Should We Use the Central Bank Balance Sheet to Control Flight-to-Safety?

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Abstract

Wholesale banking panics have driven several flight-to-safety events in recent decades, raising concerns over financial stability. I study such events using a model with retail and wholesale banks, where the severity of wholesale banking panics is endogenously determined and captures the magnitude of flight-to-safety. The central bank manages its balance sheet actively. Asset returns react endogenously to the central bank's actions, influencing the usefulness of assets in transactions and, therefore, depositors' demand for safe assets and bank deposits. I show that a central bank balance sheet expansion mitigates wholesale banking panics, but doing so may reduce welfare because it lowers asset returns, limiting the usefulness of assets in transactions. However, the central bank can mitigate wholesale banking panics and improve welfare by expanding the reach of its interest-bearing liabilities to wholesale banks, as in the case of the Fed's overnight reserves repurchase agreements.

Key Words: central bank balance sheet, flight-to-safety, wholesale bank
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1 Introduction

In periods of financial distress, investors reallocate their portfolios from riskier assets toward safer ones. This so-called flight-to-safety phenomenon captures shifts in asset demand and amplifies financial instability. In particular, panics in the wholesale banking sector, which is beyond the coverage of regulations and supervision like Basel III, have driven several flight-to-safety events and are viewed as a major source of financial instability in the last decades (Bernanke, 2012, 2018; Gorton, 2010).[1] For example, during the 2008 Financial Crisis, the failure of Lehman Brothers sparked a panic in money market mutual funds, with over $400 billion withdrawn in September 2008. More recently, during the COVID-19 pandemic, investors moved over $100 billion from prime money funds to safe government funds in March 2020 (Sengupta & Xue, 2020).

The central bank balance sheet could be a tool for financial stability (Bernanke, 2016; Greenwood, Hanson, & Stein, 2016).[2] In particular, the central bank could expand its balance sheet through quantitative easing, providing safe short-term assets to crowd out private financial intermediaries' risky behavior in creating runnable short-term liabilities. However, these policies can significantly affect asset returns, especially during flight-to-safety episodes that involve large movements in asset markets (Baele, Bekaert, Inghelbrecht, & Wei, 2020).

I study the implications of central bank balance sheet expansions in a general equilibrium framework with endogenous flight-to-safety concerns, highlighting the importance of shifts

[1] Wholesale banks are financial institutions that engage in "credit intermediation involving entities and activities outside the regular banking system" (Financial Stability Board), such as money market funds and hedge funds. They serve institutional investors rather than individual businesses and consumers. Some papers also refer to these banks as shadow banks. S&P Global estimates that, at the end of 2022, these banks held about $63 trillion in financial assets in major global jurisdictions, representing 79% of global GDP. Gertler, Kiyotaki, and Prestipino (2016) and Ordoñez (2018) use the same definition for these unregulated financial institutions.

[2] Although central banks are concerned about financial instability, existing regulatory tools are limited in their effectiveness and scope of coverage, particularly in the wholesale banking setting. Moreover, these banks also fall beyond the scope of direct central bank crisis intervention. For example, the Dodd-Frank Act removed the U.S. Federal Reserve's authority to lend to wholesale banks (Fischer, 2016).

in depositors' asset demand in response to central bank interventions. The key finding is that while the central bank can mitigate flight-to-safety, doing so may reduce welfare. For instance, by purchasing government bonds through quantitative easing, the central bank lowers the returns of these safe assets. With decreased returns, this purchase stabilizes financial markets as depositors have less incentive to seek refuge in bonds. However, this purchase harms depositors who initially demand bonds by making them scarce.

Specifically, I develop a two-sector banking model with retail and wholesale banks, where, unlike retail banks, wholesale banks are unregulated (no leverage requirement) and have no direct access to central bank reserves. Banking panics can arise in the wholesale banking sector due to the risk of insolvency as in Gertler and Kiyotaki (2015). Depositors anticipate potential losses, withdraw their deposits, and flee to safe assets. The endogenously determined likelihood that a depositor chooses to withdraw captures the severity of wholesale banking panics or the magnitude of flight-to-safety. I evaluate the general equilibrium effects of central bank balance sheet expansions that alter the supply of bank deposits and safe government bonds while also considering how these policies shift asset demand by affecting depositors' withdrawal behavior.

Starting with the baseline case where reserves are the main central bank liability, I show that an expansion in the size of the central bank's balance sheet mitigates wholesale banking panics. The central bank purchases government bonds from the private sector with the issuance of new reserves, thus expanding its balance sheet. As bonds become scarce, this puts upward pressure on their prices and downward pressure on interest rates, making bonds less attractive to private agents. Additionally, the increased supply of reserves makes it easier for banks to acquire assets — only banks have the privilege of holding reserves, retail banks hold them directly, while wholesale banks can hold them indirectly by lending to retail banks. Consequently, wholesale banks can more effectively compete for depositors because the balance sheet expansion promotes the issuance of bank liabilities like bank deposits. Overall, government bonds become less attractive relative to deposits in response to this balance sheet expansion, mitigating wholesale banking panics.

Although an expansion in the size of the central bank's balance sheet mitigates banking panics, it does so at the cost of reducing welfare for depositors. This expansion harms depositors who demand government bonds as they obtain a lower return. It also harms wholesale bank depositors even as it promotes the issuance of wholesale deposits. This occurs because, by mitigating wholesale banking panics, a large number of depositors switch to wholesale banks. I show that the increase in wholesale deposit demand dominates the increase in its supply in equilibrium. Therefore, the return on these deposits decreases, harming wholesale bank depositors. Finally, the balance sheet expansion also harms retail bank depositors. Retail banks substitute their funding source from deposits to cheaper interbank loans — wholesale banks ask for a lower interest rate on these loans because they need more assets to back their increased deposit liabilities. The result is a decreased supply of retail deposits that lowers their return, which is harmful to depositors.

Clearly, the change in depositors' withdrawal behavior plays a critical role in determining the effects of central bank balance sheet expansions. For comparison, I also evaluate the effects of a balance sheet expansion in special scenarios where depositors' withdrawal behavior remains unchanged. For instance, if depositors always expect a higher return from wholesale deposits than government bonds, a banking panic will never occur, and withdrawals will not change in response to the expansion. As in the baseline scenario, this expansion reduces government bond supply, harming depositors who demand bonds by lowering their return. However, the expansion now benefits depositors who demand bank deposits. With no shift in depositors' demand, the increased reserve supply promotes the issuance of bank deposits, generating higher returns and benefiting depositors.

I also explore the role of other central bank liabilities in addition to reserves. Recall that only retail banks can hold reserves. This restriction matters for allocative efficiency because retail banks are subject to leverage requirements that limit their liability-to-asset ratios. By contrast, wholesale banks are not subject to such requirements, which allows them to support more liabilities relative to their assets than retail ones. Therefore, the central bank can improve market efficiency by reallocating more resources to wholesale banks. To achieve this, I introduce another central bank liability that plays a role similar to the US Fed's overnight reverse repurchase agreement (ON-RRP) facility. This facility is accessible to financial institutions that do not have reserve accounts at the central bank, like wholesale banks. For convenience, I call this liability ON-RRPs henceforth.[3]

I show that, like central bank balance sheet expansions, a swap of reserves for ON-RRPs mitigates wholesale banking panics. As more ON-RRPs become available to wholesale banks, they reduce their lending to retail banks to avoid inefficient leverage requirements. As a result, wholesale banks can more effectively compete for depositors, mitigating banking panics. Crucially, this swap between liabilities does not require adjusting the central bank's asset holdings and, therefore, does not change the supply of government bonds.

Unlike central bank balance sheet expansions, a swap of reserves for ON-RRPs improves welfare. The demand for safe government bonds decreases as this swap mitigates banking panics. This decreased demand for bonds and their unchanged supply implies a higher interest rate on them, benefiting depositors. Additionally, the rise in the interest rate on bonds leads to fewer depositors switching to wholesale deposits compared to scenarios involving central bank balance sheet expansions. Wholesale bank depositors also obtain higher returns from their banks in equilibrium. Finally, as wholesale banks pay higher returns on their liabilities, they seek higher returns on their interbank lending. In response, retail banks substitute funding sources from interbank borrowing to bank deposits, increasing the supply of retail deposits and benefiting retail bank depositors.

The two-sector banking model I construct is based on Williamson (2019) with the introduction of financial instability in the form of wholesale banking panics. Unlike

[3]Unlike government bonds, ON-RRPs can only circulate among financial institutions, so introducing them will not amplify financial instability by providing another safe harbor to depositors.

Diamond and Dybvig (1983), the banking panics in my model are unrelated to sequential service (i.e., the first-come-first-served basis). Instead, panics arise from random bank failures and depositors' lack of information about which banks will fail. Andolfatto and Nosal (2020) and Huang and Keister (2024) explain how self-fulfilling wholesale bank runs can occur without the sequential service constraint. More related to my work, Gertler, Kiyotaki, and Prestipino (2016) study wholesale banking panics in a general equilibrium framework, discussing the lender-of-last-resort and macroprudential policies. However, I focus on central bank balance sheet policies that directly alter the relative attractiveness of bank deposits versus safe assets to which depositors might flee. In this way, I contribute to a growing literature on the financial stability implications of central bank balance sheet policies such as quantitative easing (Bernanke, 2016; Woodford, 2016).

The central bank can enhance the stability and resilience of the financial system by providing more high-quality liquid assets, such as reserves. Bush, Kirk, Martin, Weed, and Zobel (2019) argue that an ample reserve supply helps banks meet their outflow needs and avoid the fire-sale effects during crises. Carlson, Duygan-Bump, Natalucci, Nelson, Ochoa, Stein, and Van den Heuvel (2016) and Greenwood, Hanson, and Stein (2016) show that central bank balance sheet policies can mitigate key threats to financial stability and highlight the usefulness of reserves and ON-RRPs, which aligns with my findings. My contribution is showing that financial stability does not necessarily imply an improvement in welfare. I do so by explicitly modeling the shift in demand for safe and risky assets in response to a policy change. Williamson (2022) shows a similar relationship between financial stability and welfare in the context of central bank digital currency (CBDC), but for a different reason. CBDC engenders financial instability, encouraging flight-to-safety as it is a convenient safe harbor that can be used in a wide range of transactions. However, this convenience makes such instability less disruptive, improving welfare.

The rest of the paper is organized as follows. I present the model in section 2 and define the equilibrium in section 3. In section 4, I solve three possible baseline equilibria with only reserves as the central bank liability and show how the size of the central bank's balance sheet determines the type of equilibrium. Section 5 is an analysis of the introduction of the ON-RRPs. The final section 6 concludes.

2 Model

This is a two-sector banking model with retail and wholesale banks. There are three periods, $t = 1, 2, 3$, with no time discounting between periods. There are three sets of private agents: a measure one of depositors representing households and institutional investors, a measure one of producers, and an infinite measure of private bankers who self-select into operating retail or wholesale banks. Private agents can work (h) and consume (c). A production technology allows them to convert labor to goods one-for-one. Finally, there is also a government consisting of a fiscal authority and a central bank.

Depositors work and deposit their earnings in period 1, consume in period 2, and work to pay taxes in period 3. Their preferences are captured by $-h_1 + u(c_2) - h_3$, where u is strictly increasing, strictly concave, and twice continuously differentiable, with $u(0) = 0$, $\lim_{c\to 0} u'(c) = \infty$, $\lim_{c\to\infty} u'(c) = 0$, $\lim_{c\to 0} cu'(c) = 0$, and $-c\frac{u''(c)}{u'(c)} < 1$ for $c \geq 0$. Producers and bankers are risk-neutral and profit-maximizing. Producers work to produce consumption goods for depositors in period 2 and consume the returns in period 3. Their payoffs are $-h_2 + c_3$. Bankers can work in period 1 to raise equity (i.e., sweat equity) and in period 3 to pay off their debts, and they consume their profits. Their payoffs are $c_1^B - h_1^B + c_3^B - h_3^B$.

The trading pattern and frictions are similar to Lagos and Wright (2005), even though this is a finite horizon model without introducing currency. Trade between depositors and producers takes place in bilateral exchanges in period 2, where a depositor makes a take-it-or-leave-it offer to the producer they meet in exchange for consumption goods. Depositors cannot consume the goods they produced in period 1 because consumption goods are perishable, so none of them can be carried across periods. Moreover, depositors must acquire assets in advance, such as deposit claims, to settle their exchanges because no producer accepts depositors' own IOUs, as agents are subject to limited commitment (no one can be forced to repay debts), so unsecured credit is not accepted. Depositors acquire assets in period 1, where a centralized Walrasian market allows depositors and banks to trade goods and assets. In particular, banks write deposit contracts with depositors in this period. In period 3, all the debts are redeemed, and agents consume their returns.

Assets and Collateral Technology There are two underlying assets: central bank reserves and government bonds. Central bank reserves are private banks' account balances with the central bank, while government bonds are issued by the fiscal authority. Both assets are issued in period 1. In period 3, the central bank pays a gross interest rate of r^m on reserves, while the fiscal authority pays a gross interest of r^b on bonds. Crucially, reserves and government bonds are not perfect substitutes. The main distinction is their degree of liquidity: reserves are less liquid because they are limited to retail banks, whereas government bonds are more liquid because they can be held by anyone, including depositors. In addition to the underlying assets, banks have access to a collateral technology that allows them to use these assets to secure their liabilities, such as deposit claims and interbank loans.

2.1 Banking

Retail and wholesale banks follow the structure in Williamson (2019). Retail banks resemble highly regulated depository institutions serving a fraction $\alpha \in (0, 1)$ of depositors representing individual consumers and businesses, where parameter α also captures the size of the banking sector. These banks have access to central bank reserves but are subject to a leverage requirement that limits their liability-to-asset ratio to $\theta \in (0, 1)$. Wholesale banks resemble less regulated financial institutions serving a fraction of $1 - \alpha$

of depositors representing institutional investors. These banks do not have access to central bank reserves and are not subject to leverage requirements. An interbank market allows wholesale banks to lend to retail banks to hold reserves indirectly.

I introduce instability in the wholesale banking sector, where the underlying banking insolvency induces some depositors to withdraw from wholesale banks as in Gertler, Kiyotaki, and Prestipino (2016). An exogenous fraction $1-\delta$ of wholesale banks experience a collapse in collateral technology in period 2, thereby defaulting on their liabilities and absconding with assets in period 3. I call these banks insolvent because they are losing the collateral value of their assets. The identity of insolvent banks is publicly observed ex-post but is ex-ante unknown to depositors. Depositors may become *panicky* and flee to safe assets because they must make their withdrawal decisions with imperfect information at the end of period 1, before any bank becomes insolvent.

Retail Banks Retail banks maximize profits by choosing deposit contracts and financial portfolios. Each retail bank offers a deposit contract (k^r, d^r) at $t = 1$, which requires the deposits of k^r units of the consumption goods at $t = 1$, in return for a tradeable claim (a tradeable IOU) to d^r units of goods at $t = 3$. Besides taking in deposits, retail banks borrow ℓ^r from the interbank market and invest in m units of reserves and b^r units of government bonds at $t = 1$.[4] Because of the leverage requirement, retail banks must raise equity (e) to finance at least a fraction $1 - \theta$ of their assets, and they do so by working at $t = 1$. The leverage requirement acts as a balance sheet cost because it increases retail banks' cost of growing their balance sheet (Kim, Martin, & Nosal, 2020; Martin, McAndrews, Palida, & Skeie, 2019; Williamson, 2019). Table 1 presents the retail bank's balance sheet.

[4] In principle, retail banks could lend to wholesale banks. In such a scenario, wholesale banks would invest in government bonds and use them as collateral to secure interbank borrowing. However, because of the risk of wholesale banking failure, retail banks' expected rate of return on such lending must be lower than the rate they directly invest in government bonds. As a result, no retail bank lends to wholesale banks.

Assets	Liabilities and Equity
reserves: m	deposit claims: d^r
government bonds: b^r	interbank borrowing: ℓ^r
	equity: e

Table 1: Retail Bank's Balance Sheet

Competition among retail banks drives them to offer contracts that maximize depositors' utility. A depositor with a contract (k^r, d^r) makes a take-it-or-leave-it offer to producers in exchange for $c_2 = d^r$ units of consumption goods with their deposit claim after depositing k^r with their bank. This results in a payoff

$$-k^r + u\left(d^r\right). \tag{1}$$

The leverage constraint of the bank is

$$\theta \underbrace{\left(r^m m + r^b b^r\right)}_{\text{return on assets}} \geq \underbrace{d^r + r^\ell \ell^r}_{\text{payment on liabilities}}. \tag{2}$$

If the leverage constraint holds, retail banks prefer paying off their liabilities rather than defaulting and losing their assets, even if they are subject to limited commitment.

Finally, a retail bank must make a non-negative profit to operate, so

$$\overbrace{k^r - d^r}^{\text{return from the deposits}} \underbrace{-m - b^r + \ell^r + r^m m + r^b b^r - r^\ell \ell^r}_{\text{return from financial portfolio}} \geq 0, \tag{3}$$

where $k^r, d^r, m, b^r \geq 0$. In equilibrium, free entry ensures this holds with equality.

To sum up, competitive retail banks choose deposit contract (k^r, d^r) and financial portfolio (m, b^r, ℓ^r) to maximize a representative retail depositor's utility (1), subject to the leverage constraint (2), non-negative profit constraint (3) and nonnegative constraints $k^r, d^r, m, b^r \geq 0$. The first order conditions for the retail bank's problem give the following equilibrium conditions

$$r^m \left[1 - \theta + \theta u'\left(d^r\right)\right] = 1, \tag{4}$$

$$r^b \left[1 - \theta + \theta u'\left(d^r\right)\right] + \lambda^r = 1, \tag{5}$$

$$r^\ell u'\left(d^r\right) = 1, \tag{6}$$

where λ^r denotes the Lagrange multiplier for the nonnegative constraint $b^r \geq 0$.[5] Equations (4) and (5) determine retail banks' demands for reserves (m) and government bonds (b^r) associated with their deposit contract. Equation (6) determines the interest rate a retail bank is willing to pay on interbank borrowing.

Wholesale Banks Like retail banks, wholesale banks maximize profits by choosing deposit contracts and financial portfolios. However, wholesale bank deposits' claims are less liquid than retail ones. A fraction ρ of producers do not accept these claims and instead accept only safe government bonds. This restriction about the acceptability of means of payment captures a certain type of demand for bonds, particularly their demand for wholesale payments.[6]

Depositors withdraw when they meet producers demanding bonds or when they are concerned about the bank's solvency. I refer to wholesale bank depositors who withdraw for the second reason as *panicking depositors*. Panicking depositors withdraw with an endogenous withdrawal probability η. In equilibrium, η also represents the fraction of panicking depositors who withdraw and flee to safe government bonds, capturing the severity of wholesale banking panics. The more severe the panics are, the higher the aggregate demand for government bonds, as more depositors flee to these safe assets.

A deposit contract in the wholesale banking sector is a triple (k^w, b', d^w). As before, k^w

[5]Other nonnegative constraints never bind because of market clearing conditions and standard assumptions such as Inada conditions for depositors' utility function.

[6]In practice, overnight repurchase agreements (repos), for which government bonds are the dominant collateral, involve a large volume of transactions. The repo markets indirectly support exchanges in goods and services, especially the exchanges at the wholesale level. Directly exchanging government bonds is a convenient shortcut to capturing wholesale payments supported by repo markets.

Assets	Liabilities
government bonds: b^w	government bonds: $[\rho+(1-\rho)\eta]b'$
interbank lending: ℓ^w	deposit claims: $(1-\rho)(1-\eta)d^w$

Table 2: Wholesale Bank's Balance Sheet

is the required deposit for each depositor at $t=1$. In return, a wholesale bank depositor can withdraw b' units of government bonds at the end of $t=1$ or opt for a tradeable deposit claim to d^w consumption goods at $t=3$. Therefore, wholesale banks provide liquidity insurance to their depositors in the spirit of Diamond and Dybvig (1983). Besides deposits, wholesale banks lend ℓ^w to retail banks and purchase in b^w units of government bonds, where part of the bonds they purchased are used for depositors' withdrawal requests. In contrast to retail banks, wholesale banks cannot hold reserves directly. They also do not raise equity as they face no leverage requirements. Table 2 presents the wholesale bank's balance sheet.

As with retail banks, wholesale banks maximize depositors' utility to compete for depositors. The expected utility of a wholesale bank depositor is

$$-k^w+[\rho+(1-\rho)\,\eta]\,u\left(r^b b'\right)+(1-\rho)\,(1-\eta)\,\delta u\,(d^w)\,. \tag{7}$$

Wholesale banks diversify across depositors, considering the potential banking failure with probability $1-\delta$ and depositors' withdrawal probability η.[7] After paying the required deposits k^w, a fraction $\rho+(1-\rho)\,\eta$ of the wholesale depositors withdraw government bonds and make a take-it-or-leave-it offer that exchanges for $r^b b'$ units of consumption goods from the producer they meet. The remaining $(1-\rho)\,(1-\eta)$ of depositors trade with deposit claims and obtain d^w units of consumption goods.

Wholesale banks are not subject to leverage requirements. However, a collateral constraint is required to ensure wholesale banks pay off their liabilities. Specifically,

[7]Banks serve many depositors, while each depositor can contact only one bank. Although depositors cannot diversify across banks, they can observe all banks' contracts and choose the optimal one.

the following constraint holds for a fraction δ of wholesale banks that remain solvent:

$$\underbrace{r^b\left[b^w-\left[\rho+(1-\rho)\,\eta\right]b'\right]+r^\ell \ell^w}_{\text{collateral value of assets}} \geq \underbrace{(1-\rho)\,(1-\eta)\,d^w}_{\text{payment on liability}}. \tag{8}$$

By contrast, insolvent wholesale banks lose the collateral value of their assets. Consequently, these insolvent banks will default on their deposit claims.

Finally, wholesale banks also require a nonnegative profit to operate,

$$\overbrace{k^w-(1-\rho)\,(1-\eta)\,\delta d^w}^{\text{return from deposits}}\underbrace{-b^w-\ell^w+r^b\left[b^w-\left[\rho+(1-\rho)\,\eta\right]b'\right]+r^\ell \ell^w}_{\text{return from financial portfolio}} \geq 0, \tag{9}$$

where $k^w, b', d^w, b^w, b^w-\left[\rho+(1-\rho)\,\eta\right]b' \geq 0$. Wholesale banks only pay off their deposit claims if they remain solvent, which occurs with probability δ. They have some of their government bonds withdrawn by depositors, meaning they only earn returns on the remaining part, i.e., $b^w-\left[\rho+(1-\rho)\,\eta\right]b'$. Again, (9) holds equality in equilibrium because of free entry.

To sum up, wholesale banks choose deposit contract (k^w, b', d^w) and financial portfolio (b^w, ℓ^w) to maximize a representative wholesale depositor's expected utility (7), subject to the collateral constraint (8), non-negative profit constraint (9) and nonnegative constraints $k^w, b', d^w, b^w, b^w-\left[\rho+(1-\rho)\,\eta\right]b' \geq 0$. The first order conditions for the wholesale bank's problem give the following equilibrium conditions

$$r^b u'\left(r^b b'\right)=1, \tag{10}$$

$$r^b\left[1-\delta+\delta u'\left(d^w\right)\right]+\lambda^w=1, \tag{11}$$

$$r^\ell\left[1-\delta+\delta u'\left(d^w\right)\right]=1, \tag{12}$$

where λ^w denotes the Lagrange multiplier for $b^w-\left[\rho+(1-\rho)\,\eta\right]b' \geq 0$. Equation (10) determines wholesale banks' demand for government bonds associated with each depositor's withdrawal request $(\left[\rho+(1-\rho)\,\eta\right]b')$, while equation (11) determines their demand for government bonds to back their deposit claims $(b^w-\left[\rho+(1-\rho)\,\eta\right]b' \geq 0)$.

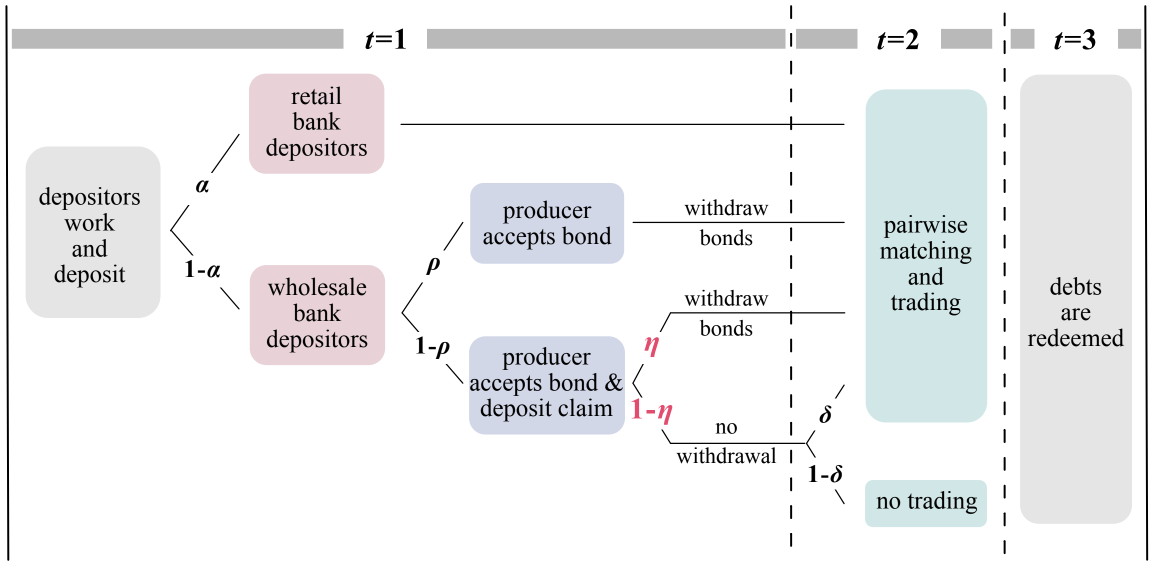

Figure 1: Timing of Events

Wholesale banks can also use claims on interbank lending to back their deposit claims, and equation (12) determines such demand.

The equilibrium conditions displayed above and those in the retail bank's problem are asset pricing kernels in consumption-based capital asset pricing models (Campbell, 1999). For example, the government bond price, i.e., the inverse of the gross interest rate on bonds $1/r^b$ in (10), is equal to wholesale bank depositors' marginal return of trading with bonds. Other equilibrium conditions, (4), (5), (6), (11) and (12), have similar interpretations by properly adjusting parameters related to the leverage requirement θ, banking failure probability $1-\delta$, and binding constraints λ^r and λ^w.

Timing of Events Figure 1 summarizes the timing of events and provides a visual guide to the pattern of meetings and exchanges among private agents.

2.2 Flight-to-Safety by Wholesale Bank Depositors

The endogenous withdrawal probability η captures the severity of wholesale banking panics or the magnitude of flight-to-safety. Depositors' expected payoffs determine withdrawal decisions and give rise to three scenarios for wholesale banking panics:[8]

$$\text{(i) No banking panic:} \qquad \eta = 0, \qquad \text{if } u\left(r^b b'\right) \leq \delta u\left(d^w\right); \tag{13}$$

$$\text{(ii) Partial banking panic:} \qquad 0 < \eta < 1, \qquad \text{if } u\left(r^b b'\right) = \delta u\left(d^w\right); \tag{14}$$

$$\text{(iii) Full banking panic:} \qquad \eta = 1, \qquad \text{if } u\left(r^b b'\right) \geq \delta u\left(d^w\right). \tag{15}$$

From (13)-(15), a no banking panic equilibrium occurs if wholesale bank depositors prefer wholesale banks' deposit claims to government bonds, a partial banking panic occurs if these depositors are indifferent between these two options, and a full banking panic occurs if government bonds are preferred.

2.3 Government

At the beginning of period 1, the fiscal authority issues $\hat{b}$ units of government bonds and transfers the revenue τ_1 to depositors:

$$\hat{b} = \tau_1. \tag{16}$$

The central bank conducts an asset swap, purchasing $\hat{b} - \bar{b}$ units of government bonds with reserves. So,

$$\underbrace{\hat{b} - \bar{b}}_{\text{asset}} = \underbrace{\bar{m}}_{\text{liability}}, \tag{17}$$

where $\bar{m}$ and $\bar{b}$ represent the amounts of reserves and government bonds circulating within the private sector, respectively. Fiscal policy determines the total supply of government bonds ($\hat{b} = \bar{m} + \bar{b}$). The central bank adjusts the relative supply of government bonds and reserves to the private sector through open market operations. The supply of reserves, $\bar{m}$, describes the central bank balance sheet policy, representing the size of its balance sheet. I focus on the effects of this balance sheet policy while holding fiscal policy constant.

[8] The tie-breaking rule will not be a concern here. The market clearing conditions will determine a unique equilibrium type for each scenario.

Government bonds and central bank reserves are redeemed in period 3. The fiscal authority taxes depositors τ_3 lump sum to payoff its debts and transfers τ^{cb} (receives, if negative) to the central bank to support its payments:

$$r^b\hat{b} + \tau^{cb} = \tau_3. \tag{18}$$

The central bank pays off its reserves, using the returns from its holdings of government bonds and the transfer from the fiscal authority:

$$r^m\bar{m} = r^b\left(\hat{b} - \bar{b}\right) + \tau^{cb}. \tag{19}$$

3 Definition of Equilibrium

In this section, I define the equilibrium and establish two important intermediate results, showing that no bank holds government bonds until maturity in equilibrium.

Definition 1 (Equilibrium)**.** *Given fiscal policy $\hat{b}$ and central bank balance sheet policy $\bar{m}$, an equilibrium consists of an allocation $\left(\bar{b}, d^r, m, b^r, \ell^r, b', d^w, b^w, \ell^w, c_2^r, c_2^b, c_2^w\right)$, Lagrange multipliers λ^r and λ^w, market-determined interest rates (r^m, r^ℓ, r^b), and a withdrawal probability η, satisfying the binding leverage constraint (2) and binding collateral constraint (8), equilibrium conditions for private banks' problems (4)-(12), one of conditions (13)-(15) to support wholesale bank depositors' withdrawal strategy, market clearing,*

$$\alpha m = \bar{m} \qquad \textit{(reserve market)}, \tag{20}$$

$$\alpha b^r + (1-\alpha)\, b^w = \bar{b} \qquad \textit{(government bond market)}, \tag{21}$$

$$\alpha \ell^r = (1-\alpha)\, \ell^w \qquad \textit{(interbank market)}, \tag{22}$$

and the complementary-slackness conditions with corresponding nonnegative constraints,

$$\lambda^r b^r = 0, \qquad \lambda^r \geq 0, \quad b^r \geq 0, \tag{23}$$

$$\lambda^w [b^w - [\rho + (1-\rho)\,\eta]\, b'] = 0, \qquad \lambda^w \geq 0, \quad b^w - [\rho + (1-\rho)\,\eta]\, b' \geq 0, \tag{24}$$

where $c_2^r = d^r$, $c_2^b = r^b b'$, and $c_2^w = d^w$ are consumption quantities for retail bank depositors, wholesale bank depositors trade with government bonds, and wholesale bank depositors trade with deposit claims.

I focus on equilibria where retail banks' leverage and wholesale banks' collateral constraints bind. Banks cannot provide deposit claims that support a satiated consumption level for depositors, i.e., $c_2 < c^\star$ with $u'(c^\star) = 1$. Equilibria, in which depositors always consume the satiated level, are trivial, as there would be no change in allocation in response to policy changes. To ensure these constraints bind, I assume a scarcity of total government bond supply as in Andolfatto and Williamson (2015) and Williamson (2019). Specifically, Assumption 1 states that even if retail banks exhaust all government liabilities, they still cannot support the satiated consumption level for their depositors.

Assumption 1 (Scarcity of Total Government Bond Supply)**.** *Total supply of government bonds is scarce, such that $\theta\hat{b} < \alpha c^\star$, where $u'(c^\star) = 1$.*

In what follows, I will solve for all equilibria under Assumption 1. I express all equilibrium conditions in terms of depositors' withdrawal probability η and their consumption quantities (c_2^r, c_2^b, c_2^w), as the center of the analysis is on depositors' withdrawal behavior and welfare in response to central bank balance sheet policy. The depositors' consumption level reflects welfare because producers and bankers make zero profits in equilibrium.[9]

Lemma 1 and Lemma 2 show that no bank holds government bonds as collateral in equilibrium and these results are independent of monetary policy. I present all proofs in the Appendix A and discuss the intuitions in the main text.

Lemma 1. *Retail banks never hold government bonds, i.e., $b^r = 0$ and $\lambda^r > 0$.*

Retail banks bear a balance sheet cost of holding assets, which is a real resource cost because they have to raise equity by working to finance their investment. They hold a

[9]Recall that producers make zero profit because depositors extract the entire trading surplus through take-it-or-leave-it offers, and bankers earn zero profit due to free entry.

positive stock of government bonds only when the interest on those bonds is higher than the interest rate they pay for interbank borrowing. However, wholesale banks always ask for a higher interest rate on interbank lending than government bonds because bonds are always available. As a result, retail banks never hold government bonds.

Lemma 2. *Wholesale banks only purchase government bonds for depositors' withdrawal requests, i.e.,* $b^w - [\rho + (1-\rho)\,\eta]\, b' = 0$.

Wholesale bank depositors prefer directly using all government bonds in exchange. Otherwise, there is a chance that their bank will default and abscond with bonds. Competition between wholesale banks drives them to assign all the government bonds for their depositors' withdrawal requests to offer better contract terms.

4 Equilibrium and Central Bank Balance Sheet

There are three types of equilibrium: no banking panic ($\eta = 0$), partial banking panic ($0 < \eta < 1$), and full banking panic ($\eta = 1$). I examine the effects of a central bank balance sheet expansion under each type and show how the size of the balance sheet determines the type of equilibrium. This balance sheet policy involves an increase in the supply of reserves, where the central bank purchases government bonds from the private sector with the issuance of new reserves. As will become clear later, this policy directly alters the relative attractiveness between bank deposits and safe government bonds, changing the severity of banking panics and shifting depositors' asset demand.

4.1 Partial Banking Panic

I begin with the partial banking panic equilibrium, where changes in the severity of wholesale banking panics (η) reflect the shifts in asset demand and play a critical role in determining the effects of a central bank balance sheet expansion.

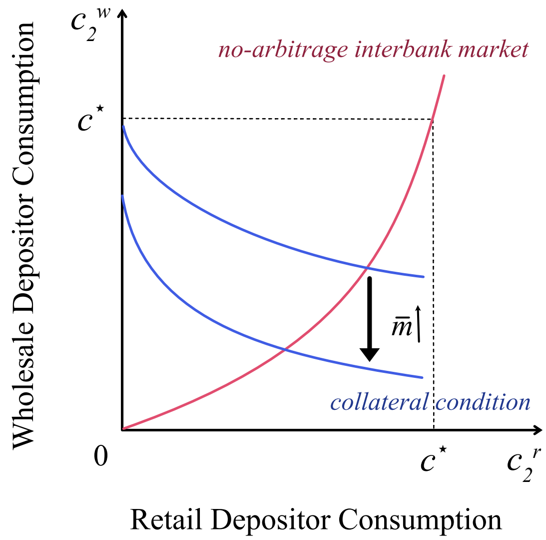

Figure 2: Partial Banking Panic Equilibrium

Existence and Uniqueness of Equilibrium In this equilibrium, wholesale bank depositors are indifferent between government bonds and deposit claims as in (14). The equilibrium is the unique solution to a system of equations consisting of condition (14), along with the no-arbitrage condition (25), collateral market clearing condition (26), and bond market clearing condition (27) explained below.

No-arbitrage Interbank Market: From asset pricing kernels (6) and (12), retail banks are willing to pay an interest rate $1/u'\left(c_2^r\right)$ on their interbank borrowing, while wholesale banks ask for an interest rate $1/\left(1-\delta+\delta u'\left(c_2^w\right)\right)$ on their interbank lending. Therefore, the following no-arbitrage condition must hold in equilibrium:

$$u'\left(c_2^r\right) = 1-\delta+\delta u'\left(c_2^w\right). \tag{25}$$

Otherwise, arbitrage opportunities exist in the interbank market. As depicted in Figure 2, this no-arbitrage condition gives an upward-sloping curve in the c_2^r–c_2^w space because of the decreasing marginal utility in consumption.

Collateral Market Clearing: The binding leverage constraint (2) and collateral constraint (8), market clearing conditions (20) and (22), condition (4) and no-arbitrage condition

(25) give the following equilibrium collateral market clearing condition:

$$\begin{aligned}\underbrace{\theta \bar{m}}_{\text{effective collateral supply (i.e., reserves)}} = & \underbrace{\alpha c_2^r \left[1-\theta+\theta u'\left(c_2^r\right)\right]}_{\text{retail banks' demand for collateral}} \\ & + \underbrace{(1-\alpha)(1-\rho)(1-\eta) c_2^w \left[1-\theta\delta+\theta\delta u'\left(c_2^w\right)\right]}_{\text{wholesale banks' demand for collateral}}.\end{aligned} \tag{26}$$

Equation (26) equates the effective collateral supply to private banks' demand.[10] Reserves work as collateral backing all private banks' deposit claims, as no bank holds government bonds as collateral (Lemmas 1 and 2). Although reserves are inaccessible to wholesale banks, they back these banks' deposit claims indirectly through the interbank market. Crucially, the severity of wholesale banking panics determines the aggregate demand for collateral because the withdrawal probability η determines the fraction of depositors who demand wholesale banks' deposit claims ultimately backed by reserves. For example, a decrease in η shifts this demand rightward.

Bond Market Clearing: The fiscal policy rule $\hat{b} = \bar{m}+\bar{b}$, equilibrium condition (10), market clearing condition (21), and binding nonnegative constraint $b^w - \left[\rho + (1-\rho)\eta\right] b' = 0$ give the following equilibrium bond market clearing condition:

$$\underbrace{\hat{b}-\bar{m}}_{\text{government bond supply}} = \underbrace{(1-\alpha)\left[\rho+(1-\rho)\eta\right] c_2^b u'\left(c_2^b\right)}_{\text{aggregate demand for government bonds}}. \tag{27}$$

The left-hand side represents the government bond supply to the private sector, which is the fiscal authority's total government bond supply minus the bonds held by the central bank to back its reserves. The right-hand side represents the private sector's aggregate demand for bonds, where wholesale bank depositors use all outstanding government bonds to settle their transactions because, again, no bank holds bonds as collateral. As with the collateral market clearing condition, the severity of wholesale banking panics determines the aggregate demand for government bonds. For example, a decrease in η shifts this

[10] Retail banks demand collateral for two reasons: to secure deposit claims or interbank borrowing. Here, the notion of "retail banks' demand for collateral" means the demand for deposit claims only.

demand leftward.

The conditions (14), (26), and (27) mentioned above implicitly define a downward-sloping *collateral condition* in the c_2^r–c_2^w space as depicted in Figure 2.[11] Reserves serve as the ultimate collateral backing both retail and wholesale bank deposits, which depositors use to exchange for consumption goods. The downward slope reflects that, with a fixed reserve supply, an increase in one type of exchange crowds out the other.

Graphically, the collateral condition and the upward-sloping no-arbitrage condition determine a unique equilibrium characterized by the consumption quantities c_2^r and c_2^w. These consumption quantities do not exceed the satiated level $c^\star$ defined in Assumption 1 because the assumption regarding the scarcity of total government bond supply constrains the collateral condition to be close to the original point. I solve for other equilibrium outcomes based on c_2^r and c_2^w. For example, (26) solves for the withdrawal probability η. Proposition 1 below summarizes these results.

Proposition 1 (Existence and Uniqueness of Equilibrium)**.** *Under Assumption 1, there exists a unique partial banking panic equilibrium characterized by the consumption allocation* $\left(c_2^r, c_2^w, c_2^b\right)$ *and the withdrawal probability* η*, where* $0 < c_2^r, c_2^w, c_2^b < c^\star$*.*

Central Bank Balance Sheet Expansion, Flight-to-Safety, and Welfare I evaluate the effects of an expansion in the size of the central bank's balance sheets ($\bar{m}$) by performing comparative statics on the system of equations (14), (25), (26), and (27).

Proposition 2. *Assume* $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ *and* $0 < \sigma < 1$*, an expansion in the size of the central bank's balance sheet mitigates wholesale banking panics, i.e.,* $\frac{\partial \eta}{\partial \bar{m}} < 0$*. However, this expansion reduces welfare, i.e.,* $\frac{\partial c_2^b}{\partial \bar{m}}, \frac{\partial c_2^r}{\partial \bar{m}}, \frac{\partial c_2^w}{\partial \bar{m}} < 0$*.*

The CRRA function form $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ satisfies earlier assumptions for utility function.

[11] See Appendix A.1 for the detailed proof. The label "collateral condition" is a slight abuse of language. I use this label because the collateral market clearing condition explains the comparative statics I study when $\bar{m}$ changes.

In particular, $0 < \sigma < 1$ is a necessary condition for $-c\frac{u''(c)}{u'(c)} < 1$, which implies that the substitution effect dominates the income effect. Consequently, the demand for assets increases with their rate of return.

Corollary 1. *An expansion in the size of the central bank's balance sheet reduces the interest rates on government bonds, reserves, and interbank loans, i.e.,* $\frac{\partial r^b}{\partial \bar{m}}, \frac{\partial r^m}{\partial \bar{m}}, \frac{\partial r^\ell}{\partial \bar{m}} < 0$.

An expansion in the size of the central bank balance sheet has three effects. First, this expansion increases the effective collateral supply by issuing more reserves, which relaxes retail banks' leverage and wholesale banks' collateral constraints, promoting the supply of bank liabilities. In particular, wholesale banks can provide more attractive deposit claims to compete for depositors. For example, they can pay higher returns or provide better liquidity insurance to depositors. Second, this expansion reduces the government bond supply to the private sector, putting pressure on their interest rate to fall. As a result, government bonds become less attractive for wholesale bank depositors. The first two direct partial equilibrium effects imply the third effect: wholesale bank depositors' withdrawal probability decreases (i.e., $\frac{\partial \eta}{\partial \bar{m}} < 0$), mitigating the severity of wholesale banking panics, as deposit claims become more attractive relative to bonds. The same logic extends when determining the type of equilibrium as illustrated in Figure 3: central bank balance sheet expansions gradually reduce the severity of panics, starting from full banking panics ($\eta = 1$) to eventually eliminate them entirely ($\eta = 0$).

Despite mitigating wholesale banking panics, expanding the central bank's balance sheet reduces welfare. This striking result follows from the interaction of the three effects mentioned above, which jointly determine the change in the consumption allocation $\left(c_2^r, c_2^w, c_2^b\right)$. A central bank balance sheet expansion puts pressure on the collateral condition in Figure 2 to shift upward by increasing the effective collateral supply as in (26). However, the withdrawal probability η decreases in response to this expansion, increasing the demand for collateral and, therefore, shifting the collateral condition downward. I show that the latter effect necessarily dominates the former one. Collateral becomes relatively more scarce, harming depositors who trade with deposit claims backed by collateral. Depositors who trade with government bonds also get worse by (14). Clearly, the endogenous shift in asset demand is crucial for these results. Different welfare implications appear when there is no such shift in response to the balance sheet expansion, as in the scenarios below with no or full banking panic.

The welfare reduction result can be more intuitively understood through individual decisions. Firstly, an expansion in the central bank's balance sheet harms depositors who trade with government bonds. Expanding the balance sheet reduces the government bond supply to the private sector as in (27), putting pressure on the interest rate on government bonds to fall. Although the decreased demand for bonds that comes from a decrease in withdrawal probability η puts pressure on bond interest rate to rise, the decreased supply of bonds dominates this decreased demand, implying a reduction in the interest rate on government bonds (i.e., $\frac{\partial r^b}{\partial \bar{m}} < 0$) and a lower trading volume for transactions settled with bonds (i.e., $\frac{\partial c_2^b}{\partial \bar{m}} < 0$).

Secondly, an expansion in the central bank's balance sheet harms depositors who trade with wholesale banks' deposit claims. The increased reserve supply increases the effective collateral supply as in (26), relaxing retail banks' leverage and wholesale banks' collateral constraints through the interbank market. Consequently, there is an increase in wholesale banks' supply of deposit claims. However, on the other side of the market, the reduction in the withdrawal probability η increases the demand for these claims. The increased demand for deposit claims dominates their increased supply. Each depositor obtains fewer claims in exchange, thereby, a lower trading volume for transactions settled with these claims (i.e., $\frac{\partial c_2^w}{\partial \bar{m}} < 0$). Furthermore, this increased demand for deposit claims intensifies banks' competition for collateral. Wholesale banks compete for interbank loans, asking for a lower interest rate on them (i.e., $\frac{\partial r^\ell}{\partial \bar{m}} < 0$). Retail banks compete for reserves to back their interbank borrowing, asking for a lower interest rate on reserve (i.e., $\frac{\partial r^m}{\partial \bar{m}} < 0$).

Finally, an expansion in the central bank's balance sheet harms depositors who trade with retail banks' deposit claims. Unlike wholesale banks, the increased reserve supply does not increase the supply of retail banks' deposit claims, even if it relaxes their leverage constraint, which works like an income effect. Besides depositors, retail banks raise funds from wholesale banks, who ask for a lower rate of return r^ℓ in response to an expansion (recall that $\frac{\partial r^\ell}{\partial \bar{m}} < 0$). Therefore, they substitute their funding source for cheaper interbank borrowing. The substitution effect from a decrease in r^ℓ dominates the income effect from a relaxing leverage constraint. Retail banks reduce their supply of deposit claims, implying a lower trading volume for transactions settled with these claims (i.e., $\frac{\partial c_2^r}{\partial \bar{m}} < 0$).

Damaging Effects of Taking Out Government Bonds In the above scenario, the decrease in supply dominates the decrease in demand for government bonds, while the increase in demand dominates the increase in supply for wholesale deposits or, more broadly, for their underlying collateral. These results arise because central bank balance sheet expansions take out government bonds from the private sector, ultimately reducing welfare. In particular, the reduction in the supply of government bonds puts downward pressure on their interest rate, playing an indirect but significant role in the markets for wholesale deposits and collateral. As bonds become less attractive, depositors switch to wholesale bank deposits backed by collateral, contributing to the dominance effects in the markets for these assets. In section 5, I demonstrate how these results can be reversed by implementing a central bank balance sheet policy that does not change the government bond supply. This alternative policy also serves as a counterfactual for isolating the role of the government bond market.

4.2 No Banking Panic and Full Banking Panic

The effects of an expansion in the size of the central bank's balance sheet are qualitatively similar in a no banking panic ($\eta = 0$) and a full banking panic ($\eta = 1$) equilibrium because

η remains constant in either case. As in the partial banking panic scenario above, the balance sheet expansion harms depositors who trade with government bonds. However, it now benefits depositors who trade with retail and wholesale banks' deposit claims. Therefore, the central bank cannot improve welfare, in the Pareto sense, by adjusting the size of its balance sheet.

No Banking Panic The market structure is similar to the one studied before, except that no wholesale bank depositor withdraws for safety concerns. Wholesale banks make sufficient revenues from interbank lending, allowing them to offer attractive enough deposit claims that prevent a wholesale banking panic from arising.

The no-arbitrage condition holds the same as before, while the collateral and bond market clearing conditions become

$$\theta \bar{m} = \alpha c_2^r \left[1 - \theta + \theta u'\left(c_2^r\right)\right] + (1-\alpha)(1-\rho) c_2^w \left[1 - \theta\delta + \theta\delta u'\left(c_2^w\right)\right], \tag{28}$$

$$\hat{b} - \bar{m} = (1-\alpha)\rho c_2^b u'\left(c_2^b\right), \tag{29}$$

respectively, by setting $\eta = 0$ in (26) and (27).

The no-arbitrage condition (25) and the collateral market clearing condition (28) jointly determine the consumption quantities (c_2^r, c_2^w) for depositors who trade with retail and wholesale banks' deposit claims. The bond market clearing condition (29) solely determines the consumption quantity c_2^b for depositors who trade with government bonds. Under Assumption 1, these conditions solve a unique equilibrium with $0 < c_2^r, c_2^w, c_2^b < c^\star$.

An expansion in the central bank's balance sheet benefits depositors who trade with retail and wholesale banks' deposit claims by relaxing retail banks' leverage and wholesale banks' collateral constraints. First of all, this expansion increases the supply of wholesale banks' deposit claims, which are ultimately backed by reserves. This increased supply implies a higher trading volume for depositors who trade with these claims (i.e., a higher c_2^w). That's because, unlike a partial banking panic equilibrium, there is no shift in demand for these claims. Furthermore, the absence of the shift in demand puts no pressure on wholesale banks to compete for interbank loans as collateral. Consequently, the interest rate on interbank loans increases (i.e., a higher r^{ℓ}), and retail banks have little incentive to substitute their funding source from deposits to interbank loans. With more slack leverage constraints, retail banks increase their supply of deposit claims, implying a higher trading volume for depositors who trade with these claims (i.e., a higher c_2^r).[12]

The central bank balance sheet expansion, however, harms depositors who trade with government bonds. The government bond supply to the private sector decreases with an increased reserve supply, while the aggregate demand for bonds remains unchanged as depositors' withdrawal probability remains constant. As in a partial banking panic equilibrium, the decreased supply of government bonds dominates their change in demand, implying a reduction in the interest rate on bonds (i.e., a lower r^b) and a lower trading volume for depositors who demand them (i.e., a lower c_2^b).

Full Banking Panic The interbank market becomes inactive in a full banking panic equilibrium because wholesale banks do not demand interbank loans to secure deposits. One interpretation of this equilibrium is that wholesale banks resemble narrow banks in that they only purchase safe government bonds for depositors' withdrawal requests. On the other hand, the equilibrium outcomes would be the same if depositors were allowed to invest in government bonds themselves, as competitive banks maximize their depositors' utility under free entry. Therefore, this full banking panic equilibrium also characterizes the feature of equilibrium with disintermediation in the wholesale banking sector.

When the withdrawal probability $\eta = 1$, only retail banks require collateral to back

[12] In this equilibrium, evaluating the effects of an expansion in the size of the central bank's balance sheets on depositors who trade with deposit claims is performing comparative statics on the system of equations (25) and (28) with respect to an increase in reserve supply $\bar{m}$. In the next paragraph, the effects of this expansion on depositors who trade with government bonds are determined by (29).

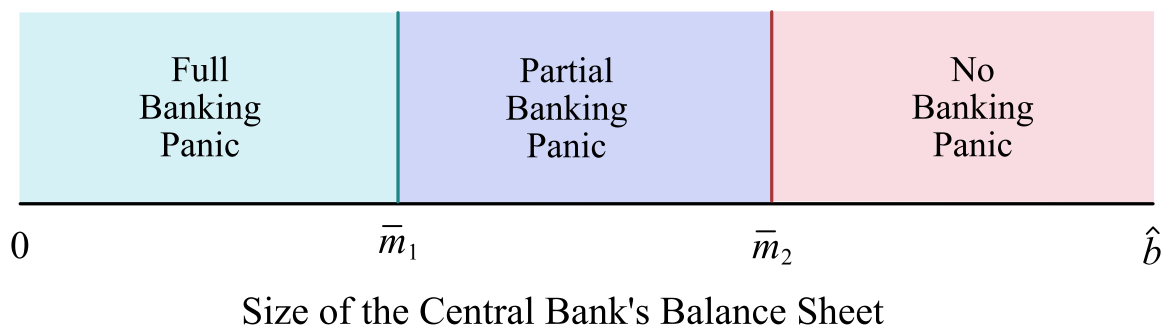

Figure 3: How To Determine the Type of Equilibrium

their deposit claims, and the collateral market clearing condition becomes

$$\theta \bar{m} = \alpha c_2^r \left[1 - \theta + \theta u'\left(c_2^r\right)\right], \tag{30}$$

which is the retail bank's leverage constraint. Similarly, the bond market clearing condition becomes

$$\hat{b} - \bar{m} = (1 - \alpha)\, c_2^b u'\left(c_2^b\right). \tag{31}$$

Conditions (30) and (31) determine the consumption allocation for retail and wholesale bank depositors, respectively. A central bank balance sheet expansion has similar effects to its no banking panic counterpart: it benefits depositors who trade with deposit claims while harming those who trade with government bonds.

4.3 How To Determine the Type of Equilibrium

As explained in the partial banking panic equilibrium, expanding the central bank's balance sheet size reduces the severity of wholesale banking panics. As in Figure 3, full banking panics occur when the size of the central bank's balance sheet, characterized by the reserves supply $\bar{m}$, is below a certain lower threshold, no banking panics occur when it is above a higher threshold, and partial banking panics occur when it lies in between.

Proposition 3. *Under Assumption 1, there are two thresholds $\bar{m}_1$ and $\bar{m}_2$ for the size of the central bank's balance sheet with $0 < \bar{m}_1 < \bar{m}_2 < \hat{b}$, where $\bar{m}_1$ and $\bar{m}_2$ solve the system* *of equations (14), (25), (26), and (27) with $\eta = 0$ and $\eta = 1$, respectively. The size of the central bank's balance sheet determines the equilibrium as follows:*

1. *a full banking panic $\eta = 1$ occurs when the size of the central bank's balance sheet is below the lower threshold such that $\bar{m} \in (0, \bar{m}_1]$;*

2. *a partial banking panic $\eta \in (0, 1)$ occurs when it lies in between the two thresholds such that $\bar{m} \in (\bar{m}_1, \bar{m}_2)$;*

3. *no banking panic $\eta = 0$ occurs when it is above the upper threshold such that $\bar{m} \in [\bar{m}_2, \hat{b})$.*

These critical values increase with the probability of wholesale banking failure $1 - \delta$, i.e., $\frac{\partial \bar{m}_1}{\partial \delta} < 0$ and $\frac{\partial \bar{m}_2}{\partial \delta} < 0$.

In addition to the equilibrium type determination, Proposition 3 shows that if wholesale banking becomes more risky, the central bank can prevent additional withdrawals by expanding its balance sheet. Specifically, swapping government bonds for reserves makes deposit claims more attractive to depositors, helping offset the increased risk of banking.

5 Expanding the Reach of Central Bank Liabilities

I now study what happens when expanding the reach of central bank interest-bearing central bank reserves to wholesale banks. There are two main reasons for doing this. Firstly, and most importantly, I will show how the central bank can mitigate wholesale banking panics and improve welfare in this case. The central bank can only achieve one of these objectives by adjusting the size of its balance sheet. Secondly, and of theoretical interest, this extension provides a counterfactual to the previous case. Mainly, I show that the increased supply of wholesale banks' deposit claims can dominate their increased demand when the central bank does not take out government bonds from the private sector. Again, this highlights the critical role of the government bond market in the consequences of central bank balance sheet policies.

Specifically, I introduce another central bank liability in addition to reserves that functions as the Fed's overnight reverse repurchase agreement (ON-RRP) facility. I call this liability ON-RRPs (o) for convenience. ON-RRPs offer better liquidity than reserves because both retail and wholesale banks can hold them, so introducing ON-RRPs works like expanding the reach of reserve accounts to wholesale banks. Unlike government bonds, ON-RRPs can only circulate among financial institutions, and depositors cannot directly use them to settle their transactions. In equilibrium, the interest rate on ON-RRPs equals the interest rate on interbank loans with $r^o = r^\ell$, reflecting their use as collateral to back wholesale banks' deposit claims.

The central bank's balance sheet policy now has two dimensions. Firstly, as before, the central bank chooses the size of its balance sheet, denoted by $s = \bar{m} + \bar{o}$. An expansion in s has the same effects as when only reserves are available. Secondly, the central bank determines the composition of the central bank's liabilities, represented by the supply of ON-RRPs $\bar{o}$. I focus on the effects of a swap of reserves ($\bar{m}$) for ON-RRPs ($\bar{o}$) while fixing the size of the central bank's balance sheet (s).[13]

Consider the scenario of a partial banking panic with an active interbank market. Retail banks invest in reserves (m) and borrow ℓ^r from wholesale banks. They do not hold ON-RRPs, for the same reason they do not hold government bonds as explained in Lemma 1. Instead, wholesale banks hold these ON-RRPs (o). They also hold government bonds (b^w) for their depositors' withdrawal requests and lend ℓ^w to retail banks. Table 3 presents private banks' balance sheets in this scenario.

A swap of reserves for ON-RRPs expands the supply of ON-RRPs that are directly

[13]In practice, the mix between ON-RRPs and reserves is determined by the interest rates the central bank sets. This swap can be viewed as a convenient shortcut to capturing such interest rate policy, and one can obtain the same results considering an experiment that fixes s while reducing the interest rate spread between reserves and ON-RRPs, i.e., reducing r^m/r^o.

Retail Bank

Assets	Liabilities and Equity
reserves: m	deposit claims: d^r
	interbank borrowing: ℓ^r
	bank capital: e

Wholesale Bank

Assets	Liabilities
government bonds: b^w	government bonds: $[\rho + (1-\rho)\eta]b'$
interbank lending: ℓ^w	deposit claims: $(1-\rho)(1-\eta)d^w$
ON-RRPs: o	

Table 3: Private Banks' Balance Sheets with Overnight Reverse Repurchase Agreements

accessible to wholesale banks. As a result, these banks reduce their lending to retail banks, which is how they gain indirect access to reserves, boosting the effective collateral supply by avoiding the inefficiency associated with retail banks' leverage requirements. To see this, notice that now the collateral market clearing condition becomes

$$\begin{aligned} \theta s + (1-\theta)\,\bar{o} \; &= \alpha c_2^r \left[1 - \theta + \theta u'\left(c_2^r\right)\right] \\ &\quad + (1-\alpha)\,(1-\rho)\,(1-\eta)\,c_2^w \left[1 - \theta\delta + \theta\delta u'\left(c_2^w\right)\right]. \end{aligned} \tag{32}$$

From (32), an increase in the supply of ON-RRPs $\bar{o}$ increases the effective collateral supply that is captured by the left-hand side of the equation. This effective collateral supply can also be written as $\theta\bar{m} + \bar{o}$ to reflect the fact that assets held by retail banks can secure fewer liabilities than those held by wholesale banks.

The bond market clearing condition (27) becomes

$$\hat{b} - s = (1-\alpha)\left[\rho + (1-\rho)\,\eta\right] c_2^b u'\left(c_2^b\right), \tag{33}$$

where the supply of government bonds to the private sector only depends on the size of the central bank's balance sheet. Crucially, this supply does not change when the central bank swaps between reserves and ON-RRPs.

Swapping Reserves for ON-RRPs, Flight-to-Safety, and Welfare I evaluate the effects of a swap of reserves for ON-RRPs by performing comparative statics on the system of equations (14), (25), (32), and (33).

Proposition 4. *A swap of reserves for ON-RRPs mitigates wholesale banking panics and improves welfare, i.e.,* $\frac{\partial \eta}{\partial \bar{o}} < 0$ *and* $\frac{\partial c_2^r}{\partial \bar{o}}, \frac{\partial c_2^w}{\partial \bar{o}}, \frac{\partial c_2^b}{\partial \bar{o}} > 0$.

Corollary 2. *A swap of reserves for ON-RRPs increases the interest rates on government bonds, reserves, interbank loans, and ON-RRPs, i.e.,* $\frac{\partial r^b}{\partial \bar{o}}, \frac{\partial r^m}{\partial \bar{o}}, \frac{\partial r^\ell}{\partial \bar{o}}, \frac{\partial r^o}{\partial \bar{o}} > 0$.

Like central bank balance sheet expansions, a swap of reserves for ON-RRPs mitigates wholesale banking panics (i.e., $\frac{\partial \eta}{\partial \bar{o}} < 0$). As in (32), this swap increases the effective collateral supply, relaxing wholesale banks' collateral constraint and enabling them to compete for depositors more effectively. Consequently, more depositors switch to deposit claims, and fewer withdraw and flee to safe government bonds.

The key difference between a swap of reserves for ON-RRPs and a central bank balance sheet expansion in Section 4 is that the swap does not take out government bonds from the private sector. While the swap of central bank liabilities also leads to depositors switching to bank deposits, the magnitude of this switch is not as large as with a central bank balance sheet expansion because, without taking out government bonds, there is no direct pressure that makes bonds less attractive. Consequently, in contrast to the balance sheet expansion, this swap shifts the collateral condition upward as in Figure 4, implying a welfare improvement.

As before, the details of the welfare improvement can be understood through individual decisions. Firstly, a swap of reserves for ON-RRPs benefits depositors who trade with government bonds. While the supply does not change, this swap reduces the aggregate demand for government bonds because, by mitigating banking panics, fewer depositors

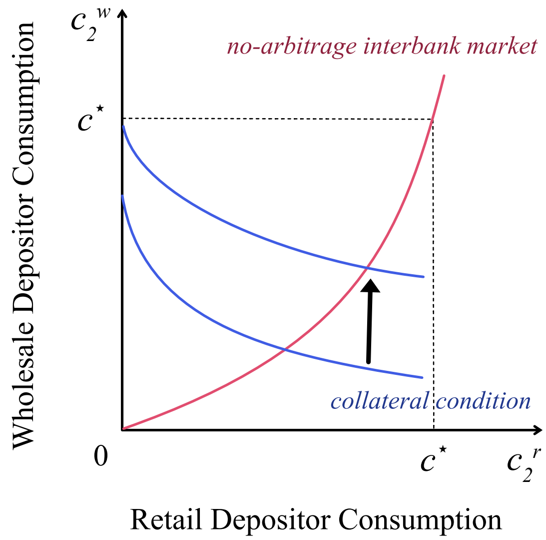

Figure 4: Partial Banking Panic Equilibrium with ON-RRPs

flee to these safe assets. As a result, the interest rate on bonds increases ($\frac{\partial r^b}{\partial \bar{o}} > 0$), implying a higher trading volume for depositors who trade with them (i.e., $\frac{\partial c_2^b}{\partial \bar{o}} > 0$).

Secondly, a swap of reserves for ON-RRPs benefits depositors who trade with wholesale banks' deposit claims. Wholesale banks supply more deposit claims because, again, this swap relaxes their collateral constraint. Although this swap also increases the demand for these claims, the increased supply dominates the increased demand, implying a higher trading volume for depositors who trade with them (i.e., $\frac{\partial c_2^w}{\partial \bar{o}} > 0$). This result suggests that the increased demand in response to the swap is more moderate than central bank balance sheet expansions. The reason is that this swap puts no direct pressure to reduce the attractiveness of bonds, while balance sheet expansions lower their interest rate by reducing their supply. Furthermore, this moderate increase in demand for deposit claims puts little pressure on banks to compete for collateral to back them. However, as explained earlier, the effective collateral supply increases, implying higher interest rates on collateral, such as reserves and interbank loans (i.e., $\frac{\partial r^\ell}{\partial \bar{o}} > 0$ and $\frac{\partial r^m}{\partial \bar{o}} > 0$).

Finally, a swap of reserves for ON-RRPs benefits retail bank depositors. The swap moves the composition of central bank liabilities away from reserves that only retail banks can hold, which could, in principle, reduce retail banking activities. Retail banks' asset holdings indeed have to fall, but so does their interbank borrowing. In fact, retail banks substitute their funding source from interbank borrowing to deposits because the former becomes more expensive in response to the swap (recall that $\frac{\partial r^\ell}{\partial \bar{o}} > 0$). As a result, the supply of retail banks' deposit claims increases, implying a higher trading volume for depositors who trade with them (i.e., $\frac{\partial c_2^r}{\partial \bar{o}} > 0$).

6 Conclusion

This paper investigates the implications of the central bank balance sheet for the severity of wholesale banking panics and welfare. Explicitly modeling the shifts in depositors' demand for safe government bonds and bank deposits is crucial for understanding the consequences of central bank balance sheet policies. An expansion in the size of the central bank's balance sheet and a swap of central bank reserves for overnight reverse repurchase agreements (ON-RRPs) mitigate wholesale banking panics. However, they have different effects on welfare.

A central bank balance sheet expansion can reduce welfare. This expansion reduces the supply of government bonds to the private sector, hindering transactions settled with bonds and lowering their interest rate. While this could promote the supply of retail and wholesale deposits by providing banks with more central bank liabilities to expand their intermediation activity, it ultimately hinders transactions settled with deposits. The reason is that, by mitigating banking panics, too many depositors shift to bank deposits backed by central bank liabilities, and the increased demand for central bank liabilities offsets the benefits from their increased supply.

By contrast, swapping reserves for ON-RRPs always improves welfare. This swap promotes transactions settled with government bonds because the interest rate on bonds rises. This rise occurs because the aggregate demand for bonds declines with the mitigation of banking panics while the supply of bonds remains unchanged. Moreover, this swap promotes transactions settled with bank deposits. Recall that the issue with central bank balance sheet expansions is that too many depositors shift to bank deposits. However, this swap addresses this issue as it does not take out government bonds from the private sector, putting no direct pressure to make bonds less attractive.

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Appendix

A Omitted Proofs

Proof of Lemma 1. Assume the contrary, i.e., $\lambda^r = 0$. Then, from the equilibrium conditions (5) and (6) in the retail bank's problem, I obtain

$$r^\ell < r^b, \tag{A.1}$$

given that $u'(d^r) > 1$ because of the assumption regarding the scarcity of total government bond supply, which implies $d^r < c^\star$. However, from the equilibrium conditions (11) and (12) in the wholesale bank's problem,

$$r^\ell \geq r^b, \tag{A.2}$$

which is a contradiction. □

Proof of Lemma 2. Assuming the contrary, i.e., $b^w - [\rho + (1-\rho)\eta]\, b' > 0$, which implies $\lambda^w = 0$. Then, from the equilibrium conditions (10) and (11),

$$u'\left(r^b b'\right) = 1 - \delta + \delta u'(d^w), \tag{A.3}$$

which implies

$$u\left(r^b b'\right) > u(d^w), \tag{A.4}$$

given that $u'(\cdot) > 0$ and $u''(\cdot) < 0$ and the fact that $u'(d^w) > 1$ because of the binding collateral constraint. Condition (A.4) implies an equilibrium with full banking panic with $\eta = 1$. However, then $b^w - [\rho + (1-\rho)\eta]\, b' > 0$, which contradicts to the binding collateral constraint (8) when $\eta = 1$. □

Lemma A.1. *Conditions (14), (26) and (27) implicitly define a collateral condition $c_2^r = h(c_2^w)$ in the c_2^r-c_2^w space, where $h'(c_2^w) < 0$, i.e., the collateral condition is downward sloping in a partial banking panic equilibrium.*

Proof of Lemma A.1. Totally differentiating (14), (26) and (27) with respect to c_2^w, I obtain

$$u'\left(c_2^b\right)\frac{\partial c_2^b}{\partial c_2^w}-\delta u'\left(c_2^w\right)=0, \tag{A.5}$$

$$\alpha F_1'\left(c_2^r\right)\frac{\partial c_2^r}{\partial c_2^w}-(1-\alpha)(1-\rho)F_2\left(c_2^w\right)\frac{\partial \eta}{\partial c_2^w}+(1-\alpha)(1-\rho)(1-\eta)F_2'\left(c_2^w\right)=0, \tag{A.6}$$

$$(1-\rho)F_3\left(c_2^b\right)\frac{\partial \eta}{\partial c_2^w}+\left[\rho+(1-\rho)\eta\right]F_3'\left(c_2^b\right)\frac{\partial c_2^b}{\partial c_2^w}=0, \tag{A.7}$$

where $F_1(c)=c\left[1-\theta+\theta u'(c)\right]$, $F_2(c)=c\left[1-\theta\delta+\theta\delta u'(c)\right]$, and $F_3(c)=cu'(c)$. Note that

$$F_i'(c)>0, \quad \text{for} \quad i\in\{1,2,3\} \tag{A.8}$$

given that $-\frac{cu''(c)}{u'(c)}<1$ for all $c\geq 0$. Then, from (A.5)-(A.7), I have

$$\frac{\partial c_2^b}{\partial c_2^w}>0, \quad \frac{\partial \eta}{\partial c_2^w}<0, \quad \frac{\partial c_2^r}{\partial c_2^w}<0. \tag{A.9}$$

The last inequality $\frac{\partial c_2^r}{\partial c_2^w}<0$ implies that conditions (14), (26) and (27) implicitly define a function $c_2^r=h\left(c_2^w\right)$ with $h'\left(c_2^w\right)<0$. This is equivalent to say these conditions implicitly define a downward sloping collateral condition in the c_2^r-c_2^w space. □

Lemma A.2. *The no-arbitrage condition (25) implicitly defines a function:*

$$c_2^r=f\left(c_2^w\right), \tag{A.10}$$

where $f'\left(c_2^w\right)>0$, $\lim_{c_2^w\to 0} f\left(c_2^w\right)=0$, *and* $f\left(c^\star\right)=c^\star$.

Proof. The proof is trivial due to the diminishing marginal utility of consumption. □

Proof of Proposition 1. The goal is to show that $c_2^r=h\left(c_2^w\right)$ and $c_2^r=f\left(c_2^w\right)$ solve for a unique allocation $\left(c_2^w,c_2^r\right)$, where the first equation characterizes a downward-sloping collateral condition (Lemma A.1) and the second equation characterizes an upward-sloping no-arbitrage interbank market condition (Lemma A.2) in the c_2^r-c_2^w space. As illustrated in Figure 5, I can finish this proof by showing that $\lim_{c_2^w\to 0} h\left(c_2^w\right)-f\left(c_2^w\right)>0$ and $h\left(c^\star\right)-f\left(c^\star\right)<0$.

Confine attention to the collateral condition $c_2^r=h\left(c_2^w\right)$ first. Taking limit of $c_2^w\to 0$

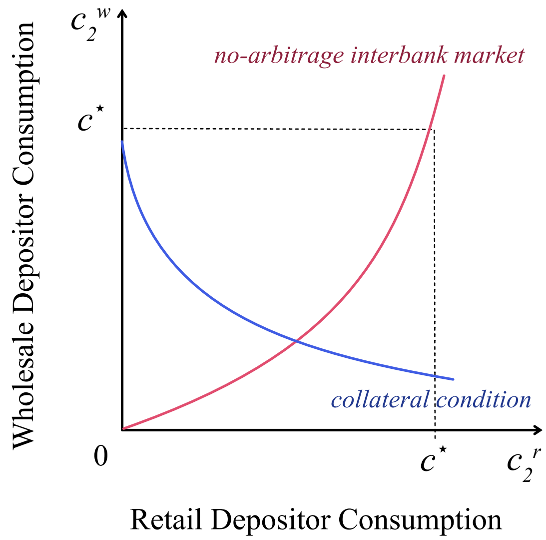

Figure 5: Existence and Uniqueness of Partial Banking Panic Equilibrium

on both side of (26), I have:

$$\theta \bar{m} = \lim_{c_2^w \to 0} \alpha c_2^r \left[1 - \theta + \theta u' \left(c_2^r\right)\right], \tag{A.11}$$

as $\lim_{c \to 0} cu'(c) = 0$. For all $\bar{m} > 0$, this further implies, implicitly,

$$c_2^r = \lim_{c_2^w \to 0} h\left(c_2^w\right) > 0, \tag{A.12}$$

because $\lim_{c \to 0} cu'(c) = 0$ and $cu'(c)$ is increasing in c as $-\frac{cu''(c)}{u'(c)} < 1$. Furthermore, when $c_2^w = c^\star$, $c_2^r = h(c^\star)$ solves the following equation:

$$\theta \bar{m} = \alpha h\left(c^\star\right)\left[1 - \theta + \theta u'\left(h\left(c^\star\right)\right)\right] + (1-\alpha)(1-\rho)(1-\eta)\, c^\star. \tag{A.13}$$

The expression $x\left[1 - \theta + \theta u'(x)\right]$ is also strictly increasing in x. Therefore, for any feasible central bank's balance sheet size $\bar{m} \in \left(0, \hat{b}\right)$ with a fiscal policy $\hat{b}$ that satisfies Assumption 1, (A.13) implies

$$h\left(c^\star\right) < c^*. \tag{A.14}$$

To conclude, $\lim_{c_2^w \to 0} h\left(c_2^w\right) > \lim_{c_2^w \to 0} f\left(c_2^w\right) = 0$ and $h\left(c^\star\right) < c^* = f\left(c^\star\right)$. That is, $\lim_{c_2^w \to 0} h\left(c_2^w\right) - f\left(c_2^w\right) > 0$ and $h\left(c^\star\right) - f\left(c^\star\right) < 0$. A unique solution that $0 < c_2^w, c_2^r < c^\star$ exists by the Intermediate Value Theorem. Finally, $0 < c_2^b < c_2^w < c^\star$ because $u(c_2^b) =$

$\delta u(c_2^w)$ as in (14). □

Proof of Proposition 2. As in Proposition 1, there is a unique partial banking panic equilibrium that can be solved by the no-arbitrage condition (25), collateral market clearing condition (26), bond market clearing condition (27), and condition (14). The proof for this proposition is performing comparative statics on the abovementioned system of equations with respect to an increase in reserve supply $\bar{m}$. For simplicity, consider a CRRA utility function

$$u(x) = \frac{x^{1-\sigma}}{1-\sigma}, \tag{A.15}$$

with $0 < \sigma < 1$, which satisfies assumptions on the utility function. Totally differentiating these functions with respect to $\bar{m}$, then solving the system of linear equations, I obtain

$$\frac{\partial c_2^w}{\partial \bar{m}} = -\frac{\Omega_3}{\delta \frac{u''(c_2^w)}{u''(c_2^r)} \Omega_1 \Omega_5 + \Omega_2 \Omega_5 + \Omega_3 \Omega_4} < 0, \tag{A.16}$$

$$\frac{\partial \eta}{\partial \bar{m}} = \frac{1}{\Omega_3} \left(\delta \frac{u''(c_2^w)}{u''(c_2^r)} \Omega_1 + \Omega_2 \right) \frac{\partial c_2^w}{\partial \bar{m}} < 0, \tag{A.17}$$

$$\frac{\partial c_2^r}{\partial \bar{m}} = \delta \frac{u''(c_2^w)}{u''(c_2^r)} \frac{\partial c_2^w}{\partial \bar{m}} < 0, \tag{A.18}$$

$$\frac{\partial c_2^b}{\partial \bar{m}} = \delta \frac{u'(c_2^w)}{u'\left(c_2^b\right)} \frac{\partial c_2^w}{\partial \bar{m}} < 0, \tag{A.19}$$

where

$$\Omega_1 = \alpha \left[1 - \theta + \theta (1-\sigma) u'(c_2^r)\right] > 0, \tag{A.20}$$

$$\Omega_2 = (1-\alpha)(1-\rho)(1-\eta)(1-\theta\delta) + (1-\alpha)(1-\sigma)\theta\delta u'(c_2^w) > 0, \tag{A.21}$$

$$\Omega_3 = (1-\alpha)(1-\rho)(1-\theta\delta) c_2^w > 0, \tag{A.22}$$

$$\Omega_4 = (1-\alpha)\left[\rho + (1-\rho)\eta\right](1-\sigma)\delta u'(c_2^w) > 0, \tag{A.23}$$

$$\Omega_5 = (1-\alpha)(1-\rho)\delta c_2^w u'(c_2^w) > 0. \tag{A.24}$$

Therefore, expanding the size of the central bank's balance sheet mitigates wholesale banking panics in a partial banking panic equilibrium. However, such a policy reduces consumption for all the depositors. □

Proof of Proposition 3. The proof takes two steps. In the first step, I show there exist two critical values $\bar{m}_1$ and $\bar{m}_2$, which solve for $\eta = 1$ and $\eta = 0$, respectively, in conditions that determines partial panic equilibria (equations (25), (26), (27), and (14)). The withdrawal probability η is strictly decreasing in $\bar{m}$ in a partial banking panic equilibrium as in Proposition 2. Therefore, if such critical value exists, they satisfy $\bar{m}_1 < \bar{m}_2$ and partial banking panic equilibria exits when $\bar{m} \in (\bar{m}_1, \bar{m}_2)$. I also show how these critical values change in response to a change in the probability of wholesale banking failure $1-\delta$. In the second step, I will show that a full banking panic exists when $\bar{m} < \bar{m}_1$ and a no banking panic equilibrium exits when $\bar{m} > \bar{m}_2$.

Step 1 First, consider a partial banking panic equilibrium with $\eta = 1$. Then, the critical value $\bar{m}_1$ and associated consumption allocation $\left(c_2^r, c_2^w, c_2^b\right)$ solve the following equations:

$$u'\left(c_2^r\right) = 1 - \delta + \delta u'\left(c_2^w\right), \tag{A.25}$$

$$\theta \bar{m}_1 = \alpha c_2^r \left[1 - \theta + \theta u'\left(c_2^r\right)\right], \tag{A.26}$$

$$\hat{b} - \bar{m}_1 = (1-\alpha) c_2^b u'\left(c_2^b\right), \tag{A.27}$$

$$u\left(c_2^b\right) - \delta u\left(c_2^w\right) = 0. \tag{A.28}$$

Taking the limit as $\bar{m}_1 \to 0$, from (A.25) and (A.26), I have $c_2^r \to 0$ and $c_2^w \to 0$ because, in particular, $\lim_{c\to 0} cu'(c) = 0$. However, from (A.27), $c_2^b > 0$. Then, the last function (A.28) does not hold with equality. In fact, $u\left(c_2^b\right) - \delta u\left(c_2^w\right) > 0$ when $\bar{m}_1 \to 0$. Similarly, taking the limit as $\bar{m}_1 \to \hat{b}$, I obtain $c_2^b \to 0$ and $c_2^w > 0$. This implies that $u\left(c_2^b\right) - \delta u\left(c_2^w\right) < 0$ when $\bar{m}_1 \to \hat{b}$. By Intermediate Value Theorem, these exists $0 < \bar{m}_1 < \hat{b}$ which solves the above system of equations, and such $\bar{m}_1$ is unique by monotonicity.

Then, totally differentiating (A.25)-(A.28), with respect to δ, I can solve for

$$\frac{\partial \bar{m}_1}{\partial \delta} = \frac{1}{\theta} \alpha F_1'\left(c_2^r\right) \delta \frac{u''\left(c_2^w\right)}{u''\left(c_2^r\right)} \frac{\partial c_2^w}{\partial \delta} < 0, \tag{A.29}$$

where $F_1'(c) > 0$ with $F_1(c) = c\left[1 - \theta + u'(c)\right]$ (recall $-c\frac{u''(c)}{u'(c)} < 1$), and

$$\frac{\partial c_2^w}{\partial \delta} = -\frac{(1-\alpha)\theta F_2'\left(c_2^b\right) \frac{u\left(c_2^w\right)}{u'\left(c_2^b\right)}}{(1-\alpha)\theta F_2'\left(c_2^b\right) \delta \frac{u'\left(c_2^w\right)}{u'\left(c_2^b\right)} + \alpha F_1'\left(c_2^r\right) \delta \frac{u''\left(c_2^w\right)}{u''\left(c_2^r\right)}} < 0, \tag{A.30}$$

given $F_2'(c) > 0$ with $F_2(c) = cu'(c)$.

Similarly, consider a partial banking panic equilibrium with $\eta = 0$. Following the same procedure, I can show there exists a unique $\bar{m}_2$ that solves the following system of equations, and such $\bar{m}_2$ is strictly decreasing in δ.

$$u'\left(c_2^r\right) = 1 - \delta + \delta u'\left(c_2^w\right), \tag{A.31}$$

$$\theta \bar{m}_2 = \alpha c_2^r \left[1 - \theta + \theta u'\left(c_2^r\right)\right] + (1-\alpha)(1-\rho) c_2^w \left[1 - \theta\delta + \theta\delta u'\left(c_2^w\right)\right], \tag{A.32}$$

$$\hat{b} - \bar{m}_2 = (1-\alpha)\rho c_2^b u'\left(c_2^b\right), \tag{A.33}$$

$$u\left(c_2^b\right) - \delta u\left(c_2^w\right) = 0. \tag{A.34}$$

Step 2 First, when $\bar{m} = \bar{m}_1$, conditions (A.25)-(A.27) in a partial banking panic equilibrium are the same as the conditions solving for a full banking panic equilibrium. Consider a decrease in $\bar{m}$, c_2^w decreases and c_2^b increases, guaranteeing a full banking panic equilibrium as in condition (15). Similarly, when $\bar{m} = \bar{m}_2$, an increase in $\bar{m}$ results in a increase in c_2^w and a decrease in c_2^b, guaranteeing a no banking panic equilibrium as in condition (13). □

Proof of Proposition 4. Totally differentiating equations (14), (25), (32) and (33) with respect to $\bar{o}$, I obtain:

$$1 - \theta = \alpha F_1'\left(c_2^r\right)\frac{\partial c_2^r}{\partial \bar{o}} - (1-\alpha)(1-\rho) F_2\left(c_2^w\right)\frac{\partial \eta}{\partial \bar{o}} + (1-\alpha)(1-\rho)(1-\eta) F_2'\left(c_2^w\right)\frac{\partial c_2^w}{\partial \bar{o}}, \tag{A.35}$$

$$0 = (1-\rho) F_3\left(c_2^b\right)\frac{\partial \eta}{\partial \bar{o}} + \left[\rho + (1-\rho)\eta\right] F_3'\left(c_2^b\right)\frac{\partial c_2^b}{\partial \bar{o}}, \tag{A.36}$$

$$u''\left(c_2^r\right)\frac{\partial c_2^r}{\partial \bar{o}} = \delta u''\left(c_2^w\right)\frac{\partial c_2^w}{\partial \bar{o}}, \tag{A.37}$$

$$u'\left(c_2^b\right)\frac{\partial c_2^b}{\partial \bar{o}} = \delta u'\left(c_2^w\right)\frac{\partial c_2^w}{\partial \bar{o}}, \tag{A.38}$$

where $F_1(c) = c\left[1 - \theta + \theta u'(c)\right]$, $F_2(c) = c\left[1 - \theta\delta + \theta\delta u'(c)\right]$, and $F_3(c) = cu'(c)$. Note that $u'(c) > 0$, $u''(c) < 0$ and $F_i'(c) > 0$ for $i \in \{1, 2, 3\}$ given $-c\frac{u''(c)}{u'(c)} < 1$. From (A.37) and (A.38), $\frac{\partial c_2^r}{\partial \bar{o}}\frac{\partial c_2^w}{\partial \bar{o}} \geq 0$ and $\frac{\partial c_2^b}{\partial \bar{o}}\frac{\partial c_2^w}{\partial \bar{o}} \geq 0$, i.e., $\frac{\partial c_2^r}{\partial \bar{o}}$, $\frac{\partial c_2^w}{\partial \bar{o}}$, and $\frac{\partial c_2^b}{\partial \bar{o}}$ have the same sign of being positive or negative. From (A.36), $\frac{\partial \eta}{\partial \bar{o}}\frac{\partial c_2^b}{\partial \bar{o}} \leq 0$, i.e., $\frac{\partial \eta}{\partial \bar{o}}$ and $\frac{\partial c_2^b}{\partial \bar{o}}$ have different sign of being positive and negative. The only possibility of making condition (A.35) and (A.36) hold is

$$\frac{\partial \eta}{\partial \bar{o}} < 0, \frac{\partial c_2^r}{\partial \bar{o}} > 0, \frac{\partial c_2^w}{\partial \bar{o}} > 0, \frac{\partial c_2^b}{\partial \bar{o}} > 0. \tag{A.39}$$

□