Endogenous Repo Rate Dispersion and Monetary Policy Transmission

Fengfan Xiang*

University of Western Ontario

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Abstract

Repurchase agreements (repos) are collateralized contracts often secured by government securities. However, even identical repo contracts can trade at different rates, and these rates only partially reflect changes in the central bank's deposit facility rate. Motivated by the fact that most repo participants trade bilaterally with dealers in the over-the-counter segment of repo markets, I develop a search-theoretic model that endogenously generates repo price distributions. A key insight is that it is not dispersion itself but the concentration of prices at the tails of the distribution that matters for central bank interventions. An increase in the deposit facility price has imperfect pass-through to repo prices, with pass-through weakening as the price rises and creating ambiguous effects on the composition of central bank liabilities. Both the central bank's borrowing and lending facilities resolve this ambiguity, but using the borrowing facility comes at the cost of raising inflation. In line with the Friedman rule, pegging the lending and deposit facility rates at the zero-lower bound achieves the most efficient use of money in exchange.

Key Words: repo rate dispersion, search friction, pass-through, monetary policy

JEL: E4, E5, G2

*Email: fxiang8@uwo.ca; Personal Website: fxiangecon.github.io

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1 Introduction

Repurchase agreements (repos) are short-term collateralized contracts, often secured by safe government securities. However, repo rates exhibit substantial dispersion. Even identical repo contracts secured by the same collateral can trade at different rates (Anbil, Anderson, & Senyuz, 2021; Eisenschmidt, Ma, & Zhang, 2024). Moreover, repo rates only partially reflect changes in central banks' policy rates, such as the U.S. Federal Reserve's interest rate on reserve balances and the European Central Bank's deposit facility rate, raising concerns about monetary policy transmission (Ballensiefen, Ranaldo, & Winterberg, 2023; Duffie & Krishnamurthy, 2016). While these features may also arise elsewhere, my analysis focuses on the over-the-counter (OTC) segment of the repo market, where participants, such as money market funds and insurance companies, trade bilaterally with dealers.[1] In particular, OTC transactions account for approximately 30% of repo trading volume in the U.S. and the euro area (European Central Bank, 2019).

I show that search frictions inherent in OTC markets give rise to endogenous repo price dispersion, and study the implications of this dispersion for monetary policy. Endogenous price dispersion implies that changes in the central bank's deposit facility price have imperfect pass-through to repo prices. As the deposit facility price increases, its pass-through to repo prices weakens, and this increased price has ambiguous effects on asset allocation, particularly on the supply of central bank liabilities. Long-standing lending and borrowing facilities provide repo customers with conduits to trade with the central bank. Both can effectively eliminate policy ambiguity, but using the borrowing facility comes at the cost of higher inflation. Consistent with the Friedman (1969) rule, pegging the lending and deposit facility rates at the zero-lower bound is optimal. It also eliminates dispersion by collapsing repo price distributions to this zero-lower bound.

[1] I consider dealers as subsidiaries of bank holding companies with commercial banking subsidiaries that are depository institutions, abstracting from stand-alone entities without affiliated commercial banks. (Afonso, Cipriani, Copeland, Kovner, La Spada, & Martin, 2021). They have exclusive access to centralized trading platforms and the central bank's deposit facility.

Specifically, I develop an infinite-horizon model with three stages of exchange in each period. In stage 1, repo customers purchase financial portfolios consisting of money and government bonds. Depending on their liquidity needs, some customers become borrowers who require money to settle transactions for consumption goods in stage 3, while others become lenders who prioritize returns on assets. These two stages follow Lagos and Wright (2005), providing micro-founded trading motives for the liquidity rearrangement in stage 2, which adopts the OTC trading framework of Duffie, Gârleanu, and Pedersen (2005). Namely, transactions in stage 2 include lending and borrowing between dealers and customers through frictional OTC repo markets, as well as a frictionless inter-dealer market that allows dealers to reallocate assets. As is standard in practice, all transactions at this stage are secured with collateral, and dealers deposit excess funds into the central bank's deposit facility, converting their money into central bank deposits.

An important element of the model is the endogenous distributions for repo *borrowing* and *lending* prices. I derive these distributions as an equilibrium outcome of frictional OTC repo markets, specifically through their inherent search frictions, as captured by the pricing framework of Burdett and Judd (1983). Namely, search frictions limit customers to contact at most two dealers for price quotations, giving dealers market power and generating price dispersion. In particular, this price quotation framework is directly motivated by the fact that the majority of repo customers rely on concentrated intermediation by *one or two* dealers (Eisenschmidt, Ma, & Zhang, 2024).

The key determinants of the price distributions are the **competitive price**, which would occur when search frictions are negligible (i.e., customers always meet two dealers, as in Bertrand competition), and the **monopoly price**, which would arise when search frictions are extremely large (i.e., customers meet one dealer). Although the monopoly price yields the highest *per-customer profit*, dealers earn the same *total profit* over the distribution's support. Lower per-customer profits at prices closer to the competitive level are offset by more customers. As a result, dealers are indifferent across the support.

I demonstrate how endogenous price dispersion leads to imperfect monetary policy pass-through, characterized by less-than-one-for-one responses of market-determined repo prices to changes in the central bank's deposit facility price.[2] When the deposit facility is active, search frictions weaken the effectiveness of pass-through by concentrating the price distributions around monopoly prices and away from competitive prices.[3] Although the competitive price is determined by the deposit facility price, the monopoly price is insensitive to policy changes. In this way, search frictions effectively limit the response of repo prices to changes in the deposit facility price. Here also comes one of the key insights of this paper: it is not the price dispersion itself, but the concentration of prices in the tails of the distributions, that matters for central bank interventions. In fact, pass-through becomes null (perfect) as search frictions become extremely large (negligible), collapsing the distribution to the monopoly (competitive) price.

The key concern about pass-through, nevertheless, is less about its imperfection and more about the predictability of monetary policy. I show that, as the deposit facility price increases, its effects on repo lending prices diminish — an immediate result of the earlier discussion on the distribution's concentration pattern. An increase in the deposit facility price, or equivalently, a lower policy rate, shifts the entire lending price distribution rightward, concentrating it around the monopoly price, which is the highest price, or the lowest interest rate, that a dealer can offer to lenders. This concentration enhances dealers' market power and reduces the effectiveness of pass-through. In particular, the pass-through becomes null in the extreme case when the increased deposit facility price pushes dealers to offer only the monopoly price.[4]

[2] In practice, changes in this price are reflected, for example, by adjustments to the European Central Bank's deposit facility rate (DFR) and the U.S. Federal Reserve's interest rate on reserve balances (IORB).

[3] An active deposit facility corresponds to the *floor system* of monetary policy implementation, currently used by major central banks like the Federal Reserve. The model also covers the *corridor system*, prevalent before the 2008 financial crisis, in which the deposit facility is inactive. In that case, changes in the deposit facility price do not affect the inter-dealer price and have no pass-through to repo prices.

[4] Regarding borrowing prices, an increase in the deposit facility price makes the repo borrowing prices less concentrated around the borrower dealer's monopoly price. However, this does not enhance the pass-through because it also leads to these prices being more dispersed around the competitive price

Crucially, when relying on the deposit facility price, monetary policy can have ambiguous and sometimes unintended implications. Raising the price, or equivalently, reducing the interest rate on the deposit facility, does not necessarily reduce the supply of central bank deposits and, thus, also has ambiguous effects on money supply. Frictions in the OTC markets play a critical role in this result, and the ambiguity disappears when search frictions are either negligible or extremely large. Nevertheless, I show that the central bank can unambiguously control the composition of its liabilities by pairing the deposit facility with its long-standing lending and borrowing facilities, such as the Federal Reserve's repurchase agreement and overnight reverse repurchase agreement facilities.

Either raising the central bank's lending facility price or lowering its borrowing facility price reduces the supply of central bank deposits while increasing the supply of money. The lending facility provides repo borrowers, rather than relying on dealers, an alternative option of borrowing from the central bank. Raising the lending facility price pushes dealers to offer higher prices to compete for customers. As a result, borrowers obtain more money with their collateral, reducing dealers' savings in the central bank's deposit facility and expanding the money supply. The borrowing facility, in turn, provides a conduit for lenders to lend to the central banks. By reducing its price, it pushes dealers to offer lower prices, or equivalently, higher interest rates to attract lenders. The increased return on lenders' money holdings raises the inflation rate, reducing the nominal bond price. Borrowers can then purchase more government bonds and borrow more money against these government securities. Therefore, lowering the borrowing facility price also increases the central bank's money supply while reducing its deposit liabilities unambiguously.

While both facilities unambiguously reallocate assets, the lending facility is more effective than the borrowing one. Despite providing repo borrowers more money for transactions, lowering the borrowing facility price also leads to higher inflation, reducing the real value of money. The cost of inflation outweighs the benefit from the increased (nominal)

determined by the deposit facility price itself.

money supply, making the borrowing facility a less desirable instrument. By contrast, raising the lending facility price does not affect inflation and still provides borrowers with additional money to support their exchanges of money for consumption goods.

Finally, I show that it is optimal to set the central bank's lending facility price arbitrarily close to its deposit facility price. This policy pushes dealers to raise the prices they offer to borrowers toward the deposit facility price, thereby reducing price dispersion. Controlling dispersion in this way enables the central bank to expand its money supply by simultaneously raising both the lending and deposit facility prices. Moreover, a higher deposit facility price, or equivalently, a lower (nominal) deposit facility rate, also reduces inflation, in line with the Fisher effect. Therefore, the optimal monetary policy is to lower the policy rates, namely, the lending and deposit facility rates, to the zero-lower bound, consistent with the Friedman (1969) rule. Under this policy, repo price dispersion disappears entirely, as all distributions collapse to the zero-lower bound.

Related Literature The model developed here adopts some elements from Lagos and Wright (2005) and Duffie, Gârleanu, and Pedersen (2005), but generalizes them because assets traded in OTC repo markets are divisible, as in Lagos and Rocheteau (2009), and because OTC transactions can be collateral-constrained. Geromichalos and Herrenbrueck (2016) and Geromichalos, Herrenbrueck, and Lee (2023) use a similar structure in their indirect liquidity approach, in which assets can be sold outright in exchange for means of payment. By contrast, I focus on repo markets, where liquidity is delivered through secured lending, given that repos are essentially collateral-backed loans. Another departure from the literature, at least from the papers mentioned above, is that I embed Burdett and Judd (1983) pricing, generalized again with divisible assets and collateral constraints, into OTC markets to generate endogenous repo rate (price) dispersion.[5]

[5] I impose unit pricing in the sense that the price is independent of the quantity traded, as in Head, Liu, Menzio, and Wright (2012). This restriction is arguably not critical for the results because collateral constraints bind, and loan quantities are always determined by the collateral values in equilibrium.

I show how search frictions inherent in OTC repo markets give rise to dealers' market power and repo rate dispersion. Huber (2023) explains dispersion in homogeneous repo contracts through dealers' (heterogeneous) identities and attributes dealers' market power with money market funds' preferences for portfolio concentration and stable funding. Eisenschmidt, Ma, and Zhang (2024) identify another channel of market power that arises from customers' costly link formation and show how it determines the magnitude of repo rate dispersion, with dispersion itself stemming from customer heterogeneity. In my paper, market power and dispersion appear simultaneously and endogenously under search frictions, thereby complementing these two recent and closely related papers. In addition to repo rates, haircuts are another important component of repo contracts. A natural concern is that dispersion in repo rates could be a consequence of dispersion in haircuts. However, Julliard, Pinter, Todorov, Wijnandts, and Yuan (2024) find no statistical association between the two, where haircuts are primarily driven by risk consideration (Chebotarev, 2025; Hempel, Kahn, Mann, & Paddrik, 2023).

I use endogenous distributions of repo lending and borrowing rates to evaluate the effectiveness of the central bank's interest-rate control and asset allocation through its standing facilities, including the deposit, borrowing, and lending facilities. This connects to the literature on monetary policy implementation, which describes how central banks set administered rates and conduct operations to transmit their policy stance to financial markets (Afonso, Armenter, & Lester, 2019; Afonso & Lagos, 2015; Armenter & Lester, 2017; Baughman & Carapella, 2024; Bianchi & Bigio, 2022; Ennis & Keister, 2008). The literature primarily focuses on unsecured credit markets, e.g., the federal funds market. I instead study secured credit markets, as activity in unsecured credit markets has declined and migrated toward secured markets in recent decades (Corradin, Eisenschmidt, Hoerova, Linzert, Schepens, & Sigaux, 2020; European Central Bank, 2021; Schnabel, 2023). My paper also complements this literature, particularly Williamson (2025), which likewise focuses on secured credit, by explicitly modeling endogenous price dispersion.

2 Environment

Time is discrete and continues forever, with three subperiods in each period. The first subperiod involves activities in a *settlement market*, where agents settle debts from the last period and rebalance their financial portfolios. The second subperiod involves borrowing and lending activities, and is labeled as *funding market*. The last subperiod involves exchanges between money and goods in a *trading market*.

There are two types of private agents, repo **customers** and **dealers**, who are infinitely-lived and discount the future between periods at a rate $\beta \in (0, 1)$. A measure s of risk-neutral dealers, which represents depository institutions, seek profits by conducting intermediation activities in the funding market. A measure one of customers, which represents money market funds and insurance companies, are also risk-neutral, and their payoffs are captured by

$$c_s - n_s + \sigma \cdot Ac_t - (1 - \sigma) \cdot n_t, \tag{1}$$

where $\sigma \in \{0, 1\}$ indicates their role of consuming (c_t) or working (n_t) in the trading market, and $A > 1$ is a parameter that guarantees the surplus from trading. Customers also consume (c_s) and work (n_s) in the settlement market. However, no one can carry goods across subperiods, which are perishable, and this is why exchanges in the trading market are essential. A linear production technology allows customers to convert labor to consumption goods one-for-one in both markets.

In each period, customers work (n_s) and produce consumption goods in exchange for assets in the settlement market, rebalancing their portfolios. They may consume (c_s) in this subperiod, which occurs if the payoff from their assets carried from the previous period exceeds their payment for current asset purchases. In this way, their payoff in the settlement market, $c_s - n_s$, works like a wealth account that allows them to freely withdraw or deposit funds, as in Duffie, Gârleanu, and Pedersen (2005). Crucially, customers are subject to an i.i.d. idiosyncratic liquidity shock that is realized at the end of the settlement market: a fraction ρ of them consume (c_t) in the trading market and want to transform their investments into money to settle their payments for goods, so that $\mathbb{P}(\sigma = 1) = \rho$. By contrast, the rest can only work (n_t) and prefer assets with higher returns. Dealers help with these asset rearrangements in the funding market, through frictional OTC repo markets where they trade with customers and a frictionless inter-dealer market where they trade with each other.

The underlying assets are money and government bonds. Assets are essential because no unsecured IOU will be accepted in any transactions under the limited commitment and lack of record-keeping technology, following Lagos and Wright (2005). The **central bank** issues *money* (m). Customers can use this divisible and portable asset to settle their transactions in the trading market, while dealers can convert money into their account balances at the central bank's deposit facility in the funding market. These central bank deposits (d), such as reserves in the U.S. Federal Reserve, can be viewed as an asset sold at a price z_d, in terms of money, and the central bank administers the nominal interest rate $1/z_d - 1$. The **fiscal authority** issues one-period nominal *government bonds* (b) that are useful collateral in the repo market. Each bond sells for z_b units of money in the settlement market and is a claim to one unit of money in the next settlement market.

I focus on stationary equilibria in which the inflation is a constant $\pi - 1$, and all real variables remain unchanged forever. I express variables in real terms, measured in units of the current-period settlement market good, including those lower-case letters, m, b, and d, in the last paragraph. Importantly, agents must adjust their nominal asset holdings for inflation when carrying across periods. For example, m units of money in the current period are worth m/π units of the settlement market good in the next period.

Fiscal Authority and Central Bank The fiscal authority issues a fixed amount of government bonds $\hat{b}$ at a price z_b in each settlement market. The exogenous value $g \equiv z_b\hat{b}$

denotes its revenue from bond issuance and describes the **fiscal policy**. The central bank then purchases $\hat{b} - \bar{b}$ units of bonds with the issuance of money $\hat{m} \in (0, g)$. Later, after dealers make their deposit decisions, the central bank's liabilities consist of money ($\bar{m}$) and central bank deposits ($z_d \bar{d}$). Therefore,

$$\underbrace{z_b \left(\hat{b} - \bar{b}\right)}_{\text{central bank assets}} = \overbrace{\bar{m} + z_d \bar{d}}^{\text{central bank liabilities, } \hat{m}}, \tag{2}$$

with $\bar{b}$ denoting the quantity of the bonds circulating in the private sector. Another way to interpret equation (2) is that it reflects the central bank's balance sheet, equating the value of its asset holdings to the value of its liabilities. Clearly, *monetary policy* has two dimensions: the size of the central bank's balance sheet, which is captured by its initial money issuance $\hat{m}$, or equivalently the ratio of central bank liabilities over the consolidated government liabilities, i.e., $\theta \equiv \hat{m}/g \in (0, 1)$; and the non-negative administered interest rate $1/z_d - 1$ that later determines the composition of central bank liabilities.

The fiscal authority uses lump-sum transfers and taxes to balance the consolidated government budget constraint period by period. This constraint is

$$\bar{m} + z_d \bar{d} + z_b \bar{b} = \frac{\bar{m} + \bar{d} + \bar{b}}{\pi} + \tau, \tag{3}$$

where τ is the real value of the lump-sum transfer (or tax if $\tau < 0$) to customers at the beginning of the settlement market. The left-hand side of (3) represents the revenue from issuing new consolidated government liabilities consisting of government bonds and central bank liabilities, which equals the revenue the fiscal authority receives from its bond issuance (g). The right-hand side is the payment of its liabilities from the previous period and the lump-sum transfer.

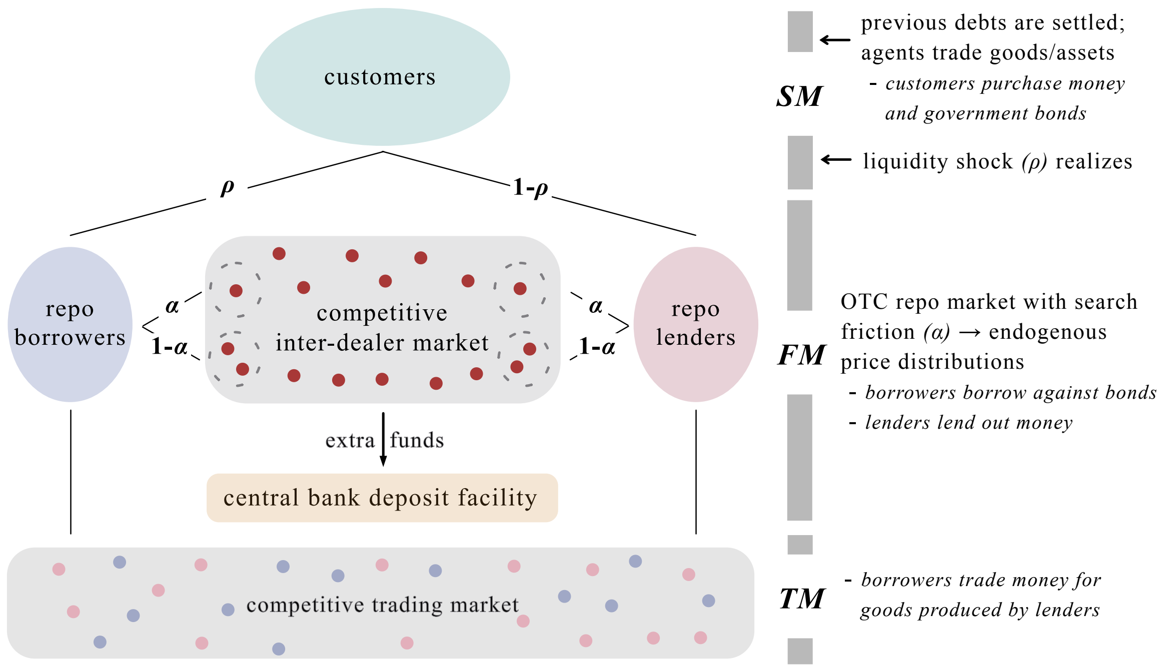

Figure 1: Timing of Events

3 Market Structure

In this section, I describe the market structure subperiod by subperiod and present the value functions of customers, taking the price distributions offered by the dealers as given. In the next section, I characterize those endogenous repo price distributions, which are determined by the customers' choices derived here. Figure 1 depicts the timing of events within a period and provides visual guidance for the transaction pattern.

3.1 Settlement Market

At the beginning of the settlement market, debts from the previous period are settled — agents pay off their outstanding liabilities and receive asset returns. Repo customers enter a Walrasian market to trade goods and assets.

Generically, consider customers who borrow in the last funding market (**borrowers** henceforth) enter the settlement market owing ℓ_B units of loans while holding m units of money and b units of government bonds. If they exit this market with $\tilde{m}$ units of money and $\tilde{b}$ units of government bonds, their value function is

$$
\begin{aligned}
U^B(m,b,\ell_B) &= \max_{c_s,n_s,\tilde{m},\tilde{b}} c_s - n_s + \mathbb{E}_i\left[V^i\left(\tilde{m},\tilde{b}\right)\right] \qquad (4)\\
\text{s.t.} \quad c_s + \tilde{m} + z_b\tilde{b} &= n_s + m + b - \ell_B + \tau, \qquad (5)
\end{aligned}
$$

where τ is the lump-sum transfer from the fiscal authority and $\mathbb{E}_i$ denotes the expectation operator over their value function V^i in the incoming funding market, considering their potential role of borrowing ($i = B$) or lending ($i = L$). Although the values of a borrower's consumption c_s and labor supply n_s are indeterminate, their difference is fixed in equilibrium, where the borrower consumes the residual funds from the last period if $c_s - n_s > 0$ and works to accumulate new funding if $c_s - n_s < 0$. Substituting $c_s - n_s$ from the budget constraint (5) into U^B yields

$$
U^B(m,b,\ell_B) = m + b - \ell_B + \tau + \max_{\tilde{m},\tilde{b}} -\tilde{m} - z_b\tilde{b} + \mathbb{E}_i\left[V^i\left(\tilde{m},\tilde{b}\right)\right]. \qquad (6)
$$

Similarly, consider customers who lend in the last funding market (**lenders** henceforth) with ℓ_L units of incoming loan payoffs. Their value function in the settlement market is

$$
U^L(m,b,\ell_L) = m + b + \ell_L + \tau + \max_{\tilde{m},\tilde{b}} -\tilde{m} - z_b\tilde{b} + \mathbb{E}_i\left[V^i\left(\tilde{m},\tilde{b}\right)\right]. \qquad (7)
$$

At the end of the settlement market, the liquidity shock is realized, reassigning customers' roles in the incoming funding market. A fraction ρ of customers become borrowers, requiring money to trade for goods in the trading market. By contrast, the remaining become lenders who lend out their money. This gives the following first-order conditions for customers' optimal portfolio choices, which are independent of their initial asset holdings:

$$
\begin{aligned}
-1 + \rho\frac{\partial}{\partial\tilde{m}}V^B\left(\tilde{m},\tilde{b}\right) + (1-\rho)\frac{\partial}{\partial\tilde{m}}V^L\left(\tilde{m},\tilde{b}\right) &= 0 \qquad \text{(money)}, \qquad (8)\\
-z_b + \rho\frac{\partial}{\partial\tilde{b}}V^B\left(\tilde{m},\tilde{b}\right) + (1-\rho)\frac{\partial}{\partial\tilde{b}}V^L\left(\tilde{m},\tilde{b}\right) &= 0 \qquad \text{(government bonds)}. \qquad (9)
\end{aligned}
$$

3.2 Funding Market

OTC Repo Markets In OTC repo markets, each dealer posts a nominal loan price z_B when they meet a borrower, taking as given the price distribution $F_B(z_B)$ posted by all other dealers, as in Burdett and Judd (1983). I refer dealers who trade with *borrower* and are likely to *borrow* from the inter-dealer market as **borrower dealers**. Meanwhile, there are **lender dealers** on the other side of the market, trading with lenders. In principle, lender and borrower dealers can be the same agents. Using two different labels to isolate their roles on different sides of the market is harmless, given that dealers are risk-neutral and operate under a competitive inter-dealer market.

Customers observe the price distribution $F_{i\in\{B,L\}}$ but trade with dealers under search frictions. With probability α, customers contact one dealer and trade with that dealer. Otherwise, they contact two dealers and trade with the one offering the better price. Specifically, borrowers borrow from the dealer who offers a higher price when they contact two dealers so that they can borrow more money against their collateral. By contrast, lenders lend to the one with a lower price so that they can receive higher interest payments. This OTC trading structure is consistent with the fact that the majority of repo customers do not have access to the inter-dealer market, but have to rely on concentrated intermediation by one or two dealers (Eisenschmidt, Ma, & Zhang, 2024). One can also follow the original Burdett and Judd (1983) to show how customers endogenously choose to contact one or two dealers randomly at certain search costs.

The fact that borrowers borrow from a random sample of dealers with random loan prices implies the following expected value of entering the funding market

$$\begin{aligned} V^B\left(\tilde{m},\tilde{b}\right) &= \alpha \int \max_{\ell_B} W^B\left(\tilde{m}+z_B\ell_B,\tilde{b},\ell_B\right)\mathrm{d}F_B(z_B) \\ &\quad + (1-\alpha)\int \max_{\ell_B} W^B\left(\tilde{m}+z_B\ell_B,\tilde{b},\ell_B\right)\mathrm{d}\left[F_B(z_B)\right]^2, \end{aligned} \tag{10}$$

where W^B is their value of entering the incoming trading market after borrowing ℓ_B units of loans under z_B, the higher loan price they are offered. Borrowers borrow against their government bonds. The following collateral constraint,

$$\ell_B \leq \tilde{b}, \tag{11}$$

guarantees that their return on bonds can cover their loan payments.

Instead of the higher price, lenders accept the lower loan price z_L they are offered, giving the following value function

$$\begin{aligned} V^L\left(\tilde{m}, \tilde{b}\right) &= \alpha \int \max_{\ell_L} W^L\left(\tilde{m} - z_L\ell_L, \tilde{b}, \ell_L\right) \mathrm{d}F_L\left(z_L\right) \\ &\quad + (1-\alpha) \int \max_{\ell_L} W^L\left(\tilde{m} - z_L\ell_L, \tilde{b}, \ell_L\right) \mathrm{d}\left(1 - \left[1 - F_L\left(z_L\right)\right]^2\right), \end{aligned} \tag{12}$$

with W^L as a lender's value of entering the trading market after lending ℓ_L at a price z_L. Lenders are subject to the following cash constraint

$$z_L\ell_L \leq \tilde{m}. \tag{13}$$

Competitive Inter-dealer Market A borrower dealer with a loan price z_B provides $z_B\ell_B(z_B)$ units of money to each borrower they serve, where $\ell_B(z_B)$ is the solution to the borrower's funding market problem (10) that captures borrowers' demand for loans. The dealer borrows from the inter-dealer market to meet their customers' borrowing needs and saves the extra in the central bank's deposit facility. As a result, the profit a borrower dealer obtains from each successful match, i.e., profit per borrower served, is

$$\begin{aligned} R_B\left(z_B\right) &= \max_{d_{BD}, \ell_{BD}} d_{BD} + \ell_B(z_B) - \ell_{BD}, \\ \text{s.t.} \quad z_B\ell_B(z_B) + z_d d_{BD} &= z_I\ell_{BD}, \quad \text{and} \quad d_{BD} \geq 0, \end{aligned} \tag{14}$$

where z_I is the loan price in the inter-dealer market, ℓ_{BD} is the quantity of their inter-dealer borrowing, and d_{BD} is their deposits at the central bank. In addition to the non-negative constraint $d_{BD} \geq 0$ that prevents dealers from borrowing from the central bank's deposit facility, dealers are also subject to the following collateral constraint

$$d_{BD} + \ell_B(z_B) \ \geq \ \ell_{BD}, \tag{15}$$

so that their returns on assets exceed the payments on their liabilities. However, this constraint will not be a concern in solving the equilibrium because dealers will make a non-negative profit.

The total profit for a borrower dealer who posts the price z_B, denoted as $\Pi_B(z_B)$, is

$$\lim_{\epsilon \to 0^+} \overbrace{\frac{\rho}{s}\left(\alpha + 2(1-\alpha) F_B(z_B - \epsilon) + (1-\alpha)\left[F_B(z_B) - F_B(z_B - \epsilon)\right] \right)}^{\text{number of borrowers served}} \underbrace{R_B(z_B)}_{\text{profit per borrower served}}, \tag{16}$$

which highlights the importance of the dealer's pricing strategy that determines the number of borrowers they serve and their total profit. Specifically, $\rho\alpha/s$ borrowers borrow from this dealer because this is their only contact. They may contact another dealer, where $\lim_{\epsilon \to 0^+} 2\rho(1-\alpha) F_B(z_B - \epsilon)/s$ borrowers still borrow from this dealer because the other one posts a price below z_B. The rest $\lim_{\epsilon \to 0^+} 2\rho(1-\alpha)\left[F_B(z_B) - F_B(z_B - \epsilon)\right]/s$ borrowers contact another dealer who post the same price z_B. They randomize according to a uniform tie-breaking rule, picking either dealer with probability 1/2.

On the other side of the inter-dealer market, lender dealers receive funds $z_L \ell_L(z_L)$ from lenders, invest $z_d d_{LD}$ into the central bank's deposit facility, and lend $z_I \ell_{LD}$ to other dealers. When posting a price z_L, a lender dealer's profit per lender served is

$$\begin{aligned} R_L(z_L) \ &= \ \max_{d_{LD}, \ell_{LD}} \ d_{LD} + \ell_{LD} - \ell_L(z_L), \\ \text{s.t.} \quad z_d d_{LD} + z_I \ell_{LD} \ &= \ z_L \ell_L(z_L), \quad \text{and} \quad d_{LD} \ \geq \ 0. \end{aligned} \tag{17}$$

As with borrower dealers, the lender dealer's collateral constraint,

$$d_{LD} + \ell_{LD} \ \geq \ \ell_L(z_L), \tag{18}$$

never binds in equilibrium.

Similarly, the total profit, $\Pi_L(z_L)$, for the lender dealer with price z_L is

$$\lim_{\epsilon \to 0^+} \overbrace{\frac{1-\rho}{s}\left(\alpha + 2(1-\alpha)\left[1 - F_L(z_L)\right] + (1-\alpha)\left[F_L(z_L) - F_L(z_L - \epsilon)\right]\right)}^{\text{number of lenders served}} \underbrace{R_L(z_L)}_{\text{profit per lender served}}. \quad (19)$$

The number of lenders served by the lender dealer is determined in a manner similar to the number in the borrower dealer's problem. The only difference is that, instead of the higher price, the lower one becomes more attractive to customers.

Lemma 1 (Dealers' Profits)**.** *Inter-dealer market problems* (14) *and* (17) *imply a higher interest rate on inter-dealer loans than the interest rate on the central bank's deposit facility, and these rates are equal if there is a positive stock of deposits in the central bank's deposit facility, i.e.,* $z_I \leq z_d$*, with equality if* $\bar{d} > 0$*. Moreover, this interest rate structure also implies the following profit functions:*

$$R_B(z_B) = \left(\frac{1}{z_B} - \frac{1}{z_I}\right) z_B \ell_B(z_B), \quad (20)$$

$$R_L(z_L) = \left(\frac{1}{z_I} - \frac{1}{z_L}\right) z_L \ell_L(z_L). \quad (21)$$

I present all the proofs in Appendix A and discuss the intuition in the main text. The nominal interest rate of the central bank's deposit facility, i.e., $1/z_d - 1$, is administered by the central bank, which acts as a floor for the inter-dealer rate because dealers can always place their funds at the central bank unless they expect a higher return from the inter-dealer market. Dealers exploit profits from the interest rate spreads between the inter-dealer rate and their repo rates with customers. For borrower dealers, the profit they obtain from each borrower is (20), which captures the profit arising from the interest rate spread, $1/z_B - 1/z_I$, after providing $z_B \ell_B$ units of money. The profit function for lender dealers has a similar interpretation in terms of the interest rate spread, $1/z_I - 1/z_L$.

3.3 Trading Market

Borrowers enter the trading market with $\tilde{m} + z_B \ell_B(z_B)$ units of money to purchase consumption goods that sell at a price p, in terms of the current-period settlement market good. This gives the following value function in the trading market

$$
\begin{aligned}
&W^B\left(\tilde{m} + z_B \ell_B(z_B), \tilde{b}, \ell_B(z_B)\right) \\
&\quad = \max_{0 \le pc_t \le \tilde{m} + z_B \ell_B(z_B)} Ac_t + \beta U^B\left(\frac{\tilde{m} + z_B \ell_B(z_B) - pc_t}{\pi}, \frac{\tilde{b}}{\pi}, \frac{\ell_B(z_B)}{\pi}\right),
\end{aligned} \tag{22}
$$

where nominal terms carried to the next period are adjusted by inflation. From (6), the value function U^B is linear in their state variables, implying that

$$
\begin{aligned}
&W^B\left(\tilde{m} + z_B \ell_B(z_B), \tilde{b}, \ell_B(z_B)\right) \\
&= \max_{0 \le pc_t \le \tilde{m} + z_B \ell_B(z_B)} Ac_t + \frac{\beta\left(\tilde{m} + z_B \ell_B(z_B) - pc_t + \tilde{b} - \ell_B(z_B)\right)}{\pi} + \beta U^B(0,0,0),
\end{aligned} \tag{23}
$$

where $U^B(0,0,0)$ is a constant that only depends on future choices.

Lenders work to produce goods, trade these goods for money, and carry all assets into the next settlement market. Their value function in the trading market can be written as

$$
\begin{aligned}
&W^L\left(\tilde{m} - z_L \ell_L(z_L), \tilde{b}, \ell_L(z_L)\right) \\
&\quad = \max_{n_t \ge 0} -n_t + \frac{\beta\left(\tilde{m} - z_L \ell_L(z_L) + pn_t + \tilde{b} + \ell_L(z_L)\right)}{\pi} + \beta U^L(0,0,0).
\end{aligned} \tag{24}
$$

Finally, the market clearing condition in the trading market is

$$
\rho \int c_t(z_B) \mathrm{d}F_B(z_B) \;=\; (1-\rho)\, n_t. \tag{25}
$$

Borrowers' consumption depends on the loan price they are offered (z_B), as this price affects their money holdings after the funding market and therefore their ability to pay for consumption goods. By contrast, lenders' asset holdings are unrelated to their production process, so n_t remains constant across leaders, at least in the symmetric equilibrium.

Lemma 2 (Trading Outcomes). *The competitive trading market gives*

$$p = \frac{\pi}{\beta}, \quad c_t(z_B) = \frac{\beta\left(\tilde{m} + z_B \ell_B(z_B)\right)}{\pi}. \tag{26}$$

Borrowers' money holding is crucial for transactions in the trading market, and they spend all their money in exchange for consumption goods. This highlights the critical role of the repo market in facilitating liquidity provision by allowing borrowers to borrow against assets to meet their liquidity needs. Lenders also benefit from repo transactions by transforming their money into higher-yielding loans. The following Lemmas 3 and 4 characterize borrowers' demand for loans and lenders' supply of loans, respectively, derived from first-order conditions from their funding market problems, i.e., equations (10) and (12), and envelope conditions from their trading market problem, i.e., (23) and (24).

Lemma 3 (Borrowers' Demand for Loans). *When $z_B > 1/A$, borrowers' collateral constraint binds, and they borrow up to their collateral value, i.e., $\ell_B(z_B) = \tilde{b}$. By contrast, when $z_B = 1/A$, their collateral constraint does not bind.*

Intuitively, risk-neutral borrowers borrow up to the value of their collateral whenever the loan price exceeds the cutoff, $1/A$. Beyond this point, they obtain enough money with their collateral, generating high trading returns that cover their interest payments. At the cutoff, borrowers are indifferent: they borrow any amount for transactions while breaking even.

Lemma 4 (Lenders' Supply of Loans). *When $z_L < 1$, lenders' cash constraint binds, and they lend out all their money holdings, i.e., $\ell_L(z_L) = \tilde{m}/z_L$. By contrast, when $z_L = 1$, their cash constraint does not bind.*

Lenders' supply of loans decreases in the loan price z_L when $z_L < 1$, that is, when the nominal interest rate they obtain is above the zero-lower bound. This is because a higher loan price implies a lower return, discouraging risk-neutral lenders from lending. At the zero-lower bound, lenders are indifferent between lending and holding money, as neither option yields a positive return. In this case, they can adjust their portfolio composition arbitrarily without affecting profits.

The envelope conditions of customers' funding market problems (10) and (12) reduce their first-order conditions (8) and (9) of their settlement market problems to the following optimal portfolio choice conditions that determine the inflation and nominal interest rate on government bonds in equilibrium.

Lemma 5 (Optimal Portfolio Choices)**.** *Customers' optimal portfolio choices give*

$$1 = \frac{\beta}{\pi}\left[\rho A + (1-\rho)\left(\alpha\int\frac{1}{z_L}\mathrm{d}F_L(z_L) + (1-\alpha)\int\frac{1}{z_L}\mathrm{d}\left(1-[1-F_L(z_L)]^2\right)\right)\right], \quad (27)$$

$$z_b = \frac{\beta}{\pi}\left[\rho A\left(\alpha\int z_B\mathrm{d}F_B(z_B) + (1-\alpha)\int z_B\mathrm{d}\left[F_B(z_B)\right]^2\right) + 1 - \rho\right]. \quad (28)$$

Conditions (27) and (28) are essentially asset pricing kernels for money and government bonds, respectively. While the nominal price of money is fixed at one, its real value, after being adjusted for inflation, is probabilistically determined by the expected payoff from its direct use in transactions, which occurs with probability ρ, or from its interest payment through lending, which occurs with probability $1-\rho$. Similarly, the nominal price of government bonds, z_b, is determined by the expected payoff from using them as collateral to support transactions or holding them until maturity to receive their returns. The former occurs with probability ρ while the latter occurs with probability $1-\rho$.

4 Endogenous Repo Price Distributions

To lay the groundwork for the following analysis, I derive the endogenous price distributions offered by both borrower and lender dealers, highlighting the critical role of search frictions in generating repo price dispersion.

Each dealer chooses price z_i to maximizes their total profit $\Pi_i(z_i)$, where, again, $i \in \{B, L\}$ refers to borrowing (B) or lending (L). In equilibrium, the price distribution $F_i(z_i)$

is consistent with dealers' profit maximization if every price z_i in its support $\mathcal{S}_i$ maximizes $\Pi_i(z_i)$, such that

$$\Pi_i\left(z_i\right) \;=\; \Pi_i^{\star} \;\equiv\; \max_{z_i} \; \Pi_i\left(z_i\right) \quad \forall z_i \in \mathcal{S}_i, \; i \in \{B, L\}. \tag{29}$$

Instead of directly solving this maximization problem, it proves helpful first to solve for the optimal prices that maximize dealers' profit per borrower served and per lender served. I refer to these optimal prices as *monopoly prices*, as each solves a standard monopoly pricing problem in which a monopoly dealer chooses a price to maximize their profit. For convenience, I rewrite the profit per customer functions in (20) and (21) as

$$R_B\left(z_B\right) \;=\; \left(1 - \frac{z_B}{z_I}\right)\tilde{b} \qquad\qquad \forall z_B \in [\frac{1}{A}, 1], \tag{30}$$

$$R_L\left(z_L\right) \;=\; \left(\frac{z_L}{z_I} - 1\right)\frac{\tilde{m}}{z_L} \qquad\qquad \forall z_L \in (0, 1], \tag{31}$$

using the demand for and supply of loans characterized in Lemmas 3 and 4. In particular, I focus on cases that favor dealers at those cutoff prices, $z_B = 1/A$ and $z_L = 1$, when there are multiple solutions to the demand and supply functions. For example, I choose $\ell_L = \tilde{m}$ under $z_L = 1$. Although lenders are indifferent to lending any amount $\ell_L \leq \tilde{m}$ under this price, dealers obtain the highest profit when $\ell_L = \tilde{m}$ and they can achieve this profit by choosing a price arbitrarily close to 1, i.e., $z_L = 1 - \epsilon$ for a vanishingly small ϵ.

Lemma 6 (Borrower Dealers' Monopoly Price)**.** *The monopoly price that maximizes borrower dealers' profit per borrower served is $z_B = 1/A$, and this price guarantees a positive profit if and only if $z_I > 1/A$.*

Proposition 1 (Borrower Dealers' Price Distribution)**.** *If $z_I > 1/A$, there exists a unique price distribution*

$$F_B\left(z_B\right) \;=\; \frac{\alpha}{2\left(1 - \alpha\right)}\left(\frac{z_B - \frac{1}{A}}{z_I - z_B}\right), \tag{32}$$

with support $\mathcal{S}_B = [1/A, \bar{z}_B]$*, where the upper bound is given by*

$$\bar{z}_B \; = \; \left(1 - \frac{\alpha}{2-\alpha}\right) z_I + \frac{\alpha}{2-\alpha}\frac{1}{A}. \tag{33}$$

The key determinants of the distribution are the monopoly price $1/A$ and the inter-dealer price z_I. The latter price also represents the *competitive price* that arises when the search friction becomes negligible, i.e., $\alpha \longrightarrow 0$, or equivalently, when dealers always have to compete with another dealer for price quotations. This works like the Bertrand competition, where competition pushes dealers to make zero profit, posting a price z_I that barely compensates for their cost of inter-dealer borrowings. In particular, the lower bound of the support $\mathcal{S}_B$ is the monopoly price, while its upper bound is a convex combination of the monopoly price and the competitive price, adjusted by the magnitude of the search friction. Intuitively, although the monopoly price yields the highest profit per borrower, dealers may also offer higher prices closer to the competitive level. In that way, they offset the loss in per-customer profit by serving more borrowers.

The price distribution satisfies standard properties in Burdett and Judd (1983). First, the distribution is continuous. Otherwise, if there was a mass point at some price, a dealer who initially posted a price at the mass point could significantly increase their profit by reducing the price slightly, as this reduction leaves the profit per borrower served almost unchanged but attracts all the borrowers who accepted their initial price. Second, the support $\mathcal{S}_B$ is connected or, say, convex in the one-dimensional case. Otherwise, if $\mathcal{S}_B$ had a gap between two prices, the lower price would yield a higher profit because, although these two prices give the same number of borrowers served, the lower price generates a higher profit per borrower. This violates the equal profit condition in (29). The two properties of the distribution mentioned above reduce the total profit (16) to

$$\Pi_B\left(z_B\right) \; = \; \frac{\rho}{s}\left[\,\alpha + 2\left(1-\alpha\right)F_B\left(z_B\right)\right]R_B\left(z_B\right). \tag{34}$$

I can then use the equal profit condition to derive the closed-form solution of F_B, given that the monopoly price $1/A$ is the lower bound of the support $\mathcal{S}_B$ so that $F(1/A) = 0$.

Lemma 7 (Lender Dealers' Monopoly Price)**.** *Under $z_d \leq 1$, the monopoly price that maximizes lender dealers' profit per lender served is $z_L = 1$, and dealers earn a nonnegative profit at this monopoly price with zero profit occurring if and only if $z_I = z_d = 1$.*

Proposition 2 (Lender Dealers' Price Distribution)**.** *If $z_I < 1$, there exists a unique price distribution*

$$F_L(z_L) \;=\; 1 - \frac{\alpha}{2(1-\alpha)} \frac{1/z_L - 1}{1/z_I - 1/z_L}, \tag{35}$$

with support $\mathcal{S}_L = [\underline{z}_L, 1]$, where the lower bound is given by

$$\underline{z}_L \;=\; \left[\left(1 - \frac{\alpha}{2-\alpha}\right)\frac{1}{z_I} + \frac{\alpha}{2-\alpha}\right]^{-1}. \tag{36}$$

The price distribution for lender dealers has similar properties and takes a similar form to the distribution for borrower dealers, F_B. The key difference is that lenders care about the interest payments from lending, which is why the lending price distribution is naturally expressed in interest rates. From equations (33) and (36), $\bar{z}_B < z_I < \underline{z}_L$, implying that any repo lending price z_L is greater than the inter-dealer price z_I, which, in turn, is greater than any repo borrowing price z_B. Thus, dealers quote bid-ask spreads to customers and earn net interest margins from intermediation services — a feature emphasized in the OTC literature, surveyed by Weill (2020), and consistent with empirical findings in Corradin and Maddaloni (2020) and Ferrari, Guagliano, and Mazzacurati (2017).

The Role of Search Frictions in Price Dispersion Search frictions play a critical role in generating repo price dispersion, and dispersion disappears in the limit cases when they become negligible or extremely large. When search frictions are negligible, such that $\alpha \longrightarrow 0$, dealers always compete with one another for customers through their price quotations. The price distributions F_B and F_L converge pointwise to the degenerate distributions $\mathbb{P}(z_B = z_I) = 1$ and $\mathbb{P}(z_L = z_I) = 1$, respectively. In other words, dealers always quote the competitive price z_I to borrowers and lenders, as in Bertrand competition. By contrast, when search frictions are extremely large, such that $\alpha \longrightarrow 1$, dealers behave as monopolists, offering the monopoly prices $1/A$ to borrowers and 1 to lenders. The price distributions F_B and F_L converge pointwise to the degenerate distributions $\mathbb{P}(z_B = 1/A) = 1$ and $\mathbb{P}(z_L = 1) = 1$, respectively.

5 Equilibrium and Central Bank Deposit Facility

I begin this section by defining and characterizing the equilibrium and establishing its existence. I then study the positive implications of changes in the central bank's deposit facility price, which is the primary policy instrument of major central banks worldwide, such as the U.S. Federal Reserve and the European Central Bank.

The following equilibrium definition focuses on equilibria with price dispersion on both lending and borrowing sides of the repo market, so that $z_I \in (1/A, 1)$. In principle, there could be equilibria that exhibit no price dispersion on one side of the market, such as when $z_I = 1$ or $z_I = 1/A$ (Propositions 1 and 2). I ignore these knife-edge equilibria and instead focus on the empirically relevant cases with price dispersion. I omit variables that are not central to the analysis in the definition. For example, the trading market price p is excluded, but it is implied by Lemma 2 once the gross inflation rate π is determined.

Definition 1 (Equilibrium with Price Dispersion)**.** *Given the fiscal policy that determines the value of consolidated government liabilities,*

$$g = \bar{m} + z_d\bar{d} + z_b\bar{b}, \tag{37}$$

and the monetary policy that determines the value of central bank liabilities,

$$\hat{m} = \bar{m} + z_d\bar{d}, \tag{38}$$

and the administered interest rate captured by $z_b \leq 1$, an equilibrium consists of an al-

location $(\bar{m}, \bar{d}, \bar{b}, \tilde{m}, \tilde{b})$*, the price distributions* F_B *and* F_L *characterized in* (32) *and* (35)*, the associated distributions of loans* $\ell_B(z_B) = \tilde{b}$ *and* $\ell_L(z_L) = \tilde{m}/z_L$ *(Lemmas 3 and 4), and market-determined prices* (z_b, π, z_I)*, satisfying customers' optimal portfolio choice decisions* (27) *and* (28)*, market clearing conditions,*

$$\hat{m} \;=\; \tilde{m} \qquad \textit{(money)}; \tag{39}$$

$$\bar{b} \;=\; \tilde{b} \qquad \textit{(government bonds)}; \tag{40}$$

$$\rho \int z_B \ell_B(z_B) \mathrm{d}F_B\left(z_B\right) + z_d \bar{d} \;=\; (1-\rho) \int z_L \ell_L(z_L) \mathrm{d}F_L\left(z_L\right) \qquad \textit{(loans)}, \tag{41}$$

where $z_I \leq z_d$ *with equality if* $\bar{d} > 0$*.*

Characterization of Equilibrium I pin down the equilibrium using the following equilibrium market clearing condition

$$z_d \bar{d} \;=\; (1-\rho)\,\hat{m} - \rho\,(g - \hat{m})\,\frac{\mu_B}{z_b}, \tag{42}$$

derived from distributions (32) and (35), fiscal and monetary policies (37) and (38), and the market clearing condition for loan (41), where

$$\mu_B \;=\; z_I + \frac{\alpha}{2\,(1-\alpha)} \ln\left(\frac{\alpha}{2-\alpha}\right)\left(z_I - \frac{1}{A}\right) \tag{43}$$

is the mean of the repo borrowing price, i.e., $\mathbb{E}[z_B]$. Condition (42) reflects the funding flows in the funding market. Lenders allocate their money holdings, $(1-\rho)\,\hat{m}$, to dealers. Dealers then lend part of the money to borrowers through repos, which is why this part is determined by the value of government bonds held by borrowers, $\frac{\rho(g-\hat{m})\mu_B}{z_b}$. Dealers also save the reminder in the central bank's deposit facility, and $z_d\bar{d}$ denotes their account balances at the central bank. Given (32) and (35), I further use Lemma 5 to derive the closed-form solution of the bond price

$$z_b \;=\; \frac{\beta}{\pi}\left[\rho A \left(\frac{\alpha}{A} + (1-\alpha)\,z_I\right) + 1 - \rho\right], \tag{44}$$

with the gross inflation rate

$$\pi \;=\; \beta\left[\rho A + (1-\rho)\left(\alpha + \frac{1-\alpha}{z_I}\right)\right]. \tag{45}$$

There are two types of equilibrium, depending on whether the central bank's deposit facility is active (i.e., $\bar{d} > 0$) or not (i.e., $\bar{d} = 0$). In an equilibrium with an active deposit facility, the deposit facility price, z_d, determines the price for inter-dealer loans z_I, such that $z_I = z_d$. The inter-dealer price z_I then determines the mean repo borrowing price μ_B and the bond price z_b through (43) and (44), respectively. Solving this equilibrium thus requires substituting μ_B and z_b into (42) to obtain a positive value of central bank deposits $\bar{d}$. By contrast, an equilibrium with an inactive deposit facility is solved by substituting $\bar{d} = 0$ in (42) to obtain an inter-dealer price z_I that clears the inter-dealer market, with $z_I \leq z_d$. For both types, once the inter-dealer price z_I is determined, all other equilibrium outcomes follow, including the endogenous repo price distributions, F_B and F_L, and the allocation of government liabilities with

$$z_b\bar{b} \;=\; g - \hat{m} \qquad \text{(central bank deposits)}, \tag{46}$$

$$\bar{m} \;=\; \rho\left[\hat{m} + (g-\hat{m})\,\frac{\mu_B}{z_b}\right] \qquad \text{(money)}. \tag{47}$$

From now on, I focus on the equilibrium with an active deposit facility throughout the analysis because, otherwise, changes in the central bank's deposit facility price would not change the inter-dealer price, thereby having no impact on market-determined repo prices. Proposition 3 establishes the existence of this type of equilibrium, requiring a large size of the central bank's balance sheet. In this way, equilibrium with an active deposit facility corresponds to the floor system of monetary policy implementation, which is currently adopted by central banks in developed economies, like the U.S. Federal Reserve, the European Central Bank, and the Bank of Canada. Equilibrium with an inactive deposit facility, instead, corresponds to the corridor system that was popular before the 2008 Financial Crisis.

Proposition 3 (Existence of Equilibrium). *For any (ρ, α, A), there exists a threshold $0 < \bar{\theta} < 1$ for the central bank's balance sheet policy such that a price-dispersed equilibrium with an active central bank deposit facility exists for any $z_d \in (1/A, 1)$ and $\theta \in (\bar{\theta}, 1)$.*

The central bank's deposit facility price must lie in the interval $(1/A, 1)$ for the equilibrium to exhibit price dispersion, under which an equilibrium with an active central bank deposit facility arises when the central bank's balance sheet is large. In this scenario, repo borrowers obtain only a small amount of government bonds as collateral because many are held by the central bank. Consequently, dealers cannot lend all their funds against collateral and deposit the excess at the central bank, consistent with the large supply of central bank liabilities implied by the large balance sheet. Although it is beyond the scope of the analysis of this paper, an equilibrium with an inactive deposit facility arises when the balance sheet becomes smaller. For future reference, I formally state conditions again in the following assumption and maintain this assumption throughout the analysis.

Assumption 1 (Large Central Bank Balance Sheet). *For any (ρ, α, A), let $z_d \in (1/A, 1)$ and $\theta \in (\bar{\theta}, 1)$, where $\bar{\theta}$ is characterized in Proposition 3.*

5.1 Imperfect Pass-through of the Deposit Facility Price

I first study how market-determined repo prices respond to changes in the central bank's deposit facility price z_d, where I find imperfect monetary policy-through, characterized by a less-than-one-for-one response of repo prices to changes in z_d. Let z_B^q and z_L^q denote the q-quantile of the price distribution F_B and F_L, respectively, such that

$$F_B\left(z_B^q\right) = \frac{\alpha}{2(1-\alpha)}\left(\frac{z_B^q - \frac{1}{A}}{z_I - z_B^q}\right) = q, \tag{48}$$

$$F_L\left(z_L^q\right) = 1 - \frac{\alpha}{2(1-\alpha)}\frac{z_I\left(1 - z_L^q\right)}{z_L^q - z_I} = q. \tag{49}$$

The following proposition and its corollary show that imperfect pass-through holds at every percentile of the price distributions as well as their mean prices.

Proposition 4 (Imperfect Pass-Through)**.** *Under Assumption 1,*

1. *The pass-through of the central bank's deposit facility price to any repo borrowing price is imperfect, such that*

$$0 \;\leq\; \eta_B^q \;\equiv\; \frac{\mathrm{d}z_B^q}{\mathrm{d}z_d} \;=\; \frac{z_B^q - \frac{1}{A}}{z_d - \frac{1}{A}} \;<\; 1, \tag{50}$$

with equality $\eta_B^q = 0$ holds at the monopoly price, $z_B^0 = 1/A$;

2. *The pass-through is imperfect for repo lending prices that are close to the zero-lower bound, such that*

$$0 \;\leq\; \eta_L^q \;\equiv\; \frac{\mathrm{d}z_L^q}{\mathrm{d}z_d} \;=\; \frac{z_L^q\,(1 - z_L^q)}{z_d\,(1 - z_d)} \;<\; 1 \quad \textit{if} \quad z_L^q > 1 - z_d, \tag{51}$$

with equality $\eta_L^q = 0$ holds at the monopoly price $z_B^1 = 1$. In particular, condition $z_L^q > 1 - z_d$ always holds when $z_d \geq 1/2$,

Corollary 1. *The pass-through of the central bank's deposit facility price to the mean repo borrowing price is imperfect, i.e., $\frac{\mathrm{d}\mu_{\mathrm{B}}}{dz_d} < 1$. Under $z_d \geq 1/2$, the pass-through to the mean repo lending price is also imperfect, i.e., $\frac{\mathrm{d}\mu_{\mathrm{L}}}{dz_d} < 1$.*

Although changes in the central bank's deposit facility price z_d transmit one-for-one to the inter-dealer price z_I (recall that $z_I = z_d$ under Assumption 1), pass-through to market-determined repo prices is imperfect, at least when $z_d \geq \frac{1}{2}$, which is a condition that holds for a wide range of interest rates above the zero-lower bound, i.e., $z_d = 1$. This concern regarding the pass-through effectiveness is consistent with the empirical findings for the U.S. (Duffie & Krishnamurthy, 2016) and the euro area (Ballensiefen, Ranaldo, & Winterberg, 2023; Eisenschmidt, Ma, & Zhang, 2024). The following lemma highlights the crucial role of search frictions in generating imperfect pass-through.

Lemma 8 (How Search Frictions Weaken Pass-Through)**.** *Higher search frictions weaken*

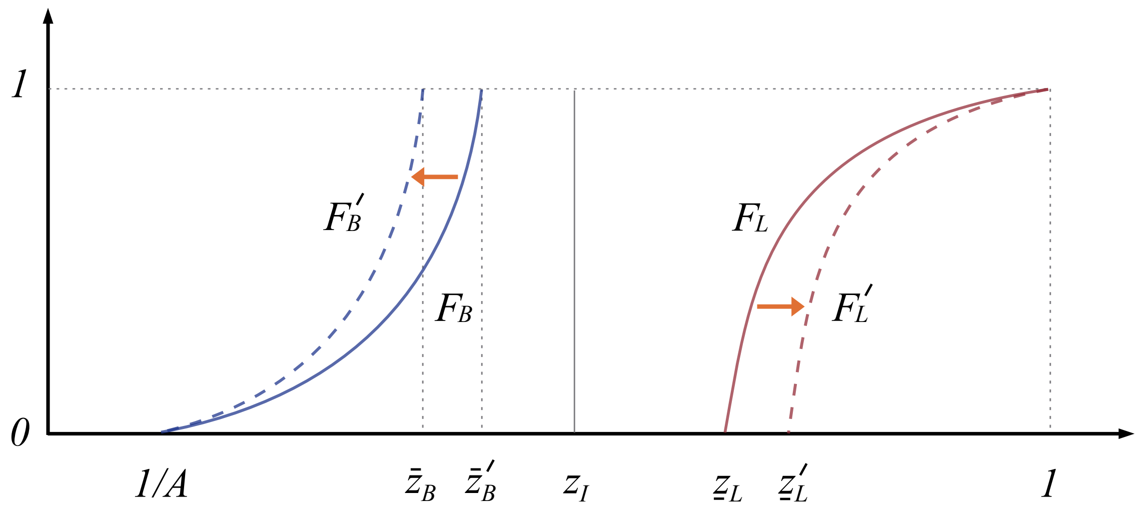

Figure 2: How Search Frictions Affect Price Distributions

the effectiveness of pass-through. Specifically,

$$\frac{\mathrm{d}\eta_B^q}{\mathrm{d}\alpha} \leq 0 \quad \forall q \in [0,1], \tag{52}$$

with equality holding only if $q = 0$*. Under* $z_d \geq 1/2$*,*

$$\frac{\mathrm{d}\eta_L^q}{\mathrm{d}\alpha} \leq 0 \quad \forall q \in [0,1], \tag{53}$$

with equality holding only if $q = 1$ *and* $z_d = 1/2$*. Moreover,* $\forall q \in [0,1]$*,*

1. *As* $\alpha \longrightarrow 0$*, then* $\eta_B^q \longrightarrow 1$ *and* $\eta_L^q \longrightarrow 1$*;*

2. *As* $\alpha \longrightarrow 1$*, then* $\eta_B^q \longrightarrow 0$ *and* $\eta_L^q \longrightarrow 0$*.*

As search frictions increase, i.e., raising α, customers are more likely to meet only one dealer. In other words, dealers face less pressure to compete for customers, strengthening their market power. In response, as illustrated in Figure 2, both borrower and lender dealers offer prices closer to the monopoly price and away from the competitive price (i.e., from (48) and (49), $\frac{\mathrm{d}z_B^q}{\mathrm{d}\alpha} < 0$ and $\frac{\mathrm{d}z_L^q}{\mathrm{d}\alpha} > 0$). In this way, the increased search frictions effectively weaken pass-through because, from (50) and (51), pass-through is more effective for prices near the competitive price that is determined by the deposit facility price, while less effective for prices near the monopoly price that is insensitive to policy changes. This highlights a key insight of this paper: the concentration of prices toward the tails of the distributions, rather than the distributions themselves, matters for policy interventions. In particular, the pass-through becomes perfect (completely imperfect) when search frictions become negligible (extremely large) because, as mentioned earlier, the price distributions collapse to the competitive prices (monopoly prices) in the limit case.

Diminishing Pass-through of the Deposit Facility Price Beyond the existence of pass-through imperfection, I show that the effectiveness of pass-through diminishes as the deposit facility price rises, or equivalently, strengthens as the central bank raises policy rates, consistent with Duffie and Krishnamurthy (2016).

Proposition 5 (Diminishing Pass-through of Deposit Facility Price)**.** *An increase in the central bank's deposit facility price z_d weakens the effectiveness of monetary policy pass-through to repo lending prices, i.e., $\frac{\mathrm{d}\eta_{\mathrm{L}}^{\mathrm{q}}}{dz_d} \leq 0$, with equality holding at $z_L^1 = 1$. By contrast, changes in z_d do not affect the pass-through to repo borrowing prices, i.e., $\frac{\mathrm{d}\eta_{\mathrm{B}}^{\mathrm{q}}}{dz_d} = 0$.*

Despite shifting both the borrowing and lending price distributions to the right in the first-order stochastic dominance sense (recall that $\eta_B^q, \eta_L^q > 0$), an increase in the central bank's deposit facility price z_d generates asymmetric effects on their concentration patterns. As illustrated in Figure 3, the lending price distribution F_L becomes more concentrated around the monopoly price $z_L = 1$, strengthening dealers' market power.[6] Consequently, lending prices become less responsive to subsequent changes in monetary policy. By contrast, the borrowing price distribution F_B becomes less concentrated around the borrower dealer's monopoly price $z_B = 1/A$. However, this does not enhance pass-through to the repo borrowing prices. This is because the increase in z_d also causes F_B to be more dispersed around the competitive price $z_I = z_d$, offsetting the potential gains from its reduced concentration at the monopoly price.

[6] For readability, I only draw the portion of the CDF where its density is strictly positive.

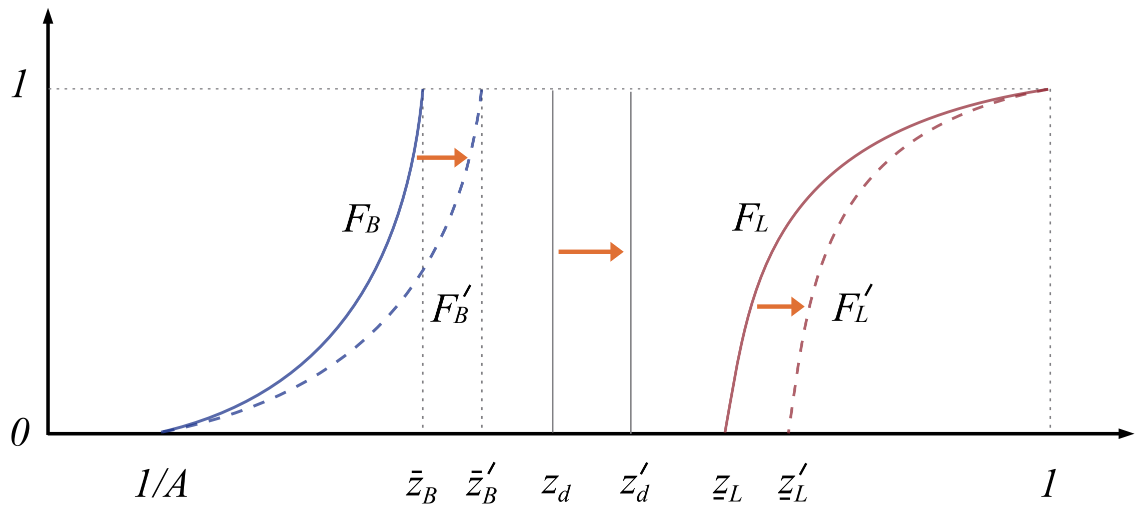

Figure 3: How An Increase in the Deposit Facility Price Shifts Price Distributions

5.2 Ambiguous Effects on Asset Allocation

I conclude this section by examining the effects of an increase in the central bank's deposit facility price on asset allocation, specifically the composition of central bank liabilities consisting of central bank deposits ($z_d\bar{d}$) and money ($\bar{m}$).

From (42) and (47),

$$\frac{\mathrm{d}\left(z_d\bar{d}\right)}{\mathrm{d}z_d} = -\rho\left(g-\hat{m}\right)\frac{\mathrm{d}\left(\mu_B/z_b\right)}{\mathrm{d}z_d}, \tag{54}$$

$$\frac{\mathrm{d}\bar{m}}{\mathrm{d}z_d} = \rho\left(g-\hat{m}\right)\frac{\mathrm{d}\left(\mu_B/z_b\right)}{\mathrm{d}z_d}. \tag{55}$$

An increase in the central bank's deposit facility price, z_d, changes the supply of central bank deposits and money in opposite directions because their sum is exogenously determined by central bank balance sheet policy (equation (38)). While the interest rate policy alters the composition of these liabilities, it does not affect their sum, which also implies that the supply of government bonds to the private sector, $z_b\bar{b}$, remains unchanged when z_d increases (equation (46)). Crucially, as shown in (54) and (55), the central bank's deposit supply decreases while its money supply increases when the mean repo borrowing price is more elastic relative to the bond price in response to an increase in z_d, i.e.,

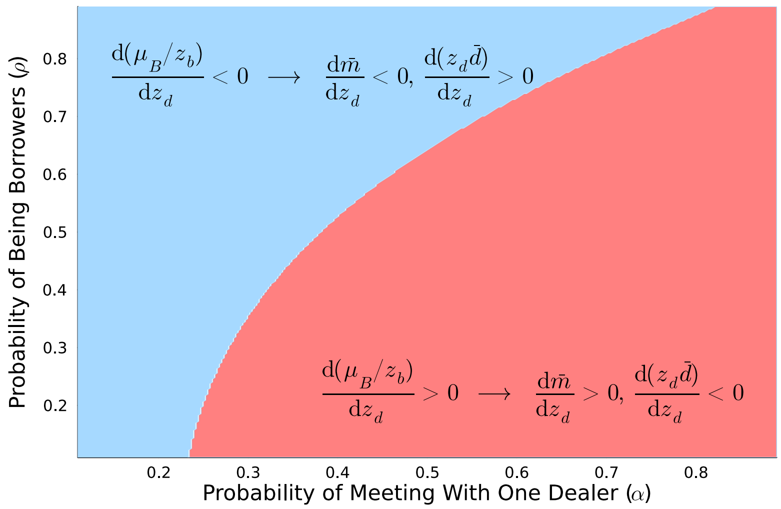

Figure 4: Ambiguous Effects of Deposit Facility Price on Asset Allocation ($z_d = 0.99$)

$\frac{\mathrm{d}(\mu_B/z_b)}{\mathrm{d}z_d} > 0$. In this scenario, borrowers borrow more money against their government bonds that become relatively cheaper in response to the increased deposit facility price, raising $\bar{m}$ and reducing $z_d\bar{d}$.

Overall, the relative elasticity of the mean repo borrowing price, μ, to the bond price, z_b, plays a crucial role in determining the effects of the central bank's deposit facility price, z_d, on asset allocation. However, changes in z_d generally have ambiguous effects on the price ratio μ_B/z_b, thereby on asset allocation. Figure 4 shows this ambiguity results with a numerical exercise, consider a deposit facility rate is close to the zero-lower bound.[7] Search frictions again play a critical role in generating these results. Lemma 9 below shows how the policy ambiguity disappears when search frictions are either vanishingly small or arbitrarily large.

Lemma 9 (How Search Frictions Matters for Ambiguous Asset Allocation)**.** *Under Assumption 1, the central bank's deposit facility is active.*

1. *As* $\alpha \longrightarrow 0$*, then* $\mu_B \longrightarrow z_d$ *and* $z_b = z_d$*;*

[7] I choose $A = 1.5$, but it does not matter because the results always hold if $A > 1$. I also choose $\theta = 0.9$, a sufficiently large central bank balance sheet that guarantees an active deposit facility for any $(z_d, \rho, \alpha) \in [0.9, 1] \times [0.1, 0.9] \times [0.1, 0.9]$.

2. As $\alpha \longrightarrow 1$, then $\mu_B \longrightarrow 1/A$ and $z_b \longrightarrow 1/(\rho A + 1 - \rho)$.

In either case, the price ratio μ_B/z_b is constant in z_d. An increase in the deposit facility price does not change the money supply $\bar{m}$ and central bank deposit supply $z_d\bar{d}$.

As explained earlier, the limit cases of $\alpha \longrightarrow 0$ and $\alpha \longrightarrow 1$ correspond to competitive and monopolistic pricing, respectively. Under competitive pricing, pass-through from the central bank deposit facility price, z_d, to the mean repo borrowing price, μ_B, and the bond price, z_b, is perfect. By contrast, under monopolistic pricing, pass-through is completely imperfect for both prices. In either case, the relative elasticity of μ_B and z_b remains constant in z_d, and monetary policy becomes neutral in the sense that the central bank cannot effectively reshuffle its liabilities. That said, a moderate degree of market power, implied by OTC trading structure with the search friction parameter $0 < \alpha < 1$, is critical for generating the policy ambiguity. Rather than requiring the central bank to assess market conditions and estimate all parameters in detail before choosing its deposit facility rate $1/z_d - 1$, I suggest pairing the deposit facility with its lending and borrowing facilities. The next section shows how this combined approach allows the central bank to reallocate assets effectively.

6 Central Bank Lending and Borrowing Facilities

I now introduce the central bank's long-standing lending and borrowing facilities and show how they can unambiguously reallocate assets. I also study their welfare implications and characterize the optimal monetary policy when all three facilities, including the lending, borrowing, and the previously introduced deposit facilities, are available. The lending facility enables the central bank to provide short-term, secured loans to commercial banks and other financial institutions, which works like the repurchase agreement facility in the U.S. or the main refinancing operations in the Euro area. The borrowing facility instead enables the central bank to borrow against collateral, similar to the overnight reverse repurchase agreement facility in the U.S..[8] Unlike the deposit facility, which is limited to highly regulated financial institutions such as banks, both lending and borrowing facilities can be accessed by a broader range of financial institutions, including mutual funds and insurance companies that are repo customers in my paper.

Let z_r and z_o denote the nominal prices for the central bank's lending and borrowing facilities, respectively. Then, impose

$$z_r \; < \; z_d \; < \; z_o \quad \longleftrightarrow \quad \frac{1}{z_o} - 1 \; < \; \frac{1}{z_d} - 1 \; < \; \frac{1}{z_r} - 1, \tag{56}$$

which is consistent with the interest structure in the U.S., where the interest rate on overnight reverse repurchase agreements is below the interest rate on reserves, and the interest rate on the repurchase agreements is the highest among the three rates. Additionally, I impose that

$$z_o \; \leq \; 1 \quad \text{and} \quad z_r \geq \frac{1}{A}, \tag{57}$$

to ensure that the market-determined repo prices do not strictly dominate the prices for these facilities. From Lemmas 3 and 4, borrowers only borrow at prices that are higher than $1/A$ and lenders only lend at prices that are lower than 1, which are the monopoly prices offered by dealers before introducing these facilities.

Long-standing borrowing and lending facilities allow customers to trade with the central bank on demand, suffering no search friction. After introducing these facilities, the profit per customer for borrower dealers and lender dealers in (20) and (21) becomes[9]

$$R_B\left(z_B\right) \; = \; \left(1 - \frac{z_B}{z_I}\right) \tilde{b} \qquad\qquad \forall z_B \in [z_r, 1], \tag{58}$$

$$R_L\left(z_L\right) \; = \; \left(\frac{z_L}{z_I} - 1\right) \frac{\tilde{m}}{z_L} \qquad\qquad \forall z_L \in (0, z_o]. \tag{59}$$

[8] The European Central Bank does not have a reverse repo facility in the way the Federal Reserve does, but relies on its deposit facility to absorb liquidity overnight.

[9] As before, I focus on cases that favor dealers when there are multiple solutions to the demand and supply: borrowers only borrow from dealers when $z_B = z_r$, and lenders only lend to dealers when $z_L = z_o$.

Although the functional form of these profit functions remains unchanged, the lending and borrowing facilities narrow their domains. Intuitively, borrowers would prefer to borrow from the central bank if their dealers provide them a price z_B that is below the price offered by the central bank's lending facility, z_r, so that they can borrow more money against their collateral. Lenders would prefer to lend to the central bank if their dealers provide them a price z_L that is above the price offered by the central bank's borrowing facility, z_o, so that they can obtain a higher return from lending.

Conditions (58) and (59) imply the following repo price distributions by affecting dealers' pricing strategies to maximize their total profits (16) and (19).

Proposition 6 (Price Distributions under Lending and Borrowing Facilities)**.** *If $z_r < z_I < z_o$, there exist unique distributions for repo borrowing and lending price, such that*

$$\begin{aligned} F_B(z_B) &= \frac{\alpha}{2(1-\alpha)}\left(\frac{z_B - z_r}{z_I - z_B}\right), \\ &\quad \textit{with} \quad \mathcal{S}_B = \left[z_r, \left(1 - \frac{\alpha}{2-\alpha}\right) z_I + \frac{\alpha}{2-\alpha} z_r\right]; \end{aligned} \tag{60}$$

$$\begin{aligned} F_L(z_L) &= 1 - \frac{\alpha}{2(1-\alpha)} \frac{1/z_L - 1/z_o}{1/z_I - 1/z_L}, \\ &\quad \textit{with} \quad \mathcal{S}_L = \left[\left(\left(1 - \frac{\alpha}{2-\alpha}\right)\frac{1}{z_I} + \frac{\alpha}{2-\alpha}\frac{1}{z_o}\right)^{-1}, z_o\right]. \end{aligned} \tag{61}$$

The price distributions are almost the same as those in Propositions 1 and 2, with the monopoly price for borrower dealers replacing from $z_B = 1/A$ to $z_B = z_r$ and the monopoly price for lender dealers replacing from $z_L = 1$ to $z_L = z_o$. For example, the borrower dealers' monopoly price z_r is still the lower bound of the support $\mathcal{S}_B$, and the upper bound of $\mathcal{S}_B$ is a convex combination of z_r and the competitive price z_I.

Under Proposition 6, the mean repo borrowing price μ_B, the gross inflation rate π, and the government bond price z_b can be written as

$$\mu_B = z_I + \frac{\alpha}{2(1-\alpha)} \ln\left(\frac{\alpha}{2-\alpha}\right)(z_I - z_r), \tag{62}$$

$$\pi = \beta\left[\rho A + (1-\rho)\left(\alpha\frac{1}{z_o} + (1-\alpha)\frac{1}{z_I}\right)\right], \tag{63}$$

$$z_b = \frac{\beta}{\pi}\left[\rho A(\alpha z_r + (1-\alpha) z_I) + 1 - \rho\right]. \tag{64}$$

As before, these endogenous, market-determined prices are key to understanding the implications of central bank interventions.

Welfare Before delving into the details, let me define welfare as the sum of the net payoffs from economic activities with equally weighted agents. The virtue of linear utility cancels out part of the utilities and disutilities, reducing total welfare to

$$\mathcal{W} = \rho \int (A-1)\, c_t(z_B)\, \mathrm{d}F_B(z_B) = \frac{\beta\rho(A-1)}{\pi}\left[\hat{m} + (g-\hat{m})\frac{\mu_B}{z_b}\right]. \tag{65}$$

The first equality in (65) indicates that welfare represents the surplus from transactions in the trading market. This highlights the crucial role of the repo market in reallocating assets across repo customers because, in particular, the consumption level depends on the loans that repo borrowers can obtain from dealers (Lemma 2). The second equality is derived from conditions in Definition 1, highlighting the central bank's critical role in steering market-determined prices (π, μ_B, z_b) that finally determine welfare.

An increase in the central bank's deposit facility price, z_d, affects welfare through two effects. First, from (45), an increase in z_d (recall that $z_I = z_d$ under Assumption 1), or equivalently, a decrease in the nominal interest rate on central bank deposits, reduces inflation through the Fisher effect and improves welfare by increasing the real value of money. Second, an increase in z_d also affects welfare by altering the relative price μ_B/z_b. As in (55), if the mean repo borrowing price μ_B reacts more elastically to z_d than the bond price z_b, an increase in z_d results in an expansion in the money supply. Consequently, borrowers obtain more money to settle their trading market transactions, resulting in a

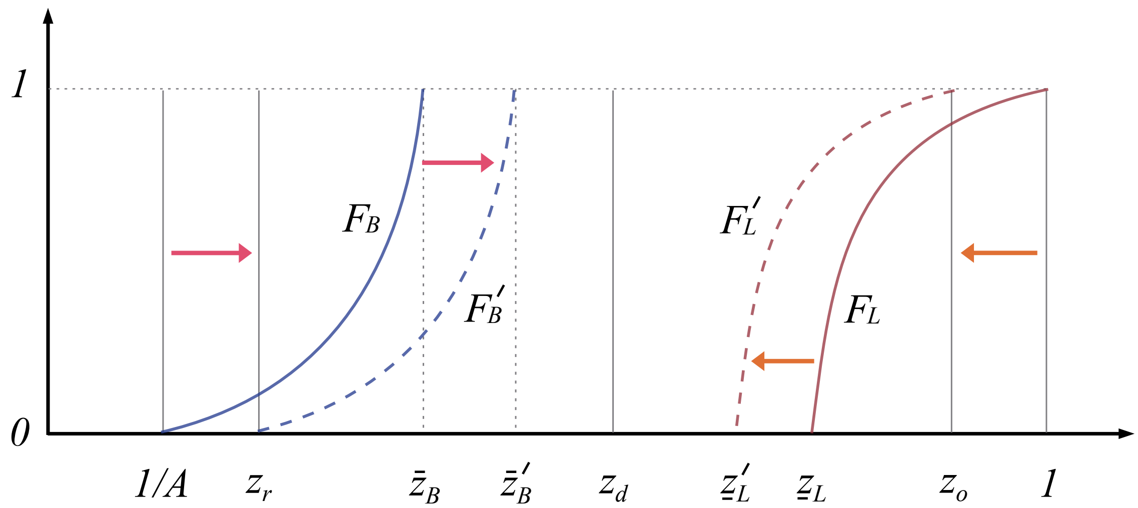

Figure 5: The Roles of Central Bank Lending and Borrowing Facilities

larger trading surplus and improving welfare. By contrast, an increase in z_d can reduce welfare if the bond price is more elastic than the mean repo borrowing price to changes in z_d. I formally summarize these two effects in the following equation

$$\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}z_d} = \frac{\beta\rho\,(A-1)}{\pi}\left(-\frac{1}{\pi}\left[\hat{m}+(g-\hat{m})\,\frac{\mu_B}{z_b}\right]\frac{\mathrm{d}\pi}{\mathrm{d}z_d}+(g-\hat{m})\,\frac{\mathrm{d}\,(\mu_B/z_b)}{\mathrm{d}z_d}\right). \tag{66}$$

6.1 Implications for Asset Allocation and Welfare

I now study the effects of raising the central bank's lending facility price while lowering its borrowing facility price. As in Figure 5, these interventions compress the price distributions, reducing dispersion. I also discuss the optimal monetary policy to provide a comprehensive understanding of the central bank's standing facilities, including the deposit facility. The results show that, although both effectively reduce dispersion and reallocate assets, the lending facility performs better than the borrowing facility, which could lead to higher inflation.

As in Figure 5, raising the central bank's lending facility price and lowering its borrowing facility price shift the entire distributions of repo lending and borrowing prices toward the competitive price z_d that dealers would offer in the absence of search frictions.

These shifts in the repo prices then lead to changes in other market-determined prices, such as the bond price, z_b, and the inflation rate, $\pi - 1$, as in the following proposition.

Proposition 7 (Implications of Lending and Borrowing Facilities for Asset Prices)**.** *Under Assumption 1, the central bank's deposit facility is active.*

1. *Despite having no impact on inflation, an **increase** in the central bank's lending facility price, z_r, increases the ratio of the mean repo borrowing price μ_B to the bond price z_b, i.e.,*

$$\forall \alpha \in (0,1) \quad \frac{\mathrm{d}\pi}{\mathrm{d}z_r} = 0, \quad \frac{\mathrm{d}\left(\mu_B/z_b\right)}{\mathrm{d}z_r} > 0; \tag{67}$$

2. *A **decrease** in the central bank's borrowing facility price, z_o, increase the relative price μ_B/z_b, and the gross inflation rate, π, i.e.,*

$$\forall \alpha \in (0,1) \quad \frac{\mathrm{d}\pi}{\mathrm{d}z_o} < 0, \quad \frac{\mathrm{d}\left(\mu_B/z_b\right)}{\mathrm{d}z_o} < 0. \tag{68}$$

Corollary 2. *Raising the lending facility price or lowering the borrowing facility price increases the supply of money, $\bar{m}$, while reducing the supply of central bank deposits, $z_d\bar{\mathrm{d}}$.*

An increase in the central bank's lending facility price shifts the borrowing price distribution rightward, raising each borrowing price as well as the ratio of mean borrowing price to the bond price, μ_B/z_b. However, it does not affect inflation, which, as in Lemma 5, is directly determined by borrowers' payoff from using money in transactions and lenders' expected interest payoffs from lending. By contrast, a decrease in the borrowing facility price shifts the lending price distribution leftward, increasing lenders' interest payoffs. This increase in the nominal payoff on money further implies higher inflation. The relative price μ_B/z_b also rises because higher inflation lowers the (nominal) bond price. Corollary 2 is an immediate result of (54), (55), and Proposition 7. I omit its intuition, as it is discussed under the earlier results.

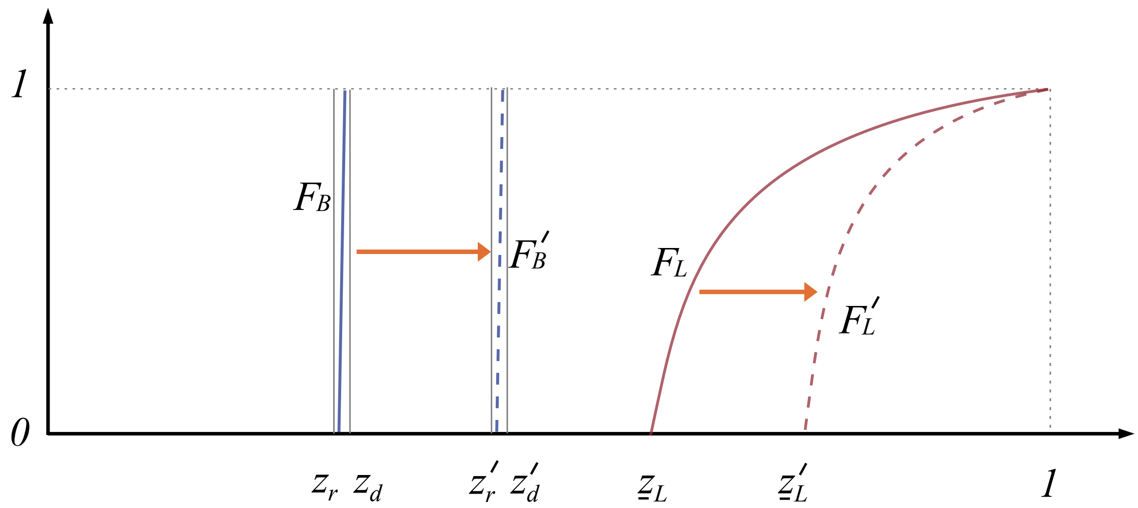

Figure 6: Price Distributions under the Optimal Lending Facility Price

Proposition 8 (Implications of Lending and Borrowing Facilities for Welfare)**.** *Raising the central bank's lending facility price improves welfare. By contrast, lowering the borrowing facility price harms welfare.*

From Proposition 8, a benevolent central bank should introduce the lending facility but not the borrowing one. As the lending facility price increases, borrowers borrow more money against their collateral (Corollary 2). The increased money supply allows them to settle a larger volume of transactions in the trading market, generating a higher trading surplus and improving welfare. However, and perhaps counterintuitively, introducing the borrowing facility is harmful, even though it can effectively increase borrowers' money holdings for transactions. Borrowers' money holdings indeed have to increase, but so does the inflation (Proposition 7). The rise in inflation reduces the real value of money, thereby lowering the trading surplus and harming welfare.

Clearly, a benevolent central bank should set the lending facility price arbitrarily close to the deposit facility price, i.e., $z_r \longrightarrow z_d$. In the limit, conditions (62) to (64) becomes

$$\mu_B = z_d, \tag{69}$$

$$\pi = \beta\left[\rho A + (1-\rho)\left(\alpha + (1-\alpha)\frac{1}{z_d}\right)\right], \tag{70}$$

$$z_b = \frac{\beta}{\pi}\left[\rho A z_d + 1 - \rho\right] \tag{71}$$

because, as illustrated in Figure 6, the borrowing price distribution, F_B, converges to a degenerate distribution with $\mathbb{P}(z_B = z_d) = 1$. With the lending facility, I show that the optimal monetary policy is to peg its nominal interest rates at the zero-lower bound, such that $1/z_r - 1 = 1/z_d - 1 = 0$, in line with the Friedman rule.

Proposition 9. *In the limit where the lending facility price z_r approaches the deposit facility price z_d, i.e., $z_r \longrightarrow z_d$, an increase in z_d lowers inflation while raising the ratio of the mean repo borrowing price to the bond price μ_B/z_b, thereby improving welfare, i.e.,*

$$\frac{\mathrm{d}\pi}{\mathrm{d}z_d} < 0 \ \textit{and} \ \frac{\mathrm{d}(\mu_B/z_b)}{\mathrm{d}z_d} > 0 \quad \longrightarrow \quad \frac{\mathrm{d}\mathcal{W}}{\mathrm{d}z_d} > 0. \tag{72}$$

As before, an increase in the central bank's deposit facility price, or equivalently, a reduction in the deposit facility rate, reduces inflation. However, it now unambiguously increases the price ratio of the mean repo borrowing price to the bond price, μ_B/z_b. As in Figure 6, an increase in the lending facility price pushes the entire borrowing price distribution toward the deposit facility price. In the limit, $\mu_B = z_d$, implying a perfect pass-through from the deposit facility price to the mean repo borrowing price, as if borrowing prices were set competitively. As a result, an increase in z_d always increases the price ratio μ_B/z_b. Finally, an increase in the deposit facility price also unambiguously improves welfare. First, it reduces inflation, increasing the real value of money and enhancing its usefulness in transactions. Second, the increased price ratio μ_B/z_b allows borrowers, who need money to settle transactions, to borrow more against their government bonds. These two effects together lead to an improvement in welfare.

7 Conclusion

I develop a search-theoretic model that embeds Burdett and Judd (1983) pricing in OTC repo markets to rationalize repo price dispersion. I show that an increase in the central bank's deposit facility price exhibits imperfect pass-through to market-determined repo prices, and the pass-through effect weakens as the deposit facility price rises. I also find an ambiguous effect of the deposit facility price on asset allocation, in particular, on the composition of central bank liabilities. Rather than exercising caution, I suggest that the central bank pair the deposit facility with its lending facility, such as the Fed's repurchase agreement facility and the European Central Bank's main refinancing operations, to reallocate assets unambiguously. In doing so, the optimal policy is to peg both the lending and deposit facility rates to the zero-lower bound, in the spirit of the Friedman rule. The borrowing facility, such as the Fed's overnight reverse repurchase agreement facility, can also effectively reallocate assets, but at the cost of raising inflation.

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Appendix

A Omitted Proofs

A.1 Proof of Lemma 1

Construct the following Lagrangian for the borrower dealer's problem (14),

$$L^{BD} \;=\; d_{BD} + \ell_B(z_B) - \ell_{BD} + \lambda_1^{BD}\left(z_I \ell_{BD} - z_B \ell_B(z_B) - z_d d_{BD}\right) + \lambda_2^{BD} d_{BD}, \tag{A.1}$$

where λ_1^{BD} and λ_2^{BD} are the Lagrange multipliers for the equality constraint and inequality constraint, respectively. The first-order conditions for the Lagrangian are

$$1 - \lambda_1^{BD} z_d + \lambda_2^{BD} \;=\; 0, \tag{A.2}$$

$$-1 + \lambda_1^{BD} z_I \;=\; 0, \tag{A.3}$$

where variables are also subject to the complementary slackness conditions

$$\lambda_2^{BD} d_{BD} = 0, \quad \lambda_2^{BD} \geq 0, \quad d_{BD} \geq 0. \tag{A.4}$$

It is then straightforward to show that $z_d \geq z_I$ and the equality holds if $d_{BD} > 0$.

When $d_{BD} = 0$, the equality constraint becomes

$$z_B \ell_B(z_B) \;=\; z_I \ell_{BD}, \tag{A.5}$$

so that

$$R_B\left(z_B\right) \;=\; \left(1 - \frac{z_B}{z_I}\right) \ell_B(z_B) \;=\; \left(\frac{1}{z_B} - \frac{1}{z_I}\right) z_B \ell_B(z_B) \tag{A.6}$$

This profit function also holds when $d^{BD} > 0$ because $z_I = z_d$ in this case.

Following the same procedure, the solution to the lender dealer's problem (17) gives a similar result, such that

$$z_d \geq z_I, \text{ with equality if } d_{LD} > 0, \quad \text{and} \quad R_L\left(z_L\right) \;=\; \left(\frac{1}{z_I} - \frac{1}{z_L}\right) z_L \ell_L(z_L). \tag{A.7}$$

Finally, aggregating all dealers' choices of central bank deposits implies that $z_d = z_I$ if $\bar{d} > 0$ in equilibrium. □

A.2 Proof of Lemma 2

Construct the following Lagrangian for the borrower's problem (23),

$$L^B = Ac_t + \frac{\beta\left(\tilde{m} + z_B\ell_B(z_B) - pc_t + \tilde{b} - \ell_B(z_B)\right)}{\pi} + \lambda_1^B pc_t + \lambda_2^B\left(\tilde{m} + z_B\ell_B(z_B) - pc_t\right). \tag{A.8}$$

The first-order condition is

$$A - \frac{\beta}{\pi}p + \lambda_1^B p - \lambda_2^B p \;=\; 0, \tag{A.9}$$

and variables are also subject to the following complementary slackness conditions

$$\lambda_1^B pc_t(z_B) = 0, \quad \lambda_1^B \geq 0, \quad pc_t(z_B) \geq 0; \tag{A.10}$$

$$\lambda_2^B\left(\tilde{m} + z_B\ell_B(z_B) - pc_t(z_B)\right) = 0, \quad \lambda_2^B \geq 0, \quad \tilde{m} + z_B\ell_B(z_B) - pc_t(z_B) \geq 0. \tag{A.11}$$

Similarly, construct the Lagrangian for the lender's problem (24), such that

$$L^L \;=\; -n_t + \frac{\beta\left(\tilde{m} - z_L\ell_L(z_L) + pn_t + \tilde{b} + \ell_L(z_L)\right)}{\pi} + \lambda^L n_t, \tag{A.12}$$

and solve for the first-order condition

$$-1 + \frac{\beta}{\pi}p + \lambda^L \;=\; 0, \tag{A.13}$$

as well as the associated complementary slackness conditions

$$\lambda^L n_t = 0, \quad \lambda^L \geq 0, \quad n_t \geq 0. \tag{A.14}$$

First, suppose that the non-negative constraint $pc_t(z_B) \geq 0$ binds so that $c_t(z_B) = 0$. This implies that $\lambda_2^B = 0$ under (A.11). However, when $\lambda_2^B = 0$, first-order conditions (A.9) and (A.13) give

$$\lambda_1^B p + \lambda^L = 1 - A < 0, \tag{A.15}$$

which contradicts to the complementary slackness conditions that require $\lambda_1^B, \lambda^L \geq 0$. As a result, there is always a positive consumption, i.e., $c_t(z_B) > 0$, as well as a positive labor supply, i.e., $n_t > 0$, under the market clearing condition (25). The associated Lagrange multipliers equal to zero, and substituting these multipliers $\lambda_1^B = 0$ and $\lambda^L = 0$ into (A.9) and (A.13) gives

$$p = \frac{\pi}{\beta}, \tag{A.16}$$

$$A - 1 = \lambda_2^B p > 0. \tag{A.17}$$

Finally, from (A.11), the fact that $\lambda_2^B > 0$ implies the binding cash constraint that solves

$$c_t(z_B) = \frac{\beta\left(\tilde{m} + z_B \ell_B(z_B)\right)}{\pi}. \tag{A.18}$$

□

A.3 Proof of Lemma 3

Borrowers' loan demand is jointly determined by the envelope condition from their trading market problem and the first-order condition from their funding market problem. The Lagrangian for the borrower's trading market problem (A.8) gives

$$\frac{\partial}{\partial \ell_B(z_B)} W^B\left(\tilde{m} + z_B \ell_B(z_B), \tilde{b}, \ell_B(z_B)\right) = \frac{\beta\left(z_B - 1\right)}{\pi} + \lambda_2^B z_B = \frac{\beta\left(A z_B - 1\right)}{\pi}, \tag{A.19}$$

where the first equation is an immediate result following the Envelope Theorem and the second one follows from equation (A.17). The first-order condition for the borrower's funding market problem (10) is

$$\frac{\partial}{\partial \ell_B(z_B)} W^B\left(\tilde{m} + z_B \ell_B(z_B), \tilde{b}, \ell_B(z_B)\right) = \lambda_W^B, \tag{A.20}$$

where λ_W^B is the Lagrange multiplier for the collateral constraint $\tilde{b} \geq \ell_B$, so that

$$\lambda_W^B\left(\tilde{b} - \ell_B(z_B)\right) = 0, \quad \lambda_W^B \geq 0, \quad \tilde{b} - \ell_B(z_B) \geq 0. \tag{A.21}$$

The fact that

$$\lambda_W^B = \frac{\beta\left(A z_B - 1\right)}{\pi} \tag{A.22}$$

in equilibrium implies a binding collateral constraint when $z_B > 1/A$. By contrast, the constraint constraint does not bind when $z_B = 1/A$. □

A.4 Proof of Lemma 4

Lenders' supply of loans is jointly determined by the envelope condition from their trading market problem and the first-order condition for the funding market problem. The Lagrangian for the lender's trading market problem (A.12) gives

$$\frac{\partial}{\partial \ell_L(z_L)} W^L\left(\tilde{m} - z_L \ell_L(z_L), \tilde{b}, \ell_L(z_L)\right) \;=\; \frac{\beta\left(-z_L + 1\right)}{\pi}, \tag{A.23}$$

an immediate result following the Envelope Theorem. The first-order condition for the lender's funding market problem (12) is

$$\frac{\partial}{\partial \ell_L(z_L)} W^L\left(\tilde{m} - z_L \ell_L(z_L), \tilde{b}, \ell_L(z_L)\right) \;=\; z_L \lambda_W^L, \tag{A.24}$$

where λ_W^L is the Lagrange multiplier for the cash constraint $\tilde{m} \geq z_L \ell_L$, so that

$$\lambda_W^L\left(\tilde{m} - z_L \ell_L(z_L)\right) = 0, \quad \lambda_W^L \geq 0, \quad \tilde{m} - z_L \ell_L(z_L) \geq 0. \tag{A.25}$$

The fact that

$$z_L \lambda_W^L \;=\; \frac{\beta\left(-z_L + 1\right)}{\pi}, \tag{A.26}$$

implies a binding cash constraint with $\ell_L(z_L) = \tilde{m}/z_L$ when $z_L < 1$. By contrast, this constraint does not bind when $z_L = 1$. □

A.5 Proof of Lemma 5

The Lagrangian for the borrower's trading market problem (A.8) gives

$$\frac{\partial}{\partial \tilde{m}} W^B\left(\tilde{m} + z_B \ell_B(z_B), \tilde{b}, \ell_B(z_B)\right) \;=\; \frac{\beta}{\pi} + \lambda_2^B \;=\; \frac{\beta}{\pi} A \tag{A.27}$$

$$\frac{\partial}{\partial \tilde{b}} W^B\left(\tilde{m} + z_B \ell_B(z_B), \tilde{b}, \ell_B(z_B)\right) \;=\; \frac{\beta}{\pi}, \tag{A.28}$$

an immediate result following the Envelope Theorem and equation (A.17). Similarly, the envelope conditions for the lender's problem (A.12) gives

$$\frac{\partial}{\partial \tilde{m}} W^L\left(\tilde{m} - z_L \ell_L(z_L), \tilde{b}, \ell_L(z_L)\right) \;=\; \frac{\partial}{\partial \tilde{b}} W^L\left(\tilde{m} - z_L \ell_L(z_L), \tilde{b}, \ell_L(z_L)\right) \;=\; \frac{\beta}{\pi}. \tag{A.29}$$

These envelope conditions on W^L and W^B help to determine the envelope conditions of the constrained optimization problems (10) and (12), such that

$$\begin{aligned}
\frac{\partial}{\partial \tilde{m}} V^B\left(\tilde{m}, \tilde{b}\right) &= A\frac{\beta}{\pi}, &\text{(A.30)}\\
\frac{\partial}{\partial \tilde{b}} V^B\left(\tilde{m}, \tilde{b}\right) &= A\frac{\beta}{\pi}\left(\alpha\int z_B \mathrm{d}F_B\left(z_B\right) + (1-\alpha)\int z_B \mathrm{d}\left[F_B\left(z_B\right)\right]^2\right), &\text{(A.31)}\\
\frac{\partial}{\partial \tilde{m}} V^L\left(\tilde{m}, \tilde{b}\right) &= \frac{\beta}{\pi}\left(\alpha\int \frac{1}{z_L}\mathrm{d}F_L\left(z_L\right) + (1-\alpha)\int \frac{1}{z_L}\mathrm{d}\left(1-\left[1-F_L\left(z_L\right)\right]^2\right)\right), &\text{(A.32)}\\
\frac{\partial}{\partial \tilde{b}} V^L\left(\tilde{m}, \tilde{b}\right) &= \frac{\beta}{\pi}. &\text{(A.33)}
\end{aligned}$$

Substituting these conditions into the first-order conditions (8) and (9) gives

$$1 = \frac{\beta}{\pi}\left[\rho A + (1-\rho)\left(\alpha\int \frac{1}{z_L}\mathrm{d}F_L\left(z_L\right) + (1-\alpha)\int \frac{1}{z_L}\mathrm{d}\left(1-\left[1-F_L\left(z_L\right)\right]^2\right)\right)\right], \tag{A.34}$$

$$z_b = \frac{\beta}{\pi}\left[\rho A\left(\alpha\int z_B \mathrm{d}F_B\left(z_B\right) + (1-\alpha)\int z_B \mathrm{d}\left[F_B\left(z_B\right)\right]^2\right) + 1 - \rho\right]. \tag{A.35}$$

□

A.6 Proof of Lemma 6

Borrower dealers' profit per borrower served is

$$R_B\left(z_B\right) = \left(1 - \frac{z_B}{z_I}\right)\tilde{b} \quad \forall z_B \in [\frac{1}{A}, 1], \tag{A.36}$$

which is strictly decreasing in z_B given that

$$\frac{\mathrm{d}}{\mathrm{d}z_B} R_B\left(z_B\right) = -\frac{\tilde{b}}{z_I} < 0. \tag{A.37}$$

The monopoly price is $z_B = 1/A$, which yields a nonnegative profit if and only if $z_I \geq 1/A$. □

A.7 Proof of Proposition 1

This proposition builds on the case in which borrower dealers could earn positive monopoly profits, such that $z_I > 1/A$. I prove this proposition by establishing the following lemmas regarding the continuity, connectedness, and boundary of the distribution F_B. The solution of F_B is an immediate result under these lemmas.

Lemma A.1. *F_B is continuous on $\mathcal{S}_B$.*

Proof. Suppose the contradictory, assume $\exists z \in \mathcal{S}_B$ such that $\xi_B(z) = \lim_{\epsilon\to 0^+} F_B(z) - F_B(z-\epsilon) > 0$, and

$$\Pi_B(z) \;=\; \Pi_B^\star \;=\; \lim_{\epsilon\to 0^+} \frac{\rho}{s}\left(\,\alpha + (1-\alpha)\left[F_B(z) + F_B(z-\epsilon)\right]\,\right) R_B(z) > 0. \tag{A.38}$$

Notice that the dealer's profit per borrower R_B is continuous. Therefore, there exists $z' > z$ such that $R_B(z') > 0$ and $\Delta \equiv R_B(z) - R_B(z') < \frac{(1-\alpha)\xi_B(z)R_B(z)}{\alpha+2(1-\alpha)F_B(z)}$. Then,

$$\begin{aligned}
\Pi_B(z') \;&=\; \lim_{\epsilon\to 0^+} \frac{\rho}{s}\left(\,\alpha + (1-\alpha)\left[F_B(z') + F_B(z'-\epsilon)\right]\,\right) R_B(z') \\
&\geq \lim_{\epsilon\to 0^+} \frac{\rho}{s}\left(\,\alpha + (1-\alpha)\left[F_B(z) + F_B(z-\epsilon) + \xi_B(z)\right]\,\right)\left(R_B(z) - \Delta\right) \\
&= \Pi_B(z) + \frac{\rho}{s}\left((1-\alpha)\,\xi_B(z)\,R_B(z) - \left[\,\alpha + 2(1-\alpha)\,F_B(z)\,\right]\Delta\right),
\end{aligned} \tag{A.39}$$

where the inequality holds because $F_B(z') \geq F_B(z)$ and $\lim_{\epsilon\to 0^+} F_B(z'-\epsilon) - F_B(z-\epsilon) \geq \xi_B(z)$. This further implies

$$\Pi_B(z') - \Pi_B(z) \geq \frac{\rho}{s}\left((1-\alpha)\,\xi_B(z)\,R_B(z) - \left[\,\alpha + 2(1-\alpha)\,F_B(z)\,\right]\Delta\right) > 0, \tag{A.40}$$

where the last inequality holds by the definition of Δ. The fact that $\Pi_B(z') > \Pi_B(z)$ contradicts with $z \in \mathcal{S}_B$. Therefore, F_B must be continuous on its support $\mathcal{S}_{\mathcal{B}}$. □

Given Lemma A.1, dealers' profit function can be rewritten as

$$\Pi_B^\star \;=\; \Pi_B(z_B) \;=\; \frac{\rho}{s}\left[\,\alpha + 2(1-\alpha)\,F_B(z_B)\,\right] R_B(z_B). \tag{A.41}$$

Lemma A.2. *The monopoly price $z_B = 1/A$ is the lowest price in $\mathcal{S}_B$.*

Proof. Suppose that $z \neq 1/A$ is the lowest price in $\mathcal{S}_B$. Then,

$$\Pi_B(z) \;=\; \frac{\rho}{s}\alpha R_B(z). \tag{A.42}$$

But now,

$$\Pi_B\left(\frac{1}{A}\right) = \frac{\rho}{s}\left[\alpha + 2(1-\alpha)\,F_B\left(\frac{1}{A}\right)\right] R_B\left(\frac{1}{A}\right) \geq \frac{\rho}{s}\alpha R_B\left(\frac{1}{A}\right) > \Pi_B(z). \tag{A.43}$$

This is a contradiction. $\square$

Lemma A.3. $\mathcal{S}_B$ *is connected.*

Proof. Suppose that $z, z' \in \mathcal{S}_B$, such that $z < z'$ and $F_B(z) = F_B(z')$. Therefore,

$$\alpha + 2(1-\alpha) F_B(z) \;=\; \alpha + 2(1-\alpha) F_B(z'), \tag{A.44}$$

which further implies that

$$\Pi_B(z') < \Pi_B(z), \tag{A.45}$$

given that $R_B(z_B)$ is strictly decreasing in z_B for all $z_B \in [1/A, 1]$. This contradicts to $z, z' \in \mathcal{S}_B$ that requires $\Pi_B(z') = \Pi_B(z)$. $\square$

The total profit is maximized at the monopoly price $1/A$ with

$$\Pi_B^\star \;=\; \frac{\rho}{s}\alpha R_B\left(\frac{1}{A}\right). \tag{A.46}$$

By equal profit among prices in the connected support, I obtain

$$\alpha R_B\left(\frac{1}{A}\right) \;=\; \left[\alpha + 2(1-\alpha) F_B(z_B)\right] R_B(z_B), \tag{A.47}$$

which solves

$$F_B(z_B) \;=\; \frac{\alpha}{2(1-\alpha)}\left(\frac{R_B\left(\frac{1}{A}\right)}{R_B(z_B)} - 1\right) \tag{A.48}$$

$$\;=\; \frac{\alpha}{2(1-\alpha)}\left(\frac{z_B - \frac{1}{A}}{z_I - z_B}\right) \quad \forall z_B \in \mathcal{S}_B. \tag{A.49}$$

Moreover, the upper bound $\bar{z}_B$ solves

$$R_B(\bar{z}_B) \;=\; \frac{\alpha}{2-\alpha} R_B\left(\frac{1}{A}\right), \tag{A.50}$$

so that

$$\bar{z}_B \;=\; \left(1 - \frac{\alpha}{2-\alpha}\right) z_I + \frac{\alpha}{2-\alpha}\frac{1}{A}. \tag{A.51}$$

$\square$

A.8 Proof of Lemma 7

Lender dealers' profit per lender served is

$$R_L\left(z_L\right) \;=\; \left(\frac{z_L}{z_I}-1\right)\frac{\tilde{m}}{z_L} \tag{A.52}$$

which is strictly increasing in z_L given that

$$\frac{\mathrm{d}}{\mathrm{d}z_L}R_L\left(z_L\right) \;=\; \frac{\tilde{m}}{\left(z_L\right)^2}>0. \tag{A.53}$$

Therefore, their monopoly loan price, which yields the highest profit, is $z_L=1$. Under $z_d\leq 1$, the monopoly profit is nonnegative, given that $z_I\leq z_d$ (Lemma 1), and the zero profit occurs if and only if $z_I=z_d=1$. □

A.9 Proof of Proposition 2

As with the borrower dealer's problem, this proposition builds on the case in which lender dealers could earn positive monopoly profits, such that $z_I<1$. I also prove this proposition by establishing the following lemmas regarding the continuity, connectedness, and boundary of the distribution F_L before solving for F_L.

Lemma A.4. *F_L is continuous on $\mathcal{S}_L$.*

Proof. Suppose the contradictory, assume $\exists z\in\mathcal{S}_L$ such that $\xi_L\left(z\right)=\lim_{\epsilon\to 0^+}F_L\left(z\right)-F_L\left(z-\epsilon\right)>0$, and

$$\Pi_L\left(z\right) \;=\; \Pi_L^{\star}=\lim_{\epsilon\to 0^+}\frac{1-\rho}{s}\left[\,\alpha+\left(1-\alpha\right)\left(2-\left[F_L\left(z\right)+F_L\left(z-\epsilon\right)\right]\right)\right]R_L\left(z\right)>0. \tag{A.54}$$

The fact that R_L is a continuous function implies the existence of $z'<z$ such that $R_L\left(z'\right)>0$ and $\Delta\equiv R_L\left(z\right)-R_L\left(z'\right)<\frac{(1-\alpha)\xi_L(z)R_L(z)}{\alpha+2(1-\alpha)[1-F_L(z)+\xi_L(z)]}$. Then,

$$\begin{aligned}\Pi_L\left(z'\right) \;&=\; \lim_{\epsilon\to 0^+}\frac{1-\rho}{s}\left[\,\alpha+\left(1-\alpha\right)\left(2-\left[F_L\left(z'\right)+F_L\left(z'-\epsilon\right)\right]\right)\right]R_L\left(z'\right)\\ &\geq \lim_{\epsilon\to 0^+}\frac{1-\rho}{s}\left[\,\alpha+\left(1-\alpha\right)\left(2-\left[F_L\left(z\right)-\xi_L\left(z\right)+F_L\left(z-\epsilon\right)\right]\right)\right]\left(R_L\left(z\right)-\Delta\right)\\ &=\Pi_L\left(z\right)+\frac{1-\rho}{s}\left(\left(1-\alpha\right)\xi_L\left(z\right)R_L\left(z\right)-\left[\alpha+2\left(1-\alpha\right)\left(1-F_L\left(z\right)+\xi_L\left(z\right)\right)\right]\Delta\right),\end{aligned} \tag{A.55}$$

where the inequality holds because $F_L\left(z\right)-\xi_L\left(z\right)\;\geq\;F_L\left(z'\right)$ and $\lim_{\epsilon\to 0^+}F\left(z-\epsilon\right)\;\geq$

$\lim_{\epsilon\to 0^+} F_L(z'-\epsilon)$. This further implies that

$$\begin{aligned}\Pi_L(z') - \Pi_L(z) \geq& \\ &\frac{1-\rho}{s}\left(\,(1-\alpha)\,\xi_L(z)\,R_L(z) - \left[\,\alpha + 2\,(1-\alpha)\,(1-F_L(z)+\xi_L(z))\,\right]\Delta\,\right) > 0\end{aligned} \tag{A.56}$$

where the last inequality holds by the definition of Δ. The fact that $\Pi_L(z') > \Pi_L(z)$ contradicts with $z \in \mathcal{S}_L$. This establishes the Lemma. $\square$

Given Lemma A.4, dealers' profit function can be rewritten as

$$\Pi_L^\star \;=\; \Pi_L(z_L) \;=\; \frac{1-\rho}{s}\left[\,\alpha + 2\,(1-\alpha)\,(1-F_L(z_L))\,\right] R_L(z_L)\,. \tag{A.57}$$

Lemma A.5. *The monopoly price $z_L = 1$ is the highest price in $\mathcal{S}_L$.*

Proof. Suppose that $z \neq 1$ is the highest price in $\mathcal{S}_L$. Then,

$$\Pi_L(z) \;=\; \frac{1-\rho}{s}\alpha R_L(z)\,. \tag{A.58}$$

But now,

$$\Pi_L(1) = \frac{1-\rho}{s}\left[\,\alpha + 2\,(1-\alpha)\,(1-F_L(1))\,\right] R_L(1) \geq \frac{1-\rho}{s}\alpha R_L(1) > \Pi_L(z)\,. \tag{A.59}$$

This is a contradiction. $\square$

Lemma A.6. *$\mathcal{S}_L$ is connected.*

Proof. Suppose that $z, z' \in \mathcal{S}_L$, such that $z < z'$ and $F_L(z) = F_L(z')$. Therefore,

$$\alpha + 2\,(1-\alpha)\,(1-F_L(z)) \;=\; \alpha + 2\,(1-\alpha)\,(1-F_L(z'))\,, \tag{A.60}$$

which further implies that

$$\Pi_L(z') > \Pi_L(z)\,, \tag{A.61}$$

given that $R_L(z)$ is strictly increasing in z for all $z \in (0,1]$. This contradicts to $z, z' \in \mathcal{S}_L$ that requires $\Pi_L(z') = \Pi_L(z)$. $\square$

At the monopoly price $z_L = 1$, profit is maximized with

$$\Pi_L^\star \;=\; \frac{1-\rho}{s}\alpha R_L(1)\,. \tag{A.62}$$

By equal profit among prices in the connected support, I obtain

$$\alpha R_L(1) = [\alpha + 2(1-\alpha)(1-F_L(z_L))]\, R_L(z_L), \tag{A.63}$$

which solves

$$F_L(z_L) = 1 - \frac{\alpha}{2(1-\alpha)}\left(\frac{R_L(1)}{R_L(z_L)} - 1\right) \tag{A.64}$$

$$= 1 - \frac{\alpha}{2(1-\alpha)}\frac{z_I(1-z_L)}{z_L - z_I} \quad \forall z_L \in \mathcal{S}_L. \tag{A.65}$$

Moreover, the lower bound $\underline{z}_L$ solves

$$R_L(\underline{z}_L) = \frac{\alpha}{2-\alpha} R_L(1), \tag{A.66}$$

so that

$$\underline{z}_L = \left(\frac{2(1-\alpha)}{2-\alpha}\frac{1}{z_I} + \frac{\alpha}{2-\alpha}\right)^{-1} = \left[\left(1 - \frac{\alpha}{2-\alpha}\right)\frac{1}{z_I} + \frac{\alpha}{2-\alpha}\right]^{-1}. \tag{A.67}$$

□

A.10 Proof of Proposition 3

Confine attention to the case with inter-dealer price $z_I \in (1/A, 1)$. Rewrite equilibrium condition (42) as

$$\frac{z_b z_d}{g}\bar{d} = z_b(1-\rho)\frac{\hat{m}}{g} - \rho\left(1 - \frac{\hat{m}}{g}\right)\mu_B. \tag{A.68}$$

Plugging in the value of bond price in (44) and $\theta \equiv \hat{m}/g \in (0,1)$ gives

$$\begin{aligned}\left[\rho A + (1-\rho)\left(\alpha + \frac{1-\alpha}{z_I}\right)\right]\frac{z_b z_d}{g}\bar{d} &= \left[\rho A\left((1-\alpha) z_I + \frac{\alpha}{A}\right) + 1 - \rho\right](1-\rho)\theta \\ &\quad -\rho\left[\rho A + (1-\rho)\left(\alpha + \frac{1-\alpha}{z_I}\right)\right]\mu_B(1-\theta).\end{aligned} \tag{A.69}$$

The last equation can be further rewritten as

$$[\rho A z_I + (1-\rho)(\alpha z_I + 1 - \alpha)]\frac{z_b z_d}{g}\bar{d} = G(z_I)\theta - H(z_I)(1-\theta), \tag{A.70}$$

where

$$G(z_I) = \left[\rho A\left((1-\alpha) z_I^2 + \frac{\alpha}{A} z_I\right) + (1-\rho) z_I\right](1-\rho) \tag{A.71}$$

and

$$H(z_I) = \rho\left[\rho A z_I + (1-\rho)(\alpha z_I + 1 - \alpha)\right]\mu_B \tag{A.72}$$

are both quadratic equations because, in particular, μ_B is linear in z_I (equation (43)).

Note that both $G(z_I)$ and $H(z_I)$ are positive and bounded on the interval $(1/A, 1)$. Consequently, the right-hand of equation (A.70) is always positive once the ratio θ is sufficiently large. This further implies a threshold $\bar{\theta}$ such that, whenever $\theta > \bar{\theta}$, equation (A.70) can solve for a positive value of central bank deposits, $\bar{d}$. The resulting equilibrium features an active central bank deposit facility, so the inter-dealer price is equal to the deposit facility price, i.e., $z_I = z_d$. To guarantee price dispersion, z_d must lie in the interval $(1/A, 1)$. By contrast, the right-hand of (A.70) becomes negative for all $z_I \in (1/A, 1)$ when the ratio θ becomes sufficiently small. As a result, there exists no equilibrium that exhibits price dispersion when $\theta < \underline{\theta}$. Finally, $\underline{\theta} < \bar{\theta}$ because the right-hand side of (A.70) is strictly increasing in θ.

Equilibria in which the central bank's deposit facility is inactive require $z_I \leq z_d$ and $\bar{d} = 0$. The equality condition reduces equation (A.70) to

$$\frac{\theta}{1-\theta} = \frac{H(z_I)}{G(z_I)}. \tag{A.73}$$

There is always a unique $\theta \in (0, 1)$ that solves the above equation as the mapping $\theta \to \frac{\theta}{1-\theta}$ is a bijection between $(0, 1)$ and $(0, +\infty)$. I can further restrict the condition regarding the ratio to $\theta \in [\underline{\theta}, \bar{\theta}]$, as I only focus on equilibria with price dispersion that limits z_I in the interval $(1/A, 1)$. To conclude, there exists $\theta \in [\underline{\theta}, \bar{\theta}]$ and $z_d \geq z_I$ that can support an equilibrium with an inactive central bank deposit facility. $\square$

A.11 Proof of Proposition 4

Inactive Central Bank Deposit Facility When the central bank's deposit facility is inactive, the inter-dealer price, z_I, is endogenously determined by equation (42) with $\bar{d} = 0$. An increase in price z_d has no impact on z_I and does not change F_B and F_L, as these distributions are directly related to the inter-dealer price z_I.

Active Central Bank Deposit Facility When the central bank's deposit facility is active, the inter-dealer price is determined by the deposit facility price, such that $z_I = z_d$. Plugging in $z_I = z_d$ and totally differentiating equation (48) with respect to z_d give

$$\frac{\alpha}{2(1-\alpha)} \frac{\frac{\mathrm{dz_B^q}}{dz_d}\left(z_d - z_B^q\right) - \left(z_B^q - \frac{1}{A}\right)\left(1 - \frac{\mathrm{dz_B^q}}{dz_d}\right)}{\left(z_d - z_B^q\right)^2} = 0 \tag{A.74}$$

$$\frac{\mathrm{dz_B^q}}{dz_d}\left(z_d - z_B^q\right) - \left(z_B^q - \frac{1}{A}\right) + \frac{\mathrm{dz_B^q}}{dz_d}\left(z_B^q - \frac{1}{A}\right) = 0 \tag{A.75}$$

$$\frac{\mathrm{dz_B^q}}{dz_d}\left(z_d - \frac{1}{A}\right) = \left(z_B^q - \frac{1}{A}\right). \tag{A.76}$$

The last equation solves

$$0 \le \eta_B^q \equiv \frac{\mathrm{dz_B^q}}{dz_d} = \frac{z_B^q - \frac{1}{A}}{z_d - \frac{1}{A}} < 1, \tag{A.77}$$

where η_B^q captures the effectiveness of monetary policy pass-through. The first inequality holds with strictly inequality unless $q = 0$ and $z_B^0 = 1/A$, and the second inequality holds for any $q \in [0,1]$ because, from (33), $z_d = z_I > \bar{z}_B > z_B^q$.

Similarly, plugging in $z_I = z_d$ and totally differentiating equation (49) with respect to z_d give

$$-\frac{\alpha}{2(1-\alpha)} \frac{1}{\left(z_L^q - z_d\right)^2}\left[\left(1 - z_L^q - z_d \frac{\mathrm{dz_L^q}}{dz_d}\right)\left(z_L^q - z_d\right) - z_d\left(1 - z_L^q\right)\left(\frac{\mathrm{dz_L^q}}{dz_d} - 1\right)\right] = 0, \tag{A.78}$$

which solves

$$\eta_L^q \equiv \frac{\mathrm{dz_L^q}}{dz_d} = \frac{z_L^q\left(1 - z_L^q\right)}{z_d\left(1 - z_d\right)} \ge 0, \tag{A.79}$$

with equality only if $q = 1$ and $z_L^1 = 1$. Again, η_L^q captures the effectiveness of monetary policy pass-through, and the pass-through is imperfect if

$$\eta_L^q \equiv \frac{z_L^q\left(1 - z_L^q\right)}{z_d\left(1 - z_d\right)} < 1 \longleftrightarrow \left(z_d + z_L^q - 1\right)\left(z_d - z_L^q\right) < 0. \tag{A.80}$$

The fact that $z_L^q > z_I = z_d$ holds for all $q \in [0,1]$ helps to reduce the last condition to

$$z_L^q > 1 - z_d. \tag{A.81}$$

Therefore, the monetary policy pass-through is imperfect for $z_L \in (1-z_d, 1]$. In particular, when $z_d \geq 1/2$, the last inequality is always satisfied. $\square$

A.12 Proof of Corollary 1

The proof for the case when the central bank's deposit facility is inactive is trivial because, as discussed in Proposition 4, an increase in z_b does not change z_I, thereby having no impact on F_B and F_L. This further implies that

$$\frac{\mathrm{d}\mu_B}{\mathrm{d}z_d} = \frac{\mathrm{d}\mu_L}{\mathrm{d}z_d} = 0. \tag{A.82}$$

Confine attention to the case when the central bank's deposit facility is active so that $z_I = z_d$. Start from the mean repo borrowing price μ_B. From the definition of the quantile function (48),

$$z_B^q = \frac{1}{\frac{\alpha}{2(1-\alpha)} + q}\left[q z_d + \frac{\alpha}{2(1-\alpha)}\frac{1}{A}\right], \tag{A.83}$$

which is integrable on $(0, 1)$. This implies that

$$\mu_B = \int_0^1 z_B^q \mathrm{d}q. \tag{A.84}$$

Therefore,

$$\frac{\mathrm{d}\mu_B}{\mathrm{d}z_d} = \int_0^1 \frac{\mathrm{d}z_B^q}{\mathrm{d}z_d}\mathrm{d}q < 1 \quad \text{given that} \quad \forall q \in [0,1]\,,\ \eta_B^q \equiv \frac{\mathrm{dz_B^q}}{dz_d} < 1, \tag{A.85}$$

no matter whether the central bank's deposit facility is active ($0 \leq \eta_B^q < 1$) or not ($\eta_B^q = 0$) Following the same procedure, I can show

$$\frac{\mathrm{d}\mu_L}{\mathrm{d}z_d} = \int_0^1 \frac{\mathrm{d}z_L^q}{\mathrm{d}z_d}\mathrm{d}q < 1 \tag{A.86}$$

holds under the condition $z_d \geq \frac{1}{2}$. $\square$

A.13 Proof of Lemma 8

I focus on equilibria with an active deposit facility with $z_I = z_d$. From the definition of the quantile function (48),

$$z_B^q \;=\; \frac{1}{\frac{\alpha}{2(1-\alpha)} + q}\left[q z_d + \frac{\alpha}{2\,(1-\alpha)}\frac{1}{A}\right]. \tag{A.87}$$

Plugging the result into $\eta_B^q = \frac{z_B^q - \frac{1}{A}}{z_d - \frac{1}{A}}$ gives

$$\eta_B^q \;=\; \frac{q\left(z_d - \frac{1}{A}\right)}{\left(\frac{\alpha}{2(1-\alpha)} + q\right)\left(z_d - \frac{1}{A}\right)} \;=\; \frac{2\,(1-\alpha)\,q}{\alpha + 2\,(1-\alpha)\,q}. \tag{A.88}$$

An increase in $\alpha \in (0,1)$ means a larger search friction and implies the following relation

$$\frac{\mathrm{d}\eta_B^q}{\mathrm{d}\alpha} \;=\; -\frac{2q}{\left[\alpha + 2q\,(1-\alpha)\right]^2} \leq 0, \tag{A.89}$$

where the equality holds only if $q = 0$. Moreover, for all $q \in [0,1]$, $\eta_B^q \longrightarrow 1$ when $\alpha \longrightarrow 0$. By contrast, $\eta_B^q \longrightarrow 0$ when $\alpha \longrightarrow 1$.

Similarly, from the definition of the quantile function (49),

$$z_L^q \;=\; \frac{\frac{\alpha}{2(1-\alpha)} + (1-q)}{\frac{\alpha}{2(1-\alpha)} z_d + (1-q)} z_d. \tag{A.90}$$

Plugging the result into $\eta_L^q = \frac{z_L^q\left(1 - z_L^q\right)}{z_d(1-z_d)}$ gives

$$\eta_L^q \;=\; \frac{\left[\frac{\alpha}{2(1-\alpha)} + (1-q)\right](1-q)}{\left[\frac{\alpha}{2(1-\alpha)} z_d + (1-q)\right]^2} \;=\; \frac{\left[2\alpha\,(1-\alpha) + 4\,(1-\alpha)^2\,(1-q)\right](1-q)}{\left[\alpha z_d + 2\,(1-\alpha)\,(1-q)\right]^2}. \tag{A.91}$$

Taking the first order derivative with respect to $t = \frac{\alpha}{2(1-\alpha)}$ gives

$$\frac{\mathrm{d}\eta_L^q}{\mathrm{d}t} \;=\; \frac{-t\,(1-q)\,z_d + (1-q)^2\,(1-2z_d)}{\left[t z_d + (1-q)\right]^3} \leq 0 \quad \text{when} \quad z_d \geq \frac{1}{2}. \tag{A.92}$$

Therefore, as an immediate result of the chain rule,

$$\frac{\mathrm{d}\eta_L^q}{\mathrm{d}\alpha} \;=\; \frac{\mathrm{d}\eta_L^q}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}\alpha} \leq 0, \tag{A.93}$$

with the equality holds only if $q = 1$ and $z_d = 1/2$. Again, for all $q \in [0,1]$, $\eta_L^q \longrightarrow 1$ when $\alpha \longrightarrow 0$. By contrast, $\eta_L^q \longrightarrow 0$ when $\alpha \longrightarrow 1$. $\square$

A.14 Proof of Proposition 5

When the central bank's deposit facility is inactive, an increase in the deposit facility price does not change OTC repo prices, implying $\frac{\mathrm{d}\eta_{\mathrm{B}}^{\mathrm{q}}}{dz_d} = \frac{\mathrm{d}\eta_{\mathrm{L}}^{\mathrm{q}}}{dz_d} = 0$. When this deposit facility is active, $z_I = z_d$. However, an increase in z_d also has no impact on η_B^q, given that

$$\frac{\mathrm{d}\eta_{\mathrm{B}}^{\mathrm{q}}}{dz_d} = \frac{\frac{\mathrm{dz}_{\mathrm{B}}^{\mathrm{q}}}{dz_d}\left(z_d - \frac{1}{A}\right) - \left(z_B^q - \frac{1}{A}\right)}{\left(z_d - \frac{1}{A}\right)^2} = \frac{\left(z_B^q - \frac{1}{A}\right) - \left(z_B^q - \frac{1}{A}\right)}{\left(z_d - \frac{1}{A}\right)^2} = 0. \tag{A.94}$$

However, an increase in z_d now reduces the effectiveness of monetary policy pass-through because

$$\frac{\mathrm{d}\eta_{\mathrm{L}}^{\mathrm{q}}}{dz_d} = \frac{z_d\left(1-z_d\right)\left(1-2z_L^q\right)\frac{\mathrm{dz}_{\mathrm{L}}^{\mathrm{q}}}{dz_d} - z_L^q\left(1-z_L^q\right)\left(1-2z_d\right)}{\left[z_d\left(1-z_d\right)\right]^2} \tag{A.95}$$

$$= \frac{\left(1-2z_L^q\right)z_L^q\left(1-z_L^q\right) - z_L^q\left(1-z_L^q\right)\left(1-2z_d\right)}{\left[z_d\left(1-z_d\right)\right]^2} \tag{A.96}$$

$$= -\frac{2z_L^q\left(1-z_L^q\right)\left(z_L^q - z_d\right)}{\left[z_d\left(1-z_d\right)\right]^2} \leq 0, \tag{A.97}$$

given that $z_L^q > z_I = z_d$. The equality holds only if $z_L^q = 1$. $\square$

A.15 Proof of Lemma 9

Consider the case when the central bank's deposit facility is active so that $z_I = z_d$. Applying L'Hôpital's rule to equation (43), I obtain

$$\begin{aligned}\lim_{\alpha\to 0}\ \mu_B &= \lim_{\alpha\to 0}\ z_d + \frac{\alpha}{2\left(1-\alpha\right)}\ln\left(\frac{\alpha}{2-\alpha}\right)\left(z_d - \frac{1}{A}\right)\\ &= \lim_{\alpha\to 0}\ z_d - \frac{\alpha}{2-\alpha}\left(z_d - \frac{1}{A}\right) = z_d;\end{aligned} \tag{A.98}$$

$$\begin{aligned}\lim_{\alpha\to 1}\ \mu_B &= \lim_{\alpha\to 1}\ z_d + \frac{\alpha}{2\left(1-\alpha\right)}\ln\left(\frac{\alpha}{2-\alpha}\right)\left(z_d - \frac{1}{A}\right)\\ &= \lim_{\alpha\to 1}\ z_d - \frac{\alpha}{2-\alpha}\left(z_d - \frac{1}{A}\right) = \frac{1}{A}.\end{aligned} \tag{A.99}$$

From (44) and (45),

$$z_b \;=\; \frac{\rho A\left(\left(1-\alpha\right)z_d+\frac{\alpha}{A}\right)+1-\rho}{\rho A+\left(1-\rho\right)\left(\alpha+\frac{1-\alpha}{z_d}\right)}. \tag{A.100}$$

As a result,

$$\lim_{\alpha\to 0} z_b \;=\; z_d \quad \lim_{\alpha\to 1} z_b \;=\; \frac{1}{\rho A+1-\rho} \tag{A.101}$$

In either case, the price ratio is constant in z_d, so that

$$\frac{\mathrm{d}\left(\mu_B/z_b\right)}{\mathrm{d}z_d}=0. \tag{A.102}$$

□

A.16 Proof of Proposition 6

Borrowing Price Distribution (F_B) I finish this proof with the same procedure as in the proof of Proposition 1. From (58), the monopoly price for borrower dealer now becomes $z_B = z_r$, which gives a positive monopoly profit when $z_I > z_r$. Under $z_I > z_r$, I can also show that, after introducing the central bank's lending facility,

1. F_B is continuous on its support $\mathcal{S}_B$;
2. The monopoly price $z_B = z_r$ is the lowest price in $\mathcal{S}_B$;
3. $\mathcal{S}_B$ is connected.

Borrower dealers' total profit can be rewritten as

$$\Pi_B^\star \;=\; \Pi_B\left(z_B\right) \;=\; \frac{\rho}{s}\left[\,\alpha+2\left(1-\alpha\right)F_B\left(z_B\right)\right]R_B\left(z_B\right), \tag{A.103}$$

which is maximized at the monopoly price z_r so that

$$\Pi_B^\star \;=\; \frac{\rho}{s}\alpha R_B\left(z_r\right). \tag{A.104}$$

By equal profit among prices in the connected support, I obtain

$$\alpha R_B\left(z_r\right) \;=\; \left[\,\alpha+2\left(1-\alpha\right)F_B\left(z_B\right)\right]R_B\left(z_B\right), \tag{A.105}$$

which solves

$$F_B(z_B) = \frac{\alpha}{2(1-\alpha)}\left(\frac{R_B(z_r)}{R_B(z_B)} - 1\right) \tag{A.106}$$

$$= \frac{\alpha}{2(1-\alpha)}\left(\frac{z_B - z_r}{z_I - z_B}\right) \quad \forall z_B \in \mathcal{S}_B. \tag{A.107}$$

Moreover, the upper bound $\bar{z}_B$ solves

$$\bar{z}_B = \left(1 - \frac{\alpha}{2-\alpha}\right) z_I + \frac{\alpha}{2-\alpha} z_r. \tag{A.108}$$

given that $F_B(\bar{z}_B) = 1$.

Lending Price Distribution (F_L) Again, this proof follows the same procedure as in the proof of Proposition 2. From (59), the monopoly price for lender dealer now becomes $z_L = z_o$, which gives a positive monopoly profit if $z_I < z_o$. Under $z_I < z_o$, I can show that, after introducing the central bank's deposit facility,

1. F_L is continuous on its support $\mathcal{S}_L$;
2. The monopoly price $z_B = z_o$ is the highest price in $\mathcal{S}_L$;
3. $\mathcal{S}_L$ is connected.

Lender dealers' total profit can be rewritten as

$$\Pi_L^\star = \Pi_L(z_L) = \frac{1-\rho}{s}\left[\alpha + 2(1-\alpha)(1 - F_L(z_L))\right] R_L(z_L), \tag{A.109}$$

which is maximized at the monopoly price z_o, so that

$$\Pi_L^\star = \frac{1-\rho}{s}\alpha R_L(z_o). \tag{A.110}$$

By equal profit among prices in the connected support, I obtain

$$\alpha R_L(z_o) = \left[\alpha + 2(1-\alpha)(1 - F_L(z_L))\right] R_L(z_L), \tag{A.111}$$

which solves

$$F_L(z_L) = 1 - \frac{\alpha}{2(1-\alpha)}\left(\frac{R_L(z_o)}{R_L(z_L)} - 1\right) \tag{A.112}$$

$$= 1 - \frac{\alpha}{2(1-\alpha)}\frac{1/z_L - 1/z_o}{1/z_I - 1/z_L} \quad \forall z_L \in \mathcal{S}_L. \tag{A.113}$$

Moreover, the lower bound $\underline{z}_L$ solves $F_L(\underline{z}_L) = 0$, so that

$$\underline{z}_L = \left[\left(1 - \frac{\alpha}{2-\alpha}\right)\frac{1}{z_I} + \frac{\alpha}{2-\alpha}\frac{1}{z_o}\right]^{-1}. \tag{A.114}$$

□

A.17 Proof of Proposition 7

When the central bank's deposit facility is active, $z_I = z_d$. Then, from (62), and (64), the price ratio

$$\frac{\mu_B}{z_b} = \frac{\pi}{\beta}\frac{z_d + \frac{\alpha}{2(1-\alpha)}\ln\left(\frac{\alpha}{2-\alpha}\right)(z_d - z_r)}{\rho A(\alpha z_r + (1-\alpha)z_d) + 1 - \rho}, \tag{A.115}$$

where, as in (63),

$$\pi = \beta\left[\rho A + (1-\rho)\left(\alpha\frac{1}{z_o} + (1-\alpha)\frac{1}{z_d}\right)\right]. \tag{A.116}$$

Lending Facility Price (z_r) I first study the effects of an increase in the lending facility price z_r, which does not change inflation, i.e.,

$$\frac{\mathrm{d}\pi}{\mathrm{d}z_r} = 0, \tag{A.117}$$

given that the gross inflation rate is constant in z_r. However, it still changes the price ratio μ_B/z_b, and I obtain

$$\frac{\mathrm{d}(\mu_B/z_b)}{\mathrm{d}z_r} = -\frac{\pi}{\beta}\frac{\left[\frac{\alpha}{2(1-\alpha)}\ln\left(\frac{\alpha}{2-\alpha}\right) + \alpha\right]\rho A z_d + (1-\rho)\frac{\alpha}{2(1-\alpha)}\ln\left(\frac{\alpha}{2-\alpha}\right)}{\left[\rho A(\alpha z_r + (1-\alpha)z_d) + 1 - \rho\right]^2}, \tag{A.118}$$

which is positive if

$$\frac{\alpha}{2\left(1-\alpha\right)}\ln\left(\frac{\alpha}{2-\alpha}\right)+\alpha<0\longleftrightarrow\ln\left(\frac{\alpha}{2-\alpha}\right)+2\left(1-\alpha\right)<0. \tag{A.119}$$

Let

$$J\left(\alpha\right) \equiv \ln\left(\frac{\alpha}{2-\alpha}\right)+2\left(1-\alpha\right), \tag{A.120}$$

and the key is to show that $\forall\alpha\in(0,1)\;\; J\left(\alpha\right)<0$. Note that

$$J''\left(\alpha\right) = \frac{4\left(\alpha-1\right)}{\left(2-\alpha\right)^{2}\alpha^{2}}<0. \tag{A.121}$$

Therefore,

$$J'\left(\alpha\right) = \frac{1}{\alpha}+\frac{1}{2-\alpha}-2 \tag{A.122}$$

is strictly decreasing in $\alpha\in(0,1)$. Consequently,

$$\forall\alpha\in(0,1)\quad J'\left(\alpha\right) > J'\left(1\right) = 0. \tag{A.123}$$

This further implies that $J\left(\alpha\right)$ is strictly increasing in $\alpha\in(0,1)$, so that

$$\forall\alpha\in(0,1)\quad J\left(\alpha\right) < J\left(1\right) = 0. \tag{A.124}$$

To conclude,

$$\forall\alpha\in(0,1)\quad \frac{\mathrm{d}\left(\mu_B/z_b\right)}{\mathrm{d}z_r} > 0. \tag{A.125}$$

Borrowing Facility Price (z_o) Taking the same procedure as before to study an increase in the lending facility price, I obtain

$$\frac{\mathrm{d}\left(\mu_B/z_b\right)}{\mathrm{d}z_o} = \frac{1}{\beta}\frac{z_d+\frac{\alpha}{2(1-\alpha)}\ln\left(\frac{\alpha}{2-\alpha}\right)\left(z_d-z_r\right)}{\rho A\left(\alpha z_r+\left(1-\alpha\right)z_d\right)+1-\rho}\frac{\mathrm{d}\pi}{\mathrm{d}z_o} < 0, \tag{A.126}$$

given that

$$\frac{\mathrm{d}\pi}{\mathrm{d}z_o} < 0. \tag{A.127}$$

□

A.18 Proof of Proposition 8

Lending Facility Price (z_r) The effects of an increase in the central bank's lending facility price on welfare are given by

$$\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}z_r} = \frac{\beta\rho\,(A-1)}{\pi}\left(-\frac{1}{\pi}\left[\hat{m}+(g-\hat{m})\,\frac{\mu_B}{z_b}\right]\frac{\mathrm{d}\pi}{\mathrm{d}z_r}+(g-\hat{m})\,\frac{\mathrm{d}\,(\mu_B/z_b)}{\mathrm{d}z_r}\right). \tag{A.128}$$

Plugging the following results from Lemma 7,

$$\frac{\mathrm{d}\pi}{\mathrm{d}z_r} = 0, \quad \text{and} \quad \frac{\mathrm{d}\,(\mu_B/z_b)}{\mathrm{d}z_r} > 0, \tag{A.129}$$

into equation (A.128) gives

$$\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}z_r} = \frac{\beta\rho\,(A-1)\,(g-\hat{m})}{\pi}\frac{\mathrm{d}\,(\mu_B/z_b)}{\mathrm{d}z_r} > 0. \tag{A.130}$$

Borrowing Facility Price (z_o) Rewrite the welfare function (65) as

$$\mathcal{W} = \frac{\beta\rho\,(A-1)}{\pi}\left[\hat{m}+(g-\hat{m})\,\frac{\mu_B}{z_b}\right] \tag{A.131}$$

$$= \beta\rho\,(A-1)\left[\hat{m}\frac{1}{\pi}+(g-\hat{m})\,\frac{z_d+\frac{\alpha}{2(1-\alpha)}\ln\left(\frac{\alpha}{2-\alpha}\right)(z_d-z_r)}{\beta\left[\rho A\,(\alpha z_r+(1-\alpha)\,z_d)+1-\rho\right]}\right]. \tag{A.132}$$

Therefore,

$$\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}z_o} = -\frac{\beta\rho\,(A-1)\,\hat{m}}{\pi^2}\frac{\mathrm{d}\pi}{\mathrm{d}z_o} > 0, \tag{A.133}$$

given that, from Lemma 7, a higher borrowing facility price reduces the gross inflation rate, π. □

A.19 Proof of Proposition 9

Totally differentiating conditions (69) to (71) with respect to z_d, I obtain

$$\frac{\mathrm{d}\left(\mu_B/z_b\right)}{\mathrm{d}z_d} = \frac{\alpha\left(1-\rho\right)\left(\rho A+1-\rho\right)}{\left(\rho A z_d+1-\rho\right)^2} > 0, \tag{A.134}$$

$$\frac{\mathrm{d}\pi}{\mathrm{d}z_d} = -\frac{\beta\left(1-\rho\right)\left(1-\alpha\right)}{\left(z_d\right)^2} < 0. \tag{A.135}$$

Plugging these results in (66) gives

$$\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}z_d} > 0. \tag{A.136}$$