

## **An optimal investment strategy that combines seasonal time series forecasting models and goal planning model**

In the decision of the investment portfolio, the rate of return of various investable assets and the rate of transaction will have a key impact on the strategy of the investment portfolio. In this paper, we build a model to **predict the price trend** of gold and Bitcoin, and analyze the impact of the rate of return on investable assets and rates on trading decisions, measure the pros and cons of investment portfolios, and explore the impact of changes in transaction costs on the rate of return. Models perform **sensitivity analysis**, reveal **optimal portfolio strategies**, and perform risk assessments on portfolios.

In question 1, we take the investment cost as a node, and comprehensively consider the factors of yield and transaction rate to calculate the investment weight of investable assets. The asset price of Bitcoin is highly volatile, so **XGBoost** regression integrated with genetic algorithm is selected for prediction to prepare for the next decision; due to the low volatility of gold price and research shows that it is seasonal, we use seasonal difference an autoregressive moving average model(**SARIMA**) and **GA-BP neural network** forecasts its five-year price. Based on this, we calculate daily returns in addition to gold and bitcoin. Finally, the value of the initial investment of \$1,000 by September 10, 2021 is calculated through the established goal planning model.

In question 2, we first conduct **sensitivity analysis** on the decision model, and the conclusion shows that the model still performs well under the changing transaction cost. After that, we selected several specific dates, combined with **Modern Portfolio Theory**, and selected five time points to conduct **Monte Carlo simulations** to obtain the effective frontier of the portfolio at different decision times. It is proved that our decision model can maximize the return under the same risk.

In question 3, we run the established **target planning model** after adjusting the transaction cost several times to obtain the final capital holdings under different transaction costs, and calculate the **total return**. By controlling the variables, establish two-dimensional curve graphs of gold transaction cost and Bitcoin transaction cost and total income respectively, and judge the impact of transaction cost on investment portfolio strategy by observing the changing trend of the two-dimensional curve graph; observe the final total income to judge The result; draw a tangent to the curve, calculate the slope, and plot it to determine how **sensitive** the investment strategy we've built is to transaction costs.

In establishing the **investment strategy model**, we fully consider the investment risks while pursuing the greatest possible benefits, and add **risk assessment** to the target planning model to avoid uncontrollable risks as much as possible. Such a risk assessment system can ensure that market traders can achieve the greatest possible return on investment with low risk. It is believed that the establishment of our model will greatly help market traders make better decisions when investing.

**Keywords:** SARIMA ; XGboost Regression Predictions ; Goal Programming ; Modern Portfolio Theory

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# 1 Introduction

## 1.1 Problem Background

Gold and Bitcoin are common volatile assets. Bitcoin has high yield, high volatility, and is easy to circulate and store; now people all over the world agree with the circulation value of gold, and its relatively stable value and good hedging characteristics. Research shows that during 2011-2021, the price of gold will be Fluctuations between \$1,100/oz and \$2,000/oz, although mostly up, no long-term trend found: Bitcoin price rose from less than \$100 to more than \$50,000 with a clear and volatile uptrend. The price trends of these two volatile assets are very different, an increase in underlying volatility will bring more risk, and all investors want to maximize their expected return. With a certain initial capital, how to predict future price changes and make portfolio investments based on previous gold and bitcoin prices is a topic worthy of study and has important practical significance.

## 1.2 Restatement of the Problem

Volatility varies widely across assets, and maximizing total returns is the ultimate goal of market traders.

Considering the background information and restricted conditions identified in the problem statement, we need to solve the following problems:

- Problem 1

In order to improve investment returns and achieve accurate decision-making, we predict the future prices of Bitcoin and gold, and calculate the predicted yield. The appendix provides the opening prices of gold and bitcoin, and we supplement a small amount of missing data according to the principle of data integrity. If gold has not opened, we take the price of the previous open instead.

In the decision of the investment portfolio, the rate of return of various investable assets and the rate of transaction will have a key impact on the strategy of the investment portfolio. Question 1 requires us to develop a decision-making model based on the daily trading price data of gold and bitcoin. The future price of gold and bitcoin is unknown at the time of decision-making, and we cannot use it.

In order to achieve a more accurate forecast of gold and bitcoin prices, we used several different models to predict their prices, taking into account the difference in their price volatility. In this article, we build a model to predict the price trend of gold and bitcoin. The asset price of Bitcoin is highly volatile, and a short-term prediction model needs to be selected. First, we perform case processing on the known data to eliminate outliers, and perform a sliding window transformation with a step size of 5 to obtain five time series variable transformations X1, X2, X3, X4, X5 and time series variable Y, and select the XGBoost regression integrated with the genetic algorithm. Predictions to prepare for the next decision; due to the low price volatility of gold and research showing that it is seasonal, we used a seasonally differential autoregressive moving average model to forecast its five-year price. At the same time, considering that the BP neural network (GABP) optimized by genetic algorithm has strong convergence ability and

generalization ability, it is selected to predict the future price of gold, and the training effects of the two are compared, and the prediction model with better effect is selected to assist The establishment of goal programming model.

Based on the results obtained from the prediction model, we establish a target programming model to set constraints and innovatively introduce risk parameters to constrain the rate of return and rate of investable assets. With full consideration of returns and risks, we finally obtain Calculate the weight of your daily investment. Finally, the pros and cons of the portfolio are measured, and finally, the value of the initial investment of \$1,000 by September 10, 2021 is calculated through the established goal planning model.

#### ● Problem 2

Problem two requires us to prove that the model is the optimal policy, first we perform a sensitivity analysis on the decision model. After that, we selected several specific dates, combined with Modern Portfolio Theory, and selected five time points to conduct Monte Carlo simulations to obtain the effective frontier of the portfolio at different decision times. And compared to the simulated investment of our decision model. The results show that at these five time points, our investment portfolio is on the efficient frontier, which shows that our decision model can maximize the return under the same risk level.

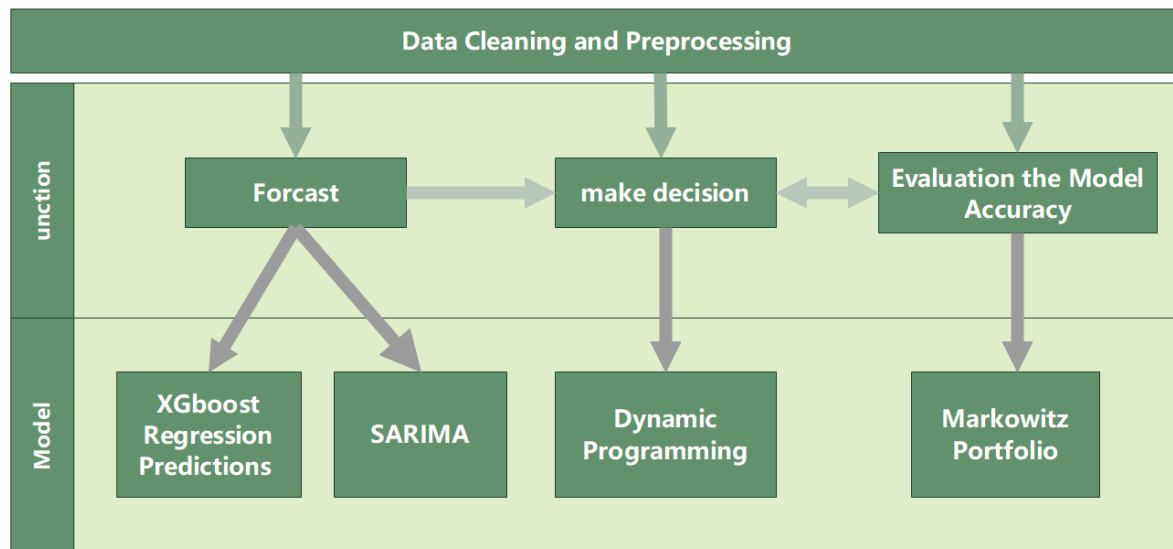
#### ● Problem 3

Question 3 requires us to analyze the sensitivity of the decision model of Question 1 to initial costs and to determine how costs affect our decisions and final outcomes. In this regard, we run the established target planning model after adjusting the transaction costs for many times, obtain the final capital holdings under different transaction costs, and calculate the total return. By controlling the variables, establish two-dimensional curve graphs of gold transaction cost, Bitcoin transaction cost and total income respectively, and judge the impact of transaction cost on investment portfolio strategy by observing the change trend of the two-dimensional curve graph; observe the final total income to judge the result; Draw a tangent to the curve, calculate the slope, and plot it to determine how sensitive the investment strategy we've built is to transaction costs.

#### ● Problem 4

Question 4 asked us to produce a memorandum, and we combined the conclusions of the first three questions to summarize and produce a memorandum.

## 1.3 Our Work



## 2 Assumptions and Justifications

**Assumption1:** Investors seek to maximize returns, and all investors have access to the same source of information and all necessary information related to investment decisions

Justification: According to financial theory, the basic behavioral characteristics of rational investors are risk aversion and the pursuit of maximizing returns. In the process of constructing the model in this paper, the premise of investment behavior that investors pursue the maximization of returns is inherited.

**Assumption2:** The investment portfolio constructed in this paper only considers the opening price on the day when the decision was made and before, ignoring other information such as trading volume and turnover rate, and does not consider the impact of major social events on investors' decision-making.

Justification: Limited by the subject requirements, each of our investment decisions is only affected by the historical opening price on the current day and before. The dataset is five years old and has a long time period, and it is reasonable to ignore the impact of major events on the price data.

**Assumption3:** The model considers buying both bitcoin and gold as investments, but does not take into account the inflation and deflation of cash in real life.

Justification: Both Bitcoin and Gold are measured in US dollars, and it is reasonable to use the US dollar as a fixed value of "1" in this article, thereby ignoring inflation and deflation.

## 3 Notations

The key mathematical notations used in this paper are listed in Table 1.

**Table 1: Notations used in this paper**

Symbol	Description	Unit
N	days	Day
C.I.	confidence interval	/
VaR	value at risk	/
R <sub>t</sub>	rate of return	/
P <sub>i</sub>	real bitcoin price	USD
p <sub>i</sub>	Bitcoin price prediction	USD
Q <sub>i</sub>	real gold price	USD
q <sub>i</sub>	Gold forecast price	USD
X <sub>i</sub>	single cycle expected return	/
x <sub>i</sub>	Bitcoin transaction volume	USD
y <sub>i</sub>	gold trading volume	USD
a <sub>i</sub>	Existing Bitcoin value	USD
b <sub>i</sub>	Existing gold value	USD

## 4 Forecast1:SARIMA

### 4.1 Data Description

The attachment to this question gives the daily closing price of a troy ounce of U.S. gold in dollars from September 11, 2021 to September 9, 2021. By making a line chart of data and dates, we found that the price volatility of Bitcoin is high, the price volatility of gold is low, and the price of gold is partially missing, we use the missing data to replace it with the previous data. . From the data files, we know that Bitcoin can be traded on a daily basis, while gold can only be traded on trading days. In order to estimate the daily gold price within the investment planning time, we obtained the daily trading closing price of gold from the official website of Yingwei Caiqing for five years for the training of the gold price prediction model.

After data preprocessing, the following figure is drawn

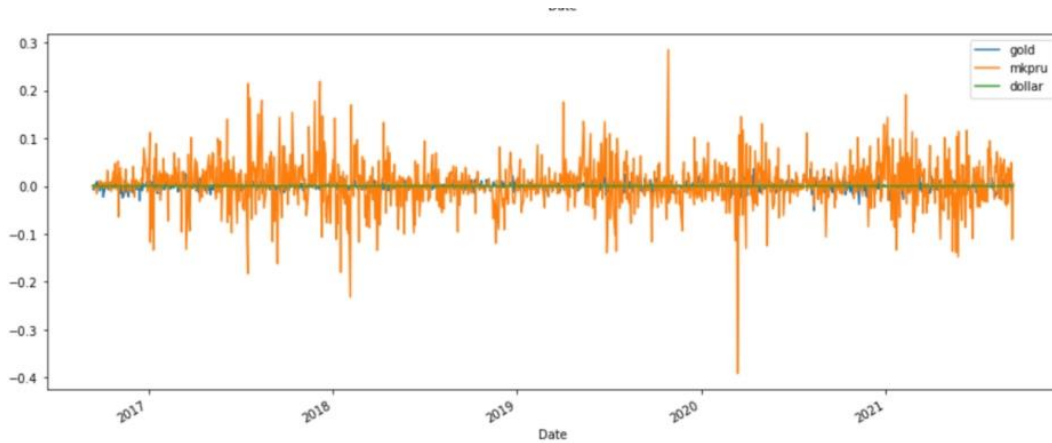


Figure1: Bitcoin and gold opening prices after data preprocessing

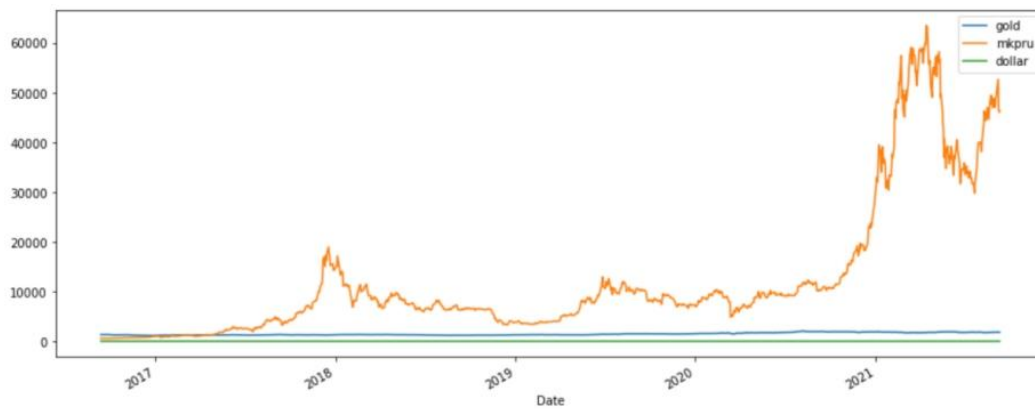


Figure2: Bitcoin and Gold daily returns after data preprocessing

## 4.2 Model building

The SARIMA model (seasonal autoregressive integrated moving average) is useful for modeling seasonal time series, where the average value and other statistics for a given season are not stationary over many years.

The SARIMA model originates from the autoregression unitary moving average model (ARIMA), and can be identified, estimated and predicted by Box-Jenkins model, so it is convenient to adjust the model in real time with the acquisition of more historical data, which can not only ensure the prediction accuracy of the model, but also can be easily applied to real-time prediction.

Common time series mostly have a certain trend, but many series can be stationary through difference. If the element series  $y_i$  is non-stationary and stationary after  $d$ -order step-wise difference, the series  $z_q$  is called a homogeneous series[13]

$$\text{Denoted as } z_t = \nabla^d y_t (t > d) \quad (1)$$

The stationary series  $z$  can build an ARMA( $p, q$ ) model. The original series  $y$  can be expressed as an ARIMA( $p, d, g$  model), denoted as

$$\text{Denoted as } z_t = \nabla^d y_t (t > d) \quad (2)$$

However, some sequences have both trend and seasonality, and seasonal difference must be carried out to eliminate the seasonality of the sequence. If the sequence  $y$  passes through the difference of the D-order period length, the seasonality basically eliminates the new sequence  $w$ , denoted as

$$w_t = \nabla_s^D y_t (t > D_s) \quad (3)$$

If the sequence is still stable after the d-order period-by-period difference is performed before the seasonal difference, the original sequence  $y$  can be compared, establish SARIMA(p,d,q)(P,D,Q)" model, denoted as

$$w_t = \nabla_s^D y_t (t > D_s) \quad (4)$$

here P is the order of seasonal autoregressive and Q is the order of seasonal moving average, and the distributions are called P-order autoregressive operators and Q-order moving average operators

### 4.3 The Results of SARIMA

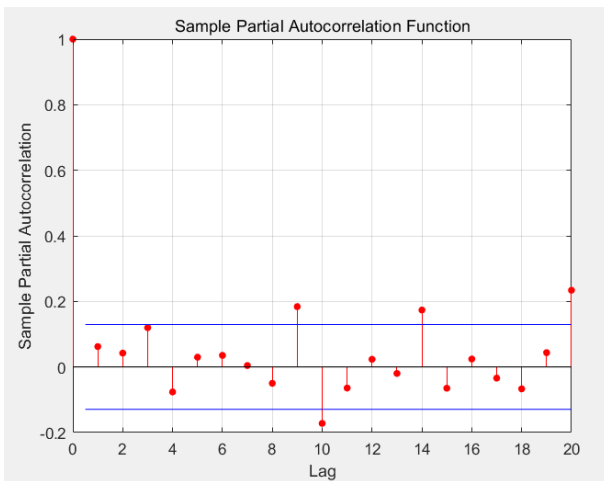


Figure 3

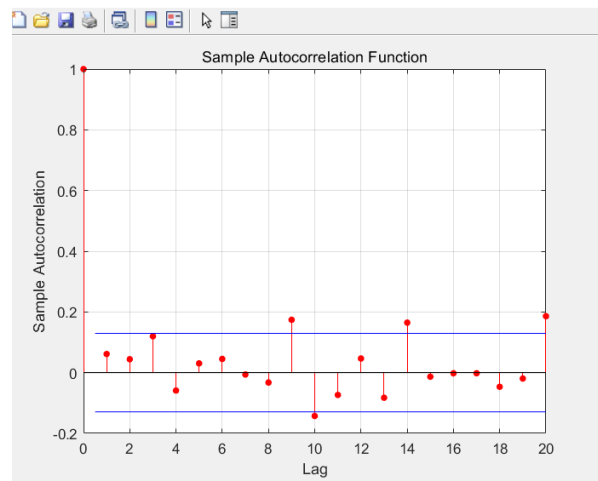


Figure 4

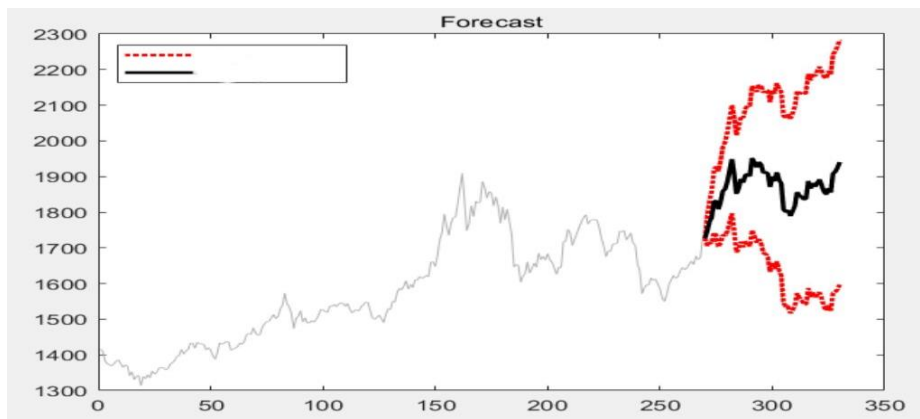


Figure 5: forecast



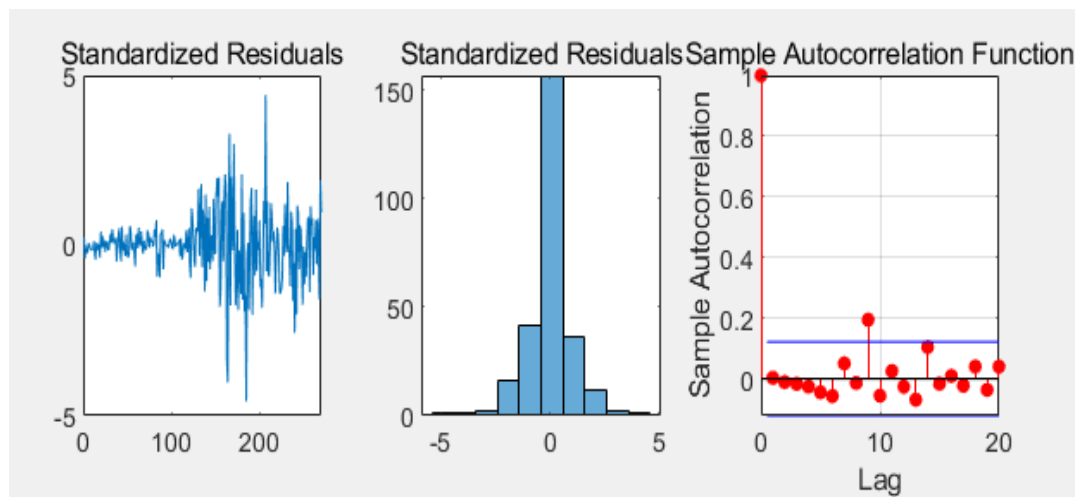


Figure 6: residual test

ARIMA(3, 1, 3) Model Seasonally Integrated with Seasonal AR(90) and MA(90) (Gaussian Distribution):

	Value	StandardError	TStatistic	PValue
Constant	0	0	NaN	NaN
AR (1)	-0.42894	0.71859	-0.59691	0.55057
AR (2)	-0.47638	0.42426	-1.1229	0.2615
AR (3)	0.20176	0.47709	0.4229	0.67237
SAR (30)	-0.53344	0.12748	-4.1846	2.8571e-05
SAR (60)	-0.48938	0.1591	-3.076	0.002098
SAR (90)	-0.76092	0.082337	-9.2415	2.4304e-20
MA (1)	0.3825	0.72615	0.52675	0.59836
MA (2)	0.58702	0.38255	1.5345	0.12491
MA (3)	-0.16106	0.54264	-0.29681	0.76661
SMA (30)	-0.46295	0.12026	-3.8494	0.00011842
SMA (60)	-0.031804	0.18662	-0.17042	0.86468
SMA (90)	0.062678	0.19616	0.31953	0.74933
Variance	454.07	27.618	16.441	9.7581e-61

DWO = 1.9890

Figure 7: Stationarity Test Results

According to research, the price of gold has seasonal characteristics, so SARIMA is used to predict this problem. We train the model on data from January 1, 2011 to September 11, 2016. However, since the SARIMA training method is multi-step forecasting and seasonal forecasting is required, the number of training sets must be at least 5 cycles. The seasonal cycle of gold can be regarded as a year, and a large number of training sets are needed to accurately predict. The number of training sets provided to the model may be too small, resulting in an unsatisfactory training effect of the model. The prediction of the bp neural network optimized by the genetic algorithm The effect is better. Forecast2:GA-BP Neural Network

#### 4.4 Model building

BP (back propagation) neural network is a multilayer feedforward neural network. Genetic algorithm optimizes BP neural network. The elements of algorithm implementation include population initialization, fitness function, selection operation, crossover operation and mutation operation.

Population initialization: individual coding consists of four parts: connection weight

between input layer and hidden layer, hidden layer threshold, connection weight between hidden layer and output layer, and output layer threshold. [12]

$$\text{Fitness function : } F = n \left( \sum_{i=1}^k |y_i - o_i| \right) \quad (5)$$

Where  $n$  is the number of network output layer nodes, the expected output of the  $i$ th node, the predicted output of the  $i$ th node, and  $N$  is the coefficient. (3) Selection operation: the selection probability of each individual is, where  $F_i$  is the fitness value of individual  $I$ ,  $N$  is the coefficient, and  $K$  is the number of individuals in the population.

$$p_i = \left( \frac{n}{F_i} \right) / \sum_{j=1}^K \left( \frac{n}{F_j} \right) \quad (6)$$

Crossover operation: the crossover method of chromosome  $N$  and chromosome  $I$  at position  $J$  is as follows, where  $B$  is the random number in  $[0,1]$

$$\begin{cases} c_{nj} = c_{nj}(1 - b) + c_{lj}b \\ c_{lj} = c_{lj}(1 - b) + c_{nj}b \end{cases} \quad (7)$$

Mutation operation: select the  $j$ th gene of the  $i$ th individual for mutation. The mutation operation method is as follows: where, is the upper bound of the gene, is the lower bound of the gene, is a random number ( $\in [0,1]$ ),  $G$  is the current iteration times and is the maximum evolution times.

$$c_{max} = \begin{cases} c_{ij} + (c_{ij} - c_{max}) \times r_2(1 - g/G_{max}), r \geq 0.5 \\ c_{ij} + (c_{min} - c_{ij}) \times r_2(1 - g/G_{max}), r < 0.5 \end{cases} \quad (8)$$

## 4.5 Model application

The model steps are as follows:

A determine BP model and input training data and prediction data: determine the structure of input layer, hidden layer and output layer, as well as the input and output data and prediction data to be trained.

B normalize the sample data: construct the network after normalizing the sample data.

C constructs the genetic algorithm and initializes the parameters of the genetic algorithm: constructs the information model of genes, chromosomes and population, constructs four genes into a chromosome, defines the population information as a structure, and the population contains two matrices, one is used to put the fitness value of all individuals, the other is used to store the chromosome information, and gives the initial value to each gene.

D generates fitness value for individual: the initial value is a random number

E select in the code: put the individual into the model of the same roulette for selection.

F. implement evolution and selection in the population: compile the cross function and

variation function to obtain the 20 generations of evolution

G training BP network: after evolution, there are clear data of weight threshold, input layer, hidden layer and input layer, and its prediction image for the next three years and the relative error between BP neural network and test data optimized by BP neural network and genetic algorithm are obtained.

We draw the above steps as a flowchart

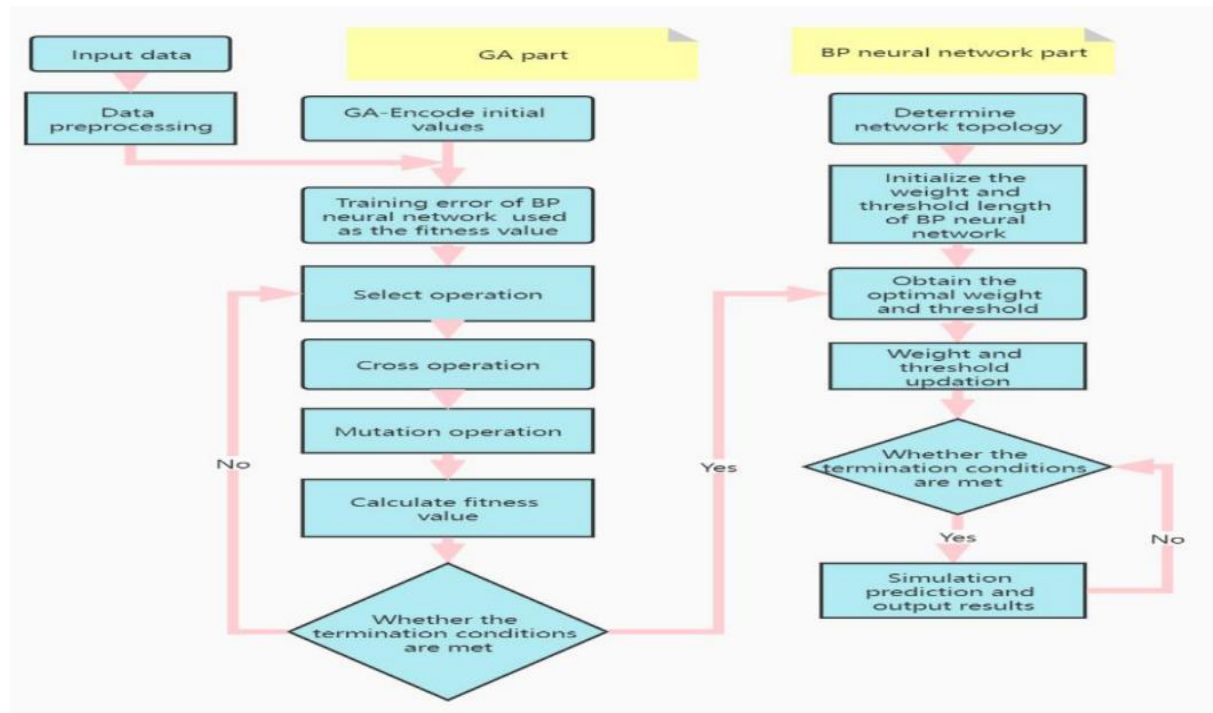


Figure 8

## 4.6 The result of GA-BP

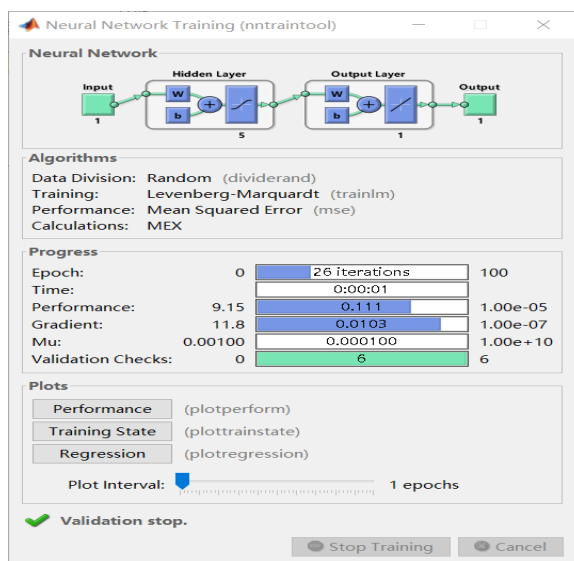


Figure 9:

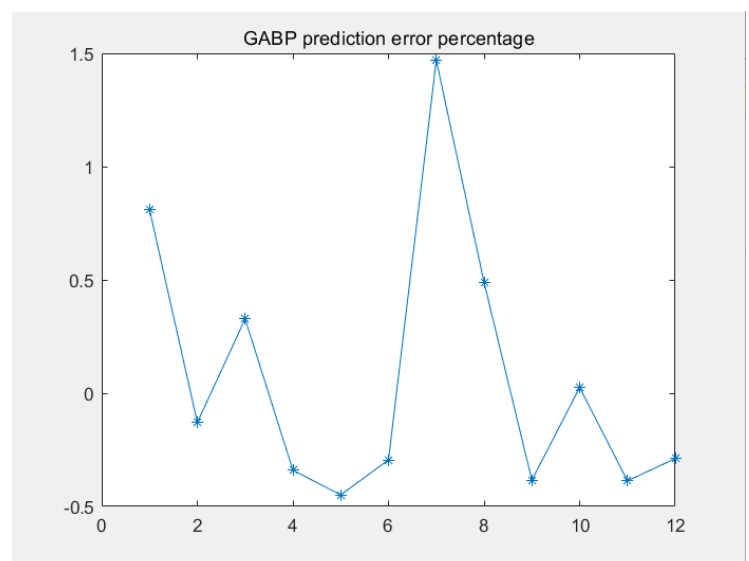


Figure 10:GABP prediction error percentage

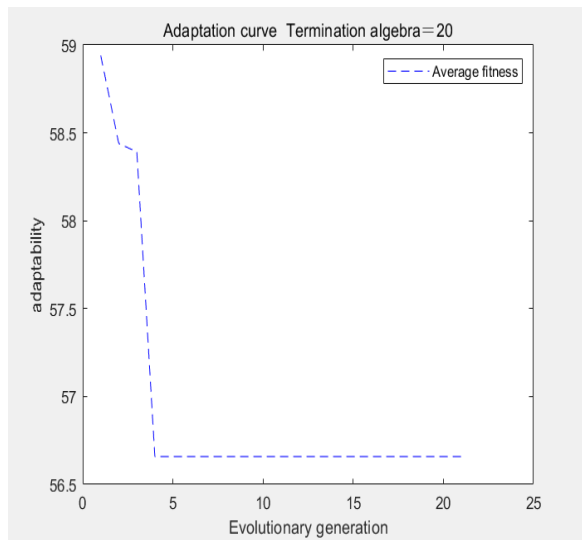


Figure 11

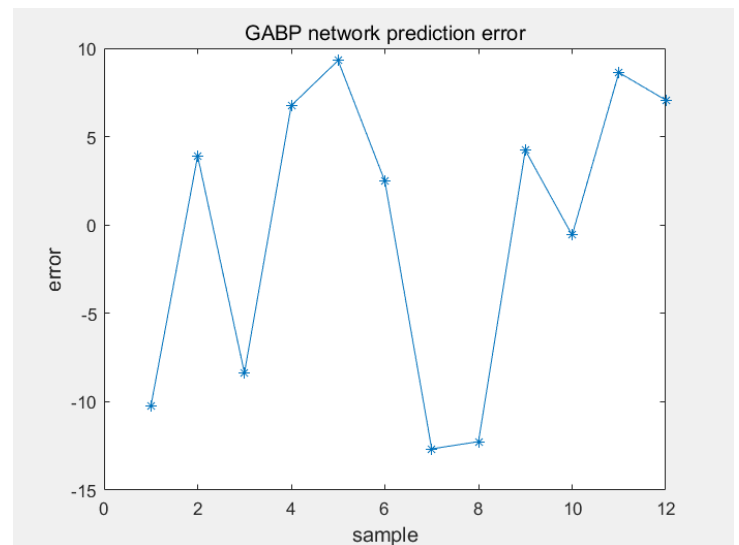


Figure 12

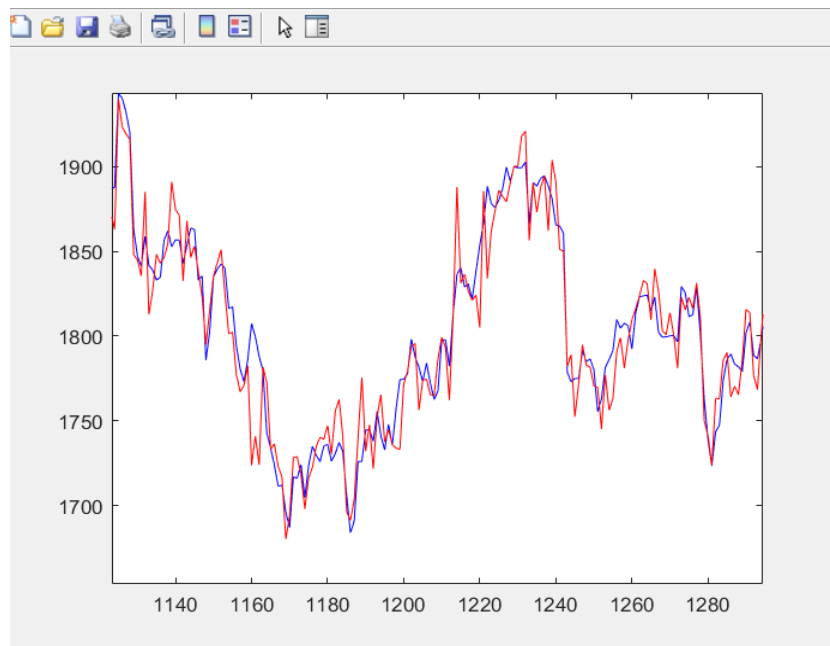


Figure 13: Gold Forecast Results

The BP neural network optimized by the genetic algorithm uses the genetic algorithm to train and evolve the weights originally randomly input to the neural network, which greatly improves the accuracy of the BP neural network and solves the problem of slow convergence speed, and saves the training of the neural network. time. In this paper, the previous data is used as the training set, and the gold price is used as the output stage, so as to predict the subsequent gold price to provide data support for the decision-making model. And the error data obtained by subtracting the predicted data from the actual data to make the error percentage is less than 2%, and the fitting effect is ideal.

## 5 Forecast3: XGboost regression predictions

### 5.1 Data Description

Attached to this question is the daily closing price of one troy ounce of gold in the United States and the daily closing price of one bitcoin. By making a line chart of data and dates, we found that the price volatility of Bitcoin is high, the price volatility of gold is low, and the price of gold is partially missing, we use the missing data to replace it with the previous data. . In order to estimate the daily gold and bitcoin prices during the investment planning time, we obtained the daily trading closing prices of bitcoin and gold from the official website of Yingwei Caiqing in the past ten years due to the training of the price prediction model.

### 5.2 The Establishment of XGboost

Considering the high volatility of Bitcoin price, which is not suitable for long-term forecasting, we adopted the XGBoost regression model integrating genetic algorithm as the optimization algorithm as the forecasting model of Bitcoin price. First, we performed case processing and missing value processing on the data, and then performed sliding window transformation. Finally, we used the XGBoost model integrating genetic algorithm as the optimization algorithm to predict the price of Bitcoin, and the prediction result was relative to the actual price of Bitcoin. The price error is small, so it can be used in decision models.

XGBoost is a tree ensemble model, which uses the sum of the predicted values of  $K$  (total  $K$ ) trees for the sample as the prediction of the corresponding sample in the XGBoost system. First, we use the collected Bitcoin price data of previous years as a training set to train the XGBoost model; secondly, we use the established XGBoost algorithm fused with the genetic algorithm as the optimization algorithm to calculate the importance of eigenvalues; The trained XGBoost model is used to predict the price of Bitcoin for five years and output the model evaluation results; in addition, considering the randomness of the XGBoost model, it means that the training results of the same data may be different. Therefore, our prediction data It can be directly input into the model operation to obtain;

Finally, we verified the accuracy of the predictions by comparing the five-year known actual price of Bitcoin with the predicted price.

We draw the above steps as a flowchart

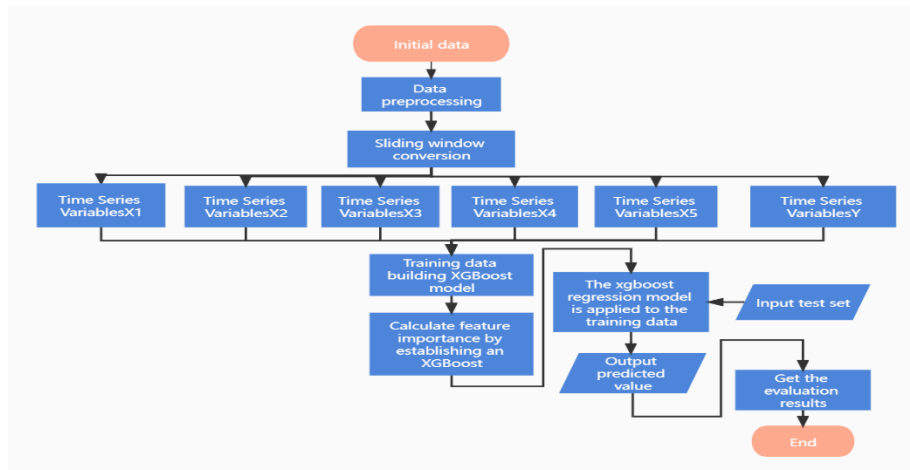


Figure 14: the mind map of XGboost

### 5.3 The Result of XGboost

We used the daily closing price data of Bitcoin in previous years as a training set to build an XGBoost model optimized by a genetic algorithm, and then compared the predicted data with the actual transaction price of Bitcoin during a five-year period (used as a test set), we found that the error rate of the forecast is within 3%, which is very invaluable compared to the large volatility of Bitcoin price, that is, the data we predict can be used for portfolio investment decisions. Through the forecast data obtained by this model, we can calculate the daily rate of return of Bitcoin, provide evidence for whether to buy Bitcoin and the purchase ratio, and finally we can use the actual price of Bitcoin to verify our investment strategy.

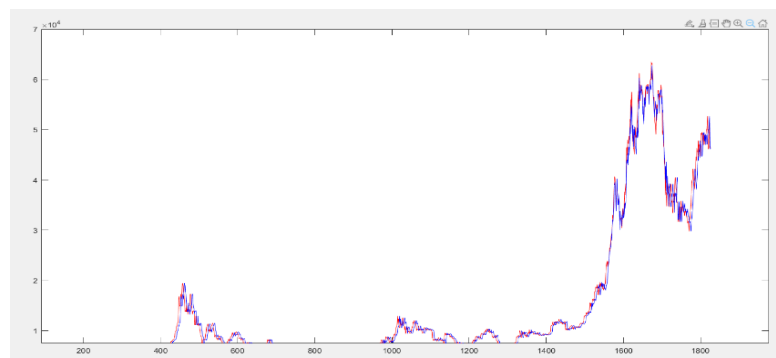


Figure 15: Gold Price Forecast Polyline

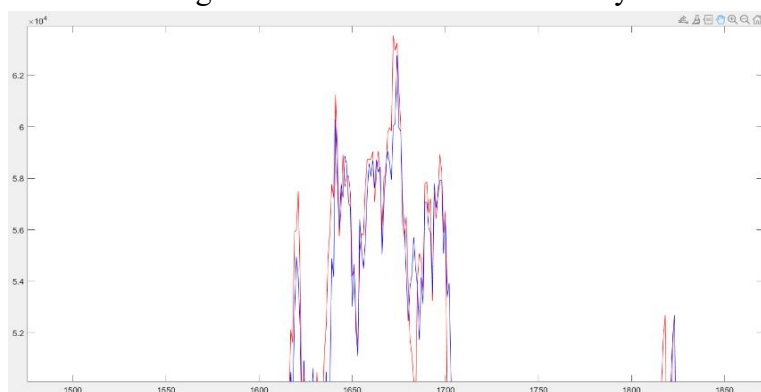


Figure 16: Partial enlargement

## 6 Decision-Making: Dynamic Programming

### 6.1 The Establishment of Dynamic Programming Model

Dynamic Programming

Make a decision and predict the return in a single cycle (the  $i$ th cycle)

$$\max X_i = \frac{p_{1+i}}{P_i} \cdot (a_i + x_i) + \frac{q_{i+1}}{P_i} \cdot (b_i + y_i) \quad (9)$$

$$\left\{ \begin{array}{l} -a_i - a_i < x_i + y_i < a_i + a_i \\ -a_i < x_i < a_i \\ -a_i < x_i < a_i \\ 0.01 \leq \frac{p_{i+1}}{P_i} - 1 \\ 0.02 \leq \frac{q_{i+1}}{P_i} - 1 \\ \left[ \left( \frac{p_{i+1}}{P_i} - 1 \right) - \alpha \right] x_i > 0 \\ \left[ \left( \frac{q_{i+1}}{Q_i} - 1 \right) - \beta \right] y_i > 0 \end{array} \right. \quad (10)$$

real accumulated capital

$$\max X_i' = \frac{P_{1+i}}{P_i} \cdot (a_i + x_i) + \frac{P_{2+i}}{P_i} \cdot (b_i + y_i) \quad (11)$$

### 6.2 The Result of Dynamic Programming Model

We program the above mathematical model to get a decision model. Considering that the potentially volatile market has more risks, combined with the actual investment situation, we limit the above model, that is, during a single cycle of trading, the transaction volume of cash to buy Bitcoin does not exceed the percentage of the cash reserve at that time 60. The transaction volume of cash purchases of gold does not exceed 30% of the cash reserves at that time.

We use the actual data from 2010 to 2016 for learning, and after many iterations, the values of  $\alpha$  and  $\beta$  are 0.03 and 0.02, respectively.

Finally, combine the predicted value of Bitcoin predicted by the XGboost model and the predicted value of gold predicted by the GA-BP model to calculate our dynamic programming decision model. In the case of holding \$1,000 in assets on September 11, 2016, by September 10, 2021, our final return was 368,383 yuan (exact to single digits).

## 7 Sensitivity Analysis

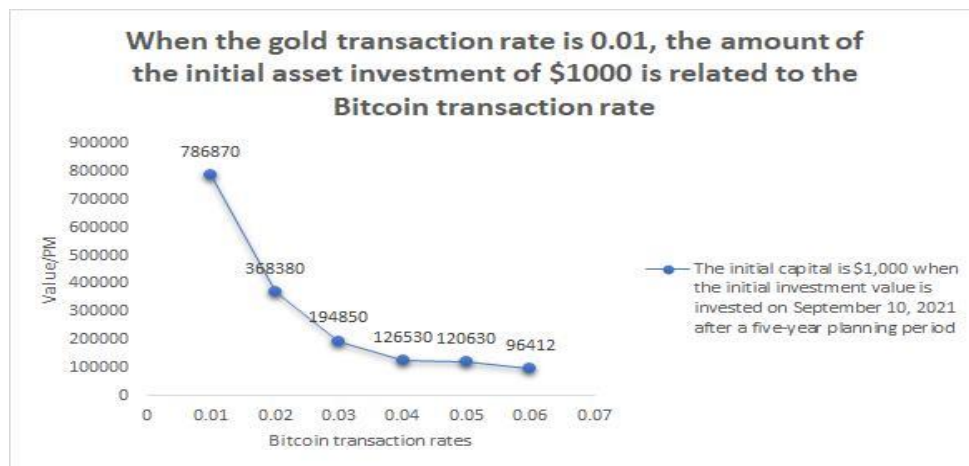


Figure 17

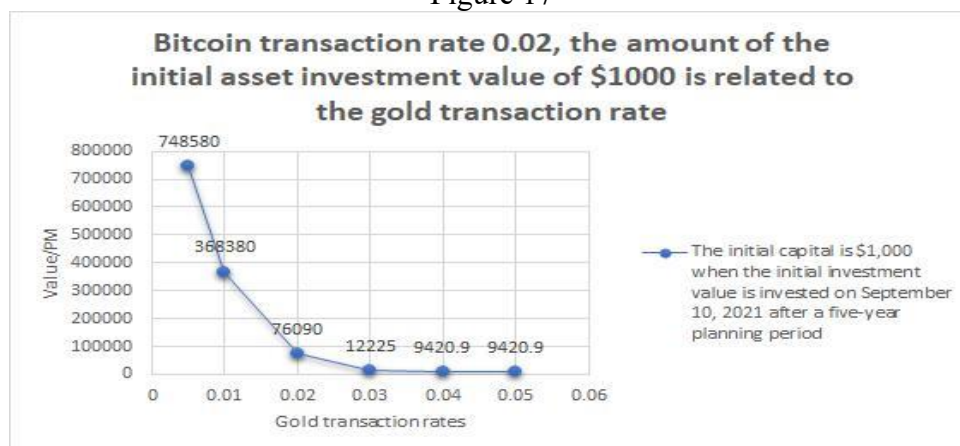


Figure 18

Sensitivity analysis is generally used to study and analyze the state of a model or system, or the sensitivity of output changes to changes in system parameters and surrounding conditions. In the optimal combination method, we often use sensitivity analysis to analyze the stability of continuing to solve the optimal solution when the given environment changes. In general, the use of sensitivity analysis can also help to determine the degree of influence of several factors on the model.

In this optimal investment strategy, we consider the sensitive relationship between transaction costs and the final return under the 5-year optimal investment strategy. Since the cost of this portfolio investment comes from the transaction fees incurred in gold and bitcoin transactions, and the two have different rates. Therefore, in order to better reflect the sensitivity of the investment strategy to the two assets of gold and Bitcoin. Here, we analyze it from two angles:

One, assuming that the gold transaction rate is unchanged, the transaction cost is only



changed by changing the transaction rate of Bitcoin. First of all, we adjust the transaction rate of Bitcoin several times, and output the initial assets of multiple optimal investments based on the model we have established to be the value of \$1,000 on September 10, 2021, and make a difference between the two. , we obtained the approximate function expression between the two by fitting the changing relationship between the two

$$Y=9E+11X-1E+11X^3+7E+09X^2-2E+08X+1E+06. \quad (12)$$

From We intuitively find that with the increase of transaction costs, the initial value of the initial \$1,000 asset after five years of optimal investment shows a significant downward trend and the decline gradually becomes smaller. The sensitivity expression  $S(Y,X)$  between the two can be written as  $X/y * dY/dX$ . We take the derivation of this function expression and take the limit to obtain the sensitivity between the two as **40%**

Second, assuming that the transaction rate of Bitcoin remains unchanged, the transaction cost is changed only by changing the transaction rate of gold. Ditto, we changed the transaction cost of gold several times by changing the transaction rate of gold. First of all, we adjust the transaction rate of gold many times and output multiple predicted values. The operation of this process is the same as that of Bitcoin. We fit the function expression between the two to get

$$Y=-2E+10X^3 +2E+09X^2-1E+08X+2E+06. \quad (13)$$

From the relationship diagram between the two, we intuitively find that with the increase of transaction cost, the initial 1000 The initial value of USD assets after five-year optimal investment shows a significant downward trend, and the decline rate is the same as the 5-year optimal investment strategy investment of \$1,000 initial assets, which gradually decreases with the transaction rate of Bitcoin. The sensitivity expression  $S(Y,X)$  between the two can be written as  $X/y * dY/dX$ . We take the derivation of this function expression and take the limit to obtain the sensitivity between the two as **67%** Sensitivity Analysis and Error Analysis.

It can be seen that the model is sensitive to changes in transaction costs

## 8 Model Evaluation and Further Discussion

### 8.1 Modern Portfolio Theory

Modern Portfolio Theory is the theory of how investors (risk-averse) can construct portfolios to maximize expected returns given certain levels of risk. The breakthrough of MPT is that it does not need to analyze the risk and return characteristics of many investments in isolation, but to study how these investments affect the performance of the portfolio. Therefore, MPT's assumptions emphasize that investors will only have additional risk when they are likely to get a higher expected return—that is, high risk, high return.

The most basic tenet of this theory is that investors can construct an efficient set of portfolios, the efficient frontier. The efficient frontier maximizes the expected return for a given level of risk. The investor's tolerance for risk will determine the efficient frontier he chooses. Investors with low tolerance will choose the portfolio that provides the greatest return with the lowest risk, and those with high tolerance will choose the portfolio with the greatest return with

high risk.

The basic content of Portfolio Theory (MPT) is simulated by Monte Carlo, and five time points of 2017-09-10, 2018-09-10, 2019-09-10, 2020-09-10, and 2021-09-10 are obtained. An efficient frontier for investment. And compared with the actual investment of our constructed decision model. The results show that our decision points are on the efficient frontier at all three time points. This proves that our decision can maximize the benefit under the same risk.

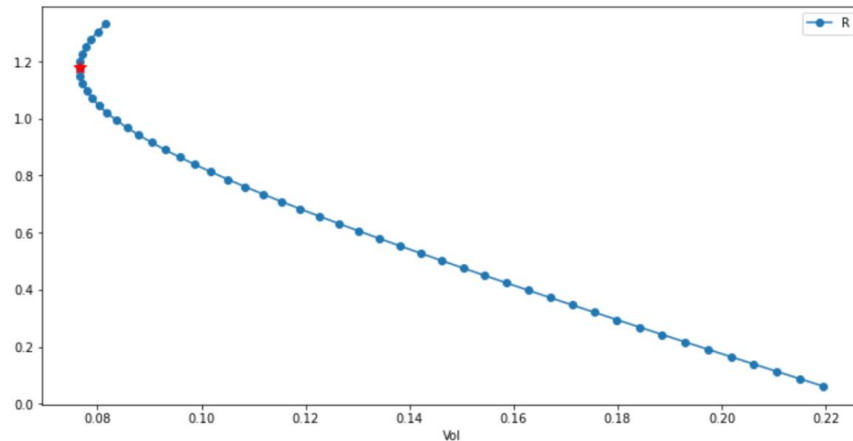


Figure 19: Minimum Variance Portfolio and Efficient Frontier for 2021-9-10

## 8.2 Strengths

1. The model building has a clear idea, from prediction-decision-evaluation to complete the entire process of portfolio investment decision-making.
2. Use multiple models and compare them to select the optimal model for subsequent decision-making.
3. After the model is established, the reliability of the model is more comprehensively verified

## 8.3 Weaknesses

1. Data is limited. The investment portfolio constructed in this paper only considers the opening price on the day when the decision was made and before, ignoring other factors such as trading volume and turnover rate. Our model still has room for improvement in actual investment.
2. We use modern investment theory to test the model, but there are other optimization theories after Modern Portfolio Theory. Since bitcoin and gold investments are not actually distributed normally, other evaluation methods with higher recognition cannot be used.

## 8.4 Further Discussion

### Model improvements:

1. The model treats the purchase of both bitcoin and gold as investments, but does not take into account the inflation and deflation of cash in real life. We can take this into account in future optimizations.
2. In the future, the training set can be appropriately expanded to make the combined model of prediction and decision more accurate.

**practical use:**

1. In addition to calculating the optimal portfolio of bitcoin and gold, the model can be extended to other forms of investment such as stocks, bonds and even insurance.
2. In addition to personal investment, after optimization, the model can also be used for regional municipal economic decision-making to achieve higher social value.

## 9 Conclusion

This paper predicts the price of gold through seasonal time series and BP neural network optimized by genetic algorithm, and uses the XGBoost time series model optimized by genetic algorithm to predict the price of bitcoin more accurately using sliding window. Based on this, a target planning model is constructed, and the daily optimal investment strategy is calculated through risk parameters and constraints, and the expected return after 5 years is obtained. Sensitivity analysis is carried out on the constructed decision-making model, and its sensitivity is calculated. Then choose five time points, draw the Markowitz efficient frontier, and prove that the investment portfolio of our decision model is the maximum return portfolio under the same risk. Finally, our advantages and disadvantages are analyzed, and future practical applications are discussed.

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## Memorandum

**To: Trader**

**From: Team # 2210729**

**Subject: The best strategy for investing in a combination of gold and bitcoin**

**Date: February 22,2021**

*Dear Sir and Madam,*

*We are all concerned about the return on investment of assets, and the largest investment report has always been the greatest pursuit of our market traders. However, we all know that achieving maximum investment returns is extremely difficult for market traders. And the great annoyance it brings to market traders. To help market traders make better portfolio investment decisions, we have built a model to help market traders invest in the market and provide them with some insights into how to avoid larger risks.*

*In short, our model can be roughly divided into three steps:*

- 1. By processing and analyzing the transaction data of Bitcoin and gold in previous years, multiple models are used to predict the future prices of gold and Bitcoin.*
- 2. Use the data obtained in the first step to calculate the respective yields of gold and bitcoin to make a better portfolio investment ratio of gold, bitcoin and US dollar.*
- 3. Use the actual price to verify our forecast results and make a risk assessment for this investment.*

*This model uses data from previous years as a training set to make more reliable guesses about the future trading prices of Bitcoin and gold. This information can help market traders make decisions and allocate investment ratios, and can also warn them about risk estimates.*

*Our model can also classify the predicted data into high-risk and low-risk, so as to judge whether a trade needs to be made, which helps to improve the efficiency of market traders in making market investments*

*Our model is also able to self-learn to update itself when given a new training dataset. Depending on the specifics of the training data, the model will change as follows:*

*If any new data comes out, we will update the training dataset;*

*Since Bitcoin is a virtual currency and the overall price of the times is constantly increasing, we will gradually discard data from older years to ensure that the predicted data is more in line with future market transaction prices.*

*Making the best portfolio investment strategy is a daunting task for market traders, but our model can be of great help, especially when unexpected situations occur, the risk assessment system in our model. It can help market traders to avoid risks in an emergency, reduce trading losses as much as possible, and maximize reporting benefits. Our tests show that the predictions made by our model are more scientific and very close to the optimal trading strategy.*

*We hope this model can serve as a valuable tool in our collective efforts to maximize market portfolio investment. Earning maximum benefit is our common goal. If you have any questions about this model or good ideas for improvement, please contact us and we will do our best to clarify or improve the model we have built.*

*We look forward to your good news.*

*Your sincerity, 2210729 team*