Particle Filtering: Practice

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1 Filtrage Particulaire

1.1 MMSE Estimation

Let us define the following dynamical system:

$$
\begin{aligned}
x(t+1) &= \frac{x(t)}{2} + \frac{25x(t)}{1+x(t)^2} + 8\cos\left(1.2(1+t)\right) + v(t) = f(x(t)) + v(t) \\
z(t) &= \frac{x(t)^2}{20} + w(t) = g(x(t)) + w(t)
\end{aligned}
$$

with $v(t) \sim \mathcal{N}(0,10)$, $x(0) \sim \mathcal{N}(0,10)$ and $w(t) \sim \mathcal{N}(0,1)$.

The file `data.mat` contains data generated according to this systems on the epoch $[0,5]$ with a sampling period $T_s = 0.01$. These data are the true state x and the measurements z. The Matlab functions `f.m` and `g.m` can also be downloaded from Moodle for your own implementation requirements.

1. Write the likelihood of the measurements ?
2. Write the expression of $p(x(t+1)|x(t))$?
3. Implement a particle filter to realize a MMSE estimate of x using the measurements of the file *data.mat*. You will first consider a bootstrap filter with a multinomial resampling step.
    - You will first consider a bootstrap filter with a multinomial resampling step.
    - Modify the previous filter to use an EKF-based importance law.

1.2 Maximum A Posteriori (MAP) Estimation

Let us introduce a new estimation criterion: the Maximum A Posteriori (MAP)

$$\hat{x}(t) = max_{x(t)} p(x(t)|z_{1:t}) \tag{1}$$

While using a particle filter we need to get a smooth estimante of the posteriori from the weighted particle representation. For this purpose, we are going to use a *Kernel Smoothing* technics with, for instance a gaussian kernel. Let us explain quickly this method. Let $K(x) = \exp\left(-\frac{1}{2}x^2\right)$ be the so-called kernel and let us suppose we have N observations (y_i, x_i) such that:

$$y_i = f(x_i) + \epsilon_i \tag{2}$$

Then we can obtain a smooth estimation of $f(x)$ using:

$$\hat{f}(x) = \frac{\sum_{i=1}^{N} K(\frac{x-x_i}{h}) y_i}{\sum_{i=1}^{N} K(\frac{x-x_i}{h})} \tag{3}$$

with h the so-called kernel bandwidth. Here we can use this method to get an approximation of the posterior.

- Use the above method and particle filter to get the MAP estimate of the state.
- Observe the evolution of the posterior with respect to time.