Endogenous Repo Price Dispersion and Monetary Policy Transmission

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Abstract

While financial arbitrage implies that repurchase agreements (repos) with the same collateral and maturity should trade at the same price, repo prices exhibit substantial dispersion. Moreover, repo prices respond only partially to changes in the central bank's deposit facility rate, raising concerns about monetary policy transmission. I develop a model in which search frictions in over-the-counter repo markets generate endogenous repo price dispersion and study the implications of this dispersion for monetary policy. The key insight is that what matters for policy is not the overall distribution of prices but their concentration in the tails — monetary policy affects only one tail of repo prices, with the other being insensitive to policy. In this way, changes in the central bank's deposit facility rate exhibit imperfect pass-through, with the pass-through weakening as repo prices shift toward their insensitive tail. The central bank's lending and borrowing facilities mitigate imperfections in monetary policy transmission, but using the borrowing facility raises inflation. Consistent with the Friedman rule, pegging lending and deposit facility rates at the zero lower bound achieves efficiency.

Key Words: repo rate dispersion, search friction, pass-through, monetary policy
JEL: E4, E5, G2

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1 Introduction

Repurchase agreements (repos) are short-term collateralized contracts, typically overnight and secured by government securities. Trading in repo markets is substantial: the gross size of the U.S. repo market stood at 11.9 trillion USD (Hempel, Kahn, & Shephard, 2025), and in the euro area, quarterly turnover reached 28 trillion euros (Arrata, Nguyen, Rahmouni-Rousseau, & Vari, 2020). As such, repo markets play a critical role in monetary policy transmission, with central banks intervening to steer repo rates toward policy targets. Nevertheless, even repos with the same collateral and maturity trade at different rates (Anbil, Anderson, & Senyuz, 2021; Eisenschmidt, Ma, & Zhang, 2024). Moreover, repo rates respond partially to changes in central banks' policy rates, such as the U.S. Federal Reserve's interest rate on reserve balances and the European Central Bank's deposit facility rate, raising concerns about the effectiveness of monetary policy transmission (Ballensiefen, Ranaldo, & Winterberg, 2023; Duffie & Krishnamurthy, 2016).

I show that search frictions inherent in OTC markets give rise to endogenous repo price (rate) dispersion, and study the implications of this dispersion for monetary policy. The key insight is that what matters for policy is not the overall distribution of prices but their concentration in the tails. Crucially, one tail of the price distribution remains insensitive to policy changes, implying imperfect pass-through from the central bank's deposit facility price to market-determined repo prices. As the deposit facility price increases, the pass-through weakens as repo prices shift toward the insensitive tail. I also find that changes in the deposit facility price have ambiguous effects on asset allocation, particularly on the composition of central bank liabilities. Long-standing central bank lending and borrowing facilities provide repo customers with direct conduits to trade with the central bank. Both can enhance the effectiveness of monetary policy, but the borrowing facility is associated with higher inflation. Consistent with the Friedman (1969) rule, pegging the lending and deposit facility rates at the zero lower bound achieves efficiency.

Specifically, I develop an infinite-horizon model with three stages of exchange in each period. In stage 1, repo customers, such as money market funds and pension funds, purchase financial portfolios consisting of money and government bonds. Depending on liquidity needs, some customers become borrowers who require money to settle transactions for consumption goods in stage 3, while others become lenders. These stages 1 and 3 follow Lagos and Wright (2005), providing micro-founded trading motives for the liquidity rearrangement in stage 2, which adopts the OTC trading framework of Duffie, Gârleanu, and Pedersen (2005). Namely, dealers, such as depository institutions, intermediate between borrowers and lenders in frictional repo markets. They also trade with each other in a frictionless inter-dealer market. As is standard in practice, all transactions at this stage are secured with collateral, and dealers deposit excess funds into the central bank's deposit facility, converting their money into central bank deposits.

An important element of the model is the endogenous distributions for repo *borrowing* and *lending* prices. I derive these distributions as an equilibrium outcome of frictional OTC repo markets, specifically through their inherent search frictions, as captured by the pricing framework of Burdett and Judd (1983). Namely, search frictions limit customers to contact at most two dealers for price quotations, giving dealers market power and generating price dispersion. In particular, this price quotation framework is directly motivated by the fact that the majority of repo customers rely on concentrated intermediation by *one or two* dealers (Eisenschmidt, Ma, & Zhang, 2024).

The equilibrium price distributions are anchored by two reference prices: the **competitive price**, which would arise when search frictions are negligible (i.e., customers always meet two dealers, as in Bertrand competition), and the **monopoly price**, which would emerge when search frictions are extremely large (i.e., customers always meet one dealer). Although the monopoly price yields the highest *per-customer profit*, dealers earn the same *total profit* over the distribution's support. Lower per-customer profits at prices near the competitive level are offset by a larger customer base.

I demonstrate how endogenous price dispersion leads to imperfect monetary policy pass-through, characterized by less-than-one-for-one responses of market-determined repo prices to changes in the central bank's deposit facility price.[1] When the deposit facility is active, search frictions weaken the effectiveness of pass-through by concentrating the price distributions around monopoly prices and away from competitive prices.[2] Although the competitive price is determined by the deposit facility price, the monopoly price is insensitive to policy changes. In this way, search frictions effectively limit the response of repo prices to changes in the deposit facility price. Here also comes one of the key insights of this paper: it is not the price dispersion itself, but the concentration of prices in the tails of the distributions, that matters for central bank interventions. In fact, pass-through becomes null (perfect) as search frictions become extremely large (negligible), collapsing the distribution to the monopoly (competitive) price.

The key concern about pass-through, perhaps, is less about imperfect interest rate responses and more about the predictability of monetary policy. As the central bank deposit facility price increases, its effect on repo lending prices diminishes — an immediate result of the earlier discussion on the distribution's concentration pattern. More precisely, a higher deposit facility price (equivalently, a lower rate) shifts the entire lending price distribution rightward, concentrating it around the monopoly price, which is the highest price, or the lowest interest rate, that a dealer can offer to lenders. This concentration strengthens dealers' market power and weakens pass-through.[3] In particular, the pass-through becomes null in the limit case when the increased deposit facility price pushes dealers to offer only the monopoly price.

[1] In practice, changes in this price are reflected, for example, by adjustments to the European Central Bank's deposit facility rate (DFR) and the U.S. Federal Reserve's interest rate on reserve balances (IORB).

[2] An active deposit facility corresponds to the floor system of monetary policy implementation, currently used by major central banks like the Federal Reserve. The model also admits equilibria with an inactive deposit facility, as in the corridor system. The corridor system prevailed before the 2008 financial crisis, under which changes in the deposit facility price have zero pass-through to repo prices.

[3] For borrowing prices, a higher deposit facility price reduces concentration around the monopoly price but simultaneously increases dispersion around the competitive price, leaving pass-through unaffected.

Crucially, when relying on the deposit facility price, monetary policy can have ambiguous and sometimes unintended implications. Raising the price, or equivalently, reducing the interest rate on the deposit facility, does not necessarily reduce the supply of central bank deposits. Thus, it also has ambiguous effects on the money supply. Search frictions in the OTC markets play a critical role in this result, and the ambiguity disappears when search frictions are either negligible or extremely large. Nevertheless, I show that the central bank can unambiguously control the composition of its liabilities by pairing the deposit facility with its long-standing lending and borrowing facilities, such as the Federal Reserve's repurchase agreement and overnight reverse repurchase agreement facilities.

Either raising the central bank's lending facility price or lowering its borrowing facility price reduces the supply of central bank deposits while increasing the supply of money. The lending facility provides repo borrowers, rather than relying on dealers, an outside option of borrowing from the central bank. Raising the lending facility price pushes dealers to offer higher prices to compete for customers. As a result, borrowers obtain more money with their collateral, reducing dealers' savings in the central bank's deposit facility and expanding the money supply. The borrowing facility, instead, provides a conduit for lenders to lend to the central banks. By reducing its price, it pushes dealers to offer lower prices, or equivalently, higher interest rates to attract lenders. The increased return on lenders' money holdings raises the inflation rate, reducing the nominal bond price. Borrowers can then purchase more government bonds and borrow more money against these government securities. Therefore, lowering the borrowing facility price also increases the central bank's money supply while reducing its deposit liabilities unambiguously.

While both facilities unambiguously reallocate assets, the lending facility is preferable to the borrowing facility. Despite providing repo borrowers more money for transactions, lowering the borrowing facility price also leads to higher inflation, reducing the real value of money. The cost of inflation outweighs the benefit from the increased (nominal) money supply, making the borrowing facility a less desirable instrument. By contrast, raising the lending facility price does not affect inflation and still provides borrowers with additional money to support their exchanges of money for consumption goods.

Finally, I show that it is optimal to set the central bank's lending facility price arbitrarily close to its deposit facility price. This policy pushes dealers to raise the prices they offer to borrowers toward the deposit facility price, thereby reducing price dispersion. Controlling dispersion in this way enables the central bank to expand its money supply by simultaneously raising both the lending and deposit facility prices. Moreover, a higher deposit facility price, or equivalently, a lower (nominal) deposit facility rate, also reduces inflation, in line with the Fisher effect. Therefore, the optimal monetary policy is to lower the policy rates, namely, the lending and deposit facility rates, to the zero lower bound, consistent with the Friedman (1969) rule. Under this policy, repo price dispersion disappears entirely, as all distributions collapse to the zero lower bound.

Related Literature The model developed here adopts some elements from Lagos and Wright (2005) and Duffie, Gârleanu, and Pedersen (2005), but generalizes them because assets traded in OTC repo markets are divisible, as in Lagos and Rocheteau (2009), and because OTC transactions can be collateral-constrained. Geromichalos and Herrenbrueck (2016) and Geromichalos, Herrenbrueck, and Lee (2023) use a similar structure in their indirect liquidity approach, in which assets can be sold outright in exchange for means of payment. By contrast, I focus on repo markets, where liquidity is delivered through secured lending, given that repos are essentially collateral-backed loans. Another departure from the literature, at least from the papers mentioned above, is that I embed Burdett and Judd (1983) pricing, generalized again with divisible assets and collateral constraints, into OTC markets to generate endogenous repo rate (price) dispersion.[4]

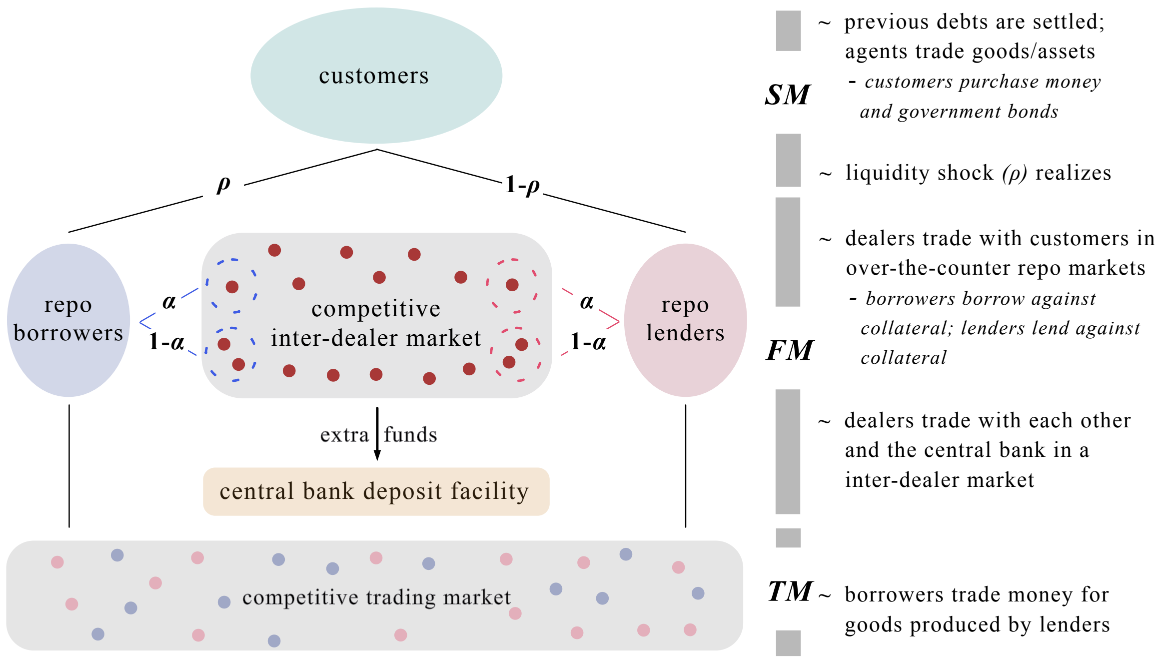
I show how search frictions inherent in OTC repo markets give rise to dealers' mar-

[4] I impose unit pricing in the sense that the price is independent of the quantity traded, as in Head, Liu, Menzio, and Wright (2012). This restriction is arguably not critical for the results because collateral constraints bind, and loan quantities are always determined by the collateral values in equilibrium.

ket power and repo rate dispersion. Huber (2023) explains dispersion in homogeneous repo contracts through dealers' (heterogeneous) identities and attributes dealers' market power with money market funds' preferences for portfolio concentration and stable funding. Eisenschmidt, Ma, and Zhang (2024) identify another channel of market power that arises from customers' costly link formation and show how it determines the magnitude of repo rate dispersion, with dispersion itself stemming from customer heterogeneity. In my paper, market power and dispersion appear simultaneously and endogenously under search frictions, thereby complementing these two recent and closely related papers. In addition to repo rates, haircuts are another important component of repo contracts. A natural concern is that dispersion in repo rates could be a consequence of dispersion in haircuts. However, Julliard, Pinter, Todorov, Wijnandts, and Yuan (2024) find no statistical association between the two, where haircuts are primarily driven by risk consideration (Chebotarev, 2025; Hempel, Kahn, Mann, & Paddrik, 2023).

I use endogenous distributions of repo lending and borrowing rates to evaluate the effectiveness of the central bank's interest-rate control and asset allocation through its standing facilities, including the deposit, borrowing, and lending facilities. This connects to the literature on monetary policy implementation, which describes how central banks set administered rates and conduct operations to transmit their policy stance to financial markets (Afonso, Armenter, & Lester, 2019; Afonso & Lagos, 2015; Armenter & Lester, 2017; Baughman & Carapella, 2024; Bianchi & Bigio, 2022; Ennis & Keister, 2008). The literature primarily focuses on unsecured credit markets, e.g., the federal funds market. I instead study secured credit markets, as activity in unsecured credit markets has declined and migrated toward secured markets in recent decades (Corradin, Eisenschmidt, Hoerova, Linzert, Schepens, & Sigaux, 2020; European Central Bank, 2021; Schnabel, 2023). My paper also complements this literature, particularly Williamson (2025), which likewise focuses on secured credit, by explicitly modeling endogenous price dispersion.

Figure 1: Timing of Events



2 Environment

Time is discrete and continues forever, with three subperiods in each period. The first subperiod involves activities in a **settlement market**, where agents settle debts from the last period and rebalance their financial portfolio. The second subperiod involves borrowing and lending activities in a **funding market**. The last subperiod involves exchanges between money and goods in a **trading market**. Figure 1 depicts the timing of events within a period, which will be clear throughout this section.

There are two types of private agents: a measure one of repo **customers** that represents money market funds, pension funds, and insurance companies; and a measure s of **dealers** that represents depository institutions.[5] Both are risk-neutral infinitely-lived agents that discount the future between periods at a rate $\beta \in (0, 1)$. There is also a government consisting of a **fiscal authority** and a **central bank**.

[5]The customer measure is normalized to one, so s reflects the dealer-to-customer ratio. This ratio does not affect the analysis unless the profitability of individual dealers is under consideration.

Customers can convert labor (n) into consumption goods (c) one-for-one in the settlement and trading market, where their instantaneous payoffs are captured by

$$c^{SM} - n^{SM} + \sigma \cdot Ac^{TM} - (1-\sigma) \cdot n^{TM}, \quad \text{with} \quad \sigma \in \{0,1\}, A > 1. \tag{1}$$

For simplicity, I drop the time subscript, as the analysis will focus on stationary equilibria. In each period, customers work (n^{SM}) in the settlement market to raise funds and rebalance their portfolios. They may consume (c^{SM}) in this subperiod, which occurs if their payoff from assets from the previous period exceeds their payment for current asset purchases.[6] Crucially, customers are subject to an i.i.d. idiosyncratic liquidity shock that is realized at the end of the settlement market. A fraction ρ of them consume (c^{TM}) in the trading market, and, therefore, borrow in the funding market to settle their payments, i.e., $\mathbb{P}(\sigma = 1) = \rho$. By contrast, the rest can work (n^{TM}) in the trading market and lend out money for higher returns. Exchanges in the trading market are essential because goods are perishable, so that no one can carry them across subperiods.

Dealers earn profits by intermediating borrowing and lending in the funding market, in the spirit of Duffie, Gârleanu, and Pedersen (2005). Dealers trade among themselves through a frictionless *inter-dealer market*. They also trade with customers through frictional *over-the-counter (OTC) repo markets*, where, with probability α, a customer contacts a single dealer. Otherwise, the customer contacts two dealers and trades with the one offering the better price. This trading structure is consistent with the fact that most repo customers do not have access to the inter-dealer market and must rely on concentrated intermediation by one or two dealers (Eisenschmidt, Ma, & Zhang, 2024).[7]

The underlying assets are one-period nominal **government bonds** (b) issued by the fiscal authority, and central bank liabilities consisting of **money** (m) and **central bank**

[6]In this way, their payoff in the settlement market, $c^{SM} - n^{SM}$, works like a wealth account that allows them to freely withdraw or deposit funds, as in Duffie, Gârleanu, and Pedersen (2005).

[7]One can also follow the original Burdett and Judd (1983) to show how customers endogenously choose to contact one or two dealers randomly at certain search costs.

deposits (d). Assets are essential because no unsecured IOU will be accepted in any transactions under the limited commitment and the lack of record-keeping technology, following Lagos and Wright (2005). Government bonds are useful collateral in the funding markets, supporting repo transactions. Each bond sells for z_b units of money in the settlement market and is a claim to one unit of money in the next settlement market. Money is used as a means of payment to settle transactions among customers in the trading market. Central bank deposits, such as reserves in the U.S. Federal Reserve, are restricted to dealers and can be viewed as an asset sold at a price z_d, in terms of money.

In steady state, real variables remain unchanged forever while nominal variables grow at a constant inflation rate $\pi - 1$. I express all variables in real terms, measured in units of current-period settlement market goods, including those lower-case letters, b, m, and d, mentioned above. Agents must adjust their nominal assets for inflation when carrying across periods. For example, m units of money in the current period are worth m/π units of settlement market goods in the next period.

Fiscal Authority and Central Bank The fiscal authority issues $\hat{b}$ units of government bonds at a price z_b in each settlement market. The exogenous value $f \equiv z_b\hat{b}$ denotes its revenue from bond issuance and describes the **fiscal policy**. The central bank purchases $\hat{b} - \bar{b}$ units of bonds with the issuance of central bank liabilities $\hat{m} \in (0, f)$. Later, after dealers make their deposit decisions, the central bank's liabilities consist of money ($\bar{m}$) and central bank deposits ($z_d\bar{d}$). Therefore,

$$\underbrace{z_b\left(\hat{b} - \bar{b}\right)}_{\text{central bank assets}} = \overbrace{\bar{m} + z_d\bar{d}}^{\text{central bank liabilities, } \hat{m}}, \tag{2}$$

with $\bar{b}$ denoting the quantity of the bonds circulating in the private sector. Equation (2) reflects the central bank's balance sheet, equating the value of its asset holdings to its liabilities. **Monetary policy** thus has two dimensions: the size of central bank balance sheet captured by central bank liabilities $\hat{m}$, or equivalently the ratio of $\hat{m}$ over consolidated government liabilities $\theta \equiv \hat{m}/f$; and the administered nominal interest rate $1/z_d - 1$ on central bank deposits that determines the composition of central bank liabilities.

The fiscal authority uses lump-sum transfers or taxes to balance the consolidated government budget constraint period by period. This constraint is

$$\bar{m} + z_d \bar{d} + z_b \bar{b} \; = \; \frac{\bar{m} + \bar{d} + \bar{b}}{\pi} + \tau, \tag{3}$$

where τ is the real value of the lump-sum transfer (or tax if $\tau < 0$) to customers at the beginning of the settlement market. The left-hand side of (3) represents the revenue from issuing new liabilities consisting of government bonds and central bank liabilities, which equals the fiscal authority's revenue from its bond issuance. The right-hand side is the payment on liabilities from the previous period and the lump-sum transfer.

3 Market Structure

In this section, I describe the market structure subperiod by subperiod and present the value functions of customers, taking the price distributions offered by the dealers as given. I then characterize those endogenous repo borrowing and lending distributions in the next section, which are determined by the customers' choices derived here.

3.1 Settlement Market

At the beginning of the settlement market, agents, including the central bank and the fiscal authority, pay off their outstanding debts and receive asset returns. Repo customers enter a Walrasian market to trade goods and assets.

Generically, consider customers who borrow (**borrowers** henceforth) in the last funding market enter the settlement market owing ℓ_B units of loans while holding m units of money and b units of government bonds. If they exit this market with $\tilde{m}$ units of money and $\tilde{b}$ units of government bonds, their value function is

$$U^B(m,b,\ell_B) = \max_{c^{SM},n^{SM},\tilde{m},\tilde{b}} c^{SM} - n^{SM} + \mathbb{E}_i\left[V^i\left(\tilde{m},\tilde{b}\right)\right] \tag{4}$$

$$\text{s.t.} \quad c^{SM} + \tilde{m} + z_b\tilde{b} = n^{SM} + m + b - \ell_B + \tau, \tag{5}$$

where τ is the lump-sum transfer and $\mathbb{E}_i$ denotes the expectation operator over their value function V^i in the incoming funding market, considering their potential role of borrowing ($i = B$) or lending ($i = L$). Although the values of a borrower's consumption c^{SM} and labor supply n^{SM} are indeterminate, their difference is fixed in equilibrium, and the borrower consumes the residual funds from the last period if $c^{SM} - n^{SM} > 0$ and works to accumulate new funding if $c^{SM} - n^{SM} < 0$. Substituting $c^{SM} - n^{SM}$ from the budget constraint (5) into U^B yields

$$U^B(m,b,\ell_B) = m + b - \ell_B + \tau + \max_{\tilde{m},\tilde{b}} -\tilde{m} - z_b\tilde{b} + \mathbb{E}_i\left[V^i\left(\tilde{m},\tilde{b}\right)\right]. \tag{6}$$

Similarly, consider customers who lend (**lenders** henceforth) in the last funding market with ℓ_L units of loan. Their value function in the settlement market is

$$U^L(m,b,\ell_L) = m + b + \ell_L + \tau + \max_{\tilde{m},\tilde{b}} -\tilde{m} - z_b\tilde{b} + \mathbb{E}_i\left[V^i\left(\tilde{m},\tilde{b}\right)\right]. \tag{7}$$

At the end of the settlement market, the liquidity shock is realized, reassigning customers' roles of borrowing or lending in the incoming funding market. A fraction ρ of customers become borrowers, requiring money to trade for goods in the trading market. The remaining become lenders. This gives the following first-order conditions for customers' optimal portfolio choices, which are independent of their initial asset holdings:

$$-1 + \rho\frac{\partial}{\partial\tilde{m}}V^B\left(\tilde{m},\tilde{b}\right) + (1-\rho)\frac{\partial}{\partial\tilde{m}}V^L\left(\tilde{m},\tilde{b}\right) = 0 \qquad \text{(money)}, \tag{8}$$

$$-z_b + \rho\frac{\partial}{\partial\tilde{b}}V^B\left(\tilde{m},\tilde{b}\right) + (1-\rho)\frac{\partial}{\partial\tilde{b}}V^L\left(\tilde{m},\tilde{b}\right) = 0 \qquad \text{(government bonds)}. \tag{9}$$

3.2 Funding Market

Over-the-counter Repo Markets In OTC repo markets, each dealer posts a nominal loan price z_i, where $i \in \{B, L\}$ indexes the type of customers served — $i = B$ for borrowers while $i = L$ for lenders — taking as given the price distribution F_i posted by other dealers, following Burdett and Judd (1983). Dealers who trade with borrowers, and thus are more likely to borrow in the inter-dealer market, are labelled as **borrower dealers**. Similarly, those who trade with lenders are labelled as **lender dealers**. In principle, lender and borrower dealers can be the same agents. Using different labels to isolate their roles on different sides of the market is harmless, as dealers are risk-neutral and operate under a competitive inter-dealer market.

Customers observe the entire price distribution F_i but trade with dealers under search frictions. With probability α, they contact one dealer and trade with that dealer. Otherwise, they contact two dealers and trade with the one offering the better price. Borrowers trade with the one posting a higher price to borrow more money against their collateral, while lenders trade with the one with a lower price to receive higher interest payments.

Borrowers borrow from a random sample of dealers and face random loan prices. Their expected value of entering the funding market is

$$\begin{aligned} V^B\left(\tilde{m}, \tilde{b}\right) &= \alpha \int \max_{\ell_B} W^B\left(\tilde{m} + z_B \ell_B, \tilde{b}, \ell_B\right) \mathrm{d}F_B\left(z_B\right) \\ &\quad + (1-\alpha) \int \max_{\ell_B} W^B\left(\tilde{m} + z_B \ell_B, \tilde{b}, \ell_B\right) \mathrm{d}\left[F_B\left(z_B\right)\right]^2, \end{aligned} \tag{10}$$

where W^B denotes their value of entering the subsequent trading market after borrowing ℓ_B units of loans under price z_B, the higher loan price they are offered. Borrowers pledge government bonds as collateral, and the following collateral constraint

$$\ell_B \leq \tilde{b}, \tag{11}$$

guarantees that their return on bonds can cover their loan payments.

Instead of the higher price, lenders accept the lower loan price z_L they are offered, giving the following value function

$$V^L\left(\tilde{m},\tilde{b}\right) = \alpha \int \max_{\ell_L} W^L\left(\tilde{m} - z_L\ell_L, \tilde{b}, \ell_L\right) \mathrm{d}F_L\left(z_L\right) + (1-\alpha)\int \max_{\ell_L} W^L\left(\tilde{m} - z_L\ell_L, \tilde{b}, \ell_L\right) \mathrm{d}\left(1-\left[1-F_L\left(z_L\right)\right]^2\right), \qquad (12)$$

with W^L denoting their value of entering the trading market after lending ℓ_L. Lenders are subject to the following cash constraint

$$z_L\ell_L \leq \tilde{m}. \qquad (13)$$

Competitive Inter-dealer Market Dealers' *total profit* depends on two factors: the number of customers served and the *profit per customer*. Any excess funds can be deposited in the central bank's deposit facility.

The total profit for a borrower dealers posting price z_B is

$$\Pi_B\left(z_B\right) = \lim_{\epsilon\to 0^+} \overbrace{\frac{\rho}{s}\left(\alpha + 2(1-\alpha)F_B\left(z_B-\epsilon\right) + (1-\alpha)\left[F_B\left(z_B\right) - F_B\left(z_B-\epsilon\right)\right]\right)}^{\text{number of borrowers served}} \underbrace{R_B\left(z_B\right)}_{\text{profit per borrower}}, \qquad (14)$$

which illustrates that a dealer's pricing strategy determines both the number of borrowers they served and their profit per borrower, R_B, explained below. In general, the borrower dealer serves three sets borrowers: borrowers who only contact them, in total $\rho\alpha/s$; borrowers contact another dealer positing a price below z_B, $\lim_{\epsilon\to 0^+} 2\rho\left(1-\alpha\right)F_B\left(z_B-\epsilon\right)/s$; and borrowers contact another dealer posting the same price z_B under the uniform tie-breaking rule, $\lim_{\epsilon\to 0^+}\rho\left(1-\alpha\right)\left[F_B\left(z_B\right) - F_B\left(z_B-\epsilon\right)\right]/s$.

Regarding profit per borrower, a borrower dealer posting price z_B provides $z_B\ell_B(z_B)$ units of money to each borrower served, where $\ell_B(z_B)$ is the borrower's loan demand that solves their funding market problem (10). The dealer finances this demand by borrowing in the inter-dealer market and may save any extra in the central bank's deposit facility at a price z_d. As a result, the dealer's profit per borrower served is

$$\begin{aligned} R_B\left(z_B\right) &= \max_{d_{BD},\ell_{BD}} \; d_{BD} + \ell_B(z_B) - \ell_{BD}, \\ \text{s.t.} \quad z_B \ell_B(z_B) + z_d d_{BD} &= z_I \ell_{BD}, \quad \text{and} \quad d_{BD} \;\geq\; 0, \end{aligned} \tag{15}$$

where z_I is the loan price in the inter-dealer market, ℓ_{BD} is their inter-dealer borrowing, and d_{BD} is their holdings of central bank deposits. In addition to the non-negative constraint $d_{BD} \geq 0$, which rules out borrowing from the central bank's deposit facility, dealers face the collateral constraint

$$d_{BD} + \ell_B(z_B) \;\geq\; \ell_{BD}, \tag{16}$$

so that their returns on assets exceed their payments on liabilities. However, this constraint will not be a concern because dealers will make a non-negative profit in equilibrium.

Similarly, the total profit for a lender dealer posting price z_L is

$$\Pi_L\left(z_L\right) \;=\; \lim_{\epsilon \to 0^+} \overbrace{\frac{1-\rho}{s}\left(\, \alpha + 2\left(1-\alpha\right)\left[1 - F_L\left(z_L\right)\right] + \left(1-\alpha\right)\left[F_L\left(z_L\right) - F_L\left(z_L - \epsilon\right)\right] \right)}^{\text{number of lenders served}} \underbrace{R_L\left(z_L\right)}_{\text{profit per lender}}. \tag{17}$$

The number of lenders served by this dealer is determined in a manner similar to the number in the borrower dealer's problem, but, instead of the higher price, the lower price becomes more attractive to lenders. The dealer receives funds $z_L \ell_L(z_L)$ from lenders, invests $z_d d_{LD}$ into the central bank's deposit facility, and lends $z_I \ell_{LD}$ in the inter-dealer market. Therefore, their profit per lender served is

$$\begin{aligned} R_L\left(z_L\right) &= \max_{d_{LD},\ell_{LD}} \; d_{LD} + \ell_{LD} - \ell_L(z_L), \\ \text{s.t.} \quad z_d d_{LD} + z_I \ell_{LD} &= z_L \ell_L(z_L), \quad \text{and} \quad d_{LD} \;\geq\; 0. \end{aligned} \tag{18}$$

As with borrower dealers, the lender dealer's collateral constraint,

$$d_{LD} + \ell_{LD} \;\geq\; \ell_L(z_L), \tag{19}$$

never binds in equilibrium.

Lemma 1 (Dealer's Profit per Customer)**.** *Inter-dealer market problems* (15) *and* (18) *imply that the inter-dealer rate is higher than the central bank's deposit facility rate, i.e.,* $z_I \leq z_d$*, with equality if the deposit facility is active, i.e.,* $\bar{d} > 0$*. This interest rate structure, in turn, yields the profit functions:*

$$R_B\left(z_B\right) = \left(\frac{1}{z_B} - \frac{1}{z_I}\right) z_B \ell_B(z_B), \tag{20}$$

$$R_L\left(z_L\right) = \left(\frac{1}{z_I} - \frac{1}{z_L}\right) z_L \ell_L(z_L). \tag{21}$$

I present all the proofs in Appendix A and discuss the intuition in the main text. The nominal interest rate of the central bank's deposit facility, i.e., $1/z_d - 1$, is administered by the central bank and acts as a floor for the inter-dealer rate because this facility is always a viable investment option for dealers. Dealers exploit profits from the interest rate spreads between the inter-dealer rate and their repo rates with customers. The profit per borrower and per lender, R_B and R_L, depends on the interest rate spreads, $1/z_B - 1/z_I$ and $1/z_I - 1/z_L$, respectively.

3.3 Trading Market

After trading with dealers, borrowers enter the trading market with $\tilde{m} + z_B\ell_B(z_B)$ units of money. They then exchange money for consumption goods sold at a price p, in terms of settlement market goods. This gives the value function in the trading market

$$\begin{aligned} &W^B\left(\tilde{m} + z_B\ell_B(z_B), \tilde{b}, \ell_B(z_B)\right) \\ &\quad = \max_{0 \leq pc_t \leq \tilde{m} + z_B\ell_B(z_B)} Ac_t + \beta U^B\left(\frac{\tilde{m} + z_B\ell_B(z_B) - pc_t}{\pi}, \frac{\tilde{b}}{\pi}, \frac{\ell_B(z_B)}{\pi}\right), \end{aligned} \tag{22}$$

where, again, nominal terms carried to the next period are adjusted by inflation. From (6), the value function U^B is linear in their state variables, implying that

$$
\begin{aligned}
&W^B\left(\tilde{m}+z_B\ell_B(z_B),\tilde{b},\ell_B(z_B)\right)\\
&=\max_{0\le pc_t\le \tilde{m}+z_B\ell_B(z_B)} Ac_t+\frac{\beta\left(\tilde{m}+z_B\ell_B(z_B)-pc_t+\tilde{b}-\ell_B(z_B)\right)}{\pi}+\beta U^B(0,0,0), \quad (23)
\end{aligned}
$$

and $U^B(0,0,0)$ is a constant.

Lenders work to produce goods, trade these goods for money, and carry all assets into the next settlement market. Their value function in the trading market is

$$
\begin{aligned}
&W^L\left(\tilde{m}-z_L\ell_L(z_L),\tilde{b},\ell_L(z_L)\right)\\
&=\max_{n_t\ge 0} -n_t+\frac{\beta\left(\tilde{m}-z_L\ell_L(z_L)+pn_t+\tilde{b}+\ell_L(z_L)\right)}{\pi}+\beta U^L(0,0,0). \quad (24)
\end{aligned}
$$

The market clearing condition in the trading market is

$$
\rho\int c_t(z_B)\mathrm{d}F_B(z_B) = (1-\rho)\,n_t. \quad (25)
$$

Borrowers' consumption depends on the loan price they are offered (z_B), as this price affects their money holdings after the funding market and therefore their ability to pay for consumption goods. By contrast, lenders' asset holdings are unrelated to their production process, so n_t remains constant across leaders, at least in the symmetric equilibrium.

Lemma 2 (Trading Outcomes)**.** *The competitive trading market gives*

$$
p = \frac{\pi}{\beta}, \quad c_t(z_B) = \frac{\beta\left(\tilde{m}+z_B\ell_B(z_B)\right)}{\pi}. \quad (26)
$$

Borrowers spend all their money on consumption goods, highlighting the critical role of repo markets in facilitating liquidity by allowing borrowers to borrow against assets. Lenders also benefit by transforming their money into higher-yielding loans. Lemmas 3 and 4 characterize borrowers' demand for loans and lenders' supply of loans, respectively, derived from first-order conditions from their funding market problems, i.e., equations (10) and (12), and envelope conditions from their trading market problem, i.e., (23) and (24).

Lemma 3 (Borrowers' Demand for Loans)**.** *If $z_B > 1/A$, the collateral constraint* (11) *binds and borrowers borrow up to their collateral value, $\ell_B(z_B) = \tilde{b}$. If $z_B = 1/A$, this constraint does not bind.*

Beyond the cutoff, $1/A$, borrowers obtain sufficient money with each unit of collateral, generating high trading returns that cover their interest payments. They borrow up to their collateral value whenever the loan price exceeds this point. At the cutoff, borrowers are indifferent: they borrow any amount for transactions while breaking even.

Lemma 4 (Lenders' Supply of Loans)**.** *If $z_L < 1$, the cash constraint* (13) *binds and lenders lend out all their money, $\ell_L(z_L) = \tilde{m}/z_L$. If $z_L = 1$, this constraint does not bind.*

When $z_L < 1$, or equivalently, when the nominal interest rate lenders obtain, $1/z_L - 1$, is above the zero lower bound, they prefer loans to money, as loans provide positive interest payments. At the zero lower bound, they are indifferent between loans and money, as neither option yields a positive return: they can adjust their portfolio composition arbitrarily without affecting profits.

The envelope conditions of customers' funding market problems (10) and (12) reduce their first-order conditions (8) and (9) of their settlement market problems to the following optimal portfolio choice conditions that determine the inflation and nominal interest rate on government bonds in equilibrium.

Lemma 5 (Optimal Portfolio Choices)**.** *Customers' optimal portfolio choices give*

$$1 = \frac{\beta}{\pi}\left[\rho A + (1-\rho)\left(\alpha \int \frac{1}{z_L} \mathrm{d}F_L\left(z_L\right) + (1-\alpha)\int \frac{1}{z_L}\mathrm{d}\left(1 - \left[1 - F_L\left(z_L\right)\right]^2\right)\right)\right], \quad (27)$$

$$z_b = \frac{\beta}{\pi}\left[\rho A \left(\alpha \int z_B \mathrm{d}F_B\left(z_B\right) + (1-\alpha)\int z_B \mathrm{d}\left[F_B\left(z_B\right)\right]^2\right) + 1 - \rho\right]. \quad (28)$$

Conditions (27) and (28) are essentially asset pricing kernels for money and government bonds, respectively. While the nominal price of money is fixed at one, its real value is determined by the expected payoff from borrowers' direct use of money in transactions and lenders' interest payments through lending. The former occurs with probability ρ, while the latter occurs with probability $1-\rho$. Similarly, the bond price reflects the expected payoff from pledging bonds as collateral to support borrowers' transactions and lenders' returns when holding them to maturity. Again, the payoff related to borrowers occurs with probability ρ, while the payoff related to lenders occurs with probability $1-\rho$.

4 Equilibrium Repo Price Distributions

To lay the groundwork for the following analysis, I now derive the endogenous price distributions offered by both borrower and lender dealers, highlighting the critical role of search frictions in generating repo price dispersion. I then define and characterize the equilibrium, establishing its existence.

Each dealer chooses price z_i to maximizes their total profit $\Pi_i(z_i)$, where, again, $i \in \{B, L\}$ refers to borrowing (B) or lending (L). In equilibrium, the price distribution $F_i(z_i)$ is consistent with dealers' profit maximization if every price z_i in its support $\mathcal{S}_i$ maximizes $\Pi_i(z_i)$, such that

$$\Pi_i(z_i) \;=\; \Pi_i^\star \;\equiv\; \max_{z_i} \Pi_i(z_i) \quad \forall z_i \in \mathcal{S}_i,\; i \in \{B, L\}. \tag{29}$$

Instead of directly solving this maximization problem, it proves helpful first to solve for the optimal prices that maximize dealers' profit per borrower served and per lender. I refer to these optimal prices as *monopoly prices*, as each solves a standard monopoly pricing problem. For convenience, I rewrite the profit per customer in (20) and (21) as

$$R_B(z_B) \;=\; \left(1 - \frac{z_B}{z_I}\right)\tilde{b} \qquad \forall z_B \in [\frac{1}{A}, 1], \tag{30}$$

$$R_L(z_L) \;=\; \left(\frac{z_L}{z_I} - 1\right)\frac{\tilde{m}}{z_L} \qquad \forall z_L \in (0, 1], \tag{31}$$

using the loan demand for and the loan supply characterized in Lemmas 3 and 4. In particular, I focus on cases that favor dealers under cutoff prices, $z_B = 1/A$ and $z_L = 1$, when there are multiple solutions to the loan demand and loan supply. For example, even though lenders are indifferent to lending any amount $\ell_L \leq \tilde{m}$ under $z_L = 1$, dealers obtain the highest profit when $\ell_L = \tilde{m}$. They can achieve this profit by choosing a price arbitrarily close to 1, i.e., $z_L = 1 - \epsilon$ for a vanishingly small ϵ.

Lemma 6 (Borrower Dealers' Monopoly Price)**.** *The monopoly price that maximizes a borrower dealer's profit per borrower is $z_B = 1/A$, which yields positive profits if and only if $z_I > 1/A$.*

The monopoly price would arise when search frictions are extremely large, such that $\alpha \longrightarrow 1$. In such a limit case, borrowers only trade with one dealer, who then behaves as a monopolist, offering a price of $1/A$ to extract the trading surplus a borrower can obtain by exchanging the money they borrow for consumption goods. Clearly, this price is insensitive to changes in monetary policy.

Besides the monopoly price, crucial for the price distribution below is the *competitive price* that would arise when the search friction becomes negligible, i.e., $\alpha \longrightarrow 0$, or equivalently, when dealers always have to compete with one another for price quotations. This works like the Bertrand competition, where competition pushes dealers to earn zero profit, offering borrowers a price that barely compensates for their cost of inter-dealer borrowings, i.e., $z_B = z_I$. Unlike the monopoly price, the competitive price will respond to central bank interventions, at least to interventions that alter the inter-dealer price.

Proposition 1 (Repo Borrowing Prices)**.** *If $z_I > 1/A$, there exists a unique distribution for repo borrowing prices characterized by the monopoly price $1/A$, the competitive price z_I, and the search friction parameter α. The cumulative distribution function is*

$$F_B(z_B) = \frac{\alpha}{2(1-\alpha)} \left(\frac{z_B - \frac{1}{A}}{z_I - z_B} \right), \tag{32}$$

with the support $\mathcal{S}_B = [1/A, \bar{z}_B]$*, where the upper bound is*

$$\bar{z}_B \;=\; \left(1 - \frac{\alpha}{2-\alpha}\right) z_I + \frac{\alpha}{2-\alpha}\frac{1}{A}. \tag{33}$$

The key determinants of the distribution are the monopoly price and the competitive price. In particular, the lower bound of the support $\mathcal{S}_B$ is the monopoly price, while its upper bound is a convex combination of the monopoly price and the competitive price, adjusted by the magnitude of the search friction. Although the monopoly price yields the highest profit per borrower, dealers earn the same total profit over the support of the distribution, where they offset the loss in per-borrower profit by serving more borrowers.

The price distribution satisfies standard properties in Burdett and Judd (1983). First, the distribution is continuous. Otherwise, if there were a mass point, a dealer who initially posted a price at the mass point could increase their profit by reducing the price slightly, as this reduction leaves the profit per borrower served almost unchanged but attracts all the borrowers who initially accepted their price. Second, the support $\mathcal{S}_B$ is connected or, say, a convex set in the one-dimensional case. Otherwise, if $\mathcal{S}_B$ had a gap between two prices, the lower price would yield a higher profit because, although these two prices give the same number of borrowers served, the lower price generates a higher profit per borrower. This violates the equal profit condition in (29). These two properties reduce the total profit (14) to

$$\Pi_B\left(z_B\right) \;=\; \frac{\rho}{s}\left[\,\alpha + 2\left(1-\alpha\right) F_B\left(z_B\right)\right] R_B\left(z_B\right). \tag{34}$$

I can then use (29) to derive the closed-form solution of F_B, given that the monopoly price $1/A$ is the lower bound of the support $\mathcal{S}_B$ so that $F(1/A) = 0$.

Lemma 7 (Lender Dealers' Monopoly Price)**.** *The monopoly price that maximizes a lender dealer's profit per lender is* $z_L = 1$*, which yields a nonnegative profit at this monopoly price with zero profit occurring if and only if* $z_I = z_d = 1$*.*

Proposition 2 (Repo Lending Prices)**.** *If $z_I < 1$, there exists a unique price distribution for repo lending prices characterized by the monopoly price* 1*, the competitive price z_I, and the search friction parameter α. The cumulative distribution function is*

$$F_L(z_L) \;=\; 1 - \frac{\alpha}{2(1-\alpha)} \frac{1/z_L - 1}{1/z_I - 1/z_L}, \tag{35}$$

with support $\mathcal{S}_L = [\underline{z}_L, 1]$, where the lower bound is

$$\underline{z}_L \;=\; \left[\left(1 - \frac{\alpha}{2-\alpha}\right)\frac{1}{z_I} + \frac{\alpha}{2-\alpha}\right]^{-1}. \tag{36}$$

The price distribution for lender dealers has similar properties and takes a similar form to the distribution for borrower dealers, F_B. However, instead of lower, lenders prefer higher prices so that they can get higher interest payments, which is also why the lending price distribution is naturally expressed in interest rates. From equations (33) and (36), $\bar{z}_B < z_I < \underline{z}_L$, implying that any repo lending price z_L is greater than the inter-dealer price z_I, which, in turn, is greater than any repo borrowing price z_B. Thus, dealers quote bid-ask spreads to customers and earn net interest margins from intermediation services — a feature emphasized in the OTC literature, surveyed by Weill (2020), and consistent with empirical findings in Corradin and Maddaloni (2020) and Ferrari, Guagliano, and Mazzacurati (2017).

Remark Search frictions play a critical role in repo price dispersion, determining how prices concentrate in the tails of the borrowing and lending distributions, such that larger search frictions concentrate prices toward the monopoly tail. Dispersion disappears in the limit cases when search frictions become negligible or extremely large. When search frictions are negligible, such that $\alpha \longrightarrow 0$, the price distributions F_B and F_L converge pointwise to the degenerate distributions $\mathbb{P}(z_B = z_I) = 1$ and $\mathbb{P}(z_L = z_I) = 1$, respectively. In other words, dealers always quote the competitive price z_I to borrowers and lenders, as in Bertrand competition. By contrast, when search frictions are extremely large, such that $\alpha \longrightarrow 1$, the price distributions F_B and F_L converge pointwise to the degenerate distributions $\mathbb{P}(z_B = 1/A) = 1$ and $\mathbb{P}(z_L = 1) = 1$, respectively. Dealers behave as monopolists, offering monopoly prices of $1/A$ to borrowers and 1 to lenders.

4.1 Definition of Equilibrium

The following equilibrium definition focuses on equilibria with price dispersion on both lending and borrowing sides of the repo market, so that $z_I \in (1/A, 1)$. In principle, there could be equilibria that exhibit no price dispersion on one side of the market, such as when $z_I = 1$ or $z_I = 1/A$ (Propositions 1 and 2). I ignore these knife-edge equilibria and instead focus on the empirically relevant cases. I also ignore variables that are not central to the analysis in the definition. For example, the trading market price p is excluded, but it is implied by Lemma 2 once the gross inflation rate π is determined.

Definition 1 (Equilibrium with Price Dispersion)**.** *Given the fiscal policy that determines the value of consolidated government liabilities,*

$$f = \bar{m} + z_d\bar{d} + z_b\bar{b}, \tag{37}$$

and the monetary policy that determines the size of the central bank's balance sheet,

$$\hat{m} = \bar{m} + z_d\bar{d}, \tag{38}$$

and the administered interest rate $1/z_b - 1 \geq 0$, an equilibrium consists of an allocation $(\bar{m}, \bar{d}, \bar{b}, \tilde{m}, \tilde{b})$, the price distributions F_B and F_L characterized in (32) and (35), the associated distributions of loans $\ell_B(z_B) = \tilde{b}$ and $\ell_L(z_L) = \tilde{m}/z_L$ (Lemmas 3 and 4), and market-determined prices (z_b, π, z_I), satisfying customers' optimal portfolio choice decisions (27) and (28), market clearing conditions,

$$\hat{m} = \tilde{m} \qquad \textit{(money)}; \tag{39}$$

$$\bar{b} = \tilde{b} \qquad \textit{(government bonds)}; \tag{40}$$

$$\rho \int z_B\ell_B(z_B)\mathrm{d}F_B(z_B) + z_d\bar{d} = (1-\rho)\int z_L\ell_L(z_L)\mathrm{d}F_L(z_L) \qquad \textit{(loans)}, \tag{41}$$

where $z_I \leq z_d$ *with equality if* $\bar{d} > 0$.

Characterization of Equilibrium I pin down the equilibrium using the following equilibrium market clearing condition for the funding market

$$z_d \bar{d} \;=\; (1-\rho)\,\hat{m} - \rho\,(f - \hat{m})\,\frac{\mu_B}{z_b}, \tag{42}$$

derived from distributions (32) and (35), fiscal and monetary policies (37) and (38), and the market clearing condition for loan (41), where

$$\mu_B \;=\; z_I + \frac{\alpha}{2\,(1-\alpha)}\ln\left(\frac{\alpha}{2-\alpha}\right)\left(z_I - \frac{1}{A}\right) \tag{43}$$

is the mean of the repo borrowing price, i.e., $\mathbb{E}[z_B]$. Condition (42) reflects the inflows and outflows of funds in the funding market. Lenders lend out their money holdings, $(1-\rho)\,\hat{m}$, to dealers. Dealers then lend part of the money to borrowers through repos, which is why this part is determined by the value of government bonds held by borrowers, $\frac{\rho(g-\hat{m})\mu_B}{z_b}$. Dealers also save the rest of the money in the central bank's deposit facility, and $z_d\bar{d}$ denotes their account balances at the central bank. Given (32) and (35), I further derive the closed-form expression of the bond price

$$z_b \;=\; \frac{\beta}{\pi}\left[\rho A\left(\frac{\alpha}{A} + (1-\alpha)\,z_I\right) + 1 - \rho\right], \tag{44}$$

in Lemma 5, with the gross inflation rate

$$\pi \;=\; \beta\left[\rho A + (1-\rho)\left(\alpha + \frac{1-\alpha}{z_I}\right)\right]. \tag{45}$$

There are two types of equilibrium, depending on whether the central bank's deposit facility is active (i.e., $\bar{d} > 0$) or not (i.e., $\bar{d} = 0$). In an equilibrium with an active deposit facility, the deposit facility price, z_d, determines the price for inter-dealer loans z_I, such that $z_I = z_d$ (Lemma 1). The inter-dealer price z_I then determines the mean repo borrowing price μ_B and the bond price z_b through (43) and (44), respectively. Solving this equilibrium thus requires substituting μ_B and z_b into (42) to obtain a positive value of central bank deposits $\bar{d}$. By contrast, an equilibrium with an inactive deposit facility is solved by substituting $\bar{d} = 0$ in (42) to obtain an inter-dealer price z_I that clears the funding market, requiring $z_I \leq z_d$. For both types, once the inter-dealer price z_I is determined, all other equilibrium outcomes follow, including the endogenous repo price distributions, F_B and F_L, and the allocation of government liabilities with

$$z_b\bar{b} = f - \hat{m} \qquad \text{(central bank deposits)}, \tag{46}$$

$$\bar{m} = \rho\left[\hat{m} + (f - \hat{m})\frac{\mu_B}{z_b}\right] \qquad \text{(money)}. \tag{47}$$

From now on, I focus on the equilibrium with an active deposit facility throughout the analysis, because otherwise, changes in the central bank's deposit facility price would not affect the inter-dealer price, thereby having no impact on repo prices. Proposition 3 establishes the existence of this type of equilibrium, requiring a large size of the central bank's balance sheet. In this way, equilibrium with an active deposit facility corresponds to the floor system of monetary policy implementation, which is currently adopted by central banks in developed economies, like the U.S. Federal Reserve, the European Central Bank, and the Bank of Canada. Equilibrium with an inactive deposit facility, instead, corresponds to the corridor system that was popular before the 2008 Financial Crisis.

Proposition 3 (Existence of Equilibrium)**.** *For any (ρ, α, A), there exists a threshold $0 < \bar{\theta} < 1$ for the central bank's balance sheet policy such that a price-dispersed equilibrium with an active central bank deposit facility exists for any $z_d \in (1/A, 1)$ and $\theta \in (\bar{\theta}, 1)$.*

The central bank's deposit facility price must lie in the interval $(1/A, 1)$ for the equilibrium to exhibit price dispersion, under which an equilibrium with an active central bank deposit facility arises when the central bank's balance sheet is large. In this scenario, repo borrowers obtain only a small amount of government bonds as collateral, as many are already held by the central bank. Consequently, dealers cannot lend all their funds against collateral and deposit the excess at the central bank, consistent with the large supply of central bank liabilities implied by the large balance sheet. Although it is beyond the scope of the analysis of this paper, an equilibrium with an inactive deposit facility arises when the balance sheet becomes smaller.

5 Central Bank Deposit Facility

In this section, I study the positive implications of changes in the central bank's deposit facility price, which is the primary policy instrument of major central banks worldwide, such as the U.S. Federal Reserve and the European Central Bank.

5.1 Imperfect Pass-through of the Deposit Facility Price

I first study how market-determined repo prices respond to changes in the central bank's deposit facility price z_d, where I find imperfect monetary policy-through, characterized by a less-than-one-for-one response of repo prices to changes in z_d. Let z_B^q and z_L^q denote the q-quantile of the price distribution F_B and F_L, respectively, such that

$$F_B\left(z_B^q\right) = \frac{\alpha}{2(1-\alpha)}\left(\frac{z_B^q - \frac{1}{A}}{z_I - z_B^q}\right) = q, \tag{48}$$

$$F_L\left(z_L^q\right) = 1 - \frac{\alpha}{2(1-\alpha)}\frac{z_I\left(1-z_L^q\right)}{z_L^q - z_I} = q. \tag{49}$$

The following proposition and its corollary show that imperfect pass-through holds at every percentile of the price distributions as well as their mean prices.

Proposition 4 (Imperfect Pass-Through)**.** *For any $z_d \in (1/A, 1)$ and $\theta \in (\bar{\theta}, 1)$*

1. *The pass-through of the central bank's deposit facility price to any percentile of repo borrowing prices is imperfect, such that*

$$0 \leq \eta_B^q \equiv \frac{\mathrm{d}z_B^q}{\mathrm{d}z_d} = \frac{z_B^q - \frac{1}{A}}{z_d - \frac{1}{A}} < 1, \tag{50}$$

with equality $\eta_B^q = 0$ at the monopoly price $z_B^0 = 1/A$;

2. *The pass-through is also imperfect for repo lending prices near the monopoly price* $z_B^1 = 1$*, such that*

$$0 \leq \eta_L^q \equiv \frac{\mathrm{d}z_L^q}{\mathrm{d}z_d} = \frac{z_L^q\,(1 - z_L^q)}{z_d\,(1 - z_d)} < 1 \quad \text{if} \quad z_L^q > 1 - z_d, \tag{51}$$

with equality $\eta_L^q = 0$ *at* z_B^1*. Moreover,* $z_L^q > 1 - z_d$ *always holds when* $z_d \geq 1/2$*.*

Corollary 1. *Pass-through of the central bank's deposit facility price is imperfect for the mean repo borrowing price and for the mean repo lending price when* $z_d \geq 1/2$*.*

Although changes in the central bank's deposit facility price z_d transmit one-for-one to the inter-dealer price z_I (recall that $z_I = z_d$), pass-through to market-determined repo prices is imperfect, at least when $z_d \geq \frac{1}{2}$, which is a condition that holds for a wide range of nominal interest rates above the zero lower bound, i.e., $z_d = 1$. This concern regarding the pass-through effectiveness is consistent with the empirical findings for the U.S. (Duffie & Krishnamurthy, 2016) and the euro area (Ballensiefen, Ranaldo, & Winterberg, 2023; Eisenschmidt, Ma, & Zhang, 2024). The following lemma highlights the crucial role of search frictions in generating imperfect pass-through.

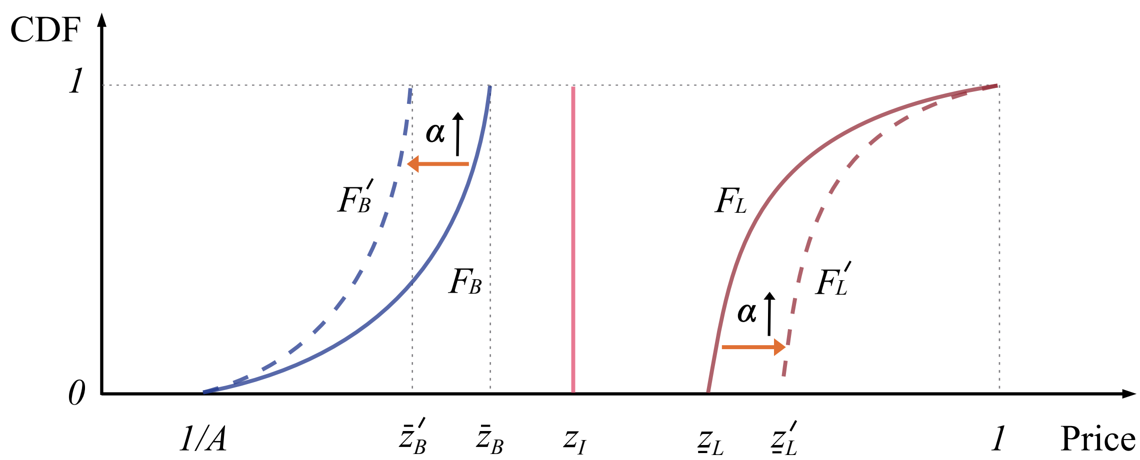
Lemma 8 (Search Frictions Weaken Pass-Through)**.** *Greater search frictions weaken pass-through:* $\frac{\mathrm{d}\eta_B^q}{\mathrm{d}\alpha} \leq 0$ *with equality only if* $q = 0$*, and, if* $z_d \geq 1/2$*,* $\frac{\mathrm{d}\eta_L^q}{\mathrm{d}\alpha} \leq 0$ *with equality holding only if* $q = 1$ *and* $z_d = 1/2$*. Moreover,*

1. *As* $\alpha \longrightarrow 0$*, then* $\eta_B^q \longrightarrow 1$ *and* $\eta_L^q \longrightarrow 1$*;*

2. *As* $\alpha \longrightarrow 1$*, then* $\eta_B^q \longrightarrow 0$ *and* $\eta_L^q \longrightarrow 0$*.*

As search frictions increase, i.e., larger α, the possibility for customers to trade with two dealers becomes lower. Dealers face less pressure to compete for customers, strengthening their market power. In response, as illustrated in Figure 2, both borrower and lender dealers offer prices closer to the monopoly price and away from the competitive price (i.e.,

Figure 2: Search Frictions Concentrate Repo Prices toward the Monopoly Price



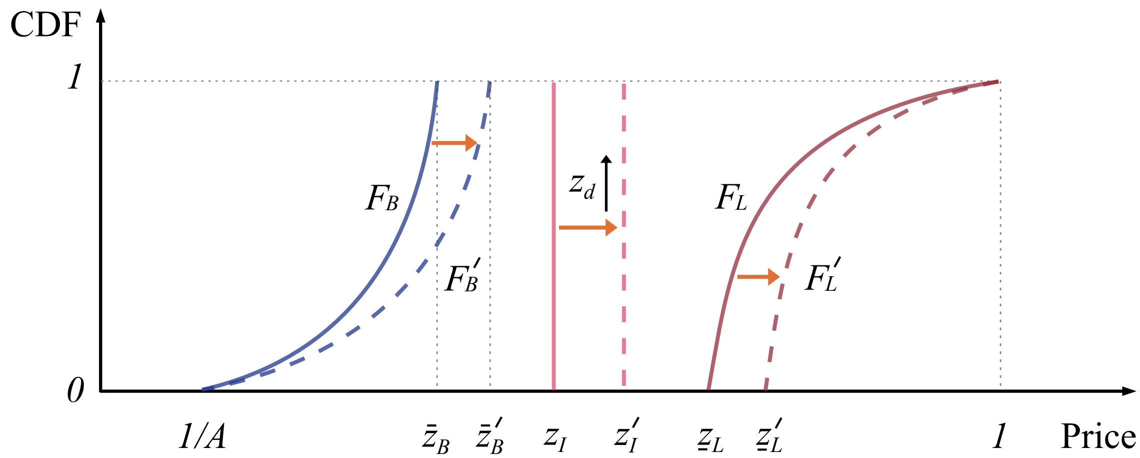
from (48) and (49), $\frac{\mathrm{d}z_B^q}{\mathrm{d}\alpha} < 0$ and $\frac{\mathrm{d}z_L^q}{\mathrm{d}\alpha} > 0$).[8] In this way, the increased search frictions effectively weaken pass-through because, from (50) and (51), pass-through is more effective for prices near the competitive price that is determined by the deposit facility price, while less effective for prices near the monopoly price that is insensitive to policy changes. This highlights a key insight of this paper: the concentration of prices toward the tails of the distributions, rather than the distributions themselves, matters for policy interventions. In particular, the pass-through becomes perfect (null) when search frictions become negligible (extremely large) because, as mentioned earlier, the price distributions collapse to the competitive prices (monopoly prices) in the limit case.

Pass-Through Weakens as the Deposit Facility Price Rises Beyond the existence of pass-through imperfection, I show that pass-through weakens as the deposit facility price rises, or equivalently, strengthens as the central bank raises this policy rate, consistent with Duffie and Krishnamurthy (2016).

Proposition 5 (Diminishing Pass-through)**.** *As the central bank's deposit facility price z_d increases, the pass-through of this price to repo lending prices weakens, i.e.,* $\frac{\mathrm{d}\eta_L^q}{dz_d} \leq 0$,

[8]For readability, I only draw the portion of the CDF where its density is strictly positive.

Figure 3: Asymmetric Effects of the Deposit Facility Price on Repo Price Distributions



with equality only at $z_L^1 = 1$. *However, pass-through to repo borrowing prices remains unchanged, i.e.,* $\frac{\mathrm{d}\eta_{\mathrm{B}}^{\mathrm{q}}}{dz_d} = 0$.

Despite shifting both the borrowing and lending price distributions to the right in the first-order stochastic dominance sense (recall that $\eta_B^q, \eta_L^q > 0$), an increase in the central bank's deposit facility price z_d generates asymmetric effects on their concentration patterns. As illustrated in Figure 3, the lending price distribution F_L becomes more concentrated around the monopoly price $z_L = 1$, strengthening dealers' market power. Consequently, lending prices become less responsive to subsequent changes in the deposit facility price. By contrast, the borrowing price distribution F_B becomes less concentrated around the borrower dealer's monopoly price $z_B = 1/A$. However, this does not enhance pass-through to the repo borrowing prices. This is because the increase in z_d also causes F_B to be more dispersed around the competitive price $z_I = z_d$, offsetting the effects from its reduced concentration at the monopoly price.

5.2 Ambiguous Effects on Asset Allocation

I conclude this section by examining the effects of an increase in the central bank's deposit facility price on asset allocation, specifically the composition of central bank liabilities consisting of central bank deposits ($z_d\bar{d}$) and money ($\bar{m}$).

From (42) and (47),

$$\frac{\mathrm{d}\left(z_d\bar{d}\right)}{\mathrm{d}z_d} = -\rho\left(f-\hat{m}\right)\frac{\mathrm{d}\left(\mu_B/z_b\right)}{\mathrm{d}z_d}, \tag{52}$$

$$\frac{\mathrm{d}\bar{m}}{\mathrm{d}z_d} = \rho\left(f-\hat{m}\right)\frac{\mathrm{d}\left(\mu_B/z_b\right)}{\mathrm{d}z_d}. \tag{53}$$

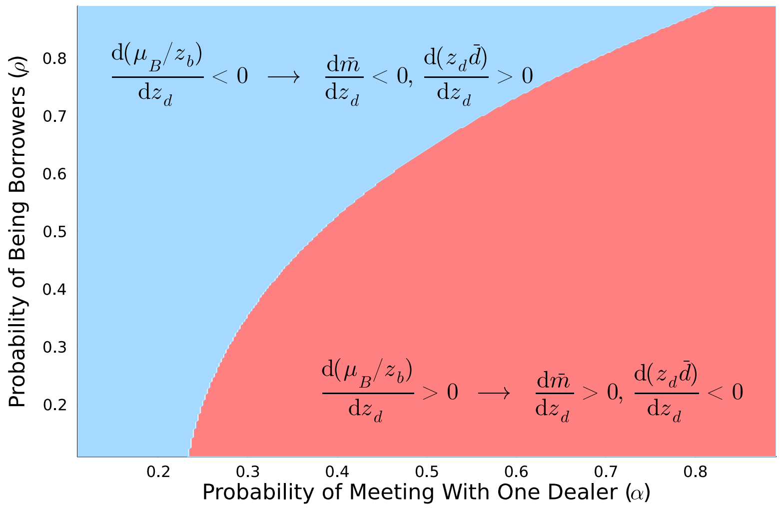
An increase in the central bank's deposit facility price, z_d, changes the supply of central bank deposits and money in opposite directions because their sum is exogenously determined by central bank balance sheet policy (equation 38). While the interest rate policy alters the composition of these liabilities, it does not affect their sum, which also implies that the supply of government bonds to the private sector, $z_b\bar{b}$, remains unchanged when z_d increases (equation 46). As shown in (52) and (53), the central bank's deposit supply decreases while its money supply increases when the mean repo borrowing price is more elastic relative to the bond price in response to an increase in z_d, i.e., $\frac{\mathrm{d}(\mu_B/z_b)}{\mathrm{d}z_d} > 0$. In this scenario, borrowers borrow more money against government bonds, which become relatively cheaper in response to the increased deposit facility price, thereby raising $\bar{m}$ and reducing $z_d\bar{d}$.

Overall, the relative elasticity of the mean repo borrowing price, μ, to the bond price, z_b, plays a crucial role in determining the effects of the central bank's deposit facility price, z_d, on asset allocation. However, changes in z_d generally have ambiguous effects on the ratio μ_B/z_b, thereby on asset allocation. Figure 4 illustrates this ambiguity through a numerical exercise, considering a deposit facility rate that is close to the zero lower bound.[9] Search frictions again play a critical role in generating these results. Lemma 9 below shows how the policy ambiguity disappears when search frictions are either vanishingly small or arbitrarily large.

Lemma 9 (How Search Frictions Matters for Ambiguous Asset Allocation)**.** *For any*

[9] I choose $A = 1.5$, but the results always hold if $A > 1$. I set $\theta = 0.9$, a sufficiently large central bank balance sheet that guarantees an active deposit facility for any $(z_d, \rho, \alpha) \in [0.9, 1] \times [0.1, 0.9] \times [0.1, 0.9]$.

Figure 4: Ambiguous Effects of Deposit Facility Price on Asset Allocation ($z_d = 0.99$)



$z_d \in (1/A, 1)$ and $\theta \in (\bar{\theta}, 1)$, there is a price-dispersed equilibrium with an active central bank deposit facility.

1. *As $\alpha \longrightarrow 0$, then $\mu_B \longrightarrow z_d$ and $z_b = z_d$;*

2. *As $\alpha \longrightarrow 1$, then $\mu_B \longrightarrow 1/A$ and $z_b \longrightarrow 1/(\rho A + 1 - \rho)$.*

In either case, the price ratio μ_B/z_b is constant in z_d. An increase in the deposit facility price does not change the money supply $\bar{m}$ and central bank deposit supply $z_d\bar{d}$.

As explained earlier, the limit cases $\alpha \longrightarrow 0$ and $\alpha \longrightarrow 1$ correspond to competitive and monopolistic pricing, respectively. Under competitive pricing, the pass-through from the central bank's deposit facility price, z_d, to both the mean repo borrowing price, μ_B, and the bond price, z_b, is perfect. By contrast, under monopolistic pricing, pass-through is null for both prices. In either case, the relative elasticity of μ_B and z_b with respect to z_d remains constant. Monetary policy is neutral in the sense that the central bank cannot reshuffle its liabilities by administering the deposit facility rate $1/z_d - 1$. Therefore, the OTC trading structure, with the search friction parameter $0 < \alpha < 1$, is critical for policy nonneutrality and ambiguity in this model. Rather than requiring the central bank to assess market conditions and estimate all parameters in detail before choosing its deposit facility rate, I suggest pairing the deposit facility with its lending and borrowing facilities. The next section shows how this combined approach allows the central bank to reallocate assets effectively.

6 Central Bank Lending and Borrowing Facilities

I introduce the central bank's long-standing lending and borrowing facilities and show how they can unambiguously reallocate assets. I also study their welfare implications and the optimal monetary policy when all three facilities, including the lending, borrowing, and the previously introduced deposit facilities, are available. The lending facility enables the central bank to provide short-term, secured loans to private financial institutions, which works like the repurchase agreement facility in the U.S. or the main refinancing operations in the Euro area. The borrowing facility instead enables the central bank to borrow against collateral, similar to the overnight reverse repurchase agreement facility in the U.S..[10] Unlike the deposit facility, which is limited to highly regulated financial institutions such as banks, both lending and borrowing facilities can be accessed by a broader range of financial institutions, including mutual funds and insurance companies that are repo customers in my paper.

Let z_r and z_o denote the nominal prices for the central bank's lending and borrowing facilities, respectively. Then, impose

$$z_r \;<\; z_d \;<\; z_o \quad \longleftrightarrow \quad \frac{1}{z_o} - 1 \;<\; \frac{1}{z_d} - 1 \;<\; \frac{1}{z_r} - 1, \tag{54}$$

which is consistent with the interest structure in the U.S., where the interest rate on overnight reverse repurchase agreements is below the interest rate on reserves, and the interest rate on the repurchase agreements is the highest among the three rates. Addi-

[10]The European Central Bank does not have a reverse repo facility in the way the Federal Reserve does, but relies on its deposit facility to absorb liquidity overnight.

tionally, I impose that

$$z_o \leq 1 \quad \text{and} \quad z_r \geq \frac{1}{A}, \tag{55}$$

to ensure that the market-determined repo prices do not strictly dominate the prices for these facilities. To see this, note that borrowers only borrow at prices that are higher than $1/A$ and lenders lend at prices that are lower than 1, which are the monopoly prices offered by dealers before introducing these facilities (Lemmas 3 and 4).

Long-standing borrowing and lending facilities enable customers to trade directly with the central bank on demand, at no search friction. After introducing these facilities, the profit per customer for borrower dealers and lender dealers in (20) and (21) becomes[11]

$$R_B(z_B) = \left(1 - \frac{z_B}{z_I}\right)\tilde{b} \qquad \forall z_B \in [z_r, 1], \tag{56}$$

$$R_L(z_L) = \left(\frac{z_L}{z_I} - 1\right)\frac{\tilde{m}}{z_L} \qquad \forall z_L \in (0, z_o]. \tag{57}$$

Although the functional form of these profit functions remains unchanged, the lending and borrowing facilities narrow their domains. Intuitively, borrowers would prefer to borrow from the central bank if their dealers provide them a price z_B that is below the price offered by the central bank's lending facility, z_r, so that they can borrow more money against their collateral. Lenders would prefer to lend to the central bank if their dealers provide them a price z_L that is above the price offered by the central bank's borrowing facility, z_o, so that they can obtain a higher return from lending.

Conditions (56) and (57) imply the following repo price distributions, as dealers choose prices to maximize their total profits (14) and (17).

Proposition 6 (Repo Price Distributions under Lending and Borrowing Facilities)**.** *If*

[11] As before, I focus on cases that favor dealers when there are multiple solutions to the demand and supply: borrowers only borrow from dealers when $z_B = z_r$, and lenders only lend to dealers when $z_L = z_o$.

$z_r < z_I < z_o$*, there exist unique distributions for repo borrowing and lending prices:*

$$
\begin{aligned}
F_B(z_B) &= \frac{\alpha}{2(1-\alpha)}\left(\frac{z_B - z_r}{z_I - z_B}\right), \\
&\quad \textit{with} \quad \mathcal{S}_B = \left[z_r, \left(1 - \frac{\alpha}{2-\alpha}\right) z_I + \frac{\alpha}{2-\alpha} z_r\right]; \qquad (58) \\
F_L(z_L) &= 1 - \frac{\alpha}{2(1-\alpha)} \frac{1/z_L - 1/z_o}{1/z_I - 1/z_L}, \\
&\quad \textit{with} \quad \mathcal{S}_L = \left[\left(\left(1 - \frac{\alpha}{2-\alpha}\right)\frac{1}{z_I} + \frac{\alpha}{2-\alpha}\frac{1}{z_o}\right)^{-1}, z_o\right]. \qquad (59)
\end{aligned}
$$

The repo price distributions above are almost identical to those in Propositions 1 and 2, except that the monopoly price for borrower dealers changes from $z_B = 1/A$ to $z_B = z_r$, and the monopoly price for lender dealers changes from $z_L = 1$ to $z_L = z_o$. For example, the borrower dealers' monopoly price z_r remains the lower bound of the support $\mathcal{S}_B$, and the upper bound of $\mathcal{S}_B$ is a convex combination of z_r and the competitive price z_I. In other words, by providing a direct conduit for customers to trade with the central bank, the borrowing and lending facilities alter the monopoly prices on both the borrowing and lending sides of the repo market, thereby reshaping the repo price distributions.

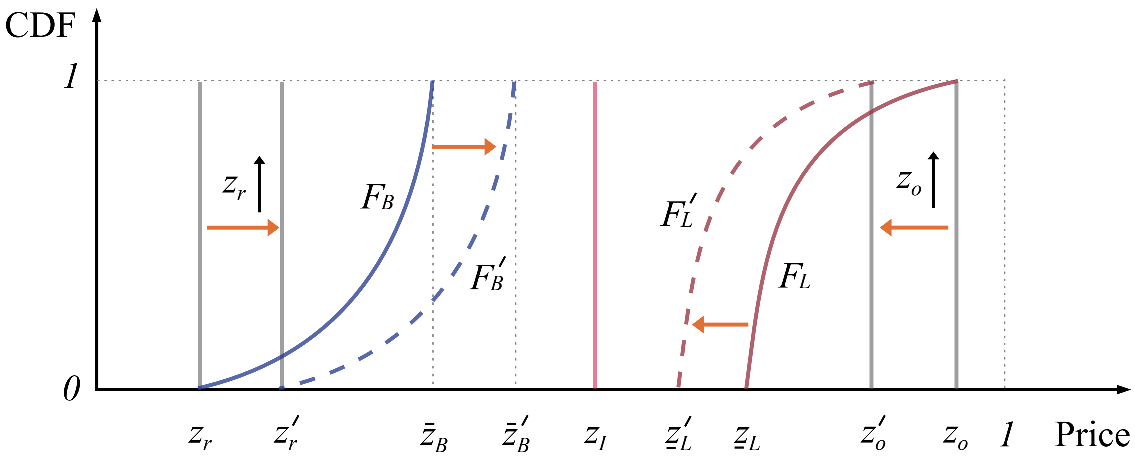
Under Proposition 6, the mean repo borrowing price μ_B, the gross inflation rate π, and the government bond price z_b can be written as

$$
\begin{aligned}
\mu_B &= z_I + \frac{\alpha}{2(1-\alpha)} \ln\left(\frac{\alpha}{2-\alpha}\right)(z_I - z_r), \qquad (60) \\
\pi &= \beta\left[\rho A + (1-\rho)\left(\alpha \frac{1}{z_o} + (1-\alpha)\frac{1}{z_I}\right)\right], \qquad (61) \\
z_b &= \frac{\beta}{\pi}\left[\rho A\left(\alpha z_r + (1-\alpha) z_I\right) + 1 - \rho\right]. \qquad (62)
\end{aligned}
$$

As before, these endogenous, market-determined prices are key to understanding the implications of central bank interventions.

Welfare Before delving into the details, let me define welfare as the sum of the net payoffs from economic activities with equally weighted agents. The virtue of linear utility

Figure 5: The Roles of Central Bank Lending and Borrowing Facilities



cancels out part of the utilities and disutilities, reducing total welfare to

$$\mathcal{W} \;=\; \rho \int (A-1)\, c_t\left(z_B\right) \mathrm{d}F_B\left(z_B\right) \;=\; \frac{\beta\rho\,(A-1)}{\pi}\left[\hat{m} + (f-\hat{m})\,\frac{\mu_B}{z_b}\right]. \qquad (63)$$

The first equality in (63) states that welfare represents the surplus from transactions in the trading market. This highlights the crucial role of the repo market in reallocating assets across repo customers, as the consumption level depends on the loans that repo borrowers can obtain from dealers (Lemma 2). The second equality is derived from the conditions in Definition 1, highlighting the central bank's critical role in steering market-determined prices (π, μ_B, z_b), which ultimately determine welfare.

6.1 Implications for Prices, Asset Allocation, and Welfare

As in Figure 5, raising the central bank's lending facility price z_r and lowering its borrowing facility price z_o shift the entire distributions of repo lending and borrowing prices toward the competitive price z_I that dealers would offer in the absence of search frictions. These shifts in the repo prices then lead to changes in other market-determined prices, such as the bond price, z_b, and the inflation rate, $\pi - 1$, as in the following proposition.

Proposition 7 (Implications of Lending and Borrowing Facilities for Prices)**.** *In equilib-*

rium with an active central bank deposit facility:

1. *An **increase** in the central bank's lending facility price z_r raises the ratio of the mean repo borrowing price μ_B to the bond price z_b, while leaving inflation unchanged, i.e.,*

$$\forall \alpha \in (0,1) \quad \frac{\mathrm{d}\pi}{\mathrm{d}z_r} = 0, \quad \frac{\mathrm{d}\left(\mu_B/z_b\right)}{\mathrm{d}z_r} > 0; \tag{64}$$

2. *A **decrease** in the central bank's borrowing facility price z_o raises both the relative price μ_B/z_b and the gross inflation rate, π, i.e.,*

$$\forall \alpha \in (0,1) \quad \frac{\mathrm{d}\pi}{\mathrm{d}z_o} < 0, \quad \frac{\mathrm{d}\left(\mu_B/z_b\right)}{\mathrm{d}z_o} < 0. \tag{65}$$

Corollary 2 (Asset Allocation)**.** *An increase in lending facility price or a decrease in the borrowing facility price raises the supply of money $\bar{m}$ and reduces the supply of central bank deposits $z_d\bar{d}$.*

An increase in the central bank's lending facility price, z_r, shifts the borrowing price distribution rightward, raising each borrowing price as well as the ratio of mean borrowing price to the bond price, μ_B/z_b. However, it does not affect inflation, which, as in Lemma 5, is exclusively determined by borrowers' payoff from using money in transactions and lenders' expected interest payoffs from lending. By contrast, a decrease in the borrowing facility price, z_o, shifts the lending price distribution to the left, increasing lenders' interest payoffs. This increase in the nominal payoff on money further implies higher inflation. The relative price μ_B/z_b also rises because higher inflation lowers the nominal bond price. Corollary 2 can be verified through (52), (53), and Proposition 7. I omit its intuition, as it is discussed under those earlier results.

Proposition 8 (Welfare Implications of Lending and Borrowing Facilities)**.** *An increase in the central bank's lending facility price improves welfare, whereas a decrease in its borrowing facility price reduces welfare.*

As the lending facility price increases, borrowers borrow more money against their collateral (Corollary 2). The increased money supply allows them to settle a larger volume of transactions in the trading market, generating a higher trading surplus and improving welfare. However, and perhaps counterintuitively, introducing the borrowing facility is harmful, even though it can effectively increase borrowers' money holdings for transactions. Borrowers' money holdings indeed have to increase, but so does the inflation (Proposition 7). The rise in inflation reduces the real value of money, thereby lowering the trading surplus and harming welfare.

6.2 Optimal Monetary Policy

Clearly, a benevolent central bank should rely on the lending facility but not the borrowing facility, as the latter raises inflation. The lending facility price should be set arbitrarily close to the deposit facility price, i.e., $z_r \longrightarrow z_d$, to achieve the efficient use of this facility. In the limit, conditions (60) to (62) reduce to

$$\mu_B = z_d, \tag{66}$$

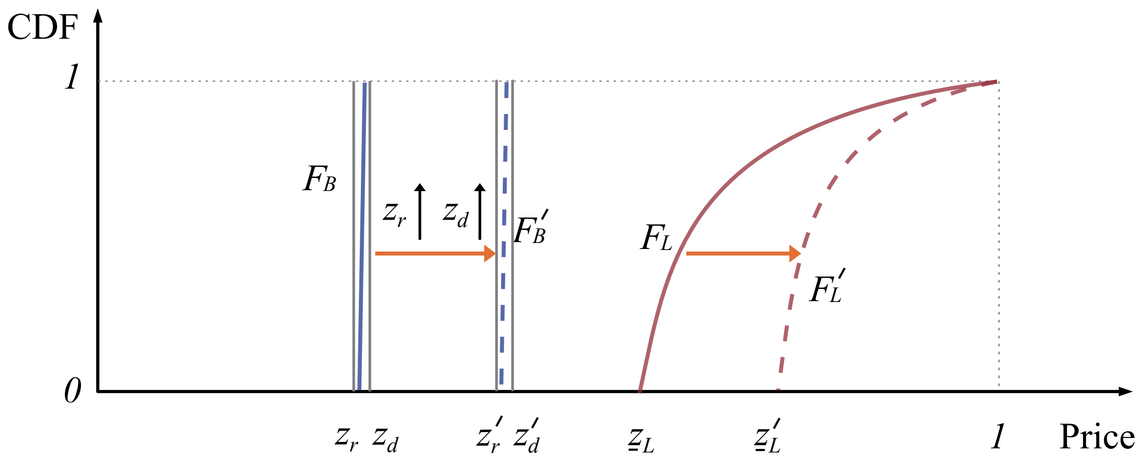
$$\pi = \beta \left[\rho A + (1-\rho) \left(\alpha + (1-\alpha) \frac{1}{z_d} \right) \right], \tag{67}$$

$$z_b = \frac{\beta}{\pi} \left[\rho A z_d + 1 - \rho \right]. \tag{68}$$

Graphically, as shown in Figure 6, the borrowing price distribution, F_B, converges to a degenerate distribution with $\mathbb{P}(z_B = z_d) = 1$. By eliminating dispersion in repo borrowing prices, the optimal monetary policy is to peg its nominal interest rates at the zero lower bound, such that $1/z_r - 1 = 1/z_d - 1 = 0$, in line with the Friedman rule.

Proposition 9 (Optimal Monetary Policy)**.** *When the lending facility price z_r approaches the deposit facility price z_d, i.e., $z_r \longrightarrow z_d$, raising these two prices together lowers inflation and increases the ratio of the mean repo borrowing price to the bond price μ_B/z_b,*

Figure 6: Price Distributions under the Optimal Lending Facility Price



thereby improving welfare, i.e.,

$$\frac{\mathrm{d}\pi}{\mathrm{d}z_d} < 0 \ \ \textit{and} \ \ \frac{\mathrm{d}\left(\mu_B/z_b\right)}{\mathrm{d}z_d} > 0 \quad \longrightarrow \quad \frac{\mathrm{d}\mathcal{W}}{\mathrm{d}z_d} > 0. \tag{69}$$

Therefore, the optimal monetary policy is to peg both the lending and deposit facility rates at the zero lower bound.

Raising the central bank's lending and deposit facility prices, or equivalently, lowering their rates, reduces the nominal returns that dealers earn on central bank deposits and that lenders earn on lending, thereby lowering inflation. Moreover, raising these facility prices also increases the price ratio of the mean repo borrowing price to the bond price, μ_B/z_b. As in Figure 6, $\mu_B = z_d$, implying perfect pass-through from the deposit facility price to the mean repo borrowing price, as if borrowing prices were set competitively. By contrast, equations (67) and (68) show that search frictions, captured by parameter α, generate an imperfect pass-through from the deposit facility price to bond price z_b. As a result, an increase in z_d always raises the price ratio μ_B/z_b.

Due to the two effects mentioned above, raising the central bank's lending and deposit facility prices jointly improves welfare. First, it reduces inflation, increasing the real value of money and enhancing its usefulness in transactions. Second, the increased price ratio μ_B/z_b allows borrowers, who need money to settle transactions, to borrow more against their government bonds. Therefore, the optimal monetary policy is to peg both the lending and deposit facility rates at the zero lower bound.

7 Conclusion

I develop a search-theoretic model that embeds Burdett and Judd (1983) pricing in OTC repo markets to rationalize repo price dispersion. I show that an increase in the central bank's deposit facility price exhibits imperfect pass-through to market-determined repo prices, and the pass-through effect weakens as the deposit facility price rises. I also find an ambiguous effect of the deposit facility price on asset allocation, in particular, on the composition of central bank liabilities. Rather than exercising caution, I suggest that the central bank pair the deposit facility with its lending facility, such as the Fed's repurchase agreement facility and the European Central Bank's main refinancing operations, to reallocate assets unambiguously. In doing so, the optimal policy is to peg both the lending and deposit facility rates to the zero lower bound, in the spirit of the Friedman rule. The borrowing facility, such as the Fed's overnight reverse repurchase agreement facility, can also effectively reallocate assets, but at the cost of raising inflation.

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Appendix

A Omitted Proofs

A.1 Proof of Lemma 1

Construct the following Lagrangian for the borrower dealer's problem (15),

$$L^{BD} \;=\; d_{BD} + \ell_B(z_B) - \ell_{BD} + \lambda_1^{BD}\left(z_I\ell_{BD} - z_B\ell_B(z_B) - z_d d_{BD}\right) + \lambda_2^{BD} d_{BD}, \tag{A.1}$$

where λ_1^{BD} and λ_2^{BD} are the Lagrange multipliers for the equality constraint and inequality constraint, respectively. The first-order conditions for the Lagrangian are

$$1 - \lambda_1^{BD} z_d + \lambda_2^{BD} \;=\; 0, \tag{A.2}$$

$$-1 + \lambda_1^{BD} z_I \;=\; 0, \tag{A.3}$$

where variables are also subject to the complementary slackness conditions

$$\lambda_2^{BD} d_{BD} = 0, \quad \lambda_2^{BD} \geq 0, \quad d_{BD} \geq 0. \tag{A.4}$$

It is then straightforward to show that $z_d \geq z_I$ and the equality holds if $d_{BD} > 0$.

When $d_{BD} = 0$, the equality constraint becomes

$$z_B\ell_B(z_B) \;=\; z_I\ell_{BD}, \tag{A.5}$$

so that

$$R_B\left(z_B\right) \;=\; \left(1 - \frac{z_B}{z_I}\right)\ell_B(z_B) \;=\; \left(\frac{1}{z_B} - \frac{1}{z_I}\right) z_B\ell_B(z_B) \tag{A.6}$$

This profit function also holds when $d^{BD} > 0$ because $z_I = z_d$ in this case.

Following the same procedure, the solution to the lender dealer's problem (18) gives a similar result, such that

$$z_d \geq z_I, \text{ with equality if } d_{LD} > 0, \quad \text{and} \quad R_L\left(z_L\right) \;=\; \left(\frac{1}{z_I} - \frac{1}{z_L}\right) z_L\ell_L(z_L). \tag{A.7}$$

Finally, aggregating all dealers' choices of central bank deposits implies that $z_d = z_I$ if $\bar{d} > 0$ in equilibrium. □

A.2 Proof of Lemma 2

Construct the following Lagrangian for the borrower's problem (23),

$$L^B = Ac_t + \frac{\beta\left(\tilde{m} + z_B\ell_B(z_B) - pc_t + \tilde{b} - \ell_B(z_B)\right)}{\pi} + \lambda_1^B pc_t + \lambda_2^B\left(\tilde{m} + z_B\ell_B(z_B) - pc_t\right). \tag{A.8}$$

The first-order condition is

$$A - \frac{\beta}{\pi}p + \lambda_1^B p - \lambda_2^B p \;=\; 0, \tag{A.9}$$

and variables are also subject to the following complementary slackness conditions

$$\lambda_1^B pc_t(z_B) = 0, \quad \lambda_1^B \geq 0, \quad pc_t(z_B) \geq 0; \tag{A.10}$$

$$\lambda_2^B\left(\tilde{m} + z_B\ell_B(z_B) - pc_t(z_B)\right) = 0, \quad \lambda_2^B \geq 0, \quad \tilde{m} + z_B\ell_B(z_B) - pc_t(z_B) \geq 0. \tag{A.11}$$

Similarly, construct the Lagrangian for the lender's problem (24), such that

$$L^L \;=\; -n_t + \frac{\beta\left(\tilde{m} - z_L\ell_L(z_L) + pn_t + \tilde{b} + \ell_L(z_L)\right)}{\pi} + \lambda^L n_t, \tag{A.12}$$

and solve for the first-order condition

$$-1 + \frac{\beta}{\pi}p + \lambda^L \;=\; 0, \tag{A.13}$$

as well as the associated complementary slackness conditions

$$\lambda^L n_t = 0, \quad \lambda^L \geq 0, \quad n_t \geq 0. \tag{A.14}$$

First, suppose that the non-negative constraint $pc_t(z_B) \geq 0$ binds so that $c_t(z_B) = 0$. This implies that $\lambda_2^B = 0$ under (A.11). However, when $\lambda_2^B = 0$, first-order conditions (A.9) and (A.13) give

$$\lambda_1^B p + \lambda^L = 1 - A < 0, \tag{A.15}$$

which contradicts to the complementary slackness conditions that require $\lambda_1^B, \lambda^L \geq 0$. As a result, there is always a positive consumption, i.e., $c_t(z_B) > 0$, as well as a positive labor supply, i.e., $n_t > 0$, under the market clearing condition (25). The associated Lagrange multipliers equal to zero, and substituting these multipliers $\lambda_1^B = 0$ and $\lambda^L = 0$ into (A.9) and (A.13) gives

$$p \;=\; \frac{\pi}{\beta}, \tag{A.16}$$

$$A - 1 \;=\; \lambda_2^B p > 0. \tag{A.17}$$

Finally, from (A.11), the fact that $\lambda_2^B > 0$ implies the binding cash constraint that solves

$$c_t(z_B) \;=\; \frac{\beta\left(\tilde{m} + z_B \ell_B(z_B)\right)}{\pi}. \tag{A.18}$$

□

A.3 Proof of Lemma 3

Borrowers' loan demand is jointly determined by the envelope condition from their trading market problem and the first-order condition from their funding market problem. The Lagrangian for the borrower's trading market problem (A.8) gives

$$\frac{\partial}{\partial \ell_B(z_B)} W^B\left(\tilde{m} + z_B \ell_B(z_B), \tilde{b}, \ell_B(z_B)\right) \;=\; \frac{\beta\left(z_B - 1\right)}{\pi} + \lambda_2^B z_B \;=\; \frac{\beta\left(A z_B - 1\right)}{\pi}, \tag{A.19}$$

where the first equation is an immediate result following the Envelope Theorem and the second one follows from equation (A.17). The first-order condition for the borrower's funding market problem (10) is

$$\frac{\partial}{\partial \ell_B(z_B)} W^B\left(\tilde{m} + z_B \ell_B(z_B), \tilde{b}, \ell_B(z_B)\right) \;=\; \lambda_W^B, \tag{A.20}$$

where λ_W^B is the Lagrange multiplier for the collateral constraint $\tilde{b} \geq \ell_B$, so that

$$\lambda_W^B\left(\tilde{b} - \ell_B(z_B)\right) = 0, \quad \lambda_W^B \geq 0, \quad \tilde{b} - \ell_B(z_B) \geq 0. \tag{A.21}$$

The fact that

$$\lambda_W^B \;=\; \frac{\beta\left(A z_B - 1\right)}{\pi} \tag{A.22}$$

in equilibrium implies a binding collateral constraint when $z_B > 1/A$. By contrast, the constraint constraint does not bind when $z_B = 1/A$. □

A.4 Proof of Lemma 4

Lenders' supply of loans is jointly determined by the envelope condition from their trading market problem and the first-order condition for the funding market problem. The Lagrangian for the lender's trading market problem (A.12) gives

$$\frac{\partial}{\partial \ell_L(z_L)} W^L\left(\tilde{m} - z_L \ell_L(z_L), \tilde{b}, \ell_L(z_L)\right) = \frac{\beta\left(-z_L + 1\right)}{\pi}, \tag{A.23}$$

an immediate result following the Envelope Theorem. The first-order condition for the lender's funding market problem (12) is

$$\frac{\partial}{\partial \ell_L(z_L)} W^L\left(\tilde{m} - z_L \ell_L(z_L), \tilde{b}, \ell_L(z_L)\right) = z_L \lambda_W^L, \tag{A.24}$$

where λ_W^L is the Lagrange multiplier for the cash constraint $\tilde{m} \geq z_L \ell_L$, so that

$$\lambda_W^L\left(\tilde{m} - z_L \ell_L(z_L)\right) = 0, \quad \lambda_W^L \geq 0, \quad \tilde{m} - z_L \ell_L(z_L) \geq 0. \tag{A.25}$$

The fact that

$$z_L \lambda_W^L = \frac{\beta\left(-z_L + 1\right)}{\pi}, \tag{A.26}$$

implies a binding cash constraint with $\ell_L(z_L) = \tilde{m}/z_L$ when $z_L < 1$. By contrast, this constraint does not bind when $z_L = 1$. □

A.5 Proof of Lemma 5

The Lagrangian for the borrower's trading market problem (A.8) gives

$$\frac{\partial}{\partial \tilde{m}} W^B\left(\tilde{m} + z_B \ell_B(z_B), \tilde{b}, \ell_B(z_B)\right) = \frac{\beta}{\pi} + \lambda_2^B = \frac{\beta}{\pi} A \tag{A.27}$$

$$\frac{\partial}{\partial \tilde{b}} W^B\left(\tilde{m} + z_B \ell_B(z_B), \tilde{b}, \ell_B(z_B)\right) = \frac{\beta}{\pi}, \tag{A.28}$$

an immediate result following the Envelope Theorem and equation (A.17). Similarly, the envelope conditions for the lender's problem (A.12) gives

$$\frac{\partial}{\partial \tilde{m}} W^L\left(\tilde{m} - z_L \ell_L(z_L), \tilde{b}, \ell_L(z_L)\right) = \frac{\partial}{\partial \tilde{b}} W^L\left(\tilde{m} - z_L \ell_L(z_L), \tilde{b}, \ell_L(z_L)\right) = \frac{\beta}{\pi}. \tag{A.29}$$

These envelope conditions on W^L and W^B help to determine the envelope conditions of the constrained optimization problems (10) and (12), such that

$$\frac{\partial}{\partial \tilde{m}} V^B\left(\tilde{m}, \tilde{b}\right) = A\frac{\beta}{\pi}, \tag{A.30}$$

$$\frac{\partial}{\partial \tilde{b}} V^B\left(\tilde{m}, \tilde{b}\right) = A\frac{\beta}{\pi}\left(\alpha \int z_B \mathrm{d}F_B\left(z_B\right) + (1-\alpha)\int z_B \mathrm{d}\left[F_B\left(z_B\right)\right]^2\right), \tag{A.31}$$

$$\frac{\partial}{\partial \tilde{m}} V^L\left(\tilde{m}, \tilde{b}\right) = \frac{\beta}{\pi}\left(\alpha \int \frac{1}{z_L} \mathrm{d}F_L\left(z_L\right) + (1-\alpha)\int \frac{1}{z_L} \mathrm{d}\left(1-\left[1-F_L\left(z_L\right)\right]^2\right)\right), \tag{A.32}$$

$$\frac{\partial}{\partial \tilde{b}} V^L\left(\tilde{m}, \tilde{b}\right) = \frac{\beta}{\pi}. \tag{A.33}$$

Substituting these conditions into the first-order conditions (8) and (9) gives

$$1 = \frac{\beta}{\pi}\left[\rho A + (1-\rho)\left(\alpha \int \frac{1}{z_L} \mathrm{d}F_L\left(z_L\right) + (1-\alpha)\int \frac{1}{z_L} \mathrm{d}\left(1-\left[1-F_L\left(z_L\right)\right]^2\right)\right)\right], \tag{A.34}$$

$$z_b = \frac{\beta}{\pi}\left[\rho A\left(\alpha \int z_B \mathrm{d}F_B\left(z_B\right) + (1-\alpha)\int z_B \mathrm{d}\left[F_B\left(z_B\right)\right]^2\right) + 1 - \rho\right]. \tag{A.35}$$

□

A.6 Proof of Lemma 6

Borrower dealers' profit per borrower served is

$$R_B\left(z_B\right) = \left(1 - \frac{z_B}{z_I}\right)\tilde{b} \quad \forall z_B \in [\frac{1}{A}, 1], \tag{A.36}$$

which is strictly decreasing in z_B given that

$$\frac{\mathrm{d}}{\mathrm{d}z_B} R_B\left(z_B\right) = -\frac{\tilde{b}}{z_I} < 0. \tag{A.37}$$

Therefore, the monopoly price is $z_B = 1/A$, which yields a nonnegative profit if and only if $z_I \geq 1/A$. □

A.7 Proof of Proposition 1

This proposition builds on the case in which borrower dealers could earn positive monopoly profits, such that $z_I > 1/A$. I prove this proposition by establishing the following lemmas regarding the continuity, connectedness, and boundary of the distribution F_B. The solution of F_B is an immediate result under these lemmas.

Lemma A.1. *F_B is continuous on $\mathcal{S}_B$.*

Proof. Suppose the contradictory, assume $\exists z \in \mathcal{S}_B$ such that $\xi_B(z) = \lim_{\epsilon\to 0^+} F_B(z) - F_B(z-\epsilon) > 0$, and

$$\Pi_B(z) \;=\; \Pi_B^\star \;=\; \lim_{\epsilon\to 0^+} \frac{\rho}{s}\left(\alpha + (1-\alpha)\left[F_B(z) + F_B(z-\epsilon)\right]\right) R_B(z) > 0. \tag{A.38}$$

Notice that the dealer's profit per borrower R_B is continuous. Therefore, there exists $z' > z$ such that $R_B(z') > 0$ and $\Delta \equiv R_B(z) - R_B(z') < \frac{(1-\alpha)\xi_B(z)R_B(z)}{\alpha+2(1-\alpha)F_B(z)}$. Then,

$$\begin{aligned}
\Pi_B(z') \;&=\; \lim_{\epsilon\to 0^+} \frac{\rho}{s}\left(\alpha + (1-\alpha)\left[F_B(z') + F_B(z'-\epsilon)\right]\right) R_B(z') \\
&\geq \lim_{\epsilon\to 0^+} \frac{\rho}{s}\left(\alpha + (1-\alpha)\left[F_B(z) + F_B(z-\epsilon) + \xi_B(z)\right]\right)\left(R_B(z) - \Delta\right) \\
&= \Pi_B(z) + \frac{\rho}{s}\left((1-\alpha)\,\xi_B(z)\,R_B(z) - \left[\,\alpha + 2(1-\alpha)\,F_B(z)\,\right]\Delta\right),
\end{aligned} \tag{A.39}$$

where the inequality holds because $F_B(z') \geq F_B(z)$ and $\lim_{\epsilon\to 0^+} F_B(z'-\epsilon) - F_B(z-\epsilon) \geq \xi_B(z)$. This further implies

$$\Pi_B(z') - \Pi_B(z) \geq \frac{\rho}{s}\left((1-\alpha)\,\xi_B(z)\,R_B(z) - \left[\,\alpha + 2(1-\alpha)\,F_B(z)\,\right]\Delta\right) > 0, \tag{A.40}$$

where the last inequality holds by the definition of Δ. The fact that $\Pi_B(z') > \Pi_B(z)$ contradicts with $z \in \mathcal{S}_B$. Therefore, F_B must be continuous on its support $\mathcal{S}_{\mathcal{B}}$. $\square$

Given Lemma A.1, dealers' profit function can be rewritten as

$$\Pi_B^\star \;=\; \Pi_B(z_B) \;=\; \frac{\rho}{s}\left[\,\alpha + 2(1-\alpha)\,F_B(z_B)\,\right] R_B(z_B). \tag{A.41}$$

Lemma A.2. *The monopoly price $z_B = 1/A$ is the lowest price in $\mathcal{S}_B$.*

Proof. Suppose that $z \neq 1/A$ is the lowest price in $\mathcal{S}_B$. Then,

$$\Pi_B(z) \;=\; \frac{\rho}{s}\alpha R_B(z). \tag{A.42}$$

But now,

$$\Pi_B\left(\frac{1}{A}\right) = \frac{\rho}{s}\left[\alpha + 2(1-\alpha)\,F_B\left(\frac{1}{A}\right)\right] R_B\left(\frac{1}{A}\right) \geq \frac{\rho}{s}\alpha R_B\left(\frac{1}{A}\right) > \Pi_B(z). \tag{A.43}$$

This is a contradiction. □

Lemma A.3. $\mathcal{S}_B$ *is connected.*

Proof. Suppose that $z, z' \in \mathcal{S}_B$, such that $z < z'$ and $F_B(z) = F_B(z')$. Therefore,

$$\alpha + 2(1-\alpha) F_B(z) = \alpha + 2(1-\alpha) F_B(z'), \tag{A.44}$$

which further implies that

$$\Pi_B(z') < \Pi_B(z), \tag{A.45}$$

given that $R_B(z_B)$ is strictly decreasing in z_B for all $z_B \in [1/A, 1]$. This contradicts to $z, z' \in \mathcal{S}_B$ that requires $\Pi_B(z') = \Pi_B(z)$. □

The total profit is maximized at the monopoly price $1/A$ with

$$\Pi_B^\star = \frac{\rho}{s} \alpha R_B\left(\frac{1}{A}\right). \tag{A.46}$$

By equal profit among prices in the connected support, I obtain

$$\alpha R_B\left(\frac{1}{A}\right) = \left[\alpha + 2(1-\alpha) F_B(z_B)\right] R_B(z_B), \tag{A.47}$$

which solves

$$F_B(z_B) = \frac{\alpha}{2(1-\alpha)}\left(\frac{R_B\left(\frac{1}{A}\right)}{R_B(z_B)} - 1\right) \tag{A.48}$$

$$= \frac{\alpha}{2(1-\alpha)}\left(\frac{z_B - \frac{1}{A}}{z_I - z_B}\right) \quad \forall z_B \in \mathcal{S}_B. \tag{A.49}$$

Moreover, the upper bound $\bar{z}_B$ solves

$$R_B(\bar{z}_B) = \frac{\alpha}{2-\alpha} R_B\left(\frac{1}{A}\right), \tag{A.50}$$

so that

$$\bar{z}_B = \left(1 - \frac{\alpha}{2-\alpha}\right) z_I + \frac{\alpha}{2-\alpha}\frac{1}{A}. \tag{A.51}$$

□

A.8 Proof of Lemma 7

Lender dealers' profit per lender served is

$$R_L(z_L) = \left(\frac{z_L}{z_I} - 1\right)\frac{\tilde{m}}{z_L} \tag{A.52}$$

which is strictly increasing in z_L given that

$$\frac{\mathrm{d}}{\mathrm{d}z_L} R_L(z_L) = \frac{\tilde{m}}{(z_L)^2} > 0. \tag{A.53}$$

Therefore, their monopoly loan price, which yields the highest profit, is $z_L = 1$. Under $z_d \leq 1$, the monopoly profit is nonnegative, given that $z_I \leq z_d$ (Lemma 1), and the zero profit occurs if and only if $z_I = z_d = 1$. □

A.9 Proof of Proposition 2

As with the borrower dealer's problem, this proposition builds on the case in which lender dealers could earn positive monopoly profits, such that $z_I < 1$. I also prove this proposition by establishing the following lemmas regarding the continuity, connectedness, and boundary of the distribution F_L before solving for F_L.

Lemma A.4. *F_L is continuous on $\mathcal{S}_L$.*

Proof. Suppose the contradictory, assume $\exists z \in \mathcal{S}_L$ such that $\xi_L(z) = \lim_{\epsilon \to 0^+} F_L(z) - F_L(z-\epsilon) > 0$, and

$$\Pi_L(z) = \Pi_L^\star = \lim_{\epsilon \to 0^+} \frac{1-\rho}{s}\left[\alpha + (1-\alpha)\left(2 - [F_L(z) + F_L(z-\epsilon)]\right)\right] R_L(z) > 0. \tag{A.54}$$

The fact that R_L is a continuous function implies the existence of $z' < z$ such that $R_L(z') > 0$ and $\Delta \equiv R_L(z) - R_L(z') < \frac{(1-\alpha)\xi_L(z) R_L(z)}{\alpha + 2(1-\alpha)[1-F_L(z)+\xi_L(z)]}$. Then,

$$\begin{aligned}
\Pi_L(z') &= \lim_{\epsilon \to 0^+} \frac{1-\rho}{s}\left[\alpha + (1-\alpha)\left(2 - [F_L(z') + F_L(z'-\epsilon)]\right)\right] R_L(z') \\
&\geq \lim_{\epsilon \to 0^+} \frac{1-\rho}{s}\left[\alpha + (1-\alpha)\left(2 - [F_L(z) - \xi_L(z) + F_L(z-\epsilon)]\right)\right] (R_L(z) - \Delta) \\
&= \Pi_L(z) + \frac{1-\rho}{s}\left((1-\alpha)\xi_L(z) R_L(z) - [\alpha + 2(1-\alpha)(1 - F_L(z) + \xi_L(z))]\Delta\right),
\end{aligned} \tag{A.55}$$

where the inequality holds because $F_L(z) - \xi_L(z) \geq F_L(z')$ and $\lim_{\epsilon \to 0^+} F(z-\epsilon) \geq$

$\lim_{\epsilon\to 0^+} F_L(z'-\epsilon)$. This further implies that

$$\Pi_L(z') - \Pi_L(z) \geq \\ \frac{1-\rho}{s}\left(\left(1-\alpha\right)\xi_L(z)\,R_L(z) - \left[\,\alpha + 2\left(1-\alpha\right)\left(1-F_L(z)+\xi_L(z)\right)\right]\Delta\,\right) > 0 \tag{A.56}$$

where the last inequality holds by the definition of Δ. The fact that $\Pi_L(z') > \Pi_L(z)$ contradicts with $z \in \mathcal{S}_L$. This establishes the Lemma. $\square$

Given Lemma A.4, dealers' profit function can be rewritten as

$$\Pi_L^\star \;=\; \Pi_L(z_L) \;=\; \frac{1-\rho}{s}\left[\,\alpha + 2\left(1-\alpha\right)\left(1-F_L(z_L)\right)\right] R_L(z_L)\,. \tag{A.57}$$

Lemma A.5. *The monopoly price $z_L = 1$ is the highest price in $\mathcal{S}_L$.*

Proof. Suppose that $z \neq 1$ is the highest price in $\mathcal{S}_L$. Then,

$$\Pi_L(z) \;=\; \frac{1-\rho}{s}\alpha R_L(z)\,. \tag{A.58}$$

But now,

$$\Pi_L(1) = \frac{1-\rho}{s}\left[\,\alpha + 2\left(1-\alpha\right)\left(1-F_L(1)\right)\right] R_L(1) \geq \frac{1-\rho}{s}\alpha R_L(1) > \Pi_L(z)\,. \tag{A.59}$$

This is a contradiction. $\square$

Lemma A.6. *$\mathcal{S}_L$ is connected.*

Proof. Suppose that $z, z' \in \mathcal{S}_L$, such that $z < z'$ and $F_L(z) = F_L(z')$. Therefore,

$$\alpha + 2\left(1-\alpha\right)\left(1-F_L(z)\right) \;=\; \alpha + 2\left(1-\alpha\right)\left(1-F_L(z')\right), \tag{A.60}$$

which further implies that

$$\Pi_L(z') > \Pi_L(z)\,, \tag{A.61}$$

given that $R_L(z)$ is strictly increasing in z for all $z \in (0,1]$. This contradicts to $z, z' \in \mathcal{S}_L$ that requires $\Pi_L(z') = \Pi_L(z)$. $\square$

At the monopoly price $z_L = 1$, profit is maximized with

$$\Pi_L^\star \;=\; \frac{1-\rho}{s}\alpha R_L(1)\,. \tag{A.62}$$

By equal profit among prices in the connected support, I obtain

$$\alpha R_L(1) \;=\; [\,\alpha + 2(1-\alpha)(1 - F_L(z_L))\,] R_L(z_L), \tag{A.63}$$

which solves

$$\begin{aligned} F_L(z_L) \;&=\; 1 - \frac{\alpha}{2(1-\alpha)} \left(\frac{R_L(1)}{R_L(z_L)} - 1 \right) & \text{(A.64)} \\ &=\; 1 - \frac{\alpha}{2(1-\alpha)} \frac{z_I(1-z_L)}{z_L - z_I} \quad \forall z_L \in \mathcal{S}_L. & \text{(A.65)} \end{aligned}$$

Moreover, the lower bound $\underline{z}_L$ solves

$$R_L(\underline{z}_L) \;=\; \frac{\alpha}{2-\alpha} R_L(1), \tag{A.66}$$

so that

$$\underline{z}_L \;=\; \left(\frac{2(1-\alpha)}{2-\alpha} \frac{1}{z_I} + \frac{\alpha}{2-\alpha} \right)^{-1} \;=\; \left[\left(1 - \frac{\alpha}{2-\alpha} \right) \frac{1}{z_I} + \frac{\alpha}{2-\alpha} \right]^{-1}. \tag{A.67}$$

$\square$

A.10 Proof of Proposition 3

Confine attention to the case with inter-dealer price $z_I \in (1/A, 1)$. Rewrite equilibrium condition (42) as

$$\frac{z_b z_d}{f} \bar{d} \;=\; z_b (1-\rho) \frac{\hat{m}}{f} - \rho \left(1 - \frac{\hat{m}}{f} \right) \mu_B. \tag{A.68}$$

Plugging in the value of bond price in (44) and $\theta \equiv \hat{m}/f \in (0,1)$ gives

$$\begin{aligned} \left[\rho A + (1-\rho) \left(\alpha + \frac{1-\alpha}{z_I} \right) \right] \frac{z_b z_d}{f} \bar{d} \;=\; & \left[\rho A \left((1-\alpha) z_I + \frac{\alpha}{A} \right) + 1 - \rho \right] (1-\rho)\theta \\ & - \rho \left[\rho A + (1-\rho) \left(\alpha + \frac{1-\alpha}{z_I} \right) \right] \mu_B (1-\theta). \end{aligned} \tag{A.69}$$

The last equation can be further rewritten as

$$[\rho A z_I + (1-\rho)(\alpha z_I + 1 - \alpha)] \frac{z_b z_d}{f} \bar{d} \;=\; G(z_I)\theta - H(z_I)(1-\theta), \tag{A.70}$$

where

$$G(z_I) = \left[\rho A\left((1-\alpha) z_I^2 + \frac{\alpha}{A} z_I\right) + (1-\rho) z_I\right](1-\rho) \tag{A.71}$$

and

$$H(z_I) = \rho\left[\rho A z_I + (1-\rho)(\alpha z_I + 1 - \alpha)\right]\mu_B \tag{A.72}$$

are both quadratic equations because, in particular, μ_B is linear in z_I (equation 43).

Note that both $G(z_I)$ and $H(z_I)$ are positive and bounded on the interval $(1/A, 1)$. Consequently, the right-hand side of equation (A.70) is positive once the ratio θ is sufficiently large. This implies the existence of a threshold $\bar{\theta}$ such that, for all $\theta > \bar{\theta}$, equation (A.70) solves for a positive value of central bank deposits, $\bar{d} > 0$, thereby an equilibrium with an active central bank deposit facility. $\square$

A.11 Proof of Proposition 4

For any $z_d \in (1/A, 1)$ and $\theta \in (\bar{\theta}, 1)$, the central bank's deposit facility is active (Proposition 3). In this scenario, the inter-dealer price is determined by the deposit facility price, such that $z_I = z_d$. Plugging in $z_I = z_d$ and totally differentiating equation (48) with respect to z_d give

$$\frac{\alpha}{2(1-\alpha)} \frac{\frac{\mathrm{dz_B^q}}{dz_d}(z_d - z_B^q) - \left(z_B^q - \frac{1}{A}\right)\left(1 - \frac{\mathrm{dz_B^q}}{dz_d}\right)}{(z_d - z_B^q)^2} = 0 \tag{A.73}$$

$$\frac{\mathrm{dz_B^q}}{dz_d}(z_d - z_B^q) - \left(z_B^q - \frac{1}{A}\right) + \frac{\mathrm{dz_B^q}}{dz_d}\left(z_B^q - \frac{1}{A}\right) = 0 \tag{A.74}$$

$$\frac{\mathrm{dz_B^q}}{dz_d}\left(z_d - \frac{1}{A}\right) = \left(z_B^q - \frac{1}{A}\right). \tag{A.75}$$

The last equation solves

$$0 \leq \eta_B^q \equiv \frac{\mathrm{dz_B^q}}{dz_d} = \frac{z_B^q - \frac{1}{A}}{z_d - \frac{1}{A}} < 1, \tag{A.76}$$

where η_B^q captures the effectiveness of monetary policy pass-through. The first inequality holds with strictly inequality unless $q = 0$ and $z_B^0 = 1/A$, and the second inequality holds for any $q \in [0, 1]$ because, from (33), $z_d = z_I > \bar{z}_B > z_B^q$.

Similarly, plugging in $z_I = z_d$ and totally differentiating equation (49) with respect to

z_d give

$$-\frac{\alpha}{2(1-\alpha)}\frac{1}{\left(z_L^q-z_d\right)^2}\left[\left(1-z_L^q-z_d\frac{\mathrm{dz_L^q}}{dz_d}\right)\left(z_L^q-z_d\right)-z_d\left(1-z_L^q\right)\left(\frac{\mathrm{dz_L^q}}{dz_d}-1\right)\right]=0, \tag{A.77}$$

which solves

$$\eta_L^q \;\equiv\; \frac{\mathrm{dz_L^q}}{dz_d} \;=\; \frac{z_L^q\left(1-z_L^q\right)}{z_d\left(1-z_d\right)} \geq 0, \tag{A.78}$$

with equality only if $q=1$ and $z_L^1=1$. Again, η_L^q captures the effectiveness of monetary policy pass-through, and the pass-through is imperfect if

$$\eta_L^q \;\equiv\; \frac{z_L^q\left(1-z_L^q\right)}{z_d\left(1-z_d\right)} < 1 \longleftrightarrow \left(z_d+z_L^q-1\right)\left(z_d-z_L^q\right)<0. \tag{A.79}$$

The fact that $z_L^q > z_I = z_d$ holds for all $q \in [0,1]$ helps to reduce the last condition to

$$z_L^q \;>\; 1-z_d. \tag{A.80}$$

Therefore, the monetary policy pass-through is imperfect for $z_L \in (1-z_d, 1]$. In particular, when $z_d \geq 1/2$, the last inequality is always satisfied. □

A.12 Proof of Corollary 1

In the case when the central bank's deposit facility is active, $z_I = z_d$. By the definition of the quantile function (48),

$$z_B^q \;=\; \frac{1}{\frac{\alpha}{2(1-\alpha)}+q}\left[qz_d+\frac{\alpha}{2(1-\alpha)}\frac{1}{A}\right], \tag{A.81}$$

which is integrable on $(0,1)$. The mean borrowing price is therefore

$$\mu_B \;=\; \int_0^1 z_B^q \mathrm{d}q. \tag{A.82}$$

Differentiating with respect to z_d gives

$$\frac{\mathrm{d}\mu_B}{\mathrm{d}z_d} \;=\; \int_0^1 \frac{\mathrm{d}z_B^q}{\mathrm{d}z_d}\mathrm{d}q<1 \quad \text{given that} \quad \forall q\in[0,1]\,,\ \eta_B^q\equiv\frac{\mathrm{dz_B^q}}{dz_d}<1. \tag{A.83}$$

Following the same procedure, I can show

$$\frac{\mathrm{d}\mu_L}{\mathrm{d}z_d} = \int_0^1 \frac{\mathrm{d}z_L^q}{\mathrm{d}z_d}\mathrm{d}q < 1 \tag{A.84}$$

holds under the condition $z_d \geq \frac{1}{2}$. □

A.13 Proof of Lemma 8

Consider equilibria with an active deposit facility with $z_I = z_d$. For repo borrowing prices, the quantile function (48) gives

$$z_B^q = \frac{1}{\frac{\alpha}{2(1-\alpha)} + q}\left[q z_d + \frac{\alpha}{2(1-\alpha)}\frac{1}{A}\right]. \tag{A.85}$$

Plugging it into $\eta_B^q = \frac{z_B^q - \frac{1}{A}}{z_d - \frac{1}{A}}$ gives

$$\eta_B^q = \frac{q\left(z_d - \frac{1}{A}\right)}{\left(\frac{\alpha}{2(1-\alpha)} + q\right)\left(z_d - \frac{1}{A}\right)} = \frac{2(1-\alpha)q}{\alpha + 2(1-\alpha)q}. \tag{A.86}$$

An increase in $\alpha \in (0,1)$ means a larger search friction and gives the following relation

$$\frac{\mathrm{d}\eta_B^q}{\mathrm{d}\alpha} = -\frac{2q}{\left[\alpha + 2q(1-\alpha)\right]^2} \leq 0, \tag{A.87}$$

where the equality only if $q = 0$. Moreover, for all $q \in [0,1]$, $\eta_B^q \longrightarrow 1$ when $\alpha \longrightarrow 0$, and $\eta_B^q \longrightarrow 0$ when $\alpha \longrightarrow 1$.

For repo lending prices, the quantile function (49) yields,

$$z_L^q = \frac{\frac{\alpha}{2(1-\alpha)} + (1-q)}{\frac{\alpha}{2(1-\alpha)}z_d + (1-q)} z_d. \tag{A.88}$$

Substituting into $\eta_L^q = \frac{z_L^q\left(1-z_L^q\right)}{z_d(1-z_d)}$ gives

$$\eta_L^q = \frac{\left[\frac{\alpha}{2(1-\alpha)} + (1-q)\right](1-q)}{\left[\frac{\alpha}{2(1-\alpha)}z_d + (1-q)\right]^2} = \frac{\left[2\alpha(1-\alpha) + 4(1-\alpha)^2(1-q)\right](1-q)}{\left[\alpha z_d + 2(1-\alpha)(1-q)\right]^2}. \tag{A.89}$$

Let

$$t = \frac{\alpha}{2(1-\alpha)}. \tag{A.90}$$

Then,

$$\frac{\mathrm{d}\eta_L^q}{\mathrm{d}t} = \frac{-t(1-q)z_d + (1-q)^2(1-2z_d)}{[tz_d + (1-q)]^3} \leq 0 \quad \text{when} \quad z_d \geq \frac{1}{2}. \tag{A.91}$$

By the chain rule,

$$\frac{\mathrm{d}\eta_L^q}{\mathrm{d}\alpha} = \frac{\mathrm{d}\eta_L^q}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}\alpha} \leq 0, \tag{A.92}$$

with the equality only if $q = 1$ and $z_d = 1/2$. As with borrowing prices, $\eta_L^q \longrightarrow 1$ when $\alpha \longrightarrow 0$, and $\eta_L^q \longrightarrow 0$ when $\alpha \longrightarrow 1$. □

A.14 Proof of Proposition 5

Consider the case when $z_d \in (1/A, 1)$ and $\theta \in (\bar{\theta}, 1)$, so that the central bank's deposit facility is active (Proposition 3) with $z_I = z_d$. First, an increase in z_d also has no impact on η_B^q, given that

$$\frac{\mathrm{d}\eta_{\mathrm{B}}^{\mathrm{q}}}{dz_d} = \frac{\frac{\mathrm{dz}_{\mathrm{B}}^{\mathrm{q}}}{dz_d}\left(z_d - \frac{1}{A}\right) - \left(z_B^q - \frac{1}{A}\right)}{\left(z_d - \frac{1}{A}\right)^2} = \frac{\left(z_B^q - \frac{1}{A}\right) - \left(z_B^q - \frac{1}{A}\right)}{\left(z_d - \frac{1}{A}\right)^2} = 0. \tag{A.93}$$

However, an increase in z_d lowers η_L^q because

$$\begin{aligned}
\frac{\mathrm{d}\eta_{\mathrm{L}}^{\mathrm{q}}}{dz_d} &= \frac{z_d(1-z_d)(1-2z_L^q)\frac{\mathrm{dz}_{\mathrm{L}}^{\mathrm{q}}}{dz_d} - z_L^q(1-z_L^q)(1-2z_d)}{[z_d(1-z_d)]^2} && \text{(A.94)}\\
&= \frac{(1-2z_L^q)z_L^q(1-z_L^q) - z_L^q(1-z_L^q)(1-2z_d)}{[z_d(1-z_d)]^2} && \text{(A.95)}\\
&= -\frac{2z_L^q(1-z_L^q)(z_L^q - z_d)}{[z_d(1-z_d)]^2} \leq 0, && \text{(A.96)}
\end{aligned}$$

given that $z_L^q > z_I = z_d$. The equality holds only if $z_L^q = 1$. □

A.15 Proof of Lemma 9

For any $z_d \in (1/A, 1)$ and $\theta \in (\bar{\theta}, 1)$, the central bank's deposit facility is active (Proposition 3). In this scenario, the inter-dealer price is determined by the deposit facility price, such that $z_I = z_d$. Applying L'Hôpital's rule to equation (43), I obtain

$$\begin{aligned}\lim_{\alpha \to 0} \mu_B &= \lim_{\alpha \to 0} z_d + \frac{\alpha}{2(1-\alpha)} \ln\left(\frac{\alpha}{2-\alpha}\right)\left(z_d - \frac{1}{A}\right) \\ &= \lim_{\alpha \to 0} z_d - \frac{\alpha}{2-\alpha}\left(z_d - \frac{1}{A}\right) = z_d;\end{aligned} \tag{A.97}$$

$$\begin{aligned}\lim_{\alpha \to 1} \mu_B &= \lim_{\alpha \to 1} z_d + \frac{\alpha}{2(1-\alpha)} \ln\left(\frac{\alpha}{2-\alpha}\right)\left(z_d - \frac{1}{A}\right) \\ &= \lim_{\alpha \to 1} z_d - \frac{\alpha}{2-\alpha}\left(z_d - \frac{1}{A}\right) = \frac{1}{A}.\end{aligned} \tag{A.98}$$

From (44) and (45),

$$z_b = \frac{\rho A\left((1-\alpha) z_d + \frac{\alpha}{A}\right) + 1 - \rho}{\rho A + (1-\rho)\left(\alpha + \frac{1-\alpha}{z_d}\right)}. \tag{A.99}$$

As a result,

$$\lim_{\alpha \to 0} z_b = z_d \quad \lim_{\alpha \to 1} z_b = \frac{1}{\rho A + 1 - \rho} \tag{A.100}$$

In either case, the price ratio is constant in z_d, so that

$$\frac{\mathrm{d}\left(\mu_B / z_b\right)}{\mathrm{d} z_d} = 0. \tag{A.101}$$

□

A.16 Proof of Proposition 6

Borrowing Price Distribution (F_B) I finish this proof with the same procedure as in the proof of Proposition 1. From (56), the monopoly price for borrower dealer now becomes $z_B = z_r$, which gives a positive monopoly profit when $z_I > z_r$. Under $z_I > z_r$, I can also show that, after introducing the central bank's lending facility,

1. F_B is continuous on its support $\mathcal{S}_B$;

2. The monopoly price $z_B = z_r$ is the lowest price in $\mathcal{S}_B$;

3. $\mathcal{S}_B$ is connected.

Borrower dealers' total profit can be rewritten as

$$\Pi_B^\star \;=\; \Pi_B\left(z_B\right) \;=\; \frac{\rho}{s}\left[\,\alpha + 2\left(1-\alpha\right)F_B\left(z_B\right)\right]R_B\left(z_B\right), \tag{A.102}$$

which is maximized at the monopoly price z_r so that

$$\Pi_B^\star \;=\; \frac{\rho}{s}\alpha R_B\left(z_r\right). \tag{A.103}$$

By equal profit among prices in the connected support, I obtain

$$\alpha R_B\left(z_r\right) \;=\; \left[\,\alpha + 2\left(1-\alpha\right)F_B\left(z_B\right)\right]R_B\left(z_B\right), \tag{A.104}$$

which solves

$$\begin{aligned} F_B\left(z_B\right) \;&=\; \frac{\alpha}{2\left(1-\alpha\right)}\left(\frac{R_B\left(z_r\right)}{R_B\left(z_B\right)}-1\right) && \text{(A.105)}\\ &=\; \frac{\alpha}{2\left(1-\alpha\right)}\left(\frac{z_B - z_r}{z_I - z_B}\right) \qquad \forall z_B \in \mathcal{S}_B. && \text{(A.106)}\end{aligned}$$

Moreover, the upper bound $\bar{z}_B$ solves

$$\bar{z}_B \;=\; \left(1-\frac{\alpha}{2-\alpha}\right)z_I + \frac{\alpha}{2-\alpha}z_r. \tag{A.107}$$

given that $F_B(\bar{z}_B) = 1$.

Lending Price Distribution (F_L) Again, this proof follows the same procedure as in the proof of Proposition 2. From (57), the monopoly price for lender dealer now becomes $z_L = z_o$, which gives a positive monopoly profit if $z_I < z_o$. Under $z_I < z_o$, I can show that, after introducing the central bank's deposit facility,

1. F_L is continuous on its support $\mathcal{S}_L$;

2. The monopoly price $z_B = z_o$ is the highest price in $\mathcal{S}_L$;

3. $\mathcal{S}_L$ is connected.

Lender dealers' total profit can be rewritten as

$$\Pi_L^\star \;=\; \Pi_L\left(z_L\right) \;=\; \frac{1-\rho}{s}\left[\,\alpha+2\left(1-\alpha\right)\left(1-F_L\left(z_L\right)\right)\right]R_L\left(z_L\right), \tag{A.108}$$

which is maximized at the monopoly price z_o, so that

$$\Pi_L^\star \;=\; \frac{1-\rho}{s}\alpha R_L\left(z_o\right). \tag{A.109}$$

By equal profit among prices in the connected support, I obtain

$$\alpha R_L\left(z_o\right) \;=\; \left[\,\alpha+2\left(1-\alpha\right)\left(1-F_L\left(z_L\right)\right)\right]R_L\left(z_L\right), \tag{A.110}$$

which solves

$$\begin{aligned} F_L\left(z_L\right) \;&=\; 1-\frac{\alpha}{2\left(1-\alpha\right)}\left(\frac{R_L\left(z_o\right)}{R_L\left(z_L\right)}-1\right) &\text{(A.111)}\\ &=\; 1-\frac{\alpha}{2\left(1-\alpha\right)}\frac{1/z_L-1/z_o}{1/z_I-1/z_L}\quad \forall z_L\in\mathcal{S}_L. &\text{(A.112)}\end{aligned}$$

Moreover, the lower bound $\underline{z}_L$ solves $F_L\left(\underline{z}_L\right)=0$, so that

$$\underline{z}_L \;=\; \left[\left(1-\frac{\alpha}{2-\alpha}\right)\frac{1}{z_I}+\frac{\alpha}{2-\alpha}\frac{1}{z_o}\right]^{-1}. \tag{A.113}$$

□

A.17 Proof of Proposition 7

When the central bank's deposit facility is active, $z_I=z_d$. Then, from (60), and (62), the price ratio

$$\frac{\mu_B}{z_b} \;=\; \frac{\pi}{\beta}\frac{z_d+\frac{\alpha}{2(1-\alpha)}\ln\left(\frac{\alpha}{2-\alpha}\right)\left(z_d-z_r\right)}{\rho A\left(\alpha z_r+\left(1-\alpha\right)z_d\right)+1-\rho}, \tag{A.114}$$

where, as in (61),

$$\pi \;=\; \beta\left[\rho A+\left(1-\rho\right)\left(\alpha\frac{1}{z_o}+\left(1-\alpha\right)\frac{1}{z_d}\right)\right]. \tag{A.115}$$

Lending Facility Price (z_r) I first study the effects of an increase in the lending facility price z_r. An increase in z_r does not change inflation, i.e.,

$$\frac{\mathrm{d}\pi}{\mathrm{d}z_r} \;=\; 0, \tag{A.116}$$

given that the gross inflation rate is constant in z_r. For price ratio μ_B/z_b, I obtain

$$\frac{\mathrm{d}\left(\mu_B/z_b\right)}{\mathrm{d}z_r} \;=\; -\frac{\pi}{\beta}\frac{\left[\frac{\alpha}{2(1-\alpha)}\ln\left(\frac{\alpha}{2-\alpha}\right)+\alpha\right]\rho A z_d+(1-\rho)\frac{\alpha}{2(1-\alpha)}\ln\left(\frac{\alpha}{2-\alpha}\right)}{\left[\rho A\left(\alpha z_r+(1-\alpha)\,z_d\right)+1-\rho\right]^2}, \tag{A.117}$$

which is positive if

$$\frac{\alpha}{2\left(1-\alpha\right)}\ln\left(\frac{\alpha}{2-\alpha}\right)+\alpha<0\longleftrightarrow\ln\left(\frac{\alpha}{2-\alpha}\right)+2\left(1-\alpha\right)<0. \tag{A.118}$$

Define

$$J\left(\alpha\right) \;\equiv\; \ln\left(\frac{\alpha}{2-\alpha}\right)+2\left(1-\alpha\right). \tag{A.119}$$

Then, I need to show that $\forall\alpha\in(0,1)\;\; J\left(\alpha\right)<0$. First note that

$$J''\left(\alpha\right) \;=\; \frac{4\left(\alpha-1\right)}{\left(2-\alpha\right)^2\alpha^2}<0. \tag{A.120}$$

Therefore,

$$J'\left(\alpha\right) \;=\; \frac{1}{\alpha}+\frac{1}{2-\alpha}-2 \tag{A.121}$$

is strictly decreasing in $\alpha\in(0,1)$. Consequently,

$$\forall\alpha\in(0,1)\quad J'\left(\alpha\right) \;>\; J'\left(1\right) \;=\; 0. \tag{A.122}$$

This further implies that $J\left(\alpha\right)$ is strictly increasing in $\alpha\in(0,1)$, so that

$$\forall\alpha\in(0,1)\quad J\left(\alpha\right) \;<\; J\left(1\right) \;=\; 0. \tag{A.123}$$

Hence, the derivative $\frac{\mathrm{d}(\mu_B/z_b)}{\mathrm{d}z_r}$ is always positive.

Borrowing Facility Price (z_o) Taking the same procedure as before to study an increase in the lending facility price, I obtain

$$\frac{\mathrm{d}\left(\mu_B/z_b\right)}{\mathrm{d}z_o} \;=\; \frac{1}{\beta}\frac{z_d+\frac{\alpha}{2(1-\alpha)}\ln\left(\frac{\alpha}{2-\alpha}\right)\left(z_d-z_r\right)}{\rho A\left(\alpha z_r+\left(1-\alpha\right)z_d\right)+1-\rho}\frac{\mathrm{d}\pi}{\mathrm{d}z_o} \;<\; 0, \tag{A.124}$$

given that, under equation (61),

$$\frac{\mathrm{d}\pi}{\mathrm{d}z_o} \;<\; 0. \tag{A.125}$$

□

A.18 Proof of Proposition 8

Lending Facility Price (z_r) The effects of an increase in the central bank's lending facility price on welfare are given by

$$\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}z_r} \;=\; \frac{\beta\rho\left(A-1\right)}{\pi}\left(-\frac{1}{\pi}\left[\hat{m}+\left(f-\hat{m}\right)\frac{\mu_B}{z_b}\right]\frac{\mathrm{d}\pi}{\mathrm{d}z_r}+\left(f-\hat{m}\right)\frac{\mathrm{d}\left(\mu_B/z_b\right)}{\mathrm{d}z_r}\right). \tag{A.126}$$

Equation (A.126) can be further rewritten as

$$\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}z_r} \;=\; \frac{\beta\rho\left(A-1\right)\left(f-\hat{m}\right)}{\pi}\frac{\mathrm{d}\left(\mu_B/z_b\right)}{\mathrm{d}z_r} \;>\; 0, \tag{A.127}$$

given that, from Lemma 7,

$$\frac{\mathrm{d}\pi}{\mathrm{d}z_r} \;=\; 0, \quad \text{and} \quad \frac{\mathrm{d}\left(\mu_B/z_b\right)}{\mathrm{d}z_r} \;>\; 0. \tag{A.128}$$

Borrowing Facility Price (z_o) Rewrite the welfare function (63) as

$$\begin{aligned}\mathcal{W} \;&=\; \frac{\beta\rho\left(A-1\right)}{\pi}\left[\hat{m}+\left(f-\hat{m}\right)\frac{\mu_B}{z_b}\right]\\ &=\; \beta\rho\left(A-1\right)\left[\hat{m}\frac{1}{\pi}+\left(f-\hat{m}\right)\frac{z_d+\frac{\alpha}{2(1-\alpha)}\ln\left(\frac{\alpha}{2-\alpha}\right)\left(z_d-z_r\right)}{\beta\left[\rho A\left(\alpha z_r+\left(1-\alpha\right)z_d\right)+1-\rho\right]}\right].\end{aligned} \tag{A.129}$$

It is then straightforward to get

$$\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}z_o} \;=\; -\frac{\beta\rho\left(A-1\right)\hat{m}}{\pi^2}\frac{\mathrm{d}\pi}{\mathrm{d}z_o} > 0, \tag{A.130}$$

because a higher borrowing facility price lowers inflation π (Lemma 7). □

A.19 Proof of Proposition 9

Totally differentiating conditions (66) to (68) with respect to z_d yields

$$\frac{\mathrm{d}\left(\mu_B/z_b\right)}{\mathrm{d}z_d} = \frac{\alpha\left(1-\rho\right)\left(\rho A+1-\rho\right)}{\left(\rho A z_d+1-\rho\right)^2} > 0, \tag{A.131}$$

$$\frac{\mathrm{d}\pi}{\mathrm{d}z_d} = -\frac{\beta\left(1-\rho\right)\left(1-\alpha\right)}{\left(z_d\right)^2} < 0. \tag{A.132}$$

Substituting these results into the welfare derivative from (63),

$$\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}z_d} = \frac{\beta\rho\left(A-1\right)}{\pi}\left(-\frac{1}{\pi}\left[\hat{m}+\left(f-\hat{m}\right)\frac{\mu_B}{z_b}\right]\frac{\mathrm{d}\pi}{\mathrm{d}z_d}+\left(f-\hat{m}\right)\frac{\mathrm{d}\left(\mu_B/z_b\right)}{\mathrm{d}z_d}\right), \tag{A.133}$$

gives $\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}z_d} > 0$. □