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Appendix A: Transverse Shear Power Spectrum

To work out the contaminations from the transverse shear components of weak lensing, we first need to find the power spectra of those shear components.

The formalism for the two-dimensional weak lensing has been carried out *e.g.* in [1]. It is quite straightforward to generalize the treatment to 3-dimensional case, where lensing maps the intrinsic image of sources on the sky, namely the source space, onto the observed sky, namely the image space. Any position on the sky can be denoted by its angular coordinates (θ_1, θ_2) and its comoving distance D or its redshift z.

Analogous to 2-dimensional weak lensing, we have

$$\theta_i^S = \theta_i + \frac{\partial\psi}{\partial\theta_i}, \; i = 1, 2, 3, \tag{A1}$$

representing that sources intrinsicly at position $(\theta_1^S, \theta_2^S, \theta_3^S)$ are related to the apparent position $(\theta_1, \theta_2, \theta_3)$ by the lensing potential ψ. Note that we assume the universe is flat, so that the two angular coordinate components satisfy

$$\theta_i = \frac{\Delta x_i}{D}, \; i = 1, 2, \tag{A2}$$

where Δx_i is the comoving coordinates on the image plane near the source. The third coordinate is defined in the similar way

$$\theta_3 \equiv \frac{\Delta x_3}{D} = \frac{\Delta z}{HD}, \tag{A3}$$

where Δx_3 is a small comoving deviation from the origin (which can be chosen arbitarily in the image space near the source) along the line of sight.

By taking the derivative of Eqn.(A1), we find the Jacobian of the coordinate transformation from the

[1] D. H. Weinberg, M. J. Mortonson, D. J. Eisenstein, C. Hirata, A. G. Riess, and E. Rozo, “Observational probes of cosmic acceleration,” *Physics Reports*, vol. 530, no. 2, pp. 87 – 255, 2013. Observational Probes of Cosmic Acceleration.
[2] T. Venumadhav, A. Oklopcic, V. Gluscevic, A. Mishra, and C. M. Hirata, “A new probe of magnetic fields in the pre-reionization epoch: I. Formalism,” 2014.

image space to the source space,

$$\begin{aligned}\frac{\partial\theta_i^S}{\partial\theta_j} &= \delta_{ij} + \frac{\partial^2\psi}{\partial\theta_i\partial\theta_j} \\ &= (1+\kappa)I_{3\times3} + \begin{pmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{12} & \gamma_{22} & \gamma_{23} \\ \gamma_{13} & \gamma_{23} & \gamma_{33} \end{pmatrix}, \end{aligned} \tag{A4}$$

where we decompose the symmetric $\partial^2\psi/\partial\theta_i\partial\theta_j$ as the sum of a diagonal and a symmetric traceless matrix, so that

$$\gamma_{11} + \gamma_{22} + \gamma_{33} = 0. \tag{A5}$$

The diagonal matrix represents the effect of magnification, while the traceless matrix contains the shear components. There are 5 independent shear components.

The shear components are related to the derivatives by

$$\gamma_{ij}(\theta_1,\theta_2,z) = \frac{\partial^2\psi(\theta_1,\theta_2,z)}{\partial\theta_i\partial\theta_j} - \kappa\delta_{ij}, \tag{A6}$$

where δ_{ij} is the Kronecker delta. Taking 2-dimensional Fourier transform for each component with $(l_1,l_2)\equiv\vec{l}\neq 0$, we obtain

$$\begin{cases} \tilde{\gamma}_{11}(\vec{l},z) = -l_1^2\tilde{\psi}(\vec{l},z) & , \ \tilde{\gamma}_{12}(\vec{l},z) = -l_1 l_2\tilde{\psi}(\vec{l},z) \\ \tilde{\gamma}_{22}(\vec{l},z) = -l_2^2\tilde{\psi}(\vec{l},z) & , \ \tilde{\gamma}_{13}(\vec{l},z) = i l_1 \frac{\tilde{\psi}(\vec{l},z)}{\partial\theta_3} \\ \tilde{\gamma}_{33}(\vec{l},z) = \frac{\partial^2\tilde{\psi}(\vec{l},z)}{\partial\theta_3^2} & , \ \tilde{\gamma}_{23}(\vec{l},z) = i l_2 \frac{\tilde{\psi}(\vec{l},z)}{\partial\theta_3} \end{cases} \tag{A7}$$

The 2D Fourier transform of the lensing potential is

$$\tilde{\psi}(\vec{l},D) = \int_\Omega \psi(D,\hat{n}) e^{-i\vec{l}\cdot\hat{n}}\, d\theta_1 d\theta_2, \tag{A8}$$

where $\hat{n}\equiv(\theta_1,\theta_2)$. The $\tilde{\gamma}_{13},\tilde{\gamma}_{23}$ components are the transverse shear components that we care about. Let's calculate the power spectrum of γ_{13} first.

Note that the lensing potential ψ is given by

$$\psi(D,\hat{n}) = -2\int_0^D [\cot_K(D_1) - \cot_K(D)]\Phi(D_1,\hat{n})dD_1, \tag{A9}$$

where Φ is the Newtonian gravitational potential and the cotangentlike function is hyperbolic for flat universe, *i.e.* $\cot_K(D) = 1/D$. Using chain rule, one can show that

$$\frac{\partial\psi(\hat{n},z)}{\partial\theta_3} = HD\frac{\partial\psi}{\partial z} = D\frac{\partial\psi(\hat{n},z)}{\partial D}, \tag{A10}$$

which, after doing 2D Fourier transform, will become

$$\frac{\partial\tilde{\psi}(\vec{l},z)}{\partial\theta_3} = D\frac{\partial\tilde{\psi}(\vec{l},z)}{\partial D}. \tag{A11}$$

Substituting Eqn.(A9) into Eqn.(A8), we end up with

$$\frac{\partial\tilde{\psi}(\vec{l},z)}{\partial\theta_3} = -\frac{2}{D}\int_0^D dD_1\tilde{\Phi}(D_1,\vec{l}), \tag{A12}$$

where $\tilde{\Phi}$ is the 2D Fourier transform of Φ. Now we obtain

$$\begin{aligned}\langle\tilde{\gamma}_{13}^*(\vec{l},z)\tilde{\gamma}_{13}(\vec{l}',z')\rangle &= \frac{4}{D(z)D(z')}l_1 l_1' \\ &\times\int_0^{D(z)} dD_1 \int_0^{D(z')} dD_1' \langle\tilde{\Phi}^*(D_1,\vec{l})\tilde{\Phi}(D_1',\vec{l}')\rangle. \end{aligned} \tag{A13}$$

Then we need to find $\langle\tilde{\Phi}^*(D_1,\vec{l})\tilde{\Phi}(D_1',\vec{l}')\rangle$.

Define

$$\tilde{\Phi}(D_1,\vec{l}) \equiv \int_{-\infty}^{\infty} \tilde{\tilde{\Phi}}(l_3,\vec{l})e^{il_3D_1}\frac{dl_3}{2\pi}, \tag{A14}$$

we can write

$$\begin{aligned}\langle\tilde{\Phi}^*(D_1,\vec{l})\tilde{\Phi}(D_1',\vec{l}')\rangle &= \int\int \frac{dl_3}{2\pi}\frac{dl_3'}{2\pi}\langle\tilde{\tilde{\Phi}}^*(l_3,\vec{l})\tilde{\tilde{\Phi}}(l_3',\vec{l}')\rangle \\ &\times e^{i(l_3'D_1' - l_3D_1)}. \end{aligned} \tag{A15}$$

Here $\tilde{\tilde{\Phi}}$ is actually the 3D Fourier transform of Φ, so we can assume that different modes are uncorrelated, which implies

$$\langle\tilde{\tilde{\Phi}}^*(l_3,\vec{l})\tilde{\tilde{\Phi}}(l_3',\vec{l}')\rangle = (2\pi)^3\delta(l_3-l_3')\delta^2(\vec{l}-\vec{l}')P_\Phi(\sqrt{l_3^2+|\vec{l}|^2}). \tag{A16}$$

Substituting into Eqn.(A15) and applying the Limber's approximation: $l_3 \ll |\vec{l}|$, we obtain

$$\langle\tilde{\Phi}^*(D_1,\vec{l})\tilde{\Phi}(D_1',\vec{l}')\rangle \simeq (2\pi)^2\delta^2(\vec{l}-\vec{l}')P_\Phi(|\vec{l}|)\delta(D_1'-D_1). \tag{A17}$$

Thus, we have

$$\begin{aligned}\langle\tilde{\gamma}_{13}^*(\vec{l},z)\tilde{\gamma}_{13}(\vec{l}',z')\rangle &= \langle l_1 l_1' \frac{\tilde{\psi}^*(\vec{l},z)}{\partial\theta_3}\frac{\tilde{\psi}(\vec{l}',z')}{\partial\theta_3}\rangle \\ &= \frac{4}{D(z)D(z')}l_1 l_1'(2\pi)^2\delta^2(\vec{l}-\vec{l}')\int_0^{D(z)} dD_1 P_\Phi(l), \end{aligned} \tag{A18}$$

where $l\equiv|\vec{l}|$ and $P_\Phi(l)$ is the angular power spectrum,

$$P_\Phi(l) = \frac{P_\Phi(k = l/D_{A_1})}{D_{A_1}^2}, \tag{A19}$$

where comoving angular diameter distance $D_{A_1}(z_1) = D_1(z_1)$ in the flat Universe.

If we define the power spectrum $P_{13}(\vec{l},z,z')$ of γ_{13} components as

$$\langle\tilde{\gamma}_{13}^*(\vec{l},z)\tilde{\gamma}_{13}(\vec{l}',z')\rangle \equiv (2\pi)^2 P_{13}(\vec{l},z,z')\delta^2(\vec{l}-\vec{l}'), \tag{A20}$$

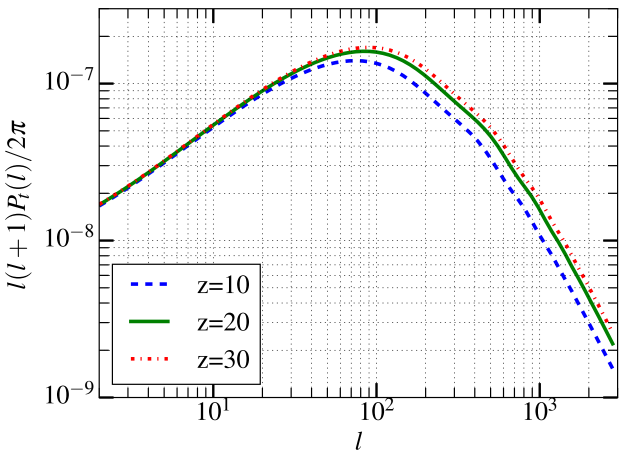

FIG. 1. The transverse power spectra for sources at redshifts $z = 10$(bottom), 20 and 30(top), predicted for the WMAP 7-year best fit cosmology ($\Omega_m = 0.265, \sigma_8 = 0.8, H_0 = 71.9$ km s^{-1}Mpc^{-1}).

then we have

$$P_{13}(\vec{l}, z, z') = \frac{4}{D(z)D(z')} l_1^2 \int_0^{D(z)} dD_1 P_\Phi(l). \quad \text{(A21)}$$

There are similar results for the power spectrum P_{23} of γ_{23} component. Furthermore, we can define the transverse power spectrum P_t as follows,

$$\begin{aligned} P_t(l, z, z') \equiv & P_{13} + P_{23} \\ = & \frac{4}{D(z)D(z')} l^2 \int_0^{D(z)} dD_1 \frac{P_\Phi(k = \frac{l}{D_1})}{D_1^2}. \end{aligned} \quad \text{(A22)}$$

This transverse power spectrum does not depend on the components but the magnitude of the angular wave number vector.

If the separation between z and z' are negligible, i.e. $z = z'$, we have

$$P_t(l, z) = \frac{4}{D^2(z)} l^2 \int_0^{D(z)} dD_1 \frac{P_\Phi(k = \frac{l}{D_1})}{D_1^2}. \quad \text{(A23)}$$

Notice that in the integral, the variable D_1 is a function of a redshift variable z_1, and in the Newtonian gravitational potential power spectrum function P_Φ, there is also an implicit dependence on the redshift variable z_1, i.e.

$$P_t(l, z) = \frac{4}{D^2(z)} l^2 \int_0^{D(z)} dD_1 \frac{P_\Phi(k = \frac{l}{D_1(z_1)}, z_1)}{D_1^2(z_1)}. \quad \text{(A24)}$$

Also notice that the power spectrum of potential is related to the matter power spectrum by

$$P_\Phi(k, z_1) = \left[\frac{3}{2}\Omega_m H_0^2(1 + z_1)\right]^2 k^{-4} P_\delta(k, z_1) \quad \text{(A25)}$$

In our calculation, we use $c = 1$, so here the present Hubble constant is taken as $100h/(3 \times 10^5)$ with unit Mpc^{-1}. The results are shown in Fig.1.

Appendix B: Lensing Magnetic Field Contamination

Venumadhav *et al.* [2] has found the expression of the brightness temperature fluctuation δT_b as a function of the magnitude of magnetic fields, *i.e.* Eqn.(1), where the parameters x_B is given by

$$x_{\rm B} = \frac{g_{\rm e}\mu_{\rm B} T_*}{2\hbar A T_\gamma} B \equiv \bar{x}_{\rm B} B \quad \text{(B1)}$$

The power spectrum of the brightness temperature is related to the matter power spectrum $P_\delta(k)$ by

$$P_{T_b}(\boldsymbol{k}) = \left|\frac{\partial \delta T_b}{\partial \delta}\right|^2 P_\delta(k). \quad \text{(B2)}$$

The transfer function $\partial\delta T_b/\partial\delta$ is given by taking derivative of (1), *i.e.*

$$\begin{aligned} \frac{\partial \delta T_b}{\partial \delta} = & A\Big[(26.4 - 2C)\left(1 + (\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{n}})^2\right) \\ & - \frac{C}{15}\sum_m \frac{4\pi}{5} \frac{Y_{2m}(\hat{\boldsymbol{k}})[Y_{2m}(\hat{\boldsymbol{n}})]^*}{1 + x_{\alpha,(2)} + x_{c,(2)} - imx_B}\Big], \end{aligned} \quad \text{(B3)}$$

where for convenience we define

$$A \equiv \left(1 - \frac{T_\gamma}{T_{\rm s}}\right) x_{1{\rm s}} \left(\frac{1+z}{10}\right)^{1/2} \quad \text{(B4)}$$

$$C \equiv 0.128 \text{ mK} \left(\frac{T_\gamma}{T_{\rm s}}\right) x_{1{\rm s}} \left(\frac{1+z}{10}\right)^{1/2} \quad \text{(B5)}$$

By expanding Eqn.(B3) for the small magnetic field B (*i.e.* small x_B), we will obtain a precession correction on the original P_{T_b} profile. The tangent of the precession angle θ_{pr} is proportional to the magnitude of the magnetic field applied.

To see that, let's first define our coordinate system as shown in Fig.2. The unit vectors then will be

$$\hat{k} = \left(\theta = \frac{\pi}{2}, \varphi\right), \; \hat{n} = \left(\theta = \frac{\pi}{2}, 0\right), \quad \text{(B6)}$$

where the first coordinate represents the angle θ between the Direction #2 and the vector. Now we are able to

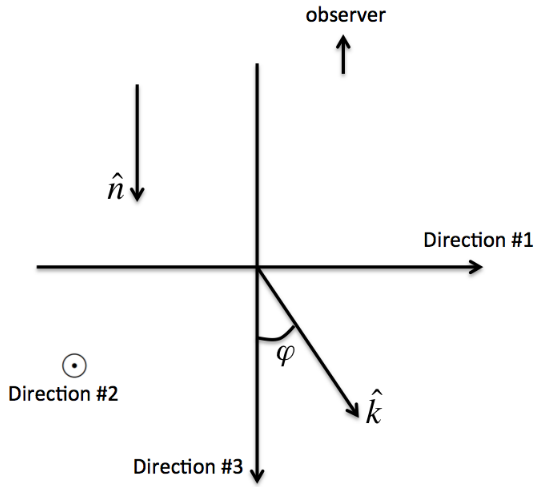

FIG. 2. The coordinate system

simplify some sums such as

$$\sum_m Y_{2m}(\hat{k})[Y_{2m}(\hat{n})]^* = \frac{15\cos 2\varphi + 5}{16\pi}, \tag{B7}$$

$$\sum_m mY_{2m}(\hat{k})[Y_{2m}(\hat{n})]^* = \frac{15i\sin 2\varphi}{8\pi}. \tag{B8}$$

When we expand the equation (B3) and simplify it, we obtain

$$\begin{aligned}\frac{\partial \delta T_b}{\partial \delta} = A\Bigg[&(39.6 - 3C) - \frac{C}{60}\frac{1}{1 + x_{\alpha,(2)} + x_{c,(2)}} \\ &+ \left(13.2 - C - \frac{C}{20}\frac{1}{1 + x_{\alpha,(2)} + x_{c,(2)}}\right)\cos 2\varphi \\ &+ \frac{C}{10}\frac{x_B}{(1 + x_{\alpha,(2)} + x_{c,(2)})^2}\sin 2\varphi\Bigg]. \end{aligned} \tag{B9}$$

For convenience, we define the following parameters,

$$\mu \equiv \frac{C}{10}\frac{x_B}{(1 + x_{\alpha,(2)} + x_{c,(2)})^2}, \tag{B10}$$

$$\lambda \equiv 13.2 - C - \frac{C}{20}\frac{1}{1 + x_{\alpha,(2)} + x_{c,(2)}}, \tag{B11}$$

$$q \equiv 39.6 - 3C - \frac{C}{60}\frac{1}{1 + x_{\alpha,(2)} + x_{c,(2)}}. \tag{B12}$$

Due to the $\sin 2\varphi$ term contributed by the small magnetic field, the original profile gets a small precession angle θ_{pr}

$$\theta_{pr} \simeq \tan\theta_{pr} = -\frac{\mu}{\lambda}, \tag{B13}$$

Thus, if we measure the precession angle θ_{pr} in the power spectra of T_b, we can determine the magnitude of the magnetic field.

Next we will show that the transverse components of the weak lensing can also produce a similar precession angle.

In the 3D Cartesian coordinate, the unit vectors are written as

$$\hat{k} = (\sin\varphi, 0, \cos\varphi),\ \hat{n} = (0, 0, 1), \tag{B14}$$

and they are distorted into

$$\begin{aligned}\hat{k}' &= \begin{pmatrix} (1 + \kappa + \gamma_{11})\sin\varphi + \gamma_{13}\cos\varphi \\ \gamma_{12}\sin\varphi + \gamma_{23}\cos\varphi \\ \gamma_{13}\sin\varphi + (1 + \kappa + \gamma_{33})\cos\varphi \end{pmatrix}, \\ \hat{n}' &= (\gamma_{13}, \gamma_{23}, 1 + \kappa + \gamma_{33}). \end{aligned} \tag{B15}$$

Since the shearing components are all very small, $\hat{k}'$ is basicly still in the original plane, we can just ignore its component in the Direction #2. Taking the first order, we obtain

$$\cos\varphi' = \hat{n}' \cdot \hat{k}' \simeq \cos\varphi + (2\kappa + 2\gamma_{33})\cos\varphi + 2\gamma_{13}\sin\varphi. \tag{B16}$$

Since

$$\cos\varphi' = \cos(\varphi + \Delta\varphi) = \cos\varphi\cos\Delta\varphi - \sin\varphi\sin\Delta\varphi, \tag{B17}$$

we have

$$\Delta\varphi \simeq \sin\Delta\varphi \simeq -2\gamma_{13}. \tag{B18}$$

Thus,

$$\Delta(\cos 2\varphi) = 4\gamma_{13}\sin 2\varphi, \tag{B19}$$

$$\Delta(\sin 2\varphi) = -4\gamma_{13}\cos 2\varphi. \tag{B20}$$

Now if we don't have magnetic field (*i.e.* $x_B = 0$), due to the weak lensing, the transfer function (B9) will become

$$\begin{aligned}\frac{\partial \delta T_b}{\partial \delta} = A\Bigg[&(39.6 - 3C) - \frac{C}{60}\frac{1}{1 + x_{\alpha,(2)} + x_{c,(2)}} \\ &+ \lambda(\cos 2\varphi + 4\gamma_{13}\sin 2\varphi)\Bigg]. \end{aligned} \tag{B21}$$

Since the weak lensing also spurs the k in $P_\delta(k)$, we need to find the relation between $P_\delta(k')$ and $P_\delta(k)$. To the first order, we can find

$$k' = k(1 + \kappa + \gamma_{11}\sin^2\varphi + \gamma_{33}\cos^2\varphi + 2\gamma_{13}\sin\varphi\cos\varphi). \tag{B22}$$

Since

$$P_\delta(k) \propto k^{n_{\rm eff}}, \tag{B23}$$

where $n_{\rm eff}$ is the effective spectral index, we have

$$\begin{aligned}P_\delta(k') =&[1 + \kappa + \gamma_{11}\sin^2\varphi + \gamma_{33}\cos^2\varphi \\ &+ 2\gamma_{13}\sin\varphi\cos\varphi]^{n_{\rm eff}} \times P_\delta(k). \end{aligned} \tag{B24}$$

Now we have

$$P_{T_b}(\vec{k}') = \left|\frac{\partial \delta T_b}{\partial \delta}[1+\kappa+\gamma_{11}\sin^2\varphi+\gamma_{33}\cos^2\varphi + 2\gamma_{13}\sin\varphi\cos\varphi]^{n_{\rm eff}/2}\right|^2 P_\delta(k), \quad \text{(B25)}$$

where we can see all the quadropole features in the power spectra of T_b come from the modified transfer function.

After expanding, combining terms and keeping the first-order terms, we can obtain the modified transfer function, since we only need the ratio of the coefficients of $\sin 2\varphi$ and $\cos 2\varphi$, we can simply write the modified transfer function as

$$\begin{aligned} &A\,[\{\text{const. terms}\} \\ &+\cos 2\varphi\left\{\lambda+\frac{n_{\rm eff}}{2}\left[\lambda\left(\kappa+\frac{\gamma_{11}+\gamma_{33}}{2}\right)+q\frac{\gamma_{33}-\gamma_{11}}{2}\right]\right\} \\ &+\sin 2\varphi\left\{4\gamma_{13}\lambda+\frac{n_{\rm eff}}{2}\left[4\gamma_{13}\lambda\left(\kappa+\frac{\gamma_{11}+\gamma_{33}}{2}\right)+q\gamma_{13}\right]\right\} \\ &+\{\sin 4\varphi, \cos 4\varphi \text{ terms}\}]\,. \end{aligned} \quad \text{(B26)}$$

The octupole terms are small and negligible. Then we have the precession angle again, given by the negative ratio of the two coefficients

$$\begin{aligned}\theta'_{pr} &\simeq -\frac{4\gamma_{13}\lambda+\frac{n_{\rm eff}}{2}\left[4\gamma_{13}\lambda\left(\kappa+\frac{\gamma_{11}+\gamma_{33}}{2}\right)+q\gamma_{13}\right]}{\lambda+\frac{n_{\rm eff}}{2}\left[\lambda\left(\kappa+\frac{\gamma_{11}+\gamma_{33}}{2}\right)+q\frac{\gamma_{33}-\gamma_{11}}{2}\right]} \\ &\simeq -\left(4+\frac{n_{\rm eff}}{2}\frac{q}{\lambda}\right)\gamma_{13}. \end{aligned} \quad \text{(B27)}$$

This precession angle is made by the transverse weak lensing component instead of the magnetic field, but the effect it makes looks like there is a (fake) magnetic field such that

$$\frac{\mu}{\lambda} = \left(4+\frac{n_{\rm eff}}{2}\frac{q}{\lambda}\right)\gamma_{13}, \quad \text{(B28)}$$

where we can solve for the (fake) comoving magnetic field

$$\begin{aligned}B_{\rm lensing,13} &= \frac{10(1+x_{\alpha,(2)}+x_{c,(2)})^2(4\lambda+\frac{n_{\rm eff}}{2}q)\gamma_{13}}{C\bar{x}_{\rm B}(1+z)^2} \\ &\equiv \alpha\gamma_{13}. \end{aligned} \quad \text{(B29)}$$

Finally, we find the power spectrum of the comoving lensing magnetic field

$$P_B^{\rm lensing,13}(l) = \left|\frac{\partial B_{\rm lensing,13}}{\partial \gamma_{13}}\right|^2 P_{\gamma_{13}}(l) = \alpha^2 P_{\gamma_{13}}(l). \quad \text{(B30)}$$

Note that this power spectrum is only the contribution from γ_{13} component because of our choice of coordinate. The contribution from γ_{23} component should have the same form, *i.e.*

$$P_B^{\rm lensing,23}(l) = \alpha^2 P_{\gamma_{23}}(l), \quad \text{(B31)}$$

so that the total power spectrum of comoving lensing magnetic field is

$$P_B^{\rm lensing}(l) = \alpha^2 P_t(l), \quad \text{(B32)}$$

$$\Delta_B^{\rm lensing}(l) = \sqrt{\frac{l(l+1)}{2\pi}P_B^{\rm lensing}(l)} \quad \text{(B33)}$$

For a survey with $\Omega_{\rm survey} = 1{\rm sr}$, the scale of interest is about $l = 6$. The scale of matter fluctuations relavant to the observed signals is determined by the resolution of the inferometers as

$$k \sim \frac{2\pi L}{\lambda_0(1+z)D} \sim 1, \quad \text{(B34)}$$

corresponding to $n_{\rm eff} \sim -2.274$. The result is shown in Fig.3.

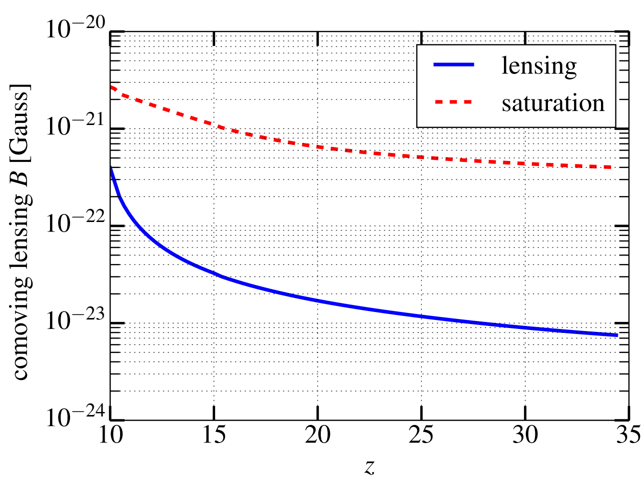

FIG. 3. The 1σ comoving lensing magnetic field that is produced by the transverse shearing effect of weak lensing, compared to the saturation limit of our method.