Project control system

System is defined by:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -10 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -2.43 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

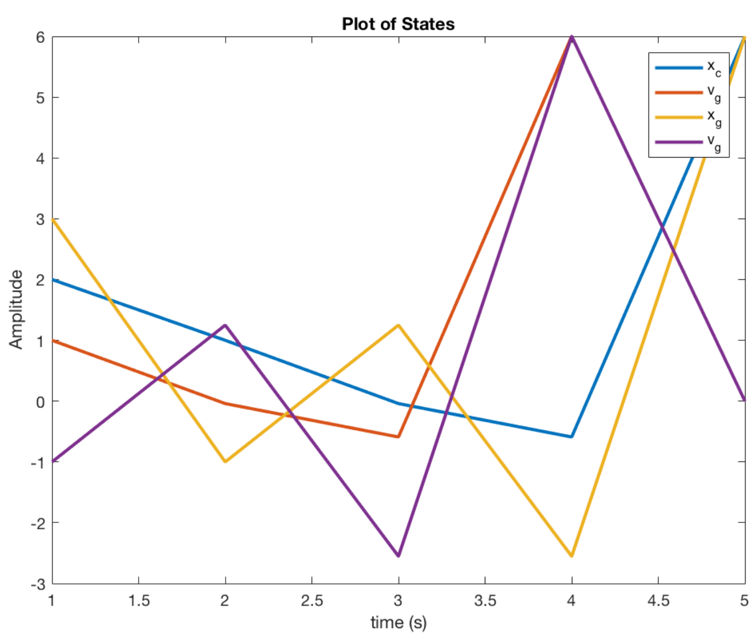
a. Finding the controllability matrix given as $C_{con} = [B \;\; AB \; \ldots \; A^{n-1}B]$ yields:

$$C_{con} = \begin{bmatrix} 0 & 0 & .001 & 0 & -.01 & 0 & .1 & 0 \\ .001 & 0 & -.01 & 0 & .1 & 0 & -1 & 0 \\ 0 & 0 & 0 & .0003 & 0 & -.0007 & 0 & .0017 \\ 0 & .0003 & 0 & -.0007 & 0 & .0017 & 0 & -.0041 \end{bmatrix}$$

rank(C_{con}) = 4, **system is fully controllable**.

b. n/a

c. Choosing x(0) = [2, 1, 3, -1] and x(T) = [6, 0, 6, 0]. Input is found as $u = V\Sigma^{+}U^{*}\, x_b$, where $x_b = x(T) - A^{T}x(0)$. Using this input yields the following plot:



d. Observability matrix is found by $O = [C \;\; CA \; \ldots \; CA^{n-1}]$, and is:

$$O = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & .1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -10 & 0 & 0 \\ 0 & 0 & 0 & -2.43 \\ 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 5.98 \end{bmatrix}$$

rank(O) = 4, **system is fully observable**.

e. n/a
f. Using the same initial condition and trajectory above, the initial condition was reconstructed using $x(0)_{pre} = V\Sigma^{+}U^{*}\, y_b$. This yields a reconstructed initial condition [2, 1, 3, -1] **which is the same as the actual initial condition.**