Adverse Selection, Private Collateral Provision, and Government Intervention

Fengfan Xiang [*]

Western University

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Abstract

A model is constructed to study the impact of adverse selection in financial markets on the efficiency of other markets. Private collateral provision matters for the transmission across markets. Adverse selection hinders collateral provision in the loan market, resulting in inefficiency in the goods market as collateral is essential to secure credit transactions. Without further intervention, when the adverse selection problem is severe, the accommodative government policy may shut down the loan market, even if it is feasible. A novel loan subsidy program is proposed, which works by addressing the adverse selection problem banks face.

Key Words: Adverse selection; Collateral provision; Government intervention

JEL: E4; E5

[*]Email: fxiang8@uwo.ca.

1 Introduction

This paper primarily concerns adverse selection problems in financial markets and their spillover effects on other markets. Adverse selection is a recognized source of friction that impedes the efficient functioning of markets. For example, in the seminal work of Akerlof (1970), adverse selection could shut down the used car market. Adverse selection can have implications not only for the inefficiency of a particular market but also for other markets through spillovers. In the used car example, buyers aware of the adverse selection problem in the used car market are more likely to opt for a new car. Consequently, the increased demand for new cars leads to higher profits for new car manufacturers, even though they are not directly involved in the used car market. Therefore, understanding the spillovers of adverse selection is vital for understanding economic activities.

This paper studies the relationship between collateral provision and adverse selection in financial markets, as well as the implications for government intervention. I study an environment where it is useful for banks to create new collateral. Adverse selection in one market hinders the ability of banks to create collateral that is useful in a second market. I also study how government intervention can mitigate the adverse selection problem and boost collateral provision in the economy.

This paper makes two main contributions. First, I examine the transmission channel by which adverse selection in one market leads to inefficiencies of collateral in another market. While a large literature[1] has studied how adverse selection causes market failure in the financial market, my focus is how adverse selection in one financial market affects economic activity in another market. To explore this issue, I study secured credit transactions that can be supported by collateralizable assets generated in the first market. I find that adverse selection hinders collateral provision, which results in inefficiency in the

[1]See for example, Gorton (2008, 2009), Heider et al. (2015), Tirole (2012), Philippon and Skreta (2012), Guerrieri and Shimer (2014) and Chiu and Koeppl (2016). I will discuss more details about these papers in the literature review part.

second market.

Second, I investigate the implications of government intervention in the presence of adverse selection. I consider a novel loan subsidy program that aims to alleviate the incentive problem faced by banks in the loan market. Unlike previous studies focusing on their target markets, such as Guerrieri and Shimer (2014) and Chiu and Koeppl (2016), I examine the spillovers between different markets. By addressing the adverse selection problem in the loan market, I find that the subsidy program facilitates private collateral provision. As a result, the lending process generates more collateralizable assets, which improves efficiency in another market.

To study how adverse selection, through collateral provision channel, affects outcomes across markets, I construct a model incorporating asymmetric information regarding assets in Williamson (2012) framework. In the model, buyers in the goods market face uncertainty over acceptable means of payment by sellers. Thus, in the spirit of Diamond and Dybvig (1983), banks are useful and serve to insure buyers against uncertainty.

Banks rely on collateral to operate effectively as banks are assumed to be untrustworthy due to limited commitment and lack of record keeping. When safe government debts are scarce, banks have the incentive to seek additional collateral from private markets. In the model, households can enjoy service flows from holding a service-generating asset. Moreover, households can use their service-generating assets as collateral to borrow from banks. Subsequently, banks can use these loans as collateral to operate effectively.

But, banks are subject to adverse selection in lending. The adverse selection problem comes from households' private information regarding their valuation of the service-generating assets. Following Wang and Williamson (1998), banks want to screen loan applicants, and they can verify if a loan applicant is truth-telling by incurring a fixed screening cost. Similar to Rothschild and Stiglitz (1978), a loan contract does not exist when the adverse selection problem is severe, and if a loan contract exists, it is a separating contract. In equilibrium, banks would like to use the screening technology and pay the screening cost if such loan contracts exist.

The adverse selection problem in the loan market hinders collateral provision for banks, which reduces efficiency in the goods market. Banks can only use government debt as collateral when the adverse selection problem is severe enough that no loan contract exists. Under such a case, adverse selection causes inefficiency for goods market exchange as it shuts down the loan market. Banks could also use loan payments as collateral when loan contracts exist. However, adverse selection still causes inefficiency in goods market exchange as banks must use part of their loan payments for the screening cost, reducing the loan's collateral value.

To address the adverse selection issue in the loan market, I consider a novel loan subsidy program, where the government commit to part of the private banks' future payments if they make loans. Under the subsidy program, households are willing to take out more loans as the competitive banks would require fewer payments. Thus, the loan subsidy program helps to facilitate the private collateral provision. As a result, this government intervention improves the efficiency of secured credit transactions in the goods market. Furthermore, it is important to note that the subsidy program could also play a role in reducing the aggregate screening cost associated with the adverse selection problem encountered by banks.

The rest of this paper is organized as follows: In section 2, I present the model economy. Section 3 characterizes the loan contract in the presence of adverse selection. Then, I present the economic agents' problems in section 4. Next, I define and characterize the benchmark equilibrium with no subsidy in section 5. Section 6 and Section 7 discuss the optimal nominal interest rate policy and the loan subsidy program, respectively. Section 8 concludes.

Related Literature This paper is closely related to a large literature studying how adverse selection causes financial market collapse and the implications for government intervention. Papers such as Gorton (2008, 2009); Heider et al. (2015) show the critical role of adverse selection in understanding the 2008 Financial Crisis. Tirole (2012) and Philippon and Skreta (2012) adopt the mechanism design approach to explore the optimal design of government intervention. More related to my paper, Guerrieri and Shimer (2014) and Chiu and Koeppl (2016) use dynamic search model to study how adverse selection affects the market liquidity and the optimal policy response to resurrect trading.

My paper contributes to this literature in the following aspects. First, previous work focuses almost exclusively on the financial market itself. However, my paper examines the transmission channel and shows that adverse selection from the loan market leads to inefficiencies in the goods market. Second, I use a general equilibrium model to investigate the policy implications for all markets instead of limiting my discussion to the target market.

My paper also contributes to the literature on secured credit exchanges and government policy following the Lagos and Wright (2005) framework, and it relates closely to Williamson (2018) and Kang (2019). In these prior works, market participants can potentially create counterfeit private assets to use as collateral, which limits the pledgeability of private assets and results in financial contracts incorporating haircuts for these assets. In contrast, my paper explores a distinct mechanism in which banks make inefficient loans due to adverse selection in lending. This adverse selection problem ultimately hinders collateral provision, as loans are collateralizable assets. Accordingly, my study focuses on government interventions that help to alleviate adverse selection. I propose a loan subsidy program which enhances market efficiency by facilitating collateral provision rather than improving the pledgeability of collateral by asset swapping as in Williamson (2018) and Kang (2019).

2 Environment

This paper studies a model of exchange under secured credit derived from Lagos and Wright (2005) and Rocheteau and Wright (2005), and I add a service-generating asset in a fashion similar to Williamson (2018). There are three sets of agents: a unit mass of buyers, a unit mass of sellers and an infinite mass of bankers. Time is indexed by $t = 0, 1, 2, \cdots$, and each period has two stages of exchange: the centralized market (CM) followed by the decentralized market (DM). A standard Walrasian market exists in the CM for agents to trade consumption goods and assets. While in the DM, there is random pairwise matching between buyers and sellers. Buyers can only produce in the CM, while sellers can only produce in the DM. Each buyer has a utility function given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[-H_t + \gamma_t a_t + u(x_t)\right], \tag{1}$$

where β is the discount rate, H_t is the buyer's labor supply in the CM, x_t is the buyer's consumption in the DM and $\gamma_t a_t$ is the service flows from holding a_t units of the service-generating assets. One interpretation of the service-generating assets is that these assets are houses, which generate ongoing service flows for the homeowners.

Assume that $u(\cdot)$ is strictly increasing, strictly concave and twice continuously differentiable with $u(0) = 0$, $u'(0) = \infty$, $u'(\infty) = 0$ and $-x\frac{u''(x)}{u'(x)} < 1$ for all $x \geq 0$. Also, assume buyers derive some time-varying and idiosyncratic service flows captured by preference shock γ_t, interpreted as agents' heterogenous hedging needs or heterogenous personal use of the assets, as in Duffie et al. (2005). Denote $i \in \{g, b\}$ as the intrinsic type of a buyer, and the preference shock, γ_t, is distributed according to a type-dependent distribution with the cumulative distribution function, $G_i(\cdot)$. Type g refers to the good type, while type b refers to the bad type. The intrinsic type is a buyer's private information, and a fraction $0 < \alpha < 1$ of buyers are type g.

All the agents have access to a production technology that can convert one unit of labor to one unit of perishable consumption goods. However, unlike the buyers, the sellers produce in the DM and consume in the CM. Each seller has a utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [X_t - h_t], \tag{2}$$

where X_t is the consumption in the CM, and h_t is the labor supply in the DM.

The utility function of each banker who can potentially run a bank is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [X_t^{bk} - H_t^{bk}], \tag{3}$$

where X_t^{bk} and H_t^{bk} are the consumption and the labor supply of a banker, respectively, in the CM. Bankers do not participate in the DM.

This economy has three underlying assets: service-generating assets, nominal government bonds, and currency. There is a supply of one unit of perfectly-divisible service-generating assets, and these assets are infinitely durable. I assume only the owners can receive the service flows from the service-generating assets, and there is no rental market for these assets, simplifying the problem as only buyers will be the owners in equilibrium. Let ψ_t denote the price of the service-generating assets. Nominal government bonds are one-period bonds issued by the fiscal authority, with a gross nominal interest rate of R_t. The central bank issues currency, where currency is a perfectly divisible and portable object that bears a nominal interest rate of zero.

I assume there is no memory or record keeping, and all the agents are subject to limited commitment. These assumptions imply that unsecured credit is impossible. However, a collateral technology exists that allows agents to secure credit activities. In this model, the primary function of government bonds is to act as collateral to secure private banks' deposit liabilities, and only banks will hold government bonds in equilibrium.

In addition to the underlying assets, the economy includes two endogenous credit contracts: loan contracts and deposit contracts. Buyers can borrow from a bank using the service-generating assets as collateral, and the borrowing procedures generate loans, which are assets of private banks. Besides government bonds, banks also use loans as collateral to back their deposit liabilities. In the CM, a buyer can contact as many banks as desired. However, assume that the buyer signs both loan and deposit contracts with one single bank for simplicity.

In the pairwise DM meetings, each buyer randomly matches with a seller and makes a take-it-or-leave-it offer to that seller. There are two types of sellers. A fraction ρ of cash-accepting sellers has no means to verify any assets other than currency, thus accepting currency only. The remaining fraction $1 - \rho$ of deposit-accepting sellers can accept the entire portfolio of financial assets. A buyer is aware of the type of matching in the following DM at the end of the CM, after productions and consumptions have occurred. Thus, in terms of deposit contracts, the banks play a liquidity insurance role by efficiently allocating assets for buyers in the spirit of Diamond and Dybvig (1983). A deposit contract grants the depositor the ability to withdraw currency if needed or to trade with higher-yielding deposit claims if feasible, in a manner related to Williamson (2012).

The timing of events. At the beginning of the CM, each buyer's preference shock γ_t realizes and is publicly observed. A bank may use screening technology to verify the buyer's type. Then, all debts from the previous period, including loans, are redeemed. After that, buyers receive the service flows from their service-generating assets. Next, a centralized Walrasian market opens where agents trade numeraire CM goods and the underlying assets. Following the Walrasian market, nature reassigns buyers' types regarding preference shock. Buyers then take out loan and deposit contracts with banks. Finally, the types of DM meetings are realized, and some buyers withdraw currency. In the DM, there are bilateral meetings in which buyers make take-it-or-leave-it offers for their corresponding cash-accepting or deposit-accepting sellers.

2.1 Consolidated Government with No Subsidy

One of the objectives of this paper is to discuss the government subsidy program. However, it will be helpful to start by understanding the baseline case with no subsidization. Once this is complete, I will discuss the subsidy program as the government's intervention in the loan market. From now on, confine attention to the stationary equilibrium where all real variables are constant. Denote π as the gross inflation rate, so nominal variables grow at the constant rate $\pi - 1$.

Consider an economy with a consolidated government consisting of fiscal authority and a central bank. Assume no unsettled government debt in the initial period. Then, the consolidated government budget constraint at $t = 0$ is given by

$$\bar{c} + \bar{b} = \tau_0, \tag{4}$$

where $\bar{c}$ and $\bar{b}$ are the real quantities of currency and government bonds outstanding, respectively. Also, τ_0 is the lump-sum transfer (or tax if $\tau_0 < 0$) to the buyers in period 0, in real term. In each subsequent period, $t = 1, 2, 3, \ldots$, besides issuing new debts, the government has to pay off the debts from the previous period, so

$$\bar{c} + \bar{b} = \frac{\bar{c} + R\bar{b}}{\pi} + \tau, \tag{5}$$

where τ is the real quantity of the lump-sum transfer to each buyer. The left-hand side of (5) represents the government's revenue from issuing new liabilities, while the right-hand side is the repayment of the government debt from the previous period and the transfer to the buyers.

As in Andolfatto and Williamson (2015) and Williamson (2018), assume that the fiscal authority fixes the supply of the consolidated government debt, v, for all the periods. Then,

$$v = \bar{c} + \bar{b}. \tag{6}$$

A key feature I want to capture through the behavior of the fiscal authority is a shortage of safe assets. As in Assumption 2, I will assume that v is sufficiently low that collateral is scarce, in a sense which I will later make precise. Given the fiscal policy with a target v, the central bank determines the composition of the consolidated government debt outstanding by setting the nominal interest rate R. The fiscal authority can manipulate lump sum transfers in response to monetary policy to hold v constant.

3 Loan Contract

For the loan market, I consider a contracting problem with adverse selection and costly screening of loan applicants. The solution concept is taken from Wang and Williamson (1998), so banks can access and commit to a costly screening technology ex-ante. Recall that γ denotes the preference shock in the subsequent period, which is ex-post public information. For each type, the corresponding probability density function $g_i(\gamma)$ is continuous and strictly positive on the support $[0, \bar{\gamma}]$. Although the preference shock is publicly observed, banks face adverse selection while lending as the intrinsic types $i \in \{g, b\}$ are buyers' private information. To characterize the loan contract, it will prove helpful to impose the following assumption on the distributions for the preference shock.

Assumption 1. *The distributions for the preference shock satisfy*

1. $\mathbb{E}_i[\gamma] > \frac{1}{\beta}$, $\forall i \in \{g, b\}$;

2. Monotone likelihood ratio property (MLRP): $\frac{g_g(x)}{g_b(x)} < \frac{g_g(y)}{g_b(y)}$, $\forall x, y \in [0, \bar{\gamma}]$ *and* $x < y$.

Let r denote the real interest rate, where $r = \frac{R}{\pi}$. This paper will focus on an environment with a low real interest rate such that $r < \frac{1}{\beta}$, as specified below. The first condition in Assumption 1 means the mean payoff from holding one unit of the service-generating asset for a type i buyer must always be strictly greater than the return on safe government debt, so there is a potential for lending against the asset as collateral. MLRP is frequently assumed in adverse selection, and it implies the first order stochastic dominance of $G_b(\cdot)$ by $G_g(\cdot)$, i.e. $G_b(x) > G_g(x) \quad \forall x \in [0, \bar{\gamma}]$. Thus, the good type is more likely to have a higher preference shock than the bad type.

As shown in Wang and Williamson (1998), the loan contract shares some common features with the insurance contract in Rothschild and Stiglitz (1978). In particular, if a loan contract exists, it is a separating contract, and a separating contract does not exist for some parameter values. Accordingly, a loan contract consists of pairs of payment schedules and screening probabilities, $(r^i(\gamma), \pi^i)$ for $i \in \{g, b\}$, for each unit of lending. Thus, if a borrower claims type i, $r^i(\gamma)$ is the payment next CM in terms of the CM goods with preference shock γ, while π^i is the probability that the bank uses the screening technology to verify if the borrower is truth-telling. Note that I assume commitment to screening probabilities for banks and that screening is costly in the sense that a bank has to pay a fixed cost of $e > 0$ to verify a buyer's type. Thus, if a separating loan contract (SMC) is optimal, i.e. there is no alternative (either separate or pooling) contract dominates it, it must meet the following conditions:

$$\begin{aligned}
&0 \leq r^i(\gamma) \leq \gamma \quad \forall \gamma \in [0, \bar{\gamma}] \quad \forall i \in \{g, b\}; \\
&\gamma_1 \leq \gamma_2 \Rightarrow r^i(\gamma_1) \leq r^i(\gamma_2) \quad \forall \gamma_1, \gamma_2 \in [0, \bar{\gamma}] \quad \forall i \in \{g, b\}; \\
&\mathbb{E}\left[r^i(\gamma)\right] \leq (1 - \pi^j) \int_0^{\bar{\gamma}} r^j(\gamma) \, \mathrm{d}G_i(\gamma) + \pi^j \mathbb{E}_i[\gamma] \quad \forall i, j \in \{g, b\} \quad \text{and} \quad i \neq j; \\
&\mathbb{E}\left[r^i(\gamma)\right] - \pi_t^i e \geq r \quad \forall i \in \{g, b\};
\end{aligned} \tag{SMC}$$

where $\mathbb{E}_i[\gamma] = \int_0^{\bar{\gamma}} \gamma \mathrm{d}G_i(\gamma)$ is the mean payoff from holding one unit of the service-generating asset for a type i buyer, $\mathbb{E}[r^i(\gamma)] = \int_0^{\bar{\gamma}} r^i(\gamma)\,\mathrm{d}G_i(\gamma)$ is the expected payment for one unit of loan to a type i buyer and $r > 0$ is the return on government bonds. The first condition in (SMC) states that the payment schedule must be feasible. The second one imposes monotonicity restrictions. The third conditions are the incentive compatibility constraints. In principle, buyers can misreport their types while borrowing. However, the incentive compatibility constraints ensure that truth-telling is always a dominant strategy. And the last one, which will hold with equality in equilibrium, guarantees non-negative payoffs for banks.

The following lemma concludes the main results for the loan contract and provides a helpful intermediate step for characterizing the economic agents' problems.

Lemma 1 (Wang and Williamson (1998)). *Given Assumption 1, if a separating loan contract exists, it satisfies the following conditions:*

1. $\pi^g > 0$ *and* $\pi^b = 0$*;*

2. *The unique equilibrium contract for a type g buyer is a debt contract with promised payment* $\bar{r}^g \in [0, \bar{\gamma}]$*;*

3. *There exists a debt contract for a type b buyer, characterized by the promised payment* $\bar{r}^b \in [0, \bar{\gamma}]$*;*

where a debt contract with promised payment $\bar{r}^i$ is defined as a payment schedule such that $r^i(\gamma) = \gamma$ if $\gamma < \bar{r}^i$, while $r^i(\gamma) = \bar{r}^i$ if $\gamma \geq \bar{r}^i$. Moreover, π^g, $\bar{r}^g$ and $\bar{r}^b$ can be determined by the following equations:

$$\bar{r}^b - \int_0^{\bar{r}^b} G_b(\gamma)\mathrm{d}\gamma = r, \tag{7}$$

$$\bar{r}^g - \int_0^{\bar{r}^g} G_g(\gamma)\mathrm{d}\gamma = \pi^g e + r, \tag{8}$$

$$(1-\pi^g)\left[\bar{r}^g - \int_0^{\bar{r}^g} G_b(\gamma)\mathrm{d}\gamma\right] + \pi^g \mathbb{E}_b\left[\gamma\right] = r. \tag{9}$$

Denote $\bar{r}$ as the solution to

$$\alpha\left[\bar{r} - \int_0^{\bar{r}} G_g(\gamma)\mathrm{d}\gamma\right] + (1-\alpha)\left[\bar{r} - \int_0^{\bar{r}} G_b(\gamma)\mathrm{d}\gamma\right] = r. \tag{10}$$

A separating loan contract exists if and only if

$$\bar{r} \geq \bar{r}^g. \tag{11}$$

4 Economic Agents' Problems

In this section, I will discuss how buyers choose to buy service-generating assets and take out loans and how banks choose their asset portfolios and deposit contracts. There are two cases, one where the loan contracts exist and the other where they do not. I denote $\chi = 0$ as the status such that no loan contract exists, while $\chi = 1$ represents loan contracts existing. Then, the economic agents' problems can be defined as follows.

4.1 Buyer's Problem

Buyers' quasi-linear preferences allow us to isolate their decisions to buy service-generating assets and take out loans from other asset allocation and consumption good trading decisions. In the CM, a buyer acquires service-generating assets at a price ψ[2] and holds them until the next CM. The direct payoffs of holding a units of that assets are from the service flows and the market value in the next CM, i.e., $a(\gamma + \psi)$. In addition to the direct payoffs, there is an indirect payoff from borrowing. Denote $\ell^{i,d}$ as the demand

[2]Note that the price for the service-generating assets is independent of buyers' types since nature will re-assign the types. Thus, the price is merely a constant ψ in stationary equilibrium.

for loans given the payment schedule $r^i(\gamma)$ if a loan contract exists, i.e., $\chi = 1$. Then, a type i buyer can also borrow $\chi\ell^{i,d}$ units of consumption goods from a bank using their service-generating assets as collateral. To ensure that no one absconds in any state, the following collateral constraints,

$$a(\gamma+\psi) \geq r^i(\gamma)\chi\ell^{i,d} \quad \forall\gamma, \tag{12}$$

must be satisfied.

I am interested in an environment with a scarcity of collateral and note that loans are collateralizable assets for banks. Thus, in equilibrium, the amount of loans $(\chi\ell^{g,d}, \chi\ell^{b,d})$ take the maximum values that satisfy collateral constraints (12). If they exist, the quantities of loans are

$$\ell^{g,d} = a\left(1+\frac{\psi}{\bar{r}^g}\right), \tag{13}$$

$$\ell^{b,d} = a\left(1+\frac{\psi}{\bar{r}^b}\right). \tag{14}$$

Then, the buyer solves

$$\max_a \left\{ \begin{array}{c} -\psi+\beta\left[\psi+\alpha\mathbb{E}_g[\gamma]+(1-\alpha)\mathbb{E}_b[\gamma]\right]+ \\ +\alpha\chi\left(1+\frac{\psi}{\bar{r}^g}\right)\left(1-\beta\mathbb{E}\left[r^g\left(\gamma\right)\right]\right)+ \\ +(1-\alpha)\chi\left(1+\frac{\psi}{\bar{r}^b}\right)\left(1-\beta\mathbb{E}\left[r^b\left(\gamma\right)\right]\right) \end{array} \right\} a, \tag{15}$$

where the payment schedules $r^g(\gamma)$, $r^b(\gamma)$ and their associated promised payments $\bar{r}^g$ and $\bar{r}^b$ are characterized in Lemma 1. The objective function (15) shows that the net payoff for each unit of service-generating asset acquired is the difference between the price paid in the current period and the direct discounted payoff from service flow in the following period and its resale value, plus the indirect discounted payoff from taking out loans.

4.2 Bank's Problem

Private banks write the deposit contracts in the CM before buyers realize the DM meeting types. Deposit contracts provide liquidity insurance to buyers as in Williamson (2012, 2016, 2018, 2019) by offering them options to withdraw currency when they learn the meeting types. Buyers who don't exercise the options can trade bank claims in the DM meetings. Thus, a deposit contract is a triple (k, c, d), where k is the quantity of CM goods deposited by the buyer, c is the currency in terms of CM goods that the buyer can withdraw at the end of CM, and d is the real quantity of claims to the subsequent period CM goods that the buyer can use to trade in the DM if the currency withdrawal option is not exercised. Similarly, let $\chi\ell^{i,s}$ represent the supply of loans by a bank contracting with a type i buyer. Besides the loans, the bank also acquires government bonds, b^i, in its asset portfolio. Then, the bank's problem is to maximize the buyer's expected utility, which can be written as

$$\max_{k,c,d,b^i,\chi\ell^{i,s}} -k + \rho u\left(\frac{\beta c}{\pi}\right) + (1-\rho)u\left(\beta d\right) \tag{16}$$

subject to

$$k - \rho c - \beta\left(1-\rho\right)d - \left(b^i + \chi\ell^{i,s}\right) + \beta\frac{R\left(b^i + \chi\ell^{i,s}\right)}{\pi} \geq 0, \tag{17}$$

$$-(1-\rho)d + \frac{R\left(b^i + \chi\ell^{i,s}\right)}{\pi} \geq 0, \tag{18}$$

$$k, c, d, b^i, \chi\ell^{i,s} \geq 0. \tag{19}$$

Given the free-entry condition of banks, banks will compete to attract buyers, and the objective function (16) is the representative buyer's expected utility. With probability ρ, a buyer realizes they will meet with a seller who only accepts currency in the following DM. Thus, the buyer withdraws currency at the end of the CM, makes a take-it-or-leave-it offer to the matched seller, and acquires $\frac{\beta c}{\pi}$ units of goods. Otherwise, with probability $1-\rho$, the buyer will meet with a seller who accepts deposits. Then, the buyer will withdraw currency, as they can use d deposit claims in exchange for βd units of goods.

Constraint (17) is a bank's participation constraint, which means that the bank earns a non-negative discounted payoff in equilibrium. In constraint (17), $k-\rho c-\beta(1-\rho)d$ refers to the return from the deposit contract, while $-\left(b^i+\chi\ell^{i,s}\right)+\beta\frac{R\left(b^i+\chi\ell^{i,s}\right)}{\pi}$ is the profit from investing in the asset portfolio. Next, note that the bank could run away with the deposits. In order to ensure they do not, they will post assets as collateral. Constraint (18) is their collateral constraint. Finally, all quantities must be positive as in constraint (19).

5 Equilibrium

5.1 Definition of Equilibrium

In equilibrium, asset markets clear in the CM in the sense that the demand for each asset is equal to the supply. Thus, the markets for government-issued assets must be clear such that

$$\rho c=\bar{c}; \tag{20}$$

$$\alpha b^g+(1-\alpha)b^b=\bar{b}. \tag{21}$$

Also, the markets for private assets must be clear, such that

$$a=1; \tag{22}$$

$$\chi\ell^{i,d}=\chi\ell^{i,s} \quad \forall i\in\{g,b\}. \tag{23}$$

The buyer's problem in (15) yields that, in equilibrium, the service-generating asset price (ψ) is characterized by

$$\left\{\begin{array}{c} -\psi + \beta\left[\psi + \alpha \mathbb{E}_g[\gamma] + (1-\alpha)\mathbb{E}_b[\gamma]\right] + \\ +\alpha\chi\left(1 + \frac{\psi}{\bar{r}^g}\right)\left(1 - \beta \mathbb{E}\left[r^g\left(\gamma\right)\right]\right) + \\ +(1-\alpha)\chi\left(1 + \frac{\psi}{\bar{r}^b}\right)\left(1 - \beta \mathbb{E}\left[r^b\left(\gamma\right)\right]\right) \end{array}\right\} = 0, \tag{24}$$

Intuitively, this states that the marginal benefit of acquiring one additional unit of service-generating asset is zero.

To solve the bank's problem, first, note that the bank's participation constraint (17) binds in equilibrium, or another bank would provide an alternative contract with a slightly lower payoff to attract the buyer. Also, the bank's collateral constraint (18) must hold with equality in equilibrium, as the collateral is assumed to be sufficiently scarce. Finally, it proves convenient, as in Williamson (2016, 2018, 2019), to rewrite the first-order conditions in terms of consumption quantities in DM meetings. Let $x^c = \frac{\beta c}{\pi}$ and $x^d = \beta d$ denote, respectively, the consumption quantities with currency and deposit claims being traded. The following are the remaining first-order conditions that solve the bank's problem, (16) subject to (17)–(19).

$$\beta u'\left(x^c\right) = \frac{R}{r}; \tag{25}$$

$$\frac{u'\left(x^c\right)}{u'\left(x^d\right)} = R. \tag{26}$$

Lastly, given the fiscal authority's policy target (6) and the market clearing conditions for the government issued assets (20)– (21), banks' aggregate collateral constraint can be written as

$$(1-\rho)\, x^d u'\left(x^d\right) + \rho x^c u'\left(x^c\right) = v + \beta r \chi\left[\alpha \ell^{g,s} + (1-\alpha)\,\ell^{b,s}\right] u'\left(x^d\right). \tag{27}$$

Scarcity of Collateral. The primary focus of this paper is to understand the impact of adverse selection on collateral provision and its resulting inefficiencies in goods market transactions. When collateral is scarce in the aggregate, mitigating the adverse selection problem in the loan market is beneficial as it helps to relax banks' collateral constraints by facilitating private collateral provision. Now, I will make precise what it means for collateral scarcity and suboptimal fiscal policy with consolidated government debt v to be small.

In the stationary equilibrium, efficiency is attained if the surplus of DM exchanges is maximized for both types of meetings, i.e., $x^c = x^d = x^*$, where $u'(x^*) = 1$. Then, from equations (25) and (26), the necessary condition for efficiency is $R = 1$ and $r = \frac{1}{\beta}$. Thus, from banks' aggregate collateral constraint (27), an efficient allocation exists if and only if

$$v + \left[\alpha \ell^{g,s} + (1-\alpha)\,\ell^{b,s}\right] \geq x^*. \tag{28}$$

Condition (28) shows that an efficient allocation can be supported with accommodative monetary policy, i.e., the central bank chooses the gross nominal interest rate $R = 1$, when the supply of collateral exceeds the efficient consumption in the DM. Thus, the following assumption guarantees sufficiently scarce collateral so that banks' collateral constraints remain binding, irrespective of the severity of adverse selection.

Assumption 2. *The fiscal policy v and the distributions for the preference shock satisfy*

$$v + \alpha\left(1 + \frac{\psi^*}{r_g^*}\right) + (1-\alpha)\left(1 + \frac{\psi^*}{r_b^*}\right) < (1-\rho)x^*$$

where $\psi^* = \frac{\beta}{1-\beta}\left[\alpha \mathbb{E}_g[\gamma] + (1-\alpha)\,\mathbb{E}_b[\gamma]\right]$, r_g^* *and* r_b^* *solve* $r_g^* - \int_0^{r_g^*} G_g(\gamma)\,\mathrm{d}\gamma = \frac{1}{\beta}$ *and* $r_b^* - \int_0^{r_b^*} G_b(\gamma)\,\mathrm{d}\gamma = \frac{1}{\beta}$*, respectively.*

Assumption 2 defines collateral scarcity as a limitation on the supply of government debt and private loans, and it results in a low real interest rate such that $r < \frac{1}{\beta}$. Note that, if the fiscal instruments are available, the fiscal authority can overcome collateral scarcity by providing an adequate supply of government debt, v. Thus, it is critical to assume that fiscal policy is suboptimal. Moreover, the assumption also restricts the distributions of preference shock to prevent excessive private collateral provision. Consequently, adverse selection in the loan market becomes crucial for secured credit transactions as it affects private collateral provision.

Definition 1. *Assume fiscal policy v and distributions for the preference shock satisfy Assumption 1 and Assumption 2, given monetary policy R, a stationary equilibrium consists of DM consumptions (x^c, x^d), the buyers' demands for service-generating assets and loans $(a, \chi\ell^{g,d}, \chi\ell^{b,d})$, the banks' supplies of loans $(\chi\ell^{g,s}, \chi\ell^{b,s})$, the prices (ψ, r) and the screening probability and the promised payments that characterize the loan contracts $(\pi^g, \bar{r}^g, \bar{r}^b)$, such that $\chi = 1$ if condition (11) holds and $\chi = 0$ otherwise, and the rest of the variables satisfy the solution to the buyer's problem (13), (14) and (24), the solution to the bank's problem (25)–(27), the service-generating asset and loan market clearing conditions (22) and (23), and loan contract conditions (7)–(9).*

5.2 Characterization of Equilibrium

Following Williamson (2018), I will pin down the equilibrium based on the demand and supply of collateral. Define the demand for collateral in (27) as

$$F^D\left(x^c, x^d\right) = (1-\rho)\, x^d u'\left(x^d\right) + \rho x^c u'\left(x^c\right). \tag{29}$$

In the right-hand side of (29), $(1-\rho)\, x^d u'\left(x^d\right)$ is the demand for collateral to support the exchanges of deposits for goods in the DM, while $\rho x^c u'\left(x^c\right)$ refers to the demand for collateral, which is the currency itself, to support the exchanges of currency for goods in the DM. The supply of collateral is

$$F^{S}\left(x^{d}\right)=v+\beta r\chi\left[\alpha\ell^{g,s}+(1-\alpha)\,\ell^{b,s}\right]u'\left(x^{d}\right). \tag{30}$$

Denote $\chi\ell^{g}$ and $\chi\ell^{b}$ as the amount of loans in equilibrium satisfying the loan market clearing conditions (23). Then, equations (25) and (26) give that, in equilibrium,

$$F^{S}\left(x^{d}\right)=v+\chi\left[\alpha\ell^{g}+(1-\alpha)\,\ell^{b}\right], \tag{31}$$

which is the supply of collateral consisting of government debt and loans. The fiscal authority fixes the supply of consolidated government debt, v, and buyers who take out loans from banks determine the supply of loans. Since banks' collateral constraints bind due to the scarcity of collateral, banks would like to issue more loans. Given that there is a fixed supply of service-generating assets, competitive banks will bid down the loan prices until zero profits from lending. Therefore, the loan supply is perfectly elastic, and the demand side dictates the loan market. Finally, the following lemma will prove helpful in pinning down the equilibrium.

Lemma 2 (Comparative Statics of the Demand Side of the loan Market)**.** *Under the case such that loan contracts exist, given the payment schedules and screening probabilities as in Lemma 1. The comparative statics of the demand for loans $\ell^{g,d}$ and $\ell^{b,d}$ with respect to real interest rate r show that*

$$\frac{\mathrm{d}\ell^{g,d}}{\mathrm{d}r}<0,\quad \frac{\mathrm{d}\ell^{b,d}}{\mathrm{d}r}<0.$$

See Appendix A for the proof. Intuitively, higher r relaxes the bank's collateral constraint and lowers the bank's demand for loans as collateral. Then, the loan contracts provided by the banks become less attractive for the buyers, which implies a decrease in loan demand. Denote the supply of collateral, $S(r) = F^S\left(x^d\right)$. The shape of $S(r)$ will depend on χ, the existence of loan contracts. If $\chi = 0$, $S(r) = v$, which is constant in r, as only government debts serve as collateral. Otherwise, if $\chi = 1$, $S(r) > v$ and $S'(r) < 0$. That is because banks also use loans to secure their credit activities. The equilibrium quantities of loans decrease, i.e., $\frac{\mathrm{d}\ell^g}{\mathrm{d}r} < 0$ and $\frac{\mathrm{d}\ell^b}{\mathrm{d}r} < 0$, as the demand for loans is decreasing in r.

Similarly, denote $D(r, R) = F^D\left(x^c, x^d\right)$. From (26), x^c is an increasing function of x^d and a decreasing function of the gross nominal interest rate, R. Equations (25) and (26) imply that $r = \frac{1}{\beta u'\left(x^d\right)}$. Thus, x^d is an increasing function of the real interest rate, r. Note that $F^D\left(x^c, x^d\right)$ is strictly increasing in each argument, which implies that $D(r, R)$ is strictly increasing in r and strictly decreasing in R. Intuitively, a higher r increase the demand for collateral as it is more profitable to hold collateral assets. However, given r, an increase in R implies an increase of the gross rate of inflation $\frac{R}{r}$, which decreases the rate of return on currency. Thus, the demand for collateral decreases.

In equilibrium, the demand for collateral will equal the supply of collateral. Thus,

$$D(r, R) = S(r). \tag{32}$$

The existence of loan contracts plays an important role in determining the provision of private liquidity, which, in this model, is backed by loans. Equation (11) in Lemma 1 provides the condition for the existence of the loan contracts. However, the following proposition discusses the existence condition more intuitively and provides a useful intermediate step in determining the general equilibrium.

Proposition 1 (Existence of loan Contracts)**.** *In an economy with a low real interest rate* $r \in (0, \frac{1}{\beta})$, $\forall \alpha \in (0, 1)$, *there exist* $0 < \bar{e}(\alpha) < \bar{\bar{e}}(\alpha)$, *such that,*

1. *if* $e \in (0, \bar{e}(\alpha)]$, *loan contracts exist* $\forall r \in (0, \frac{1}{\beta})$;

2. *if* $e \in \left(\bar{e}\left(\alpha\right), \bar{\bar{e}}\left(\alpha\right)\right]$, *there exists a cutoff real interest rate* r^* *such that loan contracts exists when* $r \in (0, r^*]$ *while loan contracts do not exist when* $r \in (r^*, \frac{1}{\beta})$;

3. *if* $e \in \left(\bar{\bar{e}}\left(\alpha\right), \infty\right)$, *no loan contract exists* $\forall r \in (0, \frac{1}{\beta})$.

Moreover, $\lim_{\alpha \to 0} \bar{e}\left(\alpha\right) = \bar{\bar{e}}\left(\alpha\right) = \infty$, $\lim_{\alpha \to 1} \bar{e}\left(\alpha\right) = \bar{\bar{e}}\left(\alpha\right) = 0$, *and* $\bar{e}\left(\alpha\right)$ *and* $\bar{\bar{e}}\left(\alpha\right)$ *are strictly decreasing in* α.

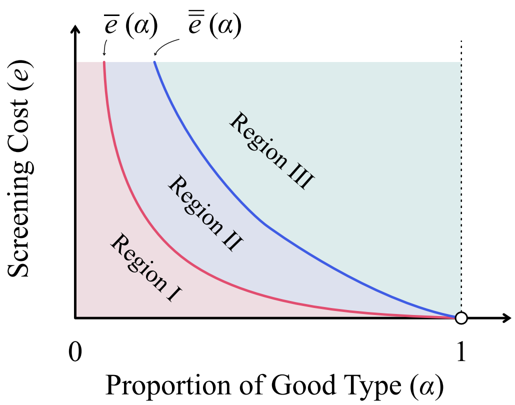


Figure 1: Existence Regions

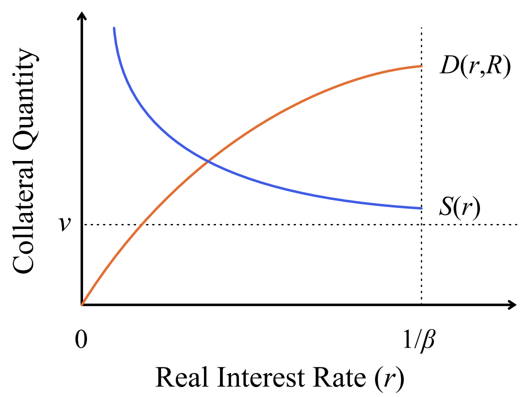


Figure 2: Type I Equilibrium

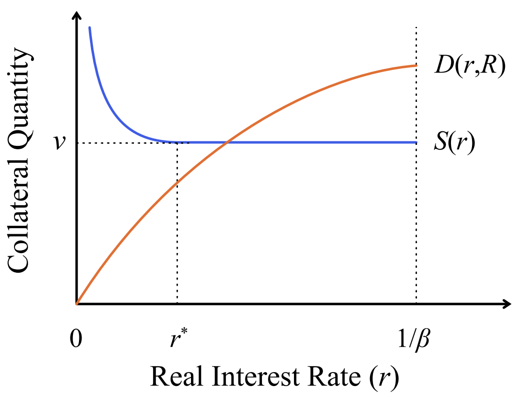


Figure 3: Type II Equilibrium

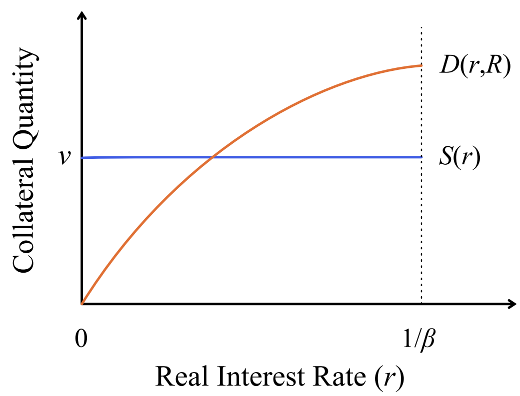


Figure 4: Type III Equilibrium

See Appendix A for the proof. The main results of Proposition 1 are depicted in Figure 1. Region I depicts the case such that loan contracts always exist. Region II depicts the case such that the loan contracts exist if the real interest rate is below the cutoff interest rate. Region III depicts the case with no loan contract. Note that if loan contracts exist, they will be separating contracts. Moreover, Figure 1 shows that lower screening cost increases the range of the proportion of type g buyers for which loans exist. The severity of adverse selection is increasing in both the cost of screening and the fraction of good buyers. For any positive screening cost ($e > 0$), the separating loan contract exists if the proportion of type g buyers, α, is small enough. That's because screening is worthwhile, no matter how high the screening cost is, when it is easy for banks to distinguish the type b buyers who misreport.

Figure 2 to Figure 4 show the three types of equilibrium corresponding to the locus of (α, e) in Region I to Region III, respectively. The type of equilibrium depends on the severity of the adverse selection. Under Type II Equilibrium, the loans exist only when the real interest rate is smaller than the cutoff real interest rate, r^*. Intuitively, a lower real interest rate (r) tightens banks' collateral constraints, which results in a higher payoff to borrowing for loan applicants. Thus, when loan contracts exist, a lower r increases the supply of collateral as buyers are more willing to take out loans. Under Type II Equilibrium, the supply of collateral is determined by the supply of the government debt when $r > r^*$ as the cost of lending, which is the screening cost in this paper, cannot be too high.

6 Nominal Interest Rate Policy

Fiscal policy is assumed to be suboptimal such that the supply of government debt is scarce, as in Assumption 2. However, the central bank can set any nominal interest rate target above the zero lower bound. This section will study the optimal nominal interest rate policy and its corresponding welfare implications. Before delving into the details, let me define the social welfare function. Define total welfare as the sum of utilities and disutilities from all economic activities in a stationary equilibrium with equally weighted economic agents. Thus,

$$W\left(x^c, x^d\right) = \rho\left[u\left(x^c\right) - x^c\right] + (1-\rho)\left[u\left(x^d\right) - x^d\right] - \chi\alpha\pi^g e, \tag{33}$$

which can also be interpreted as the weighted sum of the total surplus from the two types of DM exchange minus the aggregate screening cost. Furthermore, from equations (25) and (26):

$$u'\left(x^c\right) = \frac{R}{r\beta}, \tag{34}$$

$$u'\left(x^d\right) = \frac{1}{r\beta}. \tag{35}$$

For convenience, let $u\left(x\right) = \eta\frac{x^{1-\sigma}-1}{1-\sigma}$ with $\eta > 0$ and $0 < \sigma < 1$. Then, substituting (34) and (35) into (29) gives

$$(r\beta)^{\frac{1}{\sigma}-1}\eta^{\frac{1}{\sigma}}\left[1-\rho+\rho R^{1-\frac{1}{\sigma}}\right] = S\left(r\right), \tag{36}$$

under which the equilibrium real interest rate (r) can be solved given the gross nominal interest rate (R).

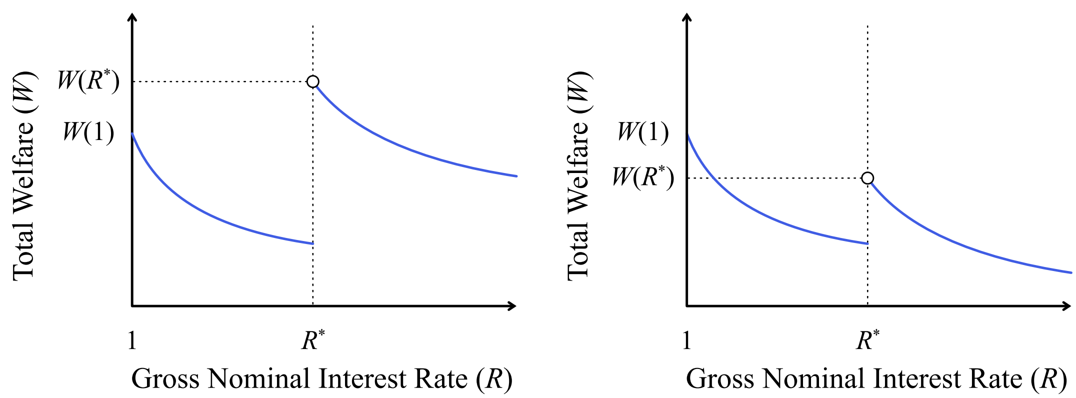
Aggregate welfare has two components, and the first is the total surplus of DM exchanges. Define the total surplus of DM exchanges as $w\left(x^c, x^d\right) = \rho\left[u\left(x^c\right) - x^c\right] + (1-\rho)\left[u\left(x^d\right) - x^d\right]$. From equations (34), (35) and (36), I can determine how $w\left(x^c, x^d\right)$ will change in response to a change in R. The result can be found in the following lemma.

Lemma 3. *When $R \geq 1$, i.e., the nominal interest rate is above the zero lower bound, an increase of the gross nominal interest rate R will decrease the total surplus of DM exchanges, i.e., $\frac{\mathrm{d}w}{\mathrm{d}R} < 0$.*

See Appendix A for the proof. In equilibrium, an increase in the nominal interest rate results in a higher real interest rate, which relaxes banks' collateral constraints. Thus, banks' demand for loans as collateral decreases, loan contracts become less attractive for buyers, and consequently, buyers tend to take out fewer loans. Since the consolidated government debt v is fixed, there is less collateral to secure DM exchanges. Thus, the total surplus of DM exchanges decreases.

The second component of welfare is the aggregate screening cost. Note that equations (8) and (9) imply $\frac{\mathrm{d}\pi^g}{\mathrm{d}r} > 0$. Also, from Figure 2 to Figure 4, $\frac{\mathrm{d}r}{\mathrm{d}R} > 0$ for all the three types of equilibrium. Then, from equations (34), (35) and (36), taking the derivative of W with respect to R, yields

$$\frac{\mathrm{d}W}{\mathrm{d}R} = \underbrace{\frac{\mathrm{d}w}{\mathrm{d}R}}_{-} - \chi\alpha e \underbrace{\frac{\mathrm{d}\pi^g}{\mathrm{d}r}}_{+} \underbrace{\frac{\mathrm{d}r}{\mathrm{d}R}}_{+} . \tag{37}$$



(a) Friedman Rule is Not Optimal (b) Friedman Rule is Optimal

Figure 5: Optimal Nominal Interest Rate Policy under a Type II Equilibrium

Under a Type I Equilibrium, as the severity of adverse selection is low, the loan contracts always exist so that $\chi = 1$ no matter which R the central bank chooses. In this case, it is optimal for the central bank to set $R = 1$, i.e., a zero nominal interest rate policy, which is just the Friedman rule. Similarly, under a Type III Equilibrium, $\chi = 0$ no matter which R is selected, the Friedman rule is also optimal. However, the optimal nominal interest rate policy is indeterminate under a Type II Equilibrium, in which the loan market shuts down for $r > r^*$. To see this, consider the case when the central bank sets $R^* > 1$ such that $r = r^* + \epsilon$, where ϵ is vanishingly small, in equilibrium. Although the total surplus of DM exchanges is higher when $R = 1$, banks do not need to pay the screening cost when $R = R^*$. Thus, the optimal nominal interest is either $R = 1$ or $R = R^*$, depending on the parameter values. Figure 5 depicts both cases, and I also provide the numerical examples in Appendix B.1.

Adverse selection matters for nominal interest rate policy, especially under a Type II Equilibrium as discussed in Figure 5. A low nominal interest rate can boost collateral supply. However, if adverse selection is severe, the expense of implementing a low nominal interest rate policy might outweigh the benefits due to the high screening cost. One way to deal with such an issue is for government to specifically address the adverse selection problem in the loan market.

7 Loan Subsidy Program

In this section, I consider a novel loan subsidy program that policymakers could implement to address adverse selection.[3] Under the loan subsidy program, the government makes extra payments to banks when the loan is repaid. These payments are financed through lump-sum taxes. I demonstrate how this subsidy program can effectively mitigate the adverse selection problem in the loan market and how it helps to increase the provision of private collateral. As a result, this program can facilitate more secure credit transactions, ultimately leading to an improvement in welfare.

Let s denote the extra payments from the government for each unit of the loan, no matter which type the buyer reports. Assume $0 < s < r$ such that $s > 0$ guarantees that it is a subsidy program, and $s < r$ ensures that there is no arbitrage for a bank to

[3] I will not differentiate between the subsidy program as a monetary or fiscal policy, as the distinction between the two has been blurry during recent crises. For instance, during the Covid-19 Crisis, the U.S. Federal Reserve introduced a series of mitigation efforts that often required Treasury involvement, as documented by Clarida et al. (2021). Thus, I consider the subsidy program implemented by the consolidated government in this paper.

short-sell government debts to issue more loans. Denote r^e as a bank's pre-subsidy return rate of loans, which comes from buyers' loan payments. With the loan subsidy program, banks could earn a return rate of $r^e + s$ by making loans or earn a return rate of r by investing in government debt. In equilibrium, it must be that $r = r^e + s$ so that banks hold both assets.

Now, instead of equations (7) – (9), the following equations solve for the promised payments and screening probability $\bar{r}^g$, $\bar{r}^b$ and π^g:

$$\bar{r}^b - \int_0^{\bar{r}^b} G_b(\gamma)\mathrm{d}\gamma = r^e, \tag{38}$$

$$\bar{r}^g - \int_0^{\bar{r}^g} G_g(\gamma)\mathrm{d}\gamma = \pi^g e + r^e, \tag{39}$$

$$(1-\pi^g)\left[\bar{r}^g - \int_0^{\bar{r}^g} G_b(\gamma)\mathrm{d}\gamma\right] + \pi^g \mathbb{E}_b\left[\gamma\right] = r^e. \tag{40}$$

As the subsidy s increases, for a given real interest rate r, the pre-subsidy loan return rate r^e must fall. The incentive compatibility constraint for type b buyer is relaxed when r^e falls. To see this, note that equation (40) is the incentive compatibility constraint for the type b buyer. The right-hand side of equation (40) is the expected payment for a type b buyer if truth-telling, while the left-hand side of equation (40) is the expected payment for a type b buyer if they misreport. With a decrease in the effective safe asset return rate (r^e), the screening probability decreases as it becomes less attractive for a type b buyer to misreport. Thus, the subsidy program mitigates the adverse selection problem in the loan market.

Again, an increase in s decreases r^e. Then, from equations (38) to (40), $\frac{\mathrm{d}\bar{r}^g}{\mathrm{d}s} < 0$, $\frac{\mathrm{d}\bar{r}^g}{\mathrm{d}s} < 0$ and $\frac{\mathrm{d}\pi^g}{\mathrm{d}s} < 0$. That is, by mitigating the adverse selection problem, the subsidy program lowers the promised payments ($\bar{r}^g$, $\bar{r}^b$) and screening probability (π^g). Consequently, from equations (13) and (14), the value of loans, $\left(\ell^g, \ell^b\right)$, increases as taking out a loan becomes more attractive for buyers due to the reduction of promised payments.

To determine the general equilibrium effects of the subsidy program, I employ the same approach as in the benchmark case. Note that the demand for collateral will not change, and the subsidy program works by increasing the supply of collateral for banks. Compared to the conditions in Definition 1, which characterizes the equilibrium with no subsidy, the difference comes from different return rates in the loan contracts. The loan contract is determined by the real interest rate (r) as in equations (7) to (9) with no subsidy, while the loan contract is determined by the pre-subsidy loan return rate (r^e) as in equations (38) to (40) under the loan subsidy program. Graphically, the intervention shifts the supply of collateral, $S(r)$, to the right by s, which is denoted by the new curve $\hat{S}(r)$.

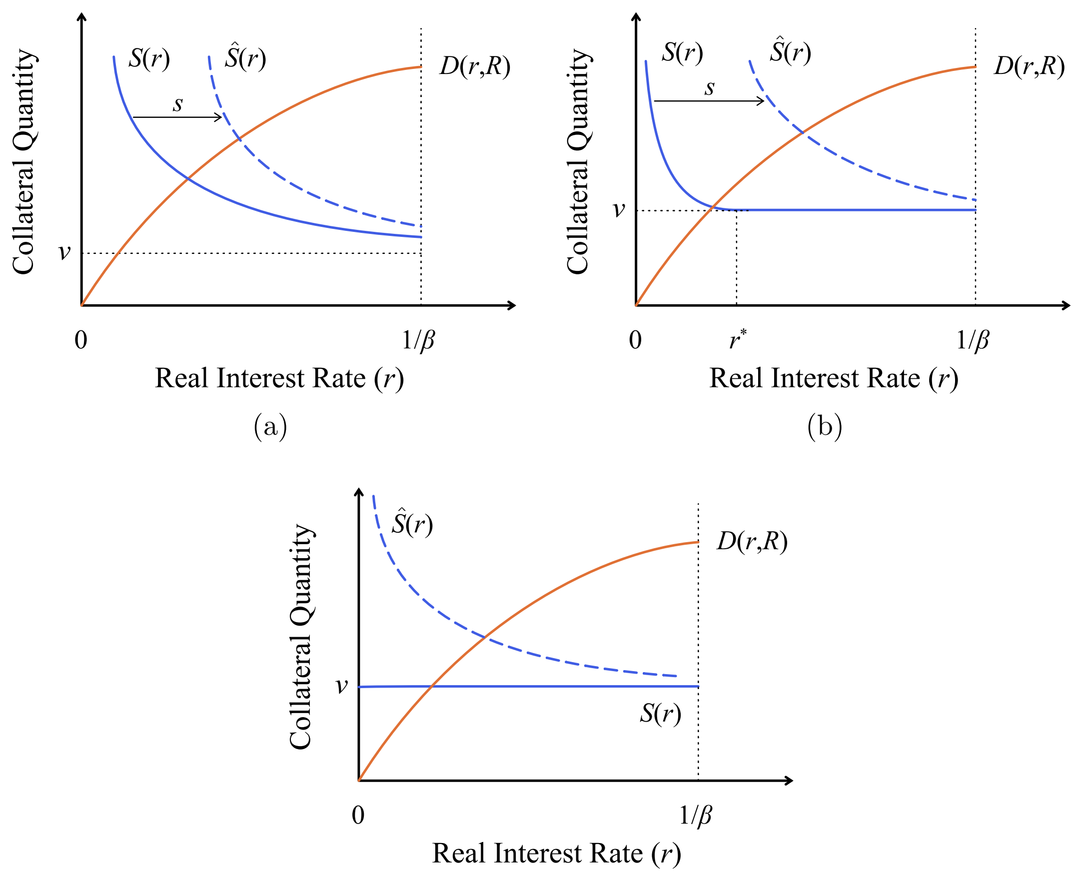


Figure 6: Equilibria under loan Subsidy Program

Figure 6 depicts the general equilibrium effects of the loan subsidy program under different types of equilibrium. Figures 6a, 6b, and 6c investigate the cases where the pre-intervention equilibrium correspond to type I, II, and III, respectively.[4] The supply of collateral increases after the intervention, while the demand for collateral remains unchanged. Thus, by holding a nominal interest rate policy constant, i.e., fixing a gross nominal interest rate R, the loan subsidy program increases the equilibrium quantity of collateral and the equilibrium real interest rate r.

What are the welfare implications of this subsidy program? Recall that welfare is measured by the total surplus of DM exchanges minus the aggregate screening cost.

First, welfare increases because the total surplus of DM exchanges increases. To see this, from equation (34) and equation (35), an increase of real interest rate implies an increase of consumption quantities x^c and x^d. Moreover, $u'(x^c)-1>0$ and $u'\left(x^d\right)-1>0$ as the real interest rate is low, i.e., $r<\frac{1}{\beta}$ and the nominal interest rate is constrained by the zero lower bound, i.e., $R\geq 1$. Consequently, the total surplus of DM exchanges, $w\left(x^c,x^d\right)=\rho\left[u\left(x^c\right)-x^c\right]+(1-\rho)\left[u\left(x^d\right)-x^d\right]$, increases by lowering the pre-subsidy loan return rate.

Second, welfare increases because of a reduction in the aggregate screening cost. From equations (39) and (40) , the screening probability π^g decreases in the pre-subsidy loan return rate r^e. Thus, the aggregate screening cost, $\chi\alpha\pi^g e$, decreases by lowering r^e.

Welfare increases by lowering the pre-subsidy loan return rate (r^e). The government should set $r^e=\epsilon$, a vanishingly small positive number, to achieve the optimal. Solving equation (39) and equation (40) jointly with $r^e=\epsilon$ results in $\bar{r}^g$ and π^g converging to zero. There is no screening cost under the optimal subsidy program, so the economy achieves an efficient level as an environment with no asymmetric information. Moreover,

[4] After the intervention, the equilibria become type III equilibria in Figure 6. As it will be clear later, these are the cases under the optimal monetary policy. However, there are other cases where the subsidy program changes a type II equilibrium to another type II equilibrium. Or the subsidy program changes a type III equilibrium to a type II or a type III equilibrium. These cases can be found in Appendix C.

loan contracts always exist when $r^e = \epsilon$ by checking condition (11). To conclude, the optimal intervention is to subsidize $s = r - \epsilon$ for each unit of the loan, and implementing the optimal loan subsidy program effectively resolves the problems caused by adverse selection, leading to an improvement in welfare.

Finally, it is optimal for the central bank to implement a zero nominal interest rate policy (i.e., the Friedman rule) in response to the optimal loan subsidy program. To see this, note that the economy achieves an efficient level as an environment with no adverse selection under the optimal subsidy program. Thus, reducing the nominal interest rate to the zero lower bound is always optimal as it increases the supply of collateral without suffering any screening cost.

8 Conclusion

I have built a model which incorporates adverse selection in the loan market. The adverse selection problem arises from households' private information about their asset valuations. When collateral is scarce in the aggregate, banks have an incentive to create additional collateral by issuing loans. However, adverse selection impedes lending and reduces banks' ability to support secured credit transactions in the goods market. Without further intervention, if the adverse selection problem is severe and the aggregate screening cost is too high, the accommodative government policy may shut down the loan market even if it is feasible. Thus, I propose a novel loan subsidy program that addresses the adverse selection problem in the loan market. This subsidy program facilitates private collateral provision, ultimately improving the goods market's efficiency. Finally, the Friedman rule is always optimal given the optimal subsidy program.

References

Akerlof, G. A. (1970). The market for "lemons": Quality uncertainty and the market mechanism. *The Quarterly Journal of Economics*, 84(3):488–500.

Andolfatto, D. and Williamson, S. (2015). Scarcity of safe assets, inflation, and the policy trap. *Journal of Monetary Economics*, 73:70–92.

Chiu, J. and Koeppl, T. V. (2016). Trading dynamics with adverse selection and search: Market freeze, intervention and recovery. *The Review of Economic Studies*, 83(3):969–1000.

Clarida, R., Duygan-Bump, B., and Scotti, C. (2021). The covid-19 crisis and the federal reserve's policy response.

Diamond, D. W. and Dybvig, P. H. (1983). Bank runs, deposit insurance, and liquidity. *Journal of Political Economy*, 91(3):401–419.

Duffie, D., Gârleanu, N., and Pedersen, L. H. (2005). Over-the-counter markets. *Econometrica*, 73(6):1815–1847.

Gorton, G. (2009). Information, liquidity, and the (ongoing) panic of 2007. *American Economic Review*, 99(2):567–572.

Gorton, G. B. (2008). The panic of 2007. Working Paper 14358, National Bureau of Economic Research.

Guerrieri, V. and Shimer, R. (2014). Dynamic adverse selection: A theory of illiquidity, fire sales, and flight to quality. *American Economic Review*, 104(7):1875–1908.

Heider, F., Hoerova, M., and Holthausen, C. (2015). Liquidity hoarding and interbank market rates: The role of counterparty risk. *Journal of Financial Economics*, 118(2):336–354.

Kang, K.-Y. (2019). Central bank purchases of private assets: An evaluation. *Review of Economic Dynamics*, 31:326–346.

Lagos, R. and Wright, R. (2005). A unified framework for monetary theory and policy analysis. *Journal of Political Economy*, 113(3):463–484.

Philippon, T. and Skreta, V. (2012). Optimal interventions in markets with adverse selection. *American Economic Review*, 102(1):1–28.

Rocheteau, G. and Wright, R. (2005). Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium. *Econometrica*, 73(1):175–202.

Rothschild, M. and Stiglitz, J. (1978). Equilibrium in competitive insurance markets: An essay on the economics of imperfect information. In *Uncertainty in economics*, pages 257–280. Elsevier.

Tirole, J. (2012). Overcoming adverse selection: How public intervention can restore market functioning. *American Economic Review*, 102(1):29–59.

Wang, C. and Williamson, S. D. (1998). Debt contracts and financial intermediation with costly screening. *Canadian Journal of Economics*, pages 573–595.

Williamson, S. D. (2012). Liquidity, monetary policy, and the financial crisis: A new monetarist approach. *American Economic Review*, 102(6):2570–2605.

Williamson, S. D. (2016). Scarce collateral, the term premium, and quantitative easing. *Journal of Economic Theory*, 164:136–165.

Williamson, S. D. (2018). Low real interest rates, collateral misrepresentation, and monetary policy. *American Economic Journal: Macroeconomics*, 10(4):202–33.

Williamson, S. D. (2019). Interest on reserves, interbank lending, and monetary policy. *Journal of Monetary Economics*, 101:14–30.

Appendices

A Proofs

Proof of Lemma 2. Note that in equilibrium, $a = 1$. Consider the case $\chi = 1$ only, as when $\chi = 0$, no loan contract implies no loan exists. Rewrite equations (13), (14) and (24) as

$$\ell^{g,d} = 1 + \frac{\psi}{\bar{r}^g}, \tag{A.1}$$

$$\ell^{b,d} = 1 + \frac{\psi}{\bar{r}^b}, \tag{A.2}$$

$$\left\{ \begin{array}{c} -\psi + \beta\left[\psi + \alpha\mathbb{E}_g\left[\gamma\right] + (1-\alpha)\mathbb{E}_b\left[\gamma\right]\right] + \\ +\alpha\ell^{g,d}\left[1 - \beta\mathbb{E}\left[r^g(\gamma)\right]\right] + \\ +(1-\alpha)\ell^{b,d}\left[1 - \beta\mathbb{E}\left[r^b(\gamma)\right]\right] \end{array} \right\} = 0. \tag{A.3}$$

I will prove this Lemma in 2 steps. First, I will show how will $\ell^{g,d}$, $\ell^{b,d}$ and ψ change in response to the change of $\bar{r}^g$ and $\bar{r}^b$, respectively. Based on the results from step 1, I will then do comparative statics for $\ell^{g,d}$, $\ell^{b,d}$ and ψ with respect to r.

Step 1. Totally differentiate equations (A.1) (A.2) and (A.3) with respect to $\bar{r}^g$ and $\bar{r}^b$, which gives

$$\begin{aligned} &\begin{bmatrix} 1 & 0 & -\frac{1}{\bar{r}^b} \\ 0 & 1 & -\frac{1}{\bar{r}^g} \\ (1-\alpha)[1-\beta\mathbb{E}[r^b(\gamma)]] & \alpha[1-\beta\mathbb{E}[r^g(\gamma)]] & -1+\beta \end{bmatrix} \begin{bmatrix} \mathrm{d}\ell^{b,d} \\ \mathrm{d}\ell^{g,d} \\ \mathrm{d}\psi \end{bmatrix} \\ = \quad & - \begin{bmatrix} \frac{\psi}{\left(\bar{r}^b\right)^2} \\ 0 \\ -\beta(1-\alpha)\ell^{b,d}\frac{\partial\mathbb{E}[r^b(\gamma)]}{\partial\bar{r}^b} \end{bmatrix} \mathrm{d}\bar{r}^b - \begin{bmatrix} 0 \\ \frac{\psi}{(\bar{r}^g)^2} \\ -\beta\alpha\ell^{g,d}\frac{\partial\mathbb{E}[r^g(\gamma)]}{\partial\bar{r}^g} \end{bmatrix} \mathrm{d}\bar{r}^g. \end{aligned} \tag{A.4}$$

Denote

$$D = \begin{bmatrix} 1 & 0 & -\frac{1}{\bar{r}^b} \\ 0 & 1 & -\frac{1}{\bar{r}^g} \\ (1-\alpha)[1-\beta\mathbb{E}[r^b(\gamma)]] & \alpha[1-\beta\mathbb{E}[r^g(\gamma)]] & -1+\beta \end{bmatrix}. \tag{A.5}$$

The determinant of matrix D is $|D| = -1+\beta+(1-\alpha)[1-\beta\mathbb{E}[r^b(\gamma)]]\frac{1}{\bar{r}^b}+\alpha[1-\beta\mathbb{E}[r^g(\gamma)]]\frac{1}{\bar{r}^g}$. Let $\mathrm{d}\bar{r}^g = 0$, equation (A.4) becomes

$$D\begin{bmatrix} \frac{\mathrm{d}\ell^{b,d}}{\mathrm{d}\bar{r}^b} \\ \frac{\mathrm{d}\ell^{g,d}}{\mathrm{d}\bar{r}^b} \\ \frac{\mathrm{d}\psi}{\mathrm{d}\bar{r}^b} \end{bmatrix} = -\begin{bmatrix} \frac{\psi}{\left(\bar{r}^b\right)^2} \\ 0 \\ -\beta(1-\alpha)\ell^{b,d}\frac{\partial\mathbb{E}[r^b(\gamma)]}{\partial\bar{r}^b} \end{bmatrix}. \tag{A.6}$$

Follow Cramer's Rule to get the following determinants,

$$\left|D_1^b\right| = -(-1+\beta)\frac{\psi}{\left(\bar{r}^b\right)^2} + \beta(1-\alpha)\ell^{b,d}\frac{\partial\mathbb{E}[r^b(\gamma)]}{\partial\bar{r}^b}\frac{1}{\bar{r}^b} - \alpha[1-\beta\mathbb{E}[r^g(\gamma)]]\frac{1}{\bar{r}^g}\frac{\psi}{\left(\bar{r}^b\right)^2}; \tag{A.7}$$

$$\left|D_2^b\right| = (1-\alpha)[1-\beta\mathbb{E}[r^b(\gamma)]]\frac{\psi}{\left(\bar{r}^b\right)^2}\frac{1}{\bar{r}^g} + \beta(1-\alpha)l^{b,d}\frac{\partial\mathbb{E}[r^b(\gamma)]}{\partial\bar{r}^b}\frac{\theta}{\bar{r}^g} > 0; \tag{A.8}$$

$$\left|D_3^b\right| = \beta(1-\alpha)\ell^{b,d}\frac{\partial\mathbb{E}[r^b(\gamma)]}{\partial\bar{r}^b} + (1-\alpha)[1-\beta\mathbb{E}[r^b(\gamma)]]\frac{\psi}{\left(\bar{r}^b\right)^2} > 0. \tag{A.9}$$

The signs of $\left|D_2^b\right|$ and $\left|D_3^b\right|$ can be determined easily, and I will later show how to determine the sign of $\left|D_1^b\right|$. Similarly, let $\mathrm{d}\bar{r}^b = 0$ and do the same manipulations, I get

$$|D_1^g| = \alpha[1-\beta\mathbb{E}[r^g(\gamma)]]\frac{1}{\bar{r}^b}\frac{\psi}{\left(\bar{r}^g\right)^2} + \beta\alpha\ell^{g,d}\frac{\partial\mathbb{E}[r^g(\gamma)]}{\partial\bar{r}^g}\frac{1}{\bar{r}^b} > 0; \tag{A.10}$$

$$|D_2^g| = -\left(-1+\beta\right)\frac{\psi}{\left(\bar{r}^g\right)^2} - (1-\alpha)[1-\beta\mathbb{E}[r^b(\gamma)]]\frac{1}{\bar{r}^b}\frac{\psi}{\left(\bar{r}^g\right)^2} + \beta\alpha\ell^{g,d}\frac{\partial\mathbb{E}[r^g(\gamma)]}{\partial\bar{r}^g}\frac{1}{\bar{r}^g}; \tag{A.11}$$

$$|D_3^g| = \beta\alpha\ell^{g,d}\frac{\partial\mathbb{E}[r^g(\gamma)]}{\partial\bar{r}^g} + \alpha[1-\beta\mathbb{E}[r^g(\gamma)]]\frac{\psi}{\left(\bar{r}^g\right)^2} > 0. \tag{A.12}$$

To determine the signs for $|D|$, $\left|D_1^b\right|$ and $|D_2^g|$, first, from equations (A.1) (A.2) and (A.3)

$$\left\{\begin{array}{c} -\psi + \beta\left[\psi + \alpha\mathbb{E}_g\left[\gamma\right] + (1-\alpha)\mathbb{E}_b\left[\gamma\right]\right] + \\ +\alpha\left(1+\frac{\psi}{\bar{r}^g}\right)\left[1-\beta\mathbb{E}\left[r^g(\gamma)\right]\right] + (1-\alpha)\left(1+\frac{\psi}{\bar{r}^b}\right)\left[1-\beta\mathbb{E}\left[r^b(\gamma)\right]\right] \end{array}\right\} = 0. \tag{A.13}$$

Rearrange it to

$$\psi\left|D\right| = -\left\{\begin{array}{c} \beta\left[\alpha\mathbb{E}_g\left[\gamma\right] + (1-\alpha)\mathbb{E}_b\left[\gamma\right]\right] + \\ +\alpha\left[1-\beta\mathbb{E}\left[r^g(\gamma)\right]\right] + (1-\alpha)\left[1-\beta\mathbb{E}\left[r^b(\gamma)\right]\right] \end{array}\right\} < 0. \tag{A.14}$$

Thus, $|D| < 0$. Take derivative with respect to $\bar{r}^b$ in both side of equation (A.13), and use the definitions of $|D|$ and $\left|D_1^b\right|$, I obtain

$$\left|D_1^b\right| = \frac{|D|}{\bar{r}^b}\left(-\frac{\psi}{\bar{r}^b} + \frac{\partial\psi}{\partial\bar{r}^b}\right), \tag{A.15}$$

which implies $\left|D_1^b\right| > 0$ since $|D| < 0$ and $\frac{\partial\psi}{\partial\bar{r}^b} = \frac{\left|D_3^b\right|}{|D|} < 0$. Similarly, I obtain $|D_2^g| = \frac{|D|}{\bar{r}^g}\left(-\frac{\psi}{\bar{r}^g} + \frac{\partial\psi}{\partial\bar{r}^g}\right) > 0$. Finally, step 1 can be concluded with the following conditions:

$$\frac{\partial\ell^{b,d}}{\partial\bar{r}^g} = \frac{|D_1^g|}{|D|} < 0 \quad \frac{\partial\ell^{g,d}}{\partial\bar{r}^g} = \frac{|D_2^g|}{|D|} > 0 \quad \frac{\partial\psi}{\partial\bar{r}^g} = \frac{|D_3^g|}{|D|} < 0; \tag{A.16}$$

$$\frac{\partial\ell^{b,d}}{\partial\bar{r}^b} = \frac{|D_1^b|}{|D|} > 0 \quad \frac{\partial\ell^{g,d}}{\partial\bar{r}^b} = \frac{|D_2^b|}{|D|} < 0 \quad \frac{\partial\psi}{\partial\bar{r}^b} = \frac{|D_3^b|}{|D|} < 0. \tag{A.17}$$

Step 2. Comparative statics for $\ell^{g,d}$, $\ell^{b,d}$ and ψ with respect to r. First, take ψ as a function of $r^g(\gamma)$ and $r^b(\gamma)$ as in equation (A.3), then

$$\frac{\mathrm{d}\psi}{\mathrm{d}r} = \frac{\partial\psi}{\partial\mathbb{E}\left[r^g(\gamma)\right]}\frac{\mathrm{d}\mathbb{E}\left[r^g(\gamma)\right]}{\mathrm{d}r} + \frac{\partial\psi}{\partial\mathbb{E}\left[r^b(\gamma)\right]}\frac{\mathrm{d}\mathbb{E}\left[r^b(\gamma)\right]}{\mathrm{d}r}. \tag{A.18}$$

With slightly abuse of notations, from equations (A.16) and (A.17), I get

$$\frac{\partial\psi}{\partial\mathbb{E}\left[r^g(\gamma)\right]} = \frac{\partial\psi}{\partial\bar{r}^g}\frac{\partial\bar{r}^g}{\partial\mathbb{E}\left[r^g(\gamma)\right]} = \frac{\partial\psi}{\partial\bar{r}^g}\frac{1}{1-G_g(\bar{r}^g)} < 0; \tag{A.19}$$

$$\frac{\partial\psi}{\partial\mathbb{E}\left[r^b(\gamma)\right]} = \frac{\partial\psi}{\partial\bar{r}^b}\frac{\partial\bar{r}^b}{\partial\mathbb{E}\left[r^b(\gamma)\right]} = \frac{\partial\psi}{\partial\bar{r}^b}\frac{1}{1-G_b(\bar{r}^b)} < 0. \tag{A.20}$$

Note that from equations (7) to (9), $\frac{\mathrm{d}\mathbb{E}[r^g(\gamma)]}{\mathrm{d}r} > 0$ and $\frac{\mathrm{d}\mathbb{E}\left[r^b(\gamma)\right]}{\mathrm{d}r} > 0$, which, together with (A.19) and (A.20) imply $\frac{\mathrm{d}\psi}{\mathrm{d}r} < 0$. Finally, taking derivative with respect to r for equations (A.1) and (A.2) gives

$$\frac{\mathrm{d}\ell^g}{\mathrm{d}r} = \frac{1}{(\bar{r}^g)^2}\left[\frac{\mathrm{d}\psi}{\mathrm{d}r}\bar{r}^g - \psi\frac{\mathrm{d}\bar{r}^g}{\mathrm{d}r}\right] < 0, \tag{A.21}$$

$$\frac{\mathrm{d}\ell^b}{\mathrm{d}r} = \frac{1}{(\bar{r}^b)^2}\left[\frac{\mathrm{d}\psi}{\mathrm{d}r}\bar{r}^b - \psi\frac{\mathrm{d}\bar{r}^b}{\mathrm{d}r}\right] < 0. \tag{A.22}$$

□

Proof of Proposition 1. I will prove Proposition 1 with two steps. First, I will provide and prove some useful conditions in this proof. Then, I will use these conditions to finish the proof in step 2.

Step 1. Define $\Phi(x) = \alpha\left[x - \int_0^x G_g(\gamma)\,\mathrm{d}\gamma\right] + (1-\alpha)\left[x - \int_0^x G_b(\gamma)\,\mathrm{d}\gamma\right]$, which is strictly increasing in x as $g_i(x)$ for $i \in \{g, b\}$ is strictly positive on the support $[0, \bar{\gamma}]$. Then, the condition for the existence of loan contracts, $\bar{r} \geq \bar{r}^g$, is equivalent to $\Phi(\bar{r}) \geq \Phi(\bar{r}^g)$. From equations (8) – (10) in Lemma 1, I obtain

$$\Phi(\bar{r}) = r; \tag{A.23}$$

$$\Phi(\bar{r}^g) = r + \pi^g\left\{\alpha e + (1-\alpha)\left[\bar{r}^g - \int_0^{\bar{r}^g} G_b(\gamma)\,\mathrm{d}\gamma\right] - (1-\alpha)\,\mathbb{E}_\mathrm{b}[\gamma]\right\}. \tag{A.24}$$

Using integration by parts, I obtain $\left[x - \int_0^x G_b(\gamma)\,\mathrm{d}\gamma\right] - \mathbb{E}_\mathrm{b}[\gamma] = x + \int_x^{\bar{\gamma}} G_b(\gamma)\,\mathrm{d}\gamma - \bar{\gamma} < 0$. For convenience, define function $\Gamma(x; \alpha, e)$, such that

$$\Gamma(x; \alpha, e) = \alpha e + (1-\alpha)\left[x + \int_x^{\bar{\gamma}} G_b(\gamma)\,\mathrm{d}\gamma - \bar{\gamma}\right], \tag{A.25}$$

where $\Gamma_x(x;\alpha,e) \geq 0$, with equality holds when $x = \bar{\gamma}$. Then, given that $\pi^g > 0$, the loan contracts exists if and only if

$$0 \geq \Gamma(\bar{r}^g;\alpha,e). \tag{A.26}$$

Totally differentiate $\Gamma(\bar{r}^g;\alpha,e)$ with respect to r, α and e, respectively. I obtain the following results given that $\frac{\mathrm{d}\bar{r}^g}{\mathrm{d}e}$ and $\frac{\mathrm{d}\bar{r}^g}{\mathrm{d}r}$ from equation (8) and (9):

$$\frac{\mathrm{d}\Gamma(\bar{r}^g;\alpha,e)}{\mathrm{d}r} = (1-\alpha)\left[1-G_b(\bar{r}^g)\right]\frac{\mathrm{d}\bar{r}^g}{\mathrm{d}r} > 0, \tag{A.27}$$

$$\frac{\mathrm{d}\Gamma(\bar{r}^g;\alpha,e)}{\mathrm{d}\alpha} = e - \left[x + \int_x^{\bar{\gamma}} G_b(\gamma)\,\mathrm{d}\gamma - \bar{\gamma}\right] > 0, \tag{A.28}$$

$$\frac{\mathrm{d}\Gamma(\bar{r}^g;\alpha,e)}{\mathrm{d}e} = \alpha + (1-\alpha)\left[1-G_b(\bar{r}^g)\right]\frac{\mathrm{d}\bar{r}^g}{\mathrm{d}e} > 0. \tag{A.29}$$

Step 2. In the low real interest rate environment, there might be three cases as $\frac{\mathrm{d}\Gamma(\bar{r}^g;\alpha,e)}{\mathrm{d}r} > 0$: no loan contract exists for $r \in (0, \frac{1}{\beta})$, loan contracts always exists for $r \in (0, \frac{1}{\beta})$ and there exists a cutoff $r^* \in (0, \frac{1}{\beta})$ such that loan contracts exist when $r < r^*$ while loan contracts do not exist when $r > r^*$. I will then so that these cases indeed exist and characterize the boundary conditions among these cases.

Fix $\alpha \in (0,1)$, and denote $\bar{r}_1^g$ as the solution to equations (8) and (9) in Lemma 1 when $r \to \frac{1}{\beta}$. Since $\frac{\mathrm{d}\Gamma(\bar{r}^g;\alpha,e)}{\mathrm{d}r} > 0$, the case that loan contracts always exist for $r \in (0, \frac{1}{\beta})$ happens if $\Gamma(\bar{r}_1^g;\alpha,e) \leq 0$. Thus,

$$\Gamma(\bar{r}_1^g;\alpha,e) \leq 0 \Longleftrightarrow \alpha e + (1-\alpha)\left[\bar{r}_1^g + \int_{\bar{r}_1^g}^{\bar{\gamma}} G_b(\gamma)\,\mathrm{d}\gamma - \bar{\gamma}\right] \leq 0. \tag{A.30}$$

Similarly, denote $\bar{r}_2^g$ as the solution to to equations (8) and (9) in Lemma 1 when $r \to 0$, so the case that no loan contract exists for $r \in (0, \frac{1}{\beta})$ happens if $\Gamma(\bar{r}_2^g;\alpha,e) > 0$. Then,

$$\Gamma(\bar{r}_2^g;\alpha,e) > 0 \Longleftrightarrow \alpha e + (1-\alpha)\left[\bar{r}_2^g + \int_{\bar{r}_2^g}^{\bar{\gamma}} G_b(\gamma)\,\mathrm{d}\gamma - \bar{\gamma}\right] > 0. \tag{A.31}$$

From (A.29) where $\frac{\mathrm{d}\Gamma(\bar{r}^g;\alpha,e)}{\mathrm{d}e} > 0$, thus I can define $\bar{e}(\alpha) = \sup E_1$ where all $e \in E_1$ satisfy (A.30) and define $\bar{\bar{e}}(\alpha) = \inf E_2$ where all $e \in E_2$ satisfy (A.31). Then, from (A.30) and (A.31) and given continuity of the functions,

$$\alpha\bar{\bar{e}}(\alpha) + (1-\alpha)\left[\bar{r}_2^g + \int_{\bar{r}_2^g}^{\bar{\gamma}} G_b(\gamma)\,\mathrm{d}\gamma - \bar{\gamma}\right] > \alpha\bar{e}(\alpha) + (1-\alpha)\left[\bar{r}_1^g + \int_{\bar{r}_1^g}^{\bar{\gamma}} G_b(\gamma)\,\mathrm{d}\gamma - \bar{\gamma}\right]. \tag{A.32}$$

Notice that $\bar{r}_1^g > \bar{r}_2^g$ since $\frac{\mathrm{d}\bar{r}^g}{\mathrm{d}r} > 0$ as in (A.27), I can conclude $\bar{\bar{e}}(\alpha) > \bar{e}(\alpha)$. Moreover,

$\bar{e}(\alpha) > 0$ by the definition of $\bar{e}(\alpha)$ and (A.30).

Given α, $\bar{e}(\alpha)$ and $\bar{\bar{e}}(\alpha)$ can also be viewed as the solution to (A.30) and (A.31) by changing inequalities with equalities. Then it is easy to get that $\lim_{\alpha\to 0}\bar{e}(\alpha) = \bar{\bar{e}}(\alpha) = \infty$ and $\lim_{\alpha\to 1}\bar{e}(\alpha) = \bar{\bar{e}}(\alpha) = 0$. Also, $\forall\alpha \in (0,1)$, $\bar{e}'(\alpha) < 0$ and $\bar{\bar{e}}'(\alpha) < 0$ as $\frac{\mathrm{d}\Gamma(\bar{r}^g;\alpha,e)}{\mathrm{d}\alpha} > 0$ and $\frac{\mathrm{d}\Gamma(\bar{r}^g;\alpha,e)}{\mathrm{d}e} > 0$ by (A.28) and (A.29).

Finally, due to the fact that $\frac{\mathrm{d}\Gamma(\bar{r}^g;\alpha,e)}{\mathrm{d}r} > 0$, for a given α and $e \in (\bar{e}(\alpha), \bar{\bar{e}}(\alpha)]$, there will be a cutoff $r^* \in (0, \frac{1}{\beta})$ such that loan contracts exist for $r \in (0, r^*]$ while loan contracts do not exist when $r \in (r^*, \frac{1}{\beta})$. □

Proof of Lemma 3. Totally differentiating equations (34), (35) and (36) with respect to R gives

$$\frac{\mathrm{d}x^c}{\mathrm{d}R} = -\eta^{\frac{1}{\sigma}}\frac{1}{\sigma}R^{-\frac{1}{\sigma}-1}\left(\frac{1}{r\beta}\right)^{-\frac{1}{\sigma}}\left(1-\frac{R}{r}\frac{\mathrm{d}r}{\mathrm{d}R}\right); \tag{A.33}$$

$$\frac{\mathrm{d}x^d}{\mathrm{d}R} = \eta^{\frac{1}{\sigma}}\frac{1}{\sigma}\left(\frac{1}{r\beta}\right)^{-\frac{1}{\sigma}-1}\frac{1}{r^2\beta}\frac{\mathrm{d}r}{\mathrm{d}R}; \tag{A.34}$$

$$\begin{gathered}\tfrac{1}{r\beta}\left[\rho R\tfrac{\mathrm{d}x^c}{\mathrm{d}R} + (1-\rho)\tfrac{\mathrm{d}x^d}{\mathrm{d}R}\right] + \rho R^{-\frac{1}{\sigma}}\left(\tfrac{1}{r\beta}\right)^{1-\frac{1}{\sigma}}\eta^{\frac{1}{\sigma}} \\ = S'(r)\tfrac{\mathrm{d}r}{\mathrm{d}R} + \tfrac{1}{r^2\beta}\left[\rho R^{1-\frac{1}{\sigma}}\left(\tfrac{1}{r\beta}\right)^{-\frac{1}{\sigma}} + (1-\rho)\left(\tfrac{1}{r\beta}\right)^{-\frac{1}{\sigma}}\right]\eta^{\frac{1}{\sigma}}\tfrac{\mathrm{d}r}{\mathrm{d}R}\end{gathered}. \tag{A.35}$$

Note that it is a system of linear equations of $\frac{\mathrm{d}x^c}{\mathrm{d}R}$, $\frac{\mathrm{d}x^d}{\mathrm{d}R}$ and $\frac{\mathrm{d}r}{\mathrm{d}R}$. Thus, by solving and substituting $\frac{\mathrm{d}x^c}{\mathrm{d}R}$ and $\frac{\mathrm{d}x^d}{\mathrm{d}R}$ into $\frac{\mathrm{d}w}{\mathrm{d}R} = \rho\left[u'(x^c) - 1\right]\frac{\mathrm{d}x^c}{\mathrm{d}R} + (1-\rho)\left[u'\left(x^d\right) - 1\right]\frac{\mathrm{d}x^d}{\mathrm{d}R}$, I obtain

$$\frac{\mathrm{d}w}{\mathrm{d}R} = \frac{\left(\frac{1}{r\beta}\right)^{1-\frac{1}{\sigma}}\rho\left[(-1+\mathrm{R})(-1+\rho)(-1+\sigma)\eta^{\frac{1}{\sigma}} - \mathrm{rS}'(\mathrm{r})\left(\frac{1}{\mathrm{r}\beta}\right)^{\frac{1}{\sigma}-1}\left(\frac{\mathrm{R}}{\mathrm{r}\beta}-1\right)\sigma\right]\eta^{\frac{1}{\sigma}}}{\mathrm{R}\sigma\left\{\frac{1}{r\beta}\eta^{\frac{1}{\sigma}}\left[-R^{\frac{1}{\sigma}}(-1+\rho) + R\rho\right](-1+\sigma) + \left(\frac{1}{\mathrm{r}\beta}\right)^{\frac{1}{\sigma}} rR^{\frac{1}{\sigma}}S'(r)\,\sigma\right\}}. \tag{A.36}$$

Given that $R \geq 1$, $r\beta < 1$,$0 < \rho, \sigma < 1$ and $S'(r) \leq 0$ in equilibrium, it is easy to conclude that $\frac{\mathrm{d}w}{\mathrm{d}R} < 0$. □

B Numerical Examples

Example B.1. *Let* $v = 42$, $\sigma = 0.4$, $\rho = 0.5$, $\beta = 0.98$, $\eta = 6$, $G_g(x) = x^{40}$ *and* $G_b(x) = x^2$, *which satisfy all the assumptions in the model. The following are two examples corresponding to Figure 5.*

1. *Let* $e = 0.345$ *and* $\alpha = 0.2$. *Then the cutoff real interest rate that shuts down the loan market is* $r^* = 0.602$ *and the associated nominal interest rate* $R^* = 1.001$. *Now* $W(1) = 32.996 < W(R^*) = 33.011$, *which is a case as in Figure 5a.*

2. *Let* $e = 0.2$ *and* $\alpha = 0.2$. *Then the cutoff real interest rate that shuts down the loan market is* $r^* = 0.646$ *and the associated nominal interest rate* $R^* = 1.179$. *Now* $W(1) = 33.024 > W(R^*) = 32.603$, *which is a case as in Figure 5b.*

C Equilibria under loan Subsidy Program

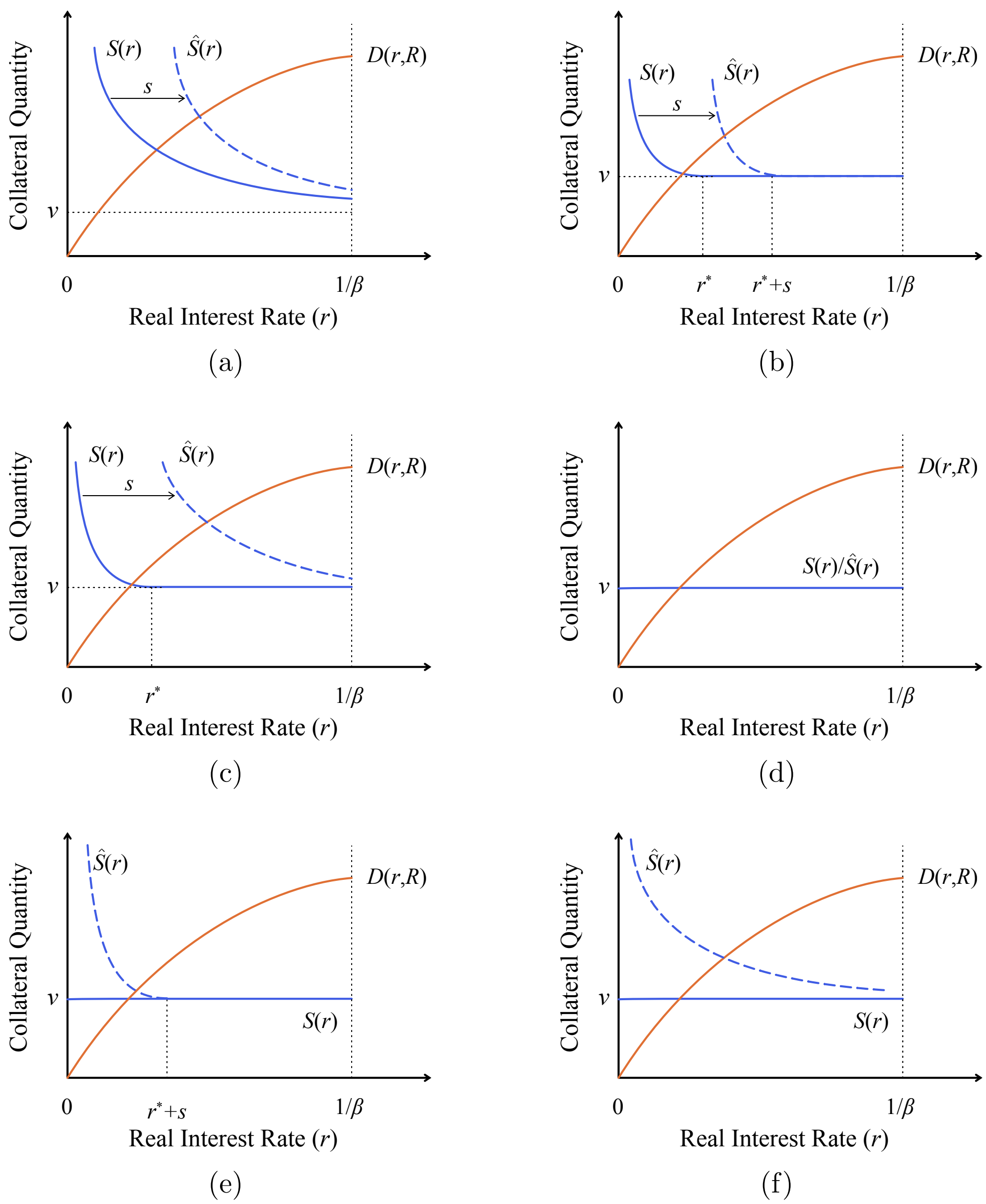


Figure 7: Equilibria under loan Subsidy Program