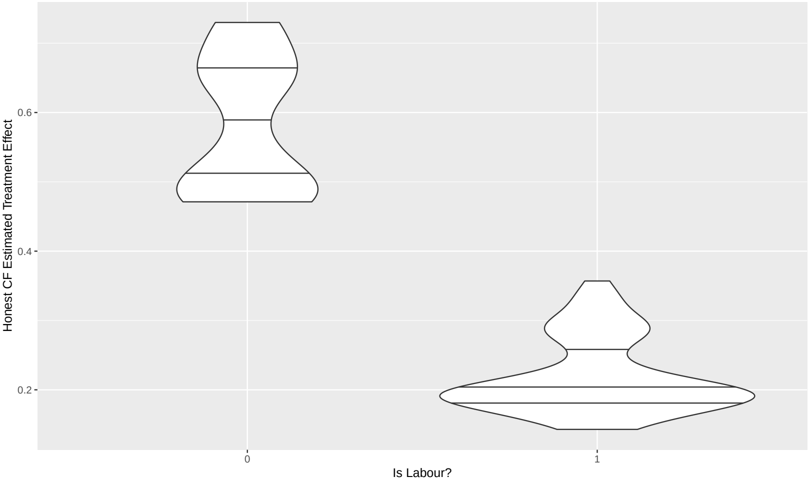
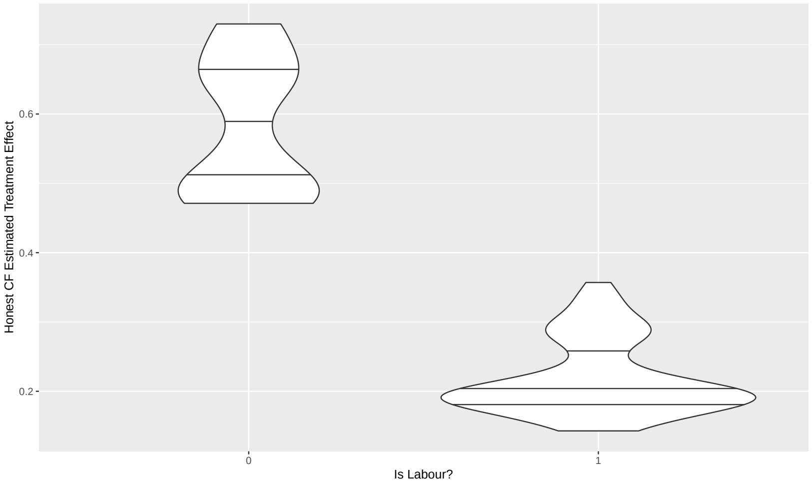


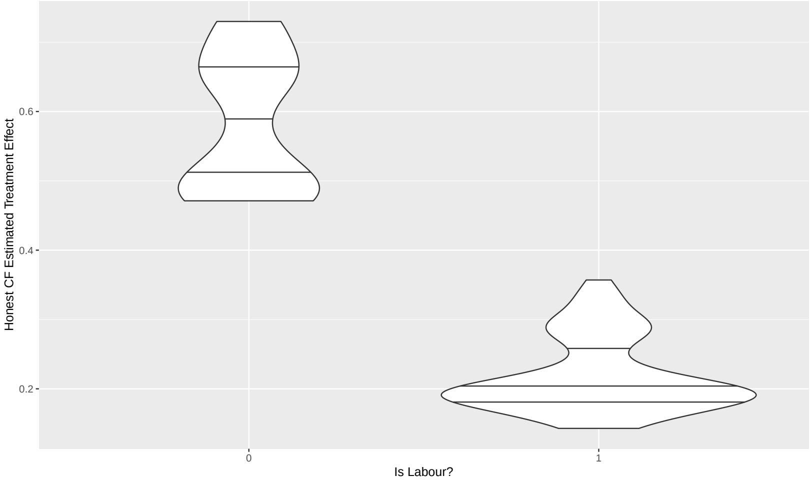
- Mean CF TE, Tory: 0.58

Example: Causal Forests Results, Party



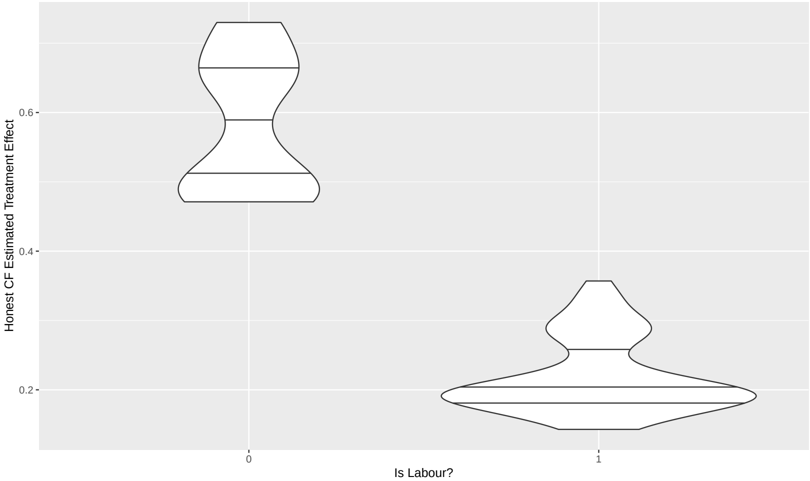
- Mean CF TE, Tory: $0.58 \rightsquigarrow £192,000$

Example: Causal Forests Results, Party



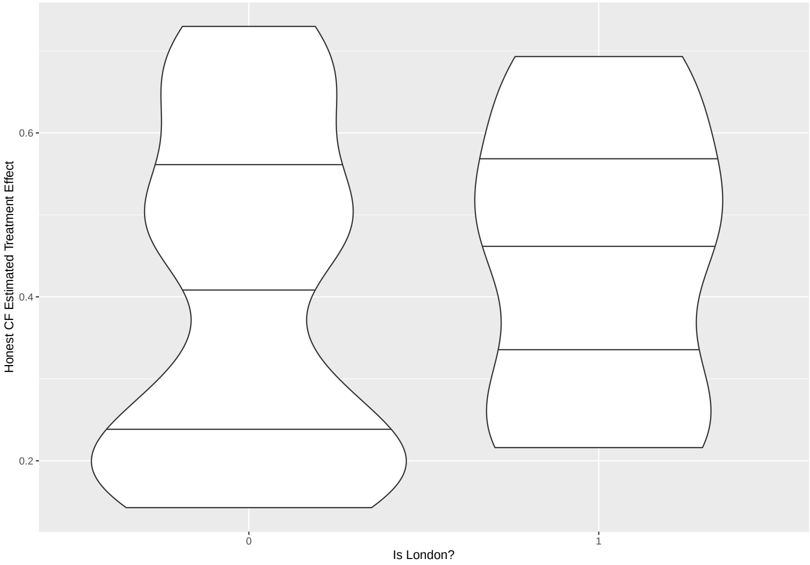
- Mean CF TE, Tory: $0.58 \rightsquigarrow$ £$192,000$
- Mean CF TE, Labour: 0.219

Example: Causal Forests Results, Party

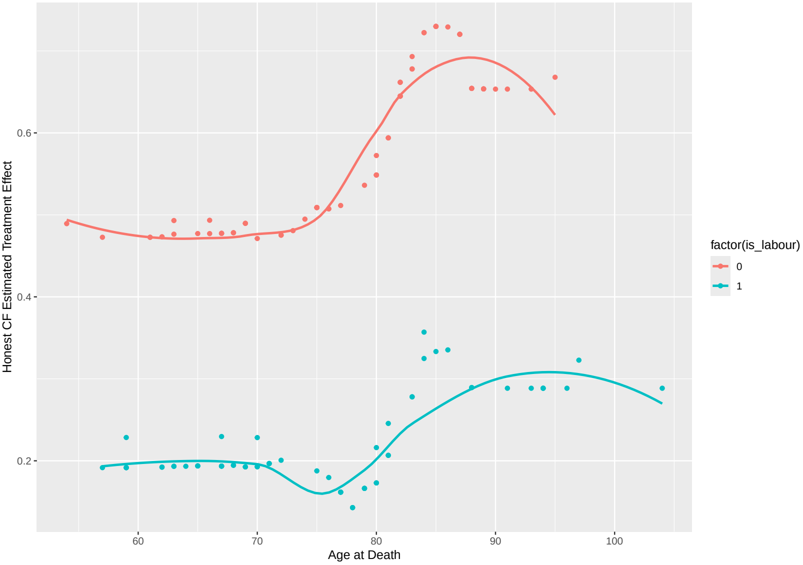


- Mean CF TE, Tory: $0.58 \rightsquigarrow$ £$192,000$
- Mean CF TE, Labour: $0.219 \rightsquigarrow$ £$60,000$

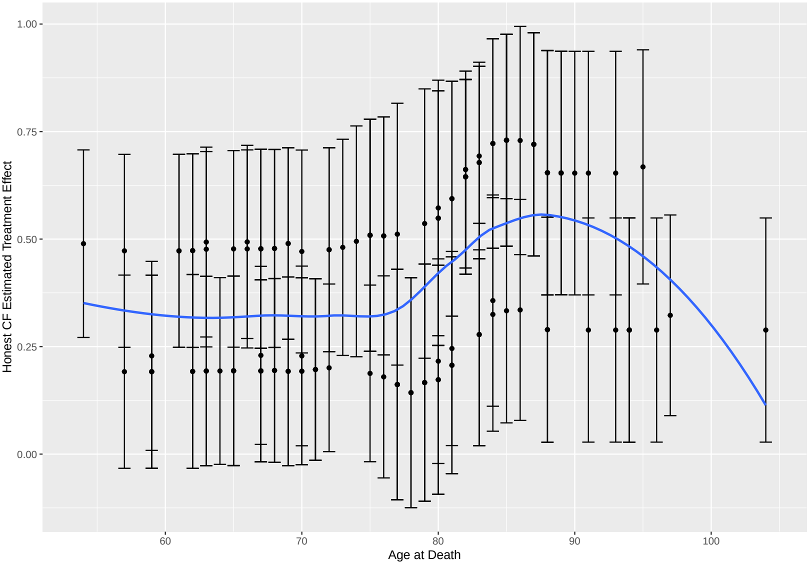
Example: Causal Forests Results, London



Example: Causal Forests Results, Age



Example: Causal Forests Results, Age



Variable Selection

Feature Selection

- Wrappers: pick subset of covars, train on data (estimate model), test on hold-out, score predictions. Keep best-scoring subset.

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- Filters: correlate covars with outcome. Keep strongest.

Feature Selection

- Wrappers: pick subset of covars, train on data (estimate model), test on hold-out, score predictions. Keep best-scoring subset.
- Filters: correlate covars with outcome. Keep strongest.
- Embeds: select features and estimate model at same time. Penalize using more predictors.

Regularization Methods

OLS reminder

Minimize SSR:

$$\mathrm{argmin}_{\beta} \sum_{i=1}^{n} \left(y_i - \hat{y}_i\right)^2$$

$$\mathrm{argmin}_{\beta} \sum_{i=1}^{n} \left(\mathbf{y} - \mathbf{X}\hat{\beta}\right)^2$$

Embedded Regularization Methods

L1 regularization: the LASSO (Least Absolute Shrinkage and Selection Operator)

$$\text{argmin}_{\beta} \left[\sum_{i=1}^{n} \left(y_i - \mathbf{X}\hat{\beta} \right)^2 + \lambda \sum_{j=1}^{k} |\hat{\beta}_j| \right]$$

Embedded Regularization Methods

L1 regularization: the LASSO (Least Absolute Shrinkage and Selection Operator)

$$\text{argmin}_\beta \left[\sum_{i=1}^{n} \left(y_i - \mathbf{X}\hat{\beta} \right)^2 + \lambda \sum_{j=1}^{k} |\hat{\beta}_j| \right]$$

L2 regularization: Ridge regression

$$\text{argmin}_\beta \left[\sum_{i=1}^{n} \left(y_i - \mathbf{X}\hat{\beta} \right)^2 + \lambda \sum_{j=1}^{k} \hat{\beta}_j^2 \right]$$

Embedded Regularization Methods

Mix L1 and L2: Elastic net

$$\operatorname{argmin}_\beta \left(\frac{\sum_{i=1}^{n} \left(y_i - \mathbf{X}\hat{\beta}\right)^2}{2n} + \lambda \left[\alpha \sum_{j=1}^{k} |\hat{\beta}_j| + \frac{1-\alpha}{2} \sum_{j=1}^{k} \hat{\beta}_j^2 \right] \right)$$

Embedded Regularization Methods

Mix L1 and L2: Elastic net

$$\operatorname{argmin}_{\beta}\left(\frac{\sum_{i=1}^{n}\left(y_i-\mathbf{X}\hat{\beta}\right)^2}{2n}+\lambda\left[\alpha\sum_{j=1}^{k}|\hat{\beta}_j|+\frac{1-\alpha}{2}\sum_{j=1}^{k}\hat{\beta}_j^2\right]\right)$$

Regularized trees, ...

Embedded Regularization Methods

How to choose λ, α?

Embedded Regularization Methods

How to choose λ, α?

The LASSO

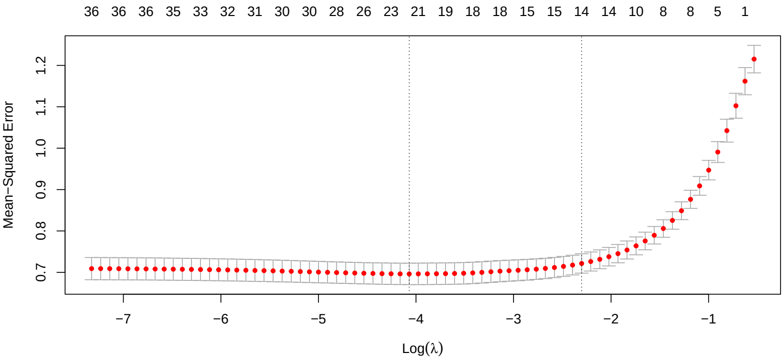
The LASSO

Cross-validation for λ:

```
df_lasso <- read_csv("~/Desktop/lasso.csv")

X <- as.matrix(df_lasso[, 2:ncol(df_lasso)])

Y <- as.matrix(df_lasso[, "y"])

library(glmnet)

cv_lasso <- cv.glmnet(X, Y, alpha = 1)
```

The LASSO

```
plot(cv_lasso)
```




```
cv_lasso$lambda.min
```

```
[1] 0.0170891
```

The LASSO

Implement:

```
lasso_out <- glmnet(X, Y, alpha = 1,
                    lambda = cv_lasso$lambda.min)

lasso_out
```

```
Call:  glmnet(x = X, y = Y, alpha = 1, lambda = cv_lasso$la

  Df  %Dev  Lambda
1 21 45.32 0.01709
```

The LASSO

Coefficients:

```r
coef_lasso <- coef(lasso_out)
round(coef_lasso, 3)
```

```
37 x 1 sparse Matrix of class "dgCMatrix"
                   s0
(Intercept)  0.000
x1           0.112
x2           0.095
x3           0.086
x4           0.147
x5           0.002
x6           0.063
x7           0.051
x8           0.074
x9           0.042
x10          .
x11
```

The LASSO

Coefficients:

```
round(coef_lasso[, ], 3)
```

```
(Intercept)          x1          x2          x3          x4
      0.000       0.112       0.095       0.086       0.147
         x6          x7          x8          x9         x10
      0.063       0.051       0.074       0.042       0.000
        x12         x13         x14         x15         x16
      0.039       0.000       0.026       0.000       0.000
        x18         x19         x20         x21         x22
      0.010       0.127      -0.015       0.030       0.000
        x24         x25         x26         x27         x28
      0.000       0.000       0.000       0.000      -0.010
        x30         x31         x32         x33         x34
      0.000       0.028       0.032       0.000      -0.041
        x36
      0.048
```

The LASSO

Implement, alternative λ:

```r
lasso_1se <- glmnet(X, Y, alpha = 1,
                    lambda = cv_lasso$lambda.1se)

coef(lasso_1se)
```

```
37 x 1 sparse Matrix of class "dgCMatrix"
                       s0
(Intercept) -0.0003034087
x1           0.1051188782
x2           0.0898842045
x3           0.0742522801
x4           0.1513883536
x5           .
x6           0.0603811184
x7           0.0389489143
x8           0.0575738993
x9           0.0374420416
```

The LASSO

Coefficients:

```r
round(coef(lasso_1se)[, ], 3)
```

```
(Intercept)          x1          x2          x3          x4
      0.000       0.105       0.090       0.074       0.15
         x6          x7          x8          x9          x1
      0.060       0.039       0.058       0.037       0.00
        x12         x13         x14         x15         x1
      0.029       0.000       0.008       0.000       0.00
        x18         x19         x20         x21         x2
      0.004       0.039       0.000       0.000       0.00
        x24         x25         x26         x27         x2
      0.000       0.000       0.000       0.000       0.00
        x30         x31         x32         x33         x3
      0.000       0.000       0.013       0.000       0.00
        x36
      0.030
```

The Double LASSO for Treatment Effects

The idea:

- covariates may $\rightsquigarrow Y$ or $\rightsquigarrow T$

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- $\approx$ "double robust", "AIPW" estimators

The Double LASSO for Treatment Effects

The idea:

- covariates may $\rightsquigarrow Y$ or $\rightsquigarrow T$
- $\approx$ "double robust", "AIPW" estimators
- (different to just "doing LASSO twice" for regularization + shrinkage)

The Double LASSO for Treatment Effects

1. Model $Y = f(X)$ using LASSO

The Double LASSO for Treatment Effects

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2. Model $T = f(X)$, using LASSO

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1. Model $Y = f(X)$ using LASSO
2. Model $T = f(X)$, using LASSO
3. Let X_{LASSO} be set of imp covariates identified s.t. each $\beta_{X_{\text{LASSO}}} > 0$

The Double LASSO for Treatment Effects

1. Model $Y = f(X)$ using LASSO
2. Model $T = f(X)$, using LASSO
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4. Model $Y = T + X_{\text{LASSO}}$

The Double LASSO for Treatment Effects

1. Model $Y = f(X)$ using LASSO
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4. Model $Y = T + X_{\text{LASSO}}$

The Double LASSO for Treatment Effects

```
library(hdm)
```

```
data(social)

df_social <- social |> mutate(is_male = if_else(sex == "mal
                              age = 2006 - yearofbirth,
                              is_neighbors = if_else(messag
  filter(messages %in% c("Neighbors", "Control"))

rlasso_out <- rlassoATE(primary2006 ~ age + is_male + prima
```

The Double LASSO for Treatment Effects

```r
summary(rlasso_out)
```

```
Estimation and significance testing of the treatment effect
Type: ATE
Bootstrap: not applicable
     coeff.      se. t-value p-value
TE 0.080091 0.002625   30.51  <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 '
```

The Double LASSO for Treatment Effects

```r
X <- as.matrix(df_social[, c("age", "is_male", "primary2004

Y <- as.matrix(df_social[, "primary2006"])

D <- as.matrix(df_social[, "is_neighbors"])

summary(rlassoEffects(X, Y, method = "double selection"))
```

```
[1] "Estimates and significance testing of the effect of ta
              Estimate. Std. Error t value Pr(>|t|)
age           0.0038449  0.0000681  56.456  < 2e-16 ***
is_male       0.0086763  0.0018889   4.593 4.36e-06 ***
primary2004   0.1474364  0.0019924  74.000  < 2e-16 ***
hhsize        0.0004260  0.0012618   0.338    0.736
is_neighbors  0.0802361  0.0026278  30.534  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 '
```

R packages for Regularization, etc.

- `glmnet`
- `caret`

See also `tidymodels`, `parsnip`, ...

Thanks!

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