Wholesale Banking Panics and Monetary Policy

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1 Introduction

Wholesale banking panics have been a major source of financial instability in the last decades. Wholesale banks, such as money market mutual funds, are subject to a panic when their depositors withdraw safe assets in large volumes, forcing them to liquidate investments.[1] This was the case during the 2008 Financial Crisis (Bernanke, 2012; Gorton, 2010). For example, the failure of Lehman Brothers in September 2008 sparked panic in money market mutual funds, with over $400 billion withdrawn in two weeks (Huang & Keister, 2023). Similarly, during the COVID-19 pandemic, investors moved over $100 billion out of prime money funds to safe government funds in March 2020 (Sengupta & Xue, 2020).

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[1]Wholesale banks refer to financial institutions engaging in “credit intermediation involving entities and activities outside the regular bank system” (Financial Stability Board). Yorulmazer (2014), Gertler, Kiyotaki, and Prestipino (2016) and Ordoñez (2018) use the same definition to study other issues related to wholesale banks, or equivalently, shadow banks in their notion.

While panics are an inherent feature of banking (Diamond & Dybvig, 1983), wholesale banks are particularly vulnerable. They lack the safeguards that retail banks have, such as deposit insurance and liquidity support from central banks. They are also not subject to the same stringent regulations on retail banks, such as Basel III, that have sought to stabilize financial markets after the 2008 Financial Crisis. This paper is concerned with wholesale banking panics and their implications for monetary policy. Specifically, I address the following questions: How do central bank balance sheet policies affect the severity of wholesale banking panics and the efficiency of financial markets? What are the welfare implications of those policies?

I develop a general equilibrium banking model, where banks self-select to operate in either the retail or wholesale banking sector. Retail banks are subject to a leverage constraint, in line with Basel III, and have direct access to central bank reserves. Wholesale banks do not face the leverage constraint and have no direct access to reserves. Therefore, the asset market is segmented. However, wholesale banks can indirectly hold reserves by lending to retail banks through an interbank market.

The main focus of this paper is financial instability, where banking panics can arise endogenously in the wholesale banking sector. Each wholesale bank faces some exogenous probability of becoming insolvent, which captures the riskiness of their activities, forcing them to default on their debts. However, depositors do not know which banks will become insolvent. A wholesale banking panic occurs if depositors execute indiscriminate large-scale withdrawals of safe government bonds because they are worried about the fate of their own bank. The severity of the banking panic reflects the probability of each depositor doing this, or equivalently, how many depositors do this.

I find that central bank balance sheet policy will determine whether wholesale banking panics happen and their severity. I start with the policy determining the size of the central bank's balance sheet, in which central bank liabilities are limited to reserves alone. Later, I will introduce the overnight reverse repurchase agreement (ON-RRP) facility to study the policy, which determines the composition of central bank liabilities.

Specifically, expanding the size of the central bank's balance sheet mitigates the severity of wholesale banking panics by adjusting the relative supply of government debt and reserves. To show this, I explicitly model depositors' and banks' liquid asset needs and how these assets are used in different banking sectors.[2] During banking panics, wholesale bank depositors flee to safe government bonds that offer greater liquidity. Their alternatives are the risky deposit liabilities issued by wholesale banks. Expanding the central bank balance sheet crowds out government bonds, lowering the interest rate on bonds as they become scarce. On the other hand, it increases the supply of reserves, which secures bank liabilities. With a lower return for holding government bonds while more reserves back bank liabilities, fewer depositors flee to government bonds.

A sufficiently large central bank balance sheet, i.e., the reserve supply is above an upper threshold, prevents any wholesale banking panic from arising. If central bank reserves are sufficiently plentiful relative to government debt, an active interbank market provides a conduit for wholesale banks to hold reserves indirectly. Therefore, supported by sufficient claims on interbank lending, wholesale banks provide deposit claims that are attractive enough to depositors compared to government bonds, and no banking panic occurs even if there is a risk of banking failure.

In this scenario, increasing reserve supply crowds out government bonds, decreasing the value of bonds. However, transactions supported by central bank liabilities increase, like the transactions of retail bank depositors. Transactions of wholesale bank depositors using deposit claims also increase because more central bank reserves ultimately back these deposits.

By contrast, a full wholesale banking panic occurs when the central bank's balance

[2]This builds upon the ideas presented in Champ, Smith, and Williamson (1996) and Allen, Carletti, and Gale (2014), where depositors and banks demand currency, which works as a medium of exchange and a unit of account. However, my framework extends beyond currency to include various liquid assets, some of which function as a store of value to back bank liabilities. These assets compete with more liquid assets during a panic. Finally, in contrast to their papers, I focus on the wholesale banking sector.

sheet size becomes sufficiently small relative to government debt, i.e., the reserve supply is below a lower threshold. All wholesale bank depositors flee to safe government bonds, so their banks have no demand for claims on interbank lending to back their deposit liabilities, and the interbank market becomes inactive. In this case, wholesale banks resemble narrow banks by investing only in safe government bonds.[3]

In this scenario, wholesale bank depositors always choose government bonds. Thus, a change in the size of the central bank's balance sheet works the same as the scenario with no banking panics, as it does not change the depositors' panicky behavior in either scenario.

Finally, when the size of the central bank's balance sheet lies in between the two thresholds, a partial banking panic occurs. In this scenario, a fraction, but not all, of the depositors who can trade higher-yielding deposit claims execute their withdrawal option for bonds. The interbank market remains active, but the volume of interbank activities is notably smaller compared to the no banking panic scenario.

In this scenario, expanding the size of the central bank's balance sheet reduces the severity of a wholesale banking panic. As before, it crowds out government bonds circulated in the private sector, decreasing their value by making them more scarce. As a result, transactions supported by government bonds decrease. Furthermore, perhaps counterintuitively, transactions supported by both retail and wholesale banks' deposit claims also decrease, even though more reserves are available to back these bank liabilities. In aggregate, increasing the reserve supply indeed reallocated more assets to back bank liabilities. However, more depositors opt for bank claims, and the value of the reserves used to support each depositor's transaction decreases. Therefore, having a large central bank balance sheet can be harmful, at least when a partial banking panic occurs.

[3]In this paper, competitive wholesale banks maximize their depositors' expected utility, so the equilibrium with wholesale banks will be the same as having wholesale bank depositors purchase government bonds directly. Thus, this full banking panic scenario also characterizes disintermediation in the wholesale banking sector.

To conclude, expanding the size of the central bank's balance sheet can effectively mitigate wholesale banking panic, but it can be harmful. By crowding out government bonds, it lowers the return on bonds as they become scarce. As a result, the trading volume for transactions settled with bonds decreases. Furthermore, the damaging effects can also be transmitted to transactions supported by bank liabilities when too many depositors switch to bank claims, exacerbating the tension in the banking sectors.

Finally, I show that the ON-RRP facility can mitigate the wholesale banking panic and promote exchanges for depositors. Unlike reserves, ON-RRPs are also available to wholesale banks. Retail banks bear "balance sheet costs" because of their leverage constraint, similar to Kim, Martin, and Nosal (2020) and Williamson (2019). The assets held by retail banks can secure fewer liabilities than wholesale banks. Therefore, replacing reserves with ON-RRPs increases the effective collateral supply, which works like an increase in the central bank's balance sheet size. However, such an asset swap between the central bank liabilities does not crowd out government bonds. In the absence of the crowding-out effect, the number of depositors who switch to bank claims is less than the number of those with expanding central bank liabilities. Therefore, the ON-RRP facility also promotes exchanges for depositors.

The rest of the paper is organized as follows: I present the environment of the model economy and the private agents' problems in section 2 and section 3, respectively. Section 4 defines the equilibrium. Then, I solve three possible baseline equilibria with only reserves as central bank liabilities in section 5. Section 6 introduces the ON-RRPs as an additional form of central bank liabilities. Section 7 concludes.

2 Environment

This is a two-sector banking model with retail and wholesale banks. There is one period with three stages $t = 1, 2, 3$. There are three sets of private agents — a measure one of depositors, a measure one of producers, and an infinite measure of private banks.

All agents have access to a production technology that can convert one unit of labor into one unit of consumption goods. There are two types of consumption goods: a *numeraire good* that everyone can produce and consume, and a *special good* that can only be produced by producers and consumed by depositors at $t = 2$. Consumption goods are perishable in the sense that none of them can be carried across stages.

The structure is similar to the one in Lagos and Wright (2005). Private agents have preferences defined over their consumption and labor supply. In particular, let $u(c_s) + c_n$ denote a depositor's utility flow, where $c_s \geq 0$ denotes the units of special good the depositor consumes, and c_n is their net utility gain from consuming the numeraire good if positive or their utility loss from producing the numeraire good if negative. Assume that u is strictly increasing, strictly concave, and twice continuously differentiable with $u(0) = 0$, $\lim_{c\to 0} u'(c) = \infty$, $\lim_{c\to\infty} u'(c) = 0$, $\lim_{c\to 0} cu'(c) = 0$, and $-c\frac{u''(c)}{u'(c)} < 1$ for all $c \geq 0$. Banks and producers are risk-neutral and receive utility flow c_n, which denotes their net consumption (if positive) or production (if negative) of numeraire goods. In this paper, banks represent financial institutions and their shareholders, who pursue profits for their shareholders' consumption and conduct equity financing from their shareholders' production.

There is a centralized meeting in stage 1, where private banks write deposit contracts with depositors and construct their financial portfolios in the financial markets. Each depositor can contact only one bank. However, they can observe all deposit contracts banks offer and choose the optimal one. In stage 2, producers become active and produce special goods. Each depositor is randomly matched with a producer and makes a take-it-or-leave-it offer to that producer to exchange for their products. Depositors are subject to limited commitment, meaning no producer accepts depositors' IOUs. Also, they can not produce during this stage. Thus, depositors must use financial assets, either bank claims or government bonds, to settle their exchanges with producers. Finally, in stage

3, all outstanding debts are redeemed, and private agents consume the returns from their asset holdings.

There are two underlying assets: central bank reserves and government bonds. Reserves are private banks' account balances with the central bank, and one unit of reserves acquired in stage 1 pays off r^m units of consumption goods in stage 3. Similarly, the fiscal authority issues government bonds, and one unit of government bonds yields a payoff of r^b units of consumption goods in stage 3. Reserves and government bonds are not perfect substitutes. The main distinctions lie in which agents can hold them and how they use them. Reserves are restricted to retail banks, while any agent can hold government bonds.

Producers There are two types of producers, distinguished by their ability to evaluate and accept the means of payment. A *safe asset producer* exclusively accepts government bonds in exchanges. Exchanges between depositors and safe asset producers capture certain wholesale payments using government bonds but not private banks' liabilities. Although government bonds are not directly used in transactions, a significant portion of trading in repurchase agreements typically employs government bonds as collateral. In this model, transactions with repurchase agreements are equivalent to the transactions exchanging government bonds for goods, as the intrinsic value of government bonds is the same across agents. Additionally, there are *all-payment producers* who accept private banks' liabilities in addition to government bonds.

Depositors There are two types of depositors. An exogenous fraction α of depositors are *retail bank depositors*, while the remaining $1-\alpha$ fraction are *wholesale bank depositors*. Retail bank depositors use deposit claims issued by retail banks as a means of payment. Wholesale bank depositors, however, use government bonds or deposit claims issued by wholesale banks to settle their transactions. For wholesale bank depositors, there is a probability ρ that they meet a safe asset producer, in which case they cannot trade unless they have brought government bonds. With probability $1-\rho$, a wholesale bank depositor meets an all-payment producer who also accepts bank liabilities. At the end of stage 1, each wholesale bank depositor learns the type of producer they will meet, and they can opt for government bonds or deposit claims based on their producer's type.

Banks Differences between retail and wholesale banking sectors come from their access to reserves and how they are regulated. All banks are active in a perfectly competitive market in stage 1 and self-select to operate in retail or wholesale banking sectors with free entry. Therefore, in equilibrium, banks are indifferent between providing retail and wholesale bank depositors intermediary services.

Retail banks resemble highly regulated depository institutions. They are subject to a leverage requirement, so their liability-to-asset ratio cannot exceed $\theta < 1$. Unlike other private agents, especially wholesale banks, retail banks can hold central bank reserves. Therefore, retail banks act as intermediaries between other private agents and the central bank to invest in reserves. Retail bank depositors demand this intermediary service and use deposit claims to settle their transactions. Wholesale banks lend to retail banks through the interbank market to hold reserves indirectly.

Wholesale banks resemble financial institutions in the unregulated sector, such as money market mutual funds. The leverage requirement does not restrict them, but they do not have reserve accounts at the central bank. Wholesale banks act as intermediaries to help their depositors invest in the interbank market. Additionally, like Williamson (2019), wholesale banks provide liquidity insurance to their depositors in the spirit of Diamond and Dybvig (1983), so depositors can use appropriate means of payment when they learn their producer's type.

Banking Panic Wholesale banking panics are driven by underlying bank insolvency in a manner similar to Gertler and Kiyotaki (2015) and Williamson (2022). As for other private agents, banks are subject to limited commitment. So, banks must back their liabilities with collateral. For simplicity, I assume that an insolvent bank experiences a collapse in its collateral technology, leading to default on its liabilities.

A wholesale banking panic arises from a lack of information among wholesale bank depositors regarding which banks will become insolvent. A fraction $1-\delta$ of wholesale banks becomes insolvent in stage 2. However, depositors have to make their withdrawal decisions at the end of stage 1, and no producer will accept deposit claims issued by an insolvent bank. As a result, depositors may execute large-scale withdrawals of safe government bonds. I allow the depositors to follow mixed withdrawal strategies and let η denote the probability of a wholesale bank depositor, who can trade with deposit claims, choosing to withdraw government bonds, which captures the likelihood of banking panic. In equilibrium, this η corresponds to the fraction of such panicky depositors executing the withdrawal option, which also captures the severity of a wholesale banking panic.

2.1 Government

There is a government consisting of a fiscal authority and a central bank, and the economy starts with no unsettled government liability. In stage 1, the fiscal authority issues $\hat{b}$ units of government bonds and transfers the revenue τ_1 to depositors, so its budget constraint in stage 1 is

$$\hat{b} = \tau_1. \tag{1}$$

Then, the central bank conducts an asset swap, where they purchase $\hat{b}-\bar{b}$ units of government bonds with reserves. So,

$$\hat{b} - \bar{b} = \bar{m}, \tag{2}$$

where $\bar{m}$ and $\bar{b}$ represent the amount of reserves and government bonds circulating within the private sector.

Government bonds and central bank reserves are redeemed in stage 3. The stage 3 budget constraint for the fiscal authority is

$$r^b \hat{b} + \tau^{cb} = \tau_3, \tag{3}$$

where τ^{cb} is a transfer to the central bank to support its payments, and τ_3 is the lump-sum tax to depositors. The central bank uses the returns on its holdings of government bonds and the transfer from the fiscal authority to pay off its reserves. Thus, in stage 3, the central bank's budget constraint is

$$r^m \bar{m} = r^b \left(\hat{b} - \bar{b} \right) + \tau^{cb}. \tag{4}$$

The following consolidated government budget constraints are the critical elements of government policy in determining an equilibrium. From (1) and (2), the consolidated government budget constraints for $t = 1$ is

$$\bar{m} + \bar{b} = \tau_1, \tag{5}$$

which states that the value of reserves and government bonds outstanding equals the fiscal authority's lump-sum transfer. Similarly, from (3) and (4), the consolidated government budget constraint at $t = 3$ is

$$r^m \bar{m} + r^b \bar{b} = \tau_3, \tag{6}$$

so that the government taxes depositors to pay off their debts circulated in the private sector.

Given consolidated government constraints, a fiscal policy is determined by the supply of total government debt ($\hat{b}$), such that

$$\hat{b} = \bar{m} + \bar{b}. \tag{7}$$

Additionally, the supply of reserves, $\bar{m}$, reflects monetary policy, with the central bank adjusting the relative supply of government bonds and reserves circulating in the private sector through open market operations.

3 Banking

3.1 Retail Bank's Problem

Retail banks offer a deposit contract (k^r, d^r) to their depositors at $t = 1$, where k^r is the required deposit from each depositor, in return for a tradeable claim to d^r units of consumption good at $t = 3$. Banking is perfectly competitive, so any bank must act to maximize its depositors' utility. Retail bank depositors make take-it-or-leave-it offers for producers at $t = 2$, so each depositor can exchange for $c_s^r = d^r$ units of special goods with their deposit claim in a stage 2 meeting. Thus, a retail bank depositor's utility is

$$-k^r + u\left(d^r\right). \tag{8}$$

A bank must make non-negative profits to operate. Besides taking in deposits, retail banks borrow ℓ^r from the interbank market in stage 1 and invest in m units of reserves and b^r government bonds. A retail bank's participation constraint is

$$k^r - d^r - m - b^r + \ell^r + r^m m + r^b b^r - r^\ell \ell^r \geq 0. \tag{9}$$

In constraint (9), $k^r - d^r$ refers to the return from the deposit contract, and $-m - b^r + \ell^r + r^m m + r^b b^r - r^\ell \ell^r$ is the return from the bank's portfolio decision on financial assets. In equilibrium, the free entry condition ensures this constraint holds with equality.

In principle, retail banks could lend to wholesale banks. In such a scenario, wholesale banks would invest the funding from borrowing in government bonds and use these bonds as collateral to secure claims on interbank lending. However, for retail banks, the expected rate of return on such lending is lower than the interest rate on government bonds because of the risk of wholesale banking failure. As a result, retail banks prefer to invest in government bonds directly rather than lending to wholesale banks.

Retail banks are subject to limited commitment, and the following leverage constraint ensures retail banks prefer paying off their deposit claims rather than defaulting:

$$\theta \left(r^m m + r^b b^r \right) \geq d^r + r^\ell \ell^r, \tag{10}$$

where the parameter θ represents the leverage requirement within the regulated retail banking sector, which sets a maximum threshold for the liability-to-asset ratio.

Finally, retail banks are subject to the following nonnegative constraints

$$k^r, d^r, m, b^r \geq 0. \tag{11}$$

3.2 Wholesale Bank's Problem

Wholesale banks offer a deposit contract (k^w, b', d^w), where k^w is the required deposit for each depositor. In return, a wholesale bank depositor can withdraw b' units of government bonds at the end of stage 1, or the depositor can opt for tradeable claims to d^w consumption goods at $t = 3$. As with retail banks, wholesale banks maximize depositors' utility to compete for depositors. The expected utility of a wholesale bank depositor is

$$-k^w + [\rho + (1-\rho)\eta] u \left(r^b b'\right) + (1-\rho)(1-\eta)\delta u(d^w). \tag{12}$$

Wholesale banks diversify across depositors considering the potential wholesale bank insolvency with probability $1-\delta$ and the likelihood of banking panic with probability η. After paying the required deposits, a fraction $\rho + (1-\rho)\eta$ of the wholesale depositors opt to withdraw government bonds b' and make a take-it-or-leave-it offer to exchange for $c_s^b = r^b b'$ of special goods from the producer they meet. The remaining $(1-\rho)(1-\eta)$ of depositors settle their transactions with deposit claims and obtain $c_s^w = d^w$ units of special goods.

Wholesale banks also purchase b^w units of government bonds and lend ℓ^w to retail banks through the interbank market to guarantee a nonnegative profit, and the following is their participation constraint,

$$k^w - (1-\rho)(1-\eta)\delta d^w - b^w - \ell^w + r^b [b^w - [\rho + (1-\rho)\eta] b'] + r^\ell \ell^w \geq 0. \tag{13}$$

Again, this inequality will hold equality in equilibrium because of free entry.

A collateral constraint is required for solvent wholesale banks to ensure they pay off their liabilities. However, unlike retail banks, wholesale banks are not subject to the leverage requirement, and the following is their collateral constraint.

$$r^b [b^w - [\rho + (1-\rho)\eta] b'] + r^\ell \ell^w \geq (1-\rho)(1-\eta) d^w. \tag{14}$$

Wholesale bank will lose their returns on interbank lending and their holdings of government bonds if they default.

Finally, the following are wholesale banks' nonnegative constraints

$$k^w, b', d^w, b^w, b^w - [\rho + (1-\rho)\eta]\, b' \geq 0. \tag{15}$$

Wholesale Banking Panic A fraction $1-\delta$ of the wholesale banks become insolvent and default. It is always optimal for depositors of these insolvent wholesale banks to withdraw government bonds because such banks' deposit claims will become valueless. However, depositors have no information on which banks are insolvent. In response, a fraction η of the depositors who want to settle transactions with bank liabilities choose to withdraw bonds from their banks.

I interpret the cases for $\eta = 0$, $0 \leq \eta \leq 1$, and $\eta = 1$ as no banking panic, partial banking panic, and full banking panic, respectively, so a wholesale banking panic describes a scenario when $\eta > 0$. The decision by those panicky depositors to withdraw government bonds reflects their payoffs on different means of payment:

$$\text{if } u\left(r^b b'\right) < \delta u\left(d^w\right), \text{ then } \eta = 0; \tag{16}$$

$$\text{if } u\left(r^b b'\right) = \delta u\left(d^w\right), \text{ then } 0 \leq \eta \leq 1; \tag{17}$$

$$\text{if } u\left(r^b b'\right) > \delta u\left(d^w\right), \text{ then } \eta = 1. \tag{18}$$

From (16)-(18), a no banking panic equilibrium occurs if wholesale bank depositors prefer wholesale banks' deposit claims to government bonds, a partial banking panic occurs if these depositors are indifferent between these two options, and a full banking panic occurs if government bonds are preferred.

4 Definition of Equilibrium

In this paper, I will focus on the effects of monetary policy given a fiscal policy $\hat{b}$. Specifically, I assume a scarcity of government debt, captured by a low value of $\hat{b}$ as specified below, resulting in binding leverage constraint (10) and collateral constraint (14), similar to the assumption in Andolfatto and Williamson (2015) and Williamson (2019). Consequently, there are not enough tradeable deposit claims to support a satiated level of exchanges between depositors and producers, i.e., $c_s^r, c_s^w < c^\star$ with $u'(c^\star) = 1$. Otherwise, with a plentiful supply of government debt, those depositors always consume $c^\star$ units of special goods, and monetary policy becomes neutral as it will not affect the equilibrium allocation.

Assumption 1 (Scarcity of Government Debt)**.** *Assume the total supply of government debt is scarce such that $\theta\hat{b} < \alpha c^\star$, where $u'(c^\star) = 1$.*

Assumption 1 provides a sufficient condition to guarantee the binding leverage and collateral constraints. The inequality states that, even if retail banks exhaust all the government bonds issued by the fiscal authority, there is not enough collateral to support efficient exchange for their own depositors.

Let λ^r denote the Lagrangian multiplier for the nonnegative constraint $b^r \geq 0$ in the retail bank's problem.[4] Then, the first order conditions for the retail bank's problem give

$$r^m \left[1 - \theta + \theta u'\left(d^r\right)\right] = 1, \tag{19}$$

$$r^b \left[1 - \theta + \theta u'\left(d^r\right)\right] + \lambda^r = 1, \tag{20}$$

$$r^\ell u'\left(d^r\right) = 1. \tag{21}$$

Equations (19) and (20) determine retail banks' demands for reserves (m) and government

[4]Other nonnegative constraints never bind because of market clearing conditions and standard assumptions such as the Inada conditions for depositors' utility function.

bonds (b^r) to back their deposit claims, while equation (21) determines the rate of return from borrowing on the interbank market (ℓ^w).

Similarly, let λ^w denote as the Lagrangian multiplier for the nonnegative constraint $b^w - [\rho + (1-\rho)\,\eta]\, b' \geq 0$ in the wholesale bank's problem. The first order conditions for wholesale bank's problem are

$$r^b u'\left(r^b b'\right) = 1, \tag{22}$$

$$r^b \left[1 - \delta + \delta u'\left(d^w\right)\right] + \lambda^w = 1, \tag{23}$$

$$r^\ell \left[1 - \delta + \delta u'\left(d^w\right)\right] = 1. \tag{24}$$

Equation (22) states the demand for government bonds associated with each depositor's withdrawal request (b'), while equation (23) states the demand for government bonds to back wholesale banks' liabilities (b^w). Wholesale banks can also use claims on interbank lending to back their deposit claims, and equation (24) determines such demand.

The first order conditions displayed above are indeed asset pricing kernels in the consumption-based capital asset pricing model, surveyed by Campbell (1999). For example, the bond price, i.e., the inverse of the gross real interest rate on bonds $\frac{1}{r^b}$, in (22) is equal to wholesale bank depositors' marginal return of withdrawing additional unit of bonds. These depositors use government bonds as a means of payment and exchange for $r^b b'$ units of goods from producers. The rest of the first order conditions have similar interpretations by properly adjusting parameters related to the regulatory restriction θ, probability of banking failure $1-\delta$, and binding constraints λ^r and λ^w.

Definition 1 (Equilibrium)**.** *Given fiscal policy* (7) *and monetary policy* $\bar{m}$*, a banking equilibrium consists of allocation* $\left(\bar{b}, d^r, m, b^r, \ell^r, b', d^w, b^w, \ell^w, c_s^r, c_s^b, c_s^w\right)$*, Lagrangian multipliers* λ^r *and* λ^w*, market-determined interest rates* (r^m, r^ℓ, r^b)*, and the severity of wholesale banking panic* η*, satisfying binding collateral constraints* (10) *and* (14)*, first order* *condition for private banks' problems* (19)-(24)*, one of conditions* (16)-(18) *to support wholesale bank depositors' withdrawal probability, market clearing,*

$$\alpha m = \bar{m} \text{ (reserve market)}, \tag{25}$$

$$\alpha b^r + (1-\alpha) b^w = \bar{b} \text{ (government bond market)}, \tag{26}$$

$$\alpha \ell^r = (1-\alpha) \ell^w \text{ (interbank market)}, \tag{27}$$

and the complementary-slackness conditions with corresponding nonnegative constraints,

$$\lambda^r b^r = 0, \lambda^r \geq 0, b^r \geq 0, \tag{28}$$

$$\lambda^w [b^w - [\rho + (1-\rho)\eta] b'] = 0, \lambda^w \geq 0, b^w - [\rho + (1-\rho)\eta] b' \geq 0, \tag{29}$$

where $c_s^r = d^r$*,* $c_s^b = r^b b'$*,* $c_s^w = d^w$ *by definition.*

Later on, I will rewrite all the equilibrium conditions in terms of consumption quantities, c_s^r, c_s^b, c_s^w for convenience.

5 Equilibrium

There can be different equilibrium cases depending on who holds assets and how these assets are used, and the following lemmas will help determine these cases.

Lemma 1. *In a banking equilibrium, retail banks never invest in government bonds, i.e., nonnegative constraint* $b^r \geq 0$ *always binds with* $\lambda^r > 0$*.*

Proof. See Appendix A. □

Retail banks hold a positive stock of government bonds only when their interest rate exceeds the rate retail banks have to pay for interbank borrowing, where the difference serves to compensate retail banks for their costs of holding assets that ultimately come from the leverage constraint. However, wholesale banks always ask for a higher return on interbank lending because they always have the option to invest in government bonds. Therefore, retail banks never invest in government bonds in equilibrium.

Lemma 2. *In a banking equilibrium with binding collateral constraint* (14)*, wholesale banks only purchase government bonds for their depositors' withdrawal requests, i.e., non-negative constraint* $b^w - [\rho + (1-\rho)\,\eta]\, b' \geq 0$ *binds.*

Proof. See Appendix A. □

In principle, wholesale banks could hold government bonds as a store of value to secure their tradeable deposit claims. However, such a banking arrangement is inefficient because wholesale bank depositors prefer to use bonds as a means of payment in transactions to avoid the risk of banking failure. Therefore, an optimal banking arrangement will assign all the government bonds for their depositors' withdrawal requests.

From Lemma 1 and Lemma 2, no bank holds government bonds as collateral to back their liabilities because of the leverage constraint on retail banks and the risk of banking failure on wholesale banks, which help to eliminate some potential equilibrium candidates. In the rest of this section, I will start solving all other possible equilibrium equilibrium cases and then provide the conditions for each case.

As it will be clear later, there will be three equilibrium cases, depending on the relative supply of central bank liabilities and government debt. When reserves are plentiful relative to government bonds, wholesale banks lend to retail banks to indirectly hold reserves. Therefore, an active interbank market permits wholesale banks to use reserves as the ultimate collateral to back their deposit claims. In this scenario, there can be a no banking panic equilibrium or a partial banking panic equilibrium, depending on the size of interbank lending. Generally, with more reserves to support interbank lending, wholesale banks can offer more attractive deposit claims, effectively mitigating the severity of wholesale banking panic. Finally, a full banking panic equilibrium occurs when the interbank market becomes inactive, and this is the case when reserves become relatively scarce.

5.1 Active Interbank Market and No Banking Panic

In this first case, retail banks invest in central bank reserves, while wholesale banks acquire government bonds to cover their depositors' withdrawal requests. The interbank market operates actively, enabling wholesale banks to generate sufficient profits from interbank lending. This profitability allows them to offer attractive deposit claims, preventing banking panic from arising. Figure 1 illustrates the balance sheets of private banks under these conditions, with the likelihood of banking panic being zero, i.e., $\eta = 0$.

(a) Retail Bank

Asset	Liability
reserves: m	deposit claims: d^r
	interbank borrowing: ℓ^r

(b) Wholesale Bank

Asset	Liability
government bonds: b^w	bond withdrawals: $[\rho + (1-\rho)\eta]b'$
interbank lending: ℓ^w	deposit claims: $(1-\rho)(1-\eta)d^w$

Figure 1: Banks' Balance Sheets with Active Interbank Market

As shown in Figure 1, retail banks use reserves to secure their deposit claims and interbank borrowing, while wholesale banks secure deposit claims with claims on interbank lending. From (21) and (24),

$$u'\left(c_s^r\right) = 1 - \delta + \delta u'\left(c_s^w\right). \tag{30}$$

Retail banks pay the same rate of return on interbank borrowing and deposit claims, which equals, $u'(c_s^r)$, the marginal utility for a retail bank depositor exchanging with such claims. Moreover, this return also reflects the marginal benefit of wholesale banks lending in the interbank market, $1-\delta+\delta u'(c_s^w)$. Otherwise, arbitrage opportunities exist, prompting banks to capitalize on them until they are fully exploited.

As depicted in Figure 2, I will use the *no-arbitrage condition* to denote the locus of equation (30) in c_s^w-c_s^r space. The upward-sloping no-arbitrage condition indicates that both retail and wholesale banks could support more exchanges with deposit claims if they are less financially constrained.

As the ultimate collateral, reserves secure deposit claims directly and indirectly in retail and wholesale sectors, and depositors use these claims when transacting with all-payment producers. Therefore, from binding leverage constraints (10) and collateral constraint (14), market clearing conditions (25) and (27), the first order condition (19) and no-arbitrage condition (30), it will be convenient to write the following aggregate collateral constraint:

$$\theta\bar{m} = \alpha c_s^r\left[1-\theta+\theta u'(c_s^r)\right] + (1-\alpha)(1-\rho)\,c_s^w\left[1-\theta\delta+\theta\delta u'(c_s^w)\right]. \tag{31}$$

Equation (31) equates the supply of reserves (on the left-hand side) to its demand (on the right-hand side), accounting for leverage requirement reflected in θ.

Similarly, I will use the *aggregate collateral constraint* to denote the locus of equation (31) in Figure 2. Retail banks can use reserves to secure their deposit claims or claims on interbank lending that finally back deposit claims issued by wholesale banks, resulting in a downward-sloping aggregate collateral constraint.

Finally, from Lemma 1 and Lemma 2, wholesale bank depositors use all outstanding government bonds as a means of payment when transacting producers who accept only government bonds. Thus, the fiscal policy rule (7), the market clearing condition (26), the first order condition (22), and the binding nonnegative constraint $b^w-\left[\rho+(1-\rho)\eta\right]b' \geq 0$

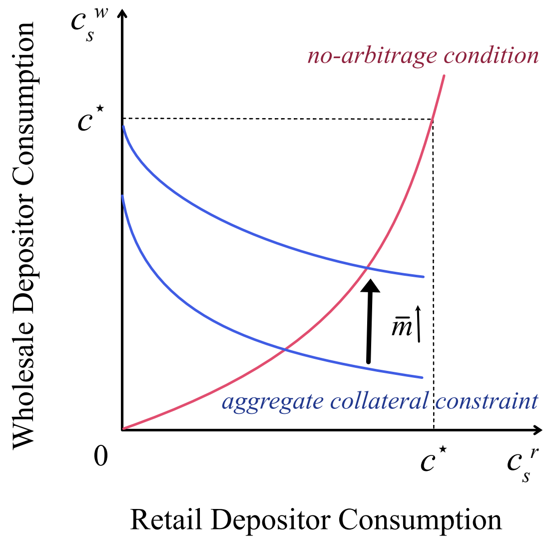

Figure 2: No Banking Panic Equilibrium

Notes: c_s^r and c_s^w are consumption for retail and wholesale bank depositors exchange with deposit claims. An increase in reserve supply works like the income effect, relaxing the aggregate collateral constraint and facilitating exchanges supported by reserves.

imply the following equilibrium bond market clearing condition:

$$\hat{b} - \bar{m} = (1 - \alpha)\, \rho c_s^b u'\left(c_s^b\right). \tag{32}$$

Here, the left-hand side represents the government bonds circulated in the private sector, which is the fiscal authority's total bond supply minus the government bonds held by the central bank to back its reserves, and the right-hand side represents private banks' demand for them.

There is market segmentation, given fiscal policy $\hat{b}$ and monetary policy $\bar{m}$, the bond market clearing condition (32) solely determines the consumption for each wholesale bank depositor trading with government bonds. Moreover, the no-arbitrage condition (30) and the aggregate collateral constraint (31) jointly determine the consumption for retail and wholesale depositors exchanging with deposit claims.

Proposition 1. *Given a fiscal policy with $\hat{b}$ satisfies Assumption 1, (30) and (31) solve for an unique no banking panic equilibrium allocation (c_s^r, c_s^w) with $0 < c_s^r < c^\star$ and*

$0 < c_s^w < c^\star$.

Proof. See Appendix A. $\square$

Graphically, the assumption regarding the scarcity of government debt constrains the downward-sloping aggregate collateral constraint to be relatively close to the original point. Combined with the upward-sloping no-arbitrage condition, this results in a unique equilibrium where consumption does not exceed its satiated level $c^\star$. Then, I can solve for other equilibrium outcomes based on the consumption allocation. For example, I can use the first order conditions for retail and wholesale banks' problems, which, as discussed before, are consumption-based asset pricing kernels, to pin down the equilibrium interest rates, which satisfy

$$r^m > r^\ell \geq r^b. \tag{33}$$

This interest rate structure provides the rationale for an active interbank market, as retail banks are willing to borrow from wholesale banks to pursue a higher interest rate on reserves.

	∂c_s^r	∂c_s^w	∂c_s^b	∂r^m	∂r^ℓ	∂r^b
$\partial \bar{m}$	$+$	$+$	$-$	$+$	$+$	$-$

Table 1: Effects of Monetary Policy with No Banking Panic

Monetary Policy Consider an increase in the size of the central bank's balance sheet, which reflects an increase in the reserve supply $\bar{m}$. As in Figure 2, c_s^r and c_s^w increase as the aggregate collateral constraint shifting upward. Expanding the size of the central bank's balance sheet works like an income effect, which improves exchanges with deposit claims by relaxing the retail bank's leverage constraint and the wholesale bank's collateral constraint. As a result, the interest rate on reserves and the interest rate on interbank lending rise. However, an increase in $\bar{m}$ crowds out government debt, so depositors who choose to withdraw can obtain fewer government bonds. In response, exchanges supported by government bonds decrease, and so does the interest rate on government bonds.

5.2 Active Interbank Market and Partial Banking Panic

In this case, the interbank bank market remains active. However, there are no adequate interbank activities to support attractive enough deposit claims issued by wholesale banks. Wholesale bank depositors are indifferent between withdrawing government bonds and exchanging with deposit claims as in (17), resulting in a partial banking panic. The market structure is similar to the one studied before, except that now a fraction η of wholesale bank depositors, who can exchange with high-yielding deposit claims, flee to safe government bonds to avoid the risk of banking failure. Figure 1 depicts banks' balance sheets.

I will follow the same procedure to solve this equilibrium as in the case with no banking panic, and the difference comes from the depositors' panicky behavior. Now, the aggregate collateral constraint becomes

$$\theta\bar{m} = \alpha c_s^r \left[1 - \theta + \theta u'\left(c_s^r\right)\right] + (1-\alpha)(1-\rho)(1-\eta)\, c_s^w \left[1 - \theta\delta + \theta\delta u'\left(c_s^w\right)\right]. \tag{34}$$

Compared to its no banking panic counterpart, a fraction η of the wholesale bank depositors who can trade with deposit claims flee to government bonds.

On the other side of the market, the government bonds clearing condition becomes

$$\hat{b} - \bar{m} = (1-\alpha)\left[\rho + (1-\rho)\eta\right] c_s^b u'\left(c_s^b\right). \tag{35}$$

Equation (35) shows that an additional fraction $(1-\rho)\eta$ of wholesale bank depositors choose to withdraw and trade with bonds.

I can use the no-arbitrage condition (30), the aggregate collateral constraint (34), the bond market clearing condition (35), and condition (17) to pin down a partial banking panic equilibrium with consumption allocation $\left(c_s^r, c_s^b, c_s^w\right)$ and the likelihood of banking panic η.

Lemma 3. (17), (34) *and* (35) *implicitly define a function* $c_s^r = h\left(c_s^w\right)$*, where* $h'\left(c_s^w\right) < 0$*, i.e., the aggregate collateral constraint is downward sloping in the* c_s^r*-*c_s^w *space.*

Proof. See Appendix A. □

Proposition 2. *Given Assumption 1, the aggregate collateral constraint characterized by function* $c_s^r = h\left(c_s^w\right)$ *in Lemma 3 and the no-arbitrage condition characterized by* (30) *solve a unique allocation* $\left(c_s^r, c_s^w\right)$ *with* $0 < c_s^r < c^\star$ *and* $0 < c_s^w < c^\star$ *in a partial banking panic equilibrium.*

Proof. See Appendix A. □

Same as before, I use the aggregate collateral constraint and the no-arbitrage condition to solve for a unique equilibrium allocation for depositors exchanging with deposit claims. Lemma 3 states that the aggregate collateral constraint is still downward sloping as illustrated in Figure 3, reflecting the fact that reserves work as the ultimate collateral to support both retail and wholesale bank depositors' exchanges with deposit claims. Finally, the interest rate structure remains the same as with the previous case as in (33).

Monetary Policy Differences between the partial banking panic equilibrium and its no banking panic counterpart come from wholesale depositors' panicky behavior, reflected in η. When a partial panic occurs, more wholesale bank depositors opt to withdraw government bonds, even if they meet a producer who accepts higher-yielding deposit claims. Therefore, wholesale banking panic affects the demand side of markets for central bank liabilities and government debt, finally affecting the equilibrium outcomes.

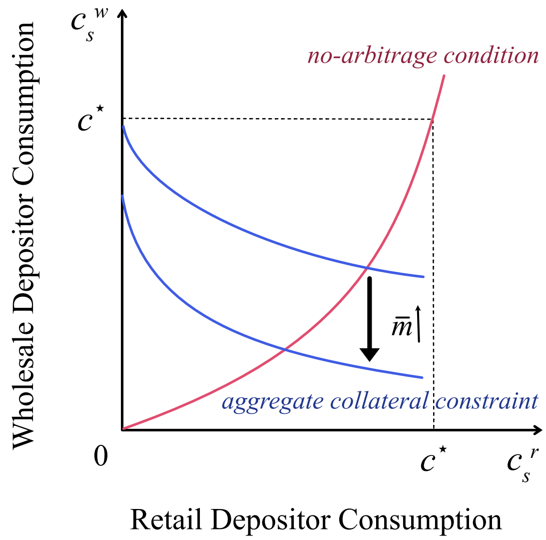

Figure 3: Partial Banking Panic Equilibrium

Notes: c_s^r and c_s^w are consumption for retail and wholesale bank depositors exchange with deposit claims. The aggregate collateral constraint shifts downward because banks have to serve more depositors, given that fewer depositors flee to government bonds in response to an increase in reserve supply. As a result, each depositor shares fewer reserves to support their transactions.

From now on, consider the utility function u satisfies the following CRRA function form $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$ with $0 < \sigma < 1$ for simplicity, which is consistent with assumptions mentioned earlier. The following proposition shows the effects of expanding the central bank's balance sheet size.

Proposition 3. *Expanding the size of the central bank's balance sheet reduces the likelihood of banking panic, i.e., $\frac{\partial \eta}{\partial \bar{m}} < 0$. However, depositors, no matter which means of payment they use, obtain fewer consumption goods in exchange with producers, i.e., $\frac{\partial c_s^r}{\partial \bar{m}} < 0$, $\frac{\partial c_s^w}{\partial \bar{m}} < 0$ and $\frac{\partial c_s^b}{\partial \bar{m}} < 0$.*

Proof. See Appendix A. □

Expanding the size of the central bank's balance sheet crows out government debt. As a result, fewer government bonds are circulating in the private sector, and each depositor obtains fewer bonds in exchange with producers. Also, government bonds become a less attractive option, and the likelihood of a banking panic η decreases. On the other side of

	∂c^r_s	∂c^w_s	∂c^b_s	$\partial \eta$	∂r^m	∂r^ℓ	∂r^b
$\partial \bar{m}$	$-$	$-$	$-$	$-$	$-$	$-$	$-$

Table 2: Effects of Monetary Policy with Partial Banking Panic

the market, although there are more reserves to back banks' liabilities, more depositors opt to exchange them with these bank liabilities, and the value of deposit claims each depositor can use in exchanges decreases. Figure 3 illustrates the effects of an increase in reserve supply for retail bank depositors and wholesale bank depositors trade with deposit claims, and banks are more financially constrained because they have to serve more depositors in this equilibrium.

5.3 Inactive Interbank Market and Full Banking Panic

From Lemma 2, wholesale banks only purchase government bonds for their depositor's withdrawal requests. Therefore, when the interbank market becomes inactive, wholesale banks hold no assets to secure deposit claims, implying a full banking panic with $\eta = 1$. Figure 4 depicts banks' balance sheets in this case.

(a) Retail Bank

Asset	Liability
reserves: m	deposit claims: d^r

(b) Wholesale Bank

Asset	Liability
government bonds: b^w	bond withdrawals: b'

Figure 4: Banks' Balance Sheets with Inactive Interbank Market

One interpretation of this equilibrium is that wholesale banks resemble narrow banks by investing only in safe government bonds. Wholesale bank depositors always request safe government bonds because they are worried about potential bank failures. In response, competitive wholesale banks self-select to be narrow. In this paper, competitive banks maximize their depositors' expected utility, and there is no financial cost for wholesale banks. The equilibrium outcomes would be the same by allowing depositors to invest in government bonds themselves. Therefore, an equilibrium with banking panic and no active interbank market also characterizes the features of disintermediation in the wholesale banking sector.

Wholesale banks demand no collateral, so reserves are only used to back retail bank's deposit claims. Thus, the aggregate collateral constraint becomes

$$\theta \bar{m} = \alpha c_s^r \left[1 - \theta + \theta u' \left(c_s^r\right)\right], \tag{36}$$

which is indeed retail banks' leverage constraint. Same as before, given reserve supply $\bar{m}$, constraint (36) solve for a unique retail bank depositors' consumption value.

From (35), take $\eta = 1$ as there is full banking panic, the bond market clearing condition becomes

$$\hat{b} - \bar{m} = (1 - \alpha) c_s^b u' \left(c_s^b\right), \tag{37}$$

because all wholesale bank depositors choose to withdraw bonds.

Finally, although there is no interbank lending and borrowing, no-arbitrage condition (30) still holds to clear the interbank market.

Monetary Policy. The effects of an increase in reserve supply in a full banking panic equilibrium are almost the same as those in its no panic counterpart, as the channel for reducing the likelihood of banking panic shuts down. By relaxing retail banks' leverage constraint, their depositors obtain more deposit claims in exchange, increasing their consumption. Also, expanding the central bank's balance sheet crowds out government debt, decreasing the value of government bonds each wholesale bank depositor can obtain. Therefore, such a policy hurts wholesale bank depositors who rely on government bonds. Interest rates respond accordingly, and Table 3 summarizes the results.

	∂c_s^r	∂c_s^w	∂c_s^b	∂r^m	∂r^ℓ	∂r^b
$\partial \bar{m}$	$+$	$+$	$-$	$+$	$+$	$-$

Table 3: Effects of Monetary Policy with Inactive Interbank Market

5.4 How Monetary Policy Determines the Type of Equilibrium

I will show how monetary policy $\bar{m}$ determines the type of equilibrium given a fiscal policy $\hat{b}$ and the probability of wholesale banking failure $1 - \delta$, and Figure 5 provides a visual guide to the results.

Proposition 4. *Given a fiscal policy $\hat{b}$ satisfying Assumption 1, there are two thresholds $\bar{m}_1$ and $\bar{m}_2$ for the supply of central bank reserves with $0 < \bar{m}_1 < \bar{m}_2 < \hat{b}$, where*

1. *a full banking panic occurs with a scarce reserve supply such that $\bar{m} \in (0, \bar{m}_1)$;*

2. *a partial banking panic occurs with a moderate reserve supply such that $\bar{m} \in [\bar{m}_1, \bar{m}_2]$;*

3. *no banking panic occurs with an abundant reserve supply such that $\bar{m} \in (\bar{m}_2, \hat{b})$.*

Moreover, these critical values increase with the probability of wholesale banking failure $1 - \delta$, i.e., $\frac{\partial \bar{m}_1}{\partial \delta} < 0$ and $\frac{\partial \bar{m}_2}{\partial \delta} < 0$.

Proof. See Appendix A. □

The central bank determines the equilibrium type by adjusting the relative supply of central bank liabilities and government debt. Generally, a more extensive reserve outstanding reduces the likelihood of wholesale banking panic. On the one hand, wholesale

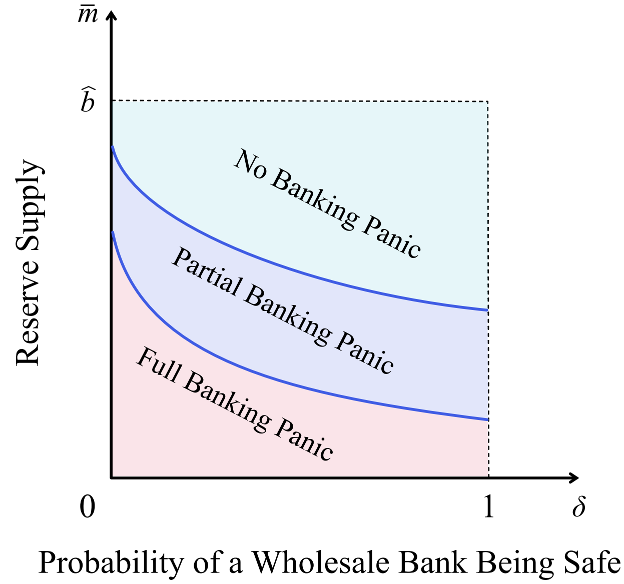

Figure 5: How Monetary Policy Determines the Type of Equilibrium

banks can offer more attractive deposit claims since reserves ultimately back these claims. On the other hand, the interest rate on government bonds decreases because they become relatively scarce, and this is a result of the central bank increasing its holding of government bonds to back its reserves. As a result, in equilibrium, fewer wholesale bank depositors flee to government bonds. Specifically, a full banking panic occurs when the reserve supply is below a certain lower threshold, no banking occurs when the reserve supply surpasses a higher threshold, and a partial banking panic occurs when the reserve supply lies between these two thresholds.

Finally, the probability of wholesale banking failure can affect the effectiveness of monetary policy, and the higher such probability is, a larger supply of reserves is required to mitigate a wholesale banking panic.

Finally, the probability of wholesale banking failure can affect the effectiveness of monetary policy. Specifically, a higher probability necessitates a larger supply of reserves to mitigate a wholesale banking panic effectively.

6 Overnight Reverse Repurchase Agreement

I have shown that expanding the central bank's balance sheet size can effectively mitigate wholesale banking panic. However, it crowds out government bonds, decreasing the trading volume for transactions settled with them. Furthermore, it can be harmful even for transactions supported by central bank reserves. The issue here is that the crowding-out effect makes government bonds scarce, resulting in a lower return on bonds. Therefore, too many depositors opt for bank claims, exacerbating the tension in the banking sector.

In this section, I will consider the Fed's existing overnight reverse repurchase agreement (ON-RRP) facility, which makes interest-bearing claims available to wholesale banks. Retail banks bear "balance sheet costs" associated with banking regulations (Kim, Martin, & Nosal, 2020; Williamson, 2019), as assets held by retail banks can secure fewer liabilities than wholesale banks because of their leverage constraint. The central bank can avoid such costs by replacing reserves with ON-RRPs, which increases the effective collateral supply. However, it does not crowd out government bonds, so the return on bonds does not necessarily decrease. As it will be clear later, in the absence of the crowding-out effect, substituting reserves with ON-RRPs can mitigate wholesale banking panic and facilitate transactions supported by government bonds and central bank liabilities.

Consider the central bank introducing the ON-RRPs, where retail and wholesale banks can invest in them and use them to back their deposit claims. However, ON-RRPs can only circulate among banks, and depositors cannot use them as a means of payment like government bonds. Let r^o denote the gross real interest rate on ON-RRPs, and the market will endogenously determine this interest rate in equilibrium. Confine attention to the case with an active interbank market and partial banking panic to study the effects of the central bank replacing reserves with ON-RRPs. Figure 6 depicts the banks' balance sheets for the case considered here. Retail banks only invest in reserves because wholesale banks prefer to hold ON-RRPs directly rather than lend to retail banks to get indirect access to them, considering retail banks face the balance costs of holding assets.

(a) Retail Bank

Asset	Liability
reserves: m	deposit claims: d^r
	interbank borrowing: ℓ^r

(b) Wholesale Bank

Asset	Liability
government bonds: b^w	bond withdrawals: $[\rho + (1-\rho)\eta]b'$
interbank lending: ℓ^w	deposit claims: $(1-\rho)(1-\eta)d^w$
ON-RRPs: o	

Figure 6: Banks' Balance Sheets with ON-RRP Facility

The procedure to solve this equilibrium is similar to the one studied before, and I will highlight the differences and interpret them in the context of ON-RRPs. For example, one more asset pricing kernel emerges in the wholesale bank's problem, where

$$r^o \left[1 - \delta + \delta u' \left(c_s^w\right)\right] = 1. \tag{38}$$

From (24), the real interest rate on ON-RRPs equals the real interest rate on interbank lending with $r^\ell = r^o$, reflecting their use as collateral to back the deposit claims, with appropriately adjusted by the risk of banking failure. The interest rate structure becomes

$$r^m > r^o = r^\ell \geq r^b, \tag{39}$$

which is consistent with private banks' portfolio decisions as in Figure 6.

After introducing ON-RRPs, monetary policy has two dimensions. First, the central bank chooses the size of its balance sheet, denoted by $s = \bar{m} + \bar{o}$, by adjusting the relative supply of central bank liabilities and government debt as the case with only reserves.

Then, it determines the composition of the central bank's liabilities, described by $\bar{o}$, the ON-RRPs outstanding.

With ON-RRPs, I can rewrite the aggregate collateral constraint (34) as

$$\theta s + (1-\theta)\,\bar{o} = \alpha c_s^r \left[1-\theta+\theta u'\left(c_s^r\right)\right] + (1-\alpha)\,(1-\rho)\,(1-\eta)\,c_s^w \left[1-\theta\delta+\theta\delta u'\left(c_s^w\right)\right]. \tag{40}$$

The right-hand side of (40) remains the same, which shows the demand for central bank liabilities as collateral to secure the transactions with deposit claims from retail and wholesale banks. The left-hand side is the effective supply of central bank liabilities as collateral, where $\theta k + (1-\theta)\,\bar{o} = \theta\bar{m} + \bar{o}$ because only reserves, which are restricted to the retail banking sector, are subject to retail bank's leverage requirement.

Finally, the government bonds clearing condition becomes

$$\hat{b} - s = (1-\alpha)\left[\rho + (1-\rho)\,\eta\right] c_s^b u'\left(c_s^b\right), \tag{41}$$

which replaces $\bar{m}$ with s, the total supply of central bank liabilities consists of reserves and ON-RRPs.

Given a fiscal policy $\hat{b}$, monetary policy s and $\bar{o}$, equations (17), (30), (40) and (41) solve for an equilibrium. An expansion in the size of the central bank's balance sheet (s) works the same as with no ON-RRP. The interesting policy experiment involves holding s constant and substituting reserves for ON-RRPs, i.e., increase $\bar{o}$ for a given s.

Proposition 5. *Fix the size of the central bank's balance sheet, an increase in the supply of ON-RRPs reduces the likelihood of wholesale banking panic, i.e.,* $\frac{\partial \eta}{\partial \bar{o}} < 0$, *and promoting exchanges for depositors, i.e.,* $\frac{\partial c_s^r}{\partial \bar{o}} > 0$, $\frac{\partial c_s^w}{\partial \bar{o}} > 0$ *and* $\frac{\partial c_s^b}{\partial \bar{o}} > 0$.

Proof. See Appendix A. □

The ON-RRP facility acts to increase the effective collateral supply, which is $\theta s +$

$(1-\theta)\,\bar{o}$ on the left-hand side of (34). When the central bank shifts its liabilities from retail to wholesale banks, it increases the effective collateral supply by avoiding the balance sheet costs associated with retail banks' leverage requirement. Like an increase in the supply of central bank liabilities s, it reduces the likelihood of banking panic. However, replacing reserves with ON-RRPs does not crowd out government debt, allowing it to promote exchanges for depositors trading with government bonds as fewer depositors flee to them. Meanwhile, although more depositors opt for bank liabilities, in the absence of the crowding-out effect, the amount is less than the one in an increase in s. As a result, retail and wholesale banks can support a larger volume of exchanges for their depositors. Therefore, unlike an increase in s, the ON-RRP facility can promote exchanges for depositors.

7 Conclusion

The key result of this paper is that expanding the size of the central bank's balance sheet mitigates wholesale banking panic, but this can harm welfare. It crowds out government bonds, resulting in a low return on bonds. As a result, not just transactions settled with government bonds, transactions supported by bank liabilities, ultimately backed by central bank liabilities, also decrease when too many depositors opt for these liabilities. I show that the overnight reverse repurchase agreement facility can avoid such a damaging crowding-out effect. Therefore, it acts to mitigate the banking panic and also promote transactions.

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Appendix

A Omitted Proofs

Proof of Lemma 1. Assume the contrary, i.e., $\lambda^r = 0$. Then, from the first order conditions (20) and (21) in the retail bank's problem, I obtain

$$r^\ell < r^b. \tag{A.1}$$

However, from the first order conditions (23) and (24) in the wholesale bank's problem,

$$r^\ell \geq r^b, \tag{A.2}$$

which is a contradiction. $\square$

Proof of Lemma 2. Assuming the contrary, i.e., $b^w - [\rho + (1-\rho)\,\eta]\, b' > 0$, which implies $\lambda^w = 0$. Then, from first order conditions (22) and (23),

$$u'\left(r^b b'\right) = 1 - \delta + \delta u'\left(d^w\right), \tag{A.3}$$

which implies

$$u\left(r^b b'\right) > u\left(d^w\right), \tag{A.4}$$

given the assumptions on utility function $u(\cdot)$ and the fact that $u'\left(d^w\right) > 1$ because of the binding collateral constraint.

Condition (A.4) implies an equilibrium with full banking panic. However, $b^w - [\rho + (1-\rho)\,\eta]\, b' > 0$ contradicts the binding collateral constraint (14) given $\eta = 1$. $\square$

Corollary A.1. *The no-arbitrage condition (30) implicitly defines a function:*

$$c_s^r = f\left(c_s^w\right), \tag{A.5}$$

where $f'\left(d^w\right) > 0$, $\lim_{d^w \to 0} f\left(d^w\right) = 0$, *and* $f\left(c^\star\right) = c^\star$.

Proof of Proposition 1. The aggregate collateral constraint (31) implicitly defines a function

$$c_s^r = g\left(c_s^w\right), \tag{A.6}$$

where $g'\left(c_s^w\right) < 0$, because the expressions $c\left[1 - \theta + \theta u'\left(c\right)\right]$ and $c\left[1 - \theta\delta + \theta\delta u'\left(c\right)\right]$ are strictly increasing in c given that $-\frac{cu''(c)}{u'(c)} < 1$.

Given that $\lim_{c \to 0} cu'\left(c\right) = 0$, for all $\bar{m} > 0$, take the limit of $c_s^w \to 0$ on both side of (31), I have:

$$\theta\bar{m} = \lim_{c_s^w \to 0} \alpha g\left(c_s^w\right)\left[1 - \theta + \theta u'\left(g\left(c_s^w\right)\right)\right], \tag{A.7}$$

which further implies $\lim_{c_s^w \to 0} g\left(c_s^w\right) > 0$, again, because the expression $c\left[1 - \theta + \theta u'\left(c\right)\right]$ is strictly increasing in c.

Furthermore, from (31), when $c_s^w = c^\star$ such that $u'(c^\star)$, $c_s^r = g(c^\star)$ solves the following equation:

$$\theta\bar{m} = \alpha g\left(c^\star\right)\left[1 - \theta + \theta u'\left(g\left(c^\star\right)\right)\right] + \left(1 - \alpha\right)\left(1 - \rho\right)c^\star. \tag{A.8}$$

For any feasible monetary policy $\bar{m} \in \left(0, \hat{b}\right)$ with a fiscal policy $\hat{b}$ that satisfies Assumption 1, (A.8) implies

$$g\left(c^\star\right) < c^*. \tag{A.9}$$

Otherwise, the right-hand side of (A.8) is greater than $\alpha c^{\star} + (1-\alpha)(1-\rho) c^{\star}$, and there is no feasible $\bar{m}$ to support such equilibrium given Assumption 1.

Finally, the aggregate collateral constraint defined by (A.6), together with the no-arbitrage condition, characterized by (A.5) as in Corollary A.1 jointly solve for a unique equilibrium allocation (c_s^w, c_s^r) given that $f'(c_s^w) > 0$ and $g'(c_s^w) < 0$,

$$\lim_{c_s^w \to 0} g(c_s^w) > \lim_{c_s^w \to 0} f(c_s^w) = 0, \tag{A.10}$$

and

$$g(c^{\star}) < c^{*} = f(c^{\star}). \tag{A.11}$$

□

Proof of Lemma 3. Totally differentiating (17), (34) and (35) with respect to c_s^w, I obtain

$$u'\left(c_s^b\right) \frac{\partial c_s^b}{\partial c_s^w} - \delta u'(c_s^w) = 0, \tag{A.12}$$

$$\alpha F_1'(c_s^r) \frac{\partial c_s^r}{\partial c_s^w} - (1-\alpha)(1-\rho) F_2(c_s^w) \frac{\partial \eta}{\partial c_s^w} + (1-\alpha)(1-\rho)(1-\eta) F_2'(c_s^w) = 0, \tag{A.13}$$

$$[\rho + (1-\rho)\eta] F_3'\left(c_s^b\right) \frac{\partial c_s^b}{\partial c_s^w} + (1-\rho)\eta F_3\left(c_s^b\right) \frac{\partial \eta}{\partial c_s^w} = 0, \tag{A.14}$$

where

$$F_1(c) = c\left[1-\theta+\theta u'(c)\right], \tag{A.15}$$

$$F_2(c) = c\left[1-\theta\delta+\theta\delta u'(c)\right], \tag{A.16}$$

$$F_3(c) = c u'(c). \tag{A.17}$$

Note that $u'(c) > 0$ for all $c \geq 0$, and $F_i'(c) > 0$, $\forall i \in \{1,2,3\}$ given $-\frac{cu''(c)}{u'(c)} < 1$.

Therefore, from (A.13)-(A.14), I have

$$\frac{\partial c_s^b}{\partial c_s^w} > 0, \quad \frac{\partial \eta}{\partial c_s^w} < 0, \quad \frac{\partial c_s^r}{\partial c_s^w} < 0. \tag{A.18}$$

That is, (17), (34) and (35) implicitly define a function $c_s^r = h(c_s^w)$ with $h'(c_s^w) < 0$, which is equivalent to say these equations implicitly define a downward sloping aggregate collateral constraint in the c_s^r-c_s^w space. □

Proof of Proposition 2. The goal is to show that $c_s^r = h(c_s^w)$ and $c_s^r = f(c_s^w)$ solve for a unique allocation (c_s^w, c_s^r) in the c_s^r-c_s^w space, where the first equation, as in Lemma 3, characterizes the aggregate collateral constraint and the second equation, as in Corollary A.1, characterizes the no-arbitrage condition.

Given that $\lim_{c\to 0} cu'(c) = 0$, take limit $c_s^w \to 0$ on both side of (34), I have:

$$\theta \bar{m} = \lim_{c_s^w \to 0} \alpha h(c_s^w)\left[1 - \theta + \theta u'(h(c_s^w))\right]. \tag{A.19}$$

For all $\bar{m} > 0$, this further implies

$$\lim_{d^w \to 0} h(c_s^w) > 0, \tag{A.20}$$

because $\lim_{c\to 0} cu'(c) = 0$ and $cu'(c) = 0$ is increasing in c.

Furthermore, when $c_s^w = c^\star$, $c_s^r = h(c^\star)$ solves the following equation:

$$\theta \bar{m} = \alpha h(c^\star)\left[1 - \theta + \theta u'(h(c^\star))\right] + (1-\alpha)(1-\rho)(1-\eta)c^\star. \tag{A.21}$$

The expression $c[1 - \theta + \theta u'(c)]$ strictly increases in c. Therefore, consider any feasible monetary policy $\bar{m} \in \left(0, \hat{b}\right)$ with a fiscal policy $\hat{b}$ that satisfies Assumption 1, (A.21) implies

$$h\left(c^{\star}\right) < c^{*}. \tag{A.22}$$

Now, I have $f'\left(c_s^w\right) > 0$, $h'\left(^w\right) < 0$, $\lim_{c_s^w \to 0} h\left(c_s^w\right) > \lim_{c_s^w \to 0} f\left(c_s^w\right) = 0$ and $h\left(c^{\star}\right) < c^{*} = f\left(c^{\star}\right)$. Therefore, a unique solution exists by the Intermediate Value Theorem. $\square$

Proof of Proposition 3. As I have shown in Proposition 2, there is a unique partial banking panic equilibrium can be solved by the no-arbitrage condition (30), the aggregate collateral constraint (34), the bond market clearing condition (35), and condition (17). Confine attention to a comparative statics analysis of such equilibrium in response to a change in monetary policy $\bar{m}$. For simplicity, consider a CRRA utility function

$$u\left(x\right) = \frac{x^{1-\sigma}}{1-\sigma}, \tag{A.23}$$

with $0 < \sigma < 1$, to be consistent with assumptions on the utility function. Totally differentiating these functions with respect to $\bar{m}$, then solving the system of linear equations, I obtain

$$\frac{\partial c_s^w}{\partial \bar{m}} = -\frac{\Omega_3}{\left[\Omega_4\Omega_3 + \Omega_5\left(\delta\frac{u''\left(c_s^w\right)}{u''\left(c_s^r\right)}\Omega_1 + \Omega_2\right)\right]} < 0, \tag{A.24}$$

$$\frac{\partial \eta}{\partial \bar{m}} = -\frac{\left(\delta\frac{u''\left(c_s^w\right)}{u''\left(c_s^r\right)}\Omega_1 + \Omega_2\right)}{\left[\Omega_4\Omega_3 + \Omega_5\left(\delta\frac{u''\left(c_s^w\right)}{u''\left(c_s^r\right)}\Omega_1 + \Omega_2\right)\right]} < 0, \tag{A.25}$$

$$\frac{\partial c_s^b}{\partial \bar{m}} = \delta\frac{u'\left(c_s^w\right)}{u'\left(c_s^b\right)}\frac{\partial c_s^w}{\partial \bar{m}} < 0, \tag{A.26}$$

$$\frac{\partial c_s^r}{\partial \bar{m}} = \delta\frac{u''\left(c_s^w\right)}{u''\left(c_s^r\right)}\frac{\partial c_s^w}{\partial \bar{m}} < 0, \tag{A.27}$$

where

$$\Omega_1 = \alpha \left[1 - \theta + \theta \left(1 - \sigma\right) u' \left(c_s^r\right)\right] > 0, \tag{A.28}$$

$$\Omega_2 = \left[\left(1 - \alpha\right)\left(1 - \rho\right)\left(1 - \eta\right)\left(1 - \theta\delta\right) + \left(1 - \alpha\right)\left(1 - \sigma\right)\theta\delta u' \left(c_s^w\right)\right] > 0, \tag{A.29}$$

$$\Omega_3 = \left(1 - \alpha\right)\left(1 - \rho\right)\left(1 - \theta\delta\right) c_s^w > 0, \tag{A.30}$$

$$\Omega_4 = \left(1 - \alpha\right)\left[\rho + \left(1 - \rho\right)\eta\right]\left(1 - \sigma\right)\delta u' \left(c_s^w\right) > 0, \tag{A.31}$$

$$\Omega_5 = \left(1 - \alpha\right)\left(1 - \rho\right)\delta c_s^w u' \left(c_s^w\right) > 0. \tag{A.32}$$

Therefore, expanding the size of the central bank's balance sheet reduces the likelihood of banking panic in a partial banking panic equilibrium. However, such a policy reduces consumption for all the depositors. □

Proof of Proposition 4. The proof takes two steps. In the first step, I will show there exist critical values $\bar{m}_1$ and $\bar{m}_2$, which solve for $\eta = 1$ and $\eta = 0$, respectively, in a partial panic equilibrium. Meanwhile, I will also show how these critical values change in response to a change in the probability of wholesale banking failure $1 - \delta$. From the comparative statics results, the likelihood of banking panic η is strictly decreasing in $\bar{m}$ in a partial banking panic equilibrium. Therefore, if such critical value exists, they satisfy $\bar{m}_1 < \bar{m}_2$. In the second step, I will show that a full banking panic exists when $\bar{m} < \bar{m}_1$ and a no banking panic equilibrium exits when $\bar{m} > \bar{m}_2$.

Step 1 First, consider a partial banking panic equilibrium with $\eta = 1$. Then, the critical value $\bar{m}_1$ and associated consumption allocation $\left(c_s^r, c_s^w, c_s^b\right)$ solve the following equations:

$$u' \left(c_s^r\right) = 1 - \delta + \delta u' \left(c_s^w\right), \tag{A.33}$$

$$\theta \bar{m}_1 = \alpha c_s^r \left[1 - \theta + \theta u' \left(c_s^r\right)\right], \tag{A.34}$$

$$\hat{b} - \bar{m}_1 = \left(1 - \alpha\right) c_s^b u' \left(c_s^b\right), \tag{A.35}$$

$$u\left(c_s^b\right) - \delta u\left(c_s^w\right) = 0. \tag{A.36}$$

Taking the limit as $\bar{m}_1 \to 0$, from (A.33) and (A.34), I have $c_s^r \to 0$ and $c_s^w \to 0$, given that $u'(c) > 0$, $u''(c) < 0$ and $\lim_{c\to 0} cu'(c) = 0$. However, from (A.35), $c_s^b > 0$. Then, the function in (A.36) does not hold with equality, as $u\left(c_s^b\right) - \delta u\left(c_s^w\right) > 0$ when $\bar{m}_1 \to 0$. Similarly, taking the limit as $\bar{m}_1 \to \hat{b}$, I obtain $c_s^b \to 0$ and $c_s^w > 0$. Then, given that $u(0) = 0$ and $u'(c) > 0$, I obtain $u\left(c_s^b\right) - \delta u\left(c_s^w\right) < 0$. By Intermediate Value Theorem, these exists $0 < \bar{m}_1 < \hat{b}$ which solves the above system of equations, and such $\bar{m}_1$ is unique by monotonicity.

Then, totally differentiating (A.33)-(A.36), with respect to δ, I can solve for

$$\frac{\partial c_s^w}{\partial \delta} = -\frac{(1-\alpha)\,\theta F_2'\left(c_s^b\right)\frac{u\left(c_s^w\right)}{u'\left(c_s^b\right)}}{(1-\alpha)\,\theta F_2'\left(c_s^b\right)\delta\frac{u'\left(c_s^w\right)}{u'\left(c_s^b\right)} + \alpha F_1'\left(c_s^r\right)\delta\frac{u''\left(c_s^w\right)}{u''\left(c_s^r\right)}} < 0, \tag{A.37}$$

$$\frac{\partial \bar{m}_1}{\partial \delta} = \frac{1}{\theta}\alpha F_1'\left(c_s^r\right)\delta\frac{u''\left(c_s^w\right)}{u''\left(c_s^r\right)}\frac{\partial c_s^w}{\partial \delta} < 0 \tag{A.38}$$

where $F_1(c) = c\left[1-\theta+u'(c)\right]$, and $F_2(c) = cu'(c)$, so that $F_1'(c) > 0$ and $F'(c) > 0$ given $-c\frac{u''(c)}{u'(c)} < 1$.

Similarly, consider a partial banking panic equilibrium with $\eta = 0$. Following the same procedure, I can show there exists a unique $\bar{m}_2$ that solves the following system of equations, and such $\bar{m}_2$ is strictly decreasing in δ.

$$u'\left(c_s^r\right) = 1 - \delta + \delta u'\left(c_s^w\right), \tag{A.39}$$

$$\theta\bar{m} = \alpha c_s^r\left[1-\theta+\theta u'\left(c_s^r\right)\right] + (1-\alpha)(1-\rho)\,c_s^w\left[1-\theta\delta+\theta\delta u'\left(c_s^w\right)\right], \tag{A.40}$$

$$\hat{b} - \bar{m} = (1-\alpha)\,\rho c_s^b u'\left(c_s^b\right), \tag{A.41}$$

$$u\left(c_s^b\right) - \delta u\left(c_s^w\right) = 0. \tag{A.42}$$

To conclude, a partial banking panic equilibrium exists when $\bar{m} \in [\bar{m}_1, \bar{m}_2]$ where

$0 < \bar{m}_1 < \bar{m}_2 < \hat{b}$, and the critical values $\bar{m}_1$ and $\bar{m}_2$ decrease in response to an increase in δ.

Step 2 First, when $\bar{m} = \bar{m}_1$, conditions (A.33)-(A.35) in a partial banking panic equilibrium are the same as the conditions solving for a full banking panic equilibrium. Consider a decrease in $\bar{m}$, from the comparative statics results as in Table 3, c_s^w decreases and c_s^b increases, which satisfying a full banking panic equilibrium as in condition (18). Similarly, when $\bar{m} = \bar{m}_2$, from the comparative statics results in Table 1, an increase in $\bar{m}$ results in a increase in c_s^w and a decrease in c_s^b, guaranteeing a no banking panic equilibrium as in condition (16). $\square$

Proof of Proposition 5. Totally differentiating equations (17), (30), (40) and (41) with respect to $\bar{o}$, I obtain:

$$1 - \theta = \alpha F_1' \left(c_s^r\right) \frac{\partial c_s^r}{\partial \bar{o}} - (1-\alpha)(1-\rho) F_2\left(c_s^w\right) \frac{\partial \eta}{\partial \bar{o}} + (1-\alpha)(1-\rho)(1-\eta) F_2'\left(c_s^w\right) \frac{\partial c_s^w}{\partial \bar{o}}, \tag{A.43}$$

$$0 = (1-\rho) F_3\left(c_s^b\right) \frac{\partial \eta}{\partial \bar{o}} + \left[\rho + (1-\rho)\eta\right] F_3'\left(c_s^b\right) \frac{\partial c_s^b}{\partial \bar{o}}, \tag{A.44}$$

$$u''\left(c_s^r\right) \frac{\partial c_s^r}{\partial \bar{o}} = \delta u''\left(c_s^w\right) \frac{\partial c_s^w}{\partial \bar{o}}, \tag{A.45}$$

$$u'\left(c_s^b\right) \frac{\partial c_s^b}{\partial \bar{o}} = \delta u'\left(c_s^w\right) \frac{\partial c_s^w}{\partial \bar{o}}, \tag{A.46}$$

where

$$F_1(c) = c\left[1 - \theta + \theta u'(c)\right], \tag{A.47}$$

$$F_2(c) = c\left[1 - \theta\delta + \theta\delta u'(c)\right], \tag{A.48}$$

$$F_3(c) = cu'(c). \tag{A.49}$$

Note that $u'(c) > 0$, $u''(c) < 0$ and $F_i'(c) > 0$ for $i \in \{1, 2, 3\}$ given $-c\frac{u''(c)}{u'(c)} < 1$. From

(A.45) and (A.46), $\frac{\partial c_s^r}{\partial \bar{o}}\frac{\partial c_s^w}{\partial \bar{o}} \geq 0$ and $\frac{\partial c_s^b}{\partial \bar{o}}\frac{\partial c_s^w}{\partial \bar{o}} \geq 0$, i.e., $\frac{\partial c_s^r}{\partial \bar{o}}$, $\frac{\partial c_s^w}{\partial \bar{o}}$, and $\frac{\partial c_s^b}{\partial \bar{o}}$ have the same sign of being positive or negative. From (A.44), $\frac{\partial \eta}{\partial \bar{o}}\frac{\partial c_s^b}{\partial \bar{o}} \leq 0$, i.e., $\frac{\partial \eta}{\partial \bar{o}}$ and $\frac{\partial c_s^b}{\partial \bar{o}}$ have different sign of being positive and negative. Therefore, it is easy to verity from (A.43), the only possibility is

$$\frac{\partial \eta}{\partial \bar{o}} < 0, \frac{\partial c_s^r}{\partial \bar{o}} > 0, \frac{\partial c_s^w}{\partial \bar{o}} > 0, \frac{\partial c_s^b}{\partial \bar{o}} > 0. \tag{A.50}$$

□