Endogenous Repo Price Dispersion, and Monetary Policy Transmission

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Abstract

Repurchase agreements (repos) are collateralized contracts often fully secured by safe government securities. However, even identical contracts can trade at different rates, and the rates only partially reflect changes in the central bank's deposit facility rate. Motivated by the fact that most repo participants trade bilaterally with dealers in the over-the-counter segment of repo markets, I develop a search-theoretic model that endogenously generates repo price dispersion and imperfect monetary policy pass-through. The deposit facility price can have ambiguous effects on the supply of central bank liabilities, and its pass-through to repo prices weakens as this price increases. Both the central bank's borrowing and lending facilities eliminate this ambiguity, but using the borrowing one is at the cost of inflation. Consistent with the Friedman rule, pegging the lending and deposit rates at the zero-lower bound achieves the most efficient use of currency in exchange. Crucial for the results is not price dispersion itself, but the concentration of prices at the tails of the distribution.

Key Words: repo price dispersion, search friction, pass-through, monetary policy
JEL: E4, E5, G2

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1 Introduction

Markets for repurchase agreements (repos) are crucial for the transmission of monetary policy to the real economy. Repos are short-term collateralized contracts often secured by safe government securities. However, repo rates exhibit large dispersion. Even identical repo contracts fully secured by the same collateral can trade at substantially different rates (Anbil, Anderson, & Senyuz, 2021; Eisenschmidt, Ma, & Zhang, 2024). Moreover, these rates do not perfectly reflect changes in central banks' policy rates, such as the European Central Bank's deposit facility rate, and this imperfect pass-through impedes the efficient transmission of monetary policy (Ballensiefen, Ranaldo, & Winterberg, 2023; Duffie & Krishnamurthy, 2016).

Participants in repo markets, such as money market funds and insurance companies, trade bilaterally with dealer banks in the over-the-counter (OTC) segment of the markets.[1] This OTC segment accounts for approximately 30% of repo trading volume in both the U.S. and the euro area (European Central Bank, 2019). I show that search frictions inherent to OTC markets give rise to dispersion in repo prices and endogenously generate imperfect pass-through from the deposit facility price to repo prices. The endogenous price dispersion implies that the deposit facility price can have ambiguous effects on asset allocation, and its pass-through to repo prices weakens as this price increases. The framework allows me to evaluate the role of central banks' lending and borrowing instruments in shaping repo price formation and to characterize optimal monetary policy. Overall, I show that it is not the price dispersion itself but the concentration of prices in the tails of the distributions that matters for central bank interventions.

Specifically, I develop an infinite-horizon model with three stages of exchange in each period. In stage 1, repo customers purchase financial portfolios consisting of currency and

[1] I consider dealer banks, or dealers, as subsidiaries of bank holding companies with commercial banking subsidiaries that are depository institutions (Afonso, Cipriani, Copeland, Kovner, La Spada, & Martin, 2021). They are repo market participants that have exclusive access to centralized trading platforms and central bank deposit facilities.

government bonds. Depending on their liquidity needs, some customers become borrowers who require currency to settle transactions for consumption goods in stage 3, while the rest become lenders who care more about asset returns. These two stages follow Lagos and Wright (2005), providing micro-founded trading motives for the liquidity rearrangement in stage 2, which adopts the OTC trading framework of Duffie, Gârleanu, and Pedersen (2005). Namely, transactions in stage 2 include lending and borrowing between dealers and customers through frictional OTC repo markets, as well as a frictionless inter-dealer market that allows dealers to reallocate assets. As is standard in practice, all transactions in this stage are secured with collateral, and dealers can deposit excess funds with the central bank's deposit facility.

An important element of the model is the endogenous distributions for repo *borrowing* and *lending* prices. I derive these distributions as an equilibrium outcome of frictional OTC repo markets, specifically, through their inherent search frictions captured by the pricing framework of Burdett and Judd (1983). Namely, search frictions limit customers to contact at most two dealers for price quotations, giving dealers market power and generating price dispersion. In particular, this price quotation framework is directly motivated by the fact that "the majority of market participants [...] rely on concentrated intermediation by *one or two* dealer banks" (Eisenschmidt, Ma, & Zhang, 2024).

The key determinants of the borrowing or lending price distribution are the **competitive price** that would occur when search frictions are negligible (i.e., customers always meet two dealers), and the **monopoly price** that would arise when search frictions are extremely large (i.e., customers always meet one dealer). The monopoly price provides dealers with the highest *profit per customer*. However, the potential competition from another dealer forces them to play a mixed strategy, offering prices closer to the competitive price to earn a higher *total profit* by attracting more customers.

Endogenous price dispersion induces imperfect monetary policy pass-through, captured by less-than-one-for-one responses of market-determined repo prices to changes in the central bank's deposit facility price.[2] I show that, when the deposit facility is active, search frictions weaken the effectiveness of pass-through by concentrating the price distribution around the monopoly prices and away from the competitive price. Although the competitive price is determined by the deposit facility price, the monopoly price is insensitive to policy changes. Therefore, by changing the price distribution's concentration pattern, search frictions effectively limit the response of repo prices to changes in the deposit facility price. Here comes one of the key insights of this paper: it is not the price dispersion itself, but its concentration pattern, that determines how markets react to central bank interventions. In fact, pass-through becomes null (perfect) as search frictions become extremely large (negligible), collapsing the distribution to the monopoly (competitive) price.

The scenario with an active deposit facility discussed above corresponds to the *floor system* of the monetary policy implementation regime, which is currently adopted by major central banks around the world, including the U.S. Federal Reserve, the European Central Bank, and the Bank of Canada. I also include the scenario with an inactive deposit facility that corresponds to the *corridor system*, which was common before the 2008 Financial Crisis. In a corridor system, changes in the deposit facility price do not affect the inter-dealer price and therefore have no pass-through to repo prices.

A natural response to imperfect monetary policy pass-through is to ask central banks to react more aggressively: adjust their policy rates by larger amounts to achieve their objectives. However, as the deposit facility price increases, its effects on repo lending prices diminish — an immediate result from the earlier discussion on the distribution's concentration pattern. An increase in the deposit facility price, or equivalently, a lower policy rate, shifts the entire lending price distribution rightward, concentrating it around the monopoly price, which is the highest price, or the lowest interest rate, a dealer can

[2] In practice, changes in this price are reflected, for example, by adjustments to the European Central Bank's deposit facility rate (DFR) and the U.S. Federal Reserve's interest rate on reserve balances (IORB).

offer to lenders. This concentration enhances dealers' market power and reduces the effectiveness of pass-through. In particular, the pass-through becomes null in the extreme case when the increased deposit facility induces dealers to offer only the monopoly price. Overall, it may not be worthwhile for central banks to compensate for this imperfect pass-through with larger efforts.[3]

Crucially, I find an ambiguous effect of the central bank's deposit facility price on asset allocation, particularly, the composition of central bank liabilities between deposits and currency. Raising its price, or equivalently, reducing the interest rate on the deposit facility, does not necessarily reduce the supply of central bank deposits, and thus also has ambiguous effects on currency supply. Frictions in the OTC markets, again, play a critical role in this result, with the ambiguity disappearing when search frictions are either negligible or extremely large. Nevertheless, I show that central banks can unambiguously control the composition of their liabilities by pairing their deposit facility with lending and borrowing instruments, such as the U.S. Federal Reserve's repurchase agreement and overnight reverse repurchase agreement facilities.

I show that either raising the central bank's lending facility price or reducing its deposit facility price reduces the supply of central bank deposits while increasing the supply of currency. First, the lending facility provides repo borrowers, instead of relying on dealers, an outside option of borrowing from the central banks. Consequently, raising the lending facility price pushes dealers to offer higher prices to compete for customers. Borrowers obtain more currency with their collateral, reducing dealers' savings in the central bank's deposit facility and increasing its currency supply. Second, the borrowing facility provides a conduit for lenders to lend to the central banks. By reducing its price, it pushes dealers to offer lower prices, or equivalently, higher interest rates to attract lenders. The increased

[3]Regarding borrowing prices, an increase in the deposit facility price makes the repo borrowing prices less concentrated around the borrower dealer's monopoly price. However, this does not enhance the pass-through to borrowing prices, as it also leads these prices to be more dispersed around the competitive price determined by the deposit facility price.

return on lenders' currency holding raises the inflation rate, reducing the nominal bond price. As a result, borrowers can purchase more government bonds and borrow more currency against these assets. This also increases the central bank's currency supply while reducing its deposit liabilities unambiguously.

While both can unambiguously reallocate assets, the lending facility performs better than the borrowing one. This is because, although a lower borrowing facility price provides repo borrowers more currency for transactions, it also leads to higher inflation, reducing the real value of currency. The cost of inflation outweighs the benefit from the increased currency supply, making the borrowing facility an undesirable instrument. By contrast, raising the lending facility price does not increase inflation, and therefore, improves welfare by providing borrowers with more currency to support their exchange of currency for consumption goods.

Finally, I show that it is optimal to set the central bank's lending facility price arbitrarily close to its deposit facility price. This policy induces dealers to raise the prices they offer to borrowers toward the deposit facility price available from the central bank, thereby reducing the price dispersion. By effectively controlling this dispersion, the central bank increases its currency supply by raising both the lending and deposit facility prices simultaneously. Moreover, a higher deposit facility price, or equivalently, a lower (nominal) deposit facility rate, also reduces inflation through the Fisher effect. As a result, the optimal monetary policy is to reduce the policy rates, namely, the lending and deposit facility rates, to the zero lower bound, in line with the Friedman (1969) rule.

2 Environment

I develop a hybrid model combining Lagos and Wright (2005) and Duffie, Gârleanu, and Pedersen (2005): the former framework provides a micro-founded trading motive, which stems from the specific liquidity demand for consumption opportunities, for the latter over-the-counter (OTC) repurchase agreement (repo) market.[4] I also embed Burdett and Judd (1983) pricing into the repo market, generating endogenous prices (interest rates) dispersion without introducing any ex-ante heterogeneity.

Time is discrete and continues forever, with three subperiods in each period. The first subperiod involves activities in a Walrasian *settlement market*, where agents settle debts from the previous period and trade goods and assets. The second subperiod is labeled as the *funding market*, consisting of an OTC repo market and a competitive inter-dealer market. Particularly, **dealer** banks trade with repo **customers** in the repo markets under search friction. Dealers also trade with each other in the frictionless inter-dealer market to rebalance their financial portfolios or save in the central bank. The last subperiod involves trading among customers in a decentralized competitive market, as in Rocheteau and Wright (2005), and is labeled as the *trading market*.

Dealers and customers are infinitely-lived and discount the future between periods at a rate $\beta \in (0, 1)$. There is a measure s of risk-neutral dealers seeking profits from the funding market. There is a measure one of customers who work (n_s) in the settlement market, producing perishable consumption goods in exchange for assets. Customers may also consume (c_s) in this subperiod, which occurs when the payoff from their assets carried from the previous period exceeds their payment for current asset purchases. This works like customers have a "deep pocket", a typical assumption in the OTC literature, as in Duffie, Gârleanu, and Pedersen (2005). Crucially, customers are subject to an i.i.d. idiosyncratic liquidity shock that is realized at the end of the settlement market: a fraction ρ of them can consume (c_t) in the trading market and want to transform their illiquid assets into liquid ones to settle their payments for goods.[5] By contrast, the rest can only

[4] Geromichalos and Herrenbrueck (2016) and Geromichalos, Herrenbrueck, and Lee (2023) have a similar structure in their indirect liquidity approach, such that assets can be sold outright in exchange for means of payment. However, in this paper, liquidity can only be delivered through secured lending, capturing transactions supported by repos, which are essentially collateral-backed loans.

[5] Repo customers are market participants, such as insurance companies and mutual funds, and their preference for consumption in the trading market reflects their clients' preferences. Elaborating the model by introducing these clients and making repo customers operate in a competitive market with free entry

work (n_t) and prefer assets with higher returns.

Customers' period utility function is

$$c_s - n_s + \sigma \cdot Ac_t - (1-\sigma) \cdot n_t, \tag{1}$$

where $\sigma \in \{0,1\}$ indicates their liquidity shock so that $\Pr(\sigma = 1) = \rho$ and $A > 1$ is a parameter that motivates trading. A linear production technology allows them to convert labor to goods one-for-one in both settlement and trading markets.

The underlying assets are currency and government bonds. Assets are essential because no unsecured IOU will be accepted in any transactions under the limited commitment and lack of record-keeping technology, following Lagos and Wright (2005). The **central bank** issues *currency* (m), a liquid asset that is used to settle trading market transactions among customers. Dealers may deposit currency in the central bank's deposit facility — for example, by holding reserve balances at the Federal Reserve in the U.S. context. These deposits (d) can be viewed as an asset sold at a price z_d, in terms of currency, and the central bank administers their nominal interest rate $1/z_d - 1$. The **fiscal authority** issues one-period nominal *government bonds* (b) that are less liquid and work as the collateral in the repo market. Each bond sells for z_b units of currency in the settlement market and is a claim to one unit of currency in the next settlement market.

I focus on stationary equilibria in which the inflation is a constant $\pi - 1$, and all real variables remain unchanged forever. Therefore, it is convenient to express variables in real terms, measured in units of the current-period settlement market good, such as those lower-case letters introduced in the last paragraph (i.e., m, b, and d). Importantly, agents must adjust their nominal asset holdings for inflation when carrying them across periods. For example, m units of currency in the current period are worth m/π units of the settlement market good in the next period.

will deliver the same result.

Fiscal Authority and Central Bank The fiscal authority issues a fixed amount of government bonds $\hat{b}$ at a price z_b in each settlement market. The exogenous value $g \equiv z_b\hat{b}$ denotes its revenue from bond issuance and describes the **fiscal policy**. The central bank then purchases $\hat{b} - \bar{b}$ units of bonds with the issuance of currency $\hat{m} \in (0, g)$. Later, after dealers make their deposit decision, the central bank's liabilities consist of currency ($\bar{m}$) and dealers' account balances in its deposit facility ($z_d\bar{d}$). Therefore,

$$\underbrace{z_b\left(\hat{b} - \bar{b}\right)}_{\text{central bank assets}} = \overbrace{\bar{m} + z_d\bar{d}}^{\text{central bank liabilities}}, \tag{2}$$

with $\bar{b}$ denoting the quantity of the bonds circulating in the private sector. Another way to interpret equation (2) is that it reflects the central bank's balance sheet, equating the value of its asset holdings to the value of its liabilities. In this way, **monetary policy** has two dimensions: the size of the central bank's balance sheet, which is captured by its initial currency issuance $\hat{m}$, or equivalently the ratio of central bank liabilities over the total consolidated government liabilities, i.e., $\theta \equiv \hat{m}/g \in (0, 1)$; and the non-negative administered interest rate $1/z_d - 1$ that later determines the composition of its liabilities.

The fiscal authority has access to lump-sum transfers and taxes to balance the *consolidated government budget constraint* period by period. This constraint is

$$\bar{m} + z_d\bar{d} + z_b\bar{b} = \frac{\bar{m} + \bar{d} + \bar{b}}{\pi} + \tau, \tag{3}$$

where τ is the real value of the lump-sum transfer (or tax if $\tau < 0$) to customers at the beginning of the settlement market. The left-hand side of (3) represents the revenue from issuing new consolidated government liabilities consisting of government bonds and central bank liabilities outstanding, which equals the revenue the fiscal authority receives from its bond issuance g. The right-hand side is the payment of its liabilities from the previous period and the lump-sum transfer.

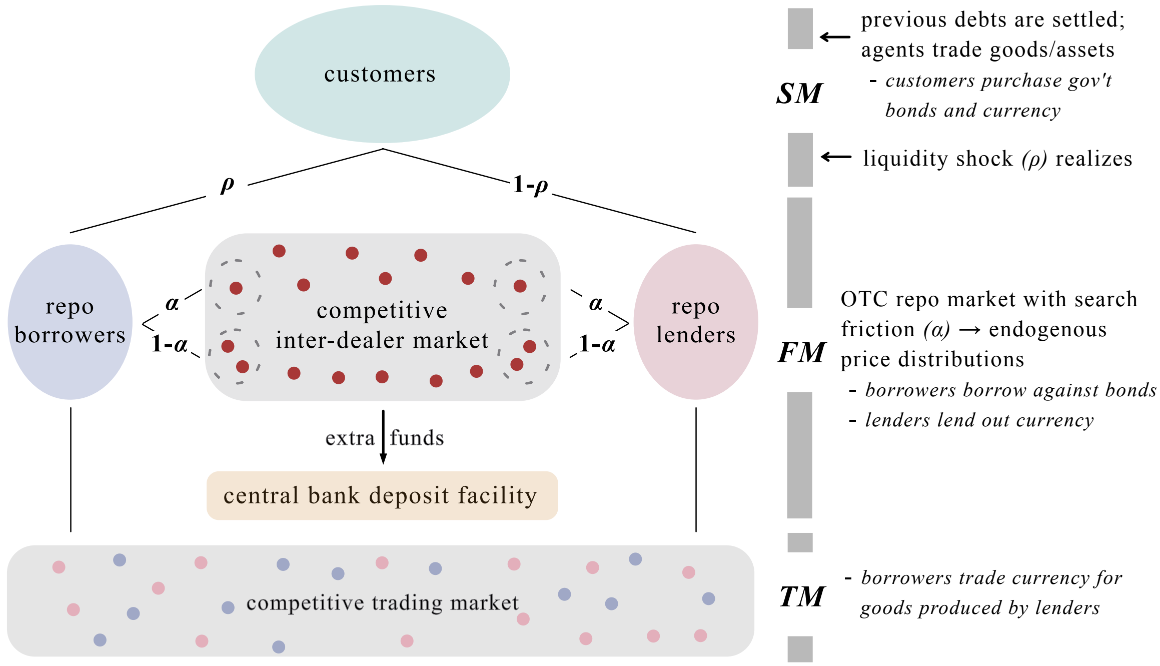

Figure 1: Timing of Events

3 Market Structure

In this section, I describe the market structure subperiod by subperiod and exhibit private agents' value functions in the steady state. Figure 1 depicts the timing of events within a period and provides a visual guidance to the pattern of transactions.

3.1 Settlement Market

At the beginning of the settlement market, debts from the previous period are settled — agents pay off their outstanding liabilities and receive asset returns. Repo customers enter a Walrasian market to trade goods and assets. As explained earlier, they may either consume the residual funds from the last period or work to accumulate new funding to rebalance their financial portfolio.

Generically, consider customers who borrow in the last funding market (**borrowers** henceforth) enter the settlement market owing ℓ_B units of loans while holding m units of currency and b units of government bonds. If they exit this market with $\tilde{m}$ units of currency and $\tilde{b}$ units of government bonds, their value function is

$$
\begin{aligned}
U^B(m,b,\ell_B) &= \max_{c_s,n_s,\tilde{m},\tilde{b}} c_s - n_s + \mathbb{E}_i\left[V^i\left(\tilde{m},\tilde{b}\right)\right] && (4)\\
\text{s.t.}\quad c_s + \tilde{m} + z_b\tilde{b} &= n_s + m + b - \ell_B + \tau, && (5)
\end{aligned}
$$

where τ is the lump-sum transfer from the fiscal authority and $\mathbb{E}_i$ denotes the expectation operator over their value function V^i in the incoming funding market, considering their potential role of borrowing ($i = B$) or lending ($i = L$). Although the values of a borrower's consumption c_s and labor supply n_s are indeterminate, the value of their difference is fixed in equilibrium. Substituting $c_s - n_s$ from the budget constraint (5) into U^B yields

$$
U^B(m,b,\ell_B) = m + b - \ell_B + \tau + \max_{\tilde{m},\tilde{b}} -\tilde{m} - z_b\tilde{b} + \mathbb{E}_i\left[V^i\left(\tilde{m},\tilde{b}\right)\right]. \tag{6}
$$

Similarly, consider customers who lend in the last funding market (**lenders** henceforth) with ℓ_L units of incoming loan payoffs. Their value function in the settlement market is

$$
U^L(m,b,\ell_L) = m + b + \ell_L + \tau + \max_{\tilde{m},\tilde{b}} -\tilde{m} - z_b\tilde{b} + \mathbb{E}_i\left[V^i\left(\tilde{m},\tilde{b}\right)\right]. \tag{7}
$$

At the end of the settlement market, the liquidity shock is realized, reassigning customers' role in the incoming funding market. A fraction ρ of customers become borrowers, requiring currency to trade for goods in the trading market. By contrast, the remaining become lenders who lend out their currency holdings. This gives the following first-order conditions for customers' optimal portfolio choices:

$$
\begin{aligned}
-1 + \rho\frac{\partial}{\partial\tilde{m}}V^B\left(\tilde{m},\tilde{b}\right) + (1-\rho)\frac{\partial}{\partial\tilde{m}}V^L\left(\tilde{m},\tilde{b}\right) &= 0, && (8)\\
-z_b + \rho\frac{\partial}{\partial\tilde{b}}V^B\left(\tilde{m},\tilde{b}\right) + (1-\rho)\frac{\partial}{\partial\tilde{b}}V^L\left(\tilde{m},\tilde{b}\right) &= 0. && (9)
\end{aligned}
$$

These conditions hold for all customers because, from (6) and (7), their optimal portfolio choice is independent of the current state due to the virtue of their quasi-linear utility (Lagos & Wright, 2005). As a result, customers enter the funding market with the same asset holdings $\left(\tilde{m}, \tilde{b}\right)$.

3.2 Funding Market

OTC Repo Market In the OTC repo market, each dealer posts a nominal loan price z_B when they meet a borrower, taking as given the price distribution $F_B(z_B)$ posted by all other dealers, as in Burdett and Judd (1983). I refer dealers who trade with *borrower* and are likely to *borrow* money from the inter-dealer market as **borrower dealers**. Meanwhile, there are **lender dealers** on the other side of the market, trading with lenders. In principle, lender and borrower dealers can be the same agents. Using two different labels to isolate their roles on different sides of the market is harmless, given that dealers are risk-neutral and operate under a competitive inter-dealer market.

Customers are aware of the distribution $F_{i\in\{B,L\}}$ but trade with dealers under search friction. With probability α, customers contact one dealer and trade with that dealer. Otherwise, they contact two dealers and trade with the one offering a better price. Specifically, borrowers borrow from the dealer who offers a higher price, z_B, when they contact two dealers, thereby incurring a lower interest payment. By contrast, lenders lend to the one with a lower price, z_L, so that they can receive a higher payoff. This OTC trading structure aligns with the empirical fact that "the majority of market participants do not have access to inter-dealer markets, and rely on concentrated intermediation by one or two dealer banks" (Eisenschmidt, Ma, & Zhang, 2024). One can also follow the original Burdett and Judd (1983) paper to show how customers endogenously choose to contact one or two dealers randomly at a certain search cost.

The fact that borrowers borrow from a random sample of dealers with random loan prices implies the following expected value of entering the funding market

$$
\begin{aligned}
V^B\left(\tilde{m},\tilde{b}\right) &= \alpha\int \max_{\ell_B}\ W^B\left(\tilde{m}+z_B\ell_B,\tilde{b},\ell_B\right)\mathrm{d}F_B\left(z_B\right)\\
&\quad+\left(1-\alpha\right)\int \max_{\ell_B}\ W^B\left(\tilde{m}+z_B\ell_B,\tilde{b},\ell_B\right)\mathrm{d}\left[F_B\left(z_B\right)\right]^2, \qquad (10)
\end{aligned}
$$

where W^B is their value of entering the incoming trading market after borrowing ℓ_B units of loans under z_B, the higher loan price they are offered. Borrowers borrow against their government bonds. The following collateral constraint,

$$
\ell_B \leq \tilde{b}, \qquad (11)
$$

guarantees that their return on bonds can cover their loan payment.

Instead of the higher price, lenders accept the lower loan price z_L they are offered. This results in a value function

$$
\begin{aligned}
V^L\left(\tilde{m},\tilde{b}\right) &= \alpha\int \max_{\ell_L}\ W^L\left(\tilde{m}-z_L\ell_L,\tilde{b},\ell_L\right)\mathrm{d}F_L\left(z_L\right)\\
&\quad+\left(1-\alpha\right)\int \max_{\ell_L}\ W^L\left(\tilde{m}-z_L\ell_L,\tilde{b},\ell_L\right)\mathrm{d}\left(1-\left[1-F_L\left(z_L\right)\right]^2\right), \qquad (12)
\end{aligned}
$$

with W^L as lenders' value of entering the trading market after lending ℓ_L at a price z_L. Lenders are not subject to collateral constraints, but the following cash constraints

$$
z_L\ell_L \leq \tilde{m}. \qquad (13)
$$

Competitive Inter-dealer Market A borrower dealer with a loan price z_B provides $z_B\ell_B(z_B)$ units of currency to each borrower they serve, where $\ell_B(z_B)$ is the solution to the borrower's funding market problem (10) that captures borrowers' demand for loans under this price. The dealer can borrow from the inter-dealer market to meet their customers' borrowing needs and save the extra in the central bank's deposit facility. As a result, the profit a borrower dealer obtains from each successful match, i.e., profit per borrower served, is

$$
\begin{aligned}
R_B\left(z_B\right) &= \max_{d_{BD}, \ell_{BD}} \; d_{BD} + \ell_B(z_B) - \ell_{BD}, \\
\text{s.t.} \quad z_B \ell_B(z_B) + z_d d_{BD} &= z_I \ell_{BD}, \quad \text{and} \quad d_{BD} \geq 0,
\end{aligned} \tag{14}
$$

where z_I is the loan price in the inter-dealer market, ℓ_{BD} is the quantity of their inter-dealer borrowing, and d_{BD} is their deposits in the central bank. In addition to the non-negative constraint $d_{BD} \geq 0$ that prevents dealers from borrowing from the central bank's deposit facility, dealers are also subject to the following collateral constraint

$$
d_{BD} + \ell_B(z_B) \geq \ell_{BD}, \tag{15}
$$

so that their returns on assets exceed the payments on their liabilities. However, this constraint will not be a concern in determining the equilibrium because dealers always make a non-negative profit to operate.

The total profit for a borrower dealer who posts the price z_B, denoted as $\Pi_B\left(z_B\right)$, is

$$
\lim_{\epsilon \to 0^+} \overbrace{\frac{\rho}{s}\left(\, \alpha + 2\left(1-\alpha\right) F_B\left(z_B - \epsilon\right) + \left(1-\alpha\right)\left[F_B\left(z_B\right) - F_B\left(z_B - \epsilon\right)\right]\right)}^{\text{number of borrowers served}} \underbrace{R_B\left(z_B\right)}_{\text{profit per borrower served}}, \tag{16}
$$

which highlights the importance of the dealer's price selection in determining the number of borrowers served and, ultimately, their total profit. Specifically, $\rho\alpha/s$ borrowers borrow from this dealer because this is their only contact. They may also contact another dealer, where $\lim_{\epsilon\to 0^+} 2\rho\left(1-\alpha\right) F_B\left(z_B - \epsilon\right)/s$ borrowers still borrow from this dealer because the other one posts a price below z_B. The rest $\lim_{\epsilon\to 0^+} \rho\left(1-\alpha\right)\left[F_B\left(z_B\right) - F_B\left(z_B-\epsilon\right)\right]/s$ borrowers contact another dealer who post the same price z_B. These borrowers randomize according to a uniform tie-breaking rule, picking either dealer with probability 1/2.

On the other side of the inter-dealer market, lender dealers receive funds $z_L\ell_L(z_L)$ from lenders, invest $z_d d_{LD}$ into the central bank's deposit facility, and lend $z_I\ell_{LD}$ to other dealers. Therefore, a lender dealer's profit per lender served is

$$R_L(z_L) = \max_{d_{LD},\ell_{LD}} d_{LD} + \ell_{LD} - \ell_L(z_L),$$
$$\text{s.t.} \quad z_d d_{LD} + z_I \ell_{LD} = z_L \ell_L(z_L), \quad \text{and} \quad d_{LD} \geq 0, \tag{17}$$

when they post a price z_L. As with borrower dealers, lender dealers' collateral constraints,

$$d_{LD} + \ell_{LD} \geq \ell_L(z_L), \tag{18}$$

never bind in equilibrium.

Similarly, the total profit, $\Pi_L(z_L)$, for the lender dealer with price z_L is

$$\lim_{\epsilon\to 0^+} \overbrace{\frac{1-\rho}{s}\left(\alpha + 2(1-\alpha)[1-F_L(z_L)] + (1-\alpha)[F_L(z_L) - F_L(z_L-\epsilon)]\right)}^{\text{number of lenders served}} \underbrace{R_L(z_L)}_{\text{profit per lender served}}. \tag{19}$$

The number of lenders served by the lender dealer is determined in a manner similar to the number in the borrower dealer's problem. The only difference is that, instead of the higher price, the lower one becomes more attractive.

Lemma 1 (Dealers' Profits)**.** *Inter-dealer market problems* (14) *and* (17) *imply a higher interest rate on inter-dealer loans than the interest rate on the central bank's deposit facility, and these rates are equal if there is a positive stock of deposits in the central bank's deposit facility, i.e.,* $z_I \leq z_d$*, with equality if* $\bar{d} > 0$*. Moreover, this interest rate structure also implies the following profit functions:*

$$R_B(z_B) = \left(\frac{1}{z_B} - \frac{1}{z_I}\right) z_B \ell_B(z_B), \tag{20}$$
$$R_L(z_L) = \left(\frac{1}{z_I} - \frac{1}{z_L}\right) z_L \ell_L(z_L). \tag{21}$$

I present all the proofs in Appendix A and discuss the intuition in the main text. The nominal interest rate of the central bank's deposit facility, i.e., $1/z_d$, is set by the central bank, which acts as a floor for the inter-dealer rate because dealers can always choose to place all their funds with the central bank unless they expect a higher return from the inter-dealer market. Dealers exploit profits from the interest rate spreads between the inter-dealer rate and their repo rates with customers. For borrower dealers, the profit they obtain from each borrower is (20), which captures the profit arising from the interest rate spread, $1/z_B - 1/z_I$, with providing $z_B\ell_B$ units of currency to the borrower. The profit function for lender dealers has a similar interpretation in terms of the interest rate spread, $1/z_I - 1/z_L$.

3.3 Trading Market

Borrowers enter the trading market with $\tilde{m} + z_B\ell_B(z_B)$ units of currency to purchase consumption goods that sell at a price p, in terms of the current-period settlement market good. This gives the following value function in the trading market

$$\begin{aligned} &W^B\left(\tilde{m} + z_B\ell_B(z_B), \tilde{b}, \ell_B(z_B)\right) \\ &\quad = \max_{0\le pc_t\le \tilde{m}+z_B\ell_B(z_B)} Ac_t + \beta U^B\left(\frac{\tilde{m} + z_B\ell_B(z_B) - pc_t}{\pi}, \frac{\tilde{b}}{\pi}, \frac{\ell_B(z_B)}{\pi}\right), \end{aligned} \tag{22}$$

where nominal terms carried to the next period are adjusted by inflation. From (6), the value function U^B is linear in their state variables, implying that

$$\begin{aligned} &W^B\left(\tilde{m} + z_B\ell_B(z_B), \tilde{b}, \ell_B(z_B)\right) \\ &= \max_{0\le pc_t\le \tilde{m}+z_B\ell_B(z_B)} Ac_t + \frac{\beta\left(\tilde{m} + z_B\ell_B(z_B) - pc_t + \tilde{b} - \ell_B(z_B)\right)}{\pi} + \beta U^B(0,0,0), \end{aligned} \tag{23}$$

where $U^B(0,0,0)$ is a constant that only depends on future choices.

Lenders work and convert their labor into consumption goods for borrowers one-for-one. They then trade goods for currency and carry all assets into the next settlement market. Their value function in the trading market can be written as

$$\begin{aligned} &W^L\left(\tilde{m} - z_L\ell_L(z_L), \tilde{b}, \ell_L(z_L)\right) \\ &\quad = \max_{n_t\ge 0} -n_t + \frac{\beta\left(\tilde{m} - z_L\ell_L(z_L) + pn_t + \tilde{b} + \ell_L(z_L)\right)}{\pi} + \beta U^L(0,0,0). \end{aligned} \tag{24}$$

Finally, the market clearing condition in the trading market is

$$\rho \int c_t(z_B) \mathrm{d}F_B(z_B) \;=\; (1-\rho)\, n_t. \tag{25}$$

Borrowers' consumption potentially depends on their loan price, z_B, because this price affects their currency holding, which ultimately determines their ability to pay for consumption goods. By contrast, lenders' asset holdings are unrelated to their production process, so n_t remains constant across leaders, at least in the symmetric equilibrium.

Lemma 2 (Trading Outcomes)**.** *The competitive trading market gives*

$$p \;=\; \frac{\pi}{\beta}, \quad c_t(z_B) \;=\; \frac{\beta\,(\tilde{m} + z_B \ell_B(z_B))}{\pi}. \tag{26}$$

Borrowers' currency holding is crucial for transactions in the trading market, where they spend all their currency in exchange for consumption goods. This highlights the critical role of the repo market in facilitating liquidity provision by allowing borrowers to borrow against assets to meet their liquidity needs. Lenders also benefit from the repo market by transforming their liquid currency to higher-yielding loans. The following Lemmas 3 and 4 characterize borrowers' demand for loans and lenders' supply of loans, respectively, derived from first-order conditions from their funding market problems, i.e., equations (10) and (12), and envelope conditions from their trading market problem, i.e., (23) and (24).

Lemma 3 (Borrowers' Demand for Loans)**.** *Borrowers' collateral constraint binds, and they borrow up to their collateral value, i.e., $\ell_B(z_B) = \tilde{b}$, when $z_B > 1/A$. By contrast, their collateral constraint does not bind when $z_B = 1/A$.*

Intuitively, risk-neutral borrowers borrow up to the value of their collateral when the loan price exceeds the cutoff, $1/A$, after which their returns from trading are sufficiently high to cover the interest payments. At the cutoff, borrowers can borrow any amount for transactions while maintaining a breakeven position.

Lemma 4 (Lenders' Supply of Loans)**.** *Lenders' cash constraint binds, and they lend out all their currency holdings, i.e.,* $\ell_L(z_L) = \tilde{m}/z_L$, *when* $z_L < 1$. *By contrast, their cash constraint does not bind when* $z_L = 1$.

Lenders' supply of loans decreases in the loan price z_L when $z_L < 1$, or equivalently when its nominal interest rate is away from the zero lower bound. This is because a higher loan price implies a lower return, impeding risk-neutral lenders from lending. At the zero lower bound, lenders are indifferent between lending and holding currency, as neither option yields a positive return. They can arbitrarily alter their portfolio's composition without changing profits.

Similarly, the envelope conditions of customers' funding market problems (10) and (12) reduce their first-order conditions (8) and (9) of their settlement market problems to the following optimal portfolio choice conditions that determine the inflation and nominal interest rate on government bonds in equilibrium.

Lemma 5 (Optimal Portfolio Choices)**.** *Customers' optimal portfolio choices give*

$$1 = \frac{\beta}{\pi}\left[\rho A + (1-\rho)\left(\alpha \int \frac{1}{z_L} \mathrm{d}F_L(z_L) + (1-\alpha)\int \frac{1}{z_L}\mathrm{d}\left(1 - [1 - F_L(z_L)]^2\right)\right)\right], \quad (27)$$

$$z_b = \frac{\beta}{\pi}\left[\rho A\left(\alpha \int z_B \mathrm{d}F_B(z_B) + (1-\alpha)\int z_B \mathrm{d}\left[F_B(z_B)\right]^2\right) + 1 - \rho\right]. \quad (28)$$

Conditions (27) and (28) are essentially asset pricing conditions for currency and government, respectively. While the nominal price of currency is fixed at one, its real value, after being adjusted for inflation, is probabilistically determined by the expected payoff from its direct use in transactions that occur with probability ρ or from its interest incomes through lending that occurs with probability $1-\rho$. Similarly, the nominal price of government bonds, z_b, is determined by the expected payoff from using them as collateral to support transactions or holding them until maturity to receive their returns. The former occurs with probability ρ while the latter occurs with probability $1-\rho$.

4 Endogenous Repo Price Distributions

To lay the groundwork for the following analysis, I first derive the endogenous price distributions for both borrower and lender dealers, thereby rationalizing the dispersion in repo rates. I then integrate these results, as well as those from previous sections, into the definition of equilibrium presented at the end of this section.

Each dealer chooses price z_i to maximizes their total profit $\Pi_i(z_i)$, where, again, $i \in \{B, L\}$ refers to borrowing (B) or lending (L). In equilibrium, the price distribution $F_i(z_i)$ is consistent with dealers' profit maximization if every price z_i in its support $\mathcal{S}_i$ maximizes $\Pi_i(z_i)$, such that

$$\Pi_i\left(z_i\right) \;=\; \Pi_i^{\star} \;=\; \max_{z_i} \; \Pi_i\left(z_i\right) \quad \forall z_i \in \mathcal{S}_i,\; i \in \{B, L\}. \tag{29}$$

Instead of directly solving this maximization problem, it is helpful to solve for the optimal price that maximizes dealers' profit per customer served first. I refer to this optimal price as the *monopoly price*, as it works similarly to a standard monopoly pricing problem, where dealers choose a price to maximize their profits. For convenience, I rewrite the profit per customer functions in (20) and (21) as

$$R_B\left(z_B\right) \;=\; \left(1 - \frac{z_B}{z_I}\right)\tilde{b} \qquad \forall z_B \in [\frac{1}{A}, 1], \tag{30}$$

$$R_L\left(z_L\right) \;=\; \left(\frac{z_L}{z_I} - 1\right)\frac{\tilde{m}}{z_L} \qquad \forall z_L \in (0, 1], \tag{31}$$

using the loan demand and supply characterized in Lemmas 3 and 4. In particular, I focus on cases that favor dealers when there are multiple solutions to the demand and supply. For example, I choose $\ell_L = \tilde{m}$ when $z_L = 1$. Although lenders are indifferent to lending any amount $\ell_L \leq \tilde{m}$ under this price, dealers obtain the highest profit if lenders lend all their currency holdings $\tilde{m}$. Dealers can achieve this profit by choosing a price arbitrarily close to 1, such that $z_L = 1 - \epsilon$ for a vanishingly small ϵ.

Lemma 6 (Borrower Dealers' Monopoly Price)**.** *The monopoly price that maximizes bor-*

rower dealers' profit per borrower served is $z_B = 1/A$, and this price guarantees a positive profit if and only if $z_I > 1/A$.

Proposition 1 (Borrower Dealers' Price Distribution)**.** *If $z_I > 1/A$, there exists a unique price distribution*

$$F_B(z_B) \;=\; \frac{\alpha}{2(1-\alpha)}\left(\frac{z_B - \frac{1}{A}}{z_I - z_B}\right), \tag{32}$$

with support $\mathcal{S}_B = [1/A, \bar{z}_B]$, where the upper bound is given by

$$\bar{z}_B \;=\; \left(1 - \frac{\alpha}{2-\alpha}\right) z_I + \frac{\alpha}{2-\alpha}\frac{1}{A}. \tag{33}$$

The price distribution satisfies the properties of the price distribution in Burdett and Judd (1983). First, the distribution is continuous. Otherwise, if there was a mass point at some price, a dealer who initially posted a price at the mass point could significantly increase their profit by reducing the price slightly, as this reduction leaves the profit per borrower served almost unchanged but attracts all the borrowers who accepted the initial price. Second, the support $\mathcal{S}_B$ is connected or, say, convex in the one-dimensional case. Otherwise, if $\mathcal{S}_B$ had a gap between two prices, the lower one would yield a higher profit because, although these two prices give the same number of borrowers served, the lower price generates a higher profit per borrower, violating the equal profit in (29).

The two properties of the distribution mentioned above simply the total profit (16) to

$$\Pi_B(z_B) \;=\; \frac{\rho}{s}\left[\,\alpha + 2(1-\alpha)\,F_B(z_B)\,\right] R_B(z_B)\,. \tag{34}$$

Then, the equal profit condition solves for the closed-form solution of F_B, given that the monopoly price $1/A$ is the lower bound of the support $\mathcal{S}_B$. Intuitively, a borrower dealer's monopoly price yields the highest profit per borrower served. Although a higher price would reduce the profit these dealers can obtain from each borrower, they make it up by serving a larger number of borrowers, as borrowers would prefer the offer with a higher price when matched with two dealers. Crucially, from (33), the upper bound of $\mathcal{S}_B$,

$\bar{z}_B$, is a convex combination of the monopoly price and the inter-dealer price z_I, where the latter price also captures the *competitive borrowing price* when the search friction becomes negligible, i.e., $\alpha \longrightarrow 0$. Consequently, the larger the search friction, the more the distribution F_B is concentrated around the monopoly price. By contrast, F_B is more concentrated around the competitive borrowing price as the search friction decreases.

Lemma 7 (Lender Dealers' Monopoly Price)**.** *Under $z_d \leq 1$, the monopoly price that maximizes lender dealers' profit per lender served is $z_L = 1$, and dealers earn a nonnegative profit at this monopoly price with zero profit occurring if and only if $z_I = z_d = 1$.*

Proposition 2 (Lender Dealers' Price Distribution)**.** *If $z_I < 1$, there exists a unique price distribution*

$$F_L\left(z_L\right) \; = \; 1 - \frac{\alpha}{2\left(1-\alpha\right)} \frac{1/z_L - 1}{1/z_I - 1/z_L} \left(= \; 1 - \frac{\alpha}{2\left(1-\alpha\right)} \frac{z_I\left(1-z_L\right)}{z_L - z_I} \right) \tag{35}$$

with support $\mathcal{S}_L = [\underline{z}_L, 1]$, where the lower bound is given by

$$\underline{z}_L \; = \; \left[\left(1 - \frac{\alpha}{2-\alpha}\right) \frac{1}{z_I} + \frac{\alpha}{2-\alpha} \right]^{-1}. \tag{36}$$

The price distribution for lender dealers has similar properties and shares a similar expression to the distribution F_B for borrower dealers. The key difference comes from the fact that lenders care about the interest rate they obtain from lending, thereby preferring the lower price when matched with two dealers. Moreover, as with the support $\mathcal{S}_B$, the inverse of the lower bound of the support $\mathcal{S}_L$, $1/\underline{z}_L$, which reflects the largest interest rate lenders could incur, can be viewed as the convex combination of the inverse of the monopoly price and the inverse of the inter-dealer price. Again, the latter inter-dealer price also captures the *competitive lending price* when the search friction becomes negligible.

The following is the formal definition of equilibria that exhibit price dispersion in both the lending and borrowing side of the repo market, such that $z_I \in (1/A, 1)$. In principle, there could be equilibria that fail to satisfy conditions in Propositions 1 and 2, resulting in no price dispersion on one side of the market, such as when $z_I = 1$ or $z_I = 1/A$. I ignore these knife-edge equilibria but focus on empirically relevant cases. Additionally, I only include endogenous variables that are necessary to pin down the equilibrium in the definition. For example, I do not include the trading market price p, but it is an immediate result once the gross inflation rate π is determined (Lemma 2).

Definition 1 (Equilibrium with Price Dispersion)**.** *Given the fiscal policy that determines the value of consolidated government liabilities,*

$$g = \bar{m} + z_d \bar{d} + z_b \bar{b}, \tag{37}$$

and the monetary policy that determines the value of central bank liabilities,

$$\hat{m} = \bar{m} + z_d \bar{d}, \tag{38}$$

and the administered interest rate captured by $z_b \leq 1$*, an equilibrium consists of an allocation* $(\bar{m}, \bar{d}, \bar{b}, \tilde{m}, \tilde{b})$*, the price distributions* F_B *and* F_L *characterized in* (32) *and* (35)*, the associated distributions of loans* $\ell_B(z_B) = \tilde{b}$ *and* $\ell_L(z_L) = \tilde{m}/z_L$ *(Lemmas 3 and 4), and market-determined prices* (z_b, π, z_I)*, satisfying customers' optimal portfolio choice decisions* (27) *and* (28)*, market clearing conditions,*

$$\hat{m} = \tilde{m} \qquad \textit{(currency)}; \tag{39}$$

$$\bar{b} = \tilde{b} \qquad \textit{(government bonds)}; \tag{40}$$

$$\rho \int z_B \ell_B(z_B) \mathrm{d}F_B(z_B) + z_d \bar{d} = (1-\rho) \int z_L \ell_L(z_L) \mathrm{d}F_L(z_L) \qquad \textit{(loans)}, \tag{41}$$

where $z_I \leq z_d$ *with equality if* $\bar{d} > 0$.

5 Equilibrium and Central Bank Deposit Facility

I begin this section by establishing the existence of equilibrium. I then study the positive implications of changes in the central bank's deposit facility price, which is the primary policy instrument of major central banks worldwide, including the U.S. Federal Reserve and the European Central Bank. The model generates imperfect monetary policy pass-through from the deposit facility price to OTC repo prices, consistent with the empirical finding in Eisenschmidt, Ma, and Zhang (2024). Moreover, the degree of pass-through diminishes as the deposit facility price increases, suggesting that central banks that rely on this instrument to raise market-determined repo prices may need to respond more aggressively than they might anticipate. Finally, I show that changes in the deposit facility price have ambiguous effects on asset allocation, and I address this ambiguity in the next section by introducing the central bank's lending and borrowing facilities.

The key to determining the equilibrium is the following condition

$$z_d \bar{d} \;=\; (1-\rho)\,\hat{m} - \rho\,(g-\hat{m})\,\frac{\mu_B}{z_b}, \tag{42}$$

derived from the market clearing condition for loans (41), where

$$\mu_B \;=\; z_I + \frac{\alpha}{2\,(1-\alpha)} \ln\left(\frac{\alpha}{2-\alpha}\right)\left(z_I - \frac{1}{A}\right) \tag{43}$$

is the mean of the borrowing price z_B. Intuitively, lenders lend out their currency, $(1-\rho)\,\hat{m}$. Part of the currency goes to borrowers through repo markets, where borrowers must borrow against collateral, so this part is determined by the value of government bonds held by borrowers, $\frac{\rho(g-\hat{m})\mu_B}{z_b}$. The remainder is transferred to the central bank through its deposit facility and becomes the dealers' account balances with the central bank, denoted as $z_d \bar{d}$. Note that I can use Lemma 5 to derive the closed-form solution of the bond price

$$z_b \;=\; \frac{\beta}{\pi}\left[\rho A\left(\frac{\alpha}{A} + (1-\alpha)\,z_I\right) + 1 - \rho\right], \tag{44}$$

with the gross inflation rate

$$\pi \;=\; \beta\left[\rho A + (1-\rho)\left(\alpha + \frac{1-\alpha}{z_I}\right)\right]. \tag{45}$$

There are two types of equilibria: equilibria when the central bank's deposit facility is active (i.e., $\bar{d} > 0$) or not (i.e., $\bar{d} = 0$), corresponding to the floor or corridor systems of the monetary policy implementation regime, respectively. First, when the central bank's deposit facility is active, the deposit facility price, z_d, determines the price for inter-dealer loans, such that $z_I = z_d$. The inter-dealer price further determines the mean repo borrowing price μ_B and the bond price z_b through equations (43) and (44), respectively. Solving this type of equilibrium is simply plugging μ_B and z_b in (42) to solve for a positive value of central bank deposits $\bar{d}$. Second, when the deposit facility is inactive, its price has no impact on the inter-dealer price. Solving this type of equilibrium is plugging $\bar{d} = 0$ in (42) to solve for the endogenously inter-dealer price, z_I, and it requires $z_d \geq z_I$ to support this type of equilibrium.

For both types of equilibrium, given the equilibrium inter-dealer price z_I, I can solve for all market-determined prices and asset allocation. In particular, I can solve for the endogenous repo price distributions, F_B and F_L, and the consolidated government liabilities outstanding with

$$z_b\bar{b} = g - \hat{m} \qquad \text{(central bank deposits)}, \tag{46}$$

$$\bar{m} = \rho\left[\hat{m} + (g - \hat{m})\frac{\mu_B}{z_b}\right] \qquad \text{(currency)}. \tag{47}$$

Proposition 3 establishes the existence of equilibrium, where I focus on equilibria exhibiting price dispersion that requires the inter-dealer price z_I to lie in the interval $(1/A, 1)$. Henceforth, I refer to these equilibria as *price-dispersed* equilibria. The type of price-dispersed equilibrium is determined by the central bank's interest rate policy z_d and its balance sheet policy θ, which is the ratio of central bank liabilities over the consolidated government liabilities.

Proposition 3 (Existence of Price-dispersed Equilibria)**.** *Fix ρ, α, and A, there exist two threshold values, $0 < \underline{\theta} < \bar{\theta} < 1$, for the central bank's balance sheet policy θ, such that*

1. *A price-dispersed equilibrium with an active central bank deposit facility exists for any $z_d \in (1/A, 1)$ and $\theta \in (\bar{\theta}, 1)$;*

2. *Any price-dispersed equilibrium with an inactive deposit facility, characterized by the endogenously inter-dealer price* $z_I \in (0, 1/A)$*, can be supported by a unique* $\theta \in [\underline{\theta}, \bar{\theta}]$ *and any* $z_d \geq z_I$.

3. *No price-dispersed equilibrium exists if* $\theta \in (0, \underline{\theta})$*;*

The result is quite intuitive. First, equilibrium with an active central bank deposit facility occurs when the central bank's balance sheet is large. In this scenario, repo borrowers obtain a few government bonds as collateral, given that the central bank holds many bonds to back its liabilities. Consequently, dealers cannot lend all their currency against collateral and deposit the extra in the central bank. Second, equilibrium with an inactive deposit facility emerges when the balance sheet becomes smaller. Dealers find it harder to get extra currency as a smaller balance sheet implies fewer central bank liabilities outstanding in the first place. Moreover, borrowers also obtain more collateral to secure their borrowing of currency, as a smaller balance sheet also implies more government bonds circulate in the private sector. Finally, no price-dispersed equilibrium exists when the central bank's balance sheet is sufficiently small, resulting in a scarcity of currency. Dealers compete for loans to satisfy borrowers' currency demand, bidding the inter-dealer price down to the level that cannot support price dispersion, i.e., $z_I \leq 1/A$.

5.1 Imperfect Pass-Through

I first study how market-determined repo price distributions, F_B and F_L, change in response to an increase in the central bank's deposit facility price, z_d. Let z_B^q and z_L^q denote the q-quantile of the price distribution F_B and F_L, respectively, such that

$$F_B\left(z_B^q\right) = \frac{\alpha}{2\left(1-\alpha\right)}\left(\frac{z_B^q - \frac{1}{A}}{z_I - z_B^q}\right) = q, \tag{48}$$

$$F_L\left(z_L^q\right) = 1 - \frac{\alpha}{2\left(1-\alpha\right)}\frac{z_I\left(1 - z_L^q\right)}{z_L^q - z_I} = q. \tag{49}$$

I define the imperfect pass-through as a less-than-one-for-one response in the endogenous market-determined repo prices to changes in the deposit facility price. The following proposition and its corollary demonstrate imperfect pass-through at every percentile of the price distributions and their mean prices.

Proposition 4 (Imperfect Pass-Through)**.** *The pass-through of the central bank's deposit facility price, z_d, to the repo prices is imperfect.*

1. *When the deposit facility is inactive, raising z_d does not change the inter-dealer price z_I, thereby having no impact on repo prices, such that $\forall q \in [0,1]$*

$$\eta_B^q \equiv \frac{\mathrm{dz_B^q}}{dz_d} = 0 \quad \textit{and} \quad \eta_L^q \equiv \frac{\mathrm{dz_L^q}}{dz_d} = 0 \tag{50}$$

2. *When the deposit facility is active*

   (a) *The pass-through is imperfect for any repo borrowing price, such that*

$$0 \leq \eta_B^q = \frac{z_B^q - \frac{1}{A}}{z_d - \frac{1}{A}} < 1, \tag{51}$$

   with the equality $\eta_B^q = 0$ holds at the monopoly price, $z_B^0 = 1/A$;

   (b) *The pass-through is imperfect for repo lending prices that are above the threshold $1 - z_d$, such that*

$$0 \leq \eta_L^q = \frac{z_L^q\,(1 - z_L^q)}{z_d\,(1 - z_d)} < 1 \quad \textit{if} \quad z_L^q > 1 - z_d, \tag{52}$$

   with the equality $\eta_L^q = 0$ at the monopoly price $z_B^1 = 1$. In particular, if $z_d \geq 1/2$, the inequality $z_L^q > 1 - z_d$ always holds.

Corollary 1. *The pass-through of the central bank's deposit facility price to the mean repo borrowing price is imperfect, i.e., $\frac{\mathrm{d}\mu_\mathrm{B}}{dz_d} < 1$. Under $z_d \geq 1/2$, the pass-through to the mean repo lending price is also imperfect, i.e., $\frac{\mathrm{d}\mu_\mathrm{L}}{dz_d} < 1$.*

The imperfect price pass-through appears in both types of equilibrium, but for different reasons. First, the pass-through is imperfect when the central bank's deposit facility is inactive simply because an increase in the deposit price, z_d, does not even change the inter-dealer price, z_I, thereby having no impact on the market-determined repo prices that are directly determined by z_I (Propositions 1 and 2). Second, although a change in z_d transmits one-for-one to z_I in the scenario when the deposit facility is active (recall that now $z_I = z_d$), the pass-through is also imperfect, at lease under $z_d \geq \frac{1}{2}$ that holds for a wide range of interest rates above the zero lower bound that requires $z_d = 1$. The lemma below illustrates the crucial role of search frictions in generating the imperfect pass-through in this scenario.

Lemma 8 (How Search Frictions Weaken Pass-Through)**.** *When the central bank's deposit facility is active, higher search frictions weaken the effectiveness of pass-through. Specifically,*

$$\frac{\mathrm{d}\eta_B^q}{\mathrm{d}\alpha} \leq 0 \quad \forall q \in [0,1]\,, \tag{53}$$

with equality holds only if $q = 0$. Under $z_d \geq 1/2$,

$$\frac{\mathrm{d}\eta_L^q}{\mathrm{d}\alpha} \leq 0 \quad \forall q \in [0,1]\,, \tag{54}$$

with equality holds only if $q = 1$ and $z_d = 1/2$. Moreover, $\forall q \in [0,1]$,

1. *As $\alpha \longrightarrow 0$, then $\eta_B^q \longrightarrow 1$ and $\eta_L^q \longrightarrow 1$;*

2. *As $\alpha \longrightarrow 1$, then $\eta_B^q \longrightarrow 0$ and $\eta_L^q \longrightarrow 0$.*

I focus on equilibria in which the central bank's deposit facility is active in Lemma 8 because, otherwise, the pass-through is constant, i.e., $\eta_B^q = \eta_L^q = 0$, and invariant to the search friction. As the search friction increases, i.e., α increases, there is less tension for dealers to compete for customers, worsening the pass-through to repo prices. This logic holds for both repo borrowing and lending prices, at least under $z_d \geq 1/2$, which serves as a sufficient condition to guarantee the imperfect pass-through to repo lending prices. When the search friction becomes negligible, such that $\alpha \longrightarrow 0$, almost every customer

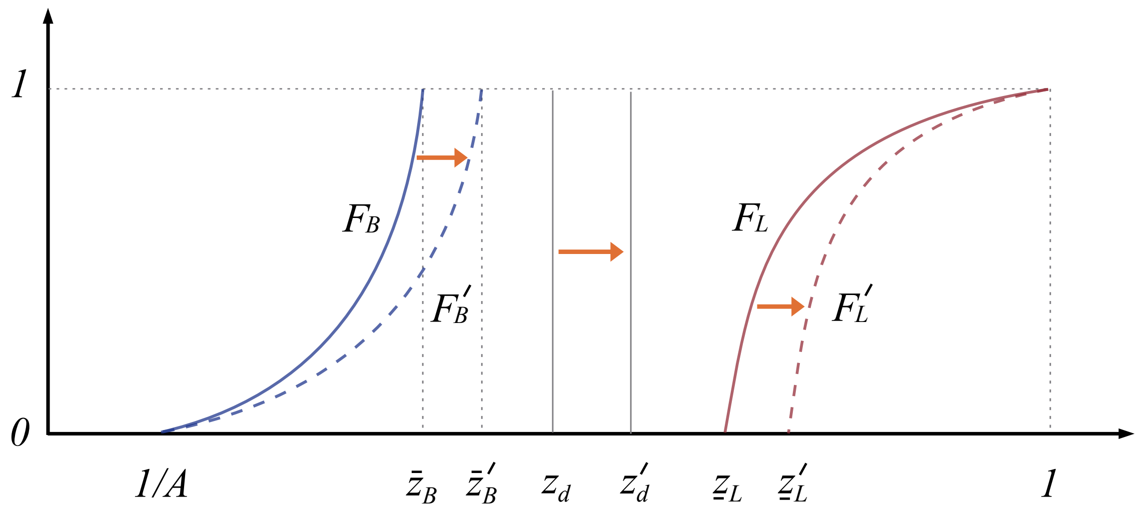

Figure 2: How An Increase in the Deposit Facility Price Shifts Price Distributions

contacts two dealers, and the intensive competition yields a perfect pass-through, as in a competitive pricing environment. By contrast, in another extreme case when $\alpha \longrightarrow 1$, almost no customer contacts two dealers, and the pass-through becomes completely imperfect, as in a monopolistic pricing environment.

Remark When $\alpha \longrightarrow 0$, price distributions F_B and F_L converge pointwise to the degenerate distributions $\mathbb{P}(z_B = z_I) = 1$ and $\mathbb{P}(z_L = z_I) = 1$, respectively. Both repo borrowing and lending prices converge to the inter-dealer price z_I, which also represents the competitive borrowing and lending prices as if there is no search friction. By contrast, when $\alpha \longrightarrow 1$, price distributions F_B and F_L converge pointwise to the degenerate distributions $\mathbb{P}(z_B = 1/A) = 1$ and $\mathbb{P}(z_L = 1) = 1$, respectively. Repo borrowing and lending prices converge to the monopoly borrowing price, $1/A$, and the monopoly lending price, 1, respectively.

5.2 Diminishing Pass-through of the Deposit Facility Price

Despite shifting both the borrowing and lending price distributions to the right in the first-order stochastic dominance sense (recall that $\eta_B^q, \eta_L^q > 0$), an increase in the central bank's deposit facility price generates asymmetric effects on their concentration patterns, at least when this facility is active. As illustrated in Figure 2, the lending price distribution F_L becomes more concentrated around the monopoly price, bolstering lender dealers' market power.[6] Consequently, lending prices become more inert to subsequent changes in monetary policy, and this diminishing effect is formally stated in Proposition 5.[7] By contrast, the borrowing price distribution F_B becomes less concentrated around the borrower dealer's monopoly price. However, this does not imply an enhanced pass-through to the repo borrowing prices. As in Figure 2, the increase in z_d also leads to the distribution F_B being more dispersed around the competitive borrowing price $z_I = z_d$, thereby offsetting the potential gains from its reduced concentration at the monopoly price.

Proposition 5 (Diminishing Pass-throgh of Deposit Facility Price)**.** *An increase in the central bank's deposit facility price z_d weakens the effectiveness of monetary policy pass-through to repo lending prices, i.e., $\frac{\mathrm{d}\eta_{\mathrm{L}}^{\mathrm{q}}}{dz_d} < 0$, except when the lending price attains its upper bound such that $z_L^q = 1$ or when this facility is inactive so that $\frac{\mathrm{d}\eta_{\mathrm{L}}^{\mathrm{q}}}{dz_d} = 0$. By contrast, changes in z_d do not affect the pass-through to repo borrowing prices, i.e., $\frac{\mathrm{d}\eta_{\mathrm{B}}^{\mathrm{q}}}{dz_d} = 0$.*

5.3 Ambiguous Effects on Asset Allocation

I conclude this section by examining the implications of changes in the central bank's deposit facility price on asset allocation. All the following analyses, again, focus on the scenario where the central bank's deposit facility is active because, otherwise, changes in the deposit facility price may have no impact on equilibrium outcomes. From (42) and

[6] For readability, I only draw the portion of the CDF where its density is strictly positive.

[7] From (51) and (52), the pass-through is more effective for prices that are closer to the competitive price and less effective for prices that are closer to the monopoly price.

(47), an increase in the deposit facility price, z_d, alters the composition of central bank deposits ($z_d\bar{d}$) and currency ($\bar{m}$) by steering the relative price μ_B/z_b, which is the mean repo borrowing price relative to the government bond price. Specifically,

$$\frac{\mathrm{d}\left(z_d\bar{d}\right)}{\mathrm{d}z_d} = -\rho\left(g-\hat{m}\right)\frac{\mathrm{d}\left(\mu_B/z_b\right)}{\mathrm{d}z_d}, \tag{55}$$

$$\frac{\mathrm{d}\bar{m}}{\mathrm{d}z_d} = \rho\left(g-\hat{m}\right)\frac{\mathrm{d}\left(\mu_B/z_b\right)}{\mathrm{d}z_d}. \tag{56}$$

An increase in the central bank's deposit facility price, z_d, moves the quantity of central bank deposits and currency in different directions because the sum of them reflects the total supply of central bank liabilities, which is exogenously determined by the central bank's balance sheet policy. The interest rate policy only alters the composition of these liabilities. This is also why the supply of government bonds to the private sector $z_b\bar{b}$ (equation (46)) remains unchanged in response to changes in the policy rate. Moreover, as in (56), the central bank's currency supply increases when the mean repo borrowing price is more elastic, relative to the bond price, in response to an increase in z_d, i.e., $\frac{\mathrm{d}(\mu_B/z_b)}{\mathrm{d}z_d} > 0$. Borrowers borrow more currency against their government bonds, which become relatively cheaper in response to the increased deposit facility price. In this scenario, there is more currency $\bar{m}$ and fewer central bank deposits $z_d\bar{d}$.

Overall, the relative elasticity of the mean repo borrowing price, μ, to the bond price, z_b, plays a crucial role in determining the effects of changes in the central bank's deposit facility price, z_d, on asset allocation and welfare. However, changes in z_d can lead to ambiguous effects on the price ratio μ_B/z_b, thereby leading to ambiguous effects on asset allocation. Figure 3 shows a numerical example when the initial deposit facility rate is close to the zero lower bound with $z_d = 0.99$.[8] The OTC trading structure, or equivalently, the search friction, is vital in generating these ambiguous effects. Lemma 9 below shows how the ambiguity disappears with vanishingly small or arbitrarily large search frictions.

[8] I choose $A = 1.5$, but it does not matter because the results always hold if $A > 1$, so that there is incentive for trading in the trading market. I also choose $\theta = 0.9$, a sufficiently large central bank balance sheet that guarantees an active deposit facility for any $(\rho, \alpha) \in [0.1, 0.9] \times [0.1, 0.9]$.

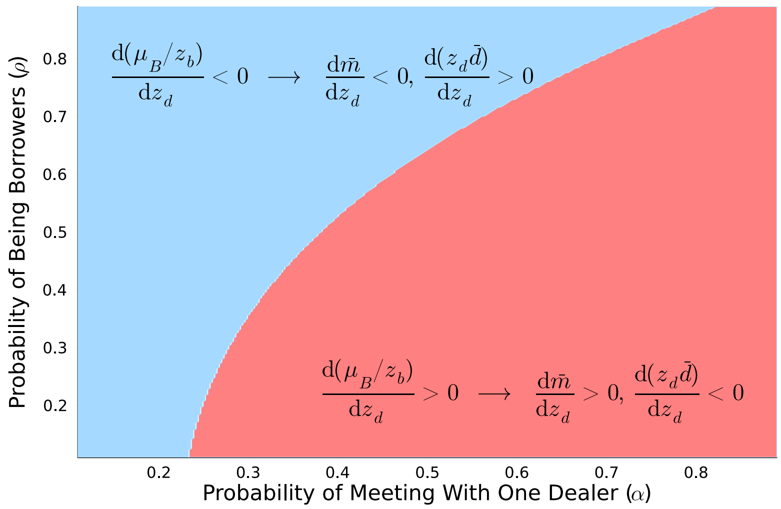

Figure 3: Ambiguous Effects of Deposit Facility Price on Asset Allocation

Lemma 9 (How Search Frictions Matters for Ambiguous Asset Allocation)**.** *When the central bank's deposit facility is active,*

1. *As* $\alpha \longrightarrow 0$*, then* $\mu_B \longrightarrow z_d$ *and* $z_b = z_d$*;*

2. *As* $\alpha \longrightarrow 1$*, then* $\mu_B \longrightarrow 1/A$ *and* $z_b \longrightarrow 1/(\rho A + 1 - \rho)$*.*

In either case, the price ratio μ_B/z_b *is constant in* z_d*, so that*

$$\frac{\mathrm{d}\left(\mu_B/z_b\right)}{\mathrm{d}z_d} = 0. \tag{57}$$

Therefore, an increase in the deposit facility price does not change the currency supply $\bar{m}$ *and central bank deposit supply* $z_d\bar{d}$*.*

As in the remark under Lemma 8, the extreme cases of $\alpha \longrightarrow 0$ and $\alpha \longrightarrow 1$ correspond to models with competitive pricing and monopolistic pricing, respectively. In the former case, pass-through from the central bank deposit facility price, z_d, to both the mean repo borrowing price, μ_B, and the bond price, z_b, is perfect. In the latter case, however, the pass-through becomes completely imperfect for both prices. Notably, in either case, the relative elasticity of μ_B and z_b remains constant in z_d. Consequently, from (55) and (55), monetary policy becomes neutral, and the central bank cannot reallocate its liabilities, either in these extreme cases or in models that do not incorporate appropriate search frictions. In this way, my model not only generates empirically relevant features such as repo price dispersion but also provides new insights into the monetary policy transmission.

Instead of exercising caution, or say, asking the central bank to assess market conditions and estimate all parameters, like the one related to search frictions, in detail before selecting its deposit facility rate $1/z_d - 1$, I suggest that the central bank pair the deposit facility with its lending and borrowing facilities. I show, in the next section, how this combined approach enables the central bank to reallocate assets effectively.

6 Central Bank Lending and Borrowing Facilities

I now introduce the central bank's lending and borrowing facilities and show how they can unambiguously reallocate assets. I also study their welfare implications and characterize the optimal monetary policy when all three instruments — the lending facility, borrowing facility, and the previously introduced deposit facility — are available. For comparison, I focus on equilibria when the central bank's deposit facility is active, which is also the focus in the baseline model. As a result, the inter-dealer price, z_I, is, again, determined by the central bank's deposit facility price, such that $z_I = z_d$.

The lending facility enables the central bank to provide short-term, secured loans to commercial banks and other financial institutions, working like the repurchase agreement facility in the U.S. or the main refinancing operations in the Euro area. Conversely, the borrowing facility enables the central bank to provide securities as collateral and borrow from these financial institutions overnight, similar to the overnight reverse repurchase agreement facility in the U.S..[9] Unlike the central bank's deposit facility, which is limited to highly regulated financial institutions such as banks, both lending and borrowing facil-

[9]The European Central Bank does not have a reverse repo facility in the way the Federal Reserve does, but relies on its deposit facility to absorb liquidity overnight.

ities can be accessed by a broader range of financial institutions, including mutual funds and insurance companies that are repo customers in my paper.

Let z_r and z_o denote the nominal prices for the central bank's lending and borrowing facilities, respectively. Then, impose

$$z_r \; < \; z_d \; < \; z_o \quad \longleftrightarrow \quad \frac{1}{z_o} - 1 \; < \; \frac{1}{z_d} - 1 \; < \; \frac{1}{z_r} - 1, \tag{58}$$

which is consistent with the interest structure in the U.S., where the interest rate on overnight reverse repurchase agreements is below the interest rate on reserves, and the interest rate on the repurchase agreements is the highest among the three rates. Additionally, I impose that

$$z_o \; \leq \; 1 \quad \text{and} \quad z_r \geq \frac{1}{A}, \tag{59}$$

to ensure that the market-determined repo prices do not strictly dominate the prices for these facilities. In particular, from Lemmas 3 and 4, borrowers only borrow at prices that are higher than $1/A$ and lenders only lend at prices that are lower than 1, which are the monopoly prices offered by dealers before introducing these facilities.

After introducing the central bank's lending and borrowing facility, the profit per customer for borrower dealers and lender dealers in (20) and (21) becomes[10]

$$R_B\left(z_B\right) \; = \; \left(1 - \frac{z_B}{z_I}\right)\tilde{b} \qquad\qquad \forall z_B \in [z_r, 1], \tag{60}$$

$$R_L\left(z_L\right) \; = \; \left(\frac{z_L}{z_I} - 1\right)\frac{\tilde{m}}{z_L} \qquad\qquad \forall z_L \in (0, z_o]. \tag{61}$$

Although the functional form of these profit functions remains unchanged, the lending and borrowing facilities narrow their domains. Intuitively, borrowers would prefer to borrow from the central bank if their dealers provide them a price z_B that is below the price offered by the central bank's lending facility, z_r, so that they can obtain more currency with their

[10] As before, I focus on cases that favor dealers when there are multiple solutions to the demand and supply. Specifically, borrowers only borrow from dealers when $z_B = z_r$ while lenders only lend to dealers when $z_L = z_o$. However, this assumption is harmless, as dealers can offer a slightly higher price for borrowers and a slightly lower price for lenders to attract all customers strategically.

collateral. Lenders would prefer to lend to the central bank if their dealers provide them a price z_L that is above the price offered by the central bank's borrowing facility, z_o, so that they can obtain a higher return on their currency holding. Consequently, these modifications in dealers' profit per customer (60) and (61) change their monopoly prices, thereby affecting their pricing strategy in determining total profits (16) and (19) and generating the following new price distributions.

Proposition 6 (Price Distributions under Lending and Borrowing Facilities)**.** *If $z_r < z_I < z_o$, there exist unique distributions for repo borrowing and lending price, such that*

$$
\begin{aligned}
F_B(z_B) &= \frac{\alpha}{2(1-\alpha)}\left(\frac{z_B - z_r}{z_I - z_B}\right), \\
&\quad \textit{with} \quad \mathcal{S}_B = \left[z_r, \left(1 - \frac{\alpha}{2-\alpha}\right) z_I + \frac{\alpha}{2-\alpha} z_r\right]; \qquad (62)\\
F_L(z_L) &= 1 - \frac{\alpha}{2(1-\alpha)} \frac{1/z_L - 1/z_o}{1/z_I - 1/z_L}, \\
&\quad \textit{with} \quad \mathcal{S}_L = \left[\left(\left(1 - \frac{\alpha}{2-\alpha}\right)\frac{1}{z_I} + \frac{\alpha}{2-\alpha}\frac{1}{z_o}\right)^{-1}, z_o\right]. \qquad (63)
\end{aligned}
$$

The price distributions are almost the same as those in Propositions 1 and 2, with the monopoly prices for borrower dealers and lender dealers replaced from the previous $z_B = 1/A$ and $z_L = 1$ to $z_B = z_r$ and $z_L = z_o$. For example, the borrower dealers' monopoly price, now z_r, is still the lower bound of the support $\mathcal{S}_B$, and the upper bound of $\mathcal{S}_B$ is, again, a convex combination of z_r and the competitive borrowing price z_I.

Under Proposition 6, the mean repo borrowing price μ_B, the gross inflation rate π, and the government bond price z_b can be written as

$$\mu_B = z_I + \frac{\alpha}{2(1-\alpha)} \ln\left(\frac{\alpha}{2-\alpha}\right)(z_I - z_r), \qquad (64)$$

$$\pi = \beta\left[\rho A + (1-\rho)\left(\alpha\frac{1}{z_o} + (1-\alpha)\frac{1}{z_I}\right)\right], \qquad (65)$$

$$z_b = \frac{\beta}{\pi}\left[\rho A\left(\alpha z_r + (1-\alpha) z_I\right) + 1 - \rho\right]. \qquad (66)$$

As before, these endogenous, market-determined prices are key to understanding the implications of the central bank's lending and borrowing facilities.

Welfare Before delving into the details, let me define welfare as the sum of the net payoffs from economic activities with equally weighted agents. The virtue of linear utility cancels out part of the utilities and disutilities, reducing total welfare to

$$\mathcal{W} \;=\; \rho \int (A-1)\, c_t\,(z_B)\, \mathrm{d}F_B\,(z_B) \;=\; \frac{\beta\rho\,(A-1)}{\pi}\left[\hat{m} + (g-\hat{m})\,\frac{\mu_B}{z_b}\right]. \tag{67}$$

The first equality in (67) indicates that welfare represents the surplus from transactions in the trading market. This highlights the crucial role of the repo market in reallocating assets across repo customers, as the consumption level there depends on the loans that repo borrowers can obtain from dealers (Lemma 2). The second equality is derived from conditions in Definition 1, highlighting the central bank's critical role in steering the endogenous, market-determined prices (π, μ_B, z_b) that finally determine welfare.

An increase in the central bank's deposit facility price, z_d, affects welfare through two effects. First, from (45), an increase in z_d (recall that $z_I = z_d$ when the deposit facility is active) means a reduction in the nominal interest rate on central bank deposits, which reduces inflation through the Fisher effect and improves welfare by increasing the real value of currency. Second, an increase in z_d also affects welfare by altering the relative price μ_B/z_b. As in (56), if the mean repo borrowing price reacts more elastically to z_d than the bond price, an increase in z_d results in an expansion in the currency supply. Consequently, borrowers obtain more currency to settle their trading market transactions, resulting in a larger trading surplus and improving welfare. By contrast, an increase in z_d can reduce welfare if the bond price is more elastic than the mean repo borrowing price to changes in z_d. These results are summarized in the following equation

$$\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}z_d} \;=\; \frac{\beta\rho\,(A-1)}{\pi}\left(-\frac{1}{\pi}\left[\hat{m} + (g-\hat{m})\,\frac{\mu_B}{z_b}\right]\frac{\mathrm{d}\pi}{\mathrm{d}z_d} + (g-\hat{m})\,\frac{\mathrm{d}\,(\mu_B/z_b)}{\mathrm{d}z_d}\right). \tag{68}$$

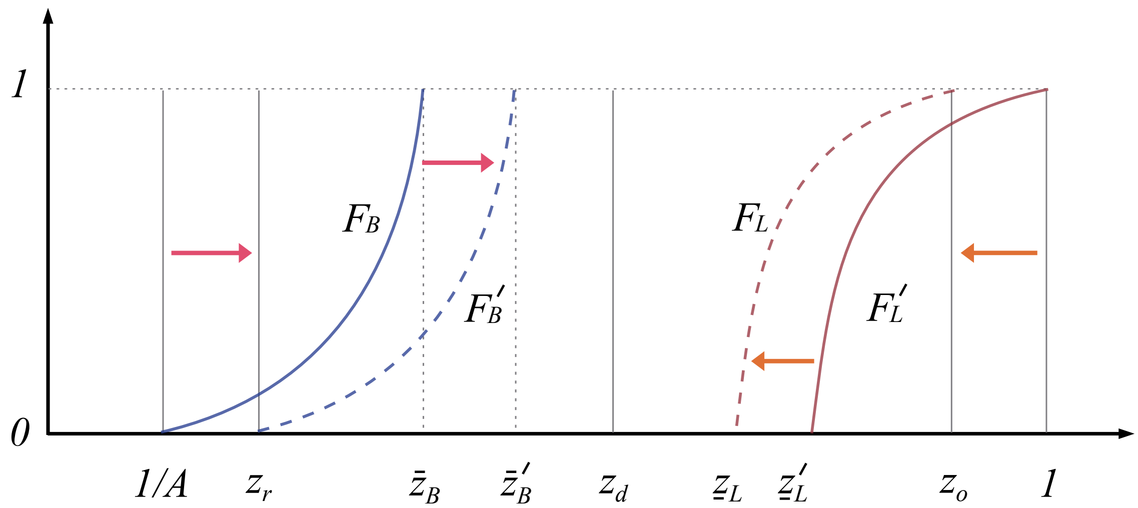

Figure 4: The Roles of Central Bank Lending and Borrowing Facilities

6.1 Implications of Central Bank Lending and Borrowing Facilities for Asset Allocation and Welfare

The exercise here is to study the effects of increasing the central bank's lending facility price and decreasing its borrowing facility price. As in Figure 4, these interventions can squeeze the price distributions and reduce the dispersion of repo prices. However, the result shows that only the lending facility can improve welfare, while the central bank would rather not introduce its borrowing facility. I also discuss the optimal monetary policy to have a comprehensive understanding of all these instruments, including the early deposit facility.

As in Figure 4, an increase in the central bank's lending facility price and a decrease in its borrowing facility price shift the monopoly prices and the entire distributions of repo lending and borrowing prices toward z_d, the competitive price that dealers would offer in the absence of search frictions. These shifts in the repo prices then lead to changes in other market-determined prices, in particular, the bond price, z_b, and the inflation rate, $\pi - 1$. I summarize the results in the following proposition.

Proposition 7 (Implications of Lending and Borrowing Facilities for Asset Prices)**.** *When* *the central bank's deposit facility is active:*

1. *Despite having no impact on inflation, an* ***increase*** *in the central bank's lending facility price, z_r, increases the ratio of the mean repo borrowing price μ_B to the bond price z_b, i.e.,*

$$\forall\alpha \in (0,1) \quad \frac{\mathrm{d}\pi}{\mathrm{d}z_r} \;=\; 0, \quad \frac{\mathrm{d}\left(\mu_B/z_b\right)}{\mathrm{d}z_r} \;>\; 0; \tag{69}$$

2. *A* ***decrease*** *in the central bank's borrowing facility price, z_o, increase the relative price μ_B/z_b, and the gross inflation rate, π, i.e.,*

$$\forall\alpha \in (0,1) \quad \frac{\mathrm{d}\pi}{\mathrm{d}z_o} \;<\; 0, \quad \frac{\mathrm{d}\left(\mu_B/z_b\right)}{\mathrm{d}z_o} \;<\; 0. \tag{70}$$

Corollary 2. *When the central bank's deposit facility is active, raising the central bank's lending facility price or reducing its borrowing facility price increases the supply of currency, $\bar{m}$, and reduces the supply of central bank deposits, $z_d\bar{d}$.*

Recall that the inflation rate is determined by borrowers' direct use of currency in transactions and lenders' expected interest payoff through currency lending, while the nominal bond price is determined by borrowers' use of bonds as collateral and lenders' use of them as a store of value (Lemma 5). Regarding the above results, first, an increase in the central bank's lending facility price does not affect inflation, as it does not impact any component that determines the value of the currency. However, it shifts the borrowing price distribution rightward. The means repo borrowing price is more elastic, relative to bond price, in response to this price increase, so the price ratio μ_B/z_b increases. Second, a decrease in the borrowing facility price shifts the lending price distribution leftward, thereby increasing the interest payoff that lenders can obtain from lending their currency. This increased nominal payoff on currency implies a higher inflation. The relative price μ_B/z_b also increases because higher inflation further implies a lower bond price.

Corollary 2 is an immediate result of (55), (56), and Proposition 7. I omit the intuition as it can be found in the discussion following those earlier results.

Proposition 8 (Welfare Implications of Lending and Borrowing Facilities)**.** *Raising the central bank's lending facility price improves welfare. By contrast, reducing the borrowing facility price harms welfare.*

It is beneficial for the central bank to introduce its lending facility, and raising the lending facility price, or equivalently, lowering its interest rate, is welfare-improving. This is because, as the lending facility price increases, borrowers borrow more currency against their collateral (Corollary 2). This increased currency holding helps them to settle a larger volume of trading market transactions, generating a higher trading surplus and improving welfare. However, perhaps counterintuitively, it is harmful for the central bank to introduce its borrowing facility, even though a decrease in its price can help borrowers obtain more currency to support their transactions. Borrowers' currency holding indeed has to increase, but so does the inflation (Proposition 7), and the effect of the increased inflation dominates, reducing the real value of currency for transactions. Therefore, introducing the borrowing facility lowers the trading surplus, reducing welfare.

Clearly, a benevolent central bank should only introduce the lending facility and set its price arbitrarily close to the deposit facility price, i.e., $z_r \longrightarrow z_d$. In the limit, conditions (64) to (66) becomes

$$\mu_B = z_d, \tag{71}$$

$$\pi = \beta \left[\rho A + (1-\rho) \left(\alpha + (1-\alpha) \frac{1}{z_d} \right) \right], \tag{72}$$

$$z_b = \frac{\beta}{\pi} \left[\rho A z_d + 1 - \rho \right] \tag{73}$$

because now, as illustrated in Figure 5, the borrowing price distribution, F_B, converges to a degenerate distribution with $\mathbb{P}\left(z_B = z_d\right) = 1$. Pairing with the optimal lending facility price, I show that it is optimal for the central bank to peg its nominal interest rates at the zero lower bound, such that $1/z_r - 1 = 1/z_d - 1 = 0$, in line with the Friedman rule.

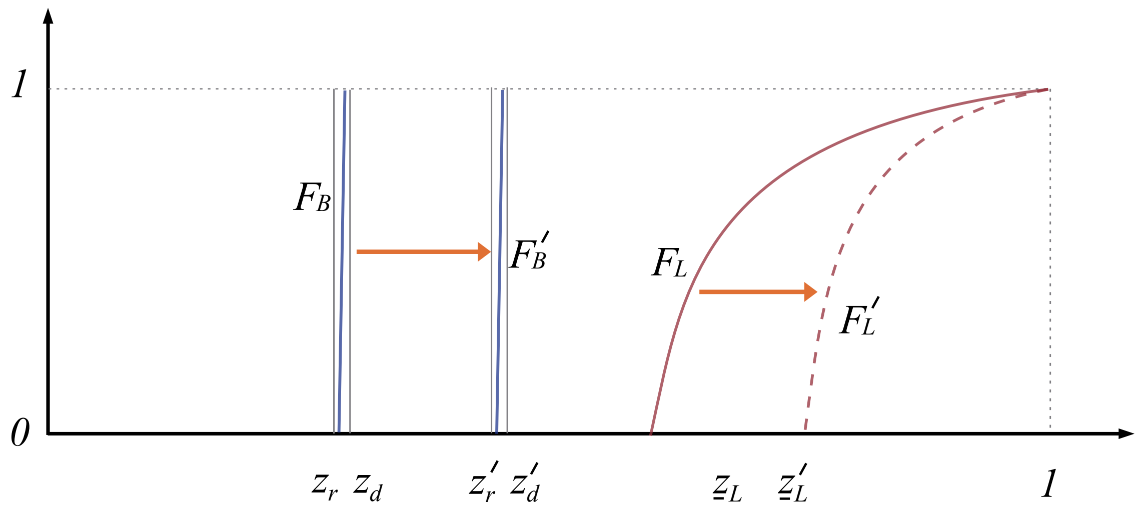

Figure 5: Price Distributions under the Optimal Lending Facility Price

Proposition 9. *When pairing with the optimal lending facility price $z_r \longrightarrow z_d$, an increase in the deposit facility price, z_d, reduces inflation and increases the ratio of the mean repo borrowing price, μ_B, to the bond price, z_b, improving welfare, i.e.,*

$$\frac{\mathrm{d}\pi}{\mathrm{d}z_d} < 0 \text{ and } \frac{\mathrm{d}\left(\mu_B/z_b\right)}{\mathrm{d}z_d} > 0 \quad \longrightarrow \quad \frac{\mathrm{d}\mathcal{W}}{\mathrm{d}z_d} > 0. \tag{74}$$

As in the baseline case, an increase in the central bank's deposit facility price, or equivalently, a reduction in the deposit facility rate, still reduces inflation. However, unlike before, it now unambiguously increases the price ratio of the mean repo borrowing price to the bond price, μ_B/z_b. As in Figure 5, when the lending facility price increases, it pushes the entire borrowing price distribution toward the deposit facility price. In the limit, $\mu_B = z_d$, implying a perfect pass-through from the deposit facility price to the mean repo borrowing price as if there is competitive pricing on borrowing prices. As a result, an increase in z_d always increases the price ratio μ_B/z_b. Finally, an increase in the deposit facility price also unambiguously improves welfare. First, it reduces inflation, thereby increasing the real value of currency and enhancing its usefulness in transactions. Second, the increased price ratio μ_B/z_b allows borrowers, who need currency to settle transactions, to borrow more against their government bonds. These two effects together lead to an improvement in welfare.

7 Conclusion

I develop a search-theoretic model that embeds Burdett and Judd (1983) pricing in OTC repo markets to explain repo price dispersion and the imperfect pass-through from the central bank's deposit facility price to market-determined repo prices. I find that pass-through is diminishing, i.e., it weakens as the deposit facility price increases, suggesting that central banks may need to act more aggressively than they might expect to overcome this imperfection. I also find an ambiguous effect of the deposit facility price on asset allocation. Rather than exercising caution, I suggest that central banks pair the deposit facility with a lending facility, such as the Fed's repurchase agreement facility and the European Central Bank's main refinancing operations, to reallocate assets unambiguously. In doing so, the optimal policy is to peg both the lending and deposit facility rates to the zero lower bound, in the spirit of the Friedman rule. A borrowing facility, such as the Fed's overnight reverse repurchase agreement facility, can also effectively reallocate assets, but at the cost of higher inflation, which reduces welfare.

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Appendix

A Omitted Proofs

A.1 Proof of Lemma 1

Construct the following Lagrangian for the borrower dealer's problem (14),

$$L^{BD} \;=\; d_{BD} + \ell_B(z_B) - \ell_{BD} + \lambda_1^{BD}\left(z_I \ell_{BD} - z_B \ell_B(z_B) - z_d d_{BD}\right) + \lambda_2^{BD} d_{BD}, \tag{A.1}$$

where λ_1^{BD} and λ_2^{BD} are the Lagrange multipliers for the equality constraint and inequality constraint, respectively. The first-order conditions for the Lagrangian are

$$1 - \lambda_1^{BD} z_d + \lambda_2^{BD} \;=\; 0, \tag{A.2}$$

$$-1 + \lambda_1^{BD} z_I \;=\; 0, \tag{A.3}$$

where variables are also subject to the complementary slackness conditions

$$\lambda_2^{BD} d_{BD} = 0, \quad \lambda_2^{BD} \geq 0, \quad d_{BD} \geq 0. \tag{A.4}$$

It is then straightforward to show that $z_d \geq z_I$ and the equality holds if $d_{BD} > 0$.

When $d_{BD} = 0$, the equality constraint becomes

$$z_B \ell_B(z_B) \;=\; z_I \ell_{BD}, \tag{A.5}$$

so that

$$R_B\left(z_B\right) \;=\; \left(1 - \frac{z_B}{z_I}\right) \ell_B(z_B) \;=\; \left(\frac{1}{z_B} - \frac{1}{z_I}\right) z_B \ell_B(z_B) \tag{A.6}$$

This profit function also holds when $d^{BD} > 0$ because $z_I = z_d$ in this case.

Following the same procedure, the solution to the lender dealer's problem (17) gives a similar result, such that

$$z_d \geq z_I, \text{ with equality if } d_{LD} > 0, \quad \text{and} \quad R_L\left(z_L\right) \;=\; \left(\frac{1}{z_I} - \frac{1}{z_L}\right) z_L \ell_L(z_L). \tag{A.7}$$

Finally, aggregating all dealers' choices of central bank deposits implies that $z_d = z_I$ if $\bar{d} > 0$ in equilibrium. $\square$

A.2 Proof of Lemma 2

Construct the following Lagrangian for the borrower's problem (23),

$$L^B = Ac_t + \frac{\beta\left(\tilde{m} + z_B\ell_B(z_B) - pc_t + \tilde{b} - \ell_B(z_B)\right)}{\pi} + \lambda_1^B pc_t + \lambda_2^B\left(\tilde{m} + z_B\ell_B(z_B) - pc_t\right). \tag{A.8}$$

The first-order condition is

$$A - \frac{\beta}{\pi}p + \lambda_1^B p - \lambda_2^B p \;=\; 0, \tag{A.9}$$

and variables are also subject to the following complementary slackness conditions

$$\lambda_1^B pc_t(z_B) = 0, \quad \lambda_1^B \geq 0, \quad pc_t(z_B) \geq 0; \tag{A.10}$$

$$\lambda_2^B\left(\tilde{m} + z_B\ell_B(z_B) - pc_t(z_B)\right) = 0, \quad \lambda_2^B \geq 0, \quad \tilde{m} + z_B\ell_B(z_B) - pc_t(z_B) \geq 0. \tag{A.11}$$

Similarly, construct the Lagrangian for the lender's problem (24), such that

$$L^L \;=\; -n_t + \frac{\beta\left(\tilde{m} - z_L\ell_L(z_L) + pn_t + \tilde{b} + \ell_L(z_L)\right)}{\pi} + \lambda^L n_t, \tag{A.12}$$

and solve for the first-order condition

$$-1 + \frac{\beta}{\pi}p + \lambda^L \;=\; 0, \tag{A.13}$$

as well as the associated complementary slackness conditions

$$\lambda^L n_t = 0, \quad \lambda^L \geq 0, \quad n_t \geq 0. \tag{A.14}$$

First, suppose that the non-negative constraint $pc_t(z_B) \geq 0$ binds so that $c_t(z_B) = 0$. This implies that $\lambda_2^B = 0$ under (A.11). However, when $\lambda_2^B = 0$, first-order conditions (A.9) and (A.13) give

$$\lambda_1^B p + \lambda^L = 1 - A < 0, \tag{A.15}$$

which contradicts to the complementary slackness conditions that require $\lambda_1^B, \lambda^L \geq 0$. As a result, there is always a positive consumption, i.e., $c_t(z_B) > 0$, as well as a positive labor supply, i.e., $n_t > 0$, under the market clearing condition (25). The associated Lagrange multipliers equal to zero, and substituting these multipliers $\lambda_1^B = 0$ and $\lambda^L = 0$ into (A.9) and (A.13) gives

$$p = \frac{\pi}{\beta}, \tag{A.16}$$

$$A - 1 = \lambda_2^B p > 0. \tag{A.17}$$

Finally, from (A.11), the fact that $\lambda_2^B > 0$ implies the binding cash constraint that solves

$$c_t(z_B) = \frac{\beta\left(\tilde{m} + z_B \ell_B(z_B)\right)}{\pi}. \tag{A.18}$$

□

A.3 Proof of Lemma 3

Borrowers' loan demand is jointly determined by the envelope condition from their trading market problem and the first-order condition from their funding market problem. The Lagrangian for the borrower's trading market problem (A.8) gives

$$\frac{\partial}{\partial \ell_B(z_B)} W^B\left(\tilde{m} + z_B \ell_B(z_B), \tilde{b}, \ell_B(z_B)\right) = \frac{\beta\left(z_B - 1\right)}{\pi} + \lambda_2^B z_B = \frac{\beta\left(A z_B - 1\right)}{\pi}, \tag{A.19}$$

where the first equation is an immediate result following the Envelope Theorem and the second one follows from equation (A.17). The first-order condition for the borrower's funding market problem (10) is

$$\frac{\partial}{\partial \ell_B(z_B)} W^B\left(\tilde{m} + z_B \ell_B(z_B), \tilde{b}, \ell_B(z_B)\right) = \lambda_W^B, \tag{A.20}$$

where λ_W^B is the Lagrange multiplier for the collateral constraint $\tilde{b} \geq \ell_B$, so that

$$\lambda_W^B\left(\tilde{b} - \ell_B(z_B)\right) = 0, \quad \lambda_W^B \geq 0, \quad \tilde{b} - \ell_B(z_B) \geq 0. \tag{A.21}$$

The fact that

$$\lambda_W^B = \frac{\beta\left(A z_B - 1\right)}{\pi} \tag{A.22}$$

in equilibrium implies a binding collateral constraint when $z_B > 1/A$. By contrast, the constraint constraint does not bind when $z_B = 1/A$. □

A.4 Proof of Lemma 4

Lenders' supply of loans is jointly determined by the envelope condition from their trading market problem and the first-order condition for the funding market problem. The Lagrangian for the lender's trading market problem (A.12) gives

$$\frac{\partial}{\partial \ell_L(z_L)} W^L\left(\tilde{m} - z_L\ell_L(z_L), \tilde{b}, \ell_L(z_L)\right) \;=\; \frac{\beta\left(-z_L+1\right)}{\pi}, \tag{A.23}$$

an immediate result following the Envelope Theorem. The first-order condition for the lender's funding market problem (12) is

$$\frac{\partial}{\partial \ell_L(z_L)} W^L\left(\tilde{m} - z_L\ell_L(z_L), \tilde{b}, \ell_L(z_L)\right) \;=\; z_L\lambda_W^L, \tag{A.24}$$

where λ_W^L is the Lagrange multiplier for the cash constraint $\tilde{m} \geq z_L\ell_L$, so that

$$\lambda_W^L\left(\tilde{m} - z_L\ell_L(z_L)\right) = 0, \quad \lambda_W^L \geq 0, \quad \tilde{m} - z_L\ell_L(z_L) \geq 0. \tag{A.25}$$

The fact that

$$z_L\lambda_W^L \;=\; \frac{\beta\left(-z_L+1\right)}{\pi}, \tag{A.26}$$

implies a binding cash constraint with $\ell_L(z_L) = \tilde{m}/z_L$ when $z_L < 1$. By contrast, this constraint does not bind when $z_L = 1$. $\square$

A.5 Proof of Lemma 5

The Lagrangian for the borrower's trading market problem (A.8) gives

$$\frac{\partial}{\partial \tilde{m}} W^B\left(\tilde{m} + z_B\ell_B(z_B), \tilde{b}, \ell_B(z_B)\right) \;=\; \frac{\beta}{\pi} + \lambda_2^B \;=\; \frac{\beta}{\pi}A \tag{A.27}$$

$$\frac{\partial}{\partial \tilde{b}} W^B\left(\tilde{m} + z_B\ell_B(z_B), \tilde{b}, \ell_B(z_B)\right) \;=\; \frac{\beta}{\pi}, \tag{A.28}$$

an immediate result following the Envelope Theorem and equation (A.17). Similarly, the envelope conditions for the lender's problem (A.12) gives

$$\frac{\partial}{\partial \tilde{m}} W^L\left(\tilde{m} - z_L\ell_L(z_L), \tilde{b}, \ell_L(z_L)\right) \;=\; \frac{\partial}{\partial \tilde{b}} W^L\left(\tilde{m} - z_L\ell_L(z_L), \tilde{b}, \ell_L(z_L)\right) \;=\; \frac{\beta}{\pi}. \tag{A.29}$$

These envelope conditions on W^L and W^B help to determine the envelope conditions of the constrained optimization problems (10) and (12), such that

$$\frac{\partial}{\partial \tilde{m}} V^B\left(\tilde{m}, \tilde{b}\right) = A\frac{\beta}{\pi}, \tag{A.30}$$

$$\frac{\partial}{\partial \tilde{b}} V^B\left(\tilde{m}, \tilde{b}\right) = A\frac{\beta}{\pi}\left(\alpha \int z_B \mathrm{d}F_B\left(z_B\right) + (1-\alpha)\int z_B \mathrm{d}\left[F_B\left(z_B\right)\right]^2\right), \tag{A.31}$$

$$\frac{\partial}{\partial \tilde{m}} V^L\left(\tilde{m}, \tilde{b}\right) = \frac{\beta}{\pi}\left(\alpha \int \frac{1}{z_L} \mathrm{d}F_L\left(z_L\right) + (1-\alpha)\int \frac{1}{z_L} \mathrm{d}\left(1-\left[1-F_L\left(z_L\right)\right]^2\right)\right), \tag{A.32}$$

$$\frac{\partial}{\partial \tilde{b}} V^L\left(\tilde{m}, \tilde{b}\right) = \frac{\beta}{\pi}. \tag{A.33}$$

Substituting these conditions into the first-order conditions (8) and (9) gives

$$1 = \frac{\beta}{\pi}\left[\rho A + (1-\rho)\left(\alpha \int \frac{1}{z_L} \mathrm{d}F_L\left(z_L\right) + (1-\alpha)\int \frac{1}{z_L} \mathrm{d}\left(1-\left[1-F_L\left(z_L\right)\right]^2\right)\right)\right], \tag{A.34}$$

$$z_b = \frac{\beta}{\pi}\left[\rho A\left(\alpha \int z_B \mathrm{d}F_B\left(z_B\right) + (1-\alpha)\int z_B \mathrm{d}\left[F_B\left(z_B\right)\right]^2\right) + 1 - \rho\right]. \tag{A.35}$$

□

A.6 Proof of Lemma 6

Borrower dealers' profit per borrower served is

$$R_B\left(z_B\right) = \left(1 - \frac{z_B}{z_I}\right)\tilde{b} \quad \forall z_B \in [\frac{1}{A}, 1], \tag{A.36}$$

which is strictly decreasing in z_B given that

$$\frac{\mathrm{d}}{\mathrm{d}z_B} R_B\left(z_B\right) = -\frac{\tilde{b}}{z_I} < 0. \tag{A.37}$$

The monopoly price is $z_B = 1/A$, which yields a nonnegative profit if and only if $z_I \geq 1/A$. □

A.7 Proof of Proposition 1

This proposition builds on the case in which borrower dealers could earn positive monopoly profits, such that $z_I > 1/A$. I prove this proposition by establishing the following lemmas regarding the continuity, connectedness, and boundary of the distribution F_B. The solution of F_B is an immediate result under these lemmas.

Lemma A.1. *F_B is continuous on $\mathcal{S}_B$.*

Proof. Suppose the contradictory, assume $\exists z \in \mathcal{S}_B$ such that $\xi_B(z) = \lim_{\epsilon\to 0^+} F_B(z) - F_B(z-\epsilon) > 0$, and

$$\Pi_B(z) \;=\; \Pi_B^\star \;=\; \lim_{\epsilon\to 0^+} \frac{\rho}{s}\left(\,\alpha + (1-\alpha)\left[F_B(z) + F_B(z-\epsilon)\right]\right) R_B(z) > 0. \tag{A.38}$$

Notice that the dealer's profit per borrower R_B is continuous. Therefore, there exists $z' > z$ such that $R_B(z') > 0$ and $\Delta \equiv R_B(z) - R_B(z') < \frac{(1-\alpha)\xi_B(z)R_B(z)}{\alpha+2(1-\alpha)F_B(z)}$. Then,

$$\begin{aligned}\Pi_B(z') \;&=\; \lim_{\epsilon\to 0^+} \frac{\rho}{s}\left(\,\alpha + (1-\alpha)\left[F_B(z') + F_B(z'-\epsilon)\right]\right) R_B(z') \\ &\geq \lim_{\epsilon\to 0^+} \frac{\rho}{s}\left(\,\alpha + (1-\alpha)\left[F_B(z) + F_B(z-\epsilon) + \xi_B(z)\right]\right)\left(R_B(z) - \Delta\right) \\ &= \Pi_B(z) + \frac{\rho}{s}\left((1-\alpha)\,\xi_B(z)\,R_B(z) - \left[\,\alpha + 2\,(1-\alpha)\,F_B(z)\,\right]\Delta\right),\end{aligned} \tag{A.39}$$

where the inequality holds because $F_B(z') \geq F_B(z)$ and $\lim_{\epsilon\to 0^+} F_B(z'-\epsilon) - F_B(z-\epsilon) \geq \xi_B(z)$. This further implies

$$\Pi_B(z') - \Pi_B(z) \geq \frac{\rho}{s}\left((1-\alpha)\,\xi_B(z)\,R_B(z) - \left[\,\alpha + 2\,(1-\alpha)\,F_B(z)\,\right]\Delta\right) > 0, \tag{A.40}$$

where the last inequality holds by the definition of Δ. The fact that $\Pi_B(z') > \Pi_B(z)$ contradicts with $z \in \mathcal{S}_B$. Therefore, F_B must be continuous on its support $\mathcal{S}_{\mathcal{B}}$. □

Given Lemma A.1, dealers' profit function can be rewritten as

$$\Pi_B^\star \;=\; \Pi_B(z_B) \;=\; \frac{\rho}{s}\left[\,\alpha + 2\,(1-\alpha)\,F_B(z_B)\,\right] R_B(z_B)\,. \tag{A.41}$$

Lemma A.2. *The monopoly price $z_B = 1/A$ is the lowest price in $\mathcal{S}_B$.*

Proof. Suppose that $z \neq 1/A$ is the lowest price in $\mathcal{S}_B$. Then,

$$\Pi_B(z) \;=\; \frac{\rho}{s}\alpha R_B(z)\,. \tag{A.42}$$

But now,

$$\Pi_B\left(\frac{1}{A}\right) = \frac{\rho}{s}\left[\alpha + 2\,(1-\alpha)\,F_B\left(\frac{1}{A}\right)\right] R_B\left(\frac{1}{A}\right) \geq \frac{\rho}{s}\alpha R_B\left(\frac{1}{A}\right) > \Pi_B(z)\,. \tag{A.43}$$

This is a contradiction. $\square$

Lemma A.3. *$\mathcal{S}_B$ is connected.*

Proof. Suppose that $z, z' \in \mathcal{S}_B$, such that $z < z'$ and $F_B(z) = F_B(z')$. Therefore,

$$\alpha + 2(1-\alpha) F_B(z) \;=\; \alpha + 2(1-\alpha) F_B(z'), \tag{A.44}$$

which further implies that

$$\Pi_B(z') < \Pi_B(z), \tag{A.45}$$

given that $R_B(z_B)$ is strictly decreasing in z_B for all $z_B \in [1/A, 1]$. This contradicts to $z, z' \in \mathcal{S}_B$ that requires $\Pi_B(z') = \Pi_B(z)$. $\square$

The total profit is maximized at the monopoly price $1/A$ with

$$\Pi_B^{\star} \;=\; \frac{\rho}{s}\alpha R_B\left(\frac{1}{A}\right). \tag{A.46}$$

By equal profit among prices in the connected support, I obtain

$$\alpha R_B\left(\frac{1}{A}\right) \;=\; \left[\alpha + 2(1-\alpha) F_B(z_B)\right] R_B(z_B), \tag{A.47}$$

which solves

$$\begin{aligned} F_B(z_B) \;&=\; \frac{\alpha}{2(1-\alpha)}\left(\frac{R_B\left(\frac{1}{A}\right)}{R_B(z_B)} - 1\right) && \text{(A.48)} \\ &=\; \frac{\alpha}{2(1-\alpha)}\left(\frac{z_B - \frac{1}{A}}{z_I - z_B}\right) \quad \forall z_B \in \mathcal{S}_B. && \text{(A.49)} \end{aligned}$$

Moreover, the upper bound $\bar{z}_B$ solves

$$R_B(\bar{z}_B) \;=\; \frac{\alpha}{2-\alpha} R_B\left(\frac{1}{A}\right), \tag{A.50}$$

so that

$$\bar{z}_B \;=\; \left(1 - \frac{\alpha}{2-\alpha}\right) z_I + \frac{\alpha}{2-\alpha}\frac{1}{A}. \tag{A.51}$$

$\square$

A.8 Proof of Lemma 7

Lender dealers' profit per lender served is

$$R_L(z_L) \;=\; \left(\frac{z_L}{z_I}-1\right)\frac{\tilde{m}}{z_L} \tag{A.52}$$

which is strictly increasing in z_L given that

$$\frac{\mathrm{d}}{\mathrm{d}z_L}R_L(z_L) \;=\; \frac{\tilde{m}}{(z_L)^2}>0. \tag{A.53}$$

Therefore, their monopoly loan price, which yields the highest profit, is $z_L=1$. Under $z_d\le 1$, the monopoly profit is nonnegative, given that $z_I\le z_d$ (Lemma 1), and the zero profit occurs if and only if $z_I=z_d=1$. □

A.9 Proof of Proposition 2

As with the borrower dealer's problem, this proposition builds on the case in which lender dealers could earn positive monopoly profits, such that $z_I<1$. I also prove this proposition by establishing the following lemmas regarding the continuity, connectedness, and boundary of the distribution F_L before solving for F_L.

Lemma A.4. *F_L is continuous on $\mathcal{S}_L$.*

Proof. Suppose the contradictory, assume $\exists z\in\mathcal{S}_L$ such that $\xi_L(z)=\lim_{\epsilon\to 0^+}F_L(z)-F_L(z-\epsilon)>0$, and

$$\Pi_L(z) \;=\; \Pi_L^{\star}=\lim_{\epsilon\to 0^+}\frac{1-\rho}{s}\left[\,\alpha+(1-\alpha)\left(2-\left[F_L(z)+F_L(z-\epsilon)\right]\right)\right]R_L(z)>0. \tag{A.54}$$

The fact that R_L is a continuous function implies the existence of $z'<z$ such that $R_L(z')>0$ and $\Delta\equiv R_L(z)-R_L(z')<\frac{(1-\alpha)\xi_L(z)R_L(z)}{\alpha+2(1-\alpha)[1-F_L(z)+\xi_L(z)]}$. Then,

$$\begin{aligned}\Pi_L(z') \;&=\; \lim_{\epsilon\to 0^+}\frac{1-\rho}{s}\left[\,\alpha+(1-\alpha)\left(2-\left[F_L(z')+F_L(z'-\epsilon)\right]\right)\right]R_L(z') \\ &\geq \lim_{\epsilon\to 0^+}\frac{1-\rho}{s}\left[\,\alpha+(1-\alpha)\left(2-\left[F_L(z)-\xi_L(z)+F_L(z-\epsilon)\right]\right)\right](R_L(z)-\Delta) \\ &=\Pi_L(z)+\frac{1-\rho}{s}\left((1-\alpha)\,\xi_L(z)\,R_L(z)-\left[\alpha+2(1-\alpha)(1-F_L(z)+\xi_L(z))\right]\Delta\right),\end{aligned} \tag{A.55}$$

where the inequality holds because $F_L(z)-\xi_L(z)\;\geq\;F_L(z')$ and $\lim_{\epsilon\to 0^+}F(z-\epsilon)\;\geq$

$\lim_{\epsilon \to 0^+} F_L(z' - \epsilon)$. This further implies that

$$\Pi_L(z') - \Pi_L(z) \geq \frac{1-\rho}{s}\left((1-\alpha)\, \xi_L(z)\, R_L(z) - [\, \alpha + 2(1-\alpha)(1 - F_L(z) + \xi_L(z))\,]\, \Delta \right) > 0 \tag{A.56}$$

where the last inequality holds by the definition of Δ. The fact that $\Pi_L(z') > \Pi_L(z)$ contradicts with $z \in \mathcal{S}_L$. This establishes the Lemma. □

Given Lemma A.4, dealers' profit function can be rewritten as

$$\Pi_L^\star \;=\; \Pi_L(z_L) \;=\; \frac{1-\rho}{s}[\, \alpha + 2(1-\alpha)(1 - F_L(z_L))\,]\, R_L(z_L)\,. \tag{A.57}$$

Lemma A.5. *The monopoly price $z_L = 1$ is the highest price in $\mathcal{S}_L$.*

Proof. Suppose that $z \neq 1$ is the highest price in $\mathcal{S}_L$. Then,

$$\Pi_L(z) \;=\; \frac{1-\rho}{s}\alpha R_L(z)\,. \tag{A.58}$$

But now,

$$\Pi_L(1) = \frac{1-\rho}{s}[\, \alpha + 2(1-\alpha)(1 - F_L(1))\,]\, R_L(1) \geq \frac{1-\rho}{s}\alpha R_L(1) > \Pi_L(z)\,. \tag{A.59}$$

This is a contradiction. □

Lemma A.6. *$\mathcal{S}_L$ is connected.*

Proof. Suppose that $z, z' \in \mathcal{S}_L$, such that $z < z'$ and $F_L(z) = F_L(z')$. Therefore,

$$\alpha + 2(1-\alpha)(1 - F_L(z)) \;=\; \alpha + 2(1-\alpha)(1 - F_L(z'))\,, \tag{A.60}$$

which further implies that

$$\Pi_L(z') > \Pi_L(z)\,, \tag{A.61}$$

given that $R_L(z)$ is strictly increasing in z for all $z \in (0,1]$. This contradicts to $z, z' \in \mathcal{S}_L$ that requires $\Pi_L(z') = \Pi_L(z)$. □

At the monopoly price $z_L = 1$, profit is maximized with

$$\Pi_L^\star \;=\; \frac{1-\rho}{s}\alpha R_L(1)\,. \tag{A.62}$$

By equal profit among prices in the connected support, I obtain

$$\alpha R_L(1) \;=\; \left[\alpha + 2(1-\alpha)(1 - F_L(z_L))\right] R_L(z_L), \tag{A.63}$$

which solves

$$\begin{aligned} F_L(z_L) \;&=\; 1 - \frac{\alpha}{2(1-\alpha)}\left(\frac{R_L(1)}{R_L(z_L)} - 1\right) && \text{(A.64)} \\ &=\; 1 - \frac{\alpha}{2(1-\alpha)}\frac{z_I(1-z_L)}{z_L - z_I} \quad \forall z_L \in \mathcal{S}_L. && \text{(A.65)} \end{aligned}$$

Moreover, the lower bound $\underline{z}_L$ solves

$$R_L(\underline{z}_L) \;=\; \frac{\alpha}{2-\alpha} R_L(1), \tag{A.66}$$

so that

$$\underline{z}_L \;=\; \left(\frac{2(1-\alpha)}{2-\alpha}\frac{1}{z_I} + \frac{\alpha}{2-\alpha}\right)^{-1} \;=\; \left[\left(1 - \frac{\alpha}{2-\alpha}\right)\frac{1}{z_I} + \frac{\alpha}{2-\alpha}\right]^{-1}. \tag{A.67}$$

□

A.10 Proof of Proposition 3

Confine attention to the case with inter-dealer price $z_I \in (1/A, 1)$. Rewrite equilibrium condition (42) as

$$\frac{z_b z_d}{g}\bar{d} \;=\; z_b(1-\rho)\frac{\hat{m}}{g} - \rho\left(1 - \frac{\hat{m}}{g}\right)\mu_B. \tag{A.68}$$

Plugging in the value of bond price in (44) and $\theta \equiv \hat{m}/g \in (0,1)$ gives

$$\begin{aligned} \left[\rho A + (1-\rho)\left(\alpha + \frac{1-\alpha}{z_I}\right)\right]\frac{z_b z_d}{g}\bar{d} \;=\; & \left[\rho A\left((1-\alpha) z_I + \frac{\alpha}{A}\right) + 1 - \rho\right](1-\rho)\theta \\ & -\rho\left[\rho A + (1-\rho)\left(\alpha + \frac{1-\alpha}{z_I}\right)\right]\mu_B(1-\theta). \end{aligned} \tag{A.69}$$

The last equation can be further rewritten as

$$\left[\rho A z_I + (1-\rho)(\alpha z_I + 1 - \alpha)\right]\frac{z_b z_d}{g}\bar{d} \;=\; G(z_I)\theta - H(z_I)(1-\theta), \tag{A.70}$$

where

$$G(z_I) = \left[\rho A\left((1-\alpha) z_I^2 + \frac{\alpha}{A} z_I\right) + (1-\rho) z_I\right](1-\rho) \tag{A.71}$$

and

$$H(z_I) = \rho\left[\rho A z_I + (1-\rho)(\alpha z_I + 1 - \alpha)\right]\mu_B \tag{A.72}$$

are both quadratic equations because, in particular, μ_B is linear in z_I (equation (43)).

Note that both $G(z_I)$ and $H(z_I)$ are positive and bounded on the interval $(1/A, 1)$. Consequently, the right-hand of equation (A.70) is always positive once the ratio θ is sufficiently large. This further implies a threshold $\bar{\theta}$ such that, whenever $\theta > \bar{\theta}$, equation (A.70) can solve for a positive value of central bank deposits, $\bar{d}$. The resulting equilibrium features an active central bank deposit facility, so the inter-dealer price is equal to the deposit facility price, i.e., $z_I = z_d$. To guarantee price dispersion, z_d must lie in the interval $(1/A, 1)$. By contrast, the right-hand of (A.70) becomes negative for all $z_I \in (1/A, 1)$ when the ratio θ becomes sufficiently small. As a result, there exists no equilibrium that exhibits price dispersion when $\theta < \underline{\theta}$. Finally, $\underline{\theta} < \bar{\theta}$ because the right-hand side of (A.70) is strictly increasing in θ.

Equilibria in which the central bank's deposit facility is inactive require $z_I \leq z_d$ and $\bar{d} = 0$. The equality condition reduces equation (A.70) to

$$\frac{\theta}{1-\theta} = \frac{H(z_I)}{G(z_I)}. \tag{A.73}$$

There is always a unique $\theta \in (0, 1)$ that solves the above equation as the mapping $\theta \to \frac{\theta}{1-\theta}$ is a bijection between $(0, 1)$ and $(0, +\infty)$. I can further restrict the condition regarding the ratio to $\theta \in [\underline{\theta}, \bar{\theta}]$, as I only focus on equilibria with price dispersion that limits z_I in the interval $(1/A, 1)$. To conclude, there exists $\theta \in [\underline{\theta}, \bar{\theta}]$ and $z_d \geq z_I$ that can support an equilibrium with an inactive central bank deposit facility. □

A.11 Proof of Proposition 4

Inactive Central Bank Deposit Facility When the central bank's deposit facility is inactive, the inter-dealer price, z_I, is endogenously determined by equation (42) with $\bar{d} = 0$. An increase in price z_d has no impact on z_I and does not change F_B and F_L, as these distributions are directly related to the inter-dealer price z_I.

Active Central Bank Deposit Facility When the central bank's deposit facility is active, the inter-dealer price is determined by the deposit facility price, such that $z_I = z_d$. Plugging in $z_I = z_d$ and totally differentiating equation (48) with respect to z_d give

$$\frac{\alpha}{2(1-\alpha)} \frac{\frac{\mathrm{dz_B^q}}{dz_d}\left(z_d - z_B^q\right) - \left(z_B^q - \frac{1}{A}\right)\left(1 - \frac{\mathrm{dz_B^q}}{dz_d}\right)}{\left(z_d - z_B^q\right)^2} = 0 \tag{A.74}$$

$$\frac{\mathrm{dz_B^q}}{dz_d}\left(z_d - z_B^q\right) - \left(z_B^q - \frac{1}{A}\right) + \frac{\mathrm{dz_B^q}}{dz_d}\left(z_B^q - \frac{1}{A}\right) = 0 \tag{A.75}$$

$$\frac{\mathrm{dz_B^q}}{dz_d}\left(z_d - \frac{1}{A}\right) = \left(z_B^q - \frac{1}{A}\right). \tag{A.76}$$

The last equation solves

$$0 \leq \eta_B^q \equiv \frac{\mathrm{dz_B^q}}{dz_d} = \frac{z_B^q - \frac{1}{A}}{z_d - \frac{1}{A}} < 1, \tag{A.77}$$

where η_B^q captures the effectiveness of monetary policy pass-through. The first inequality holds with strictly inequality unless $q = 0$ and $z_B^0 = 1/A$, and the second inequality holds for any $q \in [0, 1]$ because, from (33), $z_d = z_I > \bar{z}_B > z_B^q$.

Similarly, plugging in $z_I = z_d$ and totally differentiating equation (49) with respect to z_d give

$$-\frac{\alpha}{2(1-\alpha)} \frac{1}{\left(z_L^q - z_d\right)^2}\left[\left(1 - z_L^q - z_d \frac{\mathrm{dz_L^q}}{dz_d}\right)\left(z_L^q - z_d\right) - z_d\left(1 - z_L^q\right)\left(\frac{\mathrm{dz_L^q}}{dz_d} - 1\right)\right] = 0, \tag{A.78}$$

which solves

$$\eta_L^q \equiv \frac{\mathrm{dz_L^q}}{dz_d} = \frac{z_L^q\left(1 - z_L^q\right)}{z_d\left(1 - z_d\right)} \geq 0, \tag{A.79}$$

with equality only if $q = 1$ and $z_L^1 = 1$. Again, η_L^q captures the effectiveness of monetary policy pass-through, and the pass-through is imperfect if

$$\eta_L^q \equiv \frac{z_L^q\left(1 - z_L^q\right)}{z_d\left(1 - z_d\right)} < 1 \longleftrightarrow \left(z_d + z_L^q - 1\right)\left(z_d - z_L^q\right) < 0. \tag{A.80}$$

The fact that $z_L^q > z_I = z_d$ holds for all $q \in [0, 1]$ helps to reduce the last condition to

$$z_L^q > 1 - z_d. \tag{A.81}$$

Therefore, the monetary policy pass-through is imperfect for $z_L \in (1-z_d, 1]$. In particular, when $z_d \geq 1/2$, the last inequality is always satisfied. $\square$

A.12 Proof of Corollary 1

The proof for the case when the central bank's deposit facility is inactive is trivial because, as discussed in Proposition 4, an increase in z_b does not change z_I, thereby having no impact on F_B and F_L. This further implies that

$$\frac{\mathrm{d}\mu_B}{\mathrm{d}z_d} = \frac{\mathrm{d}\mu_L}{\mathrm{d}z_d} = 0. \tag{A.82}$$

Confine attention to the case when the central bank's deposit facility is active so that $z_I = z_d$. Start from the mean repo borrowing price μ_B. From the definition of the quantile function (48),

$$z_B^q = \frac{1}{\frac{\alpha}{2(1-\alpha)} + q}\left[qz_d + \frac{\alpha}{2(1-\alpha)}\frac{1}{A}\right], \tag{A.83}$$

which is integrable on $(0,1)$. This implies that

$$\mu_B = \int_0^1 z_B^q \mathrm{d}q. \tag{A.84}$$

Therefore,

$$\frac{\mathrm{d}\mu_B}{\mathrm{d}z_d} = \int_0^1 \frac{\mathrm{d}z_B^q}{\mathrm{d}z_d}\mathrm{d}q < 1 \quad \text{given that} \quad \forall q \in [0,1],\ \eta_B^q \equiv \frac{\mathrm{dz_B^q}}{dz_d} < 1, \tag{A.85}$$

no matter whether the central bank's deposit facility is active ($0 \leq \eta_B^q < 1$) or not ($\eta_B^q = 0$) Following the same procedure, I can show

$$\frac{\mathrm{d}\mu_L}{\mathrm{d}z_d} = \int_0^1 \frac{\mathrm{d}z_L^q}{\mathrm{d}z_d}\mathrm{d}q < 1 \tag{A.86}$$

holds under the condition $z_d \geq \frac{1}{2}$. $\square$

A.13 Proof of Lemma 8

I focus on equilibria with an active deposit facility with $z_I = z_d$. From the definition of the quantile function (48),

$$z_B^q = \frac{1}{\frac{\alpha}{2(1-\alpha)} + q}\left[q z_d + \frac{\alpha}{2(1-\alpha)}\frac{1}{A}\right]. \tag{A.87}$$

Plugging the result into $\eta_B^q = \frac{z_B^q - \frac{1}{A}}{z_d - \frac{1}{A}}$ gives

$$\eta_B^q = \frac{q\left(z_d - \frac{1}{A}\right)}{\left(\frac{\alpha}{2(1-\alpha)} + q\right)\left(z_d - \frac{1}{A}\right)} = \frac{2(1-\alpha)q}{\alpha + 2(1-\alpha)q}. \tag{A.88}$$

An increase in $\alpha \in (0,1)$ means a larger search friction and implies the following relation

$$\frac{\mathrm{d}\eta_B^q}{\mathrm{d}\alpha} = -\frac{2q}{\left[\alpha + 2q(1-\alpha)\right]^2} \leq 0, \tag{A.89}$$

where the equality holds only if $q = 0$. Moreover, for all $q \in [0,1]$, $\eta_B^q \longrightarrow 1$ when $\alpha \longrightarrow 0$. By contrast, $\eta_B^q \longrightarrow 0$ when $\alpha \longrightarrow 1$.

Similarly, from the definition of the quantile function (49),

$$z_L^q = \frac{\frac{\alpha}{2(1-\alpha)} + (1-q)}{\frac{\alpha}{2(1-\alpha)} z_d + (1-q)} z_d. \tag{A.90}$$

Plugging the result into $\eta_L^q = \frac{z_L^q\left(1-z_L^q\right)}{z_d(1-z_d)}$ gives

$$\eta_L^q = \frac{\left[\frac{\alpha}{2(1-\alpha)} + (1-q)\right](1-q)}{\left[\frac{\alpha}{2(1-\alpha)} z_d + (1-q)\right]^2} = \frac{\left[2\alpha(1-\alpha) + 4(1-\alpha)^2(1-q)\right](1-q)}{\left[\alpha z_d + 2(1-\alpha)(1-q)\right]^2}. \tag{A.91}$$

Taking the first order derivative with respect to $t = \frac{\alpha}{2(1-\alpha)}$ gives

$$\frac{\mathrm{d}\eta_L^q}{\mathrm{d}t} = \frac{-t(1-q)z_d + (1-q)^2(1-2z_d)}{\left[t z_d + (1-q)\right]^3} \leq 0 \quad \text{when} \quad z_d \geq \frac{1}{2}. \tag{A.92}$$

Therefore, as an immediate result of the chain rule,

$$\frac{\mathrm{d}\eta_L^q}{\mathrm{d}\alpha} = \frac{\mathrm{d}\eta_L^q}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}\alpha} \leq 0, \tag{A.93}$$

with the equality holds only if $q = 1$ and $z_d = 1/2$. Again, for all $q \in [0,1]$, $\eta_L^q \longrightarrow 1$ when $\alpha \longrightarrow 0$. By contrast, $\eta_L^q \longrightarrow 0$ when $\alpha \longrightarrow 1$. $\square$

A.14 Proof of Proposition 5

When the central bank's deposit facility is inactive, an increase in the deposit facility price does not change OTC repo prices, implying $\frac{\mathrm{d}\eta_{\mathrm{B}}^{\mathrm{q}}}{dz_d} = \frac{\mathrm{d}\eta_{\mathrm{L}}^{\mathrm{q}}}{dz_d} = 0$. When this deposit facility is active, $z_I = z_d$. However, an increase in z_d also has no impact on η_B^q, given that

$$\frac{\mathrm{d}\eta_{\mathrm{B}}^{\mathrm{q}}}{dz_d} = \frac{\frac{\mathrm{dz}_{\mathrm{B}}^{\mathrm{q}}}{dz_d}\left(z_d - \frac{1}{A}\right) - \left(z_B^q - \frac{1}{A}\right)}{\left(z_d - \frac{1}{A}\right)^2} = \frac{\left(z_B^q - \frac{1}{A}\right) - \left(z_B^q - \frac{1}{A}\right)}{\left(z_d - \frac{1}{A}\right)^2} = 0. \tag{A.94}$$

However, an increase in z_d now reduces the effectiveness of monetary policy pass-through because

$$\begin{aligned}
\frac{\mathrm{d}\eta_{\mathrm{L}}^{\mathrm{q}}}{dz_d} &= \frac{z_d\left(1-z_d\right)\left(1-2z_L^q\right)\frac{\mathrm{dz}_{\mathrm{L}}^{\mathrm{q}}}{dz_d} - z_L^q\left(1-z_L^q\right)\left(1-2z_d\right)}{\left[z_d\left(1-z_d\right)\right]^2} &\text{(A.95)}\\
&= \frac{\left(1-2z_L^q\right)z_L^q\left(1-z_L^q\right) - z_L^q\left(1-z_L^q\right)\left(1-2z_d\right)}{\left[z_d\left(1-z_d\right)\right]^2} &\text{(A.96)}\\
&= -\frac{2z_L^q\left(1-z_L^q\right)\left(z_L^q - z_d\right)}{\left[z_d\left(1-z_d\right)\right]^2} \leq 0, &\text{(A.97)}
\end{aligned}$$

given that $z_L^q > z_I = z_d$. The equality holds only if $z_L^q = 1$. $\square$

A.15 Proof of Lemma 9

Consider the case when the central bank's deposit facility is active so that $z_I = z_d$. Applying L'Hôpital's rule to equation (43), I obtain

$$\begin{aligned}
\lim_{\alpha\to 0}\ \mu_B &= \lim_{\alpha\to 0}\ z_d + \frac{\alpha}{2\left(1-\alpha\right)}\ln\left(\frac{\alpha}{2-\alpha}\right)\left(z_d - \frac{1}{A}\right)\\
&= \lim_{\alpha\to 0}\ z_d - \frac{\alpha}{2-\alpha}\left(z_d - \frac{1}{A}\right) = z_d; &\text{(A.98)}
\end{aligned}$$

$$\begin{aligned}
\lim_{\alpha\to 1}\ \mu_B &= \lim_{\alpha\to 1}\ z_d + \frac{\alpha}{2\left(1-\alpha\right)}\ln\left(\frac{\alpha}{2-\alpha}\right)\left(z_d - \frac{1}{A}\right)\\
&= \lim_{\alpha\to 1}\ z_d - \frac{\alpha}{2-\alpha}\left(z_d - \frac{1}{A}\right) = \frac{1}{A}. &\text{(A.99)}
\end{aligned}$$

From (44) and (45),

$$z_b \;=\; \frac{\rho A\left(\left(1-\alpha\right)z_d+\frac{\alpha}{A}\right)+1-\rho}{\rho A+\left(1-\rho\right)\left(\alpha+\frac{1-\alpha}{z_d}\right)}. \tag{A.100}$$

As a result,

$$\lim_{\alpha\to0}\, z_b \;=\; z_d \qquad \lim_{\alpha\to1}\, z_b \;=\; \frac{1}{\rho A+1-\rho} \tag{A.101}$$

In either case, the price ratio is constant in z_d, so that

$$\frac{\mathrm{d}\left(\mu_B/z_b\right)}{\mathrm{d}z_d}=0. \tag{A.102}$$

$\square$

A.16 Proof of Proposition 6

Borrowing Price Distribution (F_B) I finish this proof with the same procedure as in the proof of Proposition 1. From (60), the monopoly price for borrower dealer now becomes $z_B = z_r$, which gives a positive monopoly profit when $z_I > z_r$. Under $z_I > z_r$, I can also show that, after introducing the central bank's lending facility,

1. F_B is continuous on its support $\mathcal{S}_B$;
2. The monopoly price $z_B = z_r$ is the lowest price in $\mathcal{S}_B$;
3. $\mathcal{S}_B$ is connected.

Borrower dealers' total profit can be rewritten as

$$\Pi_B^\star \;=\; \Pi_B\left(z_B\right) \;=\; \frac{\rho}{s}\left[\,\alpha+2\left(1-\alpha\right)F_B\left(z_B\right)\right]R_B\left(z_B\right), \tag{A.103}$$

which is maximized at the monopoly price z_r so that

$$\Pi_B^\star \;=\; \frac{\rho}{s}\alpha R_B\left(z_r\right). \tag{A.104}$$

By equal profit among prices in the connected support, I obtain

$$\alpha R_B\left(z_r\right) \;=\; \left[\,\alpha+2\left(1-\alpha\right)F_B\left(z_B\right)\right]R_B\left(z_B\right), \tag{A.105}$$

which solves

$$F_B(z_B) = \frac{\alpha}{2(1-\alpha)}\left(\frac{R_B(z_r)}{R_B(z_B)} - 1\right) \tag{A.106}$$

$$= \frac{\alpha}{2(1-\alpha)}\left(\frac{z_B - z_r}{z_I - z_B}\right) \quad \forall z_B \in \mathcal{S}_B. \tag{A.107}$$

Moreover, the upper bound $\bar{z}_B$ solves

$$\bar{z}_B = \left(1 - \frac{\alpha}{2-\alpha}\right) z_I + \frac{\alpha}{2-\alpha} z_r. \tag{A.108}$$

given that $F_B(\bar{z}_B) = 1$.

Lending Price Distribution (F_L) Again, this proof follows the same procedure as in the proof of Proposition 2. From (61), the monopoly price for lender dealer now becomes $z_L = z_o$, which gives a positive monopoly profit if $z_I < z_o$. Under $z_I < z_o$, I can show that, after introducing the central bank's deposit facility,

1. F_L is continuous on its support $\mathcal{S}_L$;
2. The monopoly price $z_B = z_o$ is the highest price in $\mathcal{S}_L$;
3. $\mathcal{S}_L$ is connected.

Lender dealers' total profit can be rewritten as

$$\Pi_L^\star = \Pi_L(z_L) = \frac{1-\rho}{s}\left[\alpha + 2(1-\alpha)(1 - F_L(z_L))\right] R_L(z_L), \tag{A.109}$$

which is maximized at the monopoly price z_o, so that

$$\Pi_L^\star = \frac{1-\rho}{s}\alpha R_L(z_o). \tag{A.110}$$

By equal profit among prices in the connected support, I obtain

$$\alpha R_L(z_o) = \left[\alpha + 2(1-\alpha)(1 - F_L(z_L))\right] R_L(z_L), \tag{A.111}$$

which solves

$$F_L(z_L) = 1 - \frac{\alpha}{2(1-\alpha)}\left(\frac{R_L(z_o)}{R_L(z_L)} - 1\right) \tag{A.112}$$

$$= 1 - \frac{\alpha}{2(1-\alpha)}\frac{1/z_L - 1/z_o}{1/z_I - 1/z_L} \quad \forall z_L \in \mathcal{S}_L. \tag{A.113}$$

Moreover, the lower bound $\underline{z}_L$ solves $F_L(\underline{z}_L) = 0$, so that

$$\underline{z}_L = \left[\left(1 - \frac{\alpha}{2-\alpha}\right)\frac{1}{z_I} + \frac{\alpha}{2-\alpha}\frac{1}{z_o}\right]^{-1}. \tag{A.114}$$

□

A.17 Proof of Proposition 7

When the central bank's deposit facility is active, $z_I = z_d$. Then, from (64), and (66), the price ratio

$$\frac{\mu_B}{z_b} = \frac{\pi}{\beta}\frac{z_d + \frac{\alpha}{2(1-\alpha)}\ln\left(\frac{\alpha}{2-\alpha}\right)(z_d - z_r)}{\rho A(\alpha z_r + (1-\alpha) z_d) + 1 - \rho}, \tag{A.115}$$

where, as in (65),

$$\pi = \beta\left[\rho A + (1-\rho)\left(\alpha\frac{1}{z_o} + (1-\alpha)\frac{1}{z_d}\right)\right]. \tag{A.116}$$

Lending Facility Price (z_r) I first study the effects of an increase in the lending facility price z_r, which does not change inflation, i.e.,

$$\frac{\mathrm{d}\pi}{\mathrm{d}z_r} = 0, \tag{A.117}$$

given that the gross inflation rate is constant in z_r. However, it still changes the price ratio μ_B/z_b, and I obtain

$$\frac{\mathrm{d}(\mu_B/z_b)}{\mathrm{d}z_r} = -\frac{\pi}{\beta}\frac{\left[\frac{\alpha}{2(1-\alpha)}\ln\left(\frac{\alpha}{2-\alpha}\right) + \alpha\right]\rho A z_d + (1-\rho)\frac{\alpha}{2(1-\alpha)}\ln\left(\frac{\alpha}{2-\alpha}\right)}{\left[\rho A(\alpha z_r + (1-\alpha) z_d) + 1 - \rho\right]^2}, \tag{A.118}$$

which is positive if

$$\frac{\alpha}{2(1-\alpha)}\ln\left(\frac{\alpha}{2-\alpha}\right)+\alpha<0\longleftrightarrow\ln\left(\frac{\alpha}{2-\alpha}\right)+2(1-\alpha)<0. \tag{A.119}$$

Let

$$J(\alpha)\;\equiv\;\ln\left(\frac{\alpha}{2-\alpha}\right)+2(1-\alpha), \tag{A.120}$$

and the key is to show that $\forall\alpha\in(0,1)\;\;J(\alpha)<0$. Note that

$$J''(\alpha)\;=\;\frac{4(\alpha-1)}{(2-\alpha)^2\alpha^2}<0. \tag{A.121}$$

Therefore,

$$J'(\alpha)\;=\;\frac{1}{\alpha}+\frac{1}{2-\alpha}-2 \tag{A.122}$$

is strictly decreasing in $\alpha\in(0,1)$. Consequently,

$$\forall\alpha\in(0,1)\quad J'(\alpha)\;>\;J'(1)\;=\;0. \tag{A.123}$$

This further implies that $J(\alpha)$ is strictly increasing in $\alpha\in(0,1)$, so that

$$\forall\alpha\in(0,1)\quad J(\alpha)\;<\;J(1)\;=\;0. \tag{A.124}$$

To conclude,

$$\forall\alpha\in(0,1)\quad\frac{\mathrm{d}(\mu_B/z_b)}{\mathrm{d}z_r}\;>\;0. \tag{A.125}$$

Borrowing Facility Price (z_o) Taking the same procedure as before to study an increase in the lending facility price, I obtain

$$\frac{\mathrm{d}(\mu_B/z_b)}{\mathrm{d}z_o}\;=\;\frac{1}{\beta}\frac{z_d+\frac{\alpha}{2(1-\alpha)}\ln\left(\frac{\alpha}{2-\alpha}\right)(z_d-z_r)}{\rho A(\alpha z_r+(1-\alpha)z_d)+1-\rho}\frac{\mathrm{d}\pi}{\mathrm{d}z_o}\;<\;0, \tag{A.126}$$

given that

$$\frac{\mathrm{d}\pi}{\mathrm{d}z_o}\;<\;0. \tag{A.127}$$

$\square$

A.18 Proof of Proposition 8

Lending Facility Price (z_r) The effects of an increase in the central bank's lending facility price on welfare are given by

$$\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}z_r} = \frac{\beta\rho\,(A-1)}{\pi}\left(-\frac{1}{\pi}\left[\hat{m}+(g-\hat{m})\,\frac{\mu_B}{z_b}\right]\frac{\mathrm{d}\pi}{\mathrm{d}z_r}+(g-\hat{m})\,\frac{\mathrm{d}\,(\mu_B/z_b)}{\mathrm{d}z_r}\right). \tag{A.128}$$

Plugging the following results from Lemma 7,

$$\frac{\mathrm{d}\pi}{\mathrm{d}z_r} = 0, \quad \text{and} \quad \frac{\mathrm{d}\,(\mu_B/z_b)}{\mathrm{d}z_r} > 0, \tag{A.129}$$

into equation (A.128) gives

$$\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}z_r} = \frac{\beta\rho\,(A-1)\,(g-\hat{m})}{\pi}\frac{\mathrm{d}\,(\mu_B/z_b)}{\mathrm{d}z_r} > 0. \tag{A.130}$$

Borrowing Facility Price (z_o) Rewrite the welfare function (67) as

$$\mathcal{W} = \frac{\beta\rho\,(A-1)}{\pi}\left[\hat{m}+(g-\hat{m})\,\frac{\mu_B}{z_b}\right] \tag{A.131}$$

$$= \beta\rho\,(A-1)\left[\hat{m}\frac{1}{\pi}+(g-\hat{m})\,\frac{z_d+\frac{\alpha}{2(1-\alpha)}\ln\left(\frac{\alpha}{2-\alpha}\right)(z_d-z_r)}{\beta\left[\rho A\left(\alpha z_r+(1-\alpha)\,z_d\right)+1-\rho\right]}\right]. \tag{A.132}$$

Therefore,

$$\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}z_o} = -\frac{\beta\rho\,(A-1)\,\hat{m}}{\pi^2}\frac{\mathrm{d}\pi}{\mathrm{d}z_o} > 0, \tag{A.133}$$

given that, from Lemma 7, a higher borrowing facility price reduces the gross inflation rate, π. $\square$

A.19 Proof of Proposition 9

Totally differentiating conditions (71) to (73) with respect to z_d, I obtain

$$\frac{\mathrm{d}\left(\mu_B/z_b\right)}{\mathrm{d}z_d} = \frac{\alpha\left(1-\rho\right)\left(\rho A+1-\rho\right)}{\left(\rho A z_d+1-\rho\right)^2} > 0, \tag{A.134}$$

$$\frac{\mathrm{d}\pi}{\mathrm{d}z_d} = -\frac{\beta\left(1-\rho\right)\left(1-\alpha\right)}{\left(z_d\right)^2} < 0. \tag{A.135}$$

Plugging these results in (68) gives

$$\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}z_d} > 0. \tag{A.136}$$