How Central Banks Should Use Their Balance Sheets to Control Flight-to-Safety

Fengfan Xiang*
University of Western Ontario

April 14, 2025

Abstract

Wholesale banking panics have driven several flight-to-safety events in recent decades, raising concerns over financial stability. I study such events using a model with retail and wholesale banks, where the severity of wholesale banking panics is endogenously determined and captures the magnitude of flight-to-safety. I show that expanding the central bank's balance sheet mitigates wholesale banking panics, but doing so may reduce welfare by lowering returns on assets, thereby limiting their usefulness in transactions. However, expanding the reach of interest-bearing central bank liabilities to wholesale banks, which can be achieved through the U.S. Federal Reserve's overnight reverse repurchase agreement facility, can mitigate panics without reducing welfare.

Key Words: central bank balance sheet, flight-to-safety, wholesale bank

JEL: E4 E5

*Email: fxiang8@uwo.ca.
I am grateful to Stephen Williamson, Baxter Robinson, and Sergio Ocampo Díaz for their invaluable guidance and support. I also want to thank Juan Carlos Hatchondo, Elizabeth Caucutt, Duhyeong Kim, Jesse Perla, Eugene Tan, Julieta Caunedo, Wei Cui, Randall Wright, Luigi Pistaferri, Benjamin Pugsley, Guillermo Ordoñez, Shengxing Zhang, Wendy Morrison, Taizen Mori, as well as seminar participants at the University of Western Ontario and participants of the 2024 Rice-LEMMA Monetary Conference for their helpful comments.

1 Introduction

In periods of financial distress, investors reallocate their portfolios from riskier assets toward safer ones. These *flight-to-safety* phenomena reflect shifts in asset demand and amplify financial instability. In particular, panics in the wholesale banking sector, which is beyond the coverage of financial regulations and supervision, have driven several flight-to-safety events and are viewed as a major source of financial instability in past decades (Bernanke, 2012, 2018; Gorton, 2010).[1] For example, during the 2008 Financial Crisis, the failure of Lehman Brothers sparked a panic in money market mutual funds, with over $400 billion withdrawn in September 2008. More recently, during the COVID-19 pandemic, investors moved over $100 billion from prime money funds to safe assets backed by government securities in March 2020 (Sengupta & Xue, 2020).

I study flight-to-safety phenomena driven by wholesale banking panics and their implications for central bank balance sheet policies.[2] The key finding is that *central banks can mitigate flight-to-safety by expanding their balance sheets, but this may lower asset returns, reducing the usefulness of assets in transactions and harming welfare.* For instance, an open market operation that expands the balance sheet by purchasing Treasury bills stabilizes short-term funding markets because it lowers the returns on these safe securities, reducing investors' incentive to seek refuge in them. However, the decreased returns hinder transactions involving Treasury bills, like repo transactions. I show that the return reduction holds in general, even for risky bank liabilities. The key is the endogenous response of asset demand: by mitigating flight-to-safety, investors switch to bank liabilities, bidding down their returns and making them less useful in transactions.

[1] Wholesale (or shadow) banks, such as money market funds, are financial institutions engaging in "credit intermediation involving entities and activities outside the regular banking system" (Financial Stability Board). In 2022, these banks held $63 trillion, representing 79% of global GDP (S&P Global).

[2] Existing regulatory tools for financial stability are limited in their effectiveness and scope of coverage in wholesale banking. Moreover, wholesale banks often fall beyond the reach of direct central bank crisis interventions. For example, the Dodd-Frank Act removed the U.S. Federal Reserve's authority to lend to wholesale banks (Fischer, 2016). Recently, attention has been drawn to the financial stability implications of central bank balance sheet policies (Bernanke, 2016; Greenwood, Hanson, & Stein, 2016).

Specifically, I develop a three-period, two-sector banking model with retail and wholesale banks that conduct intermediation activities to facilitate their depositors' transactions. Unlike retail banks, wholesale banks are not subject to stringent regulations, such as the leverage requirements in Basel III, and have no reserve account at the central bank.[3] Moreover, they serve institutional investors rather than individual businesses and consumers. Linking the two sectors is an interbank market that allows wholesale banks to lend to retail banks, providing a conduit for indirect access to central bank reserves.

Crucially, panics can arise in the wholesale banking sector due to the risk of banking failure, as in Gertler and Kiyotaki (2015). Depositors anticipate potential losses, withdraw deposits, and flee to safe government bonds that are extensively used in wholesale payments. Their endogenous withdrawal behavior captures the severity of wholesale banking panics or the magnitude of flight-to-safety. I evaluate the general equilibrium effects of central bank balance sheet expansions, which alter not only the supply of bank deposits and government bonds but also depositors' withdrawal behavior, shifting their demand between these assets.

I show that an expansion in the size of the central bank's balance sheet mitigates wholesale banking panics. A balance sheet expansion, through open market operations, involves purchasing government bonds with the issuance of new reserves. This reduces the supply of government bonds to the private sector, lowering their rate of return and making bonds a less attractive safe harbor. Meanwhile, the balance sheet expansion increases the supply of reserves, facilitating transactions in the interbank market, where retail banks borrow from wholesale banks using reserves as collateral. In this way, this expansion makes it easier for wholesale banks to acquire interbank loans, allowing them to issue more attractive deposits to their depositors. Overall, the balance sheet expansion mitigates wholesale banking panics by making safe government bonds less attractive relative to wholesale bank deposits.

[3] Similar definitions are used by Gertler, Kiyotaki, and Prestipino (2016) and Ordoñez (2018).

Despite mitigating wholesale banking panics, the central bank balance sheet expansion lowers the returns obtained by retail and wholesale bank depositors, reducing welfare. First, this expansion reduces the supply of government bonds to the private sector, lowering their return and harming wholesale bank depositors who use them in wholesale payments. Second, by mitigating banking panics, a large number of wholesale bank depositors shift their asset demand from government bonds to bank deposits, bidding down their return even though the balance sheet expansion promotes the deposit issuance. Finally, this expansion also reduces the return on retail bank deposits. Retail banks pay lower returns on deposits because, otherwise, they could substitute their funding source from deposits to cheaper interbank loans. Wholesale banks ask for lower returns on interbank loans due to their higher asset demand to back increased deposit liabilities.

To better understand the crucial role of depositors' asset demand, I evaluate balance sheet expansions in special scenarios when depositors endogenously choose not to shift their asset demand. This occurs when wholesale bank depositors invariably expect a higher return on either deposits or government bonds, strictly preferring one over the other. Unlike before, the balance sheet expansion now benefits depositors, increasing welfare. The increased reserve supply promotes the issuance of bank deposits, both in the retail and wholesale banking sectors. With no shift in depositors' asset demand, this increased deposit supply necessarily implies higher (instead of lower) deposit returns.

Finally, I study a balance sheet policy that alters the composition of central bank liabilities, leaving the supply of government bonds unchanged. To do this, I introduce another central bank liability that plays a role similar to the U.S. Federal Reserve's overnight reverse repurchase agreement facility (ON-RRP henceforth), which is accessible to a broader range of financial institutions, including wholesale banks.[4]

I show that, like central bank balance sheet expansions, a swap of reserves for ON-

[4] Unlike government bonds, ON-RRPs only circulate among financial institutions, so introducing them will not amplify financial instability by providing another safe harbor to depositors.

RRPs mitigates wholesale banking panics. As more ON-RRPs become available, wholesale banks reduce their reliance on indirect access to reserves through interbank lending. This eases their burden of compensating for retail banks' balance sheet costs arising from stringent regulations.[5] As a result, wholesale banks can offer more attractive deposits to compete for depositors, mitigating banking panics.

Unlike central bank balance sheet expansions, swapping reserves for ON-RRPs increases asset returns and improves welfare. By mitigating banking panics, this swap reduces the demand for safe government bonds, raising their rate of return. In response to the increased bond return, wholesale banks offer higher returns on deposits to maintain their attractiveness. As wholesale banks incur higher payments on deposit liabilities, they seek higher returns on interbank loans. Consequently, retail banks substitute their funding sources from loans to deposits, and the increase in the supply of retail bank deposits raises their rate of return in equilibrium.

Related Literature The two-sector banking model I construct builds on Williamson (2019) with the introduction of financial instability in the form of wholesale banking panics. These panics arise from random bank failures and depositors' lack of information about which banks will fail — distinct from panics induced by sequential service constraints (i.e., first-come-first-served), as in Diamond and Dybvig (1983).[6] Gertler, Kiyotaki, and Prestipino (2016) study wholesale banking panics in a general equilibrium framework and explore the role of lender-of-last-resort and macroprudential policies. I focus on central bank balance sheet policies that directly alter the relative attractiveness of bank deposits versus safe assets to which depositors might flee. In doing so, I contribute

[5]Specifically, the balance sheet costs arise from retail banks' leverage requirements, which restrict their liability-to-asset ratios and force them to conduct costly equity financing. Kim, Martin, and Nosal (2020), Martin, McAndrews, Palida, and Skeie (2019), and Williamson (2019) also show how the costs associated with stringent banking regulations limit banks' ability to grow their balance sheet and provide useful financial intermediation activities.

[6]As shown by Andolfatto and Nosal (2020) and Huang and Keister (2024), self-fulfilling panics can occur in wholesale banking settings without such constraints.

to a growing literature on the financial stability implications of central bank balance sheet policies, including Bernanke (2016) and Woodford (2016), and others.

I show how the central bank enhances the stability and resilience of the financial system by providing more high-quality liquid assets, such as reserves. Bush, Kirk, Martin, Weed, and Zobel (2019) argue that an ample reserve supply helps banks meet their outflow needs and avoid the fire-sale effects during crises. Carlson, Duygan-Bump, Natalucci, Nelson, Ochoa, Stein, and Van den Heuvel (2016) and Greenwood, Hanson, and Stein (2016) show that central bank balance sheet policies can mitigate key threats to financial stability and highlight the usefulness of reserves and ON-RRPs, which aligns with my findings.

A key contribution of this paper is showing that financial stability does not necessarily imply an improvement in welfare. I do so by explicitly modeling the shifts in asset demand in response to a policy change. Williamson (2022) shows a similar relationship between financial stability and welfare in the context of central bank digital currency (CBDC), but for a different reason. CBDC engenders financial instability, encouraging flight-to-safety as it is a convenient safe harbor that can be used in a wide range of transactions. However, this convenience makes such instability less disruptive, improving welfare.

The welfare implications for central bank balance sheet policies are mainly determined by market liquidity, as I focus on financial instability in the *short-term funding markets* and examine the consequences of policy interventions in these markets like Gorton and Metrick (2012) and Martin, Skeie, and Thadden (2014). The positive implications, such as the interest rate structure and movements in response to central bank balance sheet expansions, are consistent with the empirical findings of Arrata, Nguyen, Rahmouni-Rousseau, and Vari (2020), who study the effects of quantitative easing on short-term interest rates. The implications of *long-term funding markets* for central bank balance sheet policies have been studied in papers that emphasize the role of quantitative easing in lowering long-term interest rates and reducing the cost of credit for production, such as Cardamone, Sims, and Wu (2023) and Cui and Sterk (2021), among others.

The rest of the paper is organized as follows. I present the model and define the equilibrium in section 2. In section 3, I solve three possible baseline equilibria with only reserves as the central bank liability and show how the size of the central bank's balance sheet determines the type of equilibrium. Section 4 is an analysis of the introduction of the ON-RRPs. The final section 5 concludes.

2 Model

This is a two-sector banking model with retail and wholesale banks. There are three periods, $t = 1, 2, 3$, with no time discounting between periods. There are three sets of private agents: a measure one of depositors, a measure one of producers, and an infinite measure of bankers who self-select into operating retail or wholesale banks. Private agents can work (h) and consume (c), and a linear production technology allows them to convert labor to goods one-for-one. Finally, there is also a government consisting of a fiscal authority and a central bank.

Depositors work and deposit their earnings in period 1, consume in period 2, and work to pay taxes in period 3. Their preferences are captured by $-h_1 + u(c_2) - h_3$, where u is strictly increasing, strictly concave, and twice continuously differentiable, with $u(0) = 0$, $\lim_{c\to 0} u'(c) = \infty$, $\lim_{c\to\infty} u'(c) = 0$, $\lim_{c\to 0} cu'(c) = 0$, and $-c\frac{u''(c)}{u'(c)} < 1$ for $c \geq 0$. Among the entire population of depositors, a fraction $\alpha \in (0, 1)$ of them are retail bank depositors that represent individual consumers and businesses, and the remaining $1 - \alpha$ of depositors are wholesale bank depositors that represent institutional investors. Producers and bankers are risk-neutral and profit-maximizing. Producers work to produce consumption goods for depositors in period 2 and consume their returns in period 3. Their payoffs are $-h_2 + c_3$. Bankers can work in period 1 to raise equity (i.e., sweat equity) and in period 3 to pay off their debts, and they consume their profits. Their payoffs are $c_1^B - h_1^B + c_3^B - h_3^B$.

Trade between depositors and producers takes place in bilateral exchanges in period 2, where depositors make a take-it-or-leave-it offer to the producer they meet in exchange for consumption goods, similar to the decentralized market in Lagos and Wright (2005). The reason depositors do not consume the goods they produced in period 1 is that all goods are perishable and cannot be carried across periods. Depositors must also acquire assets in advance, such as deposit claims, to settle their transactions because private agents are subject to limited commitment (no one can be forced to repay debts), implying that unsecured credit, like depositors' IOUs, is not accepted. They acquire assets in period 1, where a centralized Walrasian market allows agents to trade goods and assets. In particular, banks write deposit contracts with depositors in this period. In period 3, all debts are redeemed, and agents consume their returns.

Assets and Collateral Technology There are two underlying assets: central bank reserves and government bonds. Central bank reserves are private banks' account balances with the central bank, while government bonds are issued by the fiscal authority. Both assets are issued in period 1. In period 3, the central bank pays a *real interest rate* of $r^m - 1$ on reserves, while the fiscal authority pays a *real interest rate* of $r^b - 1$ on bonds.[7] Crucially, reserves and government bonds are not perfect substitutes. The main distinction is their degree of liquidity: reserves are less liquid because they are limited to retail banks, whereas government bonds are more liquid because they can be held by anyone, including depositors and producers. In addition to the underlying assets, banks have access to a collateral technology that allows them to borrow against assets, creating useful financial intermediary liabilities, such as deposit claims and interbank loans.

[7] I focus on the endogenous real interest rates exclusively throughout the analysis as this paper mainly studies the real effects of central bank balance sheet policies that alter the supply of central bank liabilities and government bonds. Although the *nominal* interest rate on reserves is administered by the central bank in practice, their real rate of return is market-determined.

2.1 Banking

The two-sector banking structure follows Williamson (2019). The sizes of these sectors are determined by α, which, as explained earlier, reflects the mass of depositors in each sector. Retail banks resemble highly regulated depository institutions. They have access to central bank reserves but are subject to a leverage requirement that limits their liability-to-asset ratio to $\theta \in (0, 1)$.[8] Wholesale banks resemble less regulated financial institutions like mutual funds and hedge funds. They do not have access to central bank reserves and are not subject to leverage requirements. An interbank market links these two types of banks, allowing wholesale banks to lend to retail banks to hold reserves indirectly.

I introduce instability in the wholesale banking sector, triggered by the risk of banking failure, as in Gertler and Kiyotaki (2015). This risk induces some wholesale bank depositors to withdraw from their banks. Specifically, an exogenous fraction $1 - \delta$ of wholesale banks fail in period 2, defaulting on their liabilities and absconding with assets in period 3.[9] The identity of these banks is ex-ante unknown to depositors. Therefore, depositors may become *panicky* and flee to safe assets because they must make their withdrawal decisions with imperfect information at the end of period 1, before any bank fails. The share of depositors that become panicky is endogenously determined and plays a crucial role in the analysis as it captures the magnitude of flight-to-safety.

Retail Banks Retail banks maximize profits by choosing deposit contracts and financial portfolios. They offer a deposit contract (k^r, d^r) to each depositor, which requires the deposit of k^r units of the consumption goods at $t = 1$, in return for a tradeable claim to d^r units of goods at $t = 3$. Besides taking in deposits, retail banks borrow ℓ^r from the

[8] I impose the leverage requirement at $t = 3$ when agents have to settle their debts, thereby directly linking this requirement to banks' incentive problem — namely, whether they would default on their liabilities and abscond with assets.

[9] Technically, this can be captured by a "collapse" in the collateral technology that allows banks to default without worrying about losing their collateral.

Assets	Liabilities and Equity
reserves: $r^m m$	deposit claims: d^r
government bonds: $r^b b^r$	interbank borrowing: $r^\ell \ell^r$
	equity: e

Table 1: Retail Bank's Balance Sheet

interbank market.[10] They must also raise equity to finance part of their assets to meet their leverage requirement, which leads to a balance sheet cost because it forces retail bankers to suffer the cost of working at $t = 1$ to accumulate (sweat) equity (Williamson, 2019). Retail banks invest their funds in m units of reserves and b^r units of government bonds at $t = 1$. Table 1 presents retail banks' balance sheets, or more precisely, the inflow of funds from their asset holdings and payment outflows on their liabilities at $t = 3$.

Competition among retail banks drives them to offer contracts that maximize depositors' utility. A depositor with a contract (k^r, d^r) makes a take-it-or-leave-it offer to the producer they meet in exchange for $c_2 = d^r$ units of consumption goods with their deposit claim after depositing k^r with their bank. This results in a payoff

$$-k^r + u\left(d^r\right). \tag{1}$$

The limited commitment problem implies that banks' return on assets should exceed their payment on liabilities. As for retail banks, the leverage requirement further restricts their balance sheet in a way such that a fraction θ of their asset returns must cover their liabilities so

$$\theta \overbrace{\left(r^m m + r^b b^r\right)}^{\text{return on assets}} \geq \underbrace{d^r + r^\ell \ell^r}_{\text{payment on liabilities}}. \tag{2}$$

[10] In principle, retail banks could lend to wholesale banks. In such a scenario, wholesale banks would hold government bonds as collateral to secure interbank borrowing. However, because of the risk of wholesale banking failure, retail banks' expected rate of return on such lending must be lower than the rate they directly invest in government bonds. As a result, no retail bank lends to wholesale banks.

Finally, a retail bank must make a nonnegative profit to operate, so

$$\overbrace{k^r - d^r}^{\text{return from the deposits}} \underbrace{-m - b^r + \ell^r + r^m m + r^b b^r - r^\ell \ell^r}_{\text{return from financial portfolios}} \ \geq \ 0, \tag{3}$$

where $k^r, d^r, m, b^r \geq 0$. In equilibrium, free entry ensures this holds with equality.[11]

To sum up, competitive retail banks choose deposit contract (k^r, d^r) and financial portfolio (m, b^r, ℓ^r) to maximize their depositors' utility (1), subject to the leverage constraint (2), nonnegative profit constraint (3) and nonnegative constraints $k^r, d^r, m, b^r \geq 0$. Before solving the retail bank's problem, notice that, except for the nonnegative constraint on government bonds $b^r \geq 0$, other nonnegative constraints never bind in equilibrium. In particular, the nonnegative constraint on reserves $m \geq 0$ does not bind because the demand for reserves must equal the supply, which is exogenously determined by the central bank's balance sheet policy. That is, the *real interest rate* on reserves adjusts so that retail banks are willing to hold a positive stock of reserves in equilibrium.

The solution to the retail bank's problem gives the following equilibrium conditions

$$r^m \left[1 - \theta + \theta u'(d^r)\right] \ = \ 1, \tag{4}$$

$$r^b \left[1 - \theta + \theta u'(d^r)\right] + \lambda^r \ = \ 1, \tag{5}$$

$$r^\ell u'(d^r) \ = \ 1, \tag{6}$$

where λ^r denotes the Lagrange multiplier for the nonnegative constraint $b^r \geq 0$. Equations (4) and (5) determine retail banks' demands for reserves (m) and government bonds (b^r) associated with their deposit contract. Equation (6) determines the interest rate a retail bank is willing to pay on interbank borrowing.

[11] From (3) with equality,

$$\underbrace{m + b^r - k^r - \ell^r}_{\text{equity raised at } t=1} = \underbrace{r^m m + r^b b^r - d^r - r^\ell \ell^r}_{\text{return on equity at } t=3},$$

where the left-hand side of the equation denotes the equity raised by retail banks at $t = 1$. This equity must be positive because, from (2), the leverage requirement pushes retail banks to earn a positive equity return at $t = 3$.

Wholesale Banks Like retail banks, wholesale banks maximize profits by choosing deposit contracts and financial portfolios. However, wholesale bank deposits' claims are less liquid than retail ones. I capture this by assuming that a fraction ρ of producers do not accept these claims and instead accept only government bonds, similar to Williamson (2019). This restriction about payment acceptance also captures certain demand for bonds by private agents, particularly their demand for wholesale payments.[12]

Depositors withdraw when they meet producers demanding bonds or when they are concerned about the bank's safety. I refer to wholesale bank depositors who withdraw for the second reason as *panicking depositors*. Panicking depositors withdraw with an endogenous withdrawal probability η. In equilibrium, η also represents the fraction of panicking depositors who withdraw and flee to safe government bonds, capturing the severity of wholesale banking panics. The more severe the panics are, the higher the aggregate demand for bonds, as more depositors flee to these safe assets.

A deposit contract in the wholesale banking sector is a triple (k^w, b', d^w). As before, k^w is the required deposit for each depositor at $t = 1$. In return, a wholesale bank depositor can withdraw b' units of government bonds at the end of $t = 1$ or opt for a tradeable deposit claim to d^w consumption goods at $t = 3$. In this way, wholesale banks provide liquidity insurance to their depositors in the spirit of Diamond and Dybvig (1983). Besides deposits, wholesale banks lend ℓ^w to retail banks and purchase in b^w units of government bonds, where part of the bonds they purchased are used for depositors' withdrawal requests. In contrast to retail banks, wholesale banks cannot hold reserves. They also do not raise equity as they face no leverage requirement. Table 2 presents wholesale banks' inflows and outflows of funds at $t = 3$.

As with retail banks, wholesale banks maximize depositors' utility to compete for

[12]In practice, overnight repurchase agreements (repos), for which government bonds are the dominant collateral, involve a large volume of transactions. The repo markets indirectly support exchanges in goods and services, especially the exchanges at the wholesale level. Directly exchanging government bonds is a convenient shortcut to capturing wholesale payments supported by repo markets.

Assets	Liabilities
government bonds: $r^b b^w$	government bonds: $r^b[\rho+(1-\rho)\eta]b'$
interbank lending: $r^\ell \ell^w$	deposit claims: $(1-\rho)(1-\eta)d^w$

Table 2: Wholesale Bank's Balance Sheet

depositors. They diversify across depositors, considering the potential banking failure with probability $1-\delta$ and depositors' withdrawal probability η.[13] The expected utility of a wholesale bank depositor is

$$-k^w + [\rho + (1-\rho)\eta]\, u\left(r^b b'\right) + (1-\rho)(1-\eta)\,\delta u\left(d^w\right). \tag{7}$$

That is, after paying the required deposit k^w, a fraction $\rho + (1-\rho)\eta$ of the wholesale depositors withdraw government bonds and make a take-it-or-leave-it offer that exchanges for $r^b b'$ units of consumption goods from the producer they meet. The remaining $(1-\rho)(1-\eta)$ of depositors trade with deposit claims and obtain d^w units of goods.

Wholesale banks are not subject to the leverage requirement. However, they must prepare assets that satisfy the following collateral constraint,

$$\overbrace{r^b\left[b^w - [\rho+(1-\rho)\eta]\, b'\right] + r^\ell \ell^w}^{\text{return on assets}} \;\geq\; \underbrace{(1-\rho)(1-\eta)\, d^w}_{\text{payment on liability}}, \tag{8}$$

to show that they are willing to pay off their liabilities. This constraint holds for all wholesale banks ex-ante. But, a fraction $1-\delta$ of wholesale banks will fail, and this potential banking failure triggers depositors' panicking behavior reflected by the endogenous withdrawal probability η.

Finally, wholesale banks also require a nonnegative profit to operate,

$$\overbrace{k^w - (1-\rho)(1-\eta)\,\delta d^w}^{\text{return from deposits}} \underbrace{- b^w - \ell^w + r^b\left[b^w - [\rho+(1-\rho)\eta]\, b'\right] + r^\ell \ell^w}_{\text{return from financial portfolio}} \;\geq\; 0, \tag{9}$$

[13]Banks serve many depositors, while each depositor can contact only one bank. Although depositors cannot diversify across banks, they can observe all banks' contracts and choose the optimal one.

where $k^w, b', d^w, b^w, b^w - [\rho + (1-\rho)\eta]\, b' \geq 0$. Wholesale banks only pay off their deposit claims if they do not fail, which occurs with probability δ. They have some of their government bonds withdrawn by depositors, meaning they only earn returns on the remaining part, i.e., $b^w - [\rho + (1-\rho)\eta]\, b'$. Again, (9) holds with equality under free entry.

To sum up, wholesale banks choose deposit contract (k^w, b', d^w) and financial portfolio (b^w, ℓ^w) to maximize a representative wholesale depositor's expected utility (7), subject to the collateral constraint (8), nonnegative profit constraint (9) and nonnegative constraints $k^w, b', d^w, b^w, b^w - [\rho + (1-\rho)\eta]\, b' \geq 0$. As with the retail bank's problem, notice that, except the nonnegative constraint on government bonds $b^w - [\rho + (1-\rho)\eta]\, b' \geq 0$, other nonnegative constraints never bind in equilibrium.

The solution to the wholesale bank's problem gives the following equilibrium conditions

$$
\begin{aligned}
r^b u'\left(r^b b'\right) &= 1, &(10)\\
r^b \left[1 - \delta + \delta u'\left(d^w\right)\right] + \lambda^w &= 1, &(11)\\
r^\ell \left[1 - \delta + \delta u'\left(d^w\right)\right] &= 1, &(12)
\end{aligned}
$$

where λ^w denotes the Lagrange multiplier for $b^w - [\rho + (1-\rho)\eta]\, b' \geq 0$. Equation (10) determines wholesale banks' demand for government bonds associated with each depositor's withdrawal request (i.e., $[\rho + (1-\rho)\eta]\, b'$), while equation (11) determines their demand for government bonds to back their deposit claims (i.e., $b^w - [\rho + (1-\rho)\eta]\, b'$). Wholesale banks can also use claims on interbank lending to back their deposit claims, and equation (12) determines such demand.

The equilibrium conditions displayed above and those in the retail bank's problem are asset pricing kernels in consumption-based capital asset pricing models, surveyed by Campbell (1999). For example, the government bond price, i.e., the inverse of the gross real interest rate on bonds $1/r^b$ in (10), is equal to wholesale bank depositors' marginal return of trading with bonds. Other conditions, (4), (5), (6), (11) and (12), have similar interpretations by properly adjusting parameters related to the leverage requirement θ,

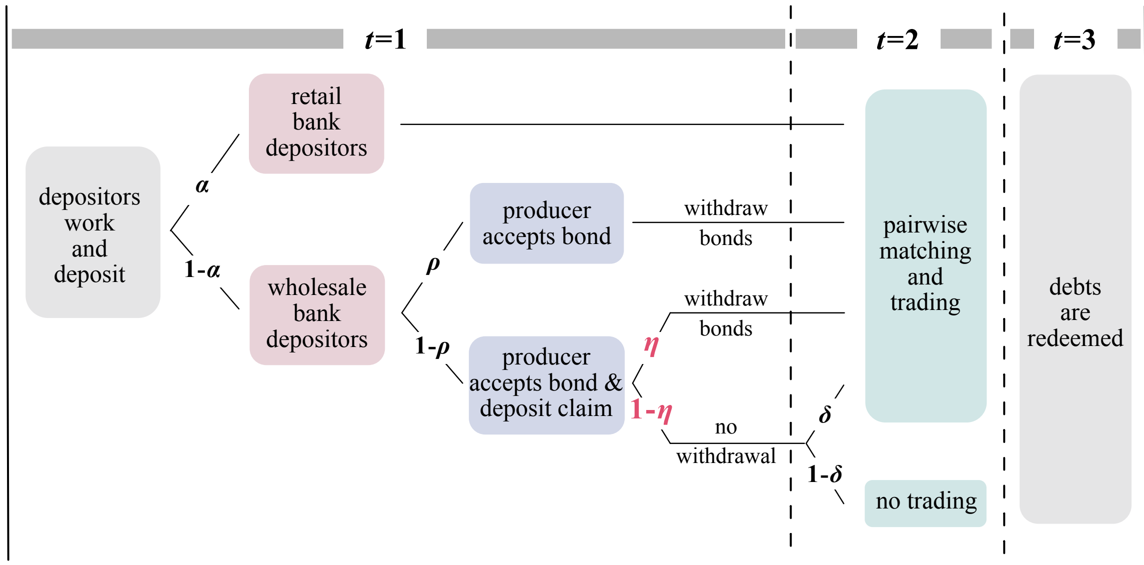

Figure 1: Timing of Events

banking failure probability $1-\delta$, and binding constraints λ^r and λ^w.

Timing of Events Figure 1 summarizes the timing of events and provides a visual guide to the pattern of meetings and exchanges among private agents.

2.2 Flight-to-Safety by Wholesale Bank Depositors

The endogenous withdrawal probability η captures the severity of wholesale banking panics or the magnitude of flight-to-safety. Depositors' expected payoffs determine their withdrawal probability and give rise to three scenarios for wholesale banking panics:

$$\text{(i) No banking panic:} \quad \eta = 0, \quad \text{if } u\left(r^b b'\right) \leq \delta u\left(d^w\right); \tag{13}$$

$$\text{(ii) Partial banking panic:} \quad 0 < \eta < 1, \quad \text{if } u\left(r^b b'\right) = \delta u\left(d^w\right); \tag{14}$$

$$\text{(iii) Full banking panic:} \quad \eta = 1, \quad \text{if } u\left(r^b b'\right) \geq \delta u\left(d^w\right). \tag{15}$$

From (13)-(15), a no banking panic equilibrium occurs if wholesale bank depositors prefer wholesale banks' deposit claims to government bonds, a partial banking panic occurs if these depositors are indifferent between these two options, and a full banking panic occurs if government bonds are preferred.

2.3 Government

At the beginning of period 1, the fiscal authority issues $\hat{b}$ units of government bonds and transfers the revenue τ_1 to depositors:

$$\hat{b} \;=\; \tau_1. \tag{16}$$

The central bank conducts an asset swap, purchasing $\hat{b} - \bar{b}$ units of government bonds with reserves. So,

$$\underbrace{\hat{b} - \bar{b}}_{\text{asset}} \;=\; \underbrace{\bar{m}}_{\text{liability}}, \tag{17}$$

where $\bar{m}$ and $\bar{b}$ represent the amounts of reserves and government bonds circulating within the private sector, respectively. Fiscal policy determines the total supply of government bonds ($\hat{b} = \bar{m} + \bar{b}$). The central bank adjusts the relative supply of government bonds and reserves to the private sector through open market operations. The supply of reserves, $\bar{m}$, describes the central bank balance sheet policy, representing the size of its balance sheet. I focus on the effects of this balance sheet policy while holding fiscal policy constant.

Government bonds and central bank reserves are redeemed in period 3. The fiscal authority taxes depositors τ_3 lump sum to payoff its debts and transfers τ^{cb} (receives, if negative) to the central bank to support its payments:

$$r^b \hat{b} + \tau^{cb} \;=\; \tau_3. \tag{18}$$

The central bank pays off its reserves, using the returns from its holdings of government bonds and the transfer from the fiscal authority:

$$r^m \bar{m} \;=\; r^b \left(\hat{b} - \bar{b}\right) + \tau^{cb}. \tag{19}$$

2.4 Definition of Equilibrium

I focus on equilibria where retail banks' leverage and wholesale banks' collateral constraints bind. As a result, banks cannot provide deposit claims that support a satiated consumption level for depositors, i.e., $c_2 < c^\star$ with $u'(c^\star) = 1$. Otherwise, depositors always consume the satiated level, and there would be no change in allocation in response to policy changes. To ensure these constraints bind, I assume a scarcity of total government bond supply as in Andolfatto and Williamson (2015) and Williamson (2019). Specifically, Assumption 1 states that even if retail banks exhaust all government bonds, they still cannot support the satiated consumption level for their depositors.

Assumption 1 (Scarcity of Total Government Bond Supply)**.** *Total supply of government bonds is scarce, such that $\theta\hat{b} < \alpha c^\star$ with $u'(c^\star) = 1$.*

Definition 1 (Equilibrium)**.** *Given fiscal policy $\hat{b}$ and central bank balance sheet policy $\bar{m}$, an equilibrium consists of an allocation $\left(\bar{b}, d^r, m, b^r, \ell^r, b', d^w, b^w, \ell^w, c_2^r, c_2^b, c_2^w\right)$, Lagrange multipliers λ^r and λ^w, market-determined real interest rates (r^m, r^ℓ, r^b), and a withdrawal probability η, satisfying the binding leverage constraint (2) and binding collateral constraint (8), equilibrium conditions for private banks' problems (4)-(12), one of conditions (13)-(15) to support wholesale bank depositors' withdrawal strategy, market clearing,*

$$\alpha m = \bar{m} \qquad \textit{(reserve market)}, \tag{20}$$

$$\alpha b^r + (1-\alpha)\, b^w = \bar{b} \qquad \textit{(government bond market)}, \tag{21}$$

$$\alpha \ell^r = (1-\alpha)\, \ell^w \qquad \textit{(interbank market)}, \tag{22}$$

and the complementary-slackness conditions with corresponding nonnegative constraints,

$$\lambda^r b^r = 0, \quad \lambda^r \geq 0, \quad b^r \geq 0, \tag{23}$$

$$\lambda^w [b^w - [\rho + (1-\rho)\,\eta]\, b'] = 0, \quad \lambda^w \geq 0, \quad b^w - [\rho + (1-\rho)\,\eta]\, b' \geq 0, \tag{24}$$

where $c_2^r = d^r$, $c_2^b = r^b b'$, and $c_2^w = d^w$ are consumption quantities for retail bank deposi-

tors, wholesale bank depositors trade with government bonds, and wholesale bank depositors trade with deposit claims.

3 Equilibrium

There will be three types of equilibria: no banking panic ($\eta = 0$), partial banking panic ($0 < \eta < 1$), and full banking panic ($\eta = 1$). For each type, I examine the effects of a central bank balance sheet expansion in the way that the central bank purchases government bonds from the private sector by issuing new reserves. As will become clear throughout this section, this policy alters the relative attractiveness between wholesale bank deposits and safe government bonds, shifting depositors' asset demand and changing the severity of banking panics.

As illustrated in Figure 2, expanding the size of the central bank's balance sheet reduces the severity of wholesale banking panics. Proposition 1 formalizes the result. I present all proofs in the Appendix A and discuss the intuition in the main text.

Proposition 1. *Under Assumption 1, there are two thresholds for the size of the central bank's balance sheet with $0 < \bar{m}_L < \bar{m}_H < \hat{b}$, where $\bar{m}_L$ and $\bar{m}_H$ solve the system of equations (14), (25), (26), and (27) with $\eta = 1$ and $\eta = 0$, respectively. The size of the central bank's balance sheet determines the equilibrium as follows:*

1. *a full banking panic, $\eta = 1$, occurs when the size of the central bank's balance sheet is below the lower threshold such that $\bar{m} \in (0, \bar{m}_L]$;*

2. *a partial banking panic, $\eta \in (0, 1)$, occurs when $\bar{m} \in (\bar{m}_L, \bar{m}_H)$;*

3. *no banking panic, $\eta = 0$, occurs when $\bar{m} \in [\bar{m}_H, \hat{b})$.*

Moreover, these thresholds increase with the probability of wholesale banking failure $1 - \delta$, i.e., $\frac{\partial \bar{m}_L}{\partial \delta} < 0$ and $\frac{\partial \bar{m}_H}{\partial \delta} < 0$.

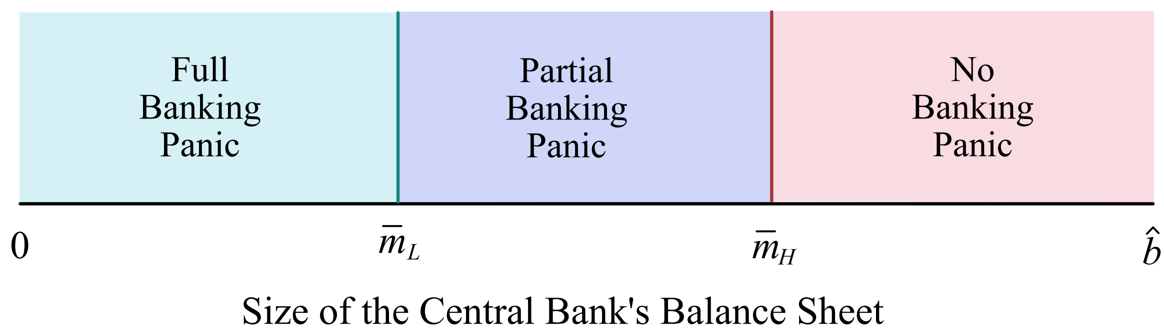

Figure 2: How To Determine the Type of Equilibrium

I prove Proposition 1 in a constructive way by first establishing conditions that solve a partial banking panic equilibrium and then establishing the role of the size of the central bank's balance sheet, $\bar{m}$, in sustaining different equilibria. Roughly speaking, compared to wholesale bank depositors, wholesale banks have a comparative advantage in holding reserves indirectly by lending to retail banks. Central bank balance sheet expansions swap government bonds that everyone can hold for reserves, making it easier for wholesale banks to compete for assets. As a result, this expansion allows wholesale banks to provide more attractive bank liabilities that help to mitigate banking panics.

In addition to the equilibrium type determination, Proposition 1 shows that if wholesale banking becomes more risky, the central bank can prevent additional withdrawals by expanding its balance sheet. Specifically, swapping government bonds for reserves makes deposit claims more attractive to depositors, helping offset the increased risk of banking.

3.1 The Role of Government Bonds

Before delving into the details of each equilibrium, I establish two important intermediate results, showing that no bank holds government bonds as collateral in equilibrium. As summarized in Lemma 1 and Lemma 2, these results are independent of monetary policy.

Lemma 1. *Retail banks never hold government bonds, i.e.,* $b^r = 0$ *and* $\lambda^r > 0$.

Retail banks bear a balance sheet cost of holding assets because they have to work to raise equity to finance their investment. Consequently, they hold a positive stock of government bonds only when the interest on bonds is strictly higher than the interest rate they pay for interbank borrowing. However, wholesale banks always ask for a higher interest rate on interbank lending than government bonds that are always available to them. As a result, retail banks never hold government bonds in equilibrium.

Lemma 2. *Wholesale banks only purchase government bonds for depositors' withdrawal requests, i.e.,* $b^w - [\rho + (1-\rho)\,\eta]\, b' = 0$.

Wholesale bank depositors prefer using government bonds in exchange directly rather than letting banks hold bonds as collateral to back tradeable deposit claims. Otherwise, there is a chance that their bank will fail and abscond. To entertain depositors' preferences, competitive wholesale banks assign all the bonds for their withdrawal requests.

Remark Under Lemma 1, conditions (4), (5), (11), and (12) imply that the interest rate on reserves is higher than the interest rate on interbank loans, which itself is higher than the interest rate on government bonds, i.e., $r^m > r^\ell \geq r^b$. This interest rate structure is consistent with the short-term interest rate structure in the U.S. since the Federal Reserve started paying interest on reserve balances in October 2008.[14] Canada also exhibits similar features. This structure can be explained by differences in asset liquidity. Reserves are the least liquid among these three assets as they can only circulate among retail banks, thereby trading at a low price and a high real interest rate. By contrast, government bonds are the most liquid, bearing a liquidity premium and a low real interest rate. The interest rate on interbank loans lies between the above two rates because both retail and wholesale banks can hold these loans.

[14] In the U.S., The interest rate on reserve balances is higher than the federal funds rate, which is higher than short-term T-bill rates. Although the federal funds rate is the overnight lending rate among depository intuitions (i.e., retail banks) and government-sponsored enterprises (GSEs), it is a good proxy of the interbank rate in this paper. The reason is that, in the last decade, most of the federal fund market activities involve lending from GSEs, which cannot directly obtain the yields on reserves, to depository institutions. Moreover, GSEs are not subject to stringent regulations like depository institutions, suffering few balance sheet costs. These features are the same as those developed in this paper.

3.2 Partial Banking Panic

I begin with the partial banking panic ($0 < \eta < 1$) equilibrium under $\bar{m} \in (\bar{m}_L, \bar{m}_H)$, where changes in the severity of wholesale banking panics (η) reflect the shifts in asset demand and play a critical role in determining the effects of a central bank balance sheet expansion. In what follows, I express all equilibrium conditions in terms of depositors' withdrawal probability η and their consumption quantities (c_2^r, c_2^b, c_2^w), as the center of the analysis is on changes in depositors' withdrawal behavior and economic welfare in response to central bank interventions. The depositors' consumption level reflects welfare because producers and bankers make zero profits in equilibrium.[15]

Existence and Uniqueness of Equilibrium In this equilibrium, wholesale bank depositors are indifferent between government bonds and deposit claims as in (14). The equilibrium is the unique solution to a system of equations consisting of condition (14), along with the no-arbitrage condition (25), collateral market clearing condition (26), and bond market clearing condition (27) explained below.

No-arbitrage Interbank Market: From asset pricing kernels (6) and (12), retail banks are willing to pay a real interest rate $1/u'(c_2^r)$ on their interbank borrowing, while wholesale banks ask for a real interest rate $1/(1-\delta+\delta u'(c_2^w))$ on their interbank lending. Therefore, the following no-arbitrage condition must hold in equilibrium:

$$u'(c_2^r) \;=\; 1-\delta+\delta u'(c_2^w)\,. \tag{25}$$

As depicted in Figure 3, this no-arbitrage condition gives an upward-sloping curve in the c_2^r–c_2^w space because of the decreasing marginal utility in consumption.

Collateral Market Clearing: The binding leverage constraint (2) and collateral constraint (8), market clearing conditions (20) and (22), condition (4) and no-arbitrage condition

[15] Recall that producers make zero profit because depositors extract the entire trading surplus through take-it-or-leave-it offers, and bankers earn zero profit due to free entry.

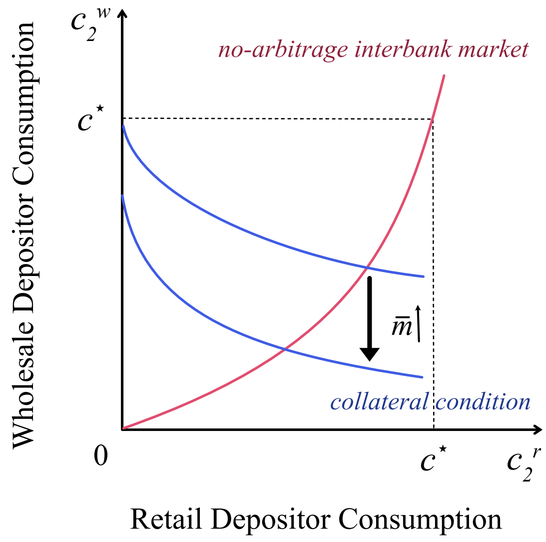

Figure 3: Partial Banking Panic Equilibrium

(25) give the following equilibrium collateral market clearing condition:

$$\underbrace{\theta\bar{m}}_{\text{effective collateral supply (i.e., reserves)}} = \underbrace{\alpha c_2^r\left[1-\theta+\theta u'\left(c_2^r\right)\right]}_{\text{retail banks' demand for collateral}} + \underbrace{(1-\alpha)(1-\rho)(1-\eta)\, c_2^w\left[1-\theta\delta+\theta\delta u'\left(c_2^w\right)\right]}_{\text{wholesale banks' demand for collateral}}. \tag{26}$$

This condition equates the effective collateral supply to private banks' demand.[16] Reserves work as collateral backing all private banks' deposit claims, as no bank holds government bonds as collateral (Lemmas 1 and 2). Although reserves are inaccessible to wholesale banks, they back these banks' deposit claims indirectly through the interbank market.

Crucially, the severity of wholesale banking panics determines the aggregate demand for collateral because the withdrawal probability η determines the fraction of depositors who demand wholesale banks' deposit claims ultimately backed by reserves. For example, a decrease in η shifts this demand rightward.

Bond Market Clearing: The fiscal policy rule $\hat{b}=\bar{m}+\bar{b}$, equilibrium condition (10), market clearing condition (21), and binding nonnegative constraint $b^w-\left[\rho+(1-\rho)\,\eta\right]b'=0$ give

[16]Retail banks demand collateral for two reasons: to secure deposit claims or interbank loans. Here, the notion of "retail banks' demand for collateral" means the demand for deposit claims only because wholesale banks further use interbank loans to back their deposit claims.

the following equilibrium bond market clearing condition:

$$\overbrace{\hat{b} - \bar{m}}^{\text{government bond supply}} = \underbrace{(1-\alpha)\left[\rho + (1-\rho)\,\eta\right] c_2^b u'\left(c_2^b\right)}_{\text{aggregate demand for government bonds}}. \tag{27}$$

The left-hand side represents the government bond supply to the private sector, which is the fiscal authority's total government bond supply minus the bonds held by the central bank to back its reserves. The right-hand side represents the private sector's aggregate demand for bonds, where wholesale bank depositors use all outstanding government bonds to settle their transactions because, again, no bank holds bonds as collateral.

As with the collateral market clearing condition, the severity of wholesale banking panics determines the aggregate demand for government bonds. For example, a decrease in η shifts this demand leftward.

The conditions (14), (26), and (27) mentioned above implicitly define a downward-sloping *collateral condition* in the c_2^r–c_2^w space as depicted in Figure 3.[17] Reserves serve as the ultimate collateral backing both retail and wholesale bank deposits, which depositors use to exchange for consumption goods. The downward slope reflects that, with a fixed reserve supply, an increase in one type of exchange crowds out the other.

Graphically, the collateral condition and the upward-sloping no-arbitrage condition determine a unique equilibrium characterized by the consumption quantities c_2^r and c_2^w. These consumption quantities do not exceed the satiated level $c^\star$ under Assumption 1 because the assumption regarding the scarcity of total government bond supply constrains the collateral condition to be close to the original point. I can then solve for other equilibrium outcomes based on c_2^r and c_2^w. For example, (26) solves for the withdrawal probability η. Proposition 2 below summarizes these results.

Proposition 2 (Unique Partial Banking Panic Equilibrium)**.** *Under Assumption 1 and*

[17] See Appendix A.1 for the detailed proof. The label "collateral condition" is a slight abuse of language. I use this label because the collateral market clearing condition explains the comparative statics I study when $\bar{m}$ changes.

$\bar{m} \in (\bar{m}_L, \bar{m}_H)$*, there exists a unique partial banking panic equilibrium characterized by the consumption allocation* $c_2^r, c_2^w, c_2^b \in (0, c^\star)$ *and the withdrawal probability* $\eta \in (0, 1)$.

Central Bank Balance Sheet Expansion, Flight-to-Safety, and Welfare I evaluate the effects of an expansion in the size of the central bank's balance sheets ($\bar{m}$) by performing comparative statics on the system of equations (14), (25), (26), and (27).

Proposition 3 (Effects of Balance Sheet Expansions)**.** *Assume* $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ *and* $0 < \sigma < 1$*, an expansion in the size of the central bank's balance sheet mitigates wholesale banking panics, i.e.,* $\frac{\partial \eta}{\partial \bar{m}} < 0$*. However, this expansion reduces welfare, i.e.,* $\frac{\partial c_2^b}{\partial \bar{m}}, \frac{\partial c_2^r}{\partial \bar{m}}, \frac{\partial c_2^w}{\partial \bar{m}} < 0$.

The CRRA function $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ satisfies earlier assumptions for utility function. In particular, $0 < \sigma < 1$ is a necessary condition for $-c\frac{u''(c)}{u'(c)} < 1$, which implies that the substitution effect dominates the income effect. Consequently, the demand for assets increases with their rate of return.

Corollary 1 (Effects on Real Interest Rates)**.** *An expansion in the size of the central bank's balance sheet reduces the real interest rates on government bonds, reserves, and interbank loans, i.e.,* $\frac{\partial r^b}{\partial \bar{m}}, \frac{\partial r^m}{\partial \bar{m}}, \frac{\partial r^\ell}{\partial \bar{m}} < 0$.

The interest rate movements in Corollary 1 are consistent with the empirical findings of Arrata, Nguyen, Rahmouni-Rousseau, and Vari (2020), who show that the central bank balance sheet expansion, through quantitative easing, lowers both interest rates on government bonds and short-term interest rates such as repo rates. As it will be clear in a moment, their empirical evidence also supports the key mechanism that generates the general equilibrium effects in this paper: by purchasing bonds, the central bank reduces the net supply of bonds available to the private market, increasing their scarcity and driving down their interest rates.

Flight-to-Safety Implication: An expansion in the size of the central bank's balance sheet has three effects. *First*, this expansion increases the effective collateral supply by issuing more reserves, which relaxes retail banks' leverage constraint and wholesale banks' collateral constraint, promoting the supply of bank liabilities. In particular, it facilitates the supply of wholesale banks' deposit claims, putting upward pressure on their interest rate. *Second*, this expansion reduces the supply of government bonds to the private sector, lowering their interest rate and making them a less attractive safe harbor for wholesale bank depositors. The first two direct partial equilibrium effects imply the *third* effect: wholesale bank depositors' withdrawal probability decreases (i.e., $\frac{\partial \eta}{\partial \bar{m}} < 0$), mitigating the severity of wholesale banking panics, as deposit claims become more attractive relative to government bonds.

The flight-to-safety implication described above under a partial banking panic equilibrium can be extended to the general case when characterizing the equilibrium type. As summarized in Proposition 1, central bank balance sheet expansions gradually reduce the severity of panics, starting from full banking panics ($\eta = 1$) to eventually eliminate them entirely ($\eta = 0$).

Welfare Implication: Despite mitigating wholesale banking panics, expanding the central bank's balance sheet reduces welfare. This striking result follows from the interaction of the three effects mentioned above, which jointly determine the change in the consumption allocation $\left(c_2^r, c_2^w, c_2^b\right)$. A central bank balance sheet expansion puts pressure on the collateral condition in Figure 3 to shift upward by increasing the effective collateral supply as in (26). However, the withdrawal probability η decreases in response to this expansion, increasing the demand for collateral and, therefore, shifting the collateral condition downward. I show that the latter effect necessarily dominates the former one. Collateral becomes relatively more scarce, harming depositors who trade with deposit claims backed by collateral. Depositors who trade with government bonds also get worse by (14). Clearly, the endogenous shift in asset demand is crucial for these results. Different welfare implications arise when there is no such shift in response to the balance sheet expansion, as in the scenarios below with no banking panic or full banking panic.

The welfare reduction result can be more intuitively understood through individual decisions. *Firstly*, an expansion in the central bank's balance sheet harms depositors who trade with government bonds. Expanding the balance sheet reduces the government bond supply to the private sector as in (27), putting pressure on the interest rate on government bonds to fall. Although the decreased demand for bonds that comes from a decrease in withdrawal probability η puts pressure on the bond interest rate to rise, the decreased supply of bonds dominates this decreased demand. This implies a reduction in the interest rate on government bonds (i.e., $\frac{\partial r^b}{\partial \bar{m}} < 0$), thereby a lower trading volume for transactions settled with these safe assets (i.e., $\frac{\partial c_2^b}{\partial \bar{m}} < 0$).

Secondly, an expansion in the central bank's balance sheet harms depositors who trade with wholesale banks' deposit claims. The increased reserve supply increases the effective collateral supply as in (26), relaxing retail banks' leverage constraint and, through the interbank market, relaxing wholesale banks' collateral constraint. Consequently, there is an increase in wholesale banks' supply of deposit claims. However, on the other side of the market, the reduction in the withdrawal probability η increases the demand for these claims. The increased demand for deposit claims dominates their increased supply. Each depositor obtains fewer claims in exchange, thereby, a lower trading volume for transactions settled with these claims (i.e., $\frac{\partial c_2^w}{\partial \bar{m}} < 0$). Furthermore, this increased demand for deposit claims intensifies banks' competition for collateral. Wholesale banks compete for interbank loans, asking for a lower interest rate on them (i.e., $\frac{\partial r^\ell}{\partial \bar{m}} < 0$). Retail banks compete for reserves to back their interbank borrowing, asking for a lower real rate of return on reserves (i.e., $\frac{\partial r^m}{\partial \bar{m}} < 0$).

Finally, an expansion in the central bank's balance sheet harms depositors who trade with retail banks' deposit claims. Unlike wholesale banks, the increased reserve supply does not increase the supply of retail banks' deposit claims, even if it relaxes their leverage constraint, which works like an income effect. That's because, besides depositors, retail banks raise funds from wholesale banks, who ask for a lower rate of return r^ℓ in response to the expansion (recall that $\frac{\partial r^\ell}{\partial \bar{m}} < 0$). They substitute their funding source for cheaper interbank borrowing. The substitution effect from a decrease in r^ℓ dominates the income effect from relaxing the leverage constraint. As a result, retail banks reduce their supply of deposit claims, implying a lower trading volume for transactions settled with these claims (i.e., $\frac{\partial c_2^r}{\partial \bar{m}} < 0$).

Damaging Effects of Central Bank Balance Sheet Expansions In the above scenario, the decrease in the supply of government bonds dominates the decrease in their demand, while the increase in the demand for wholesale deposits dominates the increase in their supply or, more broadly, the increased supply of the underlying collateral that supports these deposits. These results arise because central bank balance sheet expansions take out government bonds from the private sector, ultimately reducing welfare. In particular, the decreased government bond supply reduces their interest rate, playing an indirect but significant role in the markets for wholesale deposits and collateral. As bonds become less attractive, depositors switch to wholesale bank deposits backed by collateral, contributing to the dominance effects in the markets for these assets. In section 4, I demonstrate how these results can be reversed by implementing a central bank policy that does not change the government bond supply. This alternative policy also serves as a counterfactual for isolating the role of the government bond market.

3.3 No Banking Panic and Full Banking Panic

The effects of an expansion in the size of the central bank's balance sheet are qualitatively similar in a no banking panic ($\eta = 0$) and a full banking panic ($\eta = 1$) equilibrium because the endogenous withdrawal probability, η, remains constant in either case. As in the partial banking panic equilibrium, the balance sheet expansion harms depositors who trade with government bonds. However, it now benefits depositors who trade with retail and wholesale banks' deposit claims. That is, the central bank cannot improve welfare, in the Pareto sense, by adjusting the size of its balance sheet.

No Banking Panic ($\eta = 0$ under $\bar{m} \in [\bar{m}_H, \hat{b})$) The market structure is similar to the one studied before, except that no wholesale bank depositor withdraws for safety concerns. Wholesale banks make sufficient revenues from interbank lending, allowing them to offer attractive deposit claims that prevent a wholesale banking panic from arising.

The no-arbitrage condition holds the same as before, while the collateral and bond market clearing conditions become

$$\theta\bar{m} = \alpha c_2^r \left[1 - \theta + \theta u'\left(c_2^r\right)\right] + (1-\alpha)(1-\rho) c_2^w \left[1 - \theta\delta + \theta\delta u'\left(c_2^w\right)\right], \tag{28}$$

$$\hat{b} - \bar{m} = (1-\alpha)\rho c_2^b u'\left(c_2^b\right), \tag{29}$$

respectively, by setting $\eta = 0$ in (26) and (27).

The no-arbitrage condition (25) and the collateral market clearing condition (28) jointly determine the consumption quantities (c_2^r, c_2^w) for depositors who trade with retail and wholesale banks' deposit claims. The bond market clearing condition (29) solely determines the consumption quantity c_2^b for depositors who trade with government bonds. Under Assumption 1, these conditions solve a unique equilibrium with $0 < c_2^r, c_2^w, c_2^b < c^\star$.

An expansion in the central bank's balance sheet benefits depositors who trade with retail and wholesale banks' deposit claims by relaxing retail banks' leverage and wholesale banks' collateral constraints. First of all, this expansion increases the supply of wholesale banks' deposit claims, which are ultimately backed by reserves. This increased supply implies a higher trading volume for depositors who trade with these claims (i.e., a higher c_2^w) because, unlike a partial banking panic equilibrium, there is no shift in demand for these claims. Furthermore, the absence of the shift in demand puts no pressure on wholesale banks to compete for interbank loans as collateral. Consequently, the interest rate on interbank loans increases (i.e., a higher r^ℓ), and retail banks have little incentive to substitute their funding source from deposits to interbank loans. With more slack leverage constraints, retail banks increase their supply of deposit claims, implying a higher trading volume for their depositors (i.e., a higher c_2^r).[18]

The central bank balance sheet expansion, however, harms depositors who trade with government bonds. The supply of government bonds to the private sector decreases with an increased reserve supply, while the aggregate demand for bonds remains unchanged as depositors' withdrawal probability remains constant. As in a partial banking panic equilibrium, the decreased supply of government bonds dominates their change in demand, implying a reduction in the interest rate on bonds (i.e., a lower r^b) and a lower trading volume for depositors who demand them (i.e., a lower c_2^b).

Full Banking Panic ($\eta = 1$ under $\bar{m} \in (0, \bar{m}_L]$) The interbank market becomes inactive in a full banking panic equilibrium because wholesale banks do not demand interbank loans to secure deposits. One interpretation of this equilibrium is that wholesale banks resemble narrow banks in that they only purchase safe government bonds for depositors' withdrawal requests. On the other hand, the equilibrium outcomes would be the same if depositors were allowed to invest in government bonds themselves, as competitive banks maximize their depositors' utility under free entry. Therefore, this full banking panic equilibrium also characterizes the feature of equilibrium with disintermediation in the wholesale banking sector.

When the withdrawal probability $\eta = 1$, only retail banks require collateral to back their deposit claims, and the collateral market clearing condition becomes

$$\theta \bar{m} \ = \ \alpha c_2^r \left[1 - \theta + \theta u'\left(c_2^r\right)\right], \tag{30}$$

[18] In this equilibrium, evaluating the effects of an expansion in the size of the central bank's balance sheet on depositors who trade with deposit claims is performing comparative statics on the system of equations (25) and (28) with respect to an increase in reserve supply $\bar{m}$. In what follows, the effects of this expansion on depositors who trade with government bonds are determined by (29).

which is retail banks' leverage constraint. The bond market clearing condition becomes

$$\hat{b} - \bar{m} \ = \ (1 - \alpha)\, c_2^b u' \left(c_2^b\right). \tag{31}$$

Conditions (30) and (31) determine the consumption allocation for retail and wholesale bank depositors, respectively. A central bank balance sheet expansion has similar effects to its no banking panic counterpart: it benefits depositors who trade with deposit claims while harming those who trade with government bonds.

4 Expanding the Reach of Central Bank Liabilities

I now study what happens when expanding the reach of interest-bearing central bank reserves to wholesale banks. There are two reasons for doing this. Firstly, and most importantly, I show that this allows the central bank to mitigate wholesale banking panics and improve welfare. The central bank can only achieve one of these objectives by adjusting the size of its balance sheet. Secondly, and of theoretical interest, this extension provides a counterfactual to the previous case. Mainly, I show that the increased supply of wholesale bank deposits can dominate their increased demand when the central bank does not take out government bonds from the private sector. This highlights, again, the critical role of the government bond market in evaluating central bank balance sheet policies (see Arrata, Nguyen, Rahmouni-Rousseau, and Vari (2020) and the references therein).

Specifically, I introduce another central bank liability in addition to reserves that functions like the U.S. Federal Reserve's overnight reverse repurchase agreement (ON-RRP) facility. I call this liability ON-RRPs (o) for convenience. ON-RRPs offer better liquidity than reserves because both retail and wholesale banks can hold them, so introducing ON-RRPs works like expanding the reach of reserve accounts to wholesale banks. Unlike government bonds, ON-RRPs can only circulate among financial institutions, like interbank loans, and depositors cannot directly use them to settle their transactions. This implies that, in equilibrium, the real interest rate on ON-RRPs equals the real interest rate on interbank loans with $r^o = r^\ell$ as they have the same degree of liquidity.

The central bank's balance sheet policy now has two dimensions. Firstly, as before, the central bank determines the size of its balance sheet, now given by $s = \bar{m} + \bar{o}$. An expansion in s has the same effects as before when only reserves are available. Secondly, the central bank determines the composition of the central bank's liabilities, represented by the supply of ON-RRPs $\bar{o}$. I focus on the effects of a swap of reserves ($\bar{m}$) for ON-RRPs ($\bar{o}$) while fixing the size of the central bank's balance sheet (s).[19]

Consider the scenario of a partial banking panic with an active interbank market. Retail banks invest in reserves (m) and borrow ℓ^r from wholesale banks. They do not hold ON-RRPs, for the same reason they do not hold government bonds as explained in Lemma 1. Instead, wholesale banks hold these ON-RRPs (o). They also lend ℓ^w to retail banks and hold government bonds (b^w) for their depositors' withdrawal requests. Table 3 presents private banks' inflows and outflows of funds at $t = 3$.

A swap of reserves for ON-RRPs increases the supply of ON-RRPs that are directly accessible to wholesale banks. In response, wholesale banks reduce their lending to retail banks to gain indirect access to reserves. In this way, this swap boosts the effective collateral supply by avoiding the inefficiency associated with retail banks' leverage requirements. To see this, notice that now the collateral market clearing condition becomes

$$\begin{aligned} \theta s + (1-\theta)\,\bar{o} \ &= \alpha c_2^r \left[1 - \theta + \theta u'\left(c_2^r\right)\right] \\ &\quad + (1-\alpha)(1-\rho)(1-\eta)\, c_2^w \left[1 - \theta\delta + \theta\delta u'\left(c_2^w\right)\right]. \end{aligned} \tag{32}$$

From (32), an increase in the supply of ON-RRPs $\bar{o}$ increases the effective collateral supply that is captured by the left-hand side of the equation. This effective collateral supply can also be written as $\theta\bar{m} + \bar{o}$ to reflect the fact that assets held by retail banks can secure

[19]In practice, the mix between ON-RRPs and reserves is determined by the interest rates the central bank sets. This swap can be viewed as a convenient shortcut to capturing such interest rate policy, and one can obtain the same results considering an experiment that fixes s while reducing the interest rate spread between reserves and ON-RRPs, i.e., reducing r^m/r^o.

(a) Retail Bank

Assets	Liabilities and Equity
reserves: $r^m m$	deposit claims: d^r
	interbank borrowing: $r^\ell \ell^r$
	bank capital: e

(b) Wholesale Bank

Assets	Liabilities
government bonds: $r^b b^w$	government bonds: $r^b[\rho + (1-\rho)\eta]b'$
interbank lending: $r^\ell \ell^w$	deposit claims: $(1-\rho)(1-\eta)d^w$
ON-RRPs: $r^o o$	

Table 3: Private Banks' Balance Sheets with Overnight Reverse Repurchase Agreements

fewer liabilities than those held by wholesale banks.

The bond market clearing condition (27) becomes

$$\hat{b} - s \;=\; (1-\alpha)\left[\rho + (1-\rho)\,\eta\right] c_2^b u'\left(c_2^b\right), \tag{33}$$

where the supply of government bonds to the private sector only depends on the size of the central bank's balance sheet. Therefore, the bond supply does not change when the central bank swaps between reserves and ON-RRPs.

Swapping Reserves for ON-RRPs, Flight-to-Safety, and Welfare I evaluate the effects of a swap of reserves for ON-RRPs by performing comparative statics on the system of equations (14), (25), (32), and (33).

Proposition 4. *A swap of reserves for ON-RRPs mitigates wholesale banking panics and improves welfare, i.e.,* $\frac{\partial \eta}{\partial \bar{o}} < 0$ *and* $\frac{\partial c_2^r}{\partial \bar{o}}, \frac{\partial c_2^w}{\partial \bar{o}}, \frac{\partial c_2^b}{\partial \bar{o}} > 0$.

Corollary 2. *A swap of reserves for ON-RRPs increases the real interest rates on government bonds, reserves, interbank loans, and ON-RRPs, i.e.,* $\frac{\partial r^b}{\partial \bar{o}}, \frac{\partial r^m}{\partial \bar{o}}, \frac{\partial r^\ell}{\partial \bar{o}}, \frac{\partial r^o}{\partial \bar{o}} > 0$.

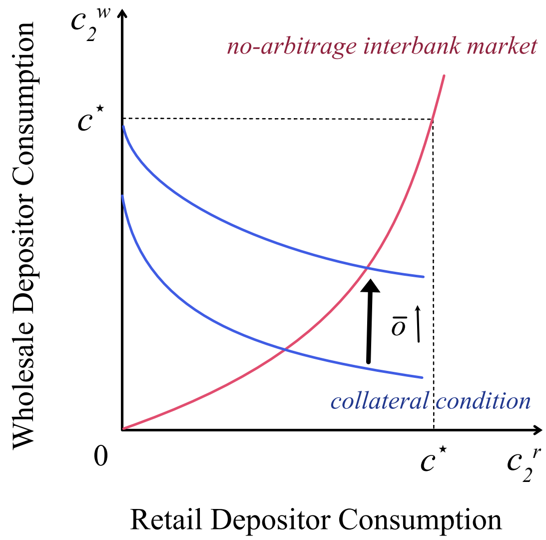

Figure 4: Partial Banking Panic Equilibrium with ON-RRPs

Like central bank balance sheet expansions, a swap of reserves for ON-RRPs mitigates wholesale banking panics (i.e., $\frac{\partial \eta}{\partial \bar{o}} < 0$). As in (32), this swap increases the effective collateral supply, relaxing wholesale banks' collateral constraint and allowing them to provide more attractive deposit claims. Consequently, more depositors switch from safe government bonds to deposit claims.

The key difference between a swap of reserves for ON-RRPs and a central bank balance sheet expansion in Section 3 is that the swap does not take out government bonds from the private sector. While the swap of central bank liabilities also leads to depositors switching to bank deposits, the magnitude of this switch is not as large as with a central bank balance sheet expansion because, without taking out government bonds, there is no direct pressure that makes bonds less attractive. Consequently, in contrast to the balance sheet expansion, this swap shifts the collateral condition upward as in Figure 4, implying a welfare improvement.

As before, the details of the welfare improvement can be understood through individual decisions. Firstly, a swap of reserves for ON-RRPs benefits depositors who trade with government bonds. While the supply does not change, this swap reduces the aggregate demand for government bonds because, by mitigating banking panics, fewer depositors flee to these safe assets. As a result, the interest rate on bonds increases ($\frac{\partial r^b}{\partial \bar{o}} > 0$), implying a higher trading volume for depositors who trade with them (i.e., $\frac{\partial c_2^b}{\partial \bar{o}} > 0$).

Secondly, a swap of reserves for ON-RRPs benefits depositors who trade with wholesale banks' deposit claims. Wholesale banks supply more deposit claims because, again, this swap relaxes their collateral constraint. Although this swap also increases the demand for these claims, the increased supply dominates the increased demand, implying a higher trading volume for depositors who trade with them (i.e., $\frac{\partial c_2^w}{\partial \bar{o}} > 0$). This result suggests that the increased demand in response to the swap is more moderate than central bank balance sheet expansions. The reason is that this swap puts no direct pressure to reduce the attractiveness of bonds, while balance sheet expansions lower their interest rate by reducing their supply. Furthermore, this moderate increase in demand for deposit claims puts little pressure on banks to compete for collateral to back them. However, as explained earlier, the effective collateral supply increases, implying higher real interest rates on collateral, such as reserves and interbank loans (i.e., $\frac{\partial r^\ell}{\partial \bar{o}} > 0$ and $\frac{\partial r^m}{\partial \bar{o}} > 0$).

Finally, a swap of reserves for ON-RRPs benefits retail bank depositors. The swap moves the composition of central bank liabilities away from reserves that only retail banks can hold, which could, in principle, reduce retail banking activities. Retail banks' asset holdings indeed have to fall, but so does their interbank borrowing. In fact, retail banks substitute their funding source from interbank borrowing to deposits because the former becomes more expensive in response to the swap (recall that $\frac{\partial r^\ell}{\partial \bar{o}} > 0$). As a result, the supply of retail banks' deposit claims increases, implying a higher trading volume for depositors who trade with them (i.e., $\frac{\partial c_2^r}{\partial \bar{o}} > 0$).

5 Conclusion

This paper studies the implications of the central bank balance sheet for the severity of wholesale banking panics and welfare. Explicitly modeling the shifts in depositors' demand for safe government bonds and bank deposits is crucial for understanding the consequences of central bank balance sheet policies. An expansion in the size of the central bank's balance sheet and a swap of central bank reserves for overnight reverse repurchase agreements (ON-RRPs) mitigate wholesale banking panics. However, they have different effects on welfare.

A central bank balance sheet expansion can reduce welfare. This expansion reduces the supply of government bonds to the private sector, lowering their interest rate and hindering transactions that rely on bonds. While this balance sheet expansion could promote the supply of retail and wholesale bank deposits by providing more central bank liabilities that help to expand useful intermediation activities, it ultimately hinders transactions settled with bank deposits. The reason is that, by mitigating banking panics, depositors shift to bank deposits backed by central bank liabilities, and the increase in the demand for central bank liabilities bids down their real returns, offsetting the benefits from their increased supply.

By contrast, swapping reserves for ON-RRPs improves welfare. This swap promotes transactions settled with government bonds because, by mitigating banking panics, the demand for bonds decreases, implying an increase in bond returns that benefits these transactions. Moreover, this swap also promotes transactions settled with bank deposits. The issue with central bank balance sheet expansions is that too many depositors shift to bank deposits. This swap addresses such an issue as it does not take out government bonds from the private sector, putting no direct pressure to make bonds less attractive, thereby alleviating the damaging effects under large shifts in asset demand.

References

Andolfatto, D., & Nosal, E. (2020). Shadow bank runs. *Working paper*.

Andolfatto, D., & Williamson, S. (2015). Scarcity of safe assets, inflation, and the policy trap. *Journal of Monetary Economics*, *73*, 70–92.

Arrata, W., Nguyen, B., Rahmouni-Rousseau, I., & Vari, M. (2020). The scarcity effect of qe on repo rates: Evidence from the euro area. *Journal of Financial Economics*, *137*(3), 837–856.

Bernanke, B. S. (2012). Some reflections on the crisis and the policy response. *Rethinking Finance: Perspectives on the Crisis conference*.

Bernanke, B. S. (2016). Should the fed keep its balance sheet large? *Ben Bernanke's Blog at Brookings*.

Bernanke, B. S. (2018). The real effects of disrupted credit: Evidence from the global financial crisis. *Brookings Papers on Economic Activity*, *2018*(2), 251–342.

Bush, R., Kirk, A., Martin, A., Weed, P., & Zobel, P. (2019). Stressed outflows and the supply of central bank reserves. Federal Reserve Bank of New York, Liberty Street Economics (20190220). https://ideas.repec.org/p/fip/fednls/87314.html

Campbell, J. Y. (1999). Asset prices, consumption, and the business cycle. In *Handbook of macroeconomics* (pp. 1231–1303, Vol. 1). Elsevier.

Cardamone, D., Sims, E., & Wu, J. C. (2023). Wall street qe vs. main street lending. *European Economic Review*, *156*, 104475.

Carlson, M., Duygan-Bump, B., Natalucci, F., Nelson, B., Ochoa, M., Stein, J., & Van den Heuvel, S. (2016). The demand for short-term, safe assets and financial stability: Some evidence and implications for central bank policies. *International Journal of Central Banking*, *12*(4), 307–333.

Cui, W., & Sterk, V. (2021). Quantitative easing with heterogeneous agents. *Journal of Monetary Economics*, *123*, 68–90.

Diamond, D. W., & Dybvig, P. H. (1983). Bank runs, deposit insurance, and liquidity. *Journal of Political Economy*, *91*(3), 401–419.

Fischer, S. (2016). The lender of last resort function in the united states. *Speech at the Committee on Capital Markets Regulation on February, 10*.

Gertler, M., & Kiyotaki, N. (2015). Banking, liquidity, and bank runs in an infinite horizon economy. *American Economic Review*, *105*(7), 2011–2043.

Gertler, M., Kiyotaki, N., & Prestipino, A. (2016). Wholesale banking and bank runs in macroeconomic modeling of financial crises. In *Handbook of macroeconomics* (pp. 1345–1425, Vol. 2). Elsevier.

Gorton, G. (2010). *Slapped by the invisible hand: The panic of 2007*. Oxford University Press.

Gorton, G., & Metrick, A. (2012). Securitized banking and the run on repo. *Journal of Financial economics*, *104*(3), 425–451.

Greenwood, R., Hanson, S., & Stein, J. (2016). The federal reserve's balance sheet as a financial-stability tool. *Innovative Federal Reserve Policies During the Great Financial Crisis*, 63–124.

Huang, X., & Keister, T. (2024). Preventing runs with redemption fees. *Working paper*.

Kim, K., Martin, A., & Nosal, E. (2020). Can the us interbank market be revived? *Journal of Money, Credit and Banking*, *52*(7), 1645–1689.

Lagos, R., & Wright, R. (2005). A unified framework for monetary theory and policy analysis. *Journal of Political Economy*, *113*(3), 463–484.

Martin, A., McAndrews, J., Palida, A., & Skeie, D. R. (2019). Federal reserve tools for managing rates and reserves. *Working paper*.

Martin, A., Skeie, D., & Thadden, E.-L. v. (2014). Repo runs. *The Review of Financial Studies*, *27*(4), 957–989.

Ordoñez, G. (2018). Sustainable shadow banking. *American Economic Journal: Macroeconomics*, *10*(1), 33–56.

Sengupta, R., & Xue, F. (2020). The global pandemic and run on shadow banks. *Main Street Views, Federal Reserve Bank of Kansas City, May, 11*.

Williamson, S. D. (2019). Interest on reserves, interbank lending, and monetary policy. *Journal of Monetary Economics*, *101*, 14–30.

Williamson, S. D. (2022). Central bank digital currency and flight to safety. *Journal of Economic Dynamics and Control*, *142*, 104146.

Woodford, M. (2016). *Quantitative easing and financial stability* (tech. rep.). National Bureau of Economic Research.

Appendix

A Omitted Proofs

Proof of Lemma 1. Assume, on the contrary, that $\lambda^r = 0$. Then, from the equilibrium conditions (5) and (6) in the retail bank's problem, I obtain

$$r^\ell < r^b, \tag{A.1}$$

given that $u'(d^r) > 1$ because of the assumption regarding the scarcity of total government bond supply, which implies $d^r < c^\star$. However, from the equilibrium conditions (11) and (12) in the wholesale bank's problem,

$$r^\ell \geq r^b, \tag{A.2}$$

which is a contradiction. □

Proof of Lemma 2. Assume, on the contrary, that $b^w - [\rho + (1-\rho)\eta]\, b' > 0$. This implies that $\lambda^w = 0$. Then, from the equilibrium conditions (10) and (11),

$$u'\left(r^b b'\right) = 1 - \delta + \delta u'(d^w), \tag{A.3}$$

which implies

$$u\left(r^b b'\right) > u(d^w), \tag{A.4}$$

given that $u'(\cdot) > 0$ and $u''(\cdot) < 0$ and the fact that $u'(d^w) > 1$ because of the binding collateral constraint. Condition (A.4) further implies an equilibrium with full banking panic with $\eta = 1$. However, under $\eta = 1$, the collateral constraint (8) can never bind with $b^w - [\rho + (1-\rho)\eta]\, b' > 0$, which is a contradiction. □

Lemma A.1. *Conditions (14), (26) and (27) implicitly define a collateral condition $c_2^r = h(c_2^w)$ in the c_2^r-c_2^w space, where $h'(c_2^w) < 0$, i.e., the collateral condition is downward sloping in a partial banking panic equilibrium.*

Proof of Lemma A.1. Totally differentiating (14), (26) and (27) with respect to c_2^w, I obtain

$$u'\left(c_2^b\right)\frac{\partial c_2^b}{\partial c_2^w}-\delta u'\left(c_2^w\right)=0, \tag{A.5}$$

$$\alpha F_1'\left(c_2^r\right)\frac{\partial c_2^r}{\partial c_2^w}-\left(1-\alpha\right)\left(1-\rho\right)F_2\left(c_2^w\right)\frac{\partial \eta}{\partial c_2^w}+\left(1-\alpha\right)\left(1-\rho\right)\left(1-\eta\right)F_2'\left(c_2^w\right)=0, \tag{A.6}$$

$$\left(1-\rho\right)F_3\left(c_2^b\right)\frac{\partial \eta}{\partial c_2^w}+\left[\rho+\left(1-\rho\right)\eta\right]F_3'\left(c_2^b\right)\frac{\partial c_2^b}{\partial c_2^w}=0, \tag{A.7}$$

where $F_1\left(c\right)=c\left[1-\theta+\theta u'\left(c\right)\right]$, $F_2\left(c\right)=c\left[1-\theta\delta+\theta\delta u'\left(c\right)\right]$, and $F_3\left(c\right)=cu'\left(c\right)$. Note that

$$F_i'\left(c\right)>0, \quad \text{for} \quad i\in\{1,2,3\} \tag{A.8}$$

given that $-\frac{cu''(c)}{u'(c)}<1$ for all $c\geq 0$. Then, from (A.5)-(A.7), I have

$$\frac{\partial c_2^b}{\partial c_2^w}>0, \quad \frac{\partial \eta}{\partial c_2^w}<0, \quad \frac{\partial c_2^r}{\partial c_2^w}<0. \tag{A.9}$$

The last inequality $\frac{\partial c_2^r}{\partial c_2^w}<0$ implies that conditions (14), (26) and (27) implicitly define a function $c_2^r=h\left(c_2^w\right)$ with $h'\left(c_2^w\right)<0$. This is equivalent to say these conditions implicitly define a downward sloping collateral condition in the c_2^r-c_2^w space. □

Lemma A.2. *The no-arbitrage condition (25) implicitly defines a function:*

$$c_2^r=f\left(c_2^w\right), \tag{A.10}$$

where $f'\left(c_2^w\right)>0$, $\lim_{c_2^w\to 0}f\left(c_2^w\right)=0$, and $f\left(c^\star\right)=c^\star$.

Proof. The proof is trivial due to the diminishing marginal utility of consumption. □

Proof of Proposition 2. The goal is to show that $c_2^r=h\left(c_2^w\right)$ and $c_2^r=f\left(c_2^w\right)$ solve for a unique allocation $\left(c_2^w,c_2^r\right)$, where the first equation characterizes a downward-sloping collateral condition (Lemma A.1) and the second equation characterizes an upward-sloping no-arbitrage interbank market condition (Lemma A.2) in the c_2^r-c_2^w space. As illustrated in Figure 5, I can finish this proof by showing that $\lim_{c_2^w\to 0}h\left(c_2^w\right)-f\left(c_2^w\right)>0$ and $h\left(c^\star\right)-f\left(c^\star\right)<0$.

Confine attention to the collateral condition $c_2^r=h\left(c_2^w\right)$ first. Taking limit of $c_2^w\to 0$

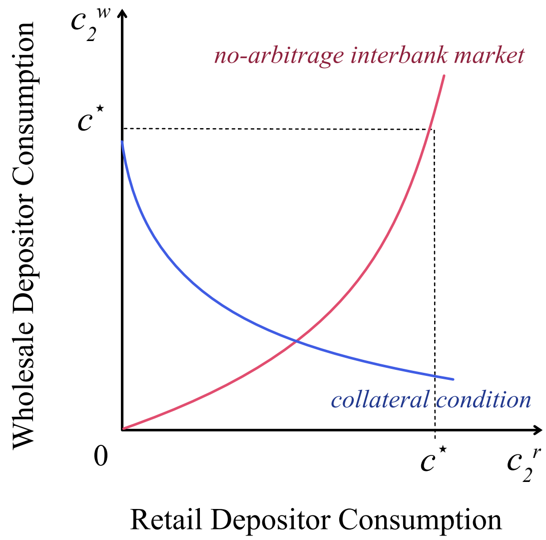

Figure 5: Existence and Uniqueness of Partial Banking Panic Equilibrium

on both side of (26), I have:

$$\theta\bar{m} = \lim_{c_2^w \to 0} \alpha c_2^r \left[1 - \theta + \theta u'\left(c_2^r\right)\right], \tag{A.11}$$

as $\lim_{c \to 0} cu'(c) = 0$. For all $\bar{m} > 0$, this further implies, implicitly,

$$c_2^r = \lim_{c_2^w \to 0} h\left(c_2^w\right) > 0, \tag{A.12}$$

because $\lim_{c \to 0} cu'(c) = 0$ and $cu'(c)$ is increasing in c as $-\frac{cu''(c)}{u'(c)} < 1$. Furthermore, when $c_2^w = c^\star$, $c_2^r = h(c^\star)$ solves the following equation:

$$\theta\bar{m} = \alpha h\left(c^\star\right)\left[1 - \theta + \theta u'\left(h\left(c^\star\right)\right)\right] + (1 - \alpha)(1 - \rho)(1 - \eta)\, c^\star. \tag{A.13}$$

The expression $x\left[1 - \theta + \theta u'(x)\right]$ is also strictly increasing in x. Therefore, for any feasible central bank's balance sheet size $\bar{m} \in \left(0, \hat{b}\right)$ with a fiscal policy $\hat{b}$ that satisfies Assumption 1, (A.13) implies

$$h\left(c^\star\right) < c^*. \tag{A.14}$$

To conclude, $\lim_{c_2^w \to 0} h\left(c_2^w\right) > \lim_{c_2^w \to 0} f\left(c_2^w\right) = 0$ and $h\left(c^\star\right) < c^* = f\left(c^\star\right)$. That is, $\lim_{c_2^w \to 0} h\left(c_2^w\right) - f\left(c_2^w\right) > 0$ and $h\left(c^\star\right) - f\left(c^\star\right) < 0$. A unique solution that $0 < c_2^w, c_2^r < c^\star$ exists by the Intermediate Value Theorem. Finally, $0 < c_2^b < c_2^w < c^\star$ because $u(c_2^b) =$

$\delta u(c_2^w)$ as in (14). $\square$

Proof of Proposition 3. As in Proposition 2, there is a unique partial banking panic equilibrium that can be solved by the no-arbitrage condition (25), collateral market clearing condition (26), bond market clearing condition (27), and condition (14). The proof for this proposition is performing comparative statics on the abovementioned system of equations with respect to an increase in reserve supply $\bar{m}$. For simplicity, consider a CRRA utility function

$$u(x) = \frac{x^{1-\sigma}}{1-\sigma}, \tag{A.15}$$

with $0 < \sigma < 1$, which satisfies assumptions on the utility function. Totally differentiating these functions with respect to $\bar{m}$, then solving the system of linear equations, I obtain

$$\frac{\partial c_2^w}{\partial \bar{m}} = -\frac{\Omega_3}{\delta \frac{u''(c_2^w)}{u''(c_2^r)}\Omega_1\Omega_5 + \Omega_2\Omega_5 + \Omega_3\Omega_4} < 0, \tag{A.16}$$

$$\frac{\partial \eta}{\partial \bar{m}} = \frac{1}{\Omega_3}\left(\delta \frac{u''(c_2^w)}{u''(c_2^r)}\Omega_1 + \Omega_2\right)\frac{\partial c_2^w}{\partial \bar{m}} < 0, \tag{A.17}$$

$$\frac{\partial c_2^r}{\partial \bar{m}} = \delta \frac{u''(c_2^w)}{u''(c_2^r)}\frac{\partial c_2^w}{\partial \bar{m}} < 0, \tag{A.18}$$

$$\frac{\partial c_2^b}{\partial \bar{m}} = \delta \frac{u'(c_2^w)}{u'\left(c_2^b\right)}\frac{\partial c_2^w}{\partial \bar{m}} < 0, \tag{A.19}$$

where

$$\Omega_1 = \alpha\left[1-\theta+\theta(1-\sigma)u'(c_2^r)\right] > 0, \tag{A.20}$$

$$\Omega_2 = (1-\alpha)(1-\rho)(1-\eta)(1-\theta\delta) + (1-\alpha)(1-\sigma)\theta\delta u'(c_2^w) > 0, \tag{A.21}$$

$$\Omega_3 = (1-\alpha)(1-\rho)(1-\theta\delta)c_2^w > 0, \tag{A.22}$$

$$\Omega_4 = (1-\alpha)\left[\rho+(1-\rho)\eta\right](1-\sigma)\delta u'(c_2^w) > 0, \tag{A.23}$$

$$\Omega_5 = (1-\alpha)(1-\rho)\delta c_2^w u'(c_2^w) > 0. \tag{A.24}$$

Therefore, expanding the size of the central bank's balance sheet mitigates wholesale banking panics in a partial banking panic equilibrium. However, such a policy reduces consumption for all the depositors. $\square$

Proof of Proposition 1. The proof takes two steps. In the first step, I show there exist two critical values $\bar{m}_L$ and $\bar{m}_H$, which solve for $\eta = 1$ and $\eta = 0$, respectively, in conditions that determines partial panic equilibria (equations (25), (26), (27), and (14)). The withdrawal probability η is strictly decreasing in $\bar{m}$ in a partial banking panic equilibrium as in Proposition 3. Therefore, if such critical value exists, they satisfy $\bar{m}_L < \bar{m}_H$ and partial banking panic equilibria exits when $\bar{m} \in (\bar{m}_L, \bar{m}_H)$. I also show how these critical values change in response to a change in the probability of wholesale banking failure $1-\delta$. In the second step, I will show that a full banking panic exists when $\bar{m} < \bar{m}_L$ and a no banking panic equilibrium exits when $\bar{m} > \bar{m}_H$.

Step 1 First, consider a partial banking panic equilibrium with $\eta = 1$. Then, the critical value $\bar{m}_L$ and associated consumption allocation $\left(c_2^r, c_2^w, c_2^b\right)$ solve the following equations:

$$u'\left(c_2^r\right) = 1 - \delta + \delta u'\left(c_2^w\right), \tag{A.25}$$

$$\theta \bar{m}_L = \alpha c_2^r \left[1 - \theta + \theta u'\left(c_2^r\right)\right], \tag{A.26}$$

$$\hat{b} - \bar{m}_L = (1-\alpha) c_2^b u'\left(c_2^b\right), \tag{A.27}$$

$$u\left(c_2^b\right) - \delta u\left(c_2^w\right) = 0. \tag{A.28}$$

Taking the limit as $\bar{m}_L \to 0$, from (A.25) and (A.26), I have $c_2^r \to 0$ and $c_2^w \to 0$ because, in particular, $\lim_{c \to 0} c u'(c) = 0$. However, from (A.27), $c_2^b > 0$. Then, the last function (A.28) does not hold with equality. In fact, $u\left(c_2^b\right) - \delta u\left(c_2^w\right) > 0$ when $\bar{m}_L \to 0$. Similarly, taking the limit as $\bar{m}_L \to \hat{b}$, I obtain $c_2^b \to 0$ and $c_2^w > 0$. This implies that $u\left(c_2^b\right) - \delta u\left(c_2^w\right) < 0$ when $\bar{m}_L \to \hat{b}$. By Intermediate Value Theorem, these exists $0 < \bar{m}_L < \hat{b}$ which solves the above system of equations, and such $\bar{m}_L$ is unique by monotonicity.

Then, totally differentiating (A.25)-(A.28), with respect to δ, I can solve for

$$\frac{\partial \bar{m}_L}{\partial \delta} = \frac{1}{\theta} \alpha F_1'\left(c_2^r\right) \delta \frac{u''\left(c_2^w\right)}{u''\left(c_2^r\right)} \frac{\partial c_2^w}{\partial \delta} < 0, \tag{A.29}$$

where $F_1'(c) > 0$ with $F_1(c) = c\left[1 - \theta + u'(c)\right]$ (recall $-c\frac{u''(c)}{u'(c)} < 1$), and

$$\frac{\partial c_2^w}{\partial \delta} = -\frac{(1-\alpha)\theta F_2'\left(c_2^b\right) \frac{u\left(c_2^w\right)}{u'\left(c_2^b\right)}}{(1-\alpha)\theta F_2'\left(c_2^b\right) \delta \frac{u'\left(c_2^w\right)}{u'\left(c_2^b\right)} + \alpha F_1'\left(c_2^r\right) \delta \frac{u''\left(c_2^w\right)}{u''\left(c_2^r\right)}} < 0, \tag{A.30}$$

given $F_2'(c) > 0$ with $F_2(c) = c u'(c)$.

Similarly, consider a partial banking panic equilibrium with $\eta = 0$. Following the same procedure, I can show there exists a unique $\bar{m}_H$ that solves the following system of equations, and such $\bar{m}_H$ is strictly decreasing in δ.

$$u'\left(c_2^r\right) = 1 - \delta + \delta u'\left(c_2^w\right), \tag{A.31}$$

$$\theta \bar{m}_H = \alpha c_2^r \left[1 - \theta + \theta u'\left(c_2^r\right)\right] + (1-\alpha)(1-\rho)\, c_2^w \left[1 - \theta\delta + \theta\delta u'\left(c_2^w\right)\right], \tag{A.32}$$

$$\hat{b} - \bar{m}_H = (1-\alpha)\, \rho c_2^b u'\left(c_2^b\right), \tag{A.33}$$

$$u\left(c_2^b\right) - \delta u\left(c_2^w\right) = 0. \tag{A.34}$$

Step 2 First, when $\bar{m} = \bar{m}_L$, conditions (A.25)-(A.27) in a partial banking panic equilibrium are the same as the conditions solving for a full banking panic equilibrium. Consider a decrease in $\bar{m}$, c_2^w decreases and c_2^b increases, guaranteeing a full banking panic equilibrium as in condition (15). Similarly, when $\bar{m} = \bar{m}_H$, an increase in $\bar{m}$ results in a increase in c_2^w and a decrease in c_2^b, guaranteeing a no banking panic equilibrium as in condition (13). □

Proof of Proposition 4. Totally differentiating equations (14), (25), (32) and (33) with respect to $\bar{o}$, I obtain:

$$1 - \theta = \alpha F_1'\left(c_2^r\right) \frac{\partial c_2^r}{\partial \bar{o}} - (1-\alpha)(1-\rho) F_2\left(c_2^w\right) \frac{\partial \eta}{\partial \bar{o}} + (1-\alpha)(1-\rho)(1-\eta) F_2'\left(c_2^w\right) \frac{\partial c_2^w}{\partial \bar{o}}, \tag{A.35}$$

$$0 = (1-\rho) F_3\left(c_2^b\right) \frac{\partial \eta}{\partial \bar{o}} + \left[\rho + (1-\rho)\eta\right] F_3'\left(c_2^b\right) \frac{\partial c_2^b}{\partial \bar{o}}, \tag{A.36}$$

$$u''\left(c_2^r\right) \frac{\partial c_2^r}{\partial \bar{o}} = \delta u''\left(c_2^w\right) \frac{\partial c_2^w}{\partial \bar{o}}, \tag{A.37}$$

$$u'\left(c_2^b\right) \frac{\partial c_2^b}{\partial \bar{o}} = \delta u'\left(c_2^w\right) \frac{\partial c_2^w}{\partial \bar{o}}, \tag{A.38}$$

where $F_1(c) = c\left[1 - \theta + \theta u'(c)\right]$, $F_2(c) = c\left[1 - \theta\delta + \theta\delta u'(c)\right]$, and $F_3(c) = cu'(c)$. Note that $u'(c) > 0$, $u''(c) < 0$ and $F_i'(c) > 0$ for $i \in \{1, 2, 3\}$ given $-c\frac{u''(c)}{u'(c)} < 1$. From (A.37) and (A.38), $\frac{\partial c_2^r}{\partial \bar{o}} \frac{\partial c_2^w}{\partial \bar{o}} \geq 0$ and $\frac{\partial c_2^b}{\partial \bar{o}} \frac{\partial c_2^w}{\partial \bar{o}} \geq 0$, i.e., $\frac{\partial c_2^r}{\partial \bar{o}}$, $\frac{\partial c_2^w}{\partial \bar{o}}$, and $\frac{\partial c_2^b}{\partial \bar{o}}$ have the same sign of being positive or negative. From (A.36), $\frac{\partial \eta}{\partial \bar{o}} \frac{\partial c_2^b}{\partial \bar{o}} \leq 0$, i.e., $\frac{\partial \eta}{\partial \bar{o}}$ and $\frac{\partial c_2^b}{\partial \bar{o}}$ have different sign of being positive and negative. The only possibility of making condition (A.35) and (A.36) hold is

$$\frac{\partial \eta}{\partial \bar{o}} < 0, \frac{\partial c_2^r}{\partial \bar{o}} > 0, \frac{\partial c_2^w}{\partial \bar{o}} > 0, \frac{\partial c_2^b}{\partial \bar{o}} > 0. \tag{A.39}$$

□