Linear Programming Research Project

Optimisation Theory and Applications

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INTRODUCTION TO THE CHOSEN PROBLEM

Edison Motors produces four variants of electric vehicles, model Y, model X, model 3, and model S. Sales forecasts indicate an expected minimum monthly sales in Shireland as shown below. It is deemed acceptable to over-produce a given model in one month as it is expected that demand will persist with relative stability through into future months and so production demands will be modified dependent on the surplus.

Table 1, Projected Demand for each model in Shireland

Model	Symbol	Monthly Sales, 000's	Profit (£), 000's
Y	x_1	6.3	4
X	x_2	4.2	6
3	x_3	7.0	5
S	x_4	5.3	2

There exist 3 production facilities serving Shireland, each specialised in a different set of models.

Facility A specialises in Model Xs and Ys. There are upper limits of 5000 Model Xs, 6000 Model Ys, and a combined maximum of 10000 vehicles.

Facility B specialises in Model Ss and Model 3s. There are upper limits of 6000 Model Ss, 10000 Model 3s, and a combined maximum of 12000 vehicles.

Facility C is optimised to manufacture Model Ss and Model Ys. Model S production at this facility is nascent, so there is a limit of 1200 per month. There is an upper limit of 4000 Model Ys, and a combined maximum of 4500 vehicles.

Model Ss yield less profit but are strategically important to the company as they are the most recent model to be released. Therefore, their value is artificially scaled by 3x in the objective.

Edison's Motors objective function can therefore be described as maximising the profit of the combined output from the 3 factories.

$$\max_x [4x_1 + 6x_2 + 5x_3 + (3 * 2)x_4] \tag{1.01}$$

where
$$x_1 = x_{1A} + x_{1C}$$
$$x_4 = x_{4B} + x_{4C}$$

Indexing is done by model and facility in some cases, where x_{1A} denotes a model Y fabricated at facility A.

Subject to the constraints on sales…

$$x_1 = x_{1A} + x_{1C} \geq 6.3 \tag{2.01}$$

$$x_2 \geq 4.2 \tag{2.02}$$

$$x_3 \geq 7.0 \tag{2.03}$$

$$x_4 = x_{4B} + x_{4C} \geq 5.3 \tag{2.04}$$

…and factory output.

Facility A

$$x_{1A} \leq 6 \tag{2.05}$$

$$x_2 \leq 5 \tag{2.06}$$

$$x_{1A} + x_2 \leq 10 \tag{2.07}$$

Facility B

$$x_3 \leq 10 \tag{2.08}$$

$$x_{4B} \leq 6 \tag{2.09}$$

$$x_3 + x_{4B} \leq 12 \tag{2.10}$$

Facility C

$$x_{1C} \leq 4 \tag{2.11}$$

$$x_{4C} \leq 1.2 \tag{2.12}$$

$$x_{1C} + x_{4C} \leq 4.5 \tag{2.13}$$

$$\max_x [4x_{1A} + 4x_{1C} + 6x_2 + 5x_3 + 6x_{4B} + 6x_{4C}] \tag{2.14}$$

<u>Standard form for input to program</u>

Where previously we had **x** = *{x_{1A}, x_{1C} …, x_{4C}}*, with some variables indexed by facility and variant, now each term is indexed by integer only: **x =** *{x_1, x_2, …, x_6}.*

Objective equation:

$$\max_x [4x_1 + 4x_2 + 6x_3 + 5x_4 + 6x_5 + 6x_6]$$

Subject to the constraints:

x1 + x2 ≥ 6.3
x3 ≥ 4.2
x4 ≥ 7.0
x5 + x6 ≥ 5.3
x1 ≤ 6.0
x3 ≤ 5.0
x1 + x3 ≤ 10.0
x4 ≤ 10.0
x5 ≤ 6.0
x4 + x5 ≤ 12.0
x2 ≤ 4.0
x6 ≤ 1.2
x2 + x6 ≤ 4.5

SOLVING THE PROBLEM

Below is an excerpt of the problem's log.txt file. To see the full log.txt file, please update the input.csv file with the entries in research_project.csv or switch the commented code sections that load the data into the program (lines 8 and 9 of linprog.py) and re-run the program.

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log.txt excerpt

```
Initial Tableau:
      x1    x2    x3    x4    x5    x6    s1    s2    s3    s4   s5   s6   s7   s8   s9  s10  s11  s12  s13   a1   a2   a3   a4  Solutions
0    1.0   1.0   0.0   0.0   0.0   0.0  -1.0  -0.0  -0.0  -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0  1.0  0.0  0.0  0.0        6.3
1    0.0   0.0   1.0   0.0   0.0   0.0  -0.0  -1.0  -0.0  -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0  0.0  1.0  0.0  0.0        4.2
2    0.0   0.0   0.0   1.0   0.0   0.0  -0.0  -0.0  -1.0  -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0  0.0  0.0  1.0  0.0        7.0
3    0.0   0.0   0.0   0.0   1.0   1.0  -0.0  -0.0  -0.0  -1.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0  0.0  0.0  0.0  1.0        5.3
4    1.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0        6.0
5    0.0   0.0   1.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0        5.0
6    1.0   0.0   1.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0       10.0
7    0.0   0.0   0.0   1.0   0.0   0.0   0.0   0.0   0.0   0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0       10.0
8    0.0   0.0   0.0   0.0   1.0   0.0   0.0   0.0   0.0   0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0        6.0
9    0.0   0.0   0.0   1.0   1.0   0.0   0.0   0.0   0.0   0.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0       12.0
10   0.0   1.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0  0.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0        4.0
11   0.0   0.0   0.0   0.0   0.0   1.0   0.0   0.0   0.0   0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0        1.2
12   0.0   1.0   0.0   0.0   0.0   1.0   0.0   0.0   0.0   0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0        4.5
13 -14.0 -14.0 -16.0 -15.0 -16.0 -16.0  10.0  10.0  10.0  10.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0     -228.0


 --- Iteration 0 ---
Column to pivot on: 2, x3 is entering the basis
Dividing solution column by pivot column...

      x1    x2    x3    x4    x5    x6    s1    s2    s3    s4   s5   s6   s7   s8   s9  s10  s11  s12  s13   a1   a2   a3   a4  Solutions  Ratios
0    1.0   1.0   0.0   0.0   0.0   0.0  -1.0  -0.0  -0.0  -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0  1.0  0.0  0.0  0.0        6.3     inf
1    0.0   0.0   1.0   0.0   0.0   0.0  -0.0  -1.0  -0.0  -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0  0.0  1.0  0.0  0.0        4.2    4.20
2    0.0   0.0   0.0   1.0   0.0   0.0  -0.0  -0.0  -1.0  -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0  0.0  0.0  1.0  0.0        7.0     inf
3    0.0   0.0   0.0   0.0   1.0   1.0  -0.0  -0.0  -0.0  -1.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0  0.0  0.0  0.0  1.0        5.3     inf
4    1.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0        6.0     inf
5    0.0   0.0   1.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0        5.0    5.00
6    1.0   0.0   1.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0       10.0   10.00
7    0.0   0.0   0.0   1.0   0.0   0.0   0.0   0.0   0.0   0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0       10.0     inf
8    0.0   0.0   0.0   0.0   1.0   0.0   0.0   0.0   0.0   0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0        6.0     inf
9    0.0   0.0   0.0   1.0   1.0   0.0   0.0   0.0   0.0   0.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0       12.0     inf
10   0.0   1.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0  0.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0        4.0     inf
11   0.0   0.0   0.0   0.0   0.0   1.0   0.0   0.0   0.0   0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0        1.2     inf
12   0.0   1.0   0.0   0.0   0.0   1.0   0.0   0.0   0.0   0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0        4.5     inf
13 -14.0 -14.0 -16.0 -15.0 -16.0 -16.0  10.0  10.0  10.0  10.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0     -228.0   14.25

Smallest non-negative ratio: 4.2
Row to pivot on: 1
Pivoting on: 1, 2

Manipulating rows...
Pivot complete, result:


      x1    x2   x3    x4    x5    x6    s1   s2    s3    s4   s5   s6   s7   s8   s9  s10  s11  s12  s13   a1    a2   a3   a4  Solutions  Ratios
0    1.0   1.0  0.0   0.0   0.0   0.0  -1.0  0.0   0.0   0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  1.0   0.0  0.0  0.0        6.3     inf
1    0.0   0.0  1.0   0.0   0.0   0.0  -0.0 -1.0  -0.0  -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0  0.0   1.0  0.0  0.0        4.2    4.20
2    0.0   0.0  0.0   1.0   0.0   0.0   0.0  0.0  -1.0   0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0   0.0  1.0  0.0        7.0     inf
3    0.0   0.0  0.0   0.0   1.0   1.0   0.0  0.0   0.0  -1.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0   0.0  0.0  1.0        5.3     inf
4    1.0   0.0  0.0   0.0   0.0   0.0   0.0  0.0   0.0   0.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0   0.0  0.0  0.0        6.0     inf
5    0.0   0.0  0.0   0.0   0.0   0.0   0.0  1.0   0.0   0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  -1.0  0.0  0.0        0.8    0.80
6    1.0   0.0  0.0   0.0   0.0   0.0   0.0  1.0   0.0   0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  -1.0  0.0  0.0        5.8    5.80
7    0.0   0.0  0.0   1.0   0.0   0.0   0.0  0.0   0.0   0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0   0.0  0.0  0.0       10.0     inf
8    0.0   0.0  0.0   0.0   1.0   0.0   0.0  0.0   0.0   0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0   0.0  0.0  0.0        6.0     inf
9    0.0   0.0  0.0   1.0   1.0   0.0   0.0  0.0   0.0   0.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0   0.0  0.0  0.0       12.0     inf
10   0.0   1.0  0.0   0.0   0.0   0.0   0.0  0.0   0.0   0.0  0.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0   0.0  0.0  0.0        4.0     inf
11   0.0   0.0  0.0   0.0   0.0   1.0   0.0  0.0   0.0   0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0   0.0  0.0  0.0        1.2     inf
12   0.0   1.0  0.0   0.0   0.0   1.0   0.0  0.0   0.0   0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0   0.0  0.0  0.0        4.5     inf
13 -14.0 -14.0  0.0 -15.0 -16.0 -16.0  10.0 -6.0  10.0  10.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  16.0  0.0  0.0     -160.8   81.45
```

RESULTS AND FURTHER WORK

Results

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result.txt excerpt

Final Tableau

	x1	x2	x3	x4	x5	x6	s1	s2	s3	s4	s5	s6	s7	s8	s9	s10	s11	s12	s13	a1	a2	a3	a4	Solutions	Ratios
0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-1.0	1.0	0.0	0.0	0.0	0.0	3.3	0.70
1	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5.0	inf
2	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	1.2	inf
3	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	-1.0	0.0	5.0	inf
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	-1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	inf
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-1.0	0.0	0.0	0.8	inf
6	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5.0	inf
7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-1.0	0.0	3.0	inf
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-1.0	0.0	0.0	0.0	0.0	0.0	1.0	-1.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	1.0	inf
9	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	-1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	7.0	inf
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	-1.0	0.0	0.0	0.0	0.0	0.7	0.70
11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	0.0	-1.0	-1.0	0.9	inf
12	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	-1.0	1.0	0.0	0.0	0.0	0.0	-1.0	1.0	-1.0	0.0	0.0	0.0	2.0	2.00
13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	2.0	4.0	0.0	0.0	6.0	0.0	2.0	4.0	10.0	10.0	9.0	10.0	135.4	-23.85

Optimal solution: x1 = 5.0, x2 = 3.3, x3 = 5.0, x4 = 7.0, x5 = 5.0, x6 = 1.2

Objective equation: max z = 4.0*x1 + 4.0*x2 + 6.0*x3 + 5.0*x4 + 6.0*x5 + 6.0*x6

... z (from Tableau) = 135.4

... z (from calculation) = 135.39999999999998

Multiple Optima? Only one optimal solution

Dividing the contribution of x5 and x6 by 3 to factor out artificial profits (see introduction):

z = 4.0*x1 + 4.0*x2 + 6.0*x3 + 5.0*x4 + 2.0*x5 + 2.0*x6

z = 4*5 + 4*3.3 + 6*5 + 5*7 + 2*5 + 2*1.2

= 110.6

For a total profit of £110,600 when pursuing the strategy of selling more model Ss.

CONCLUSION

The chosen research problem to apply the library to is an imaginary EV manufacturer that wishes to make a decision on how to allocate resources between the production of 4 vehicle models at 3 factories.

The main objective is overall profit, however the strategy of the company is to favour the production of their most recent model. This consideration is implemented by weighting the profit of the model in question, and the weight is discounted upon finding a solution to find the real profit of the strategy.

The constraints include meeting minimum forecast demand of each model, and production constraints that each facility is under.

The program written as part of this assignment, linprog.py, is capable of finding the optimal solution to this problem and calculates that the profit amounts to £110,600 when pursuing the strategy described above.

This program is capable of solving linear programming problems with both maximisation and minimisation objectives; equality, greater than/equal to, less than/equal to relationships; handles artificial variables; and is capable of identifying instances of multiple optima, but cannot list the full scope of equivalently optimal solutions.

Further work would be to write a function that could calculate the range of optimal values using a convex linear combination of basic solutions, capability to deal with unbounded variables by reformulating the problem with extra target variables, and the ability to solve linear programming problems via their Dual.